THE ROLE OF ACTIVE DANCERS IN TENSION CONTROL OF WEBS

by

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ABSTRACT

This paper investigates the role of active dancers in attenuation of web tension disturbances in a web process line. A general structure of the active dancer is considered; an input/output model is developed for analysis and controller design. Three types of control designs were investigated for the active dancer: proportional-integral-derivative controller, internal model based controller, and linear quadratic optimal controller. An open-architecture experimental platform is developed for conducting real-time control experiments using the active dancer system. Data collected from an extensive set of experiments using the three control designs validate the usefulness of the active dancers in a web process line. We present a representative sample of the experimental data in this paper.

INTRODUCTION

With the need for increased performance and productivity in the web processing industry, accurate modeling and effective controller design for web handling systems are essential for increasing the web processing speed and the quality of the processed web. Accurate tension control has always been a key element of web handling systems. An important objective of the tension control system is to maintain tension within the desired limits under a wide range of dynamic conditions such as speed changes, variations in roll sizes, and web property. Tension variations affect printing quality and tend to cause web breakage and wrinkles.

A dancer mechanism is used as a feedback element in a number of tension control systems. The tension control system is driven by the variations in the position of the dancer mechanism as opposed to the variations in actual tension from the desired tension. The requirement to maintain the desired tension within a narrow range from the unwind
zone to the first printing unit places a demand on better design of the dancer mechanism. Periodic tension disturbances arising from uneven wound rolls and misalignment of the idle rolls have to be attenuated by the dancer mechanism in order to minimize their propagation into the process section.

Currently, passive dancers are widely used as dancer mechanisms. Passive dancers have known to act as good tension feedback elements for low speed web lines; they have been known to have limitations in dealing with a wide range of dynamic conditions. In the case of inertia compensated passive dancers, the resonant frequency of the dancer roller is mainly determined by its mass. Thus to increase the tension disturbance frequency range that can be attenuated, the dancer roll mass must be reduced. However, the weight of the dancer roll is twice the reference web tension, which limits any changes to the dancer roll mass to increase the resonant frequency. It is expected that by introducing an active element into a dancer mechanism gives a control engineer more flexibility in attenuating periodic tension disturbances of wide range of frequencies and also to maintain lower tension fluctuations. The focus of this paper is on modeling, control design and experimental investigation of an active dancer mechanism for periodic tension disturbance attenuation.

**Previous Studies**

Early development of mathematical models for longitudinal dynamics of a web can be found in [9, 2, 3, 5, 1]. In [9], extensive theoretical and experimental analysis on belt drive fundamentals were reported; it studied effects such as centrifugal force, angle of contact, fixed versus floating shaft on tension in the belt. Although the mass of the belt is considerable when compared to the web during transport, similarities exist between belt drive systems and web handling systems. Early work describing the longitudinal dynamics of a web can be found in the book by Campbell [2]. Campbell developed a mathematical model for longitudinal dynamics of a web span between two pairs of pinch rolls, which are driven by two motors; the model is based on Hooke's law, i.e., the variation in web tension in the span is proportional to the positional change of the pinch rolls. Campbell's model does not predict tension transfer and does not consider tension in the entering span. A modified model that considers tension in the entering span was developed in [5].

In [1], the moving web was considered as a moving continuum and the general methods of continuum mechanics such as the conservation of mass, conservation of momentum, and conservation of energy were used in the development of a mathematical model. This comprehensive study by Brandenburg [1] included the steady state and transient behavior of tensile force, stress, strain and register error as functions of variables such as wrap angle, position and speed of the driven rollers, density, cross-sectional area, modulus of elasticity and temperature. Tension variations in pliable material due to friction between the web and guide rollers was considered in [20].

In [15], equations describing tension dynamics are derived based on the fundamentals of the web behavior and the dynamics of drives used for web transport. A simple example system was considered to compare torque control versus velocity control of a roll for the regulation of tension in a web. Tension control algorithm using optimal output feedback technique in a multi-span web transport system was reported in [22]. A decentralized observer was developed to estimate the forces due to web tension on the driven rollers;
the observer leads to improved speed response of the driven rollers in multi-span web transport systems.

Steady state and transient behavior of a moving web was reported in [18]. Non-ideal effects such as temperature and moisture change on web tension were studied. Based on the models developed, methods for distributed control of tension in multi-span web transport systems were studied. Analysis of a multi-span web system with a passive dancer for minimizing disturbances due to eccentric unwind roll was also given.

