THE LATERAL RESPONSE AND CONTROL OF A MULTI-SPAN WEB SYSTEM TO DYNAMIC CHANGES TO THE WEB AND CONVEYANCE HARDWARE

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ABSTRACT

A multi-span model that predicts the lateral response of webs to changes in the upstream position of the web and any type of motion of the included rolls is presented. The model uses a beam approximation for the deflection in the web spans. The actual, rather than idealized, web spans, roll diameters, wrap angles and sensor locations are used. Equations are derived including web span interactions, roll motion such as occur in steering devices and feedback control systems. Example cases are included showing the value of this type of analysis and accuracy through comparisons with experimental results. Web steering devices of different types are used widely both to control the web in less than ideal circumstances and to accurately position the web for key operations in processes. The design of these devices is often based on idealized configurations that don't fully match the geometry of the equipment in question. As a result problems arise such as non-stable operation, web oscillation downstream of the guide system and the formation of shear wrinkles. The work presented here describes a method of analyzing an entire web path including steering devices and evaluating the response of the system to all sorts of dynamic inputs. A computer program is described that can handle up to a 10 span system.

INTRODUCTION

Web steering devices are used widely and effectively in web processing lines. They are particularly valuable when lateral stability of the web is poor. Air flotation and arch ovens are notorious for providing poor lateral support. The arch oven has a series of rollers that could provide tracking forces, but the wrap angle on these rollers is extremely small and tracking problems often occur. Flotation ovens have no tracking at all, and the corrugated shape of the web path makes them susceptible to cross machine direction motion.

Web non-uniformities and alignment problems often cause the web to drift laterally as it is processed. Skew in the web can lead to lateral steering of the web if it is not handled properly [see reference 1]. Here, the web can have a slack edge and tends to steer in that direction. Poor alignment can cause offsets and are a particular problem for moving rolls (accumulators for example) and for cases where the web tension deflects a poorly supported roll.

Steering devices operate as an integral part of a web line, and are needed to meet requirements of downstream processing equipment. Coating, laminating, printing, and winding high quality rolls all require a consistent web path with only minor amounts of transverse direction shifts. These processes occur downstream of equipment such as unwinds, accumulators and ovens that can cause the web path to be unstable. The capability to analyze steering devices as components has existed for a number of years [see reference 2]. These calculation procedures have provided great insight to the design of these components, but currently available techniques lack the capability of analyzing steering devices as a part of a web line with its unique geometry and requirements.

Several types of steering devices are available as discussed on Figure 1. There are two basic mechanisms available for these devices, and they all use at least one. These include lateral transport of the web by moving the roll in that direction and steering of the web by rotating the roll to induce a tracking angle with the web. The guide roller system is widely used and it employs both of the above mentioned mechanisms to some advantage as pictured on Figure 2. This is an active device and requires a sensor located just downstream of the moving roll. The guide roll pivots about a point well upstream of the active roll, and usually rotates in a plane parallel to the entering web path. The resulting lateral motion physically transports the web in the cross machine direction and responds very rapidly to transverse shifts in the web. The resulting angular motion of the roll provides steering that both corrects the web position and leads to a re-centering of the device. With the preferred 90-degree wrap angle on the guide roll, the web bends upstream of the roll and twists downstream. These displacements of the web can cause substantial stress in the web and can lead to permanent distortion or wrinkling. If the line geometry requires that the wrap angle is something other than 90 degrees, a complex situation results with steering both up and down stream of the device. The response of the web to all of these motions needs to be evaluated to insure that the web is not damaged and does not wrinkle [see reference 1].

A simpler device is the pivoting roll system shown on Figure 3. It operates on one of the principals used by the guide roll system discussed above. The roll rotates about a point on the axis of the roll and the steering or tracking motion introduced corrects the web location. It does not respond to web shifts as rapidly as the guide roll, since it lacks the transport mechanism. Again, the 90-degree wrap angle is important to avoid complex steering and twisting motions and excessive forces in the web.

The offset pivot guide approach is shown on Figure 4. It only uses the transport mode of web steering by twisting the entry span and using the angular offset to direct the web to a downstream roll that moves with the entry or rotating roll. This shifts the web from side to side using feedback from a sensor located as shown. If the wrap angles are held to 90 degrees, the device twists an entry and an exit span, and the results of these distortions can be evaluated as described in reference [1]. The device is capable of rapid response due to its reliance on the transport mechanism. It has no effect on the upstream or downstream web path. Therefore, it is not normally used at the exit of an oven since it would have no beneficial effect on the upstream span of the web.

The purpose of this work is to present a design calculation procedure that can evaluate any of the steering and guiding systems in the complex operating environment of a web processing line. It evaluates the performance of both the hardware and control system, and predicts web path variations. The program [see Reference 3] is an **EXCEL** application and currently is capable of evaluating a 10-roll system with one or more active steering devices. It provides multiple locations for sensors, and allows for the evaluation of complex feedback controls.

NOMENCLATURE

F_i	Constants defined in Table 1
E	Elastic Modulus of web material
I	Moment of inertia about axis perpendicular to web in its center
k	Factor defined by $k^2 = T / EI$
L	Length of the web spans
M	Moment in web about an axis perpendicular to the sheet in its center
R_c	Guide roll radius for motion control
R_S	Radius of curvature of a cambered or skewed web
S_{NX}	Set point based on the Nth sensor location
S_w	Unstrained speed of the web
t	Time
T	Total web tension
V_T	Transverse velocity of the web
V	Web speed through spans
w	Width of the web
x	Web dimension in the machine direction
Y_{Ri}	Lateral position of the ith roll
y	Deflected web position in the transverse or width direction
y_o	Deflected position of the web at the entry roll, $x = 0$
y_{NX}	Lateral position of the web at the Nth sensor location
y_L	Deflected position of the web at the exit roll, $x = L$
z	Transverse direction coordinate starting from the web center
α	Angle defining the twist of a web span
ϕ	Angle between the web and a roller causing a transverse web motion
ε	Web strain in the machine direction
μ	Poisson's ratio
θ	Web rotation angle in derivation of equations
θ	Deflected angle of the web in a span or the wrap angle on a roll in figures
θ_o	Angle of the web at the entry roll, $x = 0$
θ_L	Angle of the web at the exiting roll, $x = L$
σ_x	Stress in the web
τ	Web passing time for a span L/V
ω	Frequency of the input disturbance in radians/second

