IMPACT OF MANDREL SUPPORT ON CORE E_C

by

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ABSTRACT

Over the past 15 years, Sonoco has conducted solid mechanics research focused on structural behavior of spirally wound, paper tubes. The scope of this program has included experimental, numerical, and analytical mechanics approaches as documented in references (1-9). As is well known from published winding models, core outside diameter stiffness (E_c) is incorporated into the analysis through a boundary condition. We have previously published proper E_c values for paper tubes (4) and, at the last OSU International Web Handling Conference, described a method to experimentally measure E_c (9). However, all published E_c data was collected on cores that were supported on the ends, but had minimal radial support in the test zone. In the field, many winding processes utilize an expandable mandrel that supports the core along its entire length. Our recent research suggests that these support conditions can have a significant impact stiffening the core wall and increasing E_c. As E_c is changed, expected stresses in the wound roll are altered. In this paper, we describe a new experimental method capable of measuring E_c data for cores supported by mandrels found in some field applications. To collect this data, we modified the test device described at the last conference. We also present a Finite Element model that quantifies core stiffening from mandrel support.

INTRODUCTION

Paper tubes for industrial packaging are fabricated using a spiral winding process. Because paper is an orthotropic material, spiral winding yields a generally anisotropic structure. To complicate matters, paper tubes are frequently loaded into the non-linear stress-strain region during use. Over the past 15 years, we have developed experimental, numerical, and analytical tools to aid in design of tubes to meet a wide variety of customer requirements (1-9).

Many researchers have computed internal stresses in web materials wound around paper or metal cores. For example, Pfeiffer (10), Yagoda (11), Hakiel (12), and Willett and Poesch (13) have considered rolls constructed by center winding. These are typically nonlinear, one-dimensional formulations in the radial direction of the roll. Roisum (14) presents an excellent review.

In these published winding models, core outside diameter stiffness (E_c) is introduced through a boundary condition. Gerhardt and Qiu (4) published E_c data for paper tubes using data collected with strain gauges on 76 mm (3 inch) diameter tubes. Analytical, numerical, and experimental results showed that core stiffness values previously published in the literature were incorrect.

As reported in previous International Web Handling Conferences, Sonoco designed and commercialized an extremely stiff paper tube to an E_c level specified by Alcoa for winding coated aluminum. Large diameter tubes, 406.4 mm (16 inch), are used in this application. At the last conference, we described a test device capable of measuring E_c for such large diameter cores (9). Becker (15) discussed the impact of increased E_c on stresses in wound, coated aluminum and its impact on roll structure. *It is important to note that the cores tested in these studies were supported on the ends, but without interior radial support.*

Many winding processes utilize an expandable mandrel that supports the core along its entire length. Such support is typical for winding metal coils. We have discovered that this mandrel support greatly stiffens the core wall and increases E_c. As E_c is changed, expected stresses in the wound roll are altered. In this paper, we describe modifications made to the test device described at the last conference to simulate field support conditions. We also present a Finite Element model to quantify the stiffening impact. Finally, experimental data is compared to finite element results.

E_c TEST DEVICE WITH MANDREL SUPPORT

The Sonoco Outside Diameter (OD) Stiffness Tester (Figure 1) consists of a pressure vessel and fixtures to hold a sample core in place inside the vessel. Inflatable seals are used to seal the ends without applying any large axial stresses to the core. The inside of the core is vented to atmosphere.

Inside the pressure vessel, laser sensors are mounted to measure deformation of the outer diameter of the core at mid-span. Two sensors are used, 180° apart. At a prescribed pressure, sensors are rotated around the sample and numerous measurements are collected. The readings are averaged to eliminate the effect of the tube position being non-concentric with the center of rotation of the sensors.

A set of sensor readings is made with no pressure applied to the outside of the sample. Then, compressed air is applied in 68.9 kPa (10 psi) increments. At each increment of pressure, another set of readings is taken from the laser sensors. When a test is complete, a plot is made of "applied pressure" versus "OD change". The slope of this line is the "OD stiffness" of the core (see Figure 6 in reference 9). By multiplying this slope by tube OD, the ratio becomes "pressure change" over "percent OD change." This value, of course, is called core stiffness, E_c, with units of kPa (psi).

