MODELING RUBBER COVERED NIP ROLLERS IN WEB LINES

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ABSTRACT

Rubber covered nip rollers have a number of applications within web lines. Rubber covered rollers are often used to nip the web against a metal surfaced roller that is driven to achieve a certain web velocity or web tension. The lamination of webs is common in web process machinery where two or more webs are bonded together under nip pressure between rubber covered roll pairs. Rubber covered rolls are also used to wring liquids from webs and prevent contamination of downstream web processes.

The complexity of rubber covered rollers is the rubber itself. Rubber is nearly incompressible. Intuitively rubber appears readily compressible but changes in shape are often mistaken for changes in volume.

The purpose of this publication include to: (1) better document some properties of rubber and (2) to examine the usefulness of two dimensional algorithms that relate the force and deformations of rubber rolls in contact with other rolls and (3) to examine the potential for extending these algorithms for use in three dimensional modeling in which the bending deformations of the roll shafts become substantial.

NOMENCLATURE

a	half width of contact
b	width of contact
D	nominal roll diameter (including cover thickness)
E	Young's modulus
E_c	Young's modulus for rubber including confinement effects
E_o	Young's modulus for rubber without confinement effects
f	transverse load acting on finite element
F	nip loading per unit width
I	area moment of inertia of metal shells or shafting
IRHD	International Rubber Hardness Degree (equivalent to Shore A)

k	material parameter for rubber, $k=0.305+0.053*IRHD-0.00135*IRHD^2+0.00000906*IRHD^3$ ($30 \leq IRHD \leq 75$)
M	concentrated moment acting on finite element
p	contact pressure
R	nominal roll radius
S	shape factor for rubber
t	rubber cover thickness
u	radial deformation of rubber
v	nodal deflection of shafting and rubber covering in finite element model
W	width of roll pair in contact
Z	stiffness of elastic foundation elements
δ	penetration of rubber covering due to nip lad, F
ε	strain
ν	Poisson's ratio
θ	nodal slope of shafting and rubber covering in finite element model
σ	stress

THE MATERIAL PROPERTIES OF RUBBER

There are several ways in which rubber properties can be characterized. The Mooney-Rivlin coefficients are often used today but are difficult to determine without compression samples. Most engineers who deal with rubber covered rolls at best may be able to discern the Shore A (i.e. IRHD) hardness of the rubber cover with a hand-held instrument. Thus the discussion here will focus on determining the material properties of rubber based upon measurements that are easily made in the field.

Compression tests were conducted on samples of Hypalon, nitrile, carboxilated nitrile, neoprene, ethylene propylene, and urethane rubbers in durometers ranging from 30 to 90 (Shore A). The results from these tests are shown in Figure 1. These results indicate that Young's modulus is highly dependent upon the Shore hardness and is independent of the rubber type. Compression data was found in the literature for natural rubber that is also shown in Figure 1 and is shown to yield similar moduli as a function of hardness [6]. All tests and specimens conformed to ASTM specification 575[1]. The specimens were nominally 28.6 mm (1.129 in) in diameter and 12.5 mm (0.5 in) high. Spotts [2] among others has previously suggested using an exponential relationship between modulus and durometer over a limited durometer range. Based upon the data presented in Figure 1 a new exponential expression was developed that is valid over a wider durometer range and based upon these tests is shown valid for natural and synthetic rubbers:

$$\begin{aligned} E_o &= 145.7\, e^{0.0564*IRHD} \quad \text{(MPa)} \\ E_o &= 20.97\, e^{0.0564*IRHD} \quad \text{(psi)} \end{aligned} \qquad \{1\}$$

Poisson's ratio is reported to approach 0.5 for rubber materials. Results presented herein show that better definition of Poisson's ratio was required. After finding no quantitative data for Poisson's ratio in the literature a set of tests were performed on the compression samples described above. Diametral expansion was measured using a strain extensiometer whilst the specimens were subjected to controlled compression. The slope of diametral strain versus the compressive strain data was used to estimate Poisson's ratio. It should be noted that such tests are difficult with specimens of the dimensions prescribed by the ASTM specification 575. If the ends of the specimen are not well lubricated the tractions due to friction will make Poisson's ratio appear to exceed 0.5. The tests become more difficult with increased durometer level as the pressure between the platens and the sample tend to exude whatever lubricants are used. The test results are given in Figure 2. There appears to be no trends between Poisson's ratio and durometer but it is clear the typical result is less than 0.5. The average result for all rubber types and durometers was 0.46.

FORCE/DEFORMATION RELATIONS

The analysis of rubber covered rolls in contact with other rollers has been addressed in several publications. Several rely upon series elasticity solutions and the finite element method to examine the force versus deformation characteristics of a nip roll pair [3-5]. Although the nonlinear finite element method is superb for studying large deformation contact problems such as this there are closed form algorithms either available or extractable from the literature that have a broad range of application for these problems. It is arguable that closed form solutions are more useful by a broad group of engineers rather than single result finite element analyses but this is not the mission of this publication. The following derivations were reviewed or extended to gain a better understanding of force/deformation relationships that exist within the literature.

Lindley [6]

Lindley developed a solution for a rubber-covered roll in contact with a plane surface, equivalent to two identical rubber covered rolls in contact. Due to a contact load F the rubber covering deforms an amount δ. This derivation is simple in that it is assumed all of the contact load is reacted by a rectangular block of material of width b. It is also assumed that the incompressibility of the rubber cover does not impact the contact width b. Using the geometry shown in the Figure 3 it can be found that:

$$R^2 - (b/2)^2 = (R-\delta)^2 \quad \{2\}$$

and for small penetrations δ,

$$b \approx \sqrt{8R\delta} = \sqrt{4D\delta} \quad \{3\}$$

Lindley then defines an incremental stiffness which is a function of the penetration, δ as:

$$\frac{dF}{d\delta} = \left[\frac{\text{modulus*area}}{\text{thickness}}\right]_{\delta} \quad \{4\}$$

It is also assumed that the contact width, b, is small compared to the width of the roller, w, and that as a result plane strain conditions exist. Under plane strain conditions the compressive modulus as affected by constraint, E_c, can be determined from:

$$E_c = \frac{4E_o}{3}\left(1 + k\,S^2\right) \quad \{5\}$$

where E_o is Young's modulus per expression {1} and S is the shape factor which is defined by Lindley as the ratio of the cross-sectional area to the force free area. k is an empirically derived factor which is a function only of Young's modulus which makes the equality {5} true. Per the definition of shape factor:

$$S = \frac{2\sqrt{D\delta}\,w}{2(t-\delta)(b+w)} = \frac{\sqrt{D\delta}}{(t-\delta)} \quad \{6\}$$

where the contact width, b, has been assumed to be small compared to roller width, w. Substituting the knowledge of modulus, area of contact, and deformed thickness into expression {4} yields an expression for stiffness:

$$\frac{dF}{d\delta}=\frac{2\sqrt{D\delta}w}{(t-\delta)}\frac{4E_o}{3}\left(1+\frac{kD\delta}{(t-\delta)^2}\right) \qquad \{7\}$$

Lindley then non-dimensionalized the deformation by dividing by the cover thickness and introducing a new variable u(=δ/t). He then integrated {7} to yield an expression for load versus the non-dimensionalized deformation u:

$$F = E_o w\sqrt{tD}\left(\alpha_R+\frac{kD}{t}\beta_R\right) \qquad \{8\}$$

where:

$$\alpha_R=\frac{8}{3}\ln\left(\frac{1+\sqrt{u}}{1-\sqrt{u}}\right)-\frac{16}{3}\sqrt{u} \qquad \{8a\}$$

and:

$$\beta_R=\ln\left(\frac{1+\sqrt{u}}{1-\sqrt{u}}\right)-\frac{10}{3}\frac{\sqrt{u}}{(1-u)}+\frac{4}{3}\frac{\sqrt{u}}{(1-u)^2} \qquad \{8b\}$$

or an equivalent form:

$$F=\frac{2}{3}\sqrt{D}E_o\left[\sqrt{\delta}\left(\frac{Dk(5\delta-3t)}{(\delta-t)^2}-8\right)+\frac{(3Dk+8t)\mathrm{ATANH}\left[\sqrt{\frac{\delta}{t}}\right]}{\sqrt{t}}\right] \qquad \{9\}$$

It should be noted that Lindley's expression would require modification to model a rubber-covered roll in contact with a metal roller. In its present form it will model the case of two identical rubber covered rollers in contact or equivalently, one rubber roller in contact with a half plane.