DERIVATION OF THE WEB STEERING RELATIONSHIPS

Guiding of a web in a processing line involves transporting and/or steering the web in the cross machine direction. These mechanisms will be handled separately and added together to get the total effect on the web. One overall equation will be derived to describe this motion. Steering involves the deflection of the web in the span upstream of the roll. These deflections will be handled using the approach described in reference [1]. Here, the web is approximated as a beam and this implies a number of limitations. These were obtained from references [1 and 4] and are:

- Homogeneous web with no mass.
- Straight or only slightly curved web (skew does not influence tracking).
- Uniform web cross section.
- Web deflections outside of the plane of the web are small (some twist allowed but no shear wrinkles).
- Maximum stress in the web does not exceed its elastic limit.
- No zones of compression are in the web (no slack edge).
- Shear stresses are low enough so that no buckling occurs.
- No shear deflection is included.
- No moment transfer is considered at any roll.

This may seem like a lot of restrictions but it is basically the assumptions that go in to describing a web. This allows us to use the relationships for web deflection in reference [1], and we can concentrate on the web behavior that causes lateral motion.

Figure 5 shows a sketch of the end of a web as it approaches a roll. Note that the web, a beam, can transmit tension, bending and shear loads. If we calculate the strain in the web that results from these loads, we get the condition pictured on the figure. Here, there is a uniform strain imparted by the tension and superimposed on that is strain introduced by the moment. Note that the shear is assumed to provide no significant strain. If it did, it would likely cause the web to form a shear wrinkle and the analysis presented here would not apply. The moment introduces tension on the low side of the web and compression on the top. It, along with the overall web tension, results in the diagonal strain variation shown.

The strain variation introduces a corresponding web speed variation. The speed at any point across the web is equal to

$$V = S_w(1 + \varepsilon_m(y) + \bar{\varepsilon}) \qquad (1)$$

where S_w is the unstrained speed of the web and $\varepsilon_m(y)$ is the strain introduced by the moment (all variables are defined in the nomenclature). However, the roll at the end of the span can turn at just one speed and we are looking at situations involving no slip or moment transfer in the web across the roll.

The roll will slow the fast or more highly strained side of the web, and, conversely, it will accelerate the slow or less strained side. The result is a rotation of the web at the interface with the roll as pictured on Figure 5 and this will cause the web span to bend or flex upwardly. Figure 6 finishes the picture of this process. Here, the increased speed on the low-tension side is given by

$$v = V\varepsilon_m(w/2) \quad \text{and} \quad d\theta/dt = V\ \varepsilon_m(w/2)/w/2 \qquad (2)$$

This relationship along with the basic beam relationship shown on the bottom of Figure 6 defines the rotation of the web and relates it to the second derivative of the web span deflection. The second derivative can be related to all of the span parameters by the beam relationships in reference [1].

The transverse velocity of the web or the basic steering motion is pictured on Figure 7. Here, both the rotation angle θ and an offset angle ϕ are shown. The transverse velocity of the web comes directly from the offset angle and is given by

$$V_T = \phi V = dy/dt = V\,[\theta_r - dy/dx] \qquad (3)$$

and $d\theta/dt = d\phi/dt$. Note that the transverse velocity is caused by a no-slip condition as the web contacts the roll in a similar way as no-slip causes the interface rotation described in equation 2. Here, each point on the web that contacts the roll tries to stay at that point on the roll. The web, however, has a transverse velocity component that pushes the web laterally and it results in the motion as shown.

The basic differential equation that describes the lateral motion of a web in a span surrounded by other spans is presented on Figure 8. It will not be reproduced in this text but will be assigned the number equation 4 and the constants F are defined in Table 1. The left hand side of this equation is the same as the classic relationship describing a spring-mass system with damping, which is discussed in detail in reference [5]. This means that the web steers like a damped oscillating system and the forcing functions (the right hand side) are in general damped and well behaved. The right hand side also introduces terms for shooting, transport, steering and offset at the entrance. Let's discuss each of these terms individually. They are:

- $F_1\, y_{Li-1}$ is the offset at the entrance of the span and is fed forward from the previous span.
- $F_2\, [\, V\theta_{rLi} + dY_{rLi}/dt\,]$ has two parts. The first is a steering term and the second is a transport term where both of these are at the span exit.
- $F_3\, [\theta_{rOi} - (\theta_{rLi-1} - dy_{Li-1}/dx\,)\,]$ is a shooting term describing the direction of the web coming off the upstream roll. The part in the bracket obtains the angle of the web at the span entry.
- d^2Y_{rLi} / dt^2 is the second derivative transport term and simply states that if the roll moves the web will move with it.

This differential equation requires two boundary conditions to describe the initial conditions of the deflection at the exit of the span or y_{Li}. Normally these are the value and the first derivative at t=0. On most occasions, we don't know the precise conditions at t=0, but we are interested in how the devices behave with some type of periodic, step or ramp input. The transient solution for the initial portion of the transient is not that significant for those cases and the exact form of the initial boundary conditions is not that important. We want the device to be stable early under a variety of starting conditions and then achieve the desired level of control. This is evaluated by letting the transient run until some type of steady state is achieved. Often this is a regular periodic response to some type of periodic input. This means that several different periodic response calculations need to be made to insure that we cover the range of problems that may be encountered in the actual device.

