A FULLY NONLINEAR SOLUTION TO THE STRESSES INSIDE WOUND ROLLS

by

A. W. Forrest Jr.
DuPont
USA

ABSTRACT

The Hakiel approach has been the accepted method for calculating stresses in wound rolls. It determines the stress in rolls by summing the stress induced by wraps of film applied to the roll surface. This approach is called linear superposition and it is strictly valid only for linear differential equations. The radial properties of film in a roll make the equations highly nonlinear and this presents a problem. Test results have supported the general accuracy of the Hakiel method. The method described here obtains a fully nonlinear solution for the stresses which occur during winding and any roll deformation that may occur afterward. The subsequent roll deformation can be a result of air leaking out or plastic deformation of the film both in the stack and in-plane directions. Only the case involving air leaking is presented. It reproduces the Hakiel approach for linearized conditions and differs to varying degree for the nonlinear cases. The approach presented here is more suitable for highly nonlinear cases and for making roll-aging calculations.

NOMENCLATURE

E	Elastic Modulus of Web
E_r	Elastic Modulus of film in the Radial or Stack Direction
G	Radial (Stack) Strain due to Radial (stack) Stress
h	Air Gap
h_{ao}	Entrained Air Gap
K_1 , K_2	First and Second Stack Compression Coefficients
P_{ao}	Pressure of Entrained Air
r	Radial Dimension
r_o	Radius of Wrap in the Unstressed State

r_s Radius of Wrap in the Stressed State
U_s Radial Deflection of layer of film
z Transverse direction dimension
ε_r, ε_θ, ε_z Strain in the Radial, Circumferential and Transverse Directions
δ_i Thickness of the i th wrap
μ Poisson's Ratio
$\sigma_r, \sigma_\theta, \sigma_z$ Stress in the Radial, Circumferential, and Transverse Directions
σ_{bf} The Body Force Apparent Tension
σ_s Pseudo Tension to Account for Radial Deflection
σ_w Winding Tension

INTRODUCTION

There are techniques available [1-3]* to solve for the stresses in wound rolls both for the plane stress and plane strain cases. Methods also are available [3] for determining whether buckles form down inside the roll. Calculation procedures also are available that include plastic deformation. All of these approaches have one thing in common, they use a layering approach to solve for stress. This method has been verified experimentally and works well for most documented cases. It does have a distinct problem that needs to be discussed.

First, the standard solution procedure incorrectly uses linear superposition to solve a nonlinear differential equation. This may appear to be of no consequence since the approach has been verified experimentally. What can be said is that it is better to avoid linear superposition entirely and use a fully nonlinear approach.

As discussed the layering approach currently used adds the stress results from each wrap as it is applied. This means that when the roll has finished winding, no more stress calculations can be made. In reference [4] plastic deformation of the roll after the winding process is considered by repeatedly rewinding the roll numerically. Here, linear superposition is used again and again. In many cases, the roll will shrink radially as air leaks out or the film plastically deforms either in the plane of the sheet or in the stack direction. This means that the radial position of a wrap can change with time. As will be shown, this affects the apparent tension in the roll and the layering approach may not consider this effect properly. This needs to be determined.

The aging of the roll can be modeled more correctly using a fully nonlinear solution. Again, linear superposition is not needed and one additional calculation is needed for each time step. The winding tension can be handled as a body force at a prescribed radius that is determined during the winding simulation. This is then used for subsequent calculations of stresses. Typically, winding operators are capable of achieving rolls that look good at the end of the winding process. Experience has shown, however, that the roll quality deteriorates for as long as 4 to 7 days. This is caused by air that was entrained in the roll during winding leaking out of the rolls and/or plastic deformation of the film in the plane of the sheet or in the stack direction.