Johnson [7]

Johnson has also developed a relationship for nip load as a function of penetration for a rigid cylinder in contact with a elastic covering on a second cylinder. First, he assumes that the rubber covering is thin and therefore plane strain conditions exist. Thus the deflections and strain in the lateral y direction are assumed to be zero. The strain in a vertical z direction is:

$$\varepsilon_z=\frac{1-\nu^2}{E_o}\sigma_z-\frac{\nu(1+\nu)}{E_o}\sigma_x \qquad \{10\}$$

The σ_z stress is assumed to be constant throughout the cover thickness and equal to the pressure of contact, p(x), refer to Figure 4. Thus expression {9} becomes:

$$\varepsilon_z=\frac{1-\nu^2}{E_o}\left[-p(x)-\frac{\nu}{1-\nu}\sigma_x\right] \qquad \{11\}$$

where the negative associated with the pressure of contact is introduced to infer a compressive σ_z stress. The deformation in the contact region in the z direction is known since the metal roll surfaces are assumed rigid. The penetration is maximum at the center of the contact region and decreases to zero at the edge of the contact zone:

$$u_z=-\left(\delta-\frac{x^2}{2R}\right) \qquad \{12\}$$

It should be noted that the radius R is an equivalent radius of the form $R=(R_1*R_2)/(R_1+R_2)$, thus allowing these expressions to be applicable to rollers in contact as well as a roller in contact with a plane. In Johnson's derivation the half width of contact is defined as *a* and is related to the maximum penetration per:

$$a^2 = 2R\delta \qquad \{13\}$$

Note that expression {13} is equivalent to expression {3} if $2a$ is substituted for b. The strain in the z direction can now be found as:

$$\varepsilon_z = \frac{\Delta t}{t} = -\frac{\left(\delta - \frac{x^2}{2R}\right)}{t} \qquad \{14\}$$

Johnson then assumes a bond between the elastic cover and the underlying rigid roll constrains the deformation in the x direction and that thereby the strain in the x direction is zero.

$$\varepsilon_x = \frac{1-\nu^2}{E_o}\sigma_x - \frac{\nu(1+\nu)}{E_o}\sigma_z = \frac{1-\nu^2}{E_o}\left[\sigma_x + \frac{\nu}{1-\nu}p(x)\right] = 0 \qquad \{15\}$$

If equations {11} and {14} are combined and then with {15} there are two equations and two unknowns, σ_x and p(x), that can solved. After eliminating σ_x the pressure distribution p(x) is found to be:

$$p(x) = \frac{(1-\nu)^2}{1-2\nu}\frac{E_o}{1-\nu^2}\frac{a^2}{2Rt}\left(1-\frac{x^2}{a^2}\right) \qquad \{16\}$$

Integrating {16} over the contact area yields the nip load:

$$F = \frac{1}{3}\frac{(1-\nu)^2}{1-2\nu}\frac{E_o}{1-\nu^2}\frac{a^3}{Rt} \qquad \{17\}$$

Expression {13} can be substituted to yield the nip load as a function of the maximum penetration:

$$F = \frac{2}{3}\frac{(1-\nu)^2}{1-2\nu}\frac{E_o}{1-\nu^2}\frac{\sqrt{D}}{t}\delta^{3/2} \qquad \{18\}$$

<u>Evans [8]</u>

Evans developed a solution on a somewhat different basis than Johnson. He as well used expression {13} to model the contact width. Evans then represents the average pressure in the contact area as:

$$p_o = \frac{F}{2\sqrt{2R\delta}} \qquad \{19\}$$

where a unit width of contact (in the y direction) has been assumed. Evans then assumes that the elastic stresses in the narrow zone of contact between the rubber covered roller and the contact plane are the same as the case in which the average pressure (p_o) acts uniformly about the circumference of the rubber covered roll. This enabled him to develop an expression between p_o and δ using Lame's solution for the elastic stresses in a cylinder subject to external (p_o) and internal (p_i) pressures. At radius r the solutions for the stresses are:

$$\sigma_r = \frac{A}{r^2} + 2C$$
$$\sigma_\theta = -\frac{A}{r^2} + 2C \qquad \{20\}$$

where:

$$A = \frac{a^2 R^2 (p_o - p_i)}{R^2 - a^2}$$

and:

$$2C = \frac{p_i a^2 - p_o R^2}{R^2 - a^2} \qquad \{21\}$$

In an axisymmetric formulation for a cylinder the general expression for the tangential strain is:

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}\left[\sigma_\theta - \nu\sigma_r\right] = \frac{1}{E}\left[\left(-\frac{A}{r^2} + 2C\right) - \nu\left(\frac{A}{r^2} + 2C\right)\right] \qquad \{22\}$$

This can be rewritten in terms of the radial deformation (u) as:

$$u = \frac{1}{E_o}\left[-\frac{(1+\nu)A}{r} + 2C(1-\nu)r\right] \qquad \{23\}$$

At the inside radius of the rubber cover it is assumed that the roller shaft restricts the deformation such that the previous expression becomes:

$$0 = \frac{1}{E_o}\left[-\frac{(1+\nu)A}{a} + 2C(1-\nu)a\right] \qquad \{24\}$$

which can be solved in terms of the inner pressure as:

$$p_i = \frac{2 p_o R^2}{a^2 + R^2 + \nu(R^2 - a^2)} \qquad \{25\}$$

The deformation at the outside of the roller cover is δ, where the outside pressure is given in expression {19}. Substituting into expression {25} and solving in terms of the nip load (F) yields:

$$F = \sqrt{\frac{2}{R}}\,\frac{2E_o\left[R^2(1+\nu) + a^2(1-\nu)\right]}{(R^2 - a^2)(1-\nu^2)}\,\delta^{3/2} \qquad \{26\}$$

COMPARISON OF FORCE/DEFORMATION RELATIONS

In Figure 5 a comparison of the theories presented by Lindley, Johnson, and Evans is shown. Johnson's expression {18} is extremely sensitive to Poisson's ratio in the .45 to 0.5 range, due to the ($1-2\nu$) term in the denominator. When Poisson's ratio is set at 0.46 the three theories yield nearly identical results for deformations up to about 10% of the cover thickness. At deformations less than 10% Lindley's expression predicts slightly less nip load at a given deformation than Johnson's or Evans' expressions which is not visible in the scale of Figure 5. At higher deformations Lindley's theory then diverges from the rest and the effect of the rubber confinement becomes notable. Johnson's and Evan's theory does not account for the confinement of the rubber as Lindley did. The effect on Young's modulus can be accounted for by including the $1+kS^2$ term as shown below in modified versions of Johnson's {27} and Evan's {28} expressions.

$$F = \frac{1}{3}\frac{(1-\nu)^2}{1-2\nu}\frac{E_o\left(1+kS^2\right)}{1-\nu^2}\frac{(2\delta)^{3/2}\sqrt{R}}{t} \tag{27}$$

$$F = \sqrt{\frac{2}{R}}\frac{E_o\left(1+kS^2\right)}{1-\nu^2}\frac{2\left[R^2(1+\nu)+a^2(1-\nu)\right]}{\left(R^2-a^2\right)}\delta^{3/2} \tag{28}$$

VERIFICATION OF THEORY

These expressions have never been thoroughly verified. A test program was setup such that the deformations of nip roll pairs could be monitored in a web line as nip load was varied. The deformations were monitored using linear variable differential transformers. Nip loads were applied using pneumatic cylinders that were controlled with regulators. Four rubber covered rollers were manufactured in pairs with various diameters, cover thickness, and durometers as shown in Table 1. In Figure 6 the results of experiments are presented that were conducted with identical rubber covered rolls in contact. In Figure 7 the results of experiments are shown that were conducted with rubber covered rolls in contact with metal rollers whose surfaces were relatively rigid compared to the covered rolls. The results of Evans expression {26} are not shown in these plots, the results would be identical to that of Johnson's expression {18} since the assumption was made to leave Poisson's ratio fixed at 0.46. Johnson's expression appears to match the experimental data quite nicely while Lindley's expression {8} appears to undershoot the data slightly in several of the cases. The maximum cover penetrations shown in these figures range from 2.5 to 3.4%. Johnson's expression {27} that was modified to account for the constraint of the rubber would fit the data as well. For low cover penetrations the $1+kS^2$ term approaches one. These tests were conducted with a 23 μm polyester web running through the web line at 30 m/min.

Rubber has some strain rate dependency. The same rubber covered rollers used in the tests whose results were shown in Figures 6 and 7 were compressed diametrally in a servohydraulic material testing system. The results are shown in Figure 8. The dynamic data acquired in the moving web line at 30 m/min is compared to the static data. The correlation is not perfect but quite reasonable and it appears at least for the cases studied that strain rate effects must not be significant, at least up to the strain rates associated with web line velocities of 30 m/min for the penetrations tested herein.

In Figure 9 experimental results from Miller [4] are compared to the expressions derived herein. Miller's experimental setup was dynamic in that measurements were taken while the rollers were turning. Three rollers with rubber coverings were tested in contact with a comparatively rigid, metal surfaced roller. The dimensional and cover hardness data for these rollers is given in Table 2. These tests were conducted at nip loads somewhat higher than those tests conducted in Figures 6 and 7. The necessity of modeling the effect of the constraint of the rubber on the modulus of elasticity is now seen. Johnson's expression modified for the constraint of the rubber {27} is now seen to best fit the test data. Both Johnson's unmodified expression {18} and Lindley's expression {8} undershoot the experimental data.

