

ROBUST CONTROL DESIGN USING H_∞ METHODS IN LARGE SCALE WEB HANDLING SYSTEMS

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ABSTRACT

This paper presents H_∞ control strategies for elastic web transport systems. The aim is to reduce the coupling between web tension and web transport velocity. First of all, a multivariable H_∞ centralized controller with or without gain scheduling is synthesized for a 3-motor plant composed of an unwinder, a tractor and a winder. This controller is then compared to a semi-decentralized weighted controller with overlapping. The influence of the weighting coefficients is shown on simulation results obtained from a non-linear model identified on an experimental bench.

Web handling systems are generally of large scale and it is not possible to synthesize a centralized controller for such scale. Therefore the global system is split in several subsystems (we have chosen 3-motor subsystems), each subsystem is controlled independently by its own H_∞ controller. The subsystems can be overlapped or not. Simulation results are given on a non-linear 9-motor model.

NOMENCLATURE

E	Young modulus
ε	web strain
J	roll inertia
K	robust controller
L	web length between two rolls
L_0	nominal web length between two rolls
R	roll radius
s	Laplace operator
S	web section

T_k	web tension between roll k and $(k+1)$
T_0	nominal web tension
V_k	web velocity on the roll k
ω	frequency

INTRODUCTION

Web systems handling material such as textile, paper, polymer or metal are very common in industry. The study of the modeling and the control of web handling systems are carried for now several decades. But the increasing requirement on the control performances and the handling of thinner web material oblige us to search for more sophisticated control strategies. Most of modern control strategies (LQG, H_∞ , ...) need the elaboration of the plant model and consequently also its own identification. The detailed model description is presented in [17][43]. In this paper only the main laws on which the model is based will be remained.

One of the objective in web handling systems is to increase web velocity as much as possible while controlling web tension over the entire weblines. Indeed, the web speed is inherently limited by web processing and treatment occurring between the unwinder and winder. Controlling such systems needs other requirements such as :

- web tension and speed decoupling so that constant web tension can be maintained during web speed changes.
- robustness with respect to variations in web elasticity. This allows us both to control the web throughout web processing and treatment, and to use the same control for different types of web.
- robustness with respect to variations in roll diameter and frictions (static and dynamic frictions). The same performance should be maintained throughout web processing even though the roll frictions vary and proper weblines startup must be assured regardless of roll diameter.

So far, most of industrial web transport systems have used decentralized PID-type controllers. However, if higher control requirements are asked, more efficient control strategies – and thus more precise models – must be used. Therefore applications focused mainly on web fabrication quality such as web composition and thickness (e.g. [39][45] for metal and [46][47] for paper) use multivariable H_∞ control. However, recently, multivariable H_∞ control strategies have been proposed for web tension and velocity control in industrial metal transport systems [11][52] and for elastic web [15][17][24].

In this paper, the first part reminds the main laws of physical mechanics used for modelization of web handling systems. These laws enabled us to build a non-linear model which has been afterwards identified on our experimental bench composed of three motors (figure 1). This bench shows the inherent difficulties of web transport systems. The non-linear model built in Matlab/Simulink environment is used as a simulator. Moreover, linearization around an operating point which corresponds to the startup phase enables to find the state space model that is useful for modern controller computing. From this linear model we synthesize a H_∞ centralized controller for our 3-motor system in order to reduce the coupling between web tension and transport velocity.

Nevertheless, winding systems are generally of large scale and it is not reasonable to use a centralized controller for such scale. So, in the second part we will present some decentralized control strategies. Validation is made by simulation on a 9-motor plant.

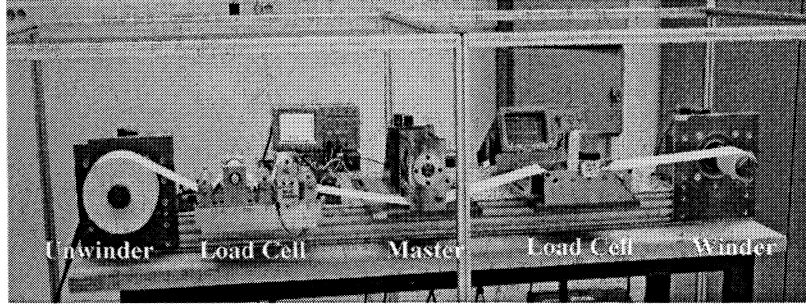


Figure 1 – Experimental setup with 3 motors and 2 load cells

PROCESS MODEL

The model of web transport systems is briefly presented. More details can be found in recent publications [17][15]. It was derived from the model of the web tension between two consecutive rolls and the model of the velocity of each roll.

