MODELING AND H_∞ ROBUST CONTROL FOR WINDING SYSTEMS

by

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ABSTRACT

This paper presents the modeling and the H_∞ robust control for an elastic web transport system including an unwinder, a winder and a traction motor. This model has been created using laws of Physics concerning elasticity of the web, friction between web and roll, and web speed between two rollers. In this paper, a novel linearization of this model allowed us to predict the relationship between web tension and speed more precisely than previous simplifications.

During the winding process the radius and inertia of the rollers change substantially. In order to eliminate system sensitivity to these variations, gain scheduling control was used. In addition, multivariable H_∞ control in our linear model allowed us to effectively decouple web speed and tension. Multivariable control performance was compared with performance of the standard industry decentralized PID control. Results were validated on our experimental webline.

NOMENCLATURE

E	Young modulus
ε	web strain
J	roll inertia
K	robust controller
L	web length between two rolls
L_0	nominal web length between two rolls
μ	friction coefficient
ν	viscosity modulus
R	roll radius
s	Laplace operator

S	web section
θ	rolling-up angle
T_k	web tension between roll k and (k+1)
T_0	nominal web tension
V_k	web velocity on the roll k
ω	frequency

INTRODUCTION

Web systems handling material such as textile, paper, polymer or metal are very common in industry. The objective is to increase web velocity as much as possible while controlling web tension over the entire webline. Web speed is inherently limited by web processing and treatment occurring between the unwinder and winder. Transport control systems should meet the following requirements:

- decouple web tension and speed so that constant web tension can be maintained during web speed changes.
- be robust with respect to variations in web elasticity. This allows us both to control the web throughout web processing and treatment, and to use the same control for different types of web.
- be robust with respect to variations in roll diameter. The same performance should be maintained throughout web processing and proper webline startup must be assured regardless of roll diameter.

Until recently, most industrial web transport systems have used decentralized PID-type controllers. However, higher control requirements and the use of thinner and thinner web material require more precise control strategies and models. Multivariable controllers have been introduced in metal [12][22] and paper [23][24] transport. These applications have focussed mainly on web fabrication quality-control factors such as web composition and thickness.

We chose a multiple variable technique with a robust control H_∞. This approach, novel for web control, allowed us to decouple web speed and tension by using multivariables, as well as guarantee robustness to variability.

The system studied has three motors (cf. figure 1) and shows the inherent difficulties of web transport systems. The main concern is to prevent web breaks, folding, and damage which lower productivity and stop the production line. Excessive tension or oscillating tension due to velocity variations may cause the entire web to be lost. Therefore, system control should minimize coupling between web velocity and tension. Another critical point is that the control system must be robust to variations in web elasticity modulus due to changes in temperature or moisture. These changes are very common in industrial processing where the web goes through a bath to improve the quality and then through a furnace to dry.

In this paper, the first part presents webline modeling derived from several laws of physical mechanics. We focussed on a novel linearization of this model which allowed us to accurately predict the relationship between web tension and velocity. The second part shows the results of the model identification by parameter optimization. A gain scheduling approach is then introduced to minimize the effect of rollers radius variations during processing. The synthesis of the H_∞ robust controller is shown and its performance is compared with that of PID controllers.

PROCESS MODEL

The model of a web transport system was derived from the model of the web tension between two consecutive rolls and the model of the velocity of each roll.

Web tension calculation

Web transport system modeling is based on three laws :

- Hooke's law which introduces the elasticity of the web.
- Coulomb's law which describes the web tension variation due to friction between web and roll.
- The Law of Conservation of Mass which describes the coupling between web velocity and web tension.

These laws allow us to calculate web tension between two rolls.

Hooke's law . The tension T of an elastic web is function of the strain ε :

$$T = ES\varepsilon = ES\frac{L - L_0}{L_0} \qquad \{1\}$$

where ε is the web strain, E is the Young modulus, S is the web section, L is the web length under stress and L_0 is the web length without stress.