Mandrel Support Mechanism

As illustrated in Figure 2, we developed an expanding fixture to simulate field support conditions. Tubes can be tested with or without this internal support. The mandrel consists of four segments fabricated from steel tubing with a 406.4 mm (16 inch) OD and a 12.7 mm (0.5 inch) wall thickness. Each of the four curved segments is bolted to a wedge at the midpoint of the arc. The bolted configuration is illustrated by the left schematic of Figure 2.

Pressure from hydraulic cylinders (right schematic of Figure 2) cause mandrel expansion. The four wedges connected to hydraulic cylinders are supported by an interior steel tube (38.1 mm thick) that is not shown in Figure 2. The force applied by the hydraulic cylinders is known, but due to friction losses, the actual force transmitted to the core is not known. We will use measurements of tube circumference change and a finite element simulation to estimate this force.

The nominal diameter of the support is 406.4 mm (16 inch) at the center of its travel. However, above and below this center point, the outside of the support is not circular. *Thus, contact with the inside diameter of the core is complex. This does duplicate field conditions.*

As test data has been compared to predictions from the finite element model, some concern has arisen over whether the existing support adequately represents actual winding mandrels. The test data seem to imply that excessive flexing of the segments may be occurring. This design will be subjected to further testing and refinement.

FINITE ELEMENT ANALYSIS

Closed-form Mechanics solutions are ideal for verifying experimental results. For the problem being considered, such a solution would be extremely difficult, if not impossible, to obtain. As described above, the contact interface between core and metal support is complex. With advancements in computing technology and commercial FEA (finite element analysis) software, complex contact problems can be addressed. We utilized FEA to better understand the influence of mandrel support on outside diameter (OD) stiffness of paper cores. We used ABAQUS software and a Sun Ultra-30 Workstation for our analysis. A typical run of a finite element model, which is described below, takes about ten minutes.

Finite Element (FE) Model

The problem can be approximated using a two-dimensional (2D) plane stress FE model because both core ends are free of loading along the axial direction. This approximation is especially true for the middle part of the core where OD deformation is measured.

A typical mesh is shown in Figure 3, where a paper core is modeled with 1440 4-noded elements and four metal leaves are modeled with 720 4-noded elements. The previously described mismatch of core inside diameter (ID) and OD of the leaf segments is depicted fully in the model. Because leaf OD is larger than core ID, the center point of a leaf segment does not coincide with the core ID center point. Each leaf will contact with the core at both edges (near 0° and 90° for the leaf in the first quadrant) initially.

Interfaces between core and leaves are simulated with contact surfaces in ABAQUS. The interface between wedges and leaves are approximated in the model. Here we avoid modeling the wedge itself by providing the correct displacements to leaf nodes that would contact the wedge surfaces.

The FE model contains four distinct steps as follows:

(1) An initial pre-load establishes contact. In order to avoid divergence, a dummy step is applied to make initial contact between leaves and core. A radial displacement boundary condition is applied to nodes on a metal leaf that is in contact with the wedge. These are nodes on ID of the leaves in the range of 30° to 60°, 120° to 150°, 210° to 240°, and 300° to 330°, respectively. A small displacement is applied to just make initial contact between the leaves and the core.

(2) Uniform radial forces are then applied on nodes described in step (1) to simulate leaf expansion. The method used to determine the magnitude of these forces is discussed later.

(3) Nodes at the wedge/leaf interfaces are fixed in the radial direction with displacement values obtained at the end of step 2. This fixes leaf position after expansion. To compute E_c, a 413.7 kPa (60 psi) pressure is applied on the core OD simultaneously.

(4) The final step is to maintain the 413.7 kPa pressure and let all leaves return to their original positions.

FE Results

Because the model is symmetric for every 45°, we need only to examine results for a 45° section of the model. Radial stress contour plots at the end of steps 2 and 3 are illustrated in Figure 4, 5, and 6, respectively.

In Figure 4, radial stresses are shown after leaf expansion (step 2). Note in the range of about 30° - 60°, higher radial stresses propagate deeper into the tube wall. Since leaf OD is greater than core ID, initial contact is near 0° and 90° in this quadrant. The core stress pattern in Figure 4 is caused by stresses resulting from leaf expansion overcoming initial contact stresses. After step 2, maximum radial deformation in the leaf OD is at 45°.

Radial stress patterns are shown in Figure 5 after application of 413.7 kPa (60 psi) external pressure (step 3). Note in the ranges of 0° – 15° and 75° – 90° core radial stress is not uniform along the hoop direction. This might seem surprising given the uniform external pressure loading. However, this non-uniform pattern results from a slight bending of leaves and gaps between leaves.