Web tension calculation

Web modeling is based essentially on three physical laws which allow us to calculate web tension between two rolls:

- Hooke's law which introduces the elasticity of the web :

The tension T of an elastic web is function of the strain ε :

$$T = ES\varepsilon = ES \frac{L - L_0}{L_0} \quad \{1\}$$

where ε is the web strain, E is the Young modulus, S is the web section, L is the web length under stress and L_0 is the web length without stress. Of course, the web viscosity can also be taken into account.

- Coulomb's law which describes the web tension variation due to friction between web and roll : the study of a web tension on a roll can be considered as a problem of friction between solids [34][43]. On the roll, the web tension is constant on a sticking zone and tension change occurs on the sliding zone. The web velocity is equal to the roll velocity on the sticking zone. [34][43].
- The law of conservation of mass which describes the coupling between web velocity and web tension. Consider an element of web of length $l = l_0 (1 + \varepsilon)$ with a weight density ρ , under an unidirectional stress. The cross section is supposed to be constant. According to the mass conservation law, the mass of the web remains constant between the state without stress and the state with stress :

$$dm = \rho S l = \rho_0 S l_0 \Rightarrow \frac{\rho}{\rho_0} = \frac{1}{1 + \varepsilon} \quad \{2\}$$

Tension-velocity relation.

The model of our experimental setup was derived from the model of the web tension between two consecutive rolls (figure 2) and the model of the velocity of each roll. This

well known equation {3} (see [31] and [33]), was obtained since web length on the wrap angle can be neglected compared to the web length without contact between two rolls (figure 2) [43]:

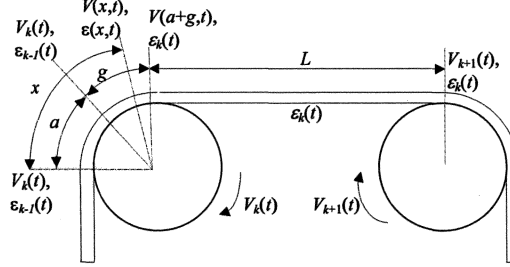


Figure 2 – Web tension on the roll

$$\frac{d}{dt} \left(\frac{L}{1 + \varepsilon_k} \right) = \frac{V_k}{1 + \varepsilon_{k-1}} - \frac{V_{k+1}}{1 + \varepsilon_k} \quad \{3\}$$

This relation can be simplified by deriving the left term and using assumption $\varepsilon \ll 1$:

$$L \frac{dT_k}{dt} = ES(V_{k+1} - V_k) + T_{k-1}V_k - T_k(2V_k - V_{k+1}) \quad \{4\}$$

Web velocity calculation

The velocity of a roll can be obtained with a torque balance:

$$J_k \frac{dV_k}{dt} = R_k^2(T_k - T_{k-1}) + R_k K_k U_k + C_f \quad \{5\}$$

where $K_k U_k$ is the motor torque (if the roll is driven) and C_f is the sum of the friction torque. We can notice at this point that at the unwinder and the winder the inertia J_k and the Radius R_k are time dependent and vary substantially during the process operation.

State-space representation

The non-linear state-space model is composed of {4} for the different web sections and {5} for the different rolls. From it, a linear parameter-varying (LPV) model can be deduced by linearization around a setting point. Under the assumption that J_k/R_k is slowly varying (which is the case for thin webs) we obtain the following linear model:

$$E(t) \frac{dX}{dt} = A(t)X + B(t)U \quad \{6\}$$

$$Y = C X$$

$$Y = [T_u \ V_3 \ T_w]^T$$

where : $X = [V_1 \ T_1 \ V_2 \ T_2 \ V_3 \ T_3 \ V_4 \ T_4 \ V_5]^T$ and $U = [u_u \ u_v \ u_w]^T$

Matrices $A(t)$, $B(t)$, $C(t)$, $D(t)$ can be found in [43][17][15].

