DYNAMICS OF A WEB ACCUMULATOR

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ABSTRACT

A conventional accumulator with no driven rollers and all rollers identical is studied for avoidance of excessive tension, slackness and slippage on the rollers. The total force imposed by the web spans on the carriage is derived for the case of a constant acceleration of the web.

For the usual case of several rollers on the carriage, the translational kinetic energy of the carriage (including the moving rollers) is shown to be negligible in comparison to the rotational kinetic energy of all the rollers in the accumulator. This discovery allows design and analysis of the control scheme of an accumulator with knowledge only of the parameters of the rollers.

The time required for a constant acceleration of the carriage is shown to be a simple function of the acceptable difference of tension across the accumulator, the total change of velocity, the number of rollers, and J/R^2 of the rollers.

Equations are derived for the ratios of tension across each roller, allowing the choice of a rate of acceleration for avoidance of slippage on a roller. The first or last roller was found to not always be the critical one for slippage.

Equations are derived relating the balancing force on the carriage to the number of rollers and an acceptable magnitude of tension.

NOMENCLATURE

KE	kinetic energy of a roller
PE	potential energy in spans of a web
E	modulus of elasticity

F	force
J	mass moment of inertia
m	mass
N	identification number of the roller on the carriage farthest from the winder or unwinder
R	radius of the rollers
T	total web tension
t	thickness of the web
Δt_2	time of acceleration or deceleration
V	velocity
W	width of the web
ε	strain of the web
ω	angular velocity-radians per second

Subscripts

c	pertaining to the carriage
i	"initial", or normal operating condition
0,1,2,...,N	numbers identifying rollers and spans

INTRODUCTION

An accumulator as shown in Figures 1 and 2, unlike a dancer, is not usually used for active control of tension; however, it may be loaded by a force in opposition to the tensions from the multiple strands of web, in which case it behaves as a multiple-roller dancer. The velocity of an accumulator may be controlled instead of the force.

Accumulators are primarily used to allow stoppage of an unwinder for splicing or (less commonly) a winder for cutoff and changing to a new core, as shown in Figure 2, while the process continues at a constant velocity. For this large change of velocity of idlers in the accumulator, the disturbances to tension by the kinetic energy of the idlers may be a major problem. The time required for acceleration of the rollers may be large, making analysis as an impulse inappropriate.

This study does not include mass/spring vibration of the roller/web dynamic system, nor does it propose schemes for controlling accumulators. Instead, limits of acceleration and deceleration for limiting differences in tension across an accumulator and preventing major slippage are derived, for facilitating the initial design of accumulators, and for programming the accelerations for filling and emptying.

The tension changes outside the accumulator, particularly those associated with acceleration and deceleration of the roll and adjacent rollers, are governed by equations of elementary physics, and are therefore not considered in this report.

Vibration of rollers caused by the sudden change in tension because of a sudden change in acceleration is not analyzed in this report. In addition to the change of tension because of acceleration, tension rises in the downstream direction because of friction of the bearings, hysteresis of the web material, and drag of the surrounding air.

ANALYSIS OF DYNAMICS

Velocities: In Figures 1 and 2, the number of moving rollers is (N + 1)/2. The number of spans to and from the moving rollers is N + 1, so that the mechanical advantage of the carriage is N + 1; that is, the force required to balance the steady-state tension is N + 1 times the tensile force, and the velocity of the carriage with a stationary web at the unwinder or winder is $V_i/(N + 1)$. In the following analysis, subscripts for tension and velocity in a span correspond to the number of the downstream roller for an unwinder, and the upstream roller for a winder. Odd-numbered rollers are on the carriage, and even-numbered are stationary.

Velocities of spans with a stationary web at the unwinder or winder are

$$
\begin{aligned}
&V_1 = 0\\
&V_2 = 2V_c\text{, or}\\
&V_2 = 2V_i/(N + 1)\\
&V_3 = 2V_i/(N + 1)\\
&V_4 = 2V_i/(N + 1) + 2V_c\text{, or}\\
&V_4 = 4V_i/(N + 1) \qquad (1)\\
&V_5 = 4V_i/(N + 1)\\
&V_6 = 6V_i/(N + 1)\\
&\cdot\\
&\cdot\\
&\cdot\\
&V_N = \frac{N-1}{N+1} V_i\\
&V_{N+1} = V_i.
\end{aligned}
$$

Angular velocities of rollers with a stationary web at the unwinder or winder when slippage is not occurring, assuming that all rollers have the same radius R, are

$$
\begin{aligned}
&\omega_1 = V_c/R\text{, or}\\
&\omega_1 = V_i/[R(N + 1)]\\
&\omega_2 = 2V_i/[R(N + 1)]\\
&\omega_3 = V_c/R + 2V_i/[R(N + 1)]\text{, or}\\
&\omega_3 = 3V_i/[R(N + 1)]\\
&\omega_4 = 4V_i/[R(N + 1)] \qquad (2)\\
&\cdot\\
&\cdot\\
&\cdot
\end{aligned}
$$