The Voigt model expresses the tension for a visco-elastic material as a function of the web strain [8]:

$$T = E.S.\varepsilon + v.S.\frac{d\varepsilon}{dt} \qquad \{2\}$$

Coulomb's law. The study of a web tension on a roll can be considered as a problem of friction between solids [7][18]. On the roll, the web tension is constant on a sticking zone of arc length g (see figure 4).

The web tension between the first contact point of a roll and the first contact point of the following roll is given by :

$$\begin{aligned} \varepsilon(x,t) &= \varepsilon_1(t) && \text{if } x \le a \\ &= \varepsilon_1(t)e^{\mu(x-a)} && \text{if } a \le x \le a+g \\ &= \varepsilon_2(t) && \text{if } a+g \le x \le L_t \end{aligned} \qquad \{3\}$$

where μ is the friction coefficient, and $L_t = a+g+L$.

The tension change occurs on the sliding zone. The web velocity is equal to the roll velocity on the sticking zone.

Mass conservation law. Consider an element of web of length $l = l_0\ (1+\varepsilon)$ with a weigth density ρ, under an unidirectional stress. The cross section is supposed to be constant. According to the mass conservation law, the mass of the web remains constant between the state without stress and the state with stress :

$$dm = \rho S l = \rho_0 S l_0 \Rightarrow \frac{\rho}{\rho_0} = \frac{1}{1+\varepsilon} \qquad \{4\}$$

Tension-velocity relation. The model of our experimental setup was derived from the model of the web tension between two consecutive rolls and the model of the velocity of each roll (see figure 4). This well known equation {5} (see [2] and [5]), was obtained since web length on the wrap angle can be neglected compared to the web length without contact between two rolls [18] : $a+g \ll L$

$$\frac{d}{dt}\left(\frac{L}{1+\varepsilon_2}\right) = \frac{V_1}{1+\varepsilon_1} - \frac{V_2}{1+\varepsilon_2} \qquad \{5\}$$

The classical simplifying consists in using the approximations

$$\varepsilon << 1 \ \ and \ \ \frac{1}{1+\varepsilon} \approx 1-\varepsilon \qquad \{6\}$$

to obtain the relation :

$$L\frac{d\varepsilon_2}{dt} = V_2 - V_1 + \varepsilon_1 V_1 - \varepsilon_2 V_2 \qquad \{7\}$$

The linearization leads us to the well known relation:

$$L\frac{d\Delta T_2}{dt} = (ES - T_0)(\Delta V_2 - \Delta V_1) + V_0(\Delta T_1 - \Delta T_2) \qquad \{8\}$$

We propose a new simplifying which consists first in deriving the left term in the relation {5} and then using the approximation {6}. We obtain also the equation {9}:

$$L\frac{d\varepsilon_2}{dt} = V_2 - V_1 + \varepsilon_1 V_1 - \varepsilon_2\left(2V_1 - V_2\right) \qquad \{9\}$$

Figure 6 shows that this new simplified equation more accurately predict the responses obtained without approximation. Moreover, linearization gives an equation {10} which differs from the equation {8} by the nominal tension sign:

$$L\frac{d\Delta T_2}{dt} = (ES + T_0)(\Delta V_2 - \Delta V_1) + V_0(\Delta T_1 - \Delta T_2) \qquad \{10\}$$

This equation {10} shows that, for a given velocity difference ΔV_2-ΔV_1, the higher is the nominal tension T_0 the higher is the tension change ΔT_2. This means that at higher tensions the web tension is more sensitive to velocity variations. This web tension sensitivity is even higher with elastic or thin web material where the approximation ES >> T_0 is not valid.

Web velocity calculation

The velocity of a roll can be obtained with a torque balance:

$$J_k \frac{dV_k}{dt} = R_k^2 (T_k - T_{k-1}) + R_k K_k U_k + C_f \qquad \{11\}$$

where $K_k U_k$ is the motor torque (if the roll is driven) and C_f is the sum of the friction torque. We can notice at this point that at the unwinder and the winder the inertia J_k and the Radius R_k are time dependent and varies largely during the process operation.