When freezing its parameters, the LPV model is converted into a linear time-invariant model that is used for controller synthesis. Since the starting phase is the most crucial, the model chosen corresponds to this phase. However, simulations presented in the sequel rely on the nonlinear equations {1}, {3} and {5}. The two models need the estimation of their parameters.

PARAMETER ESTIMATION

The off line identification is based on the model matching method. The cost function to be minimized is :

$$J_{opt} = \frac{(Y_P - Y_M)^T (Y_P - Y_M)}{Y_P \cdot Y_P} \quad \{7\}$$

where Y_M , Y_P are respectively the vectors of simulated and measured output signals (T_u , V , T_w , see figure 3). Several optimization algorithms [43] are used. The simplex method [48] gave the smallest cost function and was more robust to the initial values of the parameters. The simulations with the optimized parameters and the measurements are compared on the figure 4 for our 3-motor plant. By extending this model to a 9-motor system, a simulator was built for testing decentralized controllers. This simulator was used to obtain results shown further in this paper.

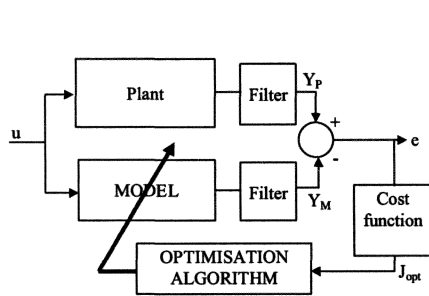


Figure 3 – Model matching method

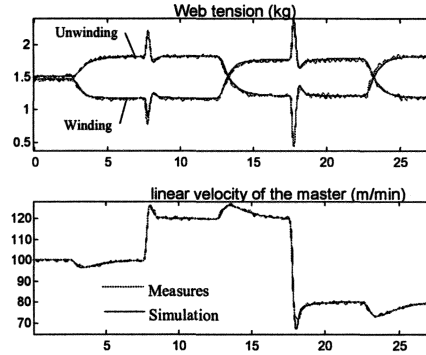


Figure 4 - identified model and measures

ROBUST CONTROL OF LOW SCALE SYSTEMS

Multivariable H_∞ robust controller synthesis

Robust H_∞ control is a powerful tool to synthesize multivariable controllers with interesting properties of robustness and disturbance rejection. We have to remain that H_∞ control theory deals with the minimization of the H_∞ -norm of the transfer matrix {8} from an exogenous input to a pertinent controlled output of a given plant. The synthesis of the multivariable controller is done using the nominal model, in our case this model corresponds to the starting phase.

The H_∞ controller is synthesized using the mixed sensitivity approach [49][37], as shown in figure 5, where w are the exogenous inputs (like tension and velocity references : T_{uref} , V_{ref} , T_{wref}) and z are the controlled signals. The frequency weighting functions W_p , W_u and W_t appear in the closed-loop transfer function matrix as following:

$$T_{wz} := \begin{bmatrix} W_p S \\ W_u K S \\ W_t T \end{bmatrix} \quad \{8\}$$

where S is the sensitivity function $S = (I + GK)^{-1}$, and T is the complementary sensitivity function $T = I - S$ [49].

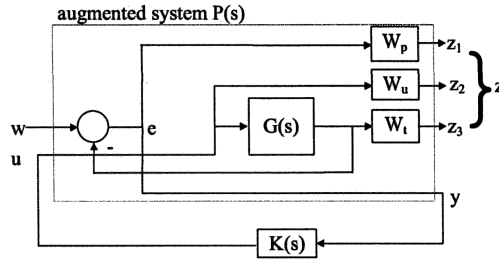


Figure 5 – Mixed sensitivity method

The controller K can be computed with LMI (linear matrix inequalities) [3][25] or with the Riccati equation [49] solved via γ -iteration (algorithm of Glover - Doyle) [37]. The controller stabilize the system such that the H_∞ -norm of the transfer function between w and z is :

$$\|Tzw\|_\infty := \sup_{\omega} \sigma_{\max}(Tzw(j\omega)) \leq \gamma \quad \{9\}$$

with γ close to γ_{\min} (the smallest value of γ). σ_{\max} denotes the maximum singular value.

