STUDY OF STRESS-STRAIN RELATION FOR PAPER ROLL

by

K. Ärölä[1], R. von Hertzen[1] and M. Jorkama[2]
[1]Helsinki University of Technology
[2]Metso Paper, Inc.
FINLAND

ABSTRACT

In the present method a wound roll of paper is loaded against a nip roller and the measured values of the nip width and the roll indentation are compared with the corresponding calculated values of the nonlinear problem. The nip width is measured by a sensitive sensor film and the roll indentation by a laser displacement sensor. The nonlinear numerical problem is solved using the Finite Element Method with four-node isoparametric quadrilateral elements and Newton-Raphson-type iteration. A suitable form of the constitutive equation and the stress state dependence of the moduli of the incremental stress-strain law will be discussed. A least squares fit to the experimental results determines the values of the paper roll elastic moduli.

NOMENCLATURE

a	nip half-width
a_i	measured nip half-width
d	roll diameter
e	residual error
E	modulus of elasticity
E_{rr}, $E_{r\theta}$, $E_{\theta\theta}$, $G_{r\theta}$	elastic moduli of the orthotropic roll
f_r, f_θ, $f_{r\theta}$	constitutive relations
K_1, K_2	Pfeiffer's constants
P	compressive load between the cylinders
R_1	paper roll radius
R_2	nip roller radius
ε_r, ε_θ	radial and tangential strains of the roll

$\gamma_{r\theta}$	shear strain of the roll
δ	indentation of the roll
δ_i	measured indentation of the roll
$\Delta\sigma_r$, $\Delta\sigma_\theta$	incremental radial and tangential stresses of the roll
$\Delta\tau_{r\theta}$	incremental shear stress of the roll
ν	Poisson's ratio
σ_r, σ_θ	radial and tangential stresses of the roll
$\tau_{r\theta}$	shear stress of the roll

INTRODUCTION

Winding a flexible web into a compact roll is a widely used process for materials such as paper, thin films and magnetic tapes. By the aid of winding models [1,2] one can analyse the internal stress state of the roll which is an important part of the defect free roll structure design. Winding models can predict several roll defects directly, and will help to understand many others.

Every winding model needs a proper constitutive law for the material wound and numerical values for the material parameters. The earliest models were based on the assumption that the wound roll could be treated as a linear, plane strain and orthotropic medium. Later the radial modulus of a wound roll was modeled as a nonlinear function of the radial stress or strain [3] and also viscoelastic effects were accounted for [4].

A common feature of modern winding is the presence of a nip in the winding path. It should be noted that the nip-induced stresses destroy the rotational symmetry of the stress field of the wound roll. Therefore, shear stresses are generated into the roll.

Since the conventional models concentrate solely on cases with rotational symmetry, the information on elastic constants in the literature is restricted to the tangential and radial moduli. However, there is an increasing interest and need for models of winding with a nip so that the values of the shear moduli and Poisson's ratios also become important. It has been shown recently, in particular, that the value of the shear modulus of the roll may have a marked effect on the nip width [2].

The aim of the present work is to present a method for the determination of the elastic moduli of the roll by comparing the results of roll compression tests with those of a nonlinear FE-model in the sense of a least squares fit.

THEORY

When determining the material parameter values from experiments one must have measured data which is diversified enough. For example, in the case of an isotropic homogeneous roll of diameter *d*, pressed against a rigid half-space by a line load *P*, one obtains for the contact half-width [5]

$$a = \sqrt{\frac{2dP}{\pi E}(1-\nu^2)} \tag{1}$$

and for the indentation of the roll

$$\delta = P\frac{1-\nu^2}{\pi E}\left[2\ln(2d/a) - \frac{\nu}{1-\nu}\right] , \tag{2}$$

where E is the modulus of elasticity and ν Poisson's ratio of the roll. It can be seen from equation (1) that measuring the contact width $2a$ as a function of the line load P fixes the value of $E/(1-\nu^2)$ only but not the values of E and ν separately. On the other hand, measuring the indentation δ for different line loads P can in principle separate the values of E and ν. In practice, however, this works purely since the term $2\ln(2d/a)$ (≈ 10 for $d = 1\,\mathrm{m}$ and $a = 1\,\mathrm{cm}$) dominates over the term $\nu/(1-\nu)$ in equation (2) so that some noise in the experimental data may annihilate the effect of the latter term. Consequently, one must in practice use the measured values of both a and δ in order to determine the values of E and ν. This is on the other hand sufficient since a fixes the value of $E/(1-\nu^2)$ and then δ fixes the value of $\nu/(1-\nu)$.