$\omega_N = NV_i/[R(N+1)]$
$\omega_{N+1} = V_i/R.$

The changes in angular velocities of the rollers as the unwinder or winder velocity is changed from zero to V_i or V_i to zero are

$$\Delta\omega_o = \frac{V_i}{R}$$
$$\Delta\omega_1 = \frac{V_i}{R}\left(\frac{N}{N+1}\right)$$
$$\Delta\omega_2 = \frac{V_i}{R}\left(\frac{N-1}{N+1}\right)$$
$$\Delta\omega_3 = \frac{V_i}{R}\left(\frac{N-2}{N+1}\right) \tag{3}$$
$$\vdots$$
$$\Delta\omega_N = \frac{V_i}{R}\left(\frac{1}{N+1}\right)$$
$$\Delta\omega_{N+1} = 0.$$

Kinetic Energy: The change in rotational kinetic energy of a roller is $J\omega_i^2/2$ - $J\omega^2/2$, where ω is the angular velocity of the specific roller as given by equations (2). For identical rollers:

$$\Delta KE_1 = \frac{1}{2}J\left(\frac{V_i}{R}\right)^2 - \frac{1}{2}J\left(\frac{V_i}{R(N+1)}\right)^2, \text{ or}$$
$$\Delta KE_1 = \frac{1}{2}\frac{J}{R^2}V_i^2\left[1-\left(\frac{1}{N+1}\right)^2\right]$$
$$\Delta KE_2 = \frac{1}{2}\frac{J}{R^2}V_i^2\left[1-\left(\frac{2}{N+1}\right)^2\right]$$
$$\Delta KE_3 = \frac{1}{2}\frac{J}{R^2}V_i^2\left[1-\left(\frac{3}{N+1}\right)^2\right] \tag{4}$$
$$\Delta KE_4 = \frac{1}{2}\frac{J}{R^2}V_i^2\left[1-\left(\frac{4}{N+1}\right)^2\right]$$
$$\vdots$$

$$\Delta KE_N = \frac{1}{2}\frac{J}{R^2}V_i^2\left[1-\left(\frac{N}{N+1}\right)^2\right].$$

In equations (4), the expression in the brackets is 0.75 for one roller (a common dancer). If N = 3 (a two-roller dancer), the sum of the terms in the brackets for the three rollers is 2.125. For a larger number of rollers in an accumulator, the total kinetic energy change in stopping or restarting is approximately (with 4 percent error for N = 5 and progressively more accurate for a larger number of rollers):

$$\Delta KE = \frac{1}{3}\frac{J}{R^2}NV_i^2. \tag{5}$$

The potential energy stored as strain energy in the length of web in the accumulator at operating conditions is

$$PE_i = \frac{1}{2}\frac{T_i^2 L_T}{EtW}. \tag{6}$$

The ratio of ΔKE to PE_i is an indicator of the permissible abruptness of stopping or restarting the web:

$$\frac{\Delta KE}{PE_i} = \frac{2}{3}N\frac{J}{R^2}\frac{V_i^2 EtW}{T_i^2 L_T},$$

or

$$\frac{\Delta KE}{PE_i} = \frac{2}{3}N\frac{J}{mR^2}\frac{mV_i^2}{L_T EtW}\frac{1}{\varepsilon_i^2}. \tag{7}$$

Note that m is the mass of one roller, but L_T is the total length of all the spans in the accumulator and between neighboring idlers. Even so, the parameter $mV_i^2/L_T EtW$ is usually larger than ε_i^2 for high-speed lines, for which case equation (7) is somewhat larger than unity, meaning that the web cannot then be abruptly stopped or restarted without a high stress in the web or slackness in spans within the accumulator, and/or slippage between the web and the rollers.

The translational kinetic energy of the carriage and its mounted rollers when $V_o = 0$ is

$$KE_c = \frac{1}{2}m_c V_c^2, \text{ or}$$

$$KE_c = \frac{1}{2} m_c \left(\frac{V_i}{N+1} \right)^2 . \tag{8}$$

If the mass of the moving rollers and the carriage framework and other traveling appurtenances is approximately equal to the total mass of all rollers, Nm, equation (8) becomes

$$KE_c \approx \frac{1}{2} m \frac{N}{(N+1)^2} V_i^2 . \tag{9}$$

With J/R^2 in equation (5) equal to approximately 0.75m for the usual roller:

$$\Delta KE \approx \frac{1}{4} mN V_i^2 . \tag{10}$$

The above approximations are usually realistic enough that the following simple ratio proves that the effect of translational energy is negligible in comparison to rotational energy, if N is greater than 5, which is the usual case except for a horizontal looper in the metals industry, which may consist of a single roller on a rail car.

$$\frac{KE_c}{\Delta KE} \approx \frac{2}{(N+1)^2} . \tag{11}$$

The permissible acceleration of the web or of the carriage is therefore primarily determined by the inertia of the rollers, not the mass of the carriage. The following derivations will aid in determining the maximum acceleration for avoidance of overstressing the web or slippage of the web on the rollers.

Time to Accelerate: If the acceleration or deceleration of the web is limited by the stress (or tension) in the web, the time period of a constant acceleration can be determined. If T_0 and T_1 are the tensions upstream and downstream from idler 0 of Figure 1, the torque is equal to $(T_1 - T_0)R$ for a positive acceleration, and the acceleration dV/dt is

$$\frac{dV}{dt} = (T_1 - T_0)\frac{R^2}{J} . \tag{12}$$

During deceleration, T_0 is greater than T_1, so that dV/dt is negative.