The performances and the robustness of the controller depend on the choice of the weighting functions. The frequency weighting function W_p is usually selected with a high gain at low frequency to reject low frequency perturbations and to reduce steady-state error. The form of W_p is as following [49]:

$$W_p(s) = \frac{\frac{s}{M} + w_B}{s + w_B \epsilon_0} \quad \{10\}$$

where M is the maximum peak magnitude of S , $\|S\|_\infty \leq M$, w_B is the required bandwidth frequency and ϵ_0 is the steady-state error allowed. The weighting function W_u is used to avoid large control signals and the weighting function W_t increases the roll-off at high frequencies.

For our 3-motor plant, the selected weighting functions are :

$$W_p(s) = \begin{bmatrix} \frac{0.7s+10}{s+0.01} & 0 & 0 \\ 0 & \frac{0.7s+6}{s+0.01} & 0 \\ 0 & 0 & \frac{0.7s+10}{s+0.01} \end{bmatrix} \quad \{11\}$$

$$W_u = 0.1 I_{3 \times 3}; \quad W_t(s) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} \quad \{12\}$$

The poles in the weighting matrix W_p are almost integrators to avoid numerical problems [49]. The order of the resulting controller is 15. It has been implemented on our experimental setup in state space representation with a sampling period of 10 ms.

The decentralized control using PID controllers (figure 6) and the multivariable H_∞ control (figure 7) are compared on figure 8.

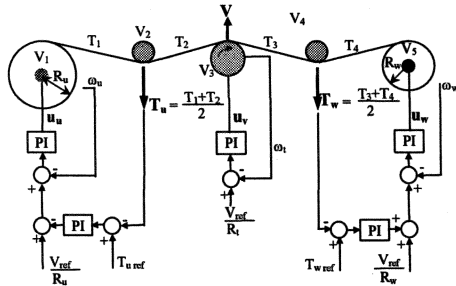


Figure 6 – industrial decentralized control scheme

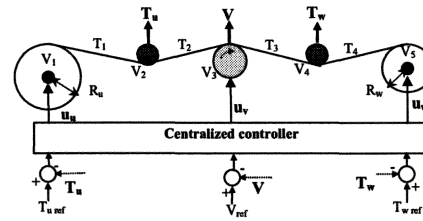


Figure 7 – Multivariable control scheme

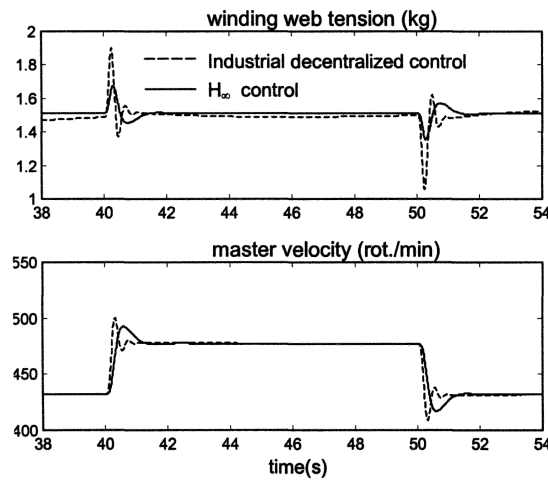


Figure 8 – Comparison industrial control – multivariable H_{∞} control

With the H_{∞} controller, the web tensions are noticeably less sensitive to velocity variations. The coupling between tension and web velocity is also reduced.

So far, we have discussed about robust control. What do we mean by “robust”? The robustness reflects the ability of the system to maintain adequate performances and particularly the stability of the closed-loop when there are variations in plant dynamics (coming e.g. from parameters variations) and errors in the plant model (nominal model) which is used for controller design. The robustness of a controlled system is therefore a fundamental requirement in designing any controller. When designing a control system *via* standard methods (PID, optimal control, ...), the robustness is not taken into account directly and is checked afterwards. The enhancement of robustness has been the major motivation for research in the area of robust control, and specifically the H_{∞} approach [7], [49], [37], [6], [5], [23]. The robustness of the controlled system can be analysed *via* μ -analysis [51], [4]. The application of this method to our 3-motor plant is presented in [18], [19].