IDENTIFICATION

The off line identification is based on the model matching method. The cost function to be minimized is :

$$J_{opt} = \frac{(Y_P - Y_S)^T (Y_P - Y_S)}{Y_P \cdot Y_P} \qquad \{12\}$$

where Ys, Ym are respectively the vectors of simulated and measured output signals (T_u, V, T_w, see figure 2). Two optimization algorithms are used : the simplex method [19] and the Quasi-Newton method [25]. The simplex method gave the smallest cost function and was more robust to the initial values of the parameters. The simulations, with the optimized parameters, and the measurements are compared on the figure 7.

CONTROL

Multivariable H∞ robust controller synthesis

The figure 2 shows the decentralized control scheme of the winding system. The control signals correspond to the torque references of synchronous motors. The web transport velocity is imposed by the traction motor. The force sensors measure the web tension at two points in the webline.

Due to the elastic web, web velocity disturbances transmitted to web tension often result in a web break or fold. Some methods have been studied to suppress this coupling in a system with two driven rolls [13]. In order to reduce the coupling between tension and velocity, we used a multivariable approach with an H_∞ robust controller [26]. This means that instead of 3 PI controllers we use one controller with 3 inputs and 3 outputs, see figure 3. This controller was synthesized using the mixed sensitivity method [26] [27]. The H_∞ controller K is computed via γ-iteration (algorithm of Glover / Doyle) [10] : the controller K is computed in order to minimize the H_∞ norm of the transfer function between exogeneous inputs (like tension and velocity references : T_{uref}, T_{wref}, V_{ref}) and weighted outputs (see figure 8), i.e. K is such that the following expression {13} is minimized :

$$\|Tzw\|_\infty := \sup_\omega \sigma_{\max}(Tzw(j\omega)) = \gamma_{\min} \qquad \{13\}$$

where σ_{max} represents the maximum of the singular value and where the transfer function matrix T_{ZW} is defined as {14} :

$$T y_1 u_1 := \begin{bmatrix} W_p S \\ W_u K S \\ W_t T \end{bmatrix} \qquad \{14\}$$

S is the sensitivity function $S = (I + GK)^{-1}$, T is the complementary sensitivity function $T = I - S$ [26] and W_p, W_u, W_t are frequency weighting functions.

The weighting function W_p is chosen to have a high gain at low frequency to reject low frequency perturbations. The form of W_p is as following [26]:

$$W_p(s) = \frac{\frac{s}{M} + w_B}{s + w_B \varepsilon_0} \qquad \{15\}$$

where M is the maximum peak magnitude of S, $\|S\|_\infty \leq M$, w_B is the required bandwidth frequency, ε_0 is the steady-state error allowed. The weighting function W_u is used to avoid large control signals and usually is a single gain. The weighting function W_t id used to increase the roll-off at high frequencies.

In our application, the selected weighting functions are :

$$W_p(s) = \begin{bmatrix} \frac{0.5s+11}{s+0.01} & 0 & 0 \\ 0 & \frac{0.5s+9}{s+0.01} & 0 \\ 0 & 0 & \frac{0.5s+11}{s+0.01} \end{bmatrix} \qquad \{16\}$$

$$W_u = I_{3x3}\,; \quad W_t(s) = \begin{bmatrix} 2s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 2s \end{bmatrix} \qquad \{17\}$$

The order of the resulting controller is 15. It has been implemented on the experimental setup in state space representation with a sampling period of 10 ms. The figure 9 shows the measurements acquired with changes in references around the nominal point. With the H_∞ controller, the web tensions are noticeably less sensitive to velocity variations. The coupling between tension and web velocity is also reduced.

