A METHOD FOR MEASURING WOUND-ON-TENSION

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ABSTRACT

The Wound On Tension (WOT) is the web machine direction stress as it has just passed the winding nip into the roll. It is the most important quantity that determines the wound roll structure and its internal radial stress distribution. All the other winding parameters can affect the WOT but they can change the internal roll stresses only through the WOT. The on-line measurement of the WOT has been possible only in laboratory winders. However, if the WOT would be known, the winding parameters could be more accurately adjusted to give optimized customer roll structure. The purpose of this work has been to find and evaluate WOT measurement method feasible in the production environment. Both the usual Small deformations model known as the Hakiel model and the more recent Large deformations model were evaluated. The WOT can be solved from the Hakiel model numerical approximation and the measured radial displacement. The numerical approximation is a large linear equation which can be reduced to a scalar equation linear for the WOT. The method is computationally effective allowing real time measurement of WOT during winding. The Density measurement is based on the roll length and diameter measurements giving the paper thickness and density when the basis weight is given. The web thickness under zero stress is needed to accurately measure the total radial displacement from this data. The results obtained with newsprint paper show that with paper grades having relatively low radial modulus the WOT can be measured at a reasonable accuracy if the free web paper thickness can be obtained from the preceding process machines.

NOMENCLATURE

a_1, a_2, a_3 Stress-strain curve parameters
C_0, C_1, C_2 Polynomial coefficients for the radial modulus curve
E_r, E_t Radial (z-direction) and tangential (machine direction) moduli
f_{eps} Stress-strain function

h	Incoming web thickness
i	Layer index in the large deformations model
l	Roll width
N	Total number of layers in the roll
R	Roll outer radius
R^m	Measured roll radii (from density measurement)
R_0	Core outer radius
r_i	Layer radius at unstressed state
r_o	Roll outer diameter during the winding
u	Radial displacement
WOT	Wound On Tension
$\delta\sigma_r,\delta\sigma_t$	Incremental radial and circumferential stress
$\varepsilon_r,\varepsilon_t$	Radial and circumferential strain
ν_{rt},ν_{tr}	Poisson ratios
ρ	Density of the web in the unstressed state
σ_r,σ_t	Radial and circumferential stress

INTRODUCTION

This work started as a simple attempt to repeat the ideas presented by David Roisum in his doctoral thesis 1990, despite the warnings Roisum had given in his later writings. I assumed that the well-proven Hakiel model from 1986 would still be the theoretical basis, and the better measurement and computer technology available today would allow improvements in accuracy and efficiency. The goal was to have a practical Wound On Tension measurement in production environment.

The work however led me deeper into the theoretical and numerical groundwork of winding modelling. The main finding was that the accuracy and even the basic correctness of the Hakiel model were in concern. The improved theoretical basis was found from the so called Large deformations model introduced in [3] , in contrast to the Hakiel model known as the Small deformations model in what follows. The earlier paper [5] had already many of the ideas needed for the better formulation, for example not using the Maxwell strain energy condition to simplify the equations.

SMALL DEFORMATIONS WINDING MODEL WITH CENTRIFUGAL EFFECT

In the customary way of modelling the winding of web material the roll is treated essentially as a solid particle since no slippage between web layers is assumed [1]. The resulting boundary value problem has to be solved for each web layer since the layers are wound onto the outer radius of the intermediate roll, which does not anymore exist in the final roll. The equations are written in cylindrical coordinates and only radial and tangential stresses/strains are accounted for. No cross-machine stress variations are included. The roll is modelled as perfect concentric cylindrical hoops.

High-speed winding requires that speed dependent factors are included in the model. Winding at the speeds over 3000 meters per minute can cause the final roll stresses to be considerably different compared to slower speed winding. The only speed related term included in the present model is the centrifugal effect. The air entrainment should also be treated even with higher permeability material as newsprint paper.

In the small deformations formulation the stress distribution is not solved directly, but instead the increments $\delta\sigma_r$ and $\delta\sigma_t$ in the stresses due to the addition of the last web layer. It is assumed that the final stresses can be found by summing up these incremental stresses as the roll is being wound. It is also assumed that the deformations are so small, that the incremental deformations can also be summed to give the total deformations. These assumptions allow the hoop radius r to be used as the independent variable of the equations, even though the radius of every hoop will deform in the process of adding the layers. The small deformations model is essentially a linearization of the underlying non-linear problem.

Equilibrium equation

The following treatment follows directly [1,2] except that the usual elimination of $\delta\sigma_t$ is not done, but both $\delta\sigma_r$ and $\delta\sigma_t$ are kept as the unknown functions to be solved. The numerical problem size grows slightly but the equations are simpler and no second order derivatives appear.