To consider parameter variations directly into the synthesis of the controller, different approaches exist, such as for example μ -synthesis [49] and the gain scheduled H_{∞} control (figure 10) for linear parameter varying systems (LPV) [1]. The first method enables to

obtain a LTI (linear time invariant) controller whereas with the second one we get a LPV controller which is more efficient when parameters vary widely.

Multivariable H_{∞} robust control with gain scheduling

Due to the wide-range variation of the roller radius during the winding process, the dynamic behavior of the system is considerably modified with time. To analyse this modification, let us consider the unwinder, respectively the winder, separately. With quasi-static assumption on radius variations, the static gains between the control signals and the web tensions appear to be proportional to the inverse of the radius [43] :

$$Gain_{DC} \left(\frac{T_u}{u_u} \right) = \frac{1}{R_u} \text{ and } Gain_{DC} \left(\frac{T_w}{u_w} \right) = \frac{1}{R_w} \quad \{13\}$$

We therefore multiply the control signals by the corresponding radius measurement or estimation (see figure 9):

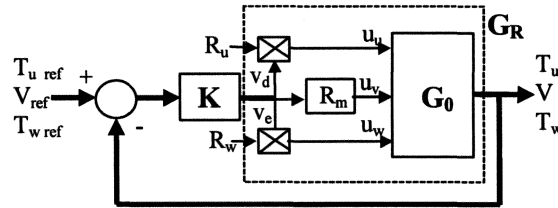


Figure 9 – Modified system

The synthesis of the controller is done using the plant G_R which includes the radii multiplication (gain scheduling). This approach allowed us to reduce web tension variations significantly despite velocity changes during processing [41], [43].

The control can be enhanced by using LPV controllers [42], [17] :

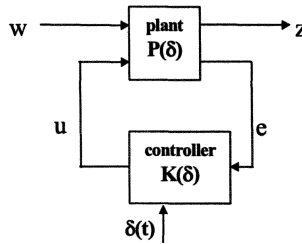


Figure 10 – Gain-scheduled problem

On figure 10, $\delta(t)$ represents the varying parameters.

Semi-decentralized H_{∞} control

Winding systems are generally of large scale (high number of motors) and it is not reasonable to use a centralized controller for such scale. Therefore these systems are a good application for the recent decentralized control theories [26], [29]. Overlapping in decentralized control gives extra degrees of freedom that allow improvements compared to

disjoint decomposition [10] and this is that approach that we are going first to apply on our 3-motor plant before studying a large scale system.

Let us consider the linear state space model S given by {6}. This system is then split into two overlapped subsystems : S_1 and S_2 mentioned by dashed lines in the matrices A , B , C :

$$\begin{aligned} \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{aligned} \quad \{14\}$$

In the *expanded space* [10][27] where S_1 and S_2 are disjoint, we obtain S_e :

$$\begin{aligned} \begin{bmatrix} \frac{dx_{1e}}{dt} \\ \frac{dx_{2e}}{dt} \\ \frac{dx_{3e}}{dt} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_{1e} \\ x_{2e} \\ x_{3e} \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 & 0 \\ 0 & B_{22} & 0 & 0 \\ 0 & 0 & B_{22} & 0 \\ 0 & 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_{1e} \\ u_{2e} \\ u_{3e} \end{bmatrix} \\ \begin{bmatrix} y_{1e} \\ y_{2e} \end{bmatrix} &= \begin{bmatrix} C_{11} & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 \\ 0 & 0 & C_{22} & 0 \\ 0 & 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} x_{1e} \\ x_{2e} \\ x_{3e} \end{bmatrix} \end{aligned} \quad \{15\}$$

with :

$$\begin{aligned} x_{1e} &= [x_1^T \ x_2^T]^T & x_{2e} &= [x_2^T \ x_3^T]^T & u_{1e} &= [u_1^T \ u_2^T]^T & u_{2e} &= [u_2^T \ u_3^T]^T & y_{1e} &= [y_1^T \ y_2^T]^T \\ y_{2e} &= [y_2^T \ y_3^T]^T \end{aligned}$$