Multivariable H∞ robust control with gain scheduling

Variations in roll radius prevented the system from maintaining the same performance throughout processing. The study of the gain with a partial model shows that the static gains between the control signals and the web tensions are roughly proportional to the inverse of the radius [18] :

$$Gain_{DC}\left(\frac{T_u}{u_u}\right) = \frac{1}{R_u} \quad and \quad Gain_{DC}\left(\frac{T_w}{u_w}\right) = \frac{1}{R_w} \qquad \{18\}$$

We therefore devised a gain scheduling control by multiplying the control signals by the corresponding radius measurement or estimation (see figure 10). We can observe on the figure 11 that the variations of the radius during the process operation have little effect on the maximum singular values of G_R whereas the singular values of G_0 change a lot.

The synthesis of controller is done using the plant G_R which includes the gain scheduling. The association of the gain scheduling with the multiple variable approach allowed us to reduce web tension variations significantly despite velocity changes during processing (see figure 12) [15][18]. Moreover, the same performances were obtained for both winding and rewinding [18].

Comparison with industrial control

Figure 13 shows an industrial control scheme of the winding system. The web transport velocity is set by the traction motor (master). The other motors are velocity controlled and the velocity reference signals for this motors depend on the web tension. For example, if the unwinding tension T_u is higher than the reference T_{uref}, the difference is added to the velocity reference in order to accelerate the unwinding process.

A comparison between the simulations of this control strategy and the H_∞ control is presented on the figure 14. We can see that the H_∞ control noticeably reduces the velocity-tension coupling. Moreover, H_∞ control with gain scheduling improves the performances.

CONCLUSIONS

The web winding systems require an efficient control of the web tension during all the process operation. The radius and the inertia of the rollers vary on a large scale, therefore, the system dynamics change considerably. The first improvement suggested is to use the radius of the rollers as a proportional gain in the control. An robust H_∞ controller with varying gains depending on the radius allows to significantly reduce the coupling between tension and web velocity. Moreover this controller keeps the same performance (overshoot and time response) during all the process run.

Robust control needs precise modelization of the system. A novel linear model allowed us to predict the relationship between web tension and velocity more precisely than the generally used approach.

ACKNOWLEDGEMENT

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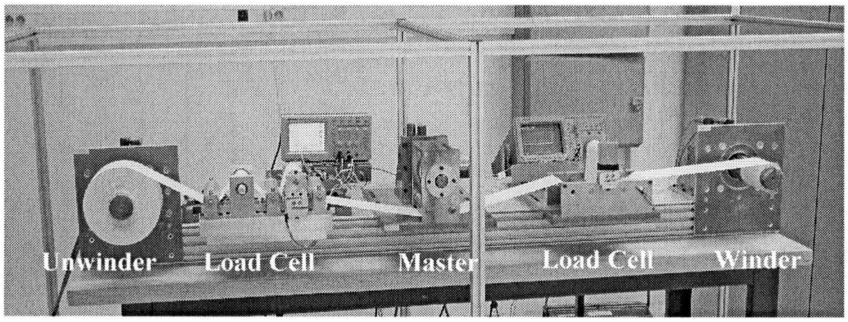