The next step is to evaluate the interfaces between the spans to insure that the proper conditions are applied there. Figure 9 shows an interface between the i^{th} and the $i^{th} + 1$ spans. The first condition is the simpler and it comes from the assumption that the web is continuous. In that case

$$y_{Li} = y_{oi+1} \tag{5}$$

where the effect of the small angle of the web at the roll over the wrap length is not included. The other required condition comes from the assumption that there is no bending of the web as it wraps a roll. This condition is complicated by the angles of the rolls and spans in a normal web transport system. This means that the angle of the web on the i^{th} roll in the i^{th} span can be different from the angle of the web exiting the same roll in the $i^{th} + 1$ span. For example, a guide roll system with a 90-degree wrap uses a roll angle to steer the upstream web. The downstream web path sees no steering angle from this motion. It is all transferred into twist. So, what we have is that the web approaches the roll at some angle to the roll in that span and it exits the roll in the next span at the same angle. If this angle is not zero, the web forms a perfect spiral around the roll at the approach angle. Mathematically, this means

$$\theta_{rLi} - dy_{Li} / dx = \theta_{rLi+1} - dy_{Li+1} / dx \tag{6}$$

The solution procedure for the differential equation is outlined on figure 10 where a marching type of solution is described. Here, everything has to be defined initially, all the web positions, and the roll angles and motions. Usually some offset, offset motion or roll motion is defined during the transient to simulate typical problems at the entrance or anywhere else in the system. The solution starts at the first span where one time step is made using a numerical technique from reference [5]. This result is used to establish the conditions at the span exit and the entrance to the next span. Then the next span is evaluated. This process continues until all spans have been processed. The approach is repeated for successive time steps until the solution is complete.

COMPARISONS WITH EXPERIMENTS AND EXACT SOLUTIONS

Experimental data [reference 2] is available for the simple guide roller system shown on Figure 11 where the roller is oscillated back and forth at an amplitude of 1. The ratio of the deflection of the web at the guide roller to that of the roller is measured. The nice thing about this is an exact solution to equation 4 can be obtained for this case. The results are plotted on Figures 12 and 13 for two values of the guide roller radius, R_c. These define systems that are referred to as under steering and over steering, respectively. The solution described above (**WEBSTEER©**) agrees with the exact solution and they both are very close to the experimental results.

FEEDBACK CONTROL TEST CASES

The feedback approach can employ constant, proportional and/or derivative control. Figure 14 describes the algorithm used in this analysis. For primary control, the guide roller motion is defined by its geometry and the lateral displacement of the roll. The speed of the lateral motion of the roll is a combination of a constant, a factor proportional to the difference between the web location at the sensor and the defined set point and an additional factor related to the time derivative of the web position. The cascade control approach uses the same three factors to modify the set point at the primary control location. The example cases are for the guide roller pictured in Figure 15 with the appropriate feedback system. Here, the first span is 5080mm long and the second is

1524mm. A sensor is located 152 mm downstream of the active guide roll and a second sensor for cascade control is located at the 1400 mm position.

The first example case included as Figure 16 uses the simple control system from Figure 15. It has a dead band of +/- 2.5mm and uses a constant roll speed correction of 5.4 mm per second. Here, three plots of deflection ratio versus time are shown and they correspond to three input frequencies. Varying the input deflection at the entrance to the 1524mm span induces the transients. This input is labeled as Y_o on the figure. Three responses to this motion also are plotted. These include the web deflection at the sensor (y_{x1}), the deflection at the exit of the 1524mm span (y_{R2}) and the motion of the guide roll (Y_{R1}), which is trying to correct for the input response. The frequency of the input response is varied from 0.2 to 5.0 radians/second. The web data is included in Table 2.

The top plot on Figure 16 is for the slowest input frequency and should give the response of the system to drifts of the web entering the guide roll. Note that the guide roll mirrors the input offset and the deflections at y_{x1} and y_{R2} are small. They do have a frequency that is related to the correction speed of the guide roll. If we alter the input frequency as shown in the middle plot, problems arise. The correction speed is not fast enough to follow the input response and y_{x1} and y_{R2} approach 0.6 and 0.4, respectively. If you need good control at the end of the exit span, you would be in trouble if the input response were around 1 rad/sec. The final plot on Figure 16 shows an even higher frequency input response. But note that the deflections are again in reasonable control. Here, the input frequency is so high that the natural damping of the system keeps the deflections in control.

Varying Guide Roll Wrap Angle and Complex Control Systems

The final series of plots are included on Figure 17 where a more complex control approach is employed. Here, proportional and derivative control performed well at all of the input frequencies and the $\omega = 0.2$ rad/sec case is shown. As an additional test of the calculation, the wrap angle of the guide roll was changed from 90 to 45 degrees and this result is shown in the center of Figure 17. There is excellent control of the web at the sensor for this case, but there is a sizable (0.25) deflection of the web at the exit roll(y_{R2}). This is due to the component of the steering angle that is transferred into the second span. The angle tends to shoot the web in the direction of roll motion and the error occurs downstream. It was thought that this could be improved by increasing the radius of the guide roller motion but this was unsuccessful.

The final plot on Figure 17 shows results for the 45 degree wrap angle and the cascade control system. The control predicted here is strikingly good. The web position at the exit roll is both accurate and stable. The amplitude of the roller motion is smaller and the web deflections in the primary span is reduced by the modified control approach. Note that systems with wrap angles that exceed 90 degrees have a similar problem but the deflection of the web at the exit roll is in the opposite direction.

The examples included are intended to demonstrate the value of a multi-span calculation that can look at a variety of control approaches and web geometries. Table 3 summarizes the need for and the application of this approach. Basically, all web lines employ multiple rolls, and most need guiding devices that have control systems. An analysis like the one described here would decrease the risk of problems with any type of guide system. It also could be used to analyze systems that are having problems to try to first duplicate these difficulties and then find a solution. Basically, the calculations included in **WEBSTEER©** provide a means to mathematically test a system design before it is installed.

SUMMARY

A model was presented that provides a means to analyze the performance of all types of steering and guide roll devices in actual line configurations. The basic differential equation and the interfacial conditions at each roll were developed for a multi-span configuration. The calculation procedure was checked by comparing results from it with experimental data obtained for a simple guide roll arrangement. Potential problems with several types of guide and steering roll systems also were discussed.

Example cases were presented that illustrates the value of performing this type of calculation. Difficulties that can be encountered with simple control systems were demonstrated and the advantage of a more complex system was shown. The effect of a non-ideal wrap angle (<90deg) in a guide roll system also was demonstrated. This example required a multi-span analysis to evaluate the steering component downstream of the guide roll. The results indicate that the web is directed or shot by the guide roll in a direction normal to it. Therefore, the guide roll misdirects the web downstream as it corrects the web position at the preferred sensor location. This is true for wrap angles either greater to or less than the preferred 90 degrees.