The force equilibriums of the cylindrical hoops building up the roll are maintained by the incremental radial stress force difference (derivative) between the inner and outer surfaces of the hoop plus the incremental hoop tension induced by the tangential stress:

$$2\pi l\left((r+dr)\delta\sigma_r(r+dr)-r\delta\sigma_r\right)-2\pi lr\left(\frac{\delta\sigma_r dr}{r}\right)=2\pi lrdr\left(f_i(r)-f_{i-1}(r)\right)\Rightarrow \quad \{1.1\}$$

$$r\frac{\partial\delta\sigma_r}{\partial r}+\delta\sigma_r-\delta\sigma_t=r\left(f_i-f_{i-1}\right) \quad \{1.2\}$$

where the volume force term f_i contains the centrifugal and Wound On Tension stresses. The subscript i refers to the roll revolutions. Since the WOT does not anymore change after the layer is got into the roll, it cancels completely in the subtraction. However, the centrifugal term does not disappear, because the roll rotational speed varies when layers are added even with constant web speed. The centrifugal force term is:

$$\begin{aligned}&\left(f_i(r)-f_{i-1}(r)\right)r=-\frac{\rho h}{dr}r\left(\omega_N(r)^2-\omega_{N-1}(r)^2\right)\frac{r^2}{r}=\\&-\frac{\rho h}{dr}\left(\omega_N(r)^2-\omega_{N-1}(r)^2\right)r^2\end{aligned} \quad \{1.3\}$$

Compatibility equation

The other equation needed for two unknowns is found from the geometrical strain condition:

$$\varepsilon_t = \frac{u}{r} \Rightarrow \frac{\partial \varepsilon_r}{\partial r} = \frac{\partial u}{\partial r} - \frac{u}{r^2} = \varepsilon_r - \frac{\varepsilon_t}{r} \qquad \{1.4\}$$

The material stress-strain behaviour is expressed in the constitutive equations:

$$\varepsilon_r = \frac{\sigma_r}{E_r} - \nu_{tr}\frac{\sigma_t}{E_t}, \ \varepsilon_t = \frac{\sigma_t}{E_t} - \nu_{rt}\frac{\sigma_r}{E_r} \qquad \{1.5\}$$

Insert the material constitutive equations into {1.4} and eliminate the derivative of σ_r with the equilibrium equation {1.2}:

$$r\frac{\partial \delta\sigma_t}{\partial r} + \left(\nu_{tr} - \nu_{rt}\frac{E_t}{E_r} + 1\right)\delta\sigma_t + \left(\frac{\nu_{rt} r}{E_r}\frac{\partial E_r}{\partial \sigma_r}\frac{\partial \sigma_r}{\partial r} - 1\right)\frac{E_t}{E_r}\delta\sigma_r = \nu_{rt}\frac{\rho h}{dr}\left(\omega_i^2 - \omega_{i-1}^2\right)\frac{r^2}{E_r} \qquad \{1.6\}$$

The equations {1.3} and {1.6} are already linear in $\delta\sigma_r$ and $\delta\sigma_t$. They need only to be discretized with a suitable method for numerical solution. The most attractive method is the simple Euler method with forward or backward differentiation, which keeps the resulting linear equation system tridiagonal. The program execution time for tridiagonal solver is short and grows only linearly with the problem size.

Boundary conditions with tension loss in the outermost layer

The roll is wound onto a supporting core that has different material properties from the roll itself. However, the stresses inside the core wall could be included as part of the problem, in which case the boundary condition at the core inner radius would be simply:

$$\delta\sigma_r(R_{coreinner}) = 0 \qquad \{1.7\}$$

The more usual inner boundary condition is to relate the core deformation and the radial stress under the bottom layer:

$$\varepsilon_t(R_0) = \frac{u}{R_0} = \frac{\delta\sigma_r}{E_c} = \frac{\delta\sigma_t}{E_t} - \nu_{rt}\frac{\delta\sigma_r}{E_r} \qquad \{1.8\}$$

The core modulus E_c can be derived from the core material parameters.

The outer boundary condition is also simply

$$\delta\sigma_r(R) = 0 \qquad \{1.9\}$$

This outer boundary condition allows the outermost layer loose part of its Wound On Tension under the first deformation process after its addition to the roll. Note that in this case the equations {1.3} and {1.6} are valid also for the last layer. Naturally all layers will go on suffering more tension losses as they are buried deeper into the roll. The Wound On Tension will appear in the equilibrium equation {1.3} for the last layer, where the cancellation will not occur since the tension was set to zero before the layer existed on the roll:

$$f_N(R)R = -\frac{\rho h}{dr}R^2\omega_N(R)^2 + \frac{WOT}{R} \qquad \{1.10\}$$

Outer boundary condition with no tension loss in the outermost layer

The outer boundary condition can be easily converted to zero tension loss, since $\delta\sigma_t$ is just the tension loss each layer suffers at every layer addition, so set it to zero at the last layer:

$$\delta\sigma_t(R) = 0 \qquad \{1.11\}$$

In this case the compatibility equation {1.6} is not valid for the last layer, because this layer does not shrink when the roll under it is deforming, so $\varepsilon_t(R) \neq \frac{u}{R}$. This is no problem for us, since equation {1.9} replaces {1.6} for the last layer.

LARGE DEFORMATIONS WINDING MODEL

The following large deformations formulation borrows its basic idea from [3] and [5], with the most important concept that the hoop radius *r* is a dependent variable, which must be solved from the boundary value problem. The main difference compared to [3] is that *r* is not solved directly, but again σ_r and σ_t are solved first and then *r* is got from the stress-strain relationship. However, the resulting equations must be written so that *r* will not appear in them, otherwise three differential equation system would be needed for the three unknowns σ_r, σ_t and *r*.