Fig 1 – Experimental setup with 3 motors and 2 load cells

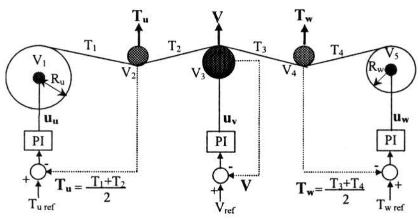

Fig 2 - *Decentralized control scheme*

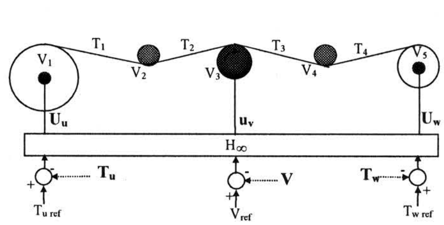

Fig 3 - *Multivariable control scheme*

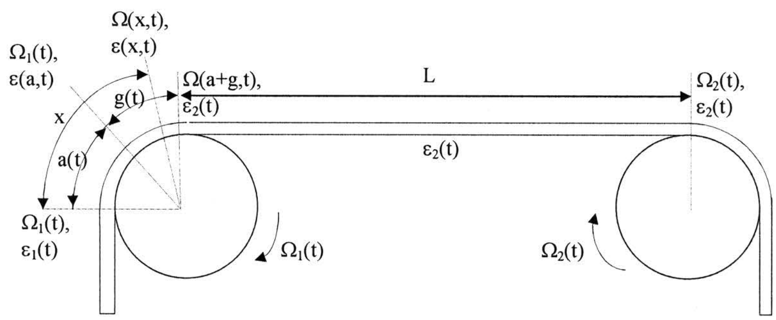

Fig 4 – Web tension on the roll

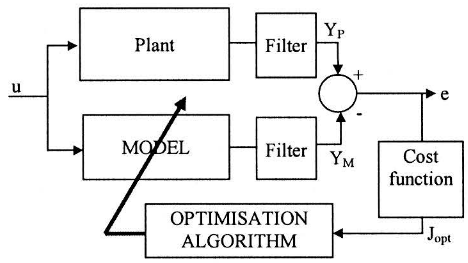

Fig 5 – Model matching method

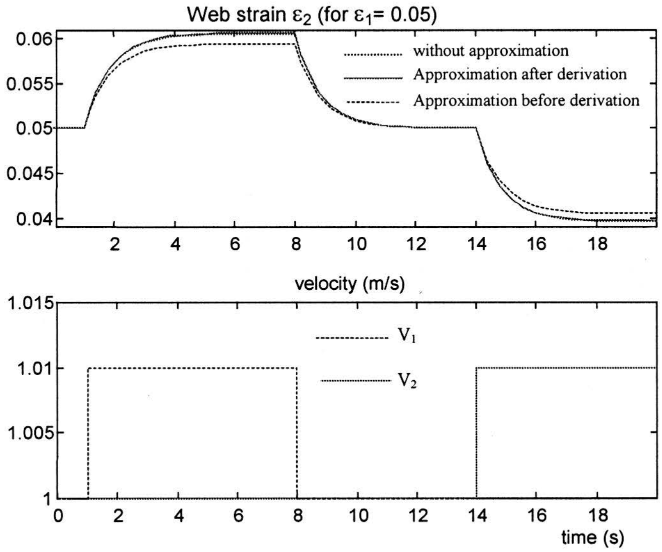

Fig 6 – Web strain under velocity variations

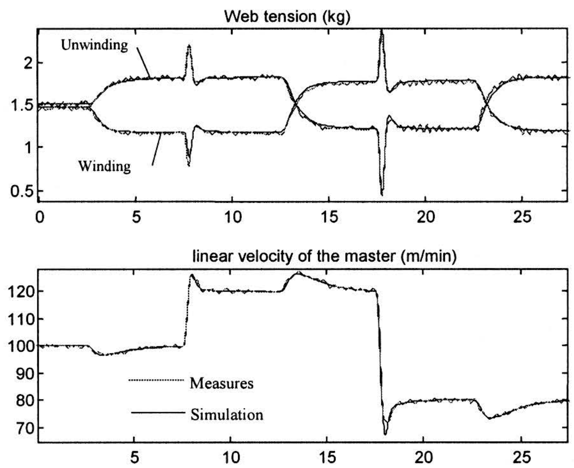

Fig 7 - Comparison identified model and measures

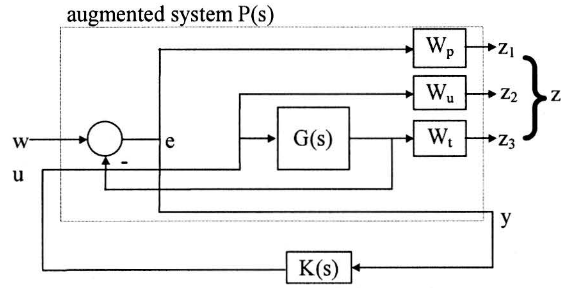

Fig 8 – Mixed sensitivity method

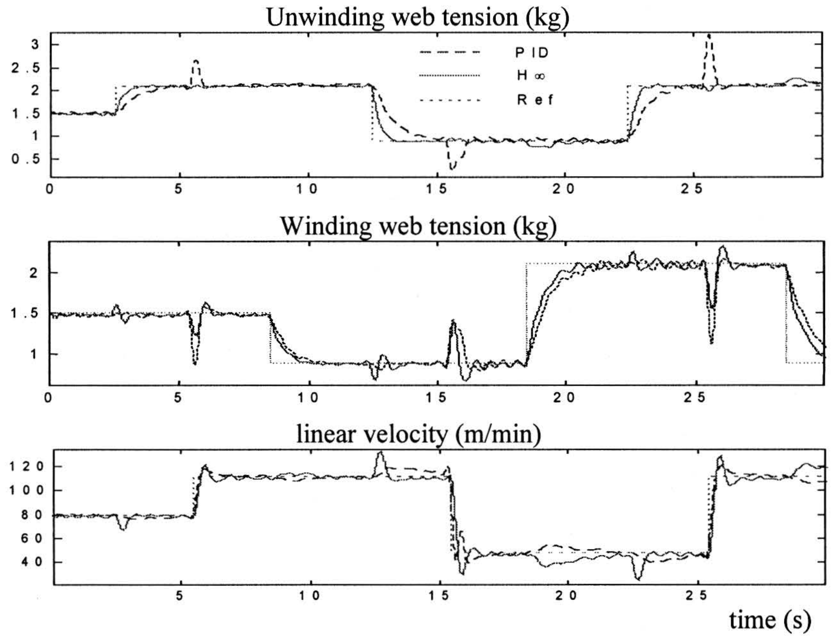

Fig 9 - H_∞ and PI controllers comparison

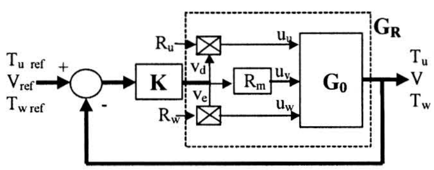

Fig 10 – Modified system

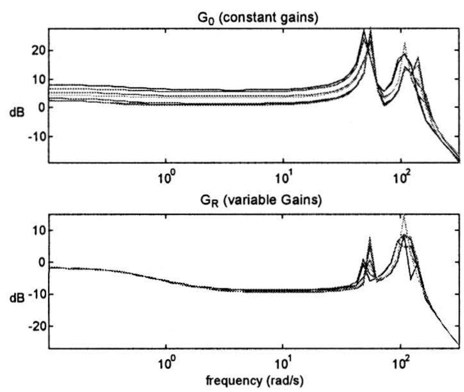

Fig 11 – Singular values for different radii

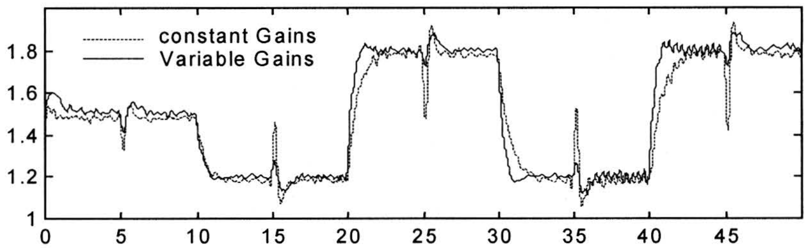

Fig 12– Comparison H_∞ and gain scheduling H_∞ control

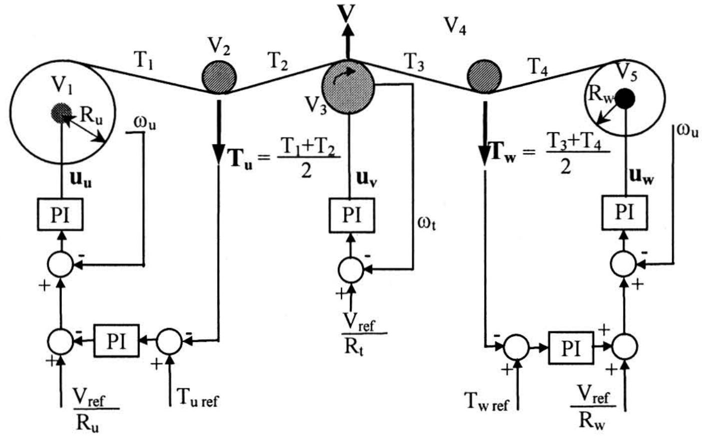

Fig 13– Industrial decentralized control

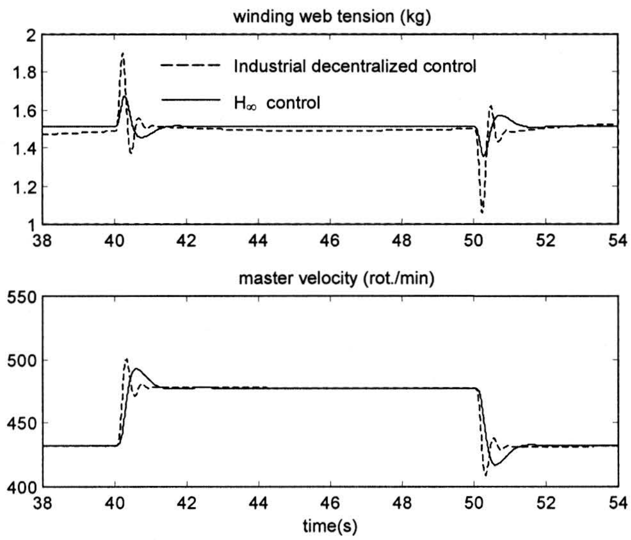

Fig 14– Comparison industrial control – multivariable H_∞ control

Modeling and H∞ Robust Control for Winding Systems