The benefits achievable by a cascade control system were presented and its value established for non-ideal steering geometries. The application of the procedure was discussed and **WEBSTEER ©**, an **EXCEL** application software program, was introduced.

REFERENCES

1. Forrest, A. W., "Web Span Analysis for Web Path Design Including Steering and Alignment Considerations", Presented at the AIMCAL 2000 Fall Conference.
2. Shelton, J, J., "Lateral Dynamics of a Moving Web", Ph. D. Thesis at Oklahoma State University, July 1968.
3. **WEBSTEER©**, an **EXCEL** application software program, written by AL Forrest, available from Advantage Web Technologies, 20 Shawnee Dr., Chillicothe Ohio, 45601, (740-773-7441).
4. Roark, R. J. and Young, W. C., Formulas for Stress and Strain, fifth edition, McGraw-Hill Book Company, 1982.
5. Kreyszig, Erwin, Advanced Engineering Mathematics, second edition, John Wiley and Sons, June 1968.

Table 1 Values of the F Constants from Equation 4

$$F_1 = (kL)^2 (\operatorname{Cosh}(kL) - 1) / \tau^2 (kL \operatorname{Sinh}(kL) - 2 \operatorname{Cosh}(kL) + 2)$$

$$F_2 = kL (kL \operatorname{Cosh}(kL) - \operatorname{Sinh}(kL)) / \tau (kL \operatorname{Sinh}(kL) - 2 \operatorname{Cosh}(kL) + 2)$$

$$F_3 = k L^2 (\operatorname{Sinh}(kL) - kL) / \tau^2 (kL \operatorname{Sinh}(kL) - 2 \operatorname{Cosh}(kL) + 2)$$

Table 2 Web Data for Example Cases

Web Width	1016 mm	Web Thickness	0.0127 mm
Poisson's Ratio	0.2	Elastic Modulus	3450 KPa
Web Speed	91.4 m/min.	Web Stress	6895 KPa

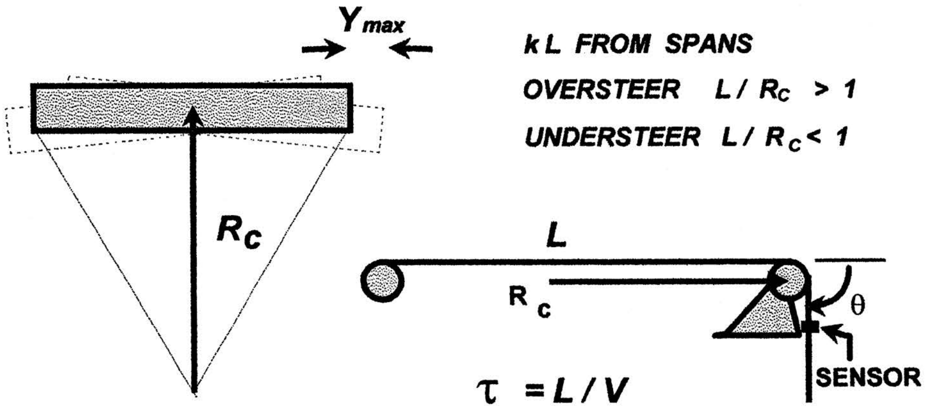
Table 3 Application of the analysis

WHEN DO I NEED TO USE THIS ANALYSIS
- WHEN THE PROCESS DOWNSTREAM OF A STEERING DEVICE REQUIRE ACCURATE POSITIONING OF THE WEB.
- WHEN A NON-IDEAL GEOMETRY IS USED FOR THE STEERING DEVICE (i.e. ANGLES OTHER THAN 90 deg ARE USED AROUND THE MOVING ROLL).
- WHEN EVALUATING CONTROL SCHEMES FOR STEERING DEVICES.
- WHEN DEVELOPING HYBRID CONTROL SCHEMES OR UNIQUE GUIDE SYSTEMS.

HOW DO I USE THIS PROCEDURE
- MODEL THE PROPOSED GUIDE SYSTEM AND SURROUNDING ROLLS. INCLUDE ROLLS DOWN TO ANY KEY DEVICES.
- RUN THE MODEL AND APPLY DISTURBANCES AT SELECTED LOCATIONS. INCLUDE A RANGE OF DISTURBANCE FREQUENCIES AND TYPES.
- TRY DIFFERENT CONTROL SCHEMES AND LOCATIONS. NOTE THE RESULTS AT ANY KEY DEVICES.
- MODIFY THE GEOMETRY, GUIDE ROLL APPROACH AND CONTROL SYSTEM AS REQUIRED.
- EVALUATE THE STRESSES AND TENDENCY TO WRINKLE USING WEBSPAN© OR EQUIVALENT.

- GUIDE ROLLER SYSTEM
  - ACTIVE METHOD THAT STEERS UPSTREAM
  - EMPLOYS BOTH WEB STEERING AND TRANSPORT
  - REQUIRES EDGE SENSOR AND CONTROL SYSTEM
- PIVOTING ROLL SYSTEMS
  - ACTIVE SYSTEM THAT STEERS UPSTREAM
  - USES WEB STEERING AND NO TRANSPORT
  - RESPONDS MORE SLOWLY
  - REQUIRES EDGE SENSOR AND CONTROL SYSTEM
- OFF-SET PIVOT GUIDE SYSTEM
  - ACTUIVE SYSTEM WITH NO UPSTREAM EFFECT
  - PRIMARILY EMPLOYS WEB TWISTING AND TRANSPORT
  - IF DESIGNED WITH ALL WEB TURNS AT 90 deg IT EMPLOYS NO STEERING
  - REQUIRES EDGE SENSOR AND CONTROL SYSTEM
- UNWIND AND WINDUP SYSTEMS
  - ACTIVE METHOD THAT MOVES UNWIND OR WINDUP
  - SYSTEM INERTIA IS KEY ISSUE
  - REQUIRES EDGE SENSOR AND CONTROL SYSTEM