The layer index *i* as the new independent variable

Since the hoop radius cannot be used as the independent variable of the equations, some other variable must be devised to replace it. The layer index *i* introduced in [3] is used as the independent variable for the equations. The layer index is a unitless variable that is defined so to have value *N* at the layer *N-1* to *N* boundary. The layer index is a continuous variable, for example the value *N*+1/2 points to the center of layer *N*. The layer index is fixed to the material positions and remains fixed to them throughout the deformation process during winding.

Equilibrium equation

The force equilibrium equation is basically the same as for the small deformations model except that it now holds for the total stresses, not for the incremental stress changes due to the addition of the last layer. The large deformations model is conceptually simpler than the small deformations model, since there is no concern about the correct form of the increments in various stress and strain terms and whether the implicit approximations are valid or not. However, still the winding problem must be solved layer by layer, since the intermediate roll outer radius r_o will appear in the equations, and it cannot be made known otherwise but by solving the winding boundary value problem for each layer successively. The stored values of the intermediate roll radii $r_o(i)$ form the "memory" of the roll, which preserves the winding history of the roll.

The force equilibrium is stated again by equating the difference in the radial force at the inner and outer surfaces of the layer plus the hoop tension by the tangential stress to the centrifugal and Wound On Tension forces. The layer thickness is now $\frac{\partial r}{\partial i}$:

$$\frac{\partial(2\pi l r \sigma_r)}{\partial i} - 2\pi l r \frac{\sigma_t}{r}\frac{\partial r}{\partial i} = -2\pi l \rho h r_o \frac{\omega^2 r^2}{r} + 2\pi l r \frac{WOT}{r} \Rightarrow \qquad \{2.1\}$$

$$r\frac{\frac{\partial \sigma_r}{\partial i}}{\frac{\partial r}{\partial i}} + \sigma_r - \sigma_t = \frac{WOT - \rho h r_o \omega^2 r}{\frac{\partial r}{\partial i}} \qquad \{2.2\}$$

All terms above are functions of the layer index i except ω that is a function of N, the number of layers wound up to now (all layers already inside the roll rotate with the same speed). Note that r_o does not deform when new layers are added to the roll in contrast to what happens to the hoop radii r.

The derivation should be continued with replacing the hoop radius r by other known variables or the unknown stresses. However, the applied iterative non-linear equation solution method converges to the correct value even though this replacement is not done.

Compatibility equation

The compatibility equation derived from the geometrical strain condition needs more rewriting. First the concept of strain must be redefined to suit the process of summing small increments of strain to give the final not so small strain. The path chosen in [3] is not followed, where a non-linear strain was used:

$$\varepsilon_{nonlinear} = \sum_{1}^{N} \frac{\Delta x_i}{x_i} \approx \int_{X_0}^{X} \frac{dx}{x} = \ln\left(\frac{X}{X_0}\right) \qquad \{2.3\}$$

where X is the final deformed size and X_0 is the initial size. Instead each successive incremental deformation is scaled with the original layer thickness, which allows addition of the incremental strains to give the correct total strain:

$$\varepsilon_r = \frac{\frac{\partial r}{\partial i} - h}{h} \qquad \{2.4\}$$

where h is the layer thickness just prior it enters the roll. This relationship is used every time new deformations are solved, and the final deformation is correctly the sum of the incremental deformations.

The tangential strain must also be defined so that the division by the deforming hoop radius r is avoided and that the strain will be zero for zero tangential stress:

$$\varepsilon_t = \frac{r - r_o}{r_i} \qquad \{2.5\}$$

where r_i is the hoop radius in the (imaginary) unstressed state:

$$r_i = \frac{r_o}{\dfrac{WOT}{E_t h} + 1} \qquad \{2.6\}$$

The compatibility equation is derived from the equations by eliminating the hoop radius r:

$$r_i \frac{\partial \varepsilon_t}{\partial i} + \frac{\partial r_i}{\partial i}\varepsilon_t - h\varepsilon_r + \frac{\partial r_o}{\partial i} - h = 0 \qquad \{2.7\}$$

The constitutive equations are now:

$$\varepsilon_r = f_{eps}(\sigma_r) - \nu_{tr}\frac{\sigma_t}{E_t}, \qquad \varepsilon_t = \frac{\sigma_t}{E_t} - \nu_{rt} f_{eps}(\sigma_r) \qquad \{2.8\}$$

where f_{eps} is the stress-strain function of the web material in the r-direction. The equation {2.7} is again changed to stress equation with the help of the constitutive equations:

$$\begin{aligned} & r_i \frac{\partial \sigma_t}{\partial i} + \left(\frac{\partial r_i}{\partial i} + \nu_{tr} h - \nu_{rt} \frac{r_i E_t}{r E_r} \frac{\partial r}{\partial i} \right) \sigma_t + \\ & E_t \left(f_{eps}(\sigma_r) \left(-\nu_{rt} \frac{\partial r_i}{\partial i} - h \right) + \frac{\partial r_o}{\partial i} - h + \nu_{rt} \frac{\partial r}{\partial i} r_i \frac{\sigma_r}{E_r r} \right) = \\ & \frac{\nu_{rt}}{E_r} \left(-\rho h r_o r_i \omega^2 + WOT \frac{r_i}{r} \right) \end{aligned} \qquad \{2.9\}$$

Boundary conditions

The inner and outer boundary conditions are otherwise the same as for the small deformations model except changes caused by the elimination of the hoop radius r:

$$\sigma_t(R_0) = \frac{E_t r_o}{E_c r_i}\sigma_r(R_0) + \nu_{rt} E_t f_{eps}(\sigma_r(R_0)), \quad \sigma_r(R) = 0 \qquad \{2.10\}$$

Calculating the displacements

The solution of the winding boundary value problem is the stresses, and the strains and displacements must be calculated from the constitutive equations. As mentioned earlier, the hoop radii *r(i)* should not appear in the stress equations, since they will change in the process of adding a layer to the roll, so they are implicitly functions of the stresses. This problem is clearly expressed in [3]. However, the non-linear solver converges well to the same solution whether the *r(i)* are eliminated or not. This naturally requires that the displacements be updated at every solver iteration step.

