A SIMPLIFIED MODEL FOR LATERAL BEHAVIOR OF SHORT WEB SPANS

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ABSTRACT

For ratios of L/W less than approximately 2.0, the effects of shear stresses on the shape of a web must be considered. In basic manufacturing and processing of webs, such ratios are very common. The analysis of a general Timoshenko beam is extremely complicated for some problems, such as the dynamic effects of the interconnection of spans, so that simplification may facilitate solution and enhance understanding.

The basis for the simplification is the fact that tension has no effect on the shape of the elastic curve if L/W is very low, so that the elastic curve of the web can be expressed as a polynomial in x instead of exponential or hyperbolic functions of x as necessary if L/W is large. Mathematical analysis thus becomes easier, and the results can be expressed in simpler, more understandable form.

Static problems, such as the critical angle of misalignment of a roller from the standpoint of slackness of an edge, are analyzed and the results are compared (for low L/W) to the solution of the Timoshenko beam (general L/W ratio) as presented in the Shelton thesis [1].

Superposition of the effects of translation and rotation of the upstream inputs and the downstream outputs allows analysis of the effects of interconnection of spans. A method of reducing "weave regeneration", the downstream reappearance of an error which was corrected by a web guide as studied by Sievers [2], is shown to be the modification of ratios of L/W to uncommon values. Specifically, the guiding span should have a large ratio of L/W, and the exiting span of the guide should have a fractional value of L/W for reduction of weave regeneration.

NOMENCLATURE

A, B, C, D	specific web spans
E	modulus of elasticity (Young's modulus)
G	modulus of elasticity in shear
K_0	open-loop gain of the control system
L	length of a free span of web

L	as a subscript, the condition at the downstream roller
M	bending moment in the web
N	shear force normal to the elastic curve of the web, in the plane of the web
O	as a subscript, the condition at the upstream roller
s	Laplace transform operator (equal to $j\omega$)
s	as a subscript denotes the effect of shear only
t	thickness of the web
V	velocity of the web
W	width of web
x	distance downweb from roller
y	lateral web deflection from its original position
ε	tensile strain of the web
θ	slope of the elastic curve (same as y')
θ_r	roller angle
τ	time constant of a web span – seconds (= L/V)
ω	frequency – radians per second

INTRODUCTION

Although the static behavior of a web span with consideration of shear stresses is not prohibitively complicated, studies of dynamics by the same methods are intimidating, at best. A simpler, more tractable model was therefore needed for the study of the dynamics of short spans, including the study of web guiding for extreme accuracy. It should be noted that all ratios of L/W studied in the Sievers thesis [2] were less than 2.0.

The new model for short spans is based on the following considerations:

(1) The effect of tension on bending is negligible for a short beam (low L/W).

(2) Although shear stresses must be considered, the angular deflection from bending is an essential basis of some problems with short spans; therefore, the deflection caused by bending must also be considered.

(3) Although the effect of tension on bending is neglected, tension can and must be considered after the deflection curve is determined for studying slackness of a edge, wrinkling, and other phenomena which are dependent on tension.

(4) Because tension is neglected in determination of the elastic curve, and shearing stresses do not affect curvature, the relationships between the second derivative and the moment as well as the third derivative and normal force are the same as for an untensioned beam, $-EIy'' = M$ and $-EIy''' = N$.

(5) When tension is neglected, the elastic curve is expressed as a polynomial in x, not exponential or hyperbolic functions of x.

(6) A disturbance to a span (an independent variable, known or assumed) is usually at the upstream roller, while the outputs (dependent variables) are primarily at the downstream roller; however, when the deflection caused by shear is considered, the angle of shear as the web leaves the upstream roller (θ_{0+}) is a dependent variable. When the upstream angle is an input, the angle of the web on the roller (θ_{0-}) must therefore be considered the independent variable.

(7) The sign conventions of Timoshenko [3] and adopted by Shelton [1] will be used.

The method of superposition of the effects of translation and rotation of the downstream end of a span when the effects of shear deflection are significant is not intuitively obvious; therefore, the short-span model will be developed and demonstrated in steps from simple static conditions to the more complicated dynamic behavior.

Example of Misaligned Downstream Roller: The new short-span model will be first illustrated by analyzing a span with a permanently misaligned downstream roller, shown in Figure 1. The fourth-order differential equation for an untensioned beam with no normal force is

$$\frac{d^4y}{dx^4} = 0, \tag{1}$$

for which the general solution is

$$y = C_1x^3 + C_2x^2 + C_3x + C_4. \tag{2}$$

In equation (2), the linear term C_3x is the deflection caused by shear stresses. The third derivative of equation (2) is always constant; therefore, the normal force ($N = -EIy'''$) is always constant in this short-span theory, in contrast to the theory for general span lengths in the Shelton thesis [1], where K_e from Timoshenko [3] is utilized.

The boundary conditions necessary for determination of the constants in equation (2) are: (1) $y_0 = 0$, (2) $y''_L = 0$, (3) $y'_L = \theta_r$, and (4) $y'_{0+} = -(EI/tWG)y'''$. The third condition is given or assumed, while the fourth expresses the angle as the web leaves the upstream roller, as caused by the normal stress ($-EIy'''/tW$).