A study on dynamic behavior of dancers in web transport systems was reported in [12]. Computer simulation studies were conducted on an example system to investigate disturbance rejection for three cases: (1) without a dancer; (2) with a classical dancer with passive elements; and (3) with an inertia compensated dancer. Simulation results show attenuation of tension disturbances in the case of both a classical dancer and an inertia compensated dancer. Control of tension during start-up/shut-down in a multi-span web transport system was considered in [13]. Since start-up or shut-down conditions involve large variations in roller velocities, nonlinear models were considered in the simulation study. Simulation results of a PID tension controller were reported. It is shown in the simulation study that a controller designed for one start-up condition when used at a substantially different operating condition could result in web breakage.

An overview of lateral and longitudinal dynamic behavior and control of moving webs was presented in [23]. A review of the problems in tension control of webs was given in [21]. A comprehensive study on tension regulation of a web was reported in [16]. Discussions on tension control versus strain control and torque control versus velocity control were given. An analysis on modeling and design of a tension control station with both inertia compensated dancers and classical passive dancers was also given. Practical recommendations for modular design of tension control stations were given.

An active dancer system for reducing the variation of tension in wire and sheet materials was proposed in [7]. This is one of the early comprehensive work on active dancers that was reported in literature. Construction of an active dancer system with a D.C. motor was discussed. A mathematical model for an active dancer system within a web transport system was derived. An output feedback controller was designed for the active dancer system for tension regulation. Experimental results were reported based on an apparatus with an active dancer system. The main drawback of the apparatus is that the results can be obtained only for a stationary web. Construction of an active dancer system that is capable of rejecting periodic cyclic process induced tension disturbances was reported in [11].
Nomenclature

\( A \) Cross-sectional area of web
\( B_f \) Bearing friction
\( E \) Modulus of elasticity
\( J \) Polar moment of inertia of roller
\( L_1, L_2 \) Length of web span
\( R \) Radius of a roller
\( T_0, T_1, T_2, T_3 \) Change in tension from reference
\( U \) Dancer translational velocity input
\( V_0, V_1, V_2, V_3 \) Change in web velocity from reference
\( X_1 \) Change in linear displacement of dancer from reference
\( v_r \) Reference web velocity
\( i_r \) Reference web tension
\( \tau_1, \tau_2 \) Time constant of a web span \((L_1/v_r, L_2/v_r)\)
\( K_1, K_2 \) Web span spring constant \((EA/L_1, EA/L_2)\)
\( K_p \) Proportional control gain
\( \alpha, \beta, \gamma \) System constants \((EA/v_r, J/R^2, B_f/R^2)\)
\( \eta \) A constant \((B/\alpha)\)

MODELING OF AN ACTIVE DANCER SYSTEM

A typical active dancer system shown in Fig. 1, is considered for deriving the input/output and state space models. This system contains web spans adjacent to the dancer roller in upstream and downstream directions and three rollers including the dancer roller. All the variables shown in Fig. 1 represent variations from their reference values. It is assumed that \( T_0 \) is the upstream tension disturbance that needs to be rejected using the active dancer. The linearized dynamics of the active dancer system shown in Fig. 1 is

Figure 1: Active dancer system
given by the following equations.

\[ \begin{align*}
\beta V_0 &= -\gamma V_0 + (T_1 - T_0) \\
\tau_1 T_1 &= -T_1 + T_0 + \alpha(V_1 - V_0) + \frac{\alpha}{\tau_1} X_1 + \alpha U \\
\beta V_1 &= -\gamma V_1 + (T_2 - T_1) \\
\tau_2 T_2 &= -T_2 + T_1 + \alpha(V_2 - V_1) + \alpha \left( \frac{1}{\tau_2} - \frac{1}{\tau_1} \right) X_1 + \alpha U \\
\beta V_2 &= -\gamma V_2 + (T_3 - T_2) \\
X_1 &= U
\end{align*} \]

where \( \beta = J/R^2 \), \( \gamma = B_f/R^2 \), \( \alpha = EA/v_r \), \( \tau_1 = L_1/v_r \), and \( \tau_2 = L_2/v_r \). The input-output dynamic model [24] for the active dancer is

\[ T_2(s) = \frac{D_{ad}(s)}{C_{ad}(s)} U(s) + \frac{A_{ad}(s)}{C_{ad}(s)} T_0(s) + \frac{B_{ad}(s)}{C_{ad}(s)} T_3(s) \]  

where the input \( U(s) \) is the dancer velocity, i.e., \( U(s) = sX_1(s) \), and

\[ \begin{align*}
A_{ad}(s) &= (\eta s + 1)^2 \\
B_{ad}(s) &= \alpha(\eta s(\tau_1 s + 1) + 2) \\
C_{ad}(s) &= (\eta s(\tau_1 s + 1) + 2)(\eta s(\tau_2 s + 1) + 2) - (\eta s + 1)) \\
D_{ad}(s) &= \beta \left( \eta s + 1 \right) \left( s + \frac{1}{\tau_1} \right) + \left( \eta s(\tau_1 s + 1) + 2 \right) \left( s + \frac{1}{\tau_2} - \frac{1}{\tau_1} \right)
\end{align*} \]

where \( \eta = J/v_r/EA R^2 \).