Fig. 1 Types of active steering systems



kL FROM SPANS

OVERSTEER $L / R_C > 1$

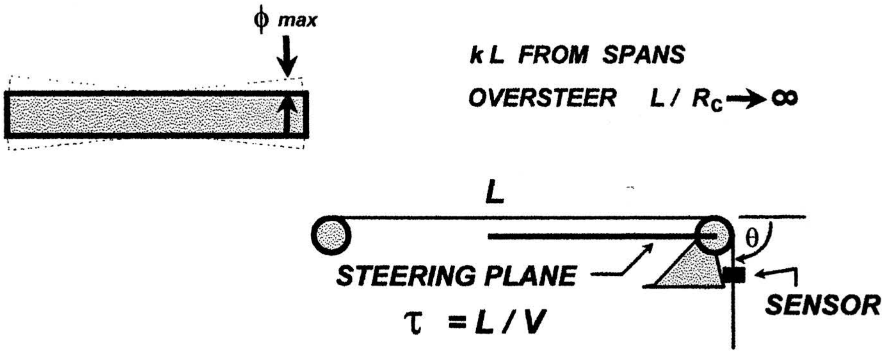
UNDERSTEER $L / R_C < 1$

DEFLECTS BOTH UPSTREAM AND DOWNSTREAM WEBS

WITH θ = 90 deg THE DOWNSTREAM WEB IS TWISTED ONLY

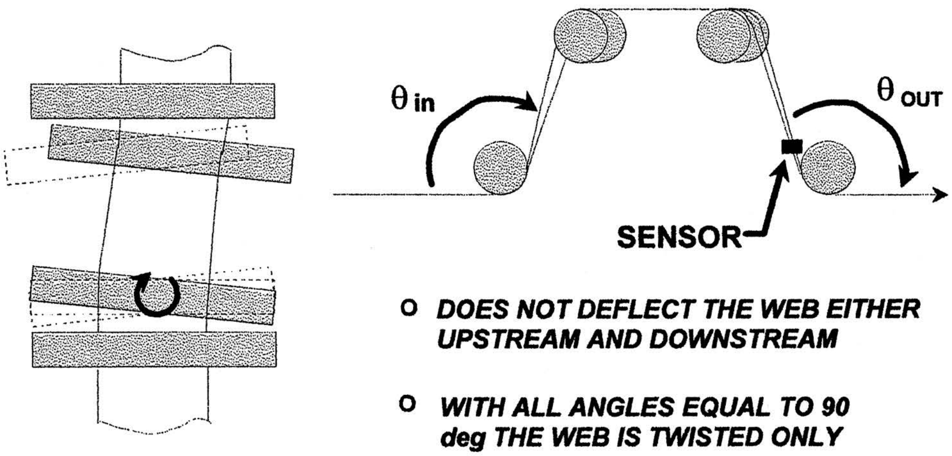
WITH θ NOT EQUAL TO 90 deg THE DOWNSTREAM WEB IS BOTH TWISTED AND BENT OR STEERED

Fig. 2 Guide roller system



- ***DEFLECTS BOTH UPSTREAM AND DOWNSTREAM WEBS***
- ***WITH θ = 90 deg THE DOWNSTREAM WEB IS TWISTED ONLY***
- ***WITH θ NOT EQUAL TO 90 deg THE DOWNSTREAM WEB IS BOTH TWISTED AND BENT OR STEERED***
- ***CORRECTS MORE SLOWLY THAN A GUIDE ROLLER SYSTEM***

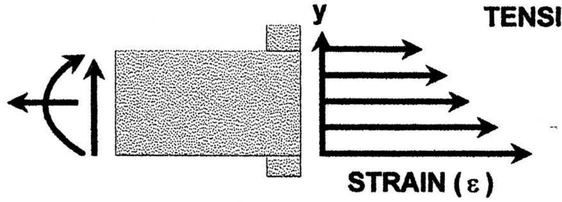
Fig. 3 Pivoting roll system



- ***DOES NOT DEFLECT THE WEB EITHER UPSTREAM AND DOWNSTREAM***
- ***WITH ALL ANGLES EQUAL TO 90 deg THE WEB IS TWISTED ONLY***
- **WITH ANY ANGLE NOT EQUAL TO 90 deg THE WEB IN THE UNIT CAN BE BOTH TWISTED AND BENT OR STEERED**

Fig. 4 Offset pivot guide system

STRAIN ON THE END OF A WEB CAUSED BY TENSION AND MOMENTS



SPEED OF ROLL = $S_w\,(1+\bar{\varepsilon})$ **sw = THE UNSTRAINED WEB SPEED**

SPEED OF WEB = $S_w\,(1+\varepsilon(y))$

$\dot{\theta}$ **= ANGULAR VELOCITY AT WEB-ROLL INTERFACE**

THE ANGULAR VELOCITY CAUSES THE END OF THE WEB TO ROTATE TOWARD THE LESS TAUT SIDE

N0-SLIP WEB SPEED VARIATION

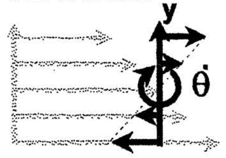


Fig. 5 Rotation of the web roll interface

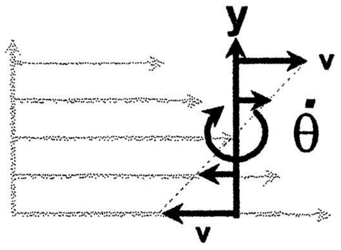
$$v = V\,\varepsilon_m(w/2)$$

$$\dot{\theta} = \frac{V\,\varepsilon_m(w/2)}{w/2} = \frac{d\theta}{dt}$$

ROTATION RELATIONSHIP

$$\boxed{\frac{\varepsilon_m(w/2)}{w/2} = \frac{1}{V}\frac{d\theta}{dt}}$$

N0-SLIP WEB SPEED VARIATION

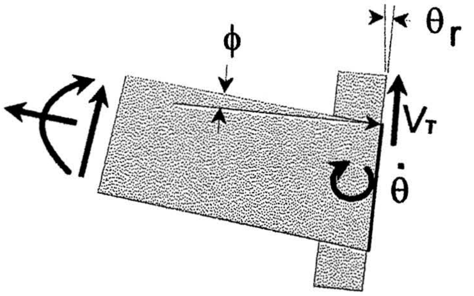


NOTE THAT TWIST IS SYMMETRICAL AND DOES NOT INTRODUCE ROTATION OF THE WEB

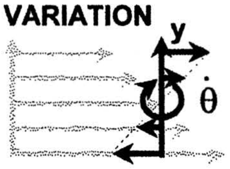
V IS THE INCREMENTAL VELOCITY DUE TO NO-SLIP AND THE STRAIN VARIATION OF THE WEB ACROSS THE ROLL DUE TO THE MOMENT IN THE WEB