Solution results in the elastic curve, which can be readily differentiated to find its slope (y'), curvature (y'') and rate of change of curvature (y'''):

$$\frac{y}{L} = \frac{\theta_r}{6(L/W)^2 + E/G}\left[\frac{E}{G}\frac{x}{L} + 6\left(\frac{L}{W}\right)^2\left(\frac{x}{L}\right)^2 - 2\left(\frac{L}{W}\right)^2\left(\frac{x}{L}\right)^3\right]. \tag{3}$$

From equation (3):

$$\frac{y_L}{L\theta_r} = \left[4(L/W)^2 + E/G\right] / \left[6(L/W)^2 + E/G\right]. \tag{4}$$

The critical roller angle from the standpoint of slackness of an edge can be derived by combining the equation for y''_0 with the relationship between y'' and the moment, $-EIy'' = M$, and the critical condition, $M = TW/6$:

$$\theta_r)_{cr} = (\varepsilon/6)(L/W)\left[6 + (E/G)/(L/W)^2\right]. \tag{5}$$

The critical angle of misalignment from the standpoint of wrinkling can be found from the equation for y''', the relationship $N = -EIy'''$, and an unpublished analysis of wrinkling caused by shear stresses, which was based on the analysis of buckling of a tensioned web span by Shelton [4] and the compressive stresses created by shear stresses as dictated by Mohr's circle. The resulting simplified equation for short spans (which have a relatively uniform distribution of shear stresses across the width) is

$$\frac{N}{tWE} = \sqrt{1.94\frac{t}{L}\varepsilon^{3/2}}\,. \tag{6}$$

The combination of equations results in

$$\theta_r)_{cr} = 1.39\,\varepsilon^{3/4}(t/W)^{1/2}\left[(E/G)(L/W)^{-1/2} + 6(L/W)^{3/2}\right]. \tag{7}$$

Equations (5) and (7) are the same as unpublished equations, derived by Shelton for spans of general length, then simplified for short spans by truncation of series expansions of hyperbolic functions. The relative simplicity of the short-span theory is thus demonstrated for a static problem, leading to hope of great simplification of analysis of the dynamics of short spans.

Example of Misaligned Upstream Roller: Figure 2 illustrates a short span with a permanently misaligned upstream roller. The general equations (1) and (2) apply to this span.

In this problem, the input (independent variable) is the roller angle, θ_r. Because of shear deflection, the angle of the web just after it leaves the roller, θ_{0+}, differs from θ_{0-}, or θ_r. These relationships of angles must be understood as a foundation for the later derivation of dynamic behavior, wherein the web is not necessarily perpendicular to the roller at any circumferential location; that is, θ_{0-} may differ from θ_r.

The boundary conditions from which the four constants in equation (2) can be determined are: (1) $y_o = 0$, (2) $y''_L = 0$, (3) $y'_L = 0$, and (4) $y'_{0+} = \theta_r + N_o/tWG$. N_o in the fourth equation is known to be negative for the chosen sign conventions, or specifically, $N_o = -EIy'''_o$, with y'''_o positive.

Determination of the constants results in the elastic curve:

$$\frac{y}{L} = \frac{\theta_r (L/W)^2}{6(L/W)^2 + E/G}\left[6\left(\frac{x}{L}\right) - 6\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3\right]. \tag{8}$$

If the angle of misalignment of the downstream roller in equation (3) is distinguished from the angle of the upstream roller in equation (8), the effects of the two angles of misalignment can be added. A special case of such superposition which is obviously correct is equal upstream and downstream angles of misalignment, for which addition of equations (3) and (8) results in the equation $y = \theta_r x$.

Because the equations for the upstream moment with upstream or downstream misalignment are identical except for the overall sign, as are the equations for normal force, equation (5) applies to the critical angle of an upstream roller from the standpoint of slackness of an edge, and equation (7) similarly applies to the critical angle from the standpoint of wrinkling.

SUPERPOSITION OF TRANSLATION AND ROTATION

In the Shelton thesis [1], the dynamic behavior of a web was determined by superposition of the effects of translation and rotation of the downstream end of the span, with the angle causing velocity and curvature causing acceleration of the web, with all parameters relative to the roller. When angles of shear deflection are considered, great care must be exercised to properly consider those angles. As a foundation for analysis of dynamics, superposition will be applied to the problems of the static behavior of a span with a downstream or upstream roller which is misaligned, for comparison to the results of the previous two articles.

Superposition Applied to a Permanently Misaligned Downstream Roller: The effects of translation and rotation of the downstream end of the span of Figure 1 will be superposed, with the goal of duplicating equation (3).

Figure 3(a) illustrates rotation of the downstream end of the span. The four boundary conditions required for evaluation of the four constants of equation (2) are (1) $y_o = 0$, (2) $y_L = 0$, (3) $y'_L = \theta_L$ (given), and (4) $y'_{0+} = -(EI/tWG)y'''_{0+}$, where y'''_{0+} is known to be positive. The result is:

$$\frac{y}{L} = \frac{\theta_r}{2(L/W)^2 + E/G}\left\{2\left(\frac{L}{W}\right)^2\left(\frac{x}{L}\right)^3 + \left[\frac{E}{G} - 2\left(\frac{L}{W}\right)^2\right]\left(\frac{x}{L}\right)^2 - \left(\frac{E}{G}\right)\frac{x}{L}\right\}. \tag{9}$$

Translation of the downstream end of the span is shown in Figure 3(b). Three boundary conditions required for solution are (1) $y_o = 0$, (2) $y'_L = 0$, and (3) $y'_{0+} = -(EI/tWG)y'''_{0+}$, where y'''_{0+} is known to be negative. The fourth boundary condition is dictated by the requirement that the overall downstream curvature (or moment) be zero; that is, that the downstream curvature in translation must be the negative of the curvature in rotation. The second boundary condition, that the downstream slope is zero, is dictated by the fact that the overall slope is equal to θ_r, and a slope of θ_r was assumed for the rotational component. For the translational component, an angle at the downstream end is caused by the shearing stress, but this angle is cancelled by the end moment. This end moment (or curvature) is a dependent variable which is dictated by the independent boundary conditions of the rotational component.