A fully nonlinear approach for calculating the roll stress has been developed that uses a body force approach to include web tension. Others [5,6] have tried to use this

* Numbers in brackets refer to references in the bibliography.

approach but the results have differed dramatically from the usually reliable layering approach. The body force method prior to this work sets the body force term based solely on the winding tension. This seems right but is not. The accurate formulation of the body forces includes both the winding tension and the radius at which the film is applied. This happens naturally with the layering approach but not so with the body force method. The current method simply corrects the body force terms to include the effects of roll compression during winding.

DISCUSSION

The body force approach is corrected by the addition of a pseudo stress. It is defined as the additional stress or winding tension that is needed so that each wrap is at the winding tension when it is the outside wrap at r_s. The body force approach normally sets the winding tension at the unstressed radius of the layer. The pseudo stress artificially sets the total stress higher to include the circumferential strain between the unstressed radius and wrap surface radius. The pseudo stress is defined as

$$\sigma_s = E\, U_s / r_s \tag{1}$$

where U_s is the difference between the actual winding at the radius r_s and the initial radius in an unstressed roll. The value of U_s can be obtained from

$$U_s = \sum_{1}^{k} \delta_i\, \varepsilon_{ri} \tag{2}$$

where k is adjusted to refer to the outside wrap of the winding roll and U_s is the sum of all the radial strains of all elements below the surface including the core deflection. The solution requires that the pseudo stress is calculated for each wrap as it is applied to the winding roll.

The body force stress then becomes

$$\sigma_{bf} = \sigma_w + \sigma_s \tag{3}$$

and the differential equation describing the roll stresses for the plane strain case is

$$d^2\sigma_r/d r^2 + (3/r)\, d\sigma_r/dr + (\sigma_r - E\,G)/r^2 (1-\mu^2) = (\sigma_w + \sigma_s)/r^2 (1-\mu^2) + (1/r)\, d(\sigma_w + \sigma_s)/dr \tag{4}$$

The parameter, G, is defined by

$$\begin{aligned} \varepsilon_r &= G - \mu\, \sigma_\theta/E - \mu\, \sigma_z/E \\ \varepsilon_\theta &= \sigma_\theta/E - \mu\, \sigma_r/E - \mu\, \sigma_z/E \\ \varepsilon_z &= \sigma_z/E - \mu\, \sigma_\theta/E - \mu\, \sigma_r/E = 0 \end{aligned} \tag{5}$$

where each of the strains is the total strain since the winding process began. For the linear portion of the stress-strain relationships, there is no difference between this approach and the layered approach used by Hakiel [1]. The radial stress-strain term, G, is different.

Instead of using the local elastic modulus (slope of the stress-strain response), we use the total strain in the radial direction. This is required to get the total stress in the roll for each calculation (no linear superposition). Using the stress-strain relationship proposed by Pfieffer [2] , this is

$$G = (1 / K_2) \ln \{ (\sigma_r + K_1) / K_1 \} \qquad (6)$$

where $\sigma_r = K_1 \exp \{ K_2 \varepsilon_r \} - K_1$. So, G is the radial strain, ε_r, introduced by only the radial stress and is determined from the stack compression test. . The formulation of G including trapped air is shown in Appendix 1 where both the physical properties of the web and the air entrained are considered.

The boundary conditions for equation 4 are

$$\sigma_r = 0 \quad \text{at} \quad r = \text{the outside radius} \qquad (7)$$
$$\sigma_r = 0 \quad \text{at} \quad r = \text{the inside radius of the core}$$

and a matching condition at the core-wound roll interface.

The solution procedure employed is not unlike the layered approach. Here, a wrap is applied to the core and equation 4 is solved with the appropriate boundary conditions. For the first wrap, r_o is the same as r_s and the pseudo stress is zero. The calculation then is repeated for each additional wrap and the value of U_s is calculated. The pseudo force assigned to each wrap is determined from U_s by using equation 1. Note that a completely new solution is obtained for each wrap that's added. No linear superposition is employed. Once the pseudo stress is determined during the winding simulation, the calculation can continue after the winding process has been completed. This allows for the modeling of roll-aging phenomena such as plastic deformation and the loss of entrapped air. Here, G is adjusted to reflect changes in the roll with time.