For each subsystem we synthesize then a multivariable H_∞ controller called C_1 and C_2 respectively. These two controllers have the following form in state space representation:

$$\begin{aligned} C_1: \quad \dot{z}_1 &= F_1 z_1 + \begin{bmatrix} G_1^1 & G_2^1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} &= \begin{bmatrix} H_1^1 \\ H_2^1 \end{bmatrix} z_1 + \begin{bmatrix} K_{11}^1 & K_{12}^1 \\ K_{21}^1 & K_{22}^1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \end{aligned} \quad \{16a\}$$

$$\begin{aligned} C_2: \quad \dot{z}_2 &= F_2 z_2 + \begin{bmatrix} G_1^2 & G_2^2 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} \\ \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} &= \begin{bmatrix} H_1^2 \\ H_2^2 \end{bmatrix} z_2 + \begin{bmatrix} K_{11}^2 & K_{12}^2 \\ K_{21}^2 & K_{22}^2 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} \end{aligned} \quad \{16b\}$$

The simple controller composed diagonally of controllers C_1 and C_2 is not *contractible* from *expanded space* to *the initial space* [26]. State model is then rearranged in order to have the *contractibility* property [26]:

$$\begin{bmatrix} \dot{Z}_{1e} \\ \dot{Z}_{2e} \end{bmatrix} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix} \begin{bmatrix} Z_{1e} \\ Z_{2e} \end{bmatrix} + \begin{bmatrix} G_1^1 & G_2^1 & 0 & 0 \\ 0 & 0 & G_1^2 & G_2^2 \end{bmatrix} \begin{bmatrix} Y_{1e} \\ Y_{2e} \end{bmatrix} \quad \{17\}$$

$$\begin{bmatrix} U_{1e} \\ U_{2e} \end{bmatrix} = \begin{bmatrix} H_1^1 & 0 \\ H_2^1 & H_1^2 \\ H_2^1 & H_1^2 \\ 0 & H_2^2 \end{bmatrix} \begin{bmatrix} Z_{1e} \\ Z_{2e} \end{bmatrix} + \begin{bmatrix} K_{11}^1 & K_{11}^1 & 0 & 0 \\ K_{21}^1 & 1/2(K_{22}^1 + K_{11}^2) & 0 & K_{12}^2 \\ K_{21}^1 & 0 & 1/2(K_{11}^2 + K_{22}^1) & K_{12}^2 \\ 0 & 0 & K_{21}^2 & K_{22}^2 \end{bmatrix} \begin{bmatrix} Y_{1e} \\ Y_{2e} \end{bmatrix}$$

This controller is then contracted into the initial space, leading to the implementation controller.

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} + \begin{bmatrix} G_1^1 & G_2^1 & 0 \\ 0 & G_1^2 & G_2^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} H_1^1 & 0 \\ H_2^1 & H_1^2 \\ 0 & H_2^2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} K_{11}^1 & K_{12}^2 & 0 \\ K_{21}^1 & (K_{22}^1 + K_{11}^2) & K_{12}^2 \\ 0 & K_{21}^2 & K_{22}^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \{18\}$$

The relation {18} notifies us that the middle actuator command results neither from the average nor from the sum of the output of each controller. However, in our case the sum (in other words, $a = 1$ and $b = 1$ in figure 11) is a good approximation as it can be seen on figures 12 to 14.