Bending in the metal leaf is shown in Figure 6, a plot of hoop stress after step 3. Bending is evident from the OD tension and ID compression pattern. Hoop stress patterns vary in the tube as well, but these are not exposed with the scale used in the contour plot.

It is interesting to note that with applied external pressure loading and mandrel support, radial compression of the core OD is greater at 0° than at 45°. This results from leaf bending as discussed above.

EXPERIMENT VS. THEORY

We expected that stiffening caused by a full length, steel mandrel would depend strongly on core thickness. For that reason, experimental vs. theoretical comparisons were conducted on cores with two different thicknesses. Both cores had inside diameters of 405.3mm (15.957 inch) and core t_1 had a 18.29 mm (0.720 inch) wall thickness while core t_2 had a 12.37 mm (0.487 inch) wall thickness. These are typical core constructions for winding metal webs. Note that leaf OD exceeds core ID.

We used material properties appropriate for the core we tested in the FE model. Qiu, et. al (7) published representative elastic properties for tube board in all directions. For these tubes, the spiral angle is 81.7° (angle between the paper machine direction and the axial direction of the core). The FE model transformed a moduli tensor of paper orthotropic material properties into an anisotropic moduli tensor in the cylindrical tube coordinate system. For the leaves, we used typical steel properties, i.e. Young's modulus of 206.8 million kPa (30 million psi) and Poisson's ratio of 0.33.

Verification of FE Model: Without Support

Gerhardt (2) and Gerhardt and Qiu (4) published closed-form elasticity solutions for paper tubes loaded with radial pressure on the core OD. Experimental data from tests on 76.2 mm (3.0 inch) ID cores (2) verified the theory within the elastic range of paper. We compared this experimentally verified elasticity solution to results from the FE model described in this paper.

In the models, both cores were loaded to a pressure of 413.7 kPa (60 psi). The FE model was used without mandrel support. For core t_1, estimated OD change was 0.401 mm (0.0158 inch) from the FE model and 0.400 mm (0.01573 inch) from the elasticity solution. For core t_2, estimated OD change was 0.498 mm (0.0196 inch) from the FE model and 0.497 mm (0.01957 inch) from the elasticity solution. *These near identical results suggest that the new FE model provides an accurate representation of the core.* As expected, computed OD deformation is smaller for the thicker core, t_1.

Experimental vs. FE Comparison of Expansion Forces

Hydraulic pressure supplied to the wedges in the expanding mandrel support (Figure 2) is known. However, due to friction loss, the actual pressure exerted on the leaves (from the wedges) is unknown. We decided to measure core OD increase with π tape before and after mandrel expansion to gain insight into the magnitude of expansion forces.

After mandrel expansion, measured circumference increase was 0.102mm (0.004 inch) for core t_1 and 0.178mm (0.007 inch) for core t_2. Thus, measured increase is roughly 75% greater for the thinner core. This large difference can be understood by the significant differences in modulus values in the core. As is well known, z-direction modulus in paperboard can be as much as 100 times less than modulus in the machine- or cross- machine directions. In a tube, paper z-direction is aligned with the radial direction. For paper tubes loaded by pressure on the ID or OD pressure, Gerhardt (2) proved the unusually small ratio of radial to circumferential modulus had a profound impact on stress distributions.

The magnitude of force exerted by a wedge on a leaf should be independent of the core being tested. From the FE model, we estimated the force needed to cause the measured tube circumference change. This required numerical integration of FE displacements along the tube circumference. To concur exactly with π tape results, the FE model required wedge forces of 23.5 kN (5274 lb) for core t_1 and 25.5 kN (5724 lb) for core t_2. *Significantly, similar wedge forces reproduced quite different (on a percentage basis) circumference changes.* Given the precision of the π tape measurements, we interpreted this finding as another verification of the model.

Impact of Mandrel Support on Core E_c

Using the described experimental procedure, we measured E_c for both core types. Compared to the unsupported case, we found that mandrel support increased E_c by 26% for core t_1 and 72% for core t_2. As suspected, the impact on Ec is highly dependent on core wall thickness.