The equation for the elastic curve of the translational component of the steady-state response to a misaligned downstream roller, with the above assumptions of sharing of boundary conditions between rotation and translation, is

$$\frac{y}{L} = \theta_r \frac{4(L/W^2) + E/G}{\left[2(L/W)^2 + E/G\right]\left[6(L/W)^2 + E/G\right]}\left[-4\left(\frac{L}{W}\right)^2\left(\frac{x}{L}\right)^3 + \left(6\left(\frac{L}{W}\right)^2 - \frac{E}{G}\right)\left(\frac{x}{L}\right)^2 + 2\left(\frac{E}{G}\right)\frac{x}{L}\right] \tag{10}$$

Addition of equations (9) and (10) results in equation (3), demonstrating success of superposition for this example of static behavior.

Superposition Applied to a Permanently Misaligned Upstream Roller: This article is included to demonstrate the method for applying the upstream input angle for the simple static case, which has a previously derived solution for verification, equation (8). The composite elastic curve is the sum of the result $\theta_r x$ of tilting the x-y axes through the angle θ_r, the result of rotation of the downstream end as expressed by the negative of equation (9), and the result of translation of the downstream end as expressed by the negative of equation (10). Figure 4(a) shows the reason that the downstream angle of the web which is tilted at the angle θ_r is negative, and Figure 4(b) similarly shows the negative translation necessary for obtaining the composite result of Figure 2.

Superposition of Dynamic Behavior: The dynamic reaction at the downstream roller was determined in the Shelton thesis [1] by considering the state of the downstream end at any instant to be a combination of translation and rotation. The lateral velocity is expressed as a function of roller angle, angle of the downstream end of the web, longitudinal velocity of the web, and lateral velocity of the roller in thesis equation (4.1.2). The lateral acceleration is expressed as a function of curvature of the downstream end of the web, lateral acceleration of the roller, longitudinal velocity of the web, and the rate of change of the downstream angle of shear (if shear deflection is significant because of a low L/W ratio and/or a large ratio of E/G) in equation (4.1.6))

Because Shelton thesis equation (4.1.5), which is applicable instead of (4.1.6) if shear deflection is negligible, is the derivative of equation (4.1.2) except for the absence of the term $\partial^2 y/\partial x \partial t$, some analysts have "discovered" this term; however, equation (4.1.5) was determined by tests shown in Figures 4.7.4, 4.7.7, and 4.7.8, in which correlation is poor if the subject term is included. Further, an intuitive discussion of the absence of the term is on page 104 of the thesis.

The dynamic translational component at the downstream roller has the four boundary conditions: (1) $y_o = 0$, (2) y_L given as the independent variable, (3) $y'_L = 0$, and (4) $y'_{o+} = (EI/tWG)y'''$. For a nonzero value of y_o, the solution from the above boundary conditions applies to the translation $y_L - y_o$, as shown in Figure 5A(a).

Although an angle at the downstream end is caused by the shear stress, this angle will be included in the rotational component, as in the previous solutions for static behavior.

The elastic curve for the translational component is:

$$y = \frac{y_L}{2(L/W)^2 + E/G}\left\{-4(L/W)^2\left(\frac{x}{L}\right)^3 + \left(6\left(\frac{L}{W}\right)^2 - \frac{E}{G}\right)\left(\frac{x}{L}\right)^2 + 2\frac{E}{G}\frac{x}{L}\right\}. \tag{11}$$

Equations required for the translational component of the dynamic response are:

$$y''_L = -2\frac{y_L}{L^2}\frac{6(L/W)^2 + E/G}{2(L/W)^2 + E/G} \tag{12}$$

and

$$\theta_{LS} = \frac{y_L}{L}\frac{2E/G}{2(L/W)^2 + E/G}. \tag{13}$$

Equation (13) may be derived from the equation for the angle of shear, $\theta_s = N/(tWG)$, or $\theta_{LS} = -EIy'''_L/(tWG)$.

The dynamic rotational component at the downstream roller has the following boundary conditions if the upstream angle θ_{o-} is zero: (1) $y_o = 0$, (2) $y_L = 0$, (3) $y'_L = \theta_L$ given as the independent variable, and (4) $y'_{o+} = -(EI/tWG)y'''$. The solution for a nonzero value of θ_{o-} will then be determined by rotation of the coordinate axes, as shown in Figure 5. The solution of this problem is based on equation (9) and its derivatives if θ_r for the fixed roller angle is replaced with θ_L for the dynamic downstream angle. With the change in notation for the downstream angle, the equation for the rotational component of the downstream curvature for dynamic analysis is

$$y''_L = \frac{2\theta_L/L}{2(L/W)^2 + E/G}\left[4\left(L/W^2 + E/G\right)\right]. \tag{14}$$

The downstream angle caused by shear is also required:

$$\theta_{LS} = -\frac{\theta_L\, E/G}{2(L/W)^2 + E/G}. \tag{15}$$

Figure 5 shows that the effective translation with nonzero values of θ_{o-} and y_o is $(y_L - y_o - L\theta_{o-})$ and the effective rotation is $(\theta_L - \theta_{o-})$. Details of substituting these superposed inputs and outputs into the superposed translational and rotational components of curvature and angle of shearing are omitted, but the steps are similar to those in the Shelton thesis [1]. The results for the downstream lateral position are:

$$\frac{Y_L(s)}{Y_o(s)} = \frac{\dfrac{E/G}{6(L/W)^2 + E/G}\tau s + 1}{\dfrac{(L/W)^2 + E/G}{6(L/W)^2 + E/G}\tau^2 s^2 + 2\dfrac{2(L/W)^2 + E/G}{6(L/W)^2 + E/G}\tau s + 1}. \tag{16}$$