The first idler after the unwinder, idler 0, must accelerate from a surface velocity of zero to a velocity of V_i. The tension difference $T_1 - T_0$ which is maintained during a constant acceleration in the time period Δt_2 is

$$T_1 - T_0 = \frac{\Delta V_0}{\Delta t_2} \frac{J}{R^2} . \tag{13}$$

The other idlers accelerate to the angular velocities given by equations (2) during the time Δt_2. The tension differences across these rollers are proportional to these lesser accelerations, and therefore are less than $T_1 - T_0$ of equation (13):

$$
\begin{aligned}
T_2 - T_1 &= \frac{\Delta V_0}{\Delta t_2}\frac{N}{N+1}\frac{J}{R^2} \\
T_3 - T_2 &= \frac{\Delta V_0}{\Delta t_2}\frac{N-1}{N+1}\frac{J}{R^2} \\
T_4 - T_3 &= \frac{\Delta V_0}{\Delta t_2}\frac{N-2}{N+1}\frac{J}{R^2} \\
&\;\;\vdots \\
T_N - T_{N-1} &= \frac{\Delta V_0}{\Delta t_2}\frac{2}{N+1}\frac{J}{R^2} \\
T_{N+1} - T_N &= \frac{\Delta V_0}{\Delta t_2}\frac{1}{N+1}\frac{J}{R^2}\,.
\end{aligned} \tag{14}
$$

The total tension difference between T_0 to T_N caused by the acceleration in the time Δt_2 is the sum of the above equations:

$$T_{N+1} - T_0 = \frac{\Delta V_0}{\Delta t_2}\frac{J}{R^2}\left[1+\frac{N}{N+1}+\frac{N-1}{N+1}+\frac{N-2}{N+1}+...+\frac{2}{N+1}+\frac{1}{N+1}\right],$$

with N + 1 terms in the brackets. The summation of the terms in the brackets is (N + 2)/2, so that the previous equation can be written as

$$T_{N+1} - T_0 = \frac{\Delta V_0}{\Delta t_2}\frac{J}{R^2}\left(\frac{N+2}{2}\right). \tag{15}$$

The simplicity of equation (15) allows calculation of Δt_2 based on the chosen total tension difference across the accumulator:

$$\Delta t_2 = \frac{\Delta V_0}{T_{N+1} - T_0}\frac{J}{R^2}\left(\frac{N+2}{2}\right). \tag{16}$$

In equations (13) through (15) for a deceleration, ΔV_0 is negative and $(T_0 - T_{N+1})$, $(T_0 - T_1)$, etc., are positive, for numerically equal results.

Equations (14) show the tension differences to decrease (for a constant acceleration of the web from an unwinder) for each span in the downstream direction.

For the accumulator at the winder end of the process (Figure 2), equations (12) through (16) are correct except for a reversal of signs, or a reversal of the modes of acceleration and deceleration. For example, equation (16) becomes

$$\Delta t_2 = \frac{\Delta V_0}{T_0 - T_{N+1}} \frac{J}{R^2}\left(\frac{N+2}{2}\right). \tag{16a}$$

<u>Slippage in Accumulator:</u> For a study of slippage between the web and the rollers, the ratios of tensions across rollers are needed, instead of the differences given by equations (13) and (14). The first step in finding the ratios of individual tensions is to express the tension differences as ratios:

$$\begin{aligned}
&\frac{T_2 - T_1}{T_1 - T_0} = \frac{N}{N+1} \\
&\frac{T_3 - T_2}{T_2 - T_1} = \frac{N-1}{N} \\
&\frac{T_4 - T_3}{T_3 - T_2} = \frac{N-2}{N-1} \\
&\cdot \\
&\cdot \\
&\cdot \\
&\frac{T_{N-1} - T_{N-2}}{T_{N-2} - T_{N-3}} = \frac{3}{4} \\
&\frac{T_N - T_{N-1}}{T_{N-1} - T_{N-2}} = \frac{2}{3} \\
&\frac{T_{N+1} - T_N}{T_N - T_{N-1}} = \frac{1}{2}.
\end{aligned} \tag{17}$$

From equations (13) and (15):

$$\frac{T_1 - T_0}{T_{N+1} - T_0} = \frac{2}{N+2}. \tag{18}$$

Equations (17) and (18), along with the trivial equation

$$T_0 = T_{N+1} - (T_{N+1} - T_0), \tag{19}$$

can be used for solving for individual tensions in terms of T_{N+1} and $(T_{N+1} - T_0)$, which would be chosen by considerations of limitations of the process and the web material. The resulting equations are:

$$T_1 = T_{N+1} - (T_{N+1} - T_0)\left(\frac{N}{N+2}\right),$$

$$T_2 = T_{N+1} - (T_{N+1} - T_0)\left[\frac{N(N-1)}{(N+1)(N+2)}\right],$$

$$T_3 = T_{N+1} - (T_{N+1} - T_0)\left[\frac{(N-1)(N-2)}{(N+1)(N+2)}\right],$$

$$T_4 = T_{N+1} - (T_{N+1} - T_0)\left[\frac{(N-2)(N-3)}{(N+1)(N+2)}\right],$$

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$$T_{N-3} = T_{N+1} - (T_{N+1} - T_0)\left[\frac{(5)(4)}{(N+1)(N+2)}\right], \tag{20}$$