$$\varepsilon_m(w/2) = \frac{M\,w}{2\,E\,I} = \frac{w}{2}\,\frac{d^2y}{dx^2}$$

Fig. 6 Rotation equation at the interface



N0-SLIP WEB SPEED VARIATION



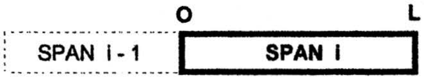
- y IS THE LATERAL POSITION OF THE WEB AT THE ROLLER
- $\dot{\theta}$ CAUSES A ROTATION OF THE WEB INTERFACE AND SETS UP AN ANGLE ϕ
- THE ANGLE ϕ AND THE WEB VELOCITY CAUSE A LATERAL SPEED V_T
- THESE VALUES ARE RELATED AS SHOWN

$$\boxed{V_T = \phi V = \frac{dy}{dt} = V\left[\theta_r - \frac{dy}{dx}\right]}$$

WHERE $\dot{\theta} = \dfrac{d\phi}{dt}$

Fig. 7 Web transport relationship

DIFFERENTIAL EQUATION



$$\frac{d^2 y_{Li}}{dt^2} + F_2 \frac{d y_{Li}}{dt} + F_1 y_{Li} = F_1 y_{Li-1} + F_2 \left[V\theta_{rLi} + \frac{d Y_{rLi}}{dt}\right]$$

$$+ F_3 \left[\theta_{rOi} - \left(\theta_{rLi-1} - \frac{d y_{Li-1}}{dx}\right)\right] + \frac{d^2 Y_{rLi}}{dt^2}$$

WHERE A VALUE OF y_{Li} AND y'_{Li} MUST BE PROVIDED FOR EACH SPAN AND ROLL 0 AND

y_{Li} = **THE UNKNOWN POSITION OF THE WEB AT THE END OF SPAN i**

Y_{rLi} = **THE LATERAL POSITION OF THE ROLL AT L IN SPAN i**

θ_{rLi-1} = **THE ANGLE OF THE ROLL AT L IN THE i-1 SPAN**

WHICH CAN BE DERIVED FROM THE ROTATION AND TRANSPORT RELATIONSHIPS WHILE ADDING IN A TERM FOR DIRECT ROLL TRANSPORT AND A RELATIONSHIP FOR A MOMENT. THESE ARE:

$$\varepsilon\,(w/2) = \frac{M\,w}{2\,EI} = (w/2)\,\frac{d^2 y}{dx^2} \qquad \left.\frac{dy_{rLi}}{dt}\right|_{\text{TRANSPORT PORTION}} = \frac{dY_{rLi}}{dt}$$

EQUATIONS FOR F's IN TABLE 1

Fig. 8 Web transport differential equation

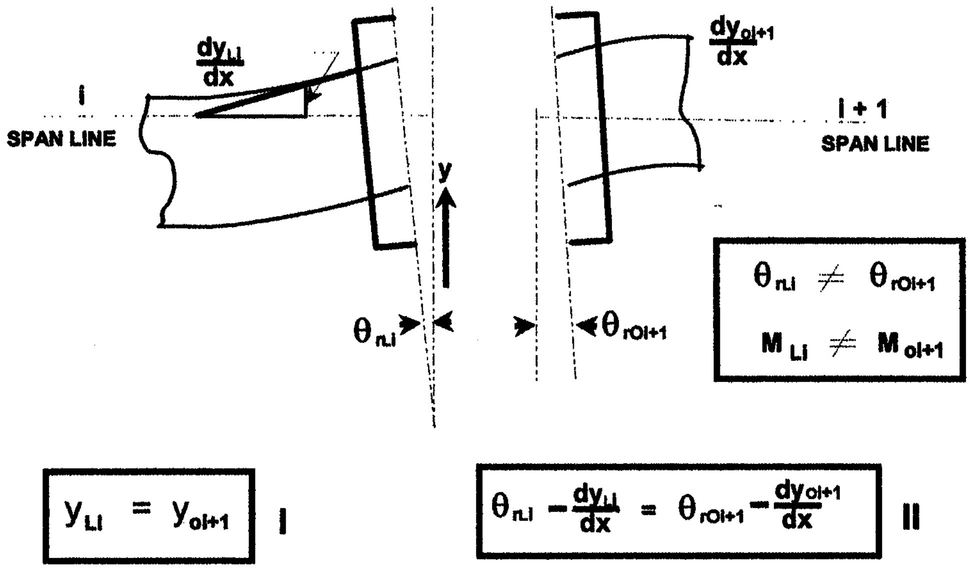
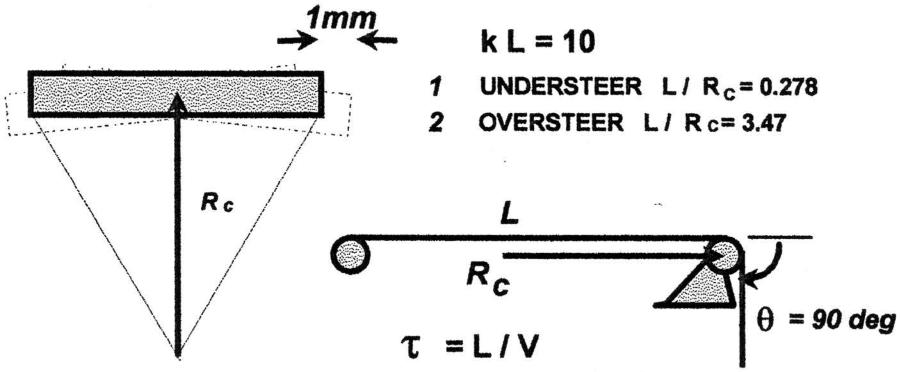