The buckling criteria, included in previous work [4], can be easily incorporated into this calculation scheme. This technique will be included to enhance the value of the calculated stresses.

Comparison Cases For Roll Stresses

The case shown in Table 1 was obtained from reference [1] and is used to compare the new body force approach with the superposition method. Here, G ($1/E_r$) is handled two ways; as a constant which makes equation 4 linear and using the exponential approach described above and in Appendix 1. Figure 2 shows the radial pressure distribution for 3 linear cases from reference [1] where E_r is treated as a constant. Here, both the body force and layered cases are identical. This is as it should be, since linear superposition can be correctly applied in this case.

One of the nonlinear cases from reference [1] is included as Figure 3 where the data used in the calculation also is included on Table 1. Here, we see a small difference for this particular web and winding condition. The pseudo tension calculated during the procedure is shown on Figure 4. Note that the magnitude of this fictitious tension is substantial and explains the significant difference between the results using this body force method as compared with previous attempts.

To test the solutions for a range of nonlinearity, the value of K_2 is varied from 100 to 1000 to approximate films with various surface designs. These comparisons are shown on Figures 5 and 6. The plots show that for high values of K_2 both techniques have similar results with minor differences in magnitude. For K_2 of 200 and below, however, the difference is substantial.

The final results include air leaking out of the roll of film after the roll has been wound. The assumption made here is that the air leaks out over a much greater time period than that required to wind the roll of film. The case is described in Table 2 and cases (1) with all the air remaining in the roll, (2) with all the air leaked out and (3) with no entrained air at all. The air entrainment for this case is held constant for the entire winding process, although this assumption is not required. Figure 7 shows the radial pressure in the wound roll and includes the air pressure for the situation immediately after winding the roll (no appreciable air leakage). Note that this was done in one step. It illustrates how the fully nonlinear technique described here can be used for roll aging calculations. Here, air leakage rates and/or plastic deformation in the wound film would be handled in a step-wise fashion.

Plots of the buckling number for the formation of MD ridges, Figure 8, are included to provide more insight. Notice that the buckling number after the air escapes is almost unchanged. In this case the prediction indicates that no MD ridges will form in the wound roll during aging. In practice MD ridges may or may not occur as air leaks out of the roll it collapses radially inward. The radial motion causes the circumferential stress to drop and by Poisson's ratio effects this causes the transverse compression to rise. This can influence the buckling number and affect the tendency to form buckles.

CONCLUSIONS

A fully nonlinear solution method for the stresses formed in winding rolls has been obtained. It eliminates the use of linear superposition employed previously to solve a highly nonlinear differential equation. The accuracy of this approach has been established by comparing it with both existing calculation procedures and experimental data.

This approach is directly applicable to calculations including roll aging. After the roll has wound it continues to deteriorate over a period of hours to days. The layering approach used previously with the form of its differential equation can not be used directly for this type of calculation. The roll has to be rewound repeatedly to include the effects of time. This area of investigation is important because many roll defects form after the roll has been wound.

The new technique is should be more accurate. The approach used both models the process and is mathematically correct. Previous calculations for roll stresses have employed linear superposition for solutions to nonlinear problems. Although accurate for many cases evaluated, cases involving a high degree of nonlinearity could cause problems. It is recommended that this approach be considered as the standard approach for roll stress calculations in the future. The computer program required is only slightly more complex and runs in a mater of several minutes on current desk-top computers.