Similarly, if y_o is assumed to be zero, the lateral response at the downstream roller to an angular disturbance at the upstream roller can be obtained:

$$\frac{Y_L(s)}{L\theta_{o-}(s)} = \frac{\dfrac{E/G}{12(L/W)^2 + 2E/G}\tau s + \dfrac{2(L/W)^2}{6(L/W)^2 + E/G}}{\dfrac{(L/W)^2 + E/G}{6(L/W)^2 + E/G}\tau^2 s^2 + 2\dfrac{2(L/W)^2 + E/G}{6(L/W)^2 + E/G}\tau s + 1}. \tag{17}$$

The transfer functions for the output $\theta_L(s)$ can be obtained from the Laplace transformation of equation (4.1.2) from the Shelton thesis (with θ_r and dz/dt equal to zero), and subsequent division by $\theta_{o-}(s)$ or $Y_o(s)$:

$$\frac{\theta_L(s)}{\theta_{o-}(s)} = -\frac{L}{V} s \frac{Y_L(s)}{L\theta_{o-}(s)} \tag{18}$$

and

$$\frac{L\theta_L(s)}{Y_o(s)} = -\frac{L}{V} s \frac{Y_L(s)}{Y_o(s)}. \tag{19}$$

From equations (16) and (19):

$$\frac{L\theta_L(s)}{Y_o(s)} = -\frac{\dfrac{E/G}{6(L/W)^2 + E/G}\tau^2 s^2 + \tau s}{\dfrac{(L/W)^2 + E/G}{6(L/W)^2 + E/G}\tau^2 s^2 + 2\dfrac{2(L/W)^2 + E/G}{6(L/W)^2 + E/G}\tau s + 1}, \tag{20}$$

and from equations (17) and (18):

$$\frac{\theta_L(s)}{\theta_{o-}(s)} = -\frac{\dfrac{E/G}{12(L/W)^2+2E/G}\tau^2 s^2 + \dfrac{2(L/W)^2}{6(L/W)^2+E/G}\tau s}{\dfrac{(L/W)^2+E/G}{6(L/W)^2+E/G}\tau^2 s^2 + 2\dfrac{2(L/W)^2+E/G}{6(L/W)^2+E/G}\tau s + 1}. \tag{21}$$

Equations (16), (17), (20), and (21) are plotted in Figure (6) for isotropic webs (Poisson's ratio assumed to be 0.35, or E/G equal to 2.7).

"Weave Regeneration" and Precise Guiding: The premise of the Sievers thesis [2] was that a web in a process line is continually forced laterally into periodic web weave, and that this lateral error will reappear at the same frequency following a web guide, with the amplitude increasing in successive spans. The four experiments reported in the Sievers thesis were run at two web velocities and two wavelengths, with small errors at low frequencies, as shown in Table 1(A).

Interest in extreme accuracy of guiding (in the range of 0.1 mm or better) for the rare cases in which an edge or printed line is more precise than this desired accuracy was an incentive for this study of interconnected spans with two inputs (y_o and θ_{o-}) and two outputs (y_L and θ_L). The complexity of the theory which applies to a general span length when deflections caused by shear must be considered (Shelton [1] and Sievers [2]), along with the fact that ratios of L/W less than 2.0 are very common, were motivations for this simplified theory of the behavior of short spans.

The basic cause of weave regeneration is that a sensor for a web guide, whether it is sensing an edge or a printed line, measures lateral position but not the angle of the web; therefore, the inherent transient angular attitude of a web on a roller is not corrected by the guide. Measurement and control of this angle of the web is not judged to be practical because of noise in the measurement of the minuscule slope.

Application of equations (16), (17), (20), and (21) requires more modeling of a multi-span system. A drawing of the configuration of the test system from the Sievers thesis is shown in Figure 7. For the web width of the reported tests, the maximum ratio of L/W is 1.371. The outputs of only the sensors at rollers R2, R3, and R4 were reported by Sievers, as shown in Table 1. Figure 8 is a block diagram for frequency-response analysis of this four-span system. The rotational components of the lateral behavior are above the dashed line and cannot be detected by the sensor, but these components influence the translational behavior at the next roller.

In Figure 7, pure translation is assumed at R_0, although the mechanism for generation of the disturbance was not reported by Sievers. [Note: Shelton [1] achieved an input of pure translation at the upstream end of the test span, and allowed measurement of this input with a wide-band transmitted-light sensor, by mechanically shifting the idler after the guide in synchronization with the downstream roller of the displacement guide.] Pure translation at one roller, however, results in rotation as well as translation at the next roller, with rotation quantified by equation (20) and translation by equation (16). The next roller then has upstream inputs of translation and rotation, the effects of which can be separately added after conversion from the usual polar coordinates of frequency-response analysis to rectangular coordinates. At R2 of Figure 7, the web is at the angle θ_{BL}, if it does not slip on the roller. This angle, undetected by the sensor and therefore unmodified by the guide, becomes an input to the next span. The translational disturbance, however, is detected by the sensor and is corrected within the frequency-response capability of the guide.