The small deformations occurring when each new layer compresses the roll are calculated from the stress-strain function f_{eps} and the total stress σ_r. This method is more accurate than the "numerical integration" when displacement increments are calculated with the linear approximation and E_r and summing. The large deformation model is more coherent than the small deformations model in that the compatibility equation derived from the cylindrical geometry and constitutive equations is valid all the way when small incremental radial strains pile up to over 10 percent final strain. This discrepancy is manifested in the error found in the sum of the incremental stresses, which does not anymore satisfy the force equilibrium in the small deformations model. This error is not seen, if only $\delta\sigma_r$ are solved layerwise and the final σ_t is solved from the force equilibrium when the roll is finished.

Non-linear boundary value problem solver

After the differential equations have been discretized, they remain as a non-linear system of equations due to the non-linear function $f_{eps}()$:

$F(\Sigma, W, \Omega) = 0$, where Σ is the unknown vector $\Sigma = \begin{bmatrix} \cdot \\ \sigma_{r,i} \\ \sigma_{t,i} \\ \cdot \end{bmatrix}$ and W the vector of Wound on Tension values for each layer and Ω the rotational speed vector. The column vector F has $2N$ rows. The chosen method for solving this non-linear system of equations is the Newton iteration:

$$\Sigma_{k+1} = \Sigma_k - \left(\frac{\partial F(\Sigma_k, W, \Omega)}{\partial \Sigma} \right)^{-1} F(\Sigma_k, W, \Omega) \qquad \{2.11\}$$

The Newton iteration is finished when the norm of the iteration step falls below a limit. In the winding modelling, the iteration can be luckily started from the previous layer Σ-vector, which is a good initial guess for the iteration. At least with the typical newsprint paper material parameters, the iteration converges well enough with only one step. After three steps per layer, the convergence is very good.

MEASURING THE RADIAL MODULUS AND STRESS-STRAIN CURVE

The stack modulus test was performed on samples consisting 500-600 sheets of web and 70mm x 70mm of footage area. The compressing/decompressing rate was 0.1 mm/s.

The compression cycle was repeated twice, and the second compression curve was used to determine the modulus E_r and stress-strain curve f_{eps} as functions of pressure.

The web thickness was measured from the same data by means of samples that contained exact number of sheets. The thickness was sampled at the pile height where the pressure first time increased over zero at the compression cycle.

The chosen parametric curve for the stress-strain curve f_{eps} fitting was

$$f_{eps}(\sigma) = a_1 \ln\left(\frac{a_2\sigma + 1}{a_3\sigma + 1}\right) \qquad \{3.1\}$$

where the parameters a_1, a_2, a_3 are determined from the curve fit. This form of fit is the same as used in most of the winding models since then

$$E_r(\sigma) = \frac{1}{\frac{\partial f_{eps}}{\partial \sigma}} = \frac{a_2 a_3 \sigma^2 + (a_2 + a_3)\sigma + 1}{a_1(a_2 - a_3)} = C_2\sigma^2 + C_1\sigma + C_0 \qquad \{3.2\}$$

which is the usual Hakiel quadratic modulus. If a_3 is set to zero, the Pfeiffer linear modulus is obtained instead. There is one restriction in the curve form: the C_0 parameter cannot be zero so $E_r(0)>0$. The E_r modulus could be also a cubic polynomial, but then the relationship between E_r and f_{eps} is more complicated as is seen from

$$f_{eps}(\sigma_r) = \int_0^{\sigma_r} \frac{d\sigma}{E_r(\sigma)} = \int_0^{\sigma_r} \frac{d\sigma}{C_3\sigma^3 + C_2\sigma^2 + C_1\sigma + C_0} = \sum_{i=1}^{3} \frac{\ln\left(1 - \frac{\sigma_r}{R_i}\right)}{\frac{\partial E_r(R_i)}{\partial \sigma}} \qquad \{3.3\}$$

$$E_r(R_i) = 0$$

The curve fit to the logarithmic function is done by means of a non-linear curve fitting software. The non-linear curve fit however requires good initial guess for the parameters sought, which can be found by first fitting the linear fit of E_r to the numerically differentiated stress-strain curve. After the non-linear fit has fine-tuned the a_i parameters, the coefficients for the E_r polynomial can be recalculated from them. The logarithmic stress-strain function is well behaving in that it always goes through the origin and it is smooth enough to have well behaving derivative E_r polynomial too, Figure 1 and Figure 2.