We also used the FE model to estimate impact on computed E_c for both core types. Compared to the unsupported case, FE results suggest that mandrel support increased E_c by 49% for core t_1 and 109% for core t_2. The FE results also substantiate that mandrel impact on Ec is highly dependent on core wall thickness. However, the FE model is overestimating experimentally measured stiffness change.

SUMMARY

Over the past 15 years, Sonoco has conducted solid mechanics research focused on structural behavior of spirally wound, paper tubes. We have published several papers on core outside diameter stiffness (E_c) of paper tubes. The cores tested in these studies had no interior radial support. Many winding processes, including metal webs, use an expandable mandrel that supports the core along its entire length. This paper presents recent research on the significant impact these support conditions have on core E_c. A new experimental method to measure E_c data for cores supported by mandrels is described. These measurements are also supplemented by results from a complex Finite Element model that quantifies core stiffening.

A significant finding is that paper core OD stiffness is significantly increased by full-length mandrel support. Both experimental data and FE calculations support this conclusion. For a given core inside diameter, degree of stiffening is a strong function of wall thickness. This is a direct consequence of the core radial modulus being nearly 100 times less than its circumferential modulus.

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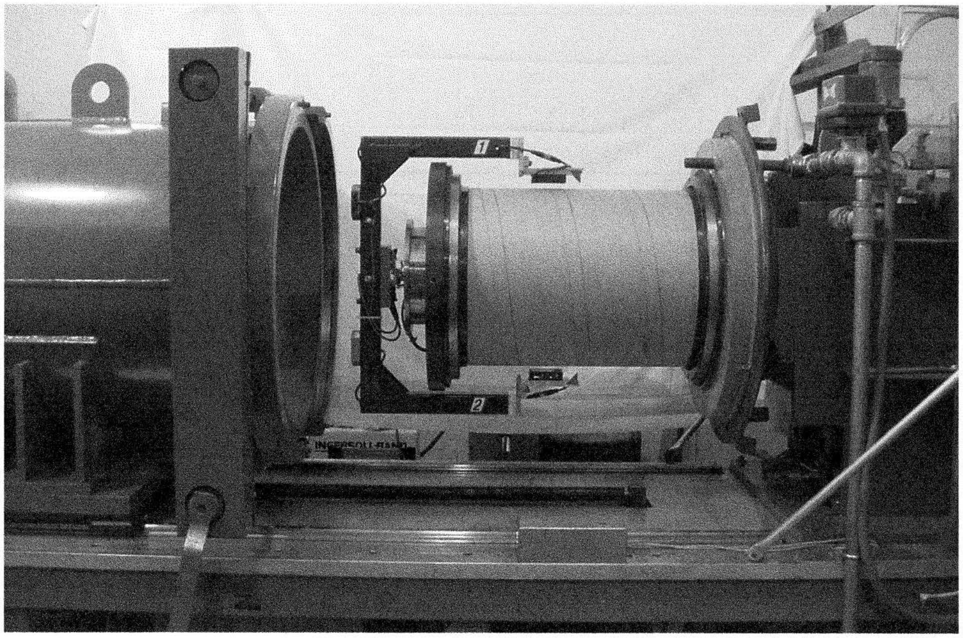