In the case of an orthotropic nonlinear material the situation is somewhat more complex. Let us consider first a general stress-strain relation. In a plane strain state, some distance away from the roll ends, one can write the general constitutive relations

$$\begin{aligned} \sigma_r &= f_r(\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}) \\ \sigma_\theta &= f_\theta(\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}) \\ \tau_{r\theta} &= f_{r\theta}(\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}) . \end{aligned} \tag{3}$$

Of what form are the functions f_r, f_θ and $f_{r\theta}$ in the case of a paper roll? No general answer exists so far. The incremented form of equations (3) is also frequently needed. One obtains

$$\begin{bmatrix} \Delta\sigma_r \\ \Delta\sigma_\theta \\ \Delta\tau_{r\theta} \end{bmatrix} = \begin{bmatrix} \partial f_r/\partial\varepsilon_r & \partial f_r/\partial\varepsilon_\theta & \partial f_r/\partial\gamma_{r\theta} \\ \partial f_\theta/\partial\varepsilon_r & \partial f_\theta/\partial\varepsilon_\theta & \partial f_\theta/\partial\gamma_{r\theta} \\ \partial f_{r\theta}/\partial\varepsilon_r & \partial f_{r\theta}/\partial\varepsilon_\theta & \partial f_{r\theta}/\partial\gamma_{r\theta} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_r \\ \Delta\varepsilon_\theta \\ \Delta\gamma_{r\theta} \end{bmatrix} \tag{4}$$

and for an orthotropic material, in particular,

$$\begin{bmatrix} \Delta\sigma_r \\ \Delta\sigma_\theta \\ \Delta\tau_{r\theta} \end{bmatrix} = \begin{bmatrix} E_{rr} & E_{r\theta} & 0 \\ E_{\theta r} & E_{\theta\theta} & 0 \\ 0 & 0 & G_{r\theta} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_r \\ \Delta\varepsilon_\theta \\ \Delta\gamma_{r\theta} \end{bmatrix} . \tag{5}$$

For a hyperelastic material one can derive the stresses from an elastic potential function so that $E_{r\theta} = E_{\theta r}$. In general, all the coefficients E_{rr}, $E_{r\theta}$, $E_{\theta r}$, $E_{\theta\theta}$ and $G_{r\theta}$ may depend on the strain components ε_r, ε_θ, and $\gamma_{r\theta}$. Due to the lack of more complete material models, one may postulate simpler models such as

$$E_{rr} = E_{rr}(\varepsilon_r) , \tag{6}$$

$$E_{\theta\theta} = E_{\theta\theta}(\varepsilon_\theta) , \tag{7}$$

$$E_{r\theta}, G_{r\theta} = const \; . \tag{8}$$

For example, Pfeiffer suggests [6]

$$E_{rr} = K_1 K_2 e^{-K_2 \varepsilon_r} = K_2 (K_1 - \sigma_r) \tag{9}$$

$$E_{r\theta} \approx E_{\theta r} \approx 0 \, . \tag{10}$$

Instead of equation (9) Hakiel prefers the relation [1]

$$E_{rr} = c_o + c_1 \sigma_r + c_2 \sigma_r^2 \; . \tag{11}$$

In the present work we consider the parameters $E_{\theta\theta}$, $E_{r\theta}$ and $G_{r\theta}$ as constants and employ equation (9) for the radial modulus.