The above input/output model has been obtained by assuming that the roller bearing friction is negligible. A full expression with non-zero bearing friction can be found in appendix B. Also, notice that the model is obtained under the assumption that the moment of inertia and radius of all the rollers in the dancer system are same, i.e., \( J_i = J \) and \( R_i = R \) for \( i = 1, 2, 3 \).

Expansion of the numerator, \( D_{ad}(s) \), and the denominator, \( C_{ad}(s) \), of the plant transfer function gives

\[ \begin{align*}
C_{ad} &= \eta^2 \tau_1 \tau_2 s^3 + \eta^2 (\tau_1 + \tau_2) s^3 + \eta(\eta + 2\tau_1 + 2\tau_2) s^2 + 3\eta s + 3 \\
D_{ad} &= \beta \eta \tau_1 s^3 + \beta \eta \left( 1 + \frac{\tau_1}{\tau_2} \right) s^2 + \beta \left( 3 + \frac{\eta}{\tau_2} \right) s + \beta \left( \frac{2}{\tau_2} - \frac{1}{\tau_1} \right)
\end{align*} \]

Notice that if \( \tau_2 > 2\tau_1 \), i.e., \( L_2 > 2L_1 \), then the constant term of the numerator polynomial, \( D_{ad}(s) \), is negative, which results in a right-half-plane zero.

**Analysis of the Active Dancer Model**

Analysis of the input/output active dancer model given by (7) is conducted by varying the parameters: \( L_1, L_2, E, A, J, R \) and \( v_r \). Since most feedback control algorithms for the dancer input involve some type of proportional action, the analysis of the basic analysis
of the active dancer system is conducted with proportional control. The closed-loop characteristic equation with proportional feedback control, i.e., \( U(s) = -K_p T_2(s) \), is

\[
1 + K_p \frac{D_{ad}(s)}{C_{ad}(s)} = 0,
\]

where \( K_p \) is the proportional gain. To investigate the effect of the four constants, \( \eta, \beta, \tau_1 \) and \( \tau_2 \), on the choice of the proportional gain, the root-locus method is employed.

**Effect of span lengths: \( L_1, L_2 \)**

To study the effect of span lengths, we plot the locus of the closed-loop poles for varying \( K_p \) for various values of \( L_1 \) and \( L_2 \). Other physical parameters of the web and the rollers are kept constant, which are given in the following table.

<table>
<thead>
<tr>
<th>( v_r )</th>
<th>( E )</th>
<th>( A )</th>
<th>( J )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>350 FPM</td>
<td>( 6 \times 10^3 ) PSI</td>
<td>( 1.27 \times 10^{-4} ) in(^2)</td>
<td>96.21 lb in(^2)</td>
<td>2.5 in</td>
</tr>
</tbody>
</table>

**Table 1:** Web and roller parameters for root-locus

Recall from classical control theory (root-locus method) that the closed-loop poles approach the open-loop zeros as \( K_p \) is increased. The following cases are considered in the investigation.

- **\( L_1 > L_2 \)**

Fig. 2 shows the root-locus plot and the location of the open-loop poles and zeros when \( L_1 = 36 \) in and \( L_2 = 9 \) in. Fig. 2 shows that the proportional gain \( K_p \) can be chosen as large as possible. Thus, the disturbance effect on the span downstream to an active dancer can be suppressed to as small a value as possible. Notice that after a certain value any increase in \( K_p \) results in moving a pair of closed-loop poles towards the imaginary axis. Hence, appropriate choice of the gain \( K_p \) must be made such that the closed-loop poles are far away from the imaginary axis.

- **\( L_1 = L_2 \)**

Fig. 3 shows the root-locus plot and the location of the open-loop poles and zeros when \( L_1 = L_2 = 9 \) in. In this case, there is a pair of complex-conjugate open-loop poles very close to a pair of complex-conjugate zeros. Thus, any choice of the gain cannot move this pair of open-loop poles further to the left-half-plane away from the imaginary axis. The control effectiveness is reduced when the span lengths are equal.