Finally some static tests were run which verify the use of these expressions at the highest nip loads which are commonly seen in the metal strip industry. These diametral compression tests were run in a servo-hydraulic testing machine, the deformations measured were divided by two such that plots of force versus radial penetration could be produced. The rollers were all nominally 102 mm in diameter with 12.7 mm thick rubber covers of varied hardness as shown in the legend (IRHD). The results are shown in Figure 10. Note that Lindley's expression {8} now fits the experimental data the best.

When does one elect to use a particular theory? Based upon the combined results presented herein it appears that Johnson's expression {27} with a modulus term accounting for the confinement of the rubber performs well up to cover strains on the order of 6-7%. At higher strains up to 16% Lindley's expression {8} performs admirably.

ANALYSIS OF WIDE NIP ROLL PAIRS

Many nip rolls are designed using pneumatic or hydraulic actuators which press upon the roll ends in an effort to provide nip pressure continuously across the width of the nip roll pairs. This can be reasonably effective if the bending stiffness of the nip roll pair is adequate. In many cases the bending stiffness is not adequate and the nip loading will be larger at the roll ends near the actuators and minimal at the center of the machine. This can cause a number of problems. If the nip pair is being used to laminate two webs together the lamination pressure variation across the web width can cause quality degradation. The rubber being nearly incompressible attempts to speed up when impinged in the contact zone between the two rolls. If the radial impingement of the covering is increased the surface velocity of the covering must increase. In cases where the nip load is higher at the roll pair edges the surface velocity of the rubber at the edges of the web is greater than the web velocity at the web center. This can cause baggy or slack edges in the web or wrinkles. In some cases these problems can be solved with "gravity nips" in which the dead weight of one of the nip rolls is sufficient to provide the uniform nip loading needed in the application. This may not be a solution if space is a constraint within the web line or if many web products must be processed at various contact pressures. In these cases the bending stiffness of the nip roll pair must be sufficient to provide the uniformity of nip loading required.

Thus it is often necessary to attempt to design a nip roll pair with sufficient bending stiffness to maintain nip load uniformity through some design range of nip loads. This is difficult as the force versus deformation relationships have already been shown to be nonlinear in two dimensions in expressions {8, 18, 26, 27, 28}. This entire problem could be approached using the finite element method to model this contact problem in three dimensions, but again the purpose of this publication is to show that a simplified approach can be used that a web line engineer could apply accurately with a prompt result.

The solution method combines the use of the force/deformation expressions previously developed with a beam finite element model. The stiffness matrix in expression {29} has been developed and used by numerous authors[9,10] for a beam loaded by concentrated forces and moments at nodes i and j as shown in Figure 11.

$$\begin{bmatrix} f_i \\ M_i \\ f_j \\ M_j \end{bmatrix} = \frac{EI}{L_e^3} \begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \begin{bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{bmatrix} \qquad \{29\}$$

This beam element will be used to model the metal core of the rubber covered roller. The rubber covering will be modeled using a Winkler foundation element, expression {30} that is commonly used to model an elastic foundation beneath a beam [10].

$$\begin{bmatrix} f_i \\ M_i \\ f_j \\ M_j \end{bmatrix} = \frac{Z\,L_e}{420}\begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\ 54 & 13L_e & 156 & -22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix}\begin{bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{bmatrix} \qquad \{30\}$$

The stiffness of the elastic foundation, Z, will be determined from the two dimensional force versus deformation relations presented earlier. Z has dimensions of stiffness per unit length. The derivation of the Winkler foundation assumes that Z is a constant that relates the force compressing the foundation to the deformation of the foundation. If Johnson's expression {18} is examined it is seen that the force of compression is related to the 3/2 power of the deformation:

$$F = \frac{2}{3}\frac{(1-\nu)^2}{1-2\nu}\frac{E_o}{1-\nu^2}\frac{\sqrt{D}}{t}\delta^{3/2} \qquad \{18\}$$

Thus Z from Johnson's expression is:

$$Z = \frac{2}{3}\frac{(1-\nu)^2}{1-2\nu}\frac{E_o}{1-\nu^2}\frac{\sqrt{D}}{t}\sqrt{\delta} \text{ or } Z = \frac{2}{3}\frac{(1-\nu)^2}{1-2\nu}\frac{E_o}{1-\nu^2}\frac{\sqrt{D}}{t}\sqrt{v} \qquad \{31\}$$

The Winkler foundation element is assumed to set atop a rigid half-space as shown in Figure 12. The beam stiffness matrix {29} and the Winkler foundation element {30} can be used to model two identical rubber covered rollers in contact. This is realistic because of the contact surface between the two identical rubber covered rollers is a plane, similar to the rigid half space that was defined at the base of the Winkler foundation. After the stiffness of all the beam and elastic foundation elements have been assembled into a model stiffness matrix [K] a set of equations of the form {F}=[K]{v} must be solved. This typically presents no problem but in this case there are Z terms in [K] which depend on the square root of nodal deformations {v} per expression {31} which are unknown prior to the solution of the equations. The solution method thus requires iteration. Trial assumptions are made for the foundation stiffness values, Z_i, for each elastic foundation element used in the model. The set of equations {F}=[K]{v} is now solved for the unknown deformations {v}. These deformations can now be substituted into expression {31} and the stiffness for each elastic foundation is calculated. An error term is now calculated in which the absolute value of the difference between the guessed and calculated values of Z_i are summed into a total. The calculated Z_i values now become the guessed values in the next iteration. The set of equations {F}=[K]{v} is now solved again for the unknown deformations {v} and the values of Z_i are recalculated from expression {31}. The error term is recalculated and this iterative procedure continues until the error term vanishes to a level acceptable to the user. A flow chart of this code is shown in Figure 12.

When modeling a rubber covered roller in contact with a metal surfaced roller the beam element {29} is used to model the metal core within the rubber covered roll and metal surfaced roller. A new Winkler foundation element was derived using the potential energy method to represent the rubber in compression. This was necessary to couple the deformations between the two rollers that were no longer identical in stiffness or covering.

$$\begin{bmatrix} f_i \\ M_i \\ f_j \\ M_j \\ f_k \\ M_k \\ f_l \\ M_l \end{bmatrix} = \frac{ZL_e}{420} \begin{bmatrix} 156 & 22L_e & 54 & -13L_e & -156 & -22L_e & -54 & 13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e^2 & -22L_e & -4L_e^2 & -13L_e & 3L_e^2 \\ 54 & 13L_e & 156 & -22L_e & -54 & -13L_e & -156 & 22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 & 13L_e & 3L_e^2 & 22L_e & -4L_e^2 \\ -156 & -22L_e & -54 & 13L_e & 156 & 22L_e & 54 & -13L_e \\ -22L_e & -4L_e^2 & -13L_e & 3L^2 & 22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\ -54 & -13L_e & -156 & 22L_e & 54 & 13L_e & 156 & -22L_e \\ 13L_e & 3L_e^2 & 22L_e & -4L_e^2 & -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix} \begin{bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \\ v_k \\ \theta_k \\ v_l \\ \theta_l \end{bmatrix} \qquad \{31\}$$

This element is meant to couple the deformations and slopes between nodes i and j on the metal core of the rubber covered roll to the deformations and slopes of nodes k and l on the metal surfaced roller as shown in Figure 13.

This solution method was verified on a pair of rubber covered rollers used in a laminating process. The rollers had live shafts and most of the dimensional data is shown in Figure 14. The metal roll bodies were composed from steel and in the central sections were hollow with an outside diameter of 80 mm and an inside diameter of 54 mm. The central sections were covered with a layer of rubber 4 mm thick and a hardness of 70 IRHD. A low and a high loading condition were tested. The nip loading was exerted by pneumatic cylinders at the roll ends as shown in Figure 14. The design of the fixture was such that the nip loading was not exactly the same from left to right. In the low loading condition 1500 N was applied at the left end of the roll pair while at the right 1515 N was applied at the right which resulted in a nominal nip load of 39.7 N/cm. In the high loading condition 2190 N was applied at the left end of the roll pair while at the right 2292 N was applied at the right that resulted in a nominal nip load of 59 N/cm. Once coded the solution method described herein easily accommodates different nip loads from left to right. In the tests local measurements of nip load were made using pull tabs similar to what are used in winding experiments. Shim steel strips 1.27 cm wide and 25.4 μm thick were inserted across the width at 5.08 cm intervals. The strips were sufficiently long that a handheld force gage could be attached to measure the force required to dislodge the strips. This force could then be divided by the friction coefficient to obtain a local estimate of nip load. These tests were repeated three times and the results were averaged. The tests were found to be highly repeatable. A comparison of the finite element analysis and experimental results are shown in Figure 15. The agreement is quite good and thus the solution method of incorporating two dimensional models of force versus deformation for the rubber compressed between the rollers into elastic foundation elements that can then be used in three dimensional studies appears to be valid and efficient.