$$T_{N-2} = T_{N+1} - (T_{N+1} - T_0)\left[\frac{(4)(3)}{(N+1)(N+2)}\right],$$

$$T_{N-1} = T_{N+1} - (T_{N+1} - T_0)\left[\frac{(3)(2)}{(N+1)(N+2)}\right],$$

$$T_N = T_{N+1} - (T_{N+1} - T_0)\left[\frac{(2)(1)}{(N+1)(N+2)}\right],$$

and obviously

$$T_{N+1} = T_{N+1} - (T_{N+1} - T_0)(0).$$

The ratios of the tensions as expressed in equations (20) are:

$$\frac{T_o}{T_1} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N+1)(N+2)\right]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N)(N+1)\right]},$$

$$\frac{T_1}{T_2} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[N(N+1)\right]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N-1)(N)\right]},$$

$$\frac{T_2}{T_3} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N-1)(N)\right]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N-2)(N-1)\right]},$$

$$\frac{T_3}{T_4} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N-2)(N-1)\right]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(N-3)(N-2)\right]},$$

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$$\frac{T_{N-3}}{T_{N-2}} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(5)(4)\right]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right]\left[(4)(3)\right]}, \tag{21}$$

$$\frac{T_{N-2}}{T_{N-1}} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right][(4)(3)]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right][(3)(2)]},$$

$$\frac{T_{N-1}}{T_N} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right][(3)(2)]}{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right][(2)(1)]},$$

and

$$\frac{T_N}{T_{N+1}} = \frac{(N+1)(N+2) - \left[\left(T_{N+1} - T_0\right)/T_{N+1}\right][(2)(1)]}{(N+1)(N+2)}.$$

If breakaway slippage is to be avoided all of the ratios in equation (21) for a decelerating web in an unwinder accumulator or an accelerating web in a winder accumulator must be less than $e^{\pi\mu}$, where μ is the coefficient of friction. For an accelerating web in an unwinder accumulator or a decelerating web in a winder accumulator, the reciprocals of equation (21) must all be less than $e^{\pi\mu}$ for prevention of slippage. The coefficients of friction vary with velocity, but all velocities are identical as the deceleration of the web is initiated, or as the acceleration is completed. Unfortunately, a cursory inspection of equations (21) does not reveal the critical (maximum) ratio, and for some values of $(T_0 - T_{N+1})/T_{N+1}$ neither the first nor the last of the accelerating rollers is the critical one for slippage.

If T_0 is to be maintained during acceleration of a winding roll for obtaining a constant wound-in tension, T_0 is a more convenient variable than T_{N+1} in equations (20) and (21). The other variable will be $(T_0 - T_{N+1})$, which is positive for a positive acceleration of the web in a winder accumulator. From equations (17) and (18):

$$T_1 = T_0 - (T_0 - T_{N+1})\left(\frac{2}{N+2}\right)$$

$$T_2 = T_0 - (T_0 - T_{N+1})\left[\frac{2(2N+1)}{(N+1)(N+2)}\right]$$

$$T_3 = T_0 - (T_0 - T_{N+1})\left[\frac{6N}{(N+1)(N+2)}\right]$$

$$T_4 = T_0 - (T_0 - T_{N+1})\left[\frac{4(2N-1)}{(N+1)(N+2)}\right]$$

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$$T_{N-3} = T_0 - (T_0 - T_{N+1})\left[\frac{(N-3)(N+6)}{(N+1)(N+2)}\right] \qquad (22)$$

$$T_{N-2} = T_0 - (T_0 - T_{N+1})\left[\frac{(N-2)(N+5)}{(N+1)(N+2)}\right]$$

$$T_{N-1} = T_0 - (T_0 - T_{N+1})\left[\frac{(N-1)(N+4)}{(N+1)(N+2)}\right]$$

and

$$T_N = T_0 - (T_0 - T_{N+1})\left[\frac{N(N+3)}{(N+1)(N+2)}\right].$$

Tension ratios similar to equations (21) are:

$$\frac{T_0}{T_1} = \frac{(N+1)(N+2)}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[2(N+1)\right]}$$

$$\frac{T_1}{T_2} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[2(N+1)\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[2(2N+1)\right]}$$

$$\frac{T_2}{T_3} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[2(2N+1)\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[6N\right]}$$

$$\frac{T_3}{T_4} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[6N\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[4(2N-1)\right]}$$

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$$\frac{T_{N-3}}{T_{N-2}} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[(N-3)(N+6)\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[(N-2)(N+5)\right]} \qquad (23)$$

$$\frac{T_{N-2}}{T_{N-1}} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[(N-2)(N+5)\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[(N-1)(N+4)\right]}$$

$$\frac{T_{N-1}}{T_N} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[(N-1)(N+4)\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[N(N+3)\right]}$$

and

$$\frac{T_N}{T_{N+1}} = \frac{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[N(N+3)\right]}{(N+1)(N+2) - \left[\left(T_0 - T_{N+1}\right)/T_0\right]\left[(N+1)(N+2)\right]}.$$

Tension ratios for an accumulator with N = 19 are shown in Tables 1 and 2. For decelerations and accelerations for which $(T_{N+1} - T_0)/T_{N+1}$ is between -1.0 and 1.0 (the latter being the impractical case in which $T_0 = 0$), the maximum ratio is T_0/T_1 or T_1/T_0. Note that the individual tension ratios in the two tables are reciprocals of each other to facilitate a simple comparison to the limiting value of $e^{\pi\mu}$.