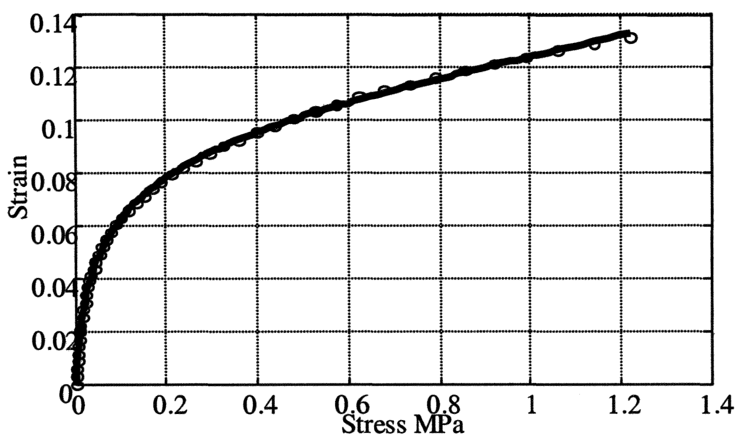

Figure 1: Newsprint stress-strain curve fit: solid line = non-linear curve fit $0.0219(\ln(0.454\sigma+1)-\ln(-157\sigma+1))$, dotted line = measurement data

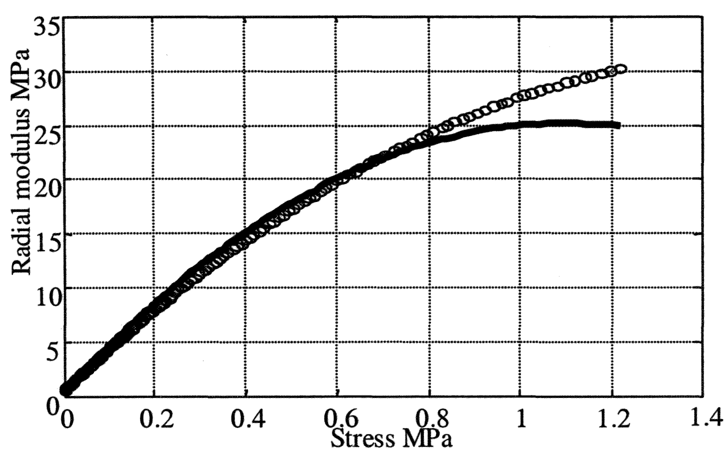

Figure 2: Newsprint stress-modulus curve fit: solid line = non-linear curve fit $E_r=-20.7\sigma^2-45.4\sigma+0.290$, dotted line = linear curve fit = $-10.5\sigma^2-43.9\sigma+0.704$

DENSITY MEASUREMENT

The density measurement has been included in a large number of production paper winders since its invention in the early 1980's by Leif Eriksson and Pekka Komulainen.

The implementation of the density measurement has been based on measuring roll diameter and web length by counting pulses from the support drum and the core chuck. The density measurement in the present work has been implemented differently from the conventional pulse counting measurement. The diameter measurement is done with a linear position sensor either measuring the roll center position or the position of the rider roller. Since there is no pulse sensor in the core chuck indicating the roll revolutions, the web thickness cannot be measured by differentiating the consecutive layerwise diameter readings. Instead an algorithm was developed which is based on diameter and length readings that are not synchronized to the roll revolutions.

The new algorithm is based on the simple linear relationship from the squared roll diameter to the web length:

$$l_i = \frac{\pi}{4d}\left(D_i^2 - D_0^2\right) \qquad \{4.1\}$$

The web thickness can be found by making a least squares curve fit to the D_i,l_i data set. This thickness would of course be the average thickness in the diameter range that the data set spans. To be able to measure varying web thickness as the roll is building up, the time varying recursive least squares method was applied, which forgets older data points and gives most of the emphasize on the most recent data. The recursive form of the algorithm means that the whole data set is not stored in memory, but only intermediate variables requiring less storage space. This algorithm is very light in terms of CPU time and storage space, and it can be easily implemented in the winder PLC. New values for the web thickness and density are updated at every new measured diameter and length, which can occur at each PLC program cycle.

Web thickness estimation variance and the buffer length

The time varying recursive least squares algorithm has the "forgetting factor" $\lambda<1$ as an input parameter. This parameter tells how quickly the older data points are forgotten and their influence on the estimation result is ended. Even though the data point weights decrease exponentially in time, an imaginary data buffer length can be associated to the forgetting factor:

$$bufferlength = \frac{1}{1-\lambda} \qquad \{4.2\}$$

The buffer length gives an approximation on how many of the last data points are included in the least squares estimation. Finding proper value for the forgetting value is not easy, good choice should be based on the knowledge of the noise contents in the measurements and on how the actual web thickness is varying with time. However, the task can be automated by means of the web thickness estimation variance. The variance of the least squares estimator can be easily calculated from the input noise estimate, and the variance estimated will be a function of the forgetting factor. Instead having the forgetting factor as an input parameter, now the desired estimation variance will be given, and the algorithm will itself adjust the forgetting factor and the associated buffer length so that the estimation variance is maintained within its limits. In the present work the variance set point was given for the measured density as 0.2 kg/m^3, which resulted buffer lengths of 50-200 samples and is about 5-10 mm thick layer on the roll surface.