Fig. 9 Span interface relationships

- **THE INITIAL CONDITIONS MUST BE DEFINED AT ALL ROLLS. THIS INCLUDES THE ORIENTATION AND MOTION OF EACH ROLL AND THE POSITION OF THE WEB ON IT.**
- **THE FIRST DERIVATIVE AND THE POSITION OF THE WEB MUST BE SET ON THE ENTRANCE ROLL.**
- **THE NUMERICAL SOLUTION IS OBTAINED BY MAKING A TIME STEP FOR THE FIRST SPAN, APPLYING THE INTERFACE CONDITIONS AND PROCEEDING TO THE NEXT SPAN. THIS CONTINUES UNTIL ALL SPANS HAVE BEEN EVALUATED.**
- **THE NEXT TIME STEP IS ACCOMPLISHED IN THE SAME MANNER AGAIN STARTING AT THE FIRST SPAN.**
- **AFTER EACH TIME STEP THE ROLL POSITIONS AND DERIVATIVES ARE RESET BASED ON DEFINED MOTION AND/OR FEEDBACK CONTROL THAT USES THE POSITION OF THE WEB AT THE DEFINED SENSOR LOCATIONS.**
- **NOTE THAT 10 SPANS MAY BE INCLUDED WITH UP TO THREE SENSOR LOCATIONS IN EACH SPAN. THIS SHOULD PROVIDE ADEQUATE MEANS TO EVALUATE LARGE SECTIONS OF A WEB LINE AT ONE TIME.**

Fig. 10 Solution procedure



TWO TEST CASES ONE FOR UNDER AND ONE FOR OVERSTEER

INPUT AT GUIDE ROLL IS SINUSOIDAL WITH A PERIOD SET BY $\omega\tau$

Fig. 11 Guide roll cases for comparison

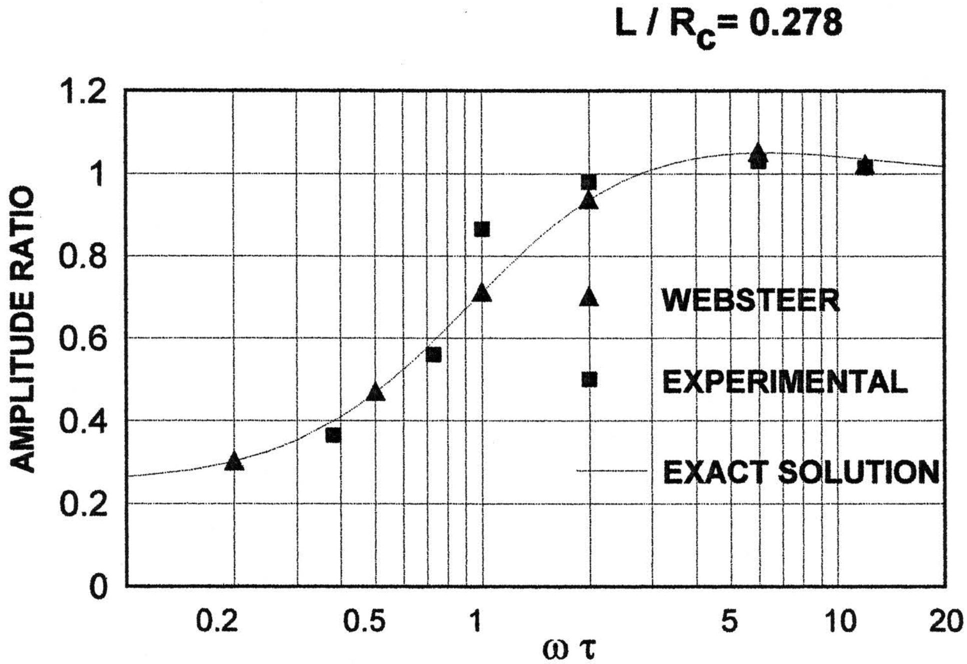


Fig. 12 Understeering guide roll case for comparison

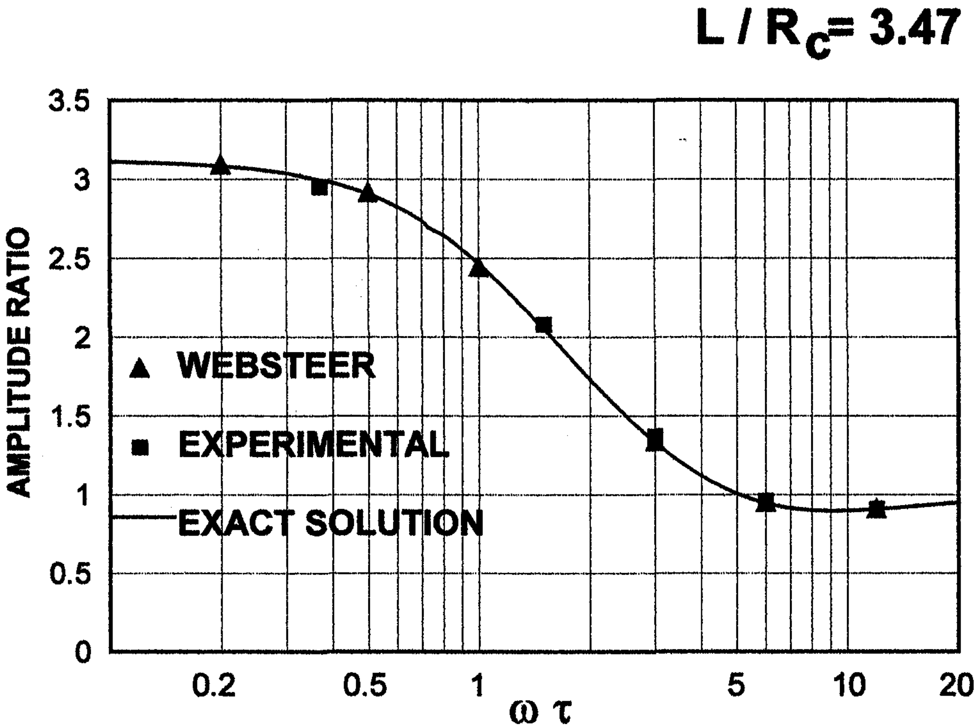
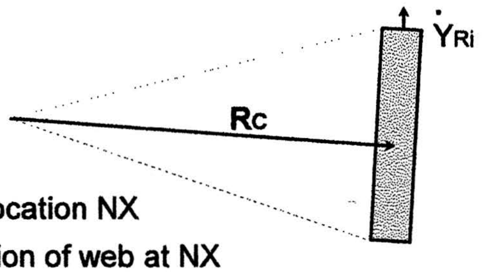