The following equations for T_0/T_1 in terms of N and $(T_{N+1} - T_0)/T_{N+1}$, and T_1/T_0 in terms of N and $(T_0 - T_{N+1})/T_0$ (including negative values of tension differences) are

$$\frac{T_0}{T_1} = \frac{\left(\frac{T_{N+1} - T_0}{T_{N+1}} - 1\right)(N+2)}{\left(\frac{T_{N+1} - T_0}{T_{N+1}} - 1\right)(N) - 2} \tag{24}$$

and

$$\frac{T_1}{T_0} = \frac{N + 2\left(1 - \frac{T_0 - T_{N+1}}{T_0}\right)}{N+2}. \tag{25}$$

Equation (24) an (25) are simpler than the first of equations (21) and (23). If $(T_{N+1} - T_0)/T_{N+1}$ is more negative than -1 (T_0/T_{N+1} greater than 2), however, T_0/T_1 may not represent the maximum tension ratio, as demonstrated in Table 1.

Table 2 shows the maximum tension ratio for an accelerating unwinder or decelerating winder is greater than for a decelerating unwinder or an accelerating winder of Table 1, if certain maximum and minimum tensions are maintained. For example, if the maximum tension is 200 newtons and the minimum is 100 newtons, T_0 = 100 newtons and T_{N+1} = 200 newtons for an accelerating unwinder or decelerating winder, and Table 2 shows the maximum tension ratio to be 1.09524. For a decelerating unwinder or accelerating winder with T_0 = 200 newtons and T_{N+1} = 100 newtons, Table 1 shows the maximum tension ratio to be 1.05000.

If the same acceleration as in above example is maintained (resulting in a tension difference of 100 newtons across the accumulator), but a constant winding tension of 200 newtons is desired, T_0 = 200 newtons and T_{N+1} = 300 newtons during deceleration. Table 2 $[(T_0 - T_{N+1})/T_0 = -1/2]$ shows a maximum tension ratio of 1.04762 during deceleration, less than the ratio of 1.05000 for T_0 = 200 newtons and T_{N+1} = 100 newtons $[(T_{N+1} - T_0)/T_{N+1} = -1]$ from Table 1 for the accelerating winder.

The above example for limits on maximum and minimum tension of 200 newtons and 100 newtons would require a coefficient of friction of 0.029 to avoid slippage, while the example of a constant winding tension of 200 newtons would require a coefficient of friction of 0.016 for avoidance of slippage.

If the time of acceleration of an unwinder or deceleration of a winder were cut in half from the first example, yet if 200 newtons were maintained as a maximum, the minimum tension would be zero and slippage would be inevitable. If $(T_0 - T_{N-1})/T_0$ were -8 in Table 2 in an attempt to reduce the period of acceleration as much as practical (T_0 = 22.22 newtons and T_{N+1} = 200 newtons), the coefficient of friction required for prevention of slippage would be 0.18, a very high value for a high-speed, non-permeable web.

Equations for conversion between the two tension variables in equations (21) and (23) are:

$$\frac{T_0 - T_{N+1}}{T_0} = \frac{\dfrac{T_{N+1} - T_0}{T_{N+1}}}{\dfrac{T_{N+1} - T_0}{T_{N+1}} - 1} \tag{26}$$

and

$$\frac{T_{N+1} - T_0}{T_{N+1}} = \frac{\dfrac{T_0 - T_{N+1}}{T_0}}{\dfrac{T_0 - T_{N+1}}{T_0} - 1}. \tag{27}$$

Forces on Carriage: The sum of the N+1 tensile forces acting on the carriage, from equations (20), is

$$F_c = (N+1)(T_{N+1}) - \left[\frac{T_{N+1} - T_0}{(N+1)(N+2)}\right][(N+1)(N) + (N)(N-1)$$
$$+ (N-1)(N-2) + (N-2)(N-3) + \ldots + (4)(3)+(3)(2) + (2)(1) + 0].$$

The above equation simplifies to

$$F_c = (N+1)(T_{N+1}) - (N/3)(T_{N+1}-T_0). \tag{28}$$

If the carriage is stationary or moving at a constant velocity, T_0 and T_{N+1} have the same magnitude, equal to $F_c/(N+1)$. If the carriage is balanced by a force (not velocity-controlled), equation (28) shows that the force on the carriage must be reduced by $(N/3)(T_{N+1}-T_0)$ during acceleration and increased by this amount during deceleration of an unwinding web if T_{N+1} is to be maintained at its steady-state value. Such management of the force on the carriage is particularly important in process lines which imply tensions from motor currents, as commonly practiced in the steel industry.