Figure 1. Sonoco OD Stiffness Tester

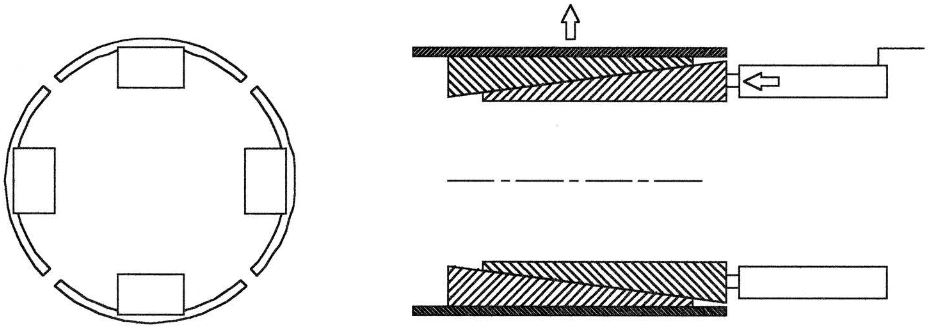

Figure 2. Expanding Support Mandrel

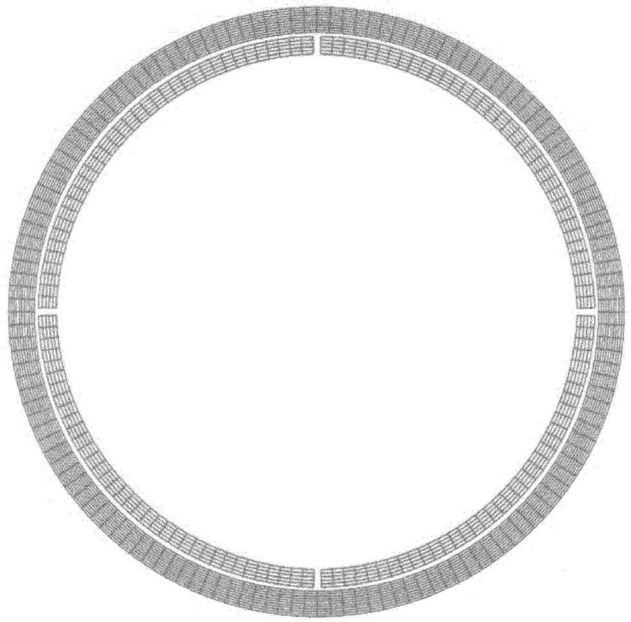

Figure 3. Finite Element Mesh.

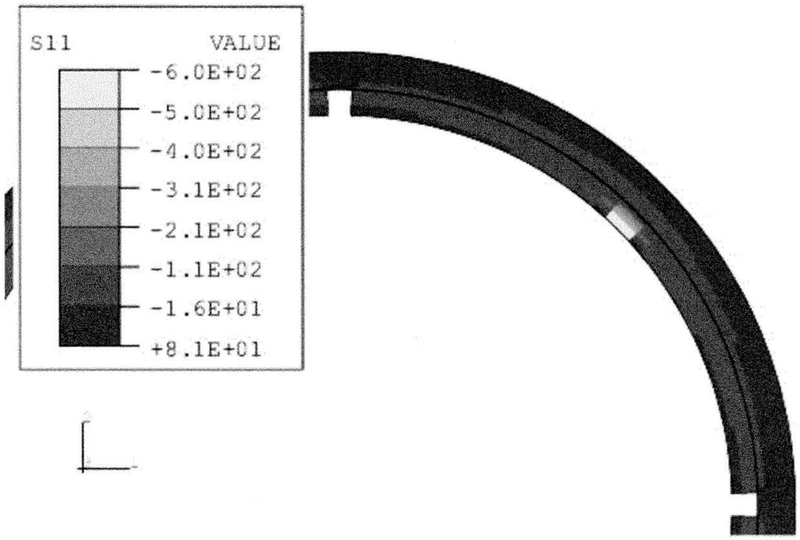

Figure 4. Radial stress after leaf expansion (at the end of step 2).

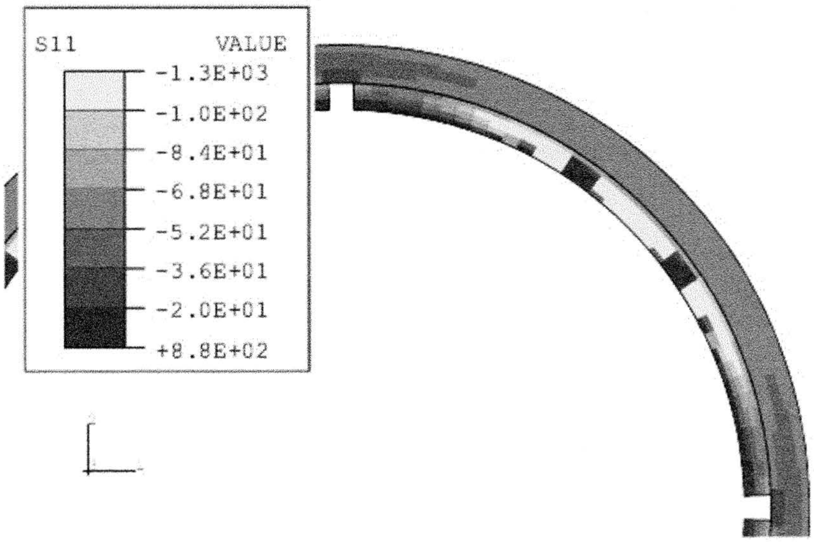

Figure 5. Radial stress after applying 413.7 kPa (60 psi) external pressure when leaf positions are maintained (at the end of step 3).