CONCLUSIONS

Experimental evidence has been provided showing that many if not all natural and synthetic rubbers have Young's moduli that are dependent only upon the hardness level of the material. Also some experimental evidence has been provided that indicates Poisson's ratio for these materials is nominally 0.46. It is not apparent why this is but it might be postulated that there must be some compressible elements in the rubber compound that result in Poisson's ratio being less than 0.5.

The two dimensional force versus deformation relationships developed by Johnson and Evans appear to be applicable up to strains of 6-7% provided the confinement of the rubber is accounted for in the modulus. Lindley's expression under predicts the load associated with a given deformation up to 6-7% strain but accurately predicts the nip loads associated with strains in the range of 7 to 16% strain.

It does appear as well that these force/deformation expressions can be used in conjunction with Winkler foundation finite elements to model roll coverings in contact between rollers accurately to greatly simplify the analysis of wide pairs of nip rollers in contact.

ACKNOWLEDGEMENTS

This publication resulted from research that was funded by the sponsors of the Web Handling Research Center at Oklahoma State University. Also the author is indebted to Mr. Ron Swanson and Mr. James Dobbs of 3M Company who provided the setup and experimental prowess that provided the dynamic force versus penetration data reported in Figures 6 and 7.

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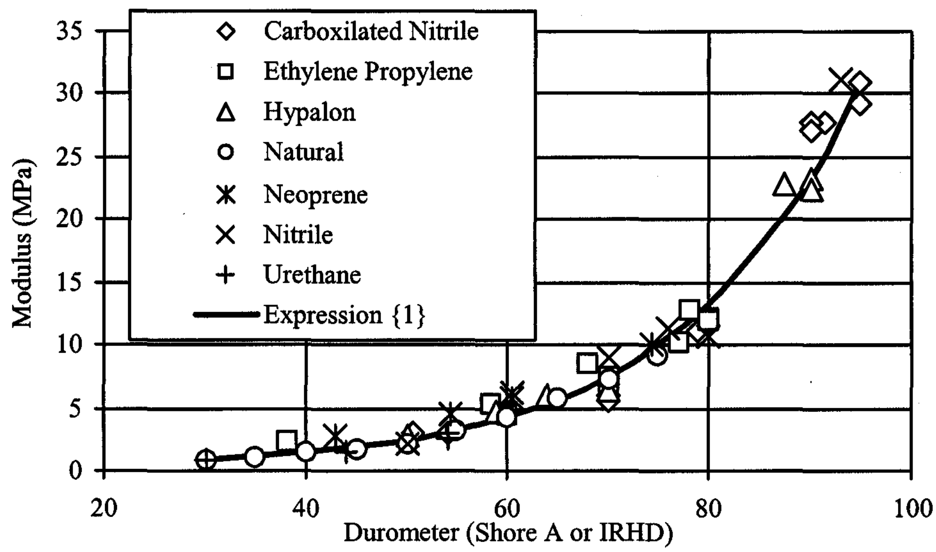