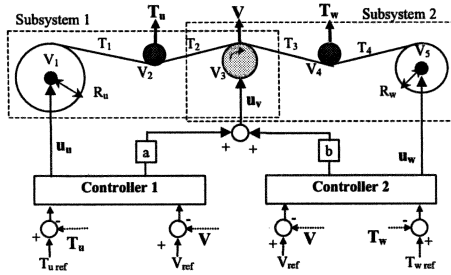


Figure 11 – semi-decentralized overlapped control for the 3-motor plant

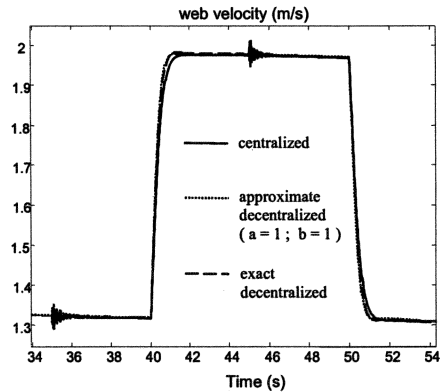


Figure 12 – simulation of V

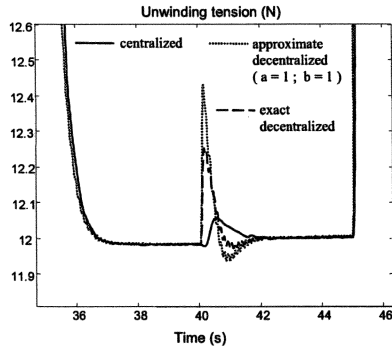


Figure 13 – simulation of T_u

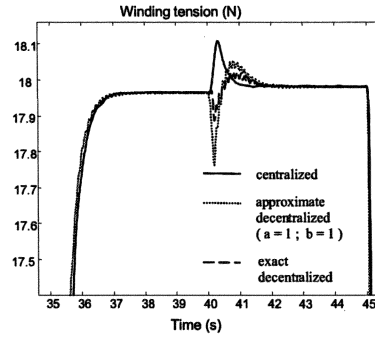


Figure 14 – simulation of T_w

On figures 12 to 14 are compared simulation results coming from the centralized multivariable control scheme (figure 7) and the semi-centralized overlapped control (figure 11) with $a = 1$ and $b = 1$. The velocity and tension references tracking are close but the decoupling between velocity and tensions is enhanced for the centralized controller, as expected.

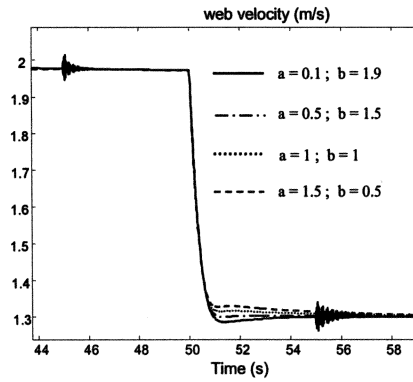


Figure 15 – simulation of V

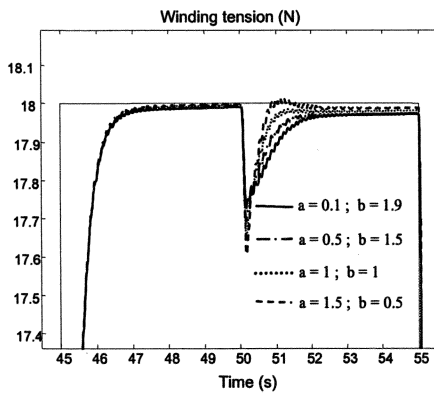


Figure 16 – simulation of T_u

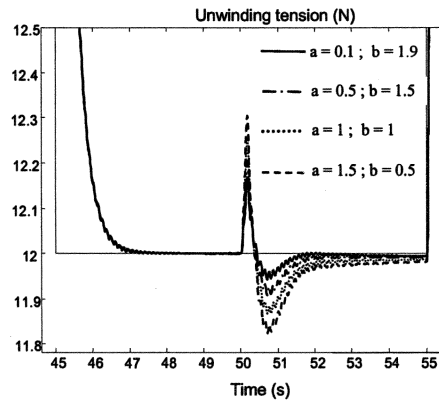


Figure 17 – simulation of T_w

Figures 15 to 17 show the influence of the weighting parameters $a = 1 - \alpha$ and $b = 1 + \alpha$ when α varies between -1 and 1 (see figure 11): the coupling effects can not significantly be reduced by α .

ROBUST CONTROL OF LARGE SCALE SYSTEMS

As industrial winding plants are generally large scale coupled systems, they appear to be a good application field for the recent improvements in decentralized control theories [27]. Seeing that it is not reasonable to synthesize a centralized controller for the global system, we first split the system in subsystems. The number of subsystems comes from a compromise of their order. For example, in the case of a 9-motor plant, each subsystem contains three motor in our case and is controlled independently by its own controller (see figure 18).