Rearrangement of equation (B38) results in equations which are useful for various schemes of control of tensions and the counterbalance force on the carriage:

$$T_0 = \frac{3}{N} F_c - \frac{2N+3}{N} T_{N+1} \tag{29}$$

$$T_{N+1} = \frac{3}{2N+3} F_c - \frac{N}{2N+3} T_0 \tag{30}$$

and

$$F_c = \left(\frac{2}{3}N + 1\right) T_{N+1} + \frac{N}{3} T_0. \tag{31}$$

<u>Example of Tensions during Acceleration and Deceleration</u>: Table 3 shows example tensions in the web spans of an accumulator with N = 9 (five rollers on the carriage), for an acceleration or deceleration such that the total tension across the accumulator is 20 percent of the operating tension. The table can be for any value of tension or units if T_0 or T_{10} of 100 is considered to be 100 percent of a reference tension, and if $(T_{N+1} - T_0)/T_{N+1}$ or $(T_0 - T_{N+1})/T_0$ are a fraction as in the table. The other numbers in the row are then percentages of the reference tension.

A specific example of a metal-handling winder accumulator demonstrated by Table 3 follows: J/R^2 = 500 kg (m probably approximately 700 kg), T_i = 8000 newtons, $T_{N+1} - T_0$ = 1600 newtons during deceleration and $T_0 - T_{N+1}$ = 16000 newtons during acceleration, and V_i = 20 meters per second. From equation (16) or (16a):

$$\Delta t_2 = (20\ \text{m/sec})(500\ \text{kg}/1600\ \text{newtons})(11/2)/(1\ \text{m - kg/newton - sec}^2)$$
$$= 34.4\ \text{sec.}$$

Table 3 shows that, for the above control of winding tension to a constant value of 8000 newtons, during acceleration the force on the carriage must be (8.60)(8000 newtons) = 68,800 newtons, and (11.40)(8000 newtons) = 91,200 newtons during deceleration, instead of the 80,000 newtons while running at a constant velocity. The force on the carriage would also be 80,000 newtons while the winder is stopped and the accumulator is filling while travelling at the constant velocity of $V_i/(N+1)$, or while the accumulator is travelling at any other constant velocity in preparation for another stoppage of the winder.

The ratio of tensions T_o/T_1 during acceleration is 100.00/96.36 = 1.0378, and during deceleration the ratio T_1/T_o is 103.64/100.00 = 1.0364. Avoidance of slippage during acceleration would require a coefficient of friction of 0.012.

The average velocity of the web during the acceleration or deceleration is 10 m per second. The carriage therefore travels (10 m/sec)(34.4 sec)/10 = 34.4 meters during acceleration or deceleration. The carriage travels 2.0 meters per second while the winder is stopped; therefore, the accumulator tower must be very tall.

The travel of the accumulator while the winder is stopped can be reduced to 1.0 meter per second by doubling the number of rollers on the carriage to 10 ($N = 19$), for which case Tables 1 and 2 give selected tension ratios. If the same 20 percent of the winder tension is maintained as the difference across the accumulator, the time for acceleration or deceleration is 65.6 seconds, nearly twice the time required for the accumulator with $N = 9$, and the travel of the carriage during acceleration or deceleration is improved only to 32.8 meters. However, the travel can be reduced to approximately 50 percent of this amount if slippage is the limiting criterion of design instead of the total difference of tension across the accumulator, by doubling the acceleration and deceleration and thereby having a tension difference of 40 percent of the winder tension.

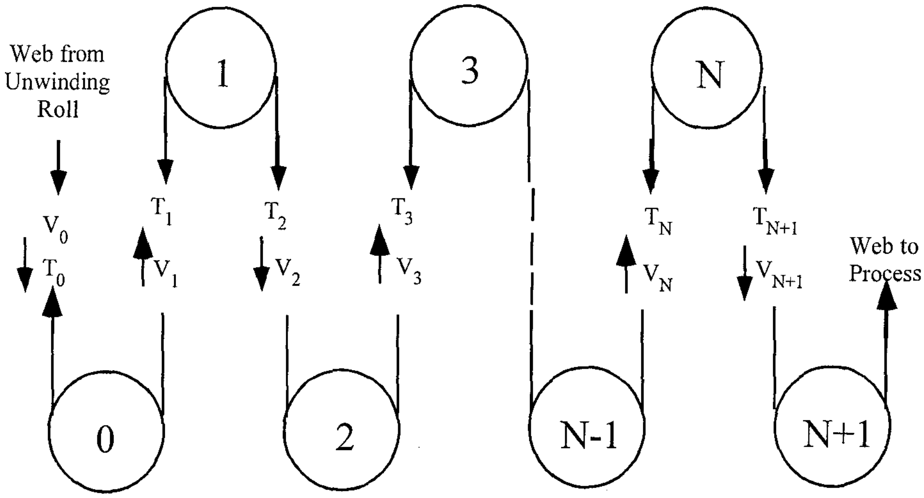
CONCLUSIONS AND RECOMMENDATIONS

Accumulators used for takeup and letoff of a web for stationary splicing at an unwinder or for changing to an empty core at a winder require a period of acceleration and deceleration for prevention of excessive tension, slackness, and web-to-roller slippage, except for low-speed lines with low-mass rollers. This report derives equations for prediction of (1) the total tension difference across the accumulator because of acceleration of all the rollers, for prevention of excessive stress in the web or unmanageably low tensions, (2) tension ratios across rollers, for prevention of slippage if the coefficient of friction is known, and (3) forces on the carriage during acceleration, for design of the control system for the carriage.

In the initial design of an accumulator or in improvement of an existing accumulator, it is recommended that charts similar to Tables 1, 2, and 3 be constructed for the specific number of rollers and the tension differences across the accumulator which are believed to be acceptable. The accumulator travel and the forces on the carriage during acceleration can then be calculated.