Figure 1 – The Relation between Young's Modulus and Durometer

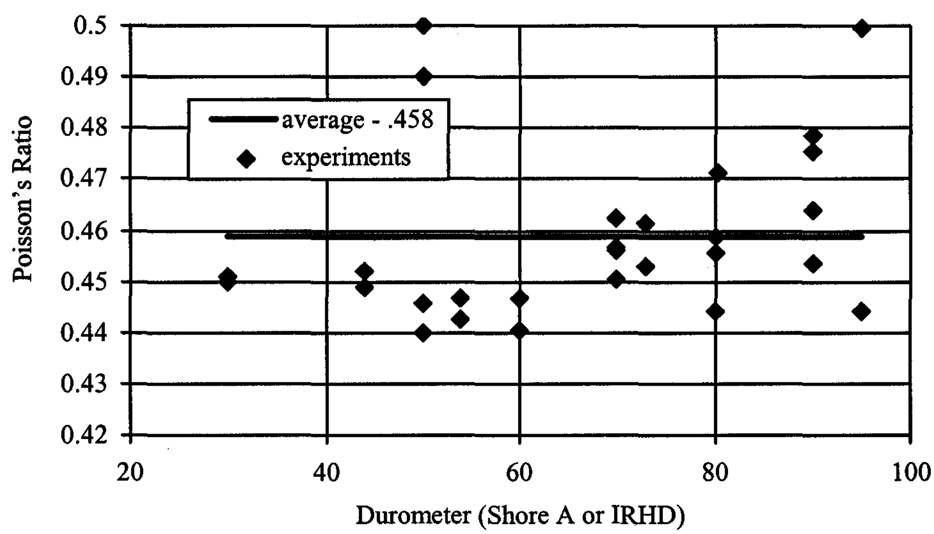

Figure 2 – Poisson's Ratio for Rubber

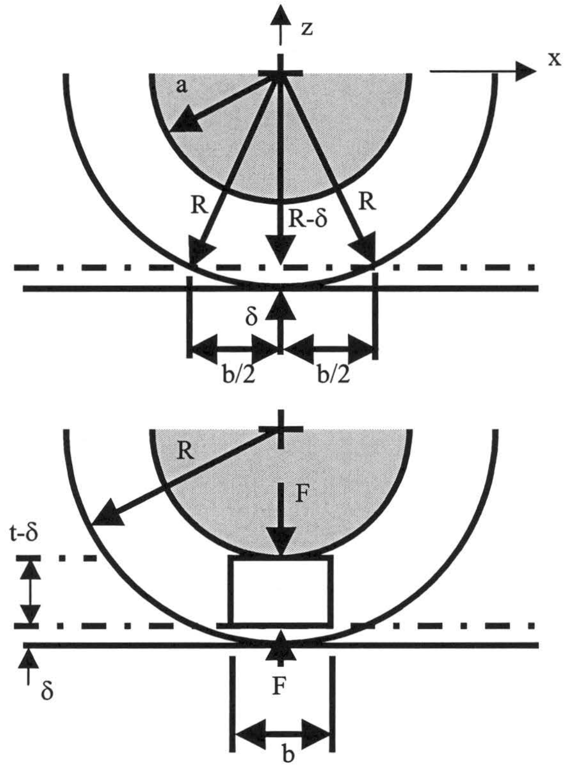

Figure 3 – Definition of Variables for Lindley's Theory

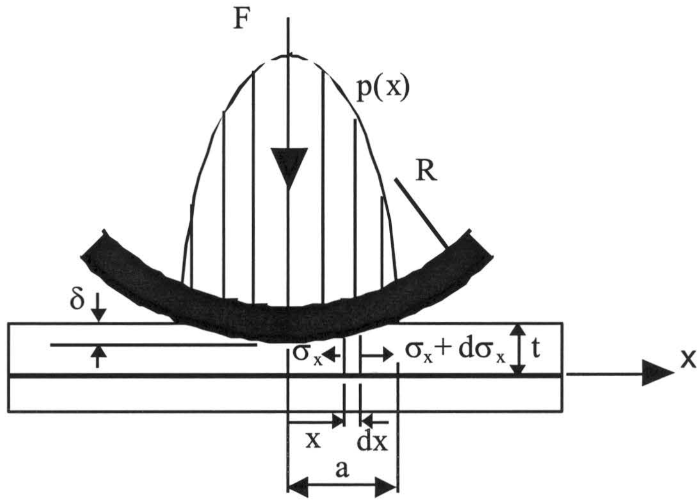

Figure 4 – Definition of Variables for Johnson's Theory

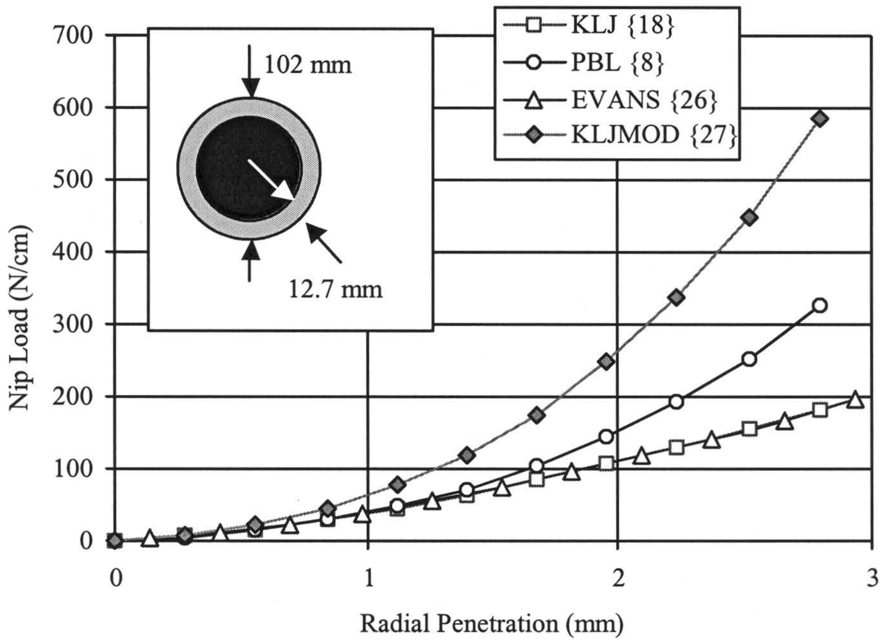

Figure 5 – Comparison of Theories

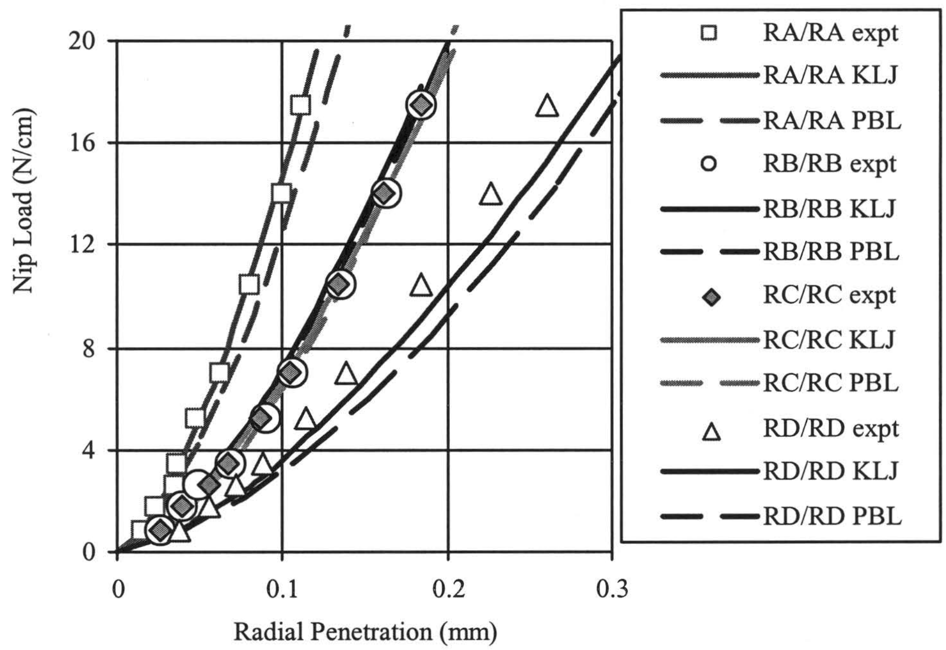

Figure 6 – Application of Johnson's and Lindley's Theories at Low Nip Load Levels for Identical Rubber Covered Rollers in Contact (Legend -Refer to Table 1).

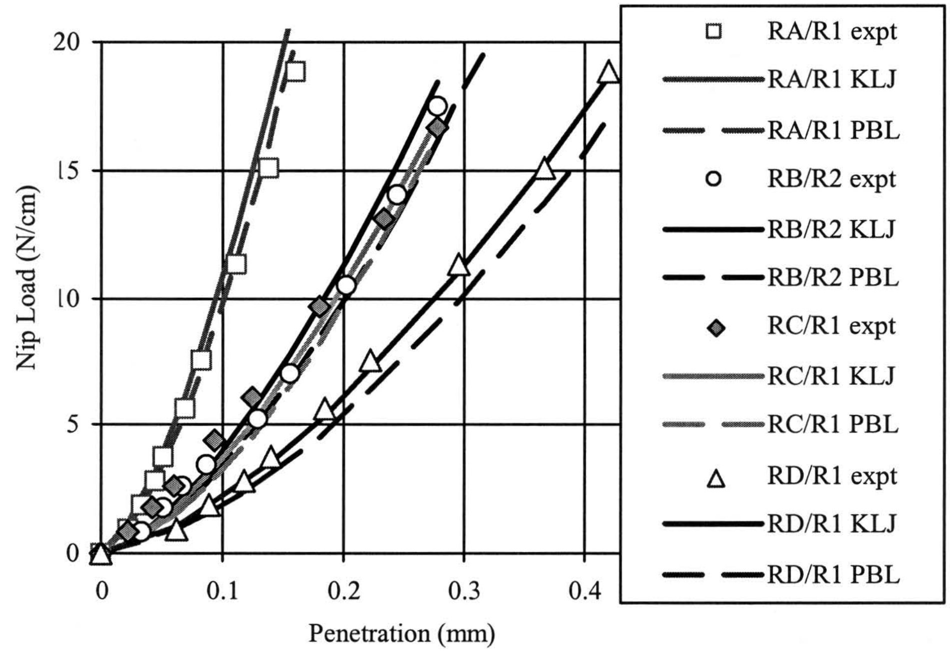

Figure 7 – Application of Johnson's and Lindley's Theories at Low Nip Load Levels for Rubber Covered Rollers in Contact with Rigid Rollers (Legend – Refer to Table 1).

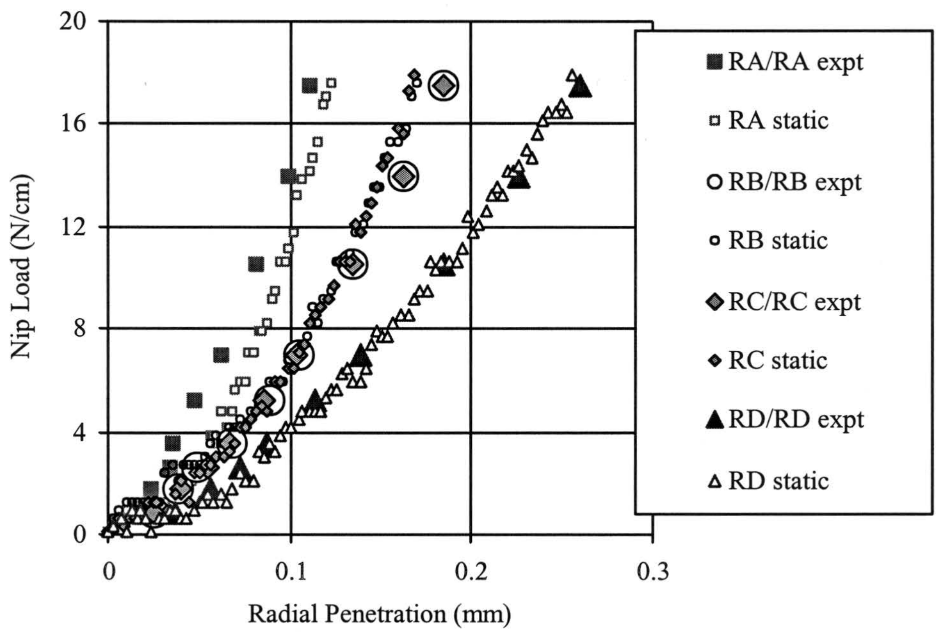

Figure 8 – Comparison of Penetrations Measured Dynamically to those Measured Statically (Legend – Refer to Table 1).

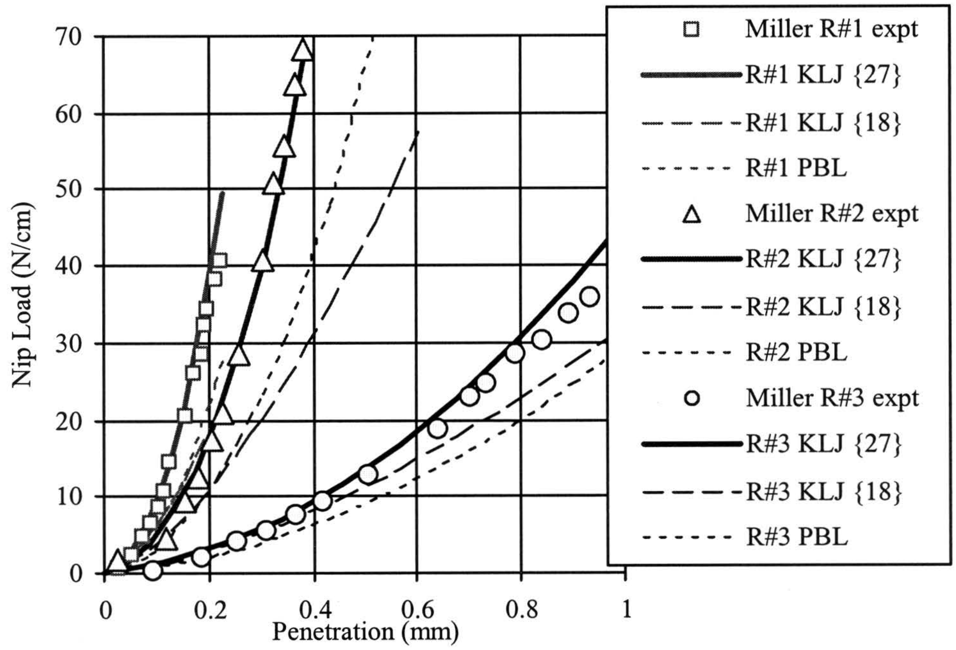

Figure 9 – Verification of Theory at Medium Nip Load Levels against Miller's Data (Legend – Refer to Table 2).