BIBLIOGRAPHIC REFERENCES

1. Hakiel, Z., "Nonlinear Model for Wound Roll Stresses", Tappi, pp. 113-17, May 1987.
2. Pfieffer, J. D., "Predictions of Roll Defects from the Structure Formulas", Tappi, Vol. 62, No. 10, pp. 83-85, 1979.
3. Forrest, A. W., "Wound Roll Stress Analysis Including Air Entrainment and The Formation of Roll Defects", Proceedings of the Third International Conference on Web Handling at Oklahoma State University, June, 1995.
4. Taylor, R. M. and Good, J. K., "Entrained Air Films in Center Wound Rolls - With and Without the Nip", Proceedings of the Fourth International Conference on Web Handling at Oklahoma State University, June, 1997.
5. Piper, C. A., A Nonlinear Model to Calculate the Stressed State of a Center-Wound Roll", Proceedings of the Third International Conference on Web Handling at Oklahoma State University, June, 1995.
6. Altmann, H. C., "Formulas for Computing the Stresses in Center-Wound Rolls", The Journal of the Technical Association of the Pulp and Paper Industry, Vol. 51, No. 4, pp. 176-179, 1968.

Table 1 Data for Comparisons with Hakiel Technique

$E_r = 1060\,\sigma_r - 0.513\,\sigma_r^2$

E	= 4137 MPa	(600000 psi)
Film Thickness	=	76 microns (3 mil)
Core Outer Radius	=	2.54 cm (1 in.)
Roll Outer Radius	=	10.16 cm (4 in.)
Winding Tension	=	230 MPa (1 pli)

Table 2 Data for Case Including Air Entrainment

Film Properties	Elastic Modulus of Film	=	4137 MPa	(600 Kpsi)
	Film Thickness	=	6 microns	(0.24 mils)
	Poisson's Ratio	=	0.15	
	OD of Roll	=	610 mm	(24 in.)
Film Surface Properties	First Stack Coefficient K_1	=	6.895 KPa	(1 psi)
	Second Stack Coef K_2	=	80	
	R_z	=	1 micron	(40 μ in.)
	Air Gap at 1 atm	=	1 micron	(40 μ in.)
Core Properties	Elastic Modulus	=	6140 MPa	(890 Kpsi)
	ID of Core	=	152.4 mm	(6 in.)
	OD of Core	=	203 mm	(8 in.)

Winding Tension	Roll Radius	Web Tension
(Linear Taper)	101 mm (4 in.)	88 N/m (0.5 pli)
	600 mm (24 in.)	17.5 N/m (0.1 pli)

Appendix 1: G Formulation Including Air Entrainment

The radial stresses inside a winding roll are supported by the contact of asperities on the web and by entrained air pressure. Mathematically this can be stated as

$$\sigma_r = \sigma_r(\text{asperities}) + \sigma_r(\text{air}) \qquad \text{(A-1)}$$

The asperity portion is handled using the stack compression results in a vacuum and the Pfieffer [2], exponential fit (see equation 6).

The air term requires input to define the amount of air entering the roll as each wrap is applied. Using the ideal gas law and assuming that the compression is isothermal leads to

$$\sigma_r(\text{air}) = P_{ao}\, h_{ao} \,/\, h \,, \quad \text{where} \quad h = R_z - \varepsilon_r(t + R_z) \qquad \text{(A-2)}$$

and

$$\sigma_r(\text{total}) = P_{ao}\, h_{ao} / \{ R_z - \varepsilon_r(t + R_z)\} - 1\ \text{atm} + K_1 \exp(K_2\, \varepsilon_r) - K_1 \qquad \text{(A-3)}$$

where G is the same as ε_r when only radial stresses are included. Equation A-3 is used along with equation 4 to solve for the radial stresses. The form of these equations demands an iterative approach.