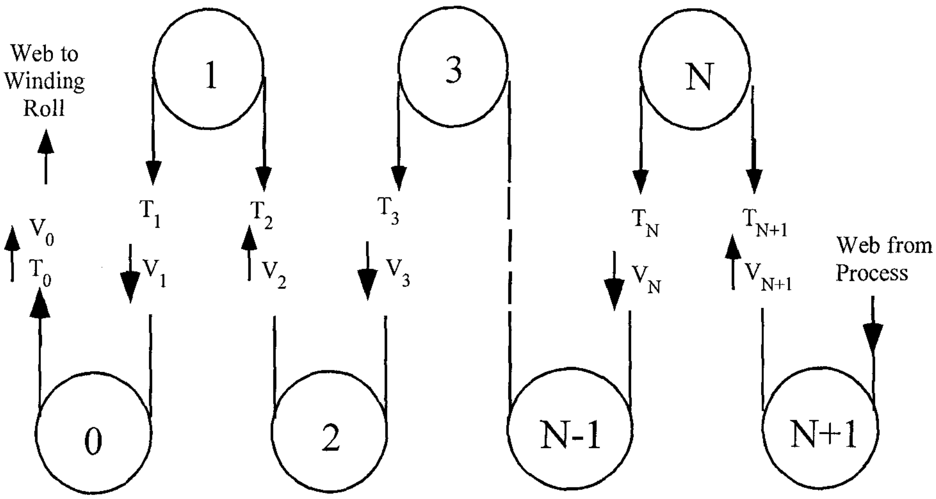
If all rollers are not identical as assumed in this report, or if several rollers are clustered either on the carriage or the ground (such as for a displacement type of web guide in the accumulator), the results of this report can be achieved by applying the same methods, but the resulting equations probably must be retained as a roller-by-roller series instead of in simplified forms, such as equations (15) and (28).

It should be noted that in the analysis of this report, as in all analyses of the acceleration of a roller by the web, the model of the inertia of the roller is its equivalent mass in translation, J/R^2, not its rotational inertia J, equal to (mass)(radius of gyration)^2.



Accumulator for Unwinder

FIGURE 1



Accumulator for Winder

FIGURE 2

TABLE 1

Tension Ratios for Accumulator with N = 19 – Deceleration of Unwinder or Acceleration of Winder (Maximum Values Underlined)

$\frac{T_{N+1} - T_0}{T_{N+1}}$	-8	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{8}$
$\frac{T_0 - T_{N+1}}{T_0}$	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{9}$
T_0/T_1	1.09249	1.08247	1.06780	$\underline{1.05000}$	$\underline{1.03279}$	$\underline{1.01942}$	$\underline{1.01070}$
T_1/T_2	1.09632	1.08501	1.06884	1.04987	1.03215	1.01879	1.01026
T_2/T_3	1.10042	1.08759	1.06977	1.04959	1.03141	1.01813	1.00982
T_3/T_4	1.10478	1.09019	1.07054	1.04913	1.03058	1.01742	1.00936
T_4/T_5	1.10940	1.09275	1.07111	1.04848	1.02963	1.01667	1.00889
T_5/T_6	1.11429	1.09524	$\underline{1.07143}$	1.04762	1.02857	1.01587	1.00840
T_6/T_7	1.11940	1.09756	$\underline{1.07143}$	1.04651	1.02740	1.01504	1.00791
T_7/T_8	1.12470	1.09962	1.07104	1.04514	1.02610	1.01416	1.00739
T_8/T_9	1.13008	1.10127	1.07018	1.04348	1.02469	1.01325	1.00687
T_9/T_{10}	1.13538	1.10233	1.06875	1.04151	1.02316	1.01229	1.00634
T_{10}/T_{11}	1.14035	$\underline{1.10256}$	1.06667	1.03922	1.02151	1.01130	1.00580
T_{11}/T_{12}	1.14458	1.10169	1.06383	1.03659	1.01974	1.01027	1.00524
T_{12}/T_{13}	1.14747	1.09938	1.06015	1.03361	1.01786	1.00922	1.00468
T_{13}/T_{14}	$\underline{1.14815}$	1.09524	1.05556	1.03030	1.01587	1.00813	1.00412
T_{14}/T_{15}	1.14545	1.08889	1.05000	1.02667	1.01379	1.00702	1.00354
T_{15}/T_{16}	1.13793	1.08000	1.04348	1.02273	1.01163	1.00588	1.00296
T_{16}/T_{17}	1.12403	1.06838	1.03604	1.01852	1.00939	1.00473	1.00237
T_{17}/T_{18}	1.10256	1.05405	1.02778	1.01408	1.00709	1.00356	1.00178
T_{18}/T_{19}	1.07339	1.03738	1.01887	1.00948	1.00475	1.00238	1.00119
T_{19}/T_{20}	1.03810	1.01905	1.00952	1.00476	1.00238	1.00119	1.00060
T_0/T_{20}	9.00000	5.00000	3.00000	2.0000	1.50000	1.25000	1.12500