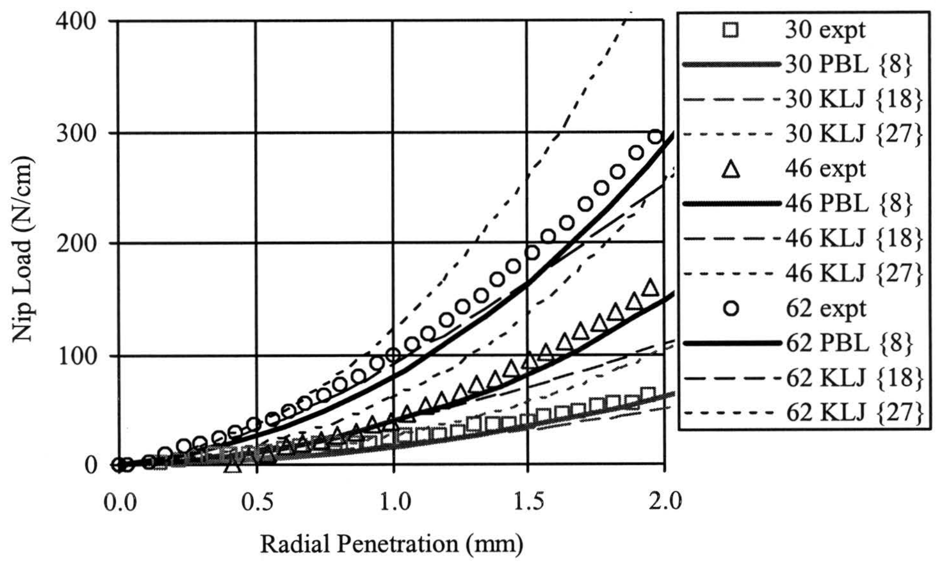

Figure 10 – Verification of Theory at Higher Nip Loads

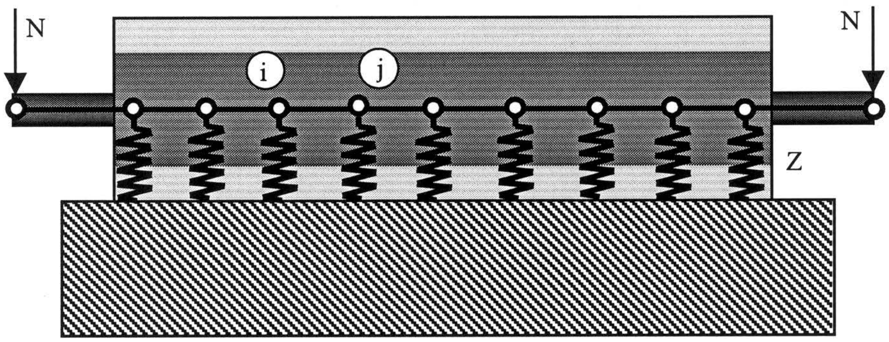

Figure 11 – Finite Element Model of a Rubber Covered Roller in Contact with an Identical Roller.

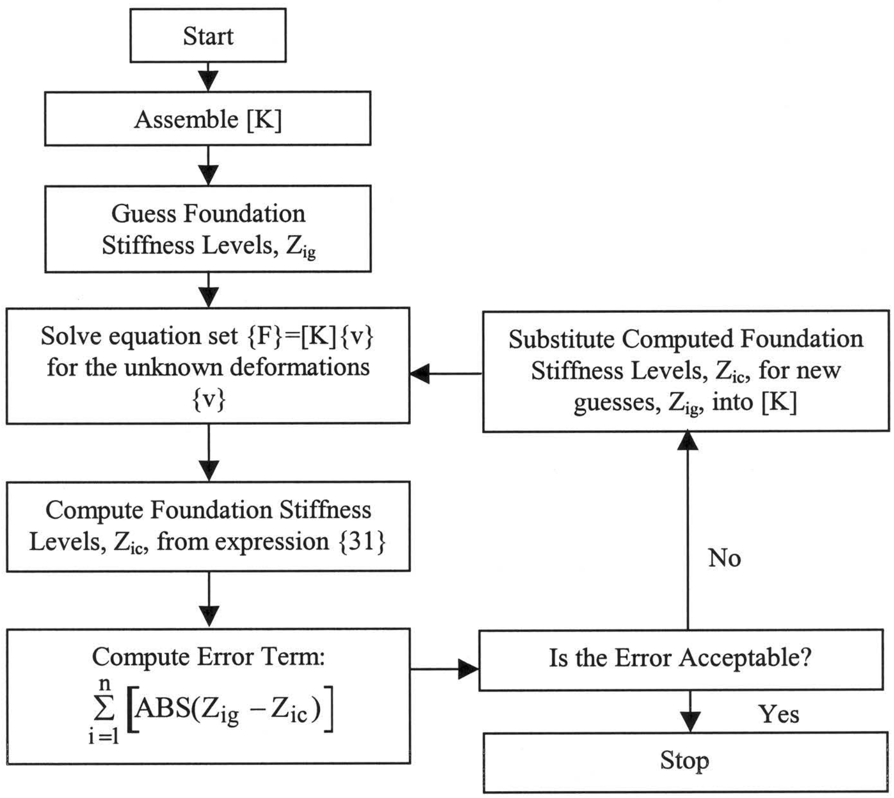

Figure 12 – Flow Chart for FEA code.