At low frequency, the output error of an ordinary type 1 web guide leads the input disturbance (distinguished from the guide-point reference input) by approximately 90 degrees, perhaps surprisingly, but easily proven by classical frequency response analysis. In Figure 8, this 90 degrees of phase lead is seen by the guide sensor on roller R_2, but approximately 90 degrees of phase lag of the rotational component relative to the condition at the upstream end of this guide span (Figure 6) cannot be detected by the sensor, and becomes a disturbance to the next span. This disturbance results in a lateral error at roller R_3, approximately 180 degrees out of phase with the error at R_2, as shown in the test results of Sievers [2]. These test results show the error at R_3 to be greater than at R_2 (weave regeneration), corresponding to the above theory. However, the further increase at R_4 is not explained by theory, but the highly distorted waveforms at R_2, apparently containing a strong third harmonic of unknown origin (input disturbance or web guide), is suspected as a major modifier of the results at R_3 and R_4. Results of modeling by Sievers [2] are tainted by her modeling of the control system as type 2 (force proportional to error acting on a mass), whereas the actual system was almost certainly type 1 (velocity proportional to error).

The usual methods of improving the performance of a guide, increasing the break frequency of the open loop and increasing the gain, do not generally reduce the weave regeneration. In fact, such "improvement" may worsen the performance, because the residual error of the guide partially cancels the rotational disturbance if the two are 180 degrees out of phase.

Figure 6 shows that, at low frequencies, the lateral response to a rotational disturbance ($Y_L/L\theta_{o-}$) usually decreases with decreasing values of L/W. Even if $Y_L/L\theta_{o-}$ were not affected by L/W, decreasing L decreases the response Y_L proportionally. Furthermore, the rotational response to a rotational input (θ_L/θ_{o-}) also decreases with decreasing L/W in most conditions of low-frequency operation. These observations lead to the conclusion that the exiting span of the guide should be reduced in length. This reduction probably should be accompanied by an increase in the span across the guide by a factor of two to four to reduce the distortion of the shortened exiting span.

Based on the test results of Table 1, the Sievers guide did not have high performance. Rough duplication of the reported accuracy was achieved with a model of a type 1 system with a first-order time constant of 1/15 second and an open-loop gain of 15 mm per second per mm of error. Table 1 shows (A) the observed amplitudes in the Sievers thesis, (B) calculations for the above model and (C) calculations for a 6.7-fold improvement of frequency response (mechanical time constant of 0.01 second) and a 6.7-fold increase in gain (100 mm per second per mm of error). The latter change worsened the weave regeneration (R_3 and R_4) for all four sets of conditions, because of the reduced residual error for cancellation of the rotational disturbance of opposite polarity, as discussed two paragraphs preceding this paragraph.

Table 1(D) shows major reductions in weave regeneration by reducing the exiting span of the guide to one-fourth of its original length, along with a doubling of the guide span for reduction of the angle of the guide. The errors at rollers R_3 and R_4 were reduced to less than 0.3 percent of the input error of 4.2 mm as in the Sievers thesis. Also shown in Table 1(E) are the results of a further reduction of the exiting span of the guide to one-eighth of its original length, or to 127 mm. The errors at R_3 and R_4 are all larger than those for the first reduction to one-fourth of the length of the exiting span, but are all less than 0.4 percent of the input error of 4.2 mm.

"Weave Regeneration" Over a Wide Range of Frequencies: The conditions of the four experiments reported in the Sievers thesis and repeated in Table 1 represent only three values of frequency, and only two values of $\omega\tau$ for each span; therefore, the benefit of shortening of the exiting span of the guide as shown in Table 1 must be questioned when the system is subjected to disturbances over a wide range of frequencies. The following study is for a system similar to that of the Sievers thesis (five rollers, four spans), with the web assumed to be isotropic (Poisson's ratio assumed to be 0.35, compared to 0.30 assumed by Sievers).

Figures 9 and 10 used the model of Figure 8, as used for the calculations of Table 1. The web guide dynamics are assumed to be near the upper limit of commercial guides, $\tau_m = 0.01$ second and $K_o = 100$ (mm/sec)/mm error, as in Table 1, conditions (C), (D), and (E).

The multiplicity of independent variables required for implementation of the model of Figure 8, arising from the machine configuration (lengths of all spans), web properties (E/G and width), the characteristics of the web guide, and operating conditions (longitudinal velocity and frequency of the disturbance) is a deterrent to finding simple rules (equations, graphs, programs, etc.) for minimizing weave regeneration. However, the results shown in Figures 9 and 10 are expected to contribute to the goal of generalization.

Figures 9 and 10 use ω, the frequency of the disturbance, as the variable of the x axis; however, the time constant of all spans except the shortened exiting span of the guide is one second. The x axis can therefore be considered to be $\omega\tau$ for these spans which have a time constant of one second. As long as $\omega\tau$ of all web spans is greater than $\omega\tau_m$ of the web guide by a large factor (but not necessarily as large as the factor of 100 of this example), the dynamics of the web guide can be neglected in a study of the web behavior. This simplification is suggested by the wide separation in Figures 9 and 10 of the peak amplitude caused by web dynamics at a value of ω just greater than unity and the small peak caused by the control system near a value of ω of 100.

In Figure 9 for all spans equal to the width, the peak regeneration from less than 1.0 percent at the downstream guide roller to greater than 18 percent at the next roller was a surprise, and is contrary to the expectation that weave regeneration would prove to be a microscopic phenomenon. In Figure 10 for all spans equal to twice the width, the regeneration is even greater, more than 25 percent at its peak. The explanation for the large regeneration of Figures 9 and 10 probably lies in the assumption of no slippage, whereas in practice the angular attitude of the web on each roller may not be sustained because of low friction, in which case the analysis presented in these figures represents the worst possible regeneration, which is valuable knowledge in light of the great variability of friction in a given process line. However, nipped rollers, vacuum rollers, and other specialized components may lock the web in its angular attitude, making this model more realistic.