Fig. 13 Oversteering guide roll case for comparison



S_{NX} Setpoint for location NX

y_{NX} Larteral position of web at NX

IF $| y_{NX} - S_{NX} | < DB_{NX}$ then $\dot{Y}_{Ri} = 0$ else

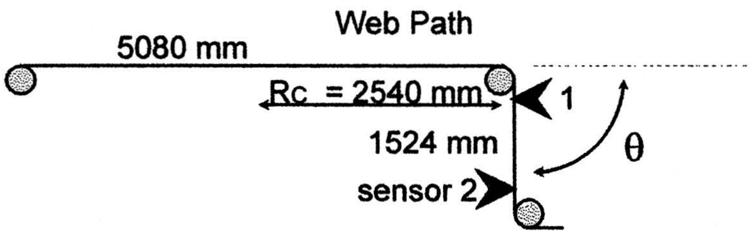
$$\dot{Y}_{Ri} = C_{iN}\,(y_{NX} - S_{NX})/\,((y_{NX} - S_{NX})^{2}\,)^{1/2} + A_{iN}\,(y_{NX} - S_{NX}) + B_{iN}\, d(y_{NX} - S_{NX})/dt$$

IF $| y_{MX} - S_{MX} | < DB_{MX}$ then $\dot{S}_{NX} = 0$ else

$$\dot{S}_{NX} = C_{iM}\,(y_{MX} - S_{MX})/\,((y_{MX} - S_{MX}^{2})\,)^{1/2} + A_{iM}\,(y_{MX} - S_{MX}) + B_{iM}\, d(y_{MX} - S_{MX})/dt$$

$$S_{NX} = S_{NX} + \dot{S}_{NX}\,\delta t \quad \text{(indexing)}$$

Figure 14 Relationships used for feedback control systems with cascading



Control Data for Example Cases

Case	A1 1/sec	B1	C1 mm/sec	A2 1/sec	B2	C2 mm/sec
Simple	0	0	5.1	0	0	0
No Cascade	3	2	0	0	0	0
Cascade	3	2	0	3	2	0

Figure 15 Descriptions of example cases and control systems used to establish the value of this analysis

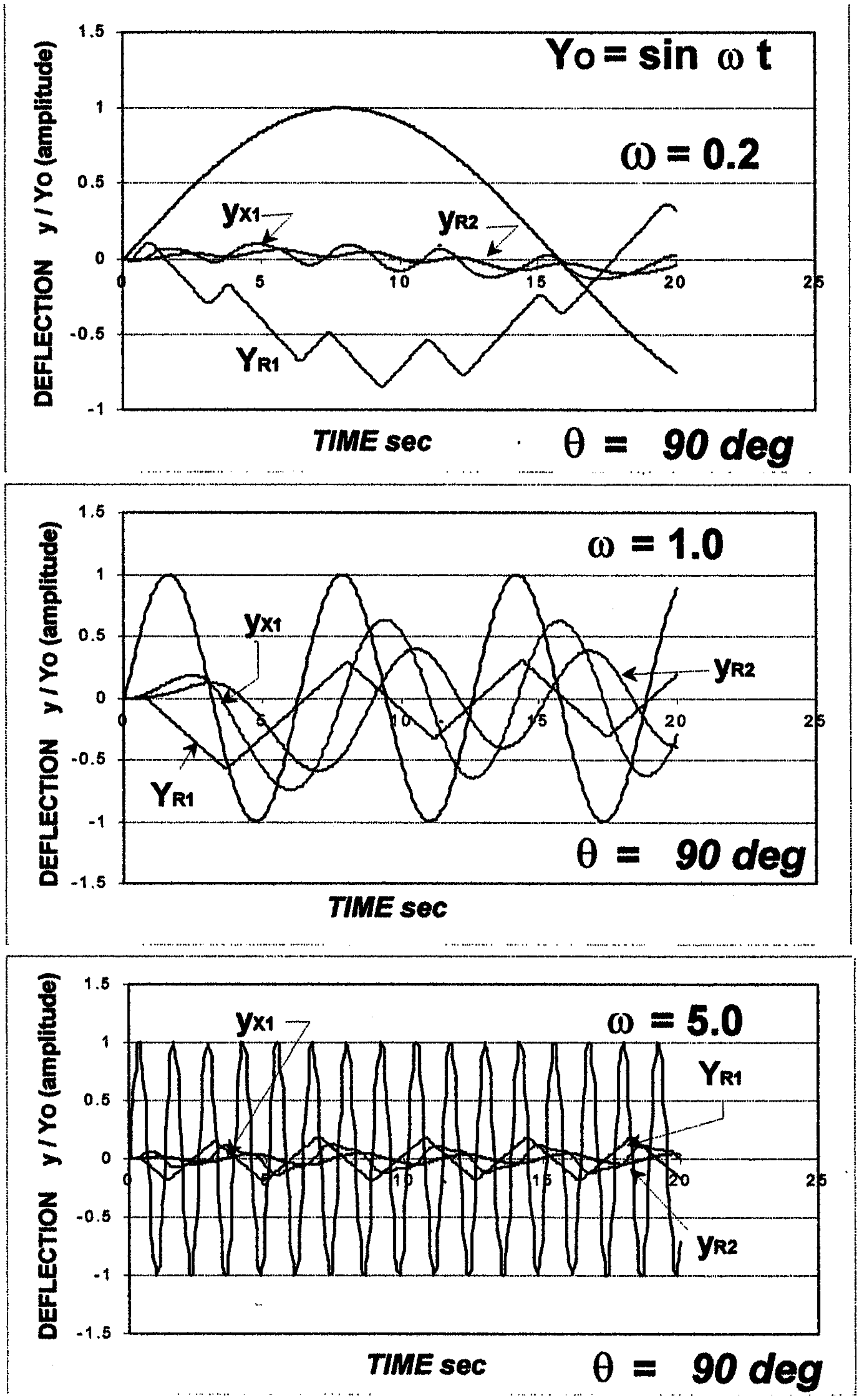


Fig. 16 Simple control approach