Density measurement accuracy

The resolution of this density measurement algorithm is well under 1 kg/m^3 in spite of low-resolution diameter measurement. At the newsprint paper basis weight 45 g/m^2 this means under 0.1 micrometer resolution in web thickness. The resolution is gained at the cost of averaging; the measurement cannot track very fast thickness variations. However, more important for the real accuracy are systematic errors in the diameter measurement. The error in density is approximately proportional to the ratio of the diameter error to the diameter. The diameter measurement can be calibrated to have less than 0.1 mm error, which results 0.1 - 1 kg/m^2 error in density.

SOLVING THE WOUND ON TENSION AS AN UNKNOWN VARIABLE FROM THE WINDING MODELS

The Wound On Tension was solved iteratively in [4] from the roll diameter deformation data. However, the small deformations model, which is linear with respect to the unknowns, can be solved directly by means of a matrix inversion for the Wound On Tension. The non-linear large deformations model needs to be solved iteratively again, as in the normal solution procedure. The large deformations solving technique is more general in that the small deformations solution can be considered as a special case of it.

The Wound On Tension is solved for each web layer from an augmented boundary value problem, which is created by adding to the original equations the roll radius deformation equation. The augmentation is needed, since now the Wound On Tension for the last web layer is an extra unknown in addition to the stresses. The solution proceeds again layer by layer so that only the last layer Wound On Tension appears as an unknown, the other layers Wound On Tensions having been solved earlier.

The deformation equation added to equations {2.2} and {2.9} is

$$R_N^m - R_{N-1}^m - h = \int_0^{N-1} \left(f_{eps}\left(\sigma_{r,N}\right) - f_{eps}\left(\sigma_{r,N-1}\right) \right) hdi \qquad \{4.3\}$$

where the left hand side measured roll radii are got from the density measurement and the web thickness is got from an off-line stack measurement or by means of an on-line caliper measurement. The right hand side of the equation is a measure of the change in the roll radius deformation due to the addition of the last layer. The subscript N refers to roll revolutions.

The solution procedure can be simplified similarly as earlier when the hoop radii where not eliminated from the stress equations without scarifying too much the accuracy. The approach is to use the earlier web layer or earlier Newton iteration values for the stresses and Wound on Tension as starting values for the next iteration step and before the next iteration step solve the new Wound on Tension approximation from the equation {4.3} and then the deformations with this tension value. This way the actual boundary value solver program code remains the same as in the original problem, which sets up the large discretized and linearized matrix equation and solves it by a tridiagonal matrix inverter. The equation {4.3} can also be simplified by means of the linearized constitutive equations. The accuracy is maintained if the Newton iteration is allowed to use enough steps to reach the solution.

NUMERICAL ACCURACY AND PROGRAM EFFICIENCY

The winding model is an exceptional boundary value problem in that it requires setting up and solving over 10000 successive boundary value problems for each layer of a larger newsprint paper roll. The final solution is got as an aggregate of the layerwise solutions and every individual layer solution depends on all the previous solutions through the roll radii deformations and the non-linear stress-strain material behaviour.

How can we be sure that the solver does not go astray due to the inevitable numerical round-off errors or model approximations? The solution might not even be unique at least in case of the Wound On Tension solver.

There is always a trade off between computation time spent and accuracy. The winding model computation time (number of floating point operations) is $O(N^2)$ because the tridiagonal solver needs order N floating point operations and total number of flops is N times the average problem size. It would be tempting to reduce the problem size N by increasing the discretation step. Doubling the step would mean reducing the computation time to one fourth without any added complexity to the program code. However, simulations show that for the 60-micron newsprint paper the accuracy is degraded considerably even if step size is increased from one layer to two layers.

Another simple program change would be reducing the floating-point word length from 64-bit double precision to 32 single precision. Simulation tests show again that the double precision accuracy is well justified for the winding modelling task.

Program optimisation

There are more or less automated software tools available for program optimisation. The simplest thing is to use an optimising compiler and let it to optimise the code for speed. This can easily give more than double speed compared to unoptimized code.

More speed can be gained by manual optimisation but it is laborious and prone to errors. The task can be eased by a software profiler tool, which can point out the "hot spots" in the code, where most of the computation time is spent. Only those sections of the code are worth streamlining where the real work is done. The winding model profiling data points out that the most critical code section is the non-linear stress-strain curve code. It is wise to have as simple parametric curve fit as possible. On the other hand the tridiagonal solver uses only 10-20 percent of the total time. This shows that solving both radial and tangential stress from the two times larger boundary value problem does not increase the computation time significantly compared to solving the radial stress only.

Matrix inversion and the condition number

Matrix inversion should always be checked for numerical problems since it can easily be very ill behaved operation even with double precision arithmetic. The reason for inaccuracy is poor scaling which could be improved by prescaling the data.

There exist a standard measure for checking the matrix before inversion, the matrix condition number. The matrix *A* condition number is defined with the matrix norm as the number $|A|*|A^{-1}|$ which can also be computed as the ratio of the matrix largest singular value to the smallest singular value. The logarithm of the condition number gives an estimate on how many significant decimal places of accuracy is lost in the matrix inversion. The closer the condition number is to one, the more accurate the inversion will be.

The condition numbers were evaluated with the help of Matlab. The test showed that the Large deformations model is over 2 digits more accurate at every matrix inversion than the Small deformations model. The logarithmic condition number for the Large deformations model was below 5.6 as for the Small deformations model the condition number grew till 8.0 for 3000 layers. The Small deformations model can loose 8 of the 15 double precision digits on each layer solution!