Figures 9 and 10 show a great reduction in regenerated error over the entire range of frequencies by means of a reduction of the exiting span of the guide to one-fourth of the web width. This span length was not determined to be optimum, but was intuitively chosen from experience with the model of the Sievers configuration. The maximum regenerated error in Figure 9 is slightly greater than 3 percent of the disturbance, and 1.2 percent in Figure 10.

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[4] Shelton, J.J., "Buckling of Webs from Lateral Compressive Forces", Proceedings of the Second International Conference on Web Handling, Oklahoma State University, Stillwater, Okla., June 6-9, 1993.

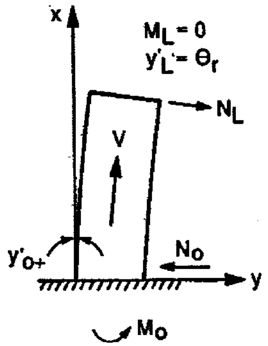


Figure 1. Misaligned Downstream Roller—Short-Span Model.

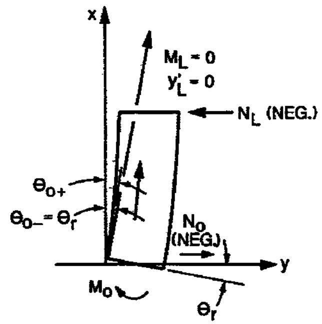
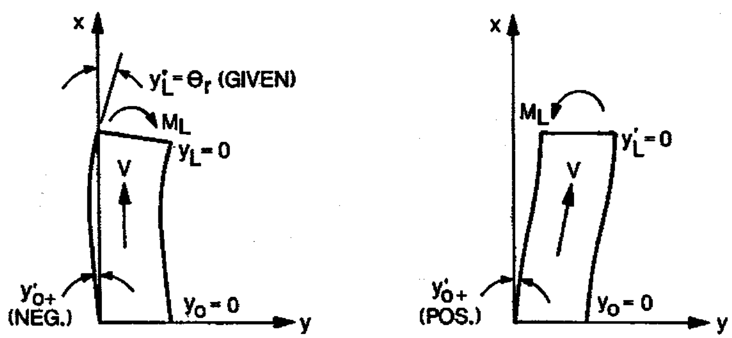
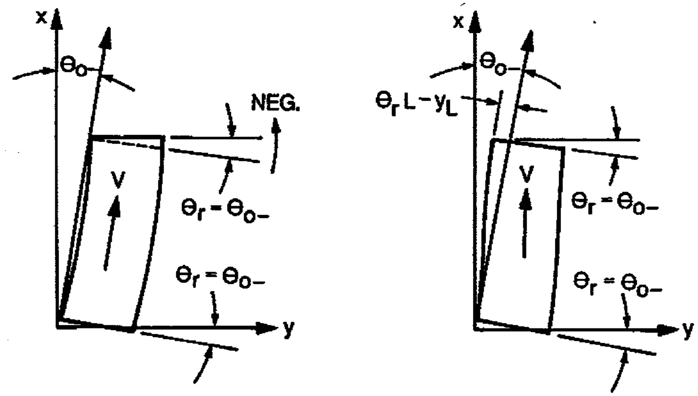


Figure 2. Misaligned Upstream Roller—Short-Span Model.



(a) ROTATION (b) TRANSLATION

Figure 3. Superposition of Static Behavior—Misaligned Downstream Roller.



(a) ROTATION (b) TRANSLATION

Figure 4. Superposition of Static Behavior—Misaligned Upstream Roller.

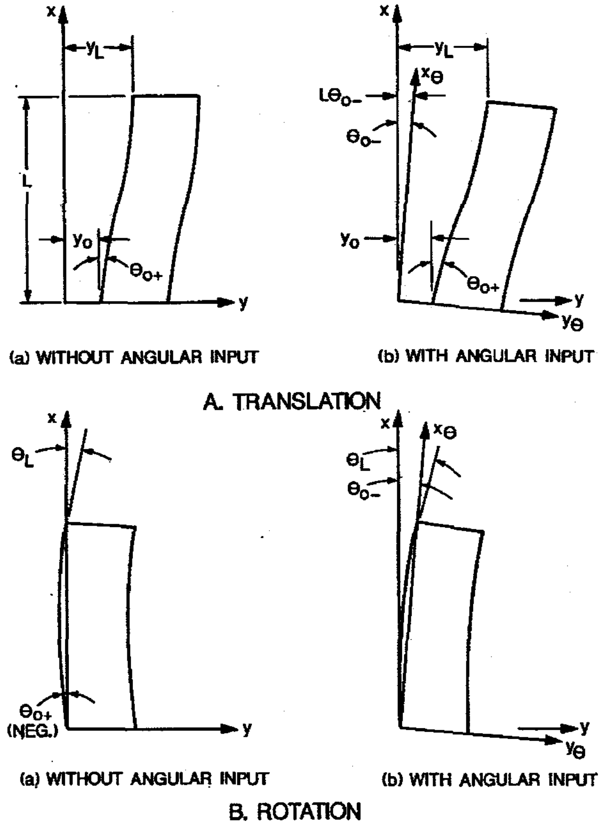


Figure 5. Superposition for Analysis of Dynamics.