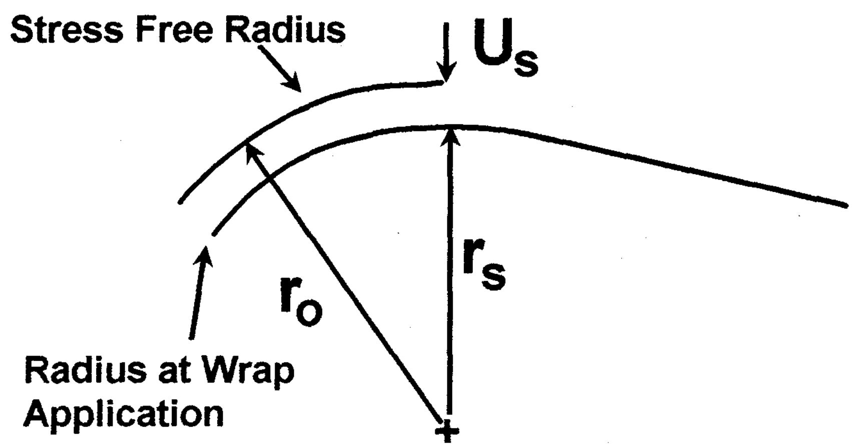

Figure 1 Geometry to Define Psuedo Stress

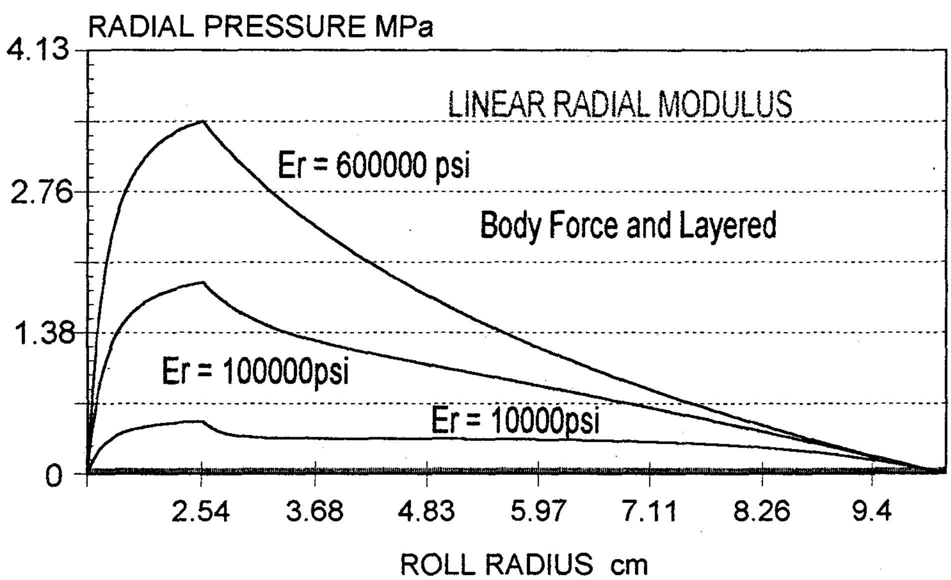

Figure 2 Comparison of Radial Pressures for Hakiel and Body Force Approaches for Linear Radial Stifness