Our numerical solution of the compression experiment is performed by using a nonlinear FE-model of the continuum combined with a modified Panagiotopoulos-iteration for the contact equations. Since in this case the contact is static the Panagiotopoulos process is much simplified. The FE-discretization of the virtual work equation yields the standard FE-formulation of small strain 2D-elasticity

$$\boldsymbol{\psi} \equiv \int_A \mathbf{B}^T \boldsymbol{\sigma} t \, dA - \mathbf{F} = \mathbf{0} \, , \tag{12}$$

where $\mathbf{B}$ is the strain-displacement matrix containing the derivatives of the shape functions, vector $\boldsymbol{\sigma}$ contains the stress components and

$$\mathbf{F} = \oint_\Gamma \mathbf{N}^T \mathbf{t} t \, d\Gamma + \int_A \mathbf{N}^T \mathbf{b} t \, dA \, . \tag{13}$$

Above $\mathbf{t}$ contains the boundary tractions and $\mathbf{b}$ the body forces. The symbols A and Γ denote the area and boundary of the plane cross section, respectively. The thickness of the considered specimen in the direction orthogonal to the plane is t. It is emphasized that no assumptions on the relation between stresses and strains have been made during the derivation of equation (12).

The introduction of a nonlinear constitutive relation renders the problem nonlinear and an iterative solution is required. Using the Newton-Rhapson method leads to the following solution algorithm for the elastic problem:

- Set $i = 0$ and specify the starting point. Usually $\mathbf{u}_0 = \mathbf{0}$.
- Compute the strains due to displacements $\boldsymbol{\varepsilon}_i = \mathbf{B}\mathbf{u}_i$
- Compute the corresponding unbalance

$$\boldsymbol{\psi}_i = \int_A \mathbf{B}^T \boldsymbol{\sigma}(\boldsymbol{\varepsilon}_i) t \, dA - \mathbf{F}$$

- Compute the tangential stiffness matrix

$$\mathbf{K}_{T,i} = \frac{\partial \psi}{\partial \mathbf{u}} = \int_A \mathbf{B}^T \frac{\partial \sigma}{\partial \mathbf{u}} t\, dA = \int_A \mathbf{B}^T \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \mathbf{u}} t\, dA = \int_A \mathbf{B}^T \mathbf{E}_T(\varepsilon_i)\mathbf{B}\, t\, dA$$

- Compute the increment of the displacement vector

$$\Delta \mathbf{u}_i = -\mathbf{K}_{T,i}^{-1} \psi_i$$

- Compute the new total displacement vector

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta \mathbf{u}_i$$

- Set $i = i + 1$ and return to step two.

Using the obtained elastic solution of the wound roll the contact equations are solved. The contact pressures are used to update the stress and strain state of the material and a new elastic solution is sought. This process is repeated until a prescribed accuracy in the distribution of contact forces is obtained.

Typically six or seven material iterations are needed to reach a relative error $\leq 10^{-6}$ in our calculations. Note that a wound roll represents a strongly prestressed body of material. Therefore, it is important to account for the initial strains in the solution procedure. If the wound-in-tension history of a roll is known, one can account for the initial strains of a finished axisymmetric roll using Hakiel's model; otherwise, experimentally determined (pull tab) values must be used.

The values of the material parameters are detemined by a least squares fitting procedure. The residual error to be minimized is

$$e(K_1, K_2, E_{\theta\theta}, E_{r\theta}, G_{r\theta}) = \sum_{i=1}^{N} \left\{ \left[a_i - a(K_1, K_2, E_{\theta\theta}, E_{r\theta}, G_{r\theta}; P_i)\right]^2 + \left[\delta_i - \delta(K_1, K_2, E_{\theta\theta}, E_{r\theta}, G_{r\theta}; P_i)\right]^2 \right\} . \quad (14)$$

Here a_i and δ_i denote the measured values of the nip half-width and the roll indentation, respectively, for the line loads P_i (i = 1,…, N) used in the compression experiment, and $a(K_1,...;P_i)$ and $\delta(K_1,...;P_i)$ denote the corresponding numerically computed values. In practice, the computational cost may be reduced by taking the values of K_1 and K_2 from a stack test and the value of $E_{\theta\theta}$ from a machine direction tensile test. Figure 1 shows the dependence of the nip width on the nip load for different values of the roll shear modulus $G_{r\theta}$. It can be seen that, especially at higher nip loads, $G_{r\theta}$ is an important elastic parameter in the non-axisymmetric nip contact models.