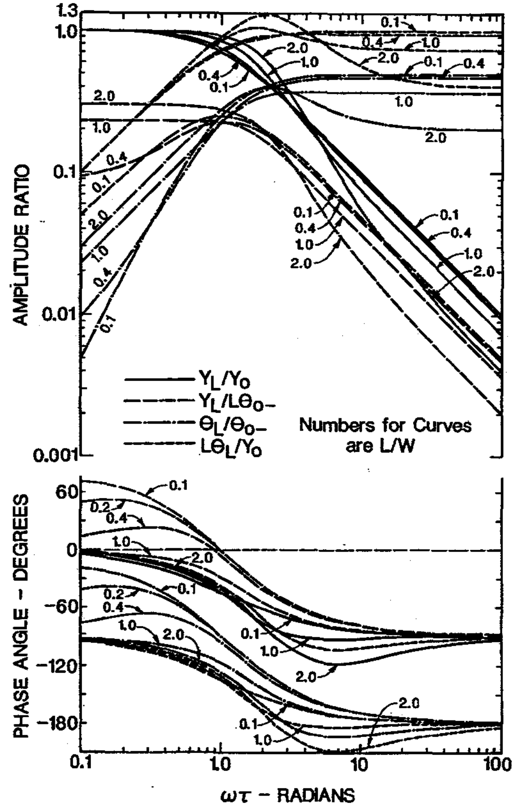


Figure 6. Frequency Response of Short Spans of Isotropic Webs (E/G = 2.7).

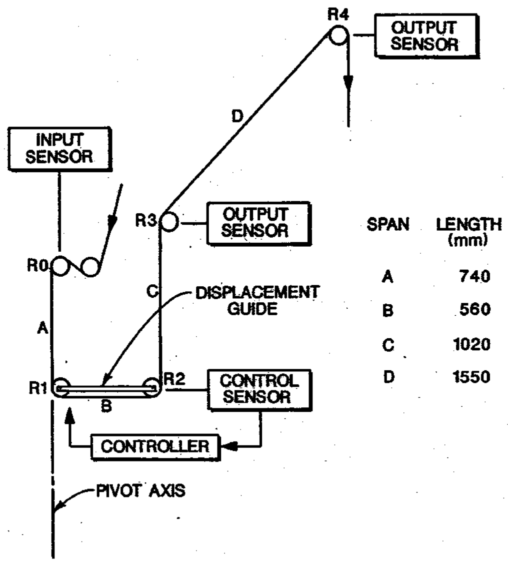


Figure 7. Five-Roller System from Thesis By Lisa Sievers [2].

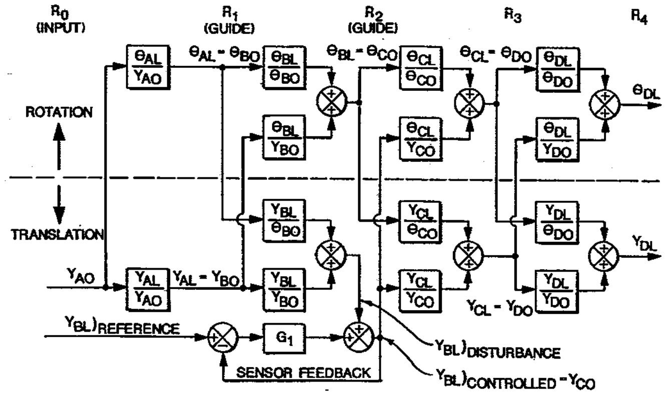


Figure 8. Block Diagram for Frequency-Response Analysis of Five-Roller System.

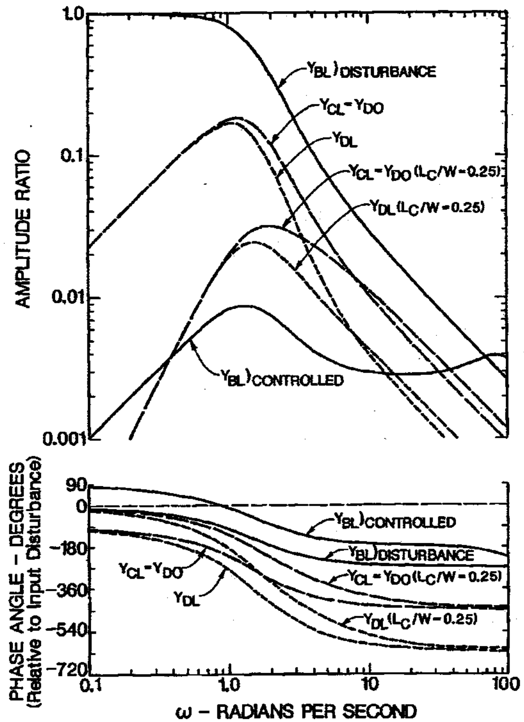


Figure 9. Frequency Response of Four-Span System. $\tau_m = 0.01$ sec, $K_o = 100$, E/G = 2.7, $\tau_A = 1.0$ sec, $L_A = 1.27$ m, L/W = 1.0 except as noted