- **\( L_1 < L_2/2 \)**

The root-locus plot and the location of open-loop poles and zeros for this case is shown in Fig. 4 where \( L_1 = 9 \) in and \( L_2 = 36 \) in. In this case, the root locus crosses the imaginary axis and enters the right-half plane when \( K_p \) exceeds a certain value. Further, the presence of an open-loop zero in the right-half-plane always results in an unstable closed-loop pole as the gain exceeds a certain value. This is true even when we choose a more advanced controller of the form \( K_p G_c(s) \), with the poles and zeros of \( G_c(s) \) in the left-half-plane. Thus, the tension disturbance can only
be suppressed to a certain extent because large control input can make the closed-loop system unstable. For the example considered, as shown in Fig. 4, a branch of the root locus moves to the right-half plane for a very small value of $K_p$. Thus the closed-loop system becomes unstable under feedback control for a very small control gain.

**Effect of web material and roller properties: $E,A,J,R$**

Each of the constants $E,A,J,R$ affect the constant parameter $\eta = (Jy_v)/(EAR^2)$ in the input/output model. So, varying the value of $\eta$ in the model reflects variations of $E,A,J,R$. We have conducted a number of numerical simulations by varying these constants and noticed that the root-locus plot essentially has the same form. Thus the closed-loop poles of the dancer system are not generally affected as much by the constants $E,A,J,R$ as it is affected by variation in upstream and downstream span lengths.

**Interpretation of the effect of span lengths on tension control**

Assuming that the web is mostly elastic, it is common practice in the web handling community to model a web span as an elastic spring with spring constant $K_n = E_nA_n/L_n$. The spring constants of the upstream and the downstream web spans to the dancer roller are $K_1 = EA/L_1$ and $K_2 = EA/L_2$, respectively.

When $L_1 \geq L_2$, $K_1 \leq K_2$, that is the upstream spring constant is smaller than the downstream spring constant. So, any motion of the dancer roller gives larger tension variation in tension $T_2$ than in $T_1$. Thus, rejection of periodic disturbances from the spans upstream of the dancer into the spans downstream of the dancer is possible in this case.

When $L_1 < L_2/2$, $K_1 < K_2/2$, that is the upstream spring constant is larger than the downstream spring constant. Periodic dancer motion induces larger tension disturbances into the upstream span than it rejects in the downstream span due to feedback of tension $T_2$.

**EXPERIMENTAL PLATFORM**

This section describes the open-architecture experimental web platform developed for conducting experiments in tension control. The platform mainly consists of an endless web line with tension control and lateral control systems as shown in Fig. 5. The term endless web line refers to a web line without unwind and rewind rolls. This type of platform mimics most of the features of a process section of a web processing line.

The experimental platform includes both the lateral control system as well as the tension control system. Mechanical components used in the platform include sixteen rollers, a master speed roller with a nip roller, an electric motor, and a passive dancer system. The main control elements are a Fife remotely pivoted guide and an active dancer mechanism as shown in Fig. 5. A functional sketch of the experimental web platform is shown in Fig. 6. Since the width of each roller is 8 inches, the maximum web width that can be used in the web line is limited to about 6 inches. The diameter of each roller is 5 inches, except for the master speed roller, which has a diameter of 10 inches. A nip roller for the master speed roller is used to reduce slip during start-up. An analog controller for the master speed roller is available to obtain the desired transport velocity of the web.
Similarly, an analog controller for air pressure in passive dancer system is available to obtain desired reference tension in the web line.

The active dancer system consists of an actuator, the dancer roll mounted on it, Fife CSP signal processor and load cells immediately downstream and upstream of the dancer roller. The control signal is generated based on control algorithm computations in the computer and the Fife CSP signal processor is used only as an amplifier. Cleveland-Kidder stationary shaft transducers are mounted on a roller downstream to the dancer roller, which measure the feedback tension. Two such transducers are used to complete the Wheatstone bridge. A pentium 450 MHz computer with a Keithley DAS 1601 digital data acquisition board is used for real-time control experiments. The real-time software is written in VC++ and implements the following functions in a modular way: data acquisition, data storage, and real-time data display and plotting, control algorithm, state observer algorithm and computing the control signal. MATLAB is used to generate hard copies of the plots and also to compute FFT of the signals.

EXPERIMENTAL RESULTS

This section presents a sample of the experimental results collected on the platform. Three types of controllers were implemented: proportional-integral-derivative controller (PID), internal model based controller (IMC), and linear quadratic optimal controller (LQR). Design and discussion of the controllers can be found in [24]. A control sampling period of 5 milli-seconds is used in all the experiments. Periodic tension disturbance upstream of the dancer is created by introducing an uneven roll surface into an idle roller in the web line. The roller with out-of-round roll surface is shown in Fig. 5. A load cell on the roll immediately downstream of the out-of-round idle roller measures the amount of tension disturbance that is being generated. The fundamental frequency of the periodic tension disturbance for a given out-of-round roll surface increases with increase in web speed.