EXPERIMENTAL SETUP

The experimental setup is shown in Figure 2. A paper roll of radius R_1 rests on a nip roller of radius R_2. A compressive load is applied from the core chucks to the roll. The resulting nip load P is calculated from the roll weigth and the core load. The distance x of the roll top from a fixed point is measured with a high precision laser sensor (resolution < 10 μm). It is assumed that the nip and core loads do not cause any significant deformation at the roll top. Hence, the nip compression can be calculated from the measured value of x. The nip width was measured using the Pressurex Micro Film. The smallest measurable pressure with this film is 13.8 kPa and, hence, also the nip edges could be quite accurately detected. An estimation based on the Hertzian pressure distribution shows that approximately a width of 0.2 - 0.35 mm at the nip edges will fall below the sensing range of the film. The evenness of the nip distribution was checked by measuring the nip width from three locations of the roll: both edges and the center. The initial internal pressure distribution of the roll was estimated by pull tab measurements. The total number of pull tabs was 10. Near the roll periphery the internal pressure was measured at 50 mm spacings. The nip roller diameter was 0.425 m and the roll diameter 1m.

CONCLUSIONS

An indirect method for determining the values of elastic moduli of a paper roll is presented. In particular, the value of the shear moduli $G_{r\theta}$ is rarely reported in existing literature, and yet it is an important parameter in modern winding models including the nip action. Our preliminary test results indicate that a strain deficiency [7] seems to exist in the top layers of the wound roll. This means that the measured nip widths must be subtracted by some value before the fitting procedure to the nonlinear solid roll model can be performed.

REFERENCES

1. Hakiel, Z., "Nonlinear Model for Wound Roll Stress", TAPPI Paper Finishing and Converting Conference Proceedings, pp. 9-15, 1986.
2. Jorkama, M., "Contact Mechanical Model for Winding Nip", Acta Polytechnica Scandinavica, Me 146, 2001.
3. Pfeiffer, J.D., "Prediction of Roll Defects from Roll Structure Formulas", TAPPI Journal, Vol. 62, No. 10, 1979.
4. Qualls, W.R. and Good, J.K., "An Orthotropic Viscoelastic Winding Model Including a Nonlinear Radial Stiffness", Journal of Applied Mechanics, Vol. 64, 1997.
5. Johnson, K.L., Contact Mechanics, Cambridge University Press, 1984.
6. Pfeiffer, J.D., "Measurement of the K2 Factor for Paper", TAPPI Journal, Vol. 64, No. 4, 1981.
7. Pfeiffer, J.D., "Surface Winding to Overcome Strain Deficiency", Proceedings of TAPPI Finishing and Converting Conference, pp. 233-236, 1990.

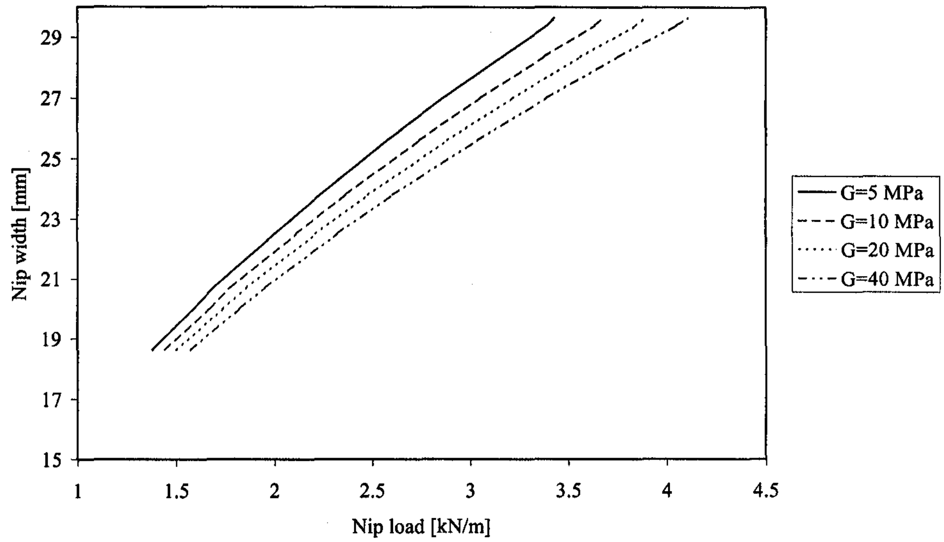

Figure 1. Effect of the shear modulus $G_{r\theta}$ on the nip width by a nonlinear FEM-calculation (K_1 = 0.275 MPa, K_2 = 12.8, $E_{\theta\theta}$= 5.1 GPa and $E_{r\theta}$= 1.0 MPa).