SIMULATION DATA

Several sets of simulation runs were performed to test the accuracy and performance of the models and their sensitivity to parameter changes. The simulations were run with the parameter values shown in the Table 1. The web material parameters are typical for the newsprint paper samples used in the experiments.

Table 1: Simulation parameter values

Parameter	Value	Units
Web direction modulus E_t	5600	MPa
Z-direction modulus E_r C_0	0.290	MPa
Z-direction modulus E_r C_1	-45.4	
Z-direction modulus E_r C_2	-20.7	1/MPa
Core modulus E_c	10000	MPa
Web thickness	0.060	mm
Basis weight	45	g/m^2
Poisson ratio ν_{rt}	0	
Poisson ratio ν_{tr}	0	
Wound On Tension	500	N/m
Web speed	0	m/s

Comparison of the Small and Large deformations models

This comparison was done to find out which one of the models would more accurately satisfy the force equilibrium and the radial direction constitutive equations in the final roll. The computation time was also compared.

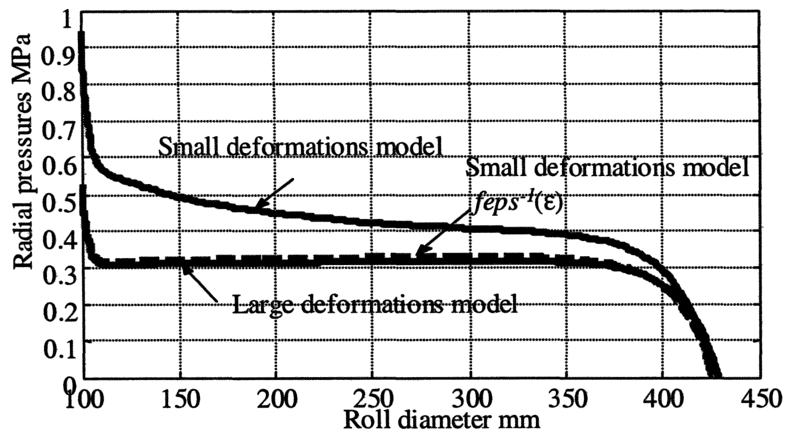

Figure 3: Comparison of the radial pressures calculated from the Small deformations and Large deformations models (solid lines) to the radial pressures calculated from the radial strains (dashed lines). The Large deformations lines are on top of each other.

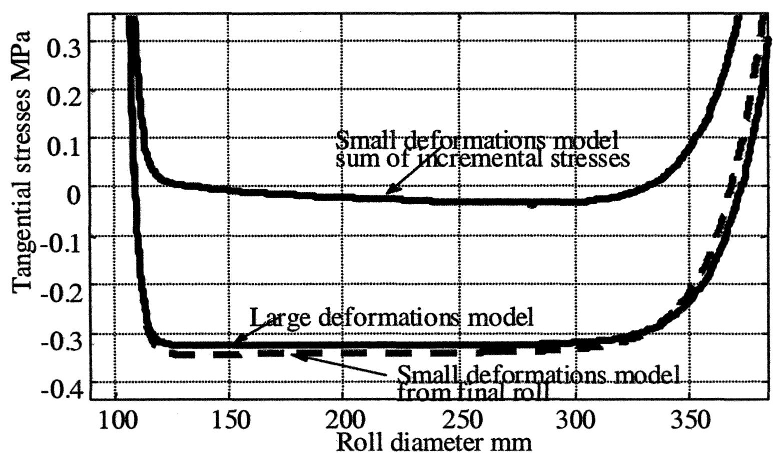

Figure 4: Comparison of the tangential stresses calculated as the sum of the layerwise incremental tangential stresses to the tangential stress calculated from the final roll force equilibrium. For the Large deformations model these are naturally the same.

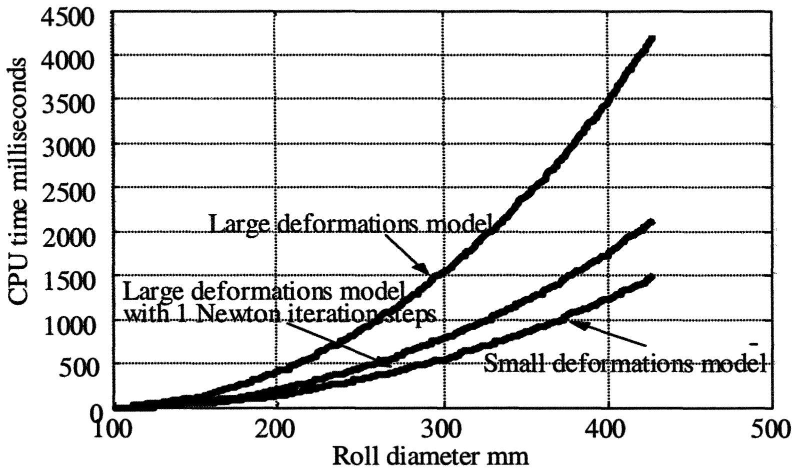

Figure 5: Comparison of the computation times of the Small and Large deformations models on a 2.5 GHz Pentium IV. The Large deformations model uses 1-3 iteration steps per layer when the iteration stop condition for the average stress increment is set to 1 kPa.