A summary of the amount of tension disturbance magnitude reduction for PID, IMC and LQR controllers for four different web speeds (200 FPM, 250 FPM, 300 FPM, and 350 FPM) is shown in Fig. 7. Experimental results for each controller for the 350 FPM web transport speed are shown in Fig. 8 and Fig. 9. In Fig. 8, the top two plots correspond to the measured periodic tension disturbance and its FFT, and the bottom two plots show the measured tension error and its FFT when the active dancer is under PID control. Notice that the fundamental frequency of disturbance is about 4 Hz. Fig. 9 shows measured tension error and its FFT for the IMC and LQR controllers.

The summary shown in Fig. 7 indicates that all three controllers give good attenuation of the tension disturbance using the active dancer. Notice that at the low speed of 200 FPM, the attenuation level of all three controllers is similar but as the speed is increased the attenuation level is more for the IMC and the LQR controllers. Experimental results indicate that the active dancer does not reject any periodic disturbances above the 10 Hz level; this is due to the bandwidth limitation of the actuator. Further, the actuator saturates above a certain value of the controller gains in each case, which negates any further tension attenuation.

In summary, experimental results show that the active dancer system is highly effective in tension disturbance attenuation. The disturbance rejection capability of the active
dancer system is limited only by the bandwidth limitation of the actuator as opposed to the passive dancer or an inertia compensated dancer which have considerable resonance problems.

CONCLUSIONS

Analysis of the model and experimental results show that considerable periodic tension disturbance attenuation is possible using an active dancer system. Further, analysis of the model reveals a critical structural constraint on the design of the active dancer system; the ratio of the downstream to upstream web span length with respect to the active dancer roller must be less than two for tension attenuation using an active dancer. Data collected from an extensive set of experiments validate the usefulness of the active dancers in a web process line.

ACKNOWLEDGEMENTS

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REFERENCES


Figure 2: Root locus plot for $L_1 > L_2$

Figure 3: Root locus plot for $L_1 = L_2$
Open-loop zeros:
-3.61 + 502i
-3.61 - 502i
0.85

Open-loop poles:
-2.74 + 425i
-2.74 - 425i
-0.45 + 171i
-0.45 - 171i

Figure 4: Root locus plot for $L_1 < L_2/2$

Figure 5: Sketch of the experimental web platform
Figure 6: Functional block of the experimental web platform

Figure 7: Summary of tension disturbance reduction
Figure 8: Tension with out-of-round idle roller (Disturbance and PID control); $v_r = 350$ FPM, $t_r = 36$ lb
Figure 9: Tension with out-of-round idle roller (IMC and LQR); \( v_r = 350 \text{ FPM}, t_r = 36 \text{ lb} \)
The Role of Active Dancers in Tension Control of Webs  

P. R. Pagilla, L. P. Perera, and R. V. Dwivedula – Oklahoma State University, USA

<table>
<thead>
<tr>
<th>Name &amp; Affiliation</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>J. Brown – Essex Systems</td>
<td>It is not clear to me how the active dancer it is constructed. I see the spring and the damper and this box with an arrow in it in your figure. Could you describe this better?</td>
</tr>
<tr>
<td>P. Pagilla – OSU</td>
<td>The active dancer roller sits on a linear guide block, whose translation is actuated by a motor. We do not have any passive elements in the actual construction of the active dancer. The dancer roller translational velocity is controlled by a DC motor, which gets feedback from the load cell mounted on the fixed roller that is downstream of the dancer roller. The sketch shown in the figure is generic and it includes passive elements also.</td>
</tr>
<tr>
<td>D. Pfeiffer – JDP Innovations</td>
<td>There were discussions at IWEB5 concerning the benefit of having the inertia all near the shell of the dancer roll. In this case, your formulas didn't show any difference between the radius of gyration of inertia and the outer radius of the roll. Do you see any advantage moving the inertia to the outside of the roll?</td>
</tr>
<tr>
<td>P. Pagilla – OSU</td>
<td>External energy is supplied to the active dancer roller by the DC motor that provides the translational movement. The roll mass and the size are determined by the type of actuator. The construction of the active dancer roller need not be similar to that of the passive dancer. In general, the active dancer roller should have low mass so that one can choose a smaller actuator.</td>
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