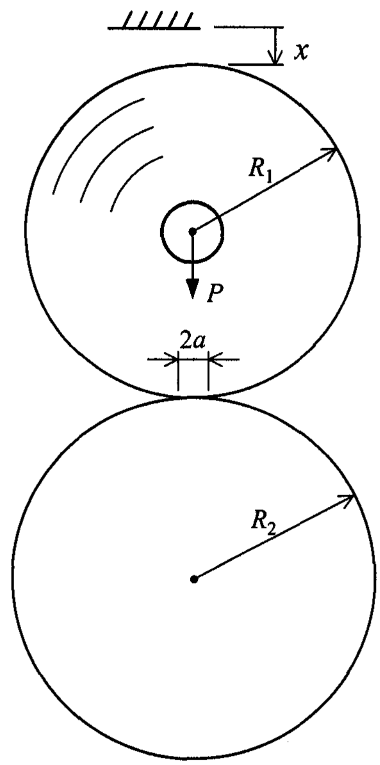

Figure 2. Setup of the compression experiment.

Study of Stress-Strain Relation for Paper Roll

K.Arola[1], R. von Hertzen[1], and M. Jorkama[2] – Helsinki University of Technology[1] and Metso Paper, Inc.[2], Finland

Name & Affiliation	Question
D. Pfeiffer – JDP Innovations	Pfeiffer wants to defend his K1-K2 model and explain why you might have trouble with it to explain the compression of a roll. The K1-K2 model applies to a stack under uniform applied pressure. In a wound roll a pressure gradient exists starting from very low pressure between the outer layers of paper or so into an increasing pressure gradient. In your model somehow you would need to incorporate that ramp up of pressure between the layers and the impact on the radial modulus. My second comment is that you have a very high value of K1 – much higher than stack measurement would normally predict in the metric system. You would normally find a K1 about 40 KPa instead of 1.9 MPa.
Name & Affiliation	**Answer**
R. von Hertzen – Helsinki University of Technology	Yes, thank you for your comments. As to the first comment, we thought the same thing -- this was not an attack against your model. In this case a more complex material model should have been used. I think that the main reason for the pretty high values of the coefficients is that the material model in this case was not describing the real behavior of the roll.
Name & Affiliation	**Reply**
M. Jorkama – Metso Paper	We did measure the internal stress pressure distribution inside the wound roll and that was accounted for in Mr. Arola's finite element calculation.
Name & Affiliation	**Answer**
R. von Hertzen – Helsinki University of Technology	The pre-stresses of the roll were measured by pull tabs and were included in the simulation.
Name & Affiliation	**Question**
K. Good – OSU	The new parameter that you introduced, a_0. How did you come up with a_0?
Name & Affiliation	**Answer**
R. von Hertzen – Helsinki University of Technology	We added an additional parameter there to test the optimization strategy to see if it could find values for the parameters so that the calculated results would fit the measured data.
Name & Affiliation	**Question**
K. Good – OSU	So it was arbitrary - the setting of a_0?

Name & Affiliation	Answer
R. von Hertzen – Helsinki University of Technology	It was optimized by the optimization strategy. It was calculated.
Name & Affiliation	**Question**
Y. Wang – Sonoco	I noticed that you used a two-parameter model and actually used the 2-D model and then reduced to the model with only a diagonal element in that constitutive question. So it's totally decoupled, then you used the elastic model for the stress-strain relationship.
Name & Affiliation	**Answer**
R. von Hertzen – Helsinki University of Technology	The equations were coupled. A full orthotropic material model was used. Of course, the linear orthotropic material parameters include three elastic moduli, three Poisson's ratios and three shear moduli. But we are not actually interested in calculating each one of these separately. What we are interested in are values of the combinations of these parameters in the elastic coefficient matrix. These combinations can be calculated using the introduced method. This way we have only four material parameters to determine for the linear orthotropic case.