The coupling existing between two consecutive subsystems can be reduced by introducing an overlapping strategy: the command signal for the tractor located at the boundary of two subsystems comes from two controllers (see figure 19).

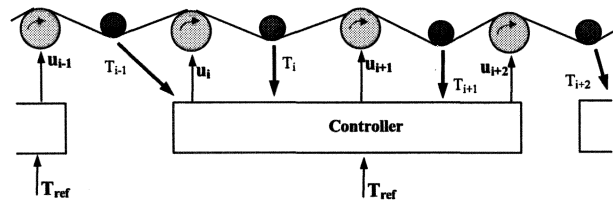


Figure 18 – Semi-decentralized control strategy for a large scale system

On figure 20 are shown simulation results of the web tension at the middle of the plant in the case of two control strategies: the first one do not include any overlapping whereas the second one presents a light overlapping as explained in figure 19. The tension reference remains constant; web velocity reference changes every 10 seconds, leading to web tension perturbations. It appears clearly that the coupling between tension and velocity is attenuated with an overlapping control strategy.

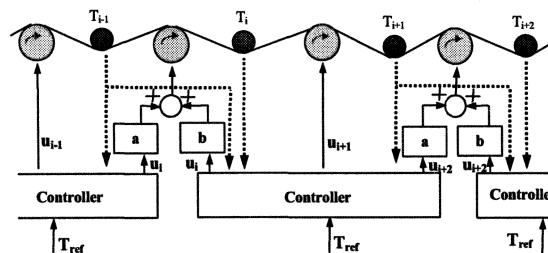


Figure 19 – Semi-decentralized weighted overlapped control strategy for a large scale system

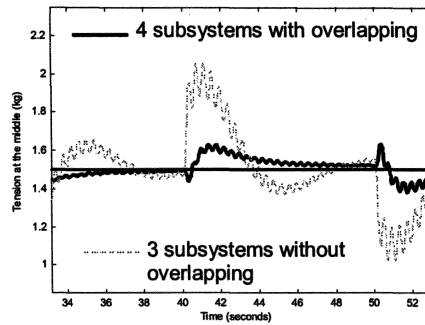


Figure 20 – Comparison between responses with and without overlapping control

On figure 21 is shown the velocity reference tracking for the semi-decentralized overlapping control and figure 22 presents the influence of the velocity changes on the web tension. Unfortunately, the coupling effects cannot significantly be reduced by α , like for the semi-decentralized control applied on the 3-motor plant.

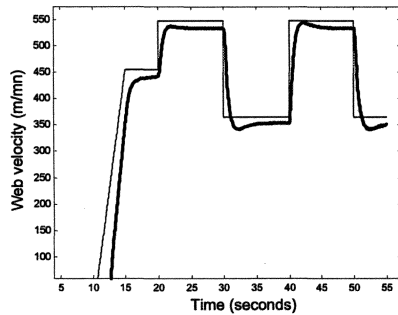


Figure 21 – Velocity variations for the semi-decentralized overlapped scheme

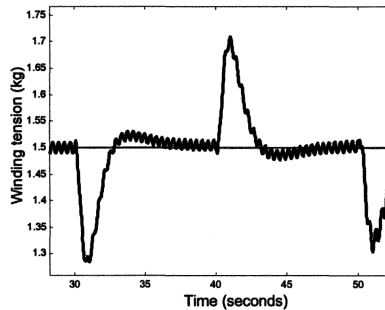


Figure 22 – Web tension in the system-middle for the semi-decentralized overlapped scheme

CONCLUSION

Compared to decentralized PID control strategies classically used in web handling systems, multivariable H_∞ control has shown improved performances in decoupling between speed and web tension for low scale systems (that means with a limited number of actuators). Nevertheless, a centralized controller cannot be used in large scale winding systems. Therefore the global system is split in several subsystems, each subsystem is controlled independently by its own H_∞ controller. The controllers can be overlapped or not. Simulation results are given, based on a non-linear 9-motor model, and have shown that overlapping improves the performances. The next step consists in robustness analysis of the global controlled system *via* μ -analysis by using linear fractional representation [22][19].

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