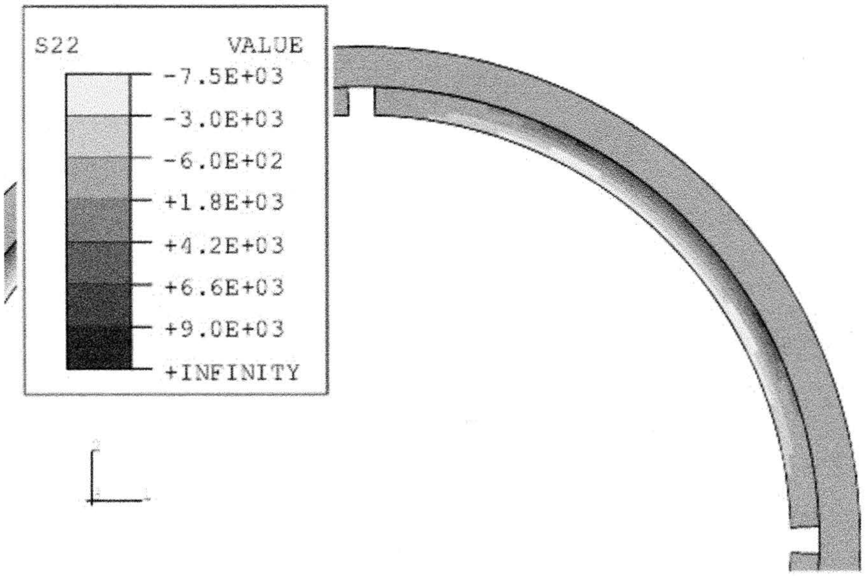

Figure 6. Hoop stress after applying 413.7 kPa (60 psi) external pressure when leaf positions are maintained (at the end of step 3).

Impact of Mandrel Support on Core Ec

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Name & Affiliation	**Question**
W. Y. Hamad – International Paper	What assumptions have you made for your finite elements for material properties and for the boundary conditions?
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	We measured the material properties for our paper board. It is an orthotropic material and we have 9 independent parameters. We did not model the wedge in the finite elements model. We could model the wedge, including the contact between the wedge and the steel leaf, but we believe that the wedge itself is stiff enough to make a good assumption of a boundary at the contact surface between the wedge and the metal surface.
Name & Affiliation	**Question**
W. Y. Hamad – International Paper	You may want to consider plain strain analysis. You can get plain strain conditions on both the core and mandrel and make the assumptions about your boundary conditions and use cylindrical isotropy for your material properties. It might prove to be closer to the experimental results.
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	We used plane stress elements in this particular study. We tried the plane strain condition but it did not yield results significantly different than plane stress in this case.
Name & Affiliation	**Question**
D. Pfeiffer – JDP Innovations	I wondered if you had measured without the mandrel support the change of the ID, as you exerted the 60 psi pressure on the core? Did you happen to measure that at the same time?
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	No.
Name & Affiliation	**Question**
D. Pfeiffer – JDP Innovations	You mentioned getting more support (more increase of E_C) with a thin wall tube compared to a thick wall tube. This is less obvious for us who are thinking about 3 inch ID cores and 4 inch OD cores, because in your case the radius, the tube wall is only 9% of you radius and in the paper winding industry where the core diameter is smaller, the tube wall is 25-30% .
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	Yes, I agree. This depends on the wall thickness as a percentage of the diameter.
Name & Affiliation	**Question**
D. Pfeiffer – JDP	Should you normalize the core radius when you do the

Innovations	comparisons?
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	Yes
Name & Affiliation	**Question**
Curt Bronkhorst – Weyerhaeuser	It was not clear to me what constitutive model you used. Did you use a linear orthotropic theory or a non-linear orthotropic theory?
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	We used a linear orthotropic constitutive equation with 9 different parameters.
Name & Affiliation	**Question**
Curt Bronkhorst – Weyerhaeuser	Do you think if you used a non-linear orthotropic theory, since paper is a cellular material, that it would improve your finite element prediction?
Name & Affiliation	**Answer**
Y. Wang – Sonoco Products Co.	Yes. This is an approach we might consider in the future. We have used a non-linear constitutive model for some other applications. By combining a nonlinear material model with contact modeling, it is very difficult to obtain convergence.