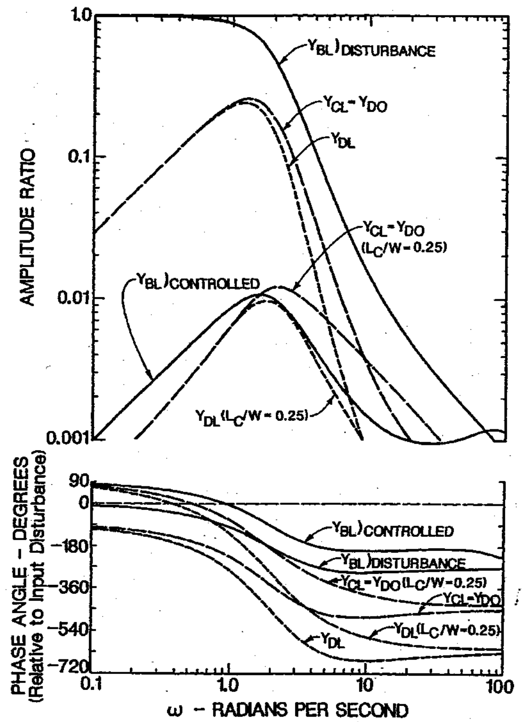


Figure 10. Frequency Response of Four-Span System. $\tau_m = 0.01$ sec, $K_o = 100$, E/G = 2.7, $\tau_A = 1.0$ sec, $L_A = 2.54$ m, L/W = 2.0 except as noted

(A) Sievers' Experiments: Note: See configuration and span lengths in Figure 7

Experiment No.		**1**	**2**	**3**	**4**
Velocity – m/Sec.		1.0	1.0	2.0	2.0
Weave Freq. – Rad/Sec		0.2094	0.1047	0.4189	0.2094
Wavelength – m		30.5	61.0	30.5	61.0
Observed Amplitudes – mm:	R_2	0.08	0.03-0.10	0.05-0.08	0.08-0.10
	R_3	0.15	0.13	0.15	0.15
	R_4	0.20-0.28	0.25	0.22-0.28	0.25-0.30

(B) Calculated Amplitude Ratio and Phase for Above Conditions, with τ_m = 1/15 Second and K_o = 15 (mm/sec)/mm Error:

R_1	0.99 /-13.9°	1.00 /-6.9°	0.99 /-13.9°	1.00 /-6.9°
R_2	0.014 /+76.1°	0.007 /+83.1°	0.028 /+76.1°	0.014 /+83.1°
R_3	0.032 /-105.6°	0.016 /-97.7°	0.018 /-99.7°	0.009 /-94.6°
R_4	0.032 /-123.9°	0.016 /-106.9°	0.018 /-118.1°	0.009 /-103.8°

(C) Calculated Amplitude Ratio and Phase for Above Conditions, with τ_m = 0.01 Second and K_o = 100 (mm/sec)/mm Error:

R_2	0.002 /+76.1°	0.001 /+83.1°	0.004 /+76.1°	0.002 /+83.1°
R_3	0.044 /-107.7°	0.022 /-98.8°	0.041 /-107.4°	0.021 /-98.6°
R_4	0.044 /-126.0°	0.022 /-107.9°	0.041 /-125.7°	0.021 /-107.8°

(D) Calculated Amplitude Ratio and Phase at Conditions of (C), Except Span B Doubled and Span C Reduced to One-Fourth

R_2	0.002 /+69.5°	0.001 /+79.8°	0.004 /+69.5°	0.002 /+79.8°
R_3	0.001 /-12.5°	0.0003 /+12.8°	0.003 /+40.4°	0.001 /+64.7°
R_4	0.001 /-30.9°	0.0003 /+3.6°	0.003 /+22.0°	0.001 /+55.5°

(E) Same as (D) Except Span C Reduced to 1/8 of Original:

R_2	0.002 /+69.5°	0.001 /+79.8°	0.004 /+69.5°	0.002 /+79.8°
R_3	0.002 /+58.0°	0.001 /+74.0°	0.004 /+63.3°	0.002 /+76.6°
R_4	0.002 /+39.6°	0.001 /+64.8°	0.004 /+44.9°	0.002 /+67.5°

Table 1. Experiments in Thesis by Lisa Sievers, and Improvements by Changing Lengths of Spans in Short-Span Model
Web Width = 1130 mm Input Weave Amplitude 4.19 mm Poisson's Ratio = 0.3

Name & Affiliation	Question
D. Pfeiffer – JDP Innovations	Dr. Shelton, do you really think that the stiffness of the web would be $EtW^3/12$ if the web refuses to maintain a planar shape in shear? Because if the web goes out of plane as your demonstration model shows, it probably goes to a minimum energy state.
Name & Affiliation	**Answer**
J. Shelton – OSU	There are many analyses which neglect Poisson's ratio and its effect on width. A web guide usually measures only one edge, so that the location of the other edge and the center are affected by Poisson's ratio and by corrugations. You can have corrugations that appear large but haven't decreased the width very much, and therefore have not greatly decreased the bending stiffness.
Name & Affiliation	**Additional Comment**
K. Good – OSU	Following on David's question. We've run beam bending experiments before at 3M as part of our wrinkling work, as reported in the multi-span paper that I wrote for the 1997 IWEB. But even with troughs in the web, John's equation for bending stiffness is still pretty good. It does explain the deflection behavior. You see the ripples, but the stress variation across the width is not that much affected. I think his bending stiffness equation is good over quite a large range even when the web has become nonplanar.
Name & Affiliation	**Additional Comment**
J. Shelton – OSU	Evidently, I misinterpreted David's question to be about the reduction in width in the stress condition, not the failure of the bending stiffness expression for a beam when applied to a web which is tensioned across its entire width. Evidence from analyses and testing of the post-buckling behavior of thin plates in framed aircraft structures (Timoshenko [3], page 434) points to continued structural integrity after buckling. For a web span, the structural integrity as a beam is provided primarily by the tension and secondarily by the supporting rollers at the two ends of the span. Corrugations are the result of lateral compressive stresses caused by shear stresses, as explained by Mohr's circle.