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Name & Affiliation	Question
D. Carlson – 3M	Did you compare the PI controller where the gain was scheduled based on inertia to the H_∞ controller? Did you ever make a comparison of a PI controller where the gains were adapted based on spindle inertias? Compensated gains instead of fixed gains?
Name & Affiliation	**Answer**
H. Koc – Siemens AG	Yes. I compared PI with varying gains with the H_∞ with varying gains. The gain was scheduled based on the radius not the inertia.
Name & Affiliation	**Question**
K. Reid – OSU	I saw the PI with fixed gains and the PI with variable gains based on the radius of the unwind roll. Then I saw the H_∞ with and without gain scheduling. Did you compare H_∞ with gain scheduling with PI with gain scheduling? I guess we both missed that. Can you return to that figure?
Name & Affiliation	**Answer**
H. Koc – Siemens AG	Yes. Here I compared PI with varying gains with H_∞ with varying gains. I don't have any figure here, but I have done this. We can see on the video that the control with H_∞ controller is much better than the PI controller. The corresponding figure will be added to the original text.
Name & Affiliation	**Question**
K. Shin – KonKuk University	How did you synchronize the speed of those three motors when you carried out the experiment?
Name & Affiliation	**Answer**
H. Koc – Siemens AG	It's naturally done, because when you want to have constant tension you must have the same velocity. So, I don't need to have this information of velocity. If you recall the presentation of P. Bourgin and M. Boutaous, we desire a control that can follow the tension set point very fast. When you use controls that include the velocity regulation, your tension control is not very fast. So you can control the tension very well or you can control both (tension and velocity) not very well. The goal is to control the tension (for winder and unwinder) not the velocity.
Name & Affiliation	**Question**
K. Shin - KonKuk University	What was your sampling rate?

Name & Affiliation	Answer
H. Koc – Siemens AG	It was 10 milliseconds.
Name & Affiliation	**Question**
K. Shin - KonKuk University	Did you use one controller? Did you use PC or PLC?
Name & Affiliation	**Answer**
H. Koc – Siemens AG	PC. It was not a sophisticated PC. It was a Pentium running at 230 MHz using DOS 32 bit.