The comparisons demonstrate that the Large deformations model is more consistent with the equations it was derived from. The computation cost is reasonable for both models.

Sensitivity to parameter changes

Changing each parameter from –50% to +200% while keeping other parameters constant tested the sensitivity to parameter changes. The sensitivity was measured as the relative change in the average radial stress. For example 100 % change in tangential modulus from 5600 MPa to 11200 MPa would change the average radial pressure by –0.26 MPa. The web speed was varied from 10 to 40 m/s. The change in Wound On Tension means the relative change in the calculated WOT when the parameter is changed from the value it had in the simulation run. For example if the tangential modulus is changed from 5600 to 11200 MPa, the WOT will be 240 N/m in error, or neglecting 40 m/s web speed causes 72 N/m error. The web thickness was varied in this case from 59.5 to 61.5 micrometers.

Table 2: Parameter sensitivity data.

Parameter	Change in average radial pressure	Change in Wound On Tension
Tangential modulus E_t	-0.26	240
Radial modulus E_r	0.24	650
Web thickness	-0.52	12000
Web speed	-0.044	-72

Web thickness is the most critical parameter for the Wound On Tension measurement. Only 1 micrometer error in the incoming web thickness means that the Wound On Tension would be 12000/60=200 N/m erroneous.

EXPERIMENTAL DATA

The experiments were run on the Metso Paper pilot winder. They were run in surface winding mode with only one support drum, which was speed controlled. The rolls were center supported but no center torque was applied. The rolls were newsprint paper with the material parameters in Table 1. The Wound On Tension was directly measured with the WOT drum. The rolls were instrumented with FlexiForce™ pressure sensors.

Three runs were performed with web tensions 300,500 and 700 N/m and respective nip loads 3000, 5000 and 7000 N/m. These winding parameters produced approximately 300, 500 and 700 N/m of Wound On Tension.

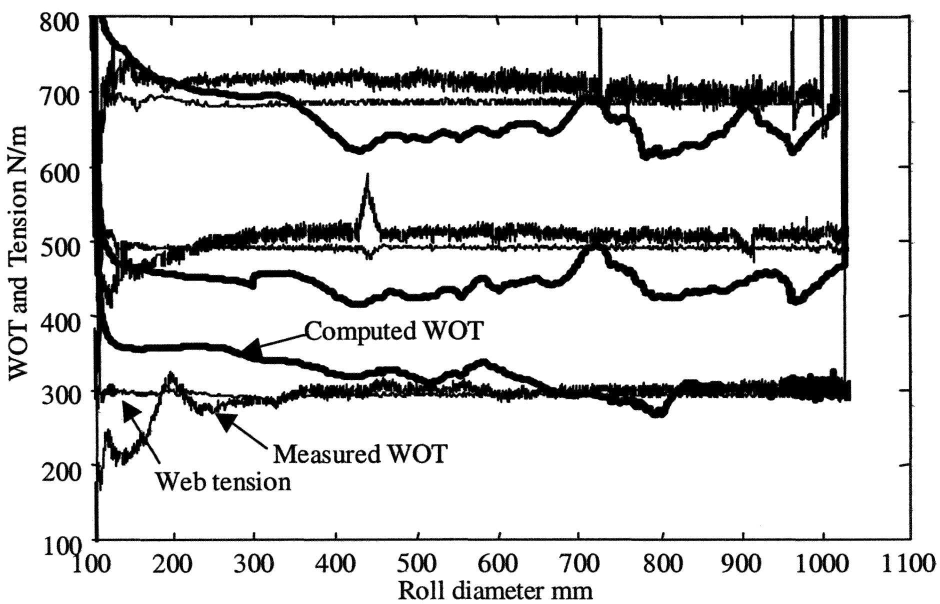

Figure 6: Comparison of computed WOT:s (thick lines) to the measured WOT:s in three winding runs.

CONCLUSIONS

The simulations and experiments showed that it is possible to measure the Wound On Tension from the density measurement by means of the winding model. The most difficult calibration problem is to determine the free web thickness, which can cause the largest error in the measurement. It would be worth of further study to see if the unwind density measurement could be used to detect the changes in the web thickness. The equation {4.3} can be used to solve the original web thickness from the unwind density measurement data if the unwind Wound Off Tension is also measured.

The Large deformations model is clearly theoretically superior to the standard Hakiel model. However, for lower Wound On Tension levels, both models seem to find the correct answer. For over 500 N/m levels the Large deformations model is more accurate.

The difficulty of determining the web thickness, especially in case of paper like newsprint, leads to viscoelastic extensions of the winding model. The paper thickness will change every time it is compressed either in the stack-testing machine or in the winding nip or under pressure in the roll. And in the short time between the winder unwind and windup the paper has not enough time to regain its original caliper.

Viscoelasticity in the large deformations model

The large deformations model is especially attractive in that it easily allows the introduction of time dependent factors and relaxation into the winding model

The viscoelasticity is modelled by time and pressure dependent material parameters. These parameters are explicitly visible in the large deformations model as $f_{eps}()$ and h. The steps needed for modelling viscoelasticity are simple: after every layer addition change the material parameters according to the σ_r and time step and solve the boundary value problem again. When the roll is finished, these time steps can be continued. The changes in the material parameters must be small enough for the non-linear boundary value problem solver to converge.

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