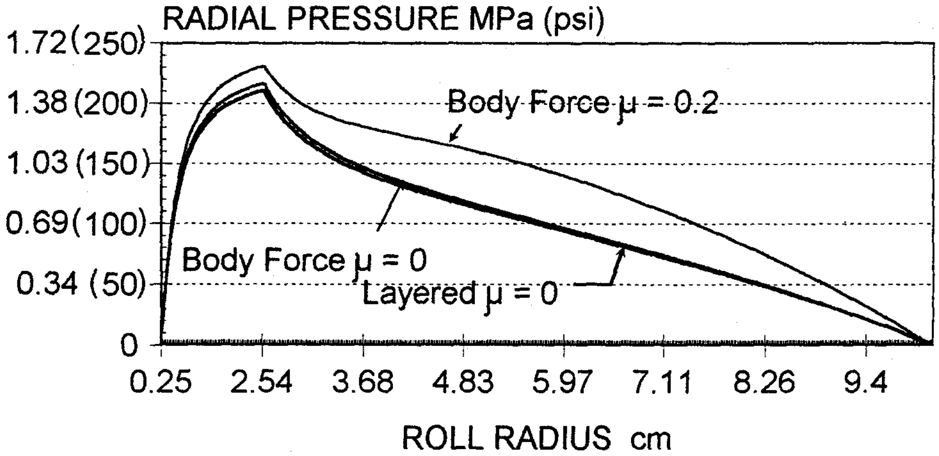

Figure 3 Comparison of Radial Pressures with Hakiel Results

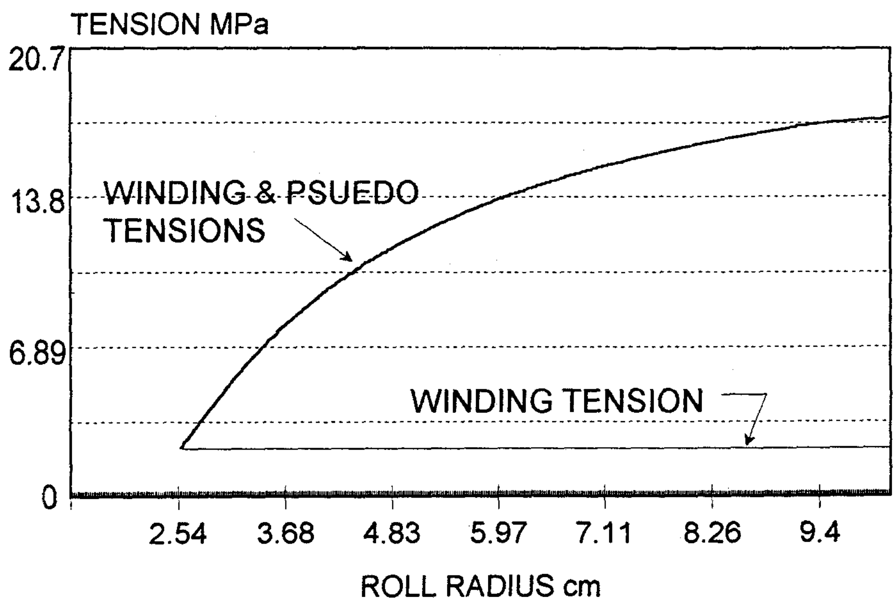

Figure 4 Winding and Psuedo Tensions for Hakiel Comparison

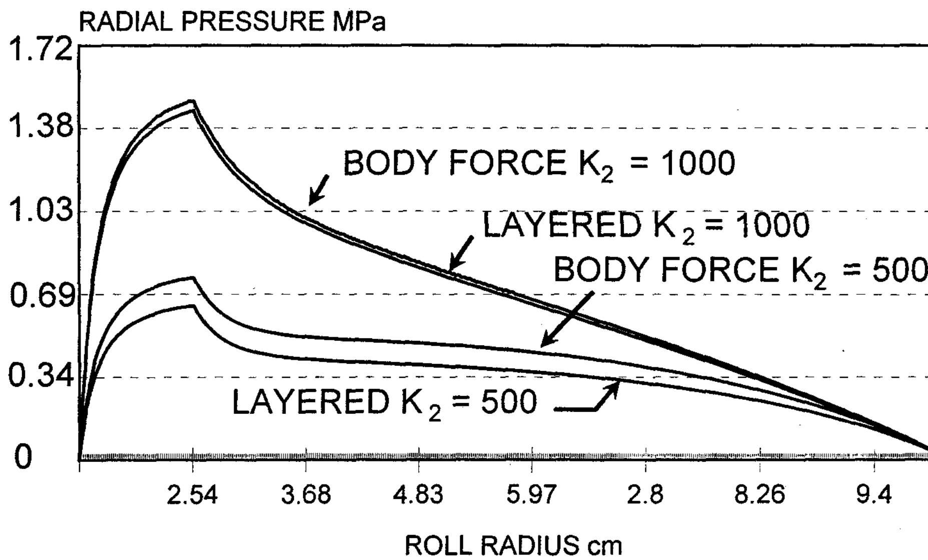

Figure 5 Radial Pressure Comparison for Cases with Differing Radial Stiffness

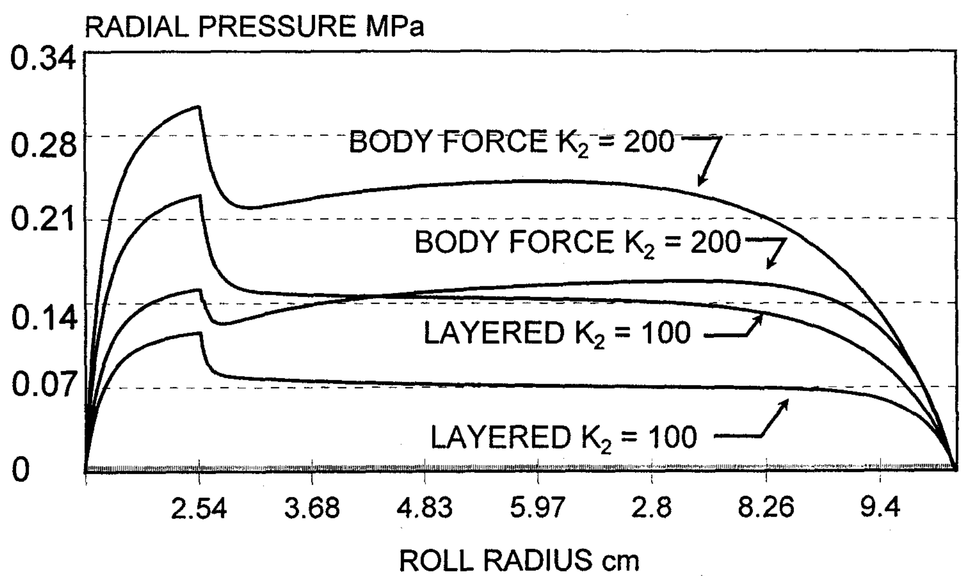

Figure 6 Radial Pressure Comparison for Cases with Differing Radial Stiffness

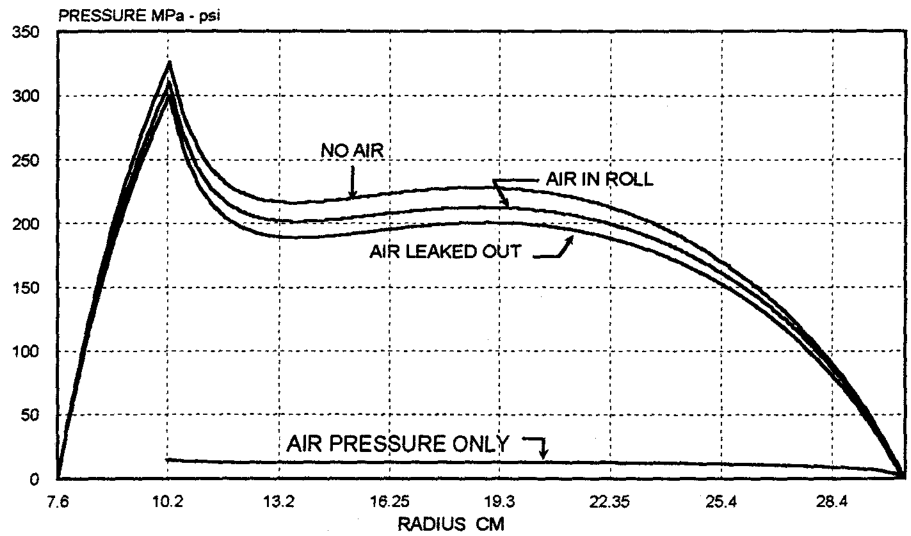

Figure 7 Radial Pressure Distribution for 6 micron Example Film

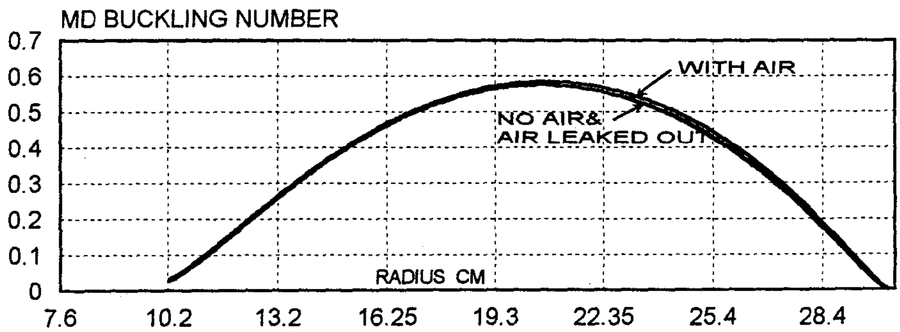

Figure 8 MD Buckling Number for 6 micron Example Film

A. W. Forrest Jr.

A Fully Nonlinear Solution to the Stresses Inside Wound Rolls

6/7/99 Session 1 11:25 – 11:50 a.m.

Question:
Did you present the radial data of the core center?

Answer – Al Forrest, Dupont
It shouldn't go below the core ID, the stress distribution peaks at the interface between the core and roll. I have a stress distribution of the roll and the core which is split out by the solution of the differential equation.

Question:
When you compare to Hakiel's model and you saw the difference at least for low K2. Did you try to compare it with a modified Hakiel's model, such as the idea of Pfeiffer and Good to introduce the radial deformation in the outer boundary condition?

Answer – Al Forrest, Dupont
No, I haven't

Questions:
Maybe its not the super-position that's the problem but the differential to the outer layer boundary condition.

Answer – Al Forrest, Dupont
Basically I was comparing apples to apples, there was no modifications made to any of the tensions of the outer wraps for either case and you would expect them to be identical under those circumstances.

Comment - David Pfieffer, JDP Innovations Inc.
My thought would be that you don't have to apologize for that initial distortion or displacement on the circumference in your model as it has nothing to do with your situation. The paper Keith Good, Matt Giachetto, and I worked on was something different. It tried to compare actual wound roll results to theoretical models, and we were always 50% short of stress, compressive stress, over what the models did predict. This loss of tension was associated only with center winding in which there was no rider or lay-on roller.

Question - Bob Lucas, Beloit Corporation
You showed several attempts with different K2 factors, in some of the areas I work we may deal with k2 factors of 25 or 30. This would mean that your suggested numbers are wildly different with a layered approach. The reason I bring up the question is in light of many of pull tab tests that many people have done which have yielded a moderately good fit. There seems to be a large discrepancy, so with respect to your model have you done any pull tab test or other tests to verify conformance with the real world?

Answer – Al Forrest, Dupont
No I haven't done any of these tests. I took the basic problem which Zig Hakiel had included in his paper and I changed the K2 parameter. If you have some data to volunteer

that includes the winding materials and winding parameters, I would then run the model to verify it for other materials.

Questions – Zig Hakiel, Eastman Kodak
Have you tried varying the number steps in the layered solution? A suggestion is that if that number is high enough in that position there shouldn't be an issue.

Answer – Al Forrest, Dupont
Computer time has become very cheap and I computed each wrap.

Questions - Keith Good, Oklahoma State University
I was surprised by the slide where you showed results for Poisson's ratio of 0 to.2, there was a marked difference in the results. In my experience with Zig Hakiel's model the influence of Poisson's ratio is very minimal, I see very little impact of Poisson's ratio and it usually shows up in the core more than in any other position.

Questions - Wolfermann, Technical University of Munich
What is the reason of the difference the body forced model and layered model so big, is K2 is getting low?

Answer – Al Forrest, Dupont
The linear position and difference in calculation procedure.
This is a new model, it is relatively untested, there still might be some bugs in it.