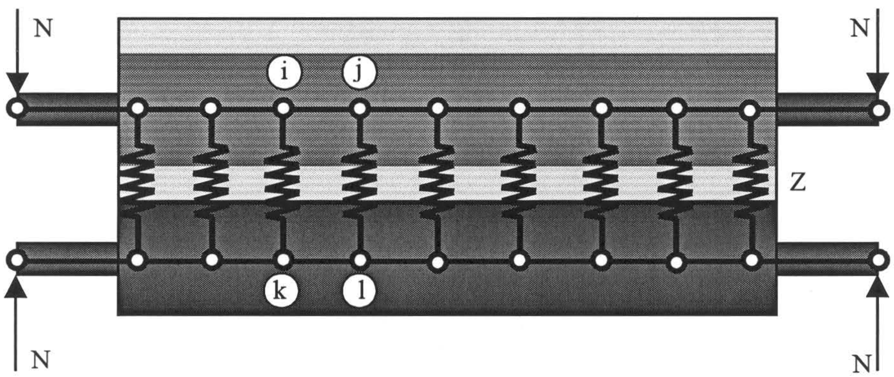

Figure 13 – A Rubber Covered Roller in Contact with a Metal Surfaced Roller

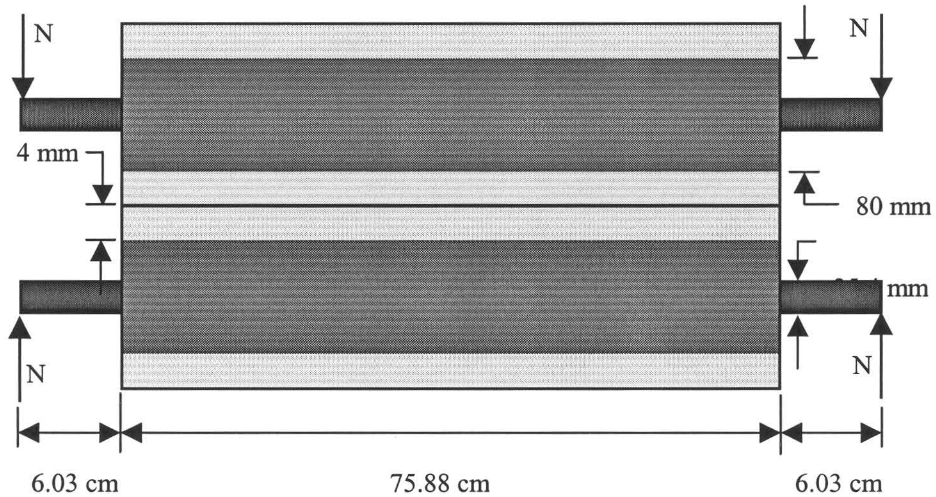

Figure 14 – Dimensions for Rubber Covered Roller Pair Tested

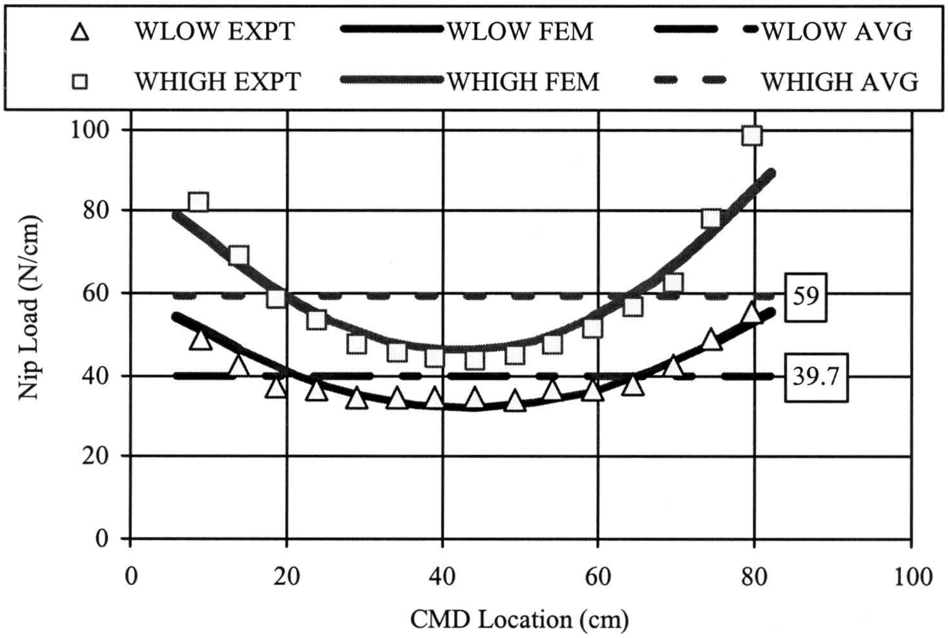

Figure 15 – Comparison of Finite Element and Experimental Results.

Roller	IRHD	Cover Thickness (mm)	Roller Diameter (mm)
A	76	5.1	62.7
B	78	20.2	190.5
C	60	7.5	165.1
D	61	12.7	127
1	Rigid	--	73.7
2	Rigid	--	87.6

Table 1 – Data for Rollers Tested

Roller	IRHD	Cover Thickness (mm)	Roller Diameter (mm)
1	57	3.3	152.7
2	52	4.4	152.1
3	45	12.7	152.2
Rigid	--	--	152.4

Table 2 – Data for Miller's Rollers [4]

Modeling Rubber Covered Nip Rollers in Web Lines **J. K. Good** – Oklahoma State University, USA

Name & Affiliation	Comment
M. Jorkama – Metso Paper	We have used a similar procedure that K. Arola described in his paper for determining the elastic modulus and Poisson's ratio of for rubber covers and this appears to work well. We measured the indentation and the nip width and then performed a fit.
Name & Affiliation	**Reply**
K. Good – OSU	Did you assume 0.5 for Poisson's ratio of rubber? What did you use?
Name & Affiliation	**Answer**
M. Jorkama – Metso Paper	No. The procedure yields both the elastic modulus and the Poisson's ratio. We calculated Poisson's ratios of about 0.6. Poisson's ratio is very important in winding applications. When you order a rubber cover you are allowed to specify the hardness but not the Poisson's ratio.
Name & Affiliation	**Comment**
K. Good – OSU	There is a certain amount of uncertainty in those tests because rubber is not dimensionally stable. Hence compression specimens or roller covers composed of rubber are by no means geometrically perfect.
Name & Affiliation	**Question**
N. Vaidyanathan – Presstek	Now that you've shown us this pressure profile that exists across the width of two rubber rolls that are in contact, which we normally encounter during lamination and similar applications, how do we improve the profile?

Name & Affiliation	Answer
K. Good – OSU	I have shown you a method by which the pressure and nip load profile can be determined across the width of the roll set. The results from the method are affected by the bending stiffness of the rolls and shafting, load level, rubber cover thickness and hardness. When you ask how you can improve your load profile I would answer first by proper selection of those parameters. Variation in nip loads is often the result of the manner in which we apply the load from pneumatic or hydraulic cylinders at the edges of nip rolls. The result is that we inject moments in addition to the loads at the edges of the roll sets. Thus a good design will attempt to minimize these moments by decreasing the length of shafting between the point of load application and the edge of the nip roller. When all of the above fail in yielding the uniformity in nip loading that you desire you may have to employ other technologies. If you are laminating you might consider a gravity nip in which the nip loading is due to the dead weight of a nip roller supported by a second roller. The complication of a gravity nip is that they are typically designed to yield one load level and if you handle multiple products on one web line which require different nip load levels to achieve lamination this will not be a solution. If the nip roll set is being used mainly as a drive roll set nip load variation may vest itself in the form of web wrinkles. The nip load variation results in the rubber speeding up differently across the roll set and thus you are attempting to transport web faster at some CMD locations than others and wrinkles result. In such a case you could consider a vacuum pull roll as an alternative technology.
Name & Affiliation	**Comment**
J. Shelton – OSU	You also could employ a cantilever nip roller where you have a shaft supporting the shell ideally at the center. The two ends then are cantilevered from the center so that the nip roller at least bends the right direction. Through calculations you can predict how to vary the shell thickness from the center to the edge to achieve nip load uniformity.
Name & Affiliation	**Answer**
K. Good – OSU	John, whether we're talking about gravity loaded nips or cantilevered rolls we are introducing nips that are very application specific. If it is required that a number of different laminated products be run through the same web line, we may need different designs.
Name & Affiliation	**Question**
J. Shelton – OSU	With a gravity nip, you are barely going to get enough force for any lamination. Gravity nips are potential solutions for treaters and such things. It will either

	conform or partially conform to the deflecting roller that it is mated to. It will not make it worse, but will make the nip impression more uniform, because it is sagging into the deflection of the mating roller.
Name & Affiliation	**Comment**
K. Good – OSU	Remember Nanda, you don't even know if you've got a problem. Use the model described herein first to determine if the nip load non-uniformity is unacceptable. Then use the model again to see if altering bending stiffness, etc. can yield an acceptable solution. If the non-uniformity is still unacceptable then you should consider an alternate technology.
Name & Affiliation	**Question**
D. Roisum – Finishing Technologies, Inc.	I think we're making this way to complicated. I think all you really need to do is crown your rolls. You could use a little skew to correct for load variations that you need for different grades. That's an easier way to do that.
Name & Affiliation	**Answer**
K. Good – OSU	This is often attempted with mixed results. Why do we want uniformity in nip load? Often it is because the web process demands it but often we also have wrinkling problems in the vicinity of nips, and this is probably one of the worst locations for wrinkling to occur, as it will result in a permanent web defect or a web break when passing through the nip. Crowning any roller in web line should be done with care because crowned rolls generate velocity differences that tend to wrinkle webs. Paper webs in general may not have as much problem with this type of wrinkling because paper webs are often 3 to 4 times thicker in caliper than film webs with nominally the same Young's modulus. Thus the paper web has a larger buckling strength and can absorb larger velocity differences across the width prior to wrinkling. If you skew two rubber covered nip rollers you could be setting the line up for web weave problems. Designing a nip roll set properly to begin with is probably less complicated than operators having to deal with intermittent wrinkling and weave problems through the life of a machine.

Discussion – Session 1

J. D. Pfeiffer – JDP Innovations, Canada

Name & Affiliation	Question
D. Pfeiffer – JDP Innovations	We've had quite a few spectacular papers. Let's recap the day. There was a keynote address by Dr. Howard. Mr. Lei from Eastman Kodak talked about modeling roll structure and thermo and temperature effects. We had questions on that about cinching and how do you test cinching. Wadood Hamed gave a coefficient of restitution paper and some stress strain models that were questionable from my point of view. Koshi Tanimoto gave an interesting paper on winding and permeability and the effect of air entrainment. Patrick Bourgin and Ahmed Boutaous talked to you about online control of tension in winding systems based on programming your winding tension to get the structure based on sampling methods. The impact of mandrel support on core Ec was presented by Mr. Wang from Sonoco and he was talking about the problem of winding aluminum on large diameter fiber tube cores. Keith Good talked about modeling nip-induced tension in wound rolls and some more theories on what happens in the nip. Then development of web tension in a winding nip by Dr. Jorkama. That was another explanation of what goes on in a nip. I think rather than my standing here and reading what you can read in your program for the order of papers, we'll open it up to some questions from the floor on any of these papers presented so far and some of the controversies that were raised over these presentations and whether you want to have further explanations from the authors or suggestions for further work or perhaps what ought to be talked about in IWEB 2003. Now is the time to think about what you didn't get a chance to ask the authors before and draw them out in discussions.
Name & Affiliation	**Question**
A. Thill – Exxon Mobil	There is one thing I am looking forward to see at the next IWEB. It is the effect of air leakage on the final roll for winding wide non-permeable film rolls. With constant nip pressure the middle of the roll winds at a larger diameter than the edges?
Name & Affiliation	**Comment**
D. Pfeiffer – JDP Innovations	The effect you notice is that the roll diameter significantly larger in the center of the roll than on the edges and you attribute this mostly to the fact that the entrained air cannot get out in the center sections of the roll?
Name & Affiliation	**Comment**
A. Thill – Exxon Mobil	It gets out in the underlying layers generating a pressure differential center to edge, which means that the roll is

	bigger in diameter in the center than on the edges.
Name & Affiliation	**Comment**
D. Pfeiffer – JDP Innovations	The edges leak so they wind more tightly. So, is the observation from the rest of you that this is a problem, a noticeable one that doesn't go away with time?
Name & Affiliation	**Comment**
A. Thill – Exxon Mobil	Do you apply the uniform contact pressure with your nip roll to the wound roll? The underlying layers leak to the outside and after a certain winding time you have a roll diameter that is bigger in the center than on the edges. With the wound roll larger at the center it has become crowned. This leads to pulling the web towards the center. This effect makes the roll narrower than what it would like to be and generates after aging the tin canning.
Name & Affiliation	**Answer**
D. Pfeiffer – JDP Innovations	Small waves in the center wouldn't necessarily be due to that. You might have diagonal ridges coming out. Bruce Feiertag has suggested that these are called corrugations in other parts of the industry.
Name & Affiliation	**Question**
A. Thill – Exxon Mobil	If you have close to a uniform thickness they get located exactly in the center. Some people say there must be a defect in the center. That is not true because it also happens on slit rolls. It is a defect that you can see on any type of material whether it's aluminum foil, polyester, or polypropylene. It is visible on any high modulus material.
Name & Affiliation	**Answer**
D. Pfeiffer – JDP Innovations	So we have a high modulus material so that the tensile strains can't relax the problem and that would give rise more to set-in wrinkling and surface wrinkling.
Name & Affiliation	**Answer**
D. Roisum – Finishing Technologies	Yes, this is actually quite a common problem in the film industry called buckles, but other people have it as well. The challenge is, you make a perfectly uniform web and it winds up reasonably uniform but after time air leaks out the edges. Then as it leaks out the edges, the edges seal themselves so that the air becomes trapped in the middle. Also the air entrapped in the middle has so much farther to go so you end up with buckles on the edges as this roll collapses. There is nothing you can do after you have wound the roll. You have to manage the air as you wind it. That's one way to do that. Some people make football shaped gauge profiles to squeeze the air out and give it a path to go. There are many, many tricks. You can also slow the line down by half. It's a tough cross to carry.

Name & Affiliation	Question
W. Qualls – Imation	First a comment. I am very familiar with this problem. I see it in high modulus thin films. Not only do we see it in the center we see it take on 80% of the roll width. But my question is do we believe air entrainment is the forcing function here? Without air entrainment we would not have this problem?
Name & Affiliation	**Comment**
A. Thill – Exxon Mobil	The air entrainment gets more and more complicated while the speed is increasing. That's why the solution is to drop the speed by half and the problem is gone.
Name & Affiliation	**Answer**
D. Pfeiffer – JDP Innovations	I think this is one of those problems where the problem increases with a cube of the speed, not just linearly with speed. I have a solution for it though – wind your rolls in a vacuum.
Name & Affiliation	**Comment**
Andre Thill – Exxon Mobil	Some do that, metalizers for instance. But look what happens when you take the roll back into the atmosphere.
Name & Affiliation	**Comment**
R. Swanson – 3M	I don't remember what IWEB it was, but Al Forrest had a model on how to predict it, buckling number.
Name & Affiliation	**Question**
L. Kindel – Eastman Chemical Co.	Question for the gentleman with the paper from Metso. It's a nomenclature question. Soft roll bottom, hard roll bottom? Other people probably know that answer to that.
Name & Affiliation	**Comment**
D. Pfeiffer – JDP Innovations	Roll bottom would be the onset of winding right after the transfer onto the new reel spool and in his paper he has a program for making a hard roll bottom. You start with a very high nip load and perhaps increase tension, but I think the graph is for tension versus diameter and the tension goes down – I'm sorry, I mean nip load versus diameter, it goes down quickly and becomes a constant nip load. For the soft roll bottom, the nip load starts low and comes up to a constant value and stays constant.
Name & Affiliation	**Question**
P. Bourgin – Ecole Supérieure de Plasturgie	I have questions dealing with the air entrainment. The first one is concerned with the paper by Lei and Cole from Eastman Kodak, who proposed a thermoelastic winding model with air entrainment. I am a little puzzled that there are no thermal expansion coefficients involved in the model. The second question would be what is the effect of humidity change and would the model be able to take into account the effect of humidity change in the roll?

Name & Affiliation	Answer
H. Lei – Kodak	The model consists of two parts. In the first part we predict the internal pressure during the winding, and in that part we assume the temperature is constant, so there's no temperature variation there. But in the second part of the model we predict the internal pressure change after the winding so in that part there no winding involved. That's why we do not have the coefficient of thermal expansion during the first part of the model. And for the second question – the humidity effect, well, we know that roll quality could change significantly due to the humidity of the paper rolls when you dry those rolls they get very soft and we did not include that into the model yet.
Name & Affiliation	**Question**
P. Bourgin – Ecole Supérieure de Plasturgie	Thank you. My next question is from Tanimoto's paper, which is concerned with air entrainment and permeation. Did you investigate the relation to relative importance of radial air migration due to permeation and lateral air migration due to the compressive force of the oncoming layers? Apparently you only considered the radial air migration only due to permeation. Would you comment on this?
Name & Affiliation	**Answer**
K. Tanimoto – Mitsubishi Heavy Industries	The paper is several meters wide and the air film thickness is less than several micrometers. At both edges there must exist some lateral movement of air but it is small
Name & Affiliation	**Comment**
P. Bourgin – Ecole Supérieure de Plasturgie	Yes, but if you have plastic film instead of paper then you don't have any permeation and still you have air migration and that must be all lateral migration. So you assume that there is no lateral recreation as compared to the effect of permeation. That's the assumption?
Name & Affiliation	**Answer**
K. Tanimoto – Mitsubishi Heavy Industries	Yes.
Name & Affiliation	**Question**
P. Bourgin – Ecole Supérieure de Plasturgie	I have the last question for Dr. Jorkama. Do you propose to take into account the air entrainment in the stick and slip model? Don't you think the air entrainment effect would affect the stick and slip regions?
Name & Affiliation	**Answer**
M. Jorkama – Metso	To be honest, I haven't thought that much what will happen when air comes in the picture. I will not be surprised if the stick and slip pattern would remain basically the same.
Name & Affiliation	**Comment**
K. Good – OSU	My remark to is to Patrick's last question. I think it comes in stages. As you begin to become partially airborne, but you still have asperity contact, I think the coefficient of

	friction that Marco and I and others use in slip models have to change. But at the point to where the nip roll becomes completely airborne, now we're talking about traction terms that would be related to viscosity of air. And so they wouldn't be very large and it is difficult for a nip to induce any additional tension with no available traction. Certainly there is a point at which air entrainment would affect our models. The second thing that I really wanted to say to begin with was look at how much time we spent today on the calculation of stresses in wound rolls compared to how little time we spent on calculating or trying to predict models of roll defects. I mean why are we in this business of calculating stresses and rolls if we aren't also in defect models? I think there needs to be a lot more work there. That's my comment. Thank you.
Name & Affiliation	**Question**
G. Homan – Westvaco	I have a question to the people from Sunoco. In the process of manufacturing spiral wound cores, moisture is introduced. At what moisture were the moduli calculated and what moisture were the cores tested? Have they looked at the effects of moisture? This has and continues to be a problem within our organization - moisture variation with cores.
Name & Affiliation	**Answer**
Y. Wang – Sonoco	When we measure the properties, it is around 80% of the relative moisture content. When we make a tube we try to avoid the moisture as much as possible. After we make a tube, we use some plastic bag or some other material to wrap it up to ship to our customers. What's the second part of the questions?
Name & Affiliation	**Question**
G. Holman – Westvaco	What moisture content were the properties measured at, what moisture content was the testing conducted, and whether you have looked at various moistures in this particular evaluation?
Name & Affiliation	**Answer**
Y. Wang – Sonoco	I think I answered the first question that we measured usually around 80%. It's in the condition of the room, the lab. We did some study on the moisture content influence on the material property and we do have the adjustment effect for the material properties with the changing of moisture content. If the moisture content is higher then the material properties will be lower. But we do have a quantitative measurement on that.
Name & Affiliation	**Comment**
D. Pfeiffer – JDP Innovations	I'd like to make one more comment to people who work on models of nip-induced tension. I think there are two factors that you have to consider. One of those is the rejected paper moving backward against the oncoming

	feed of the drum that's feeding it in and the other factor is the transfer of torque across the winding nip. So if you haven't considered those two, you haven't got a complete model. Thanks to all the authors for a very challenging first day of sessions.