TABLE 2

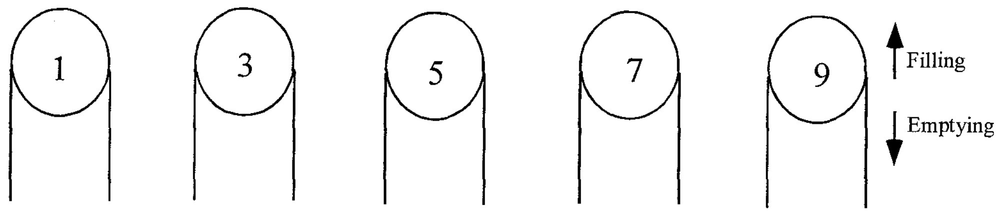
Tension Ratios for Accumulator with N = 19 – Acceleration of Unwinder or Deceleration of Winder (Maximum Values Underlined)

$\frac{T_0 - T_{N+1}}{T_0}$ $\frac{T_{N+1} - T_0}{T_{N+1}}$	-8 $\frac{8}{9}$	-4 $\frac{4}{5}$	-2 $\frac{2}{3}$	-1 $\frac{1}{2}$	$-\frac{1}{2}$ $\frac{1}{3}$	$-\frac{1}{4}$ $\frac{1}{5}$	$-\frac{1}{8}$ $\frac{1}{9}$
T_{20}/T_{19}	1.00425	1.00382	1.00318	1.00239	1.00159	1.00095	1.00053
T_{19}/T_{18}	1.00857	1.00771	1.00641	1.00480	1.00319	1.00191	1.00106
T_{18}/T_{17}	1.01303	1.01170	1.00971	1.00725	1.00481	1.00287	1.00159
T_{17}/T_{16}	1.01768	1.01584	1.01311	1.00976	1.00645	1.00385	1.00213
T_{16}/T_{15}	1.02260	1.02020	1.01667	1.01235	1.00813	1.00483	1.00267
T_{15}/T_{14}	1.02787	1.02484	1.02041	1.01504	1.00985	1.00583	1.00321
T_{14}/T_{13}	1.03361	1.02985	1.02439	1.01786	1.01163	1.00685	1.00376
T_{13}/T_{12}	1.03995	1.03532	1.02867	1.02083	1.01347	1.00789	1.00431
T_{12}/T_{11}	1.04706	1.04138	1.03333	1.02400	1.01538	1.00896	1.00488
T_{11}/T_{10}	1.05517	1.04819	1.03846	1.02740	1.01739	1.01005	1.00545
T_{10}/T_9	1.06461	1.05598	1.04418	1.03107	1.01950	1.01118	1.00603
T_9/T_8	1.07583	1.06504	1.05063	1.03509	1.02174	1.01235	1.00662
T_8/T_7	1.08950	1.07580	1.05804	1.03951	1.02412	1.01356	1.00723
T_7/T_6	1.10667	1.08889	1.06667	1.04444	1.02667	1.01481	1.00784
T_6/T_5	1.12903	1.10526	1.07692	1.05000	1.02941	1.01613	1.00847
T_5/T_4	1.15960	1.12648	1.08939	1.05634	1.03239	1.01751	1.00912
T_4/T_3	1.20420	1.15525	1.10494	1.06367	1.03564	1.01895	1.00979
T_3/T_2	1.27586	1.19672	1.12500	1.07229	1.03922	1.02048	1.01047
T_2/T_1	1.41081	1.26207	1.15200	1.08261	1.04318	1.02209	1.01118
T_1/T_0	<u>1.76190</u>	<u>1.38095</u>	<u>1.19048</u>	<u>1.09524</u>	<u>1.04762</u>	<u>1.02381</u>	<u>1.01190</u>
T_{20}/T_0	9.0000	5.00000	3.00000	2.00000	1.50000	1.25000	1.12500

TABLE 3

Example of Tensions during Constant Acceleration or Deceleration of Accumulator with N = 9
(A) Acceleration of Winder or Deceleration of Unwinder (B) Deceleration of Winder or Acceleration of Unwinder

Carriage Rollers

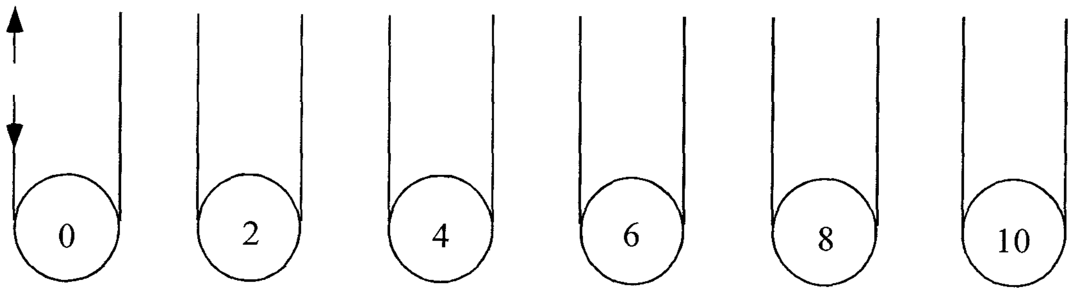


$\frac{T_{N+1} - T_0}{T_{N+1}}$	$\frac{T_0 - T_{N+1}}{T_0}$	Summation of Tensions on Carriage	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	
-1/5	1/6	1060.0	120.00	116.36	113.09	110.18	107.64	105.45	103.64	102.18	101.09	100.36	100.00	(A)
-1/4	1/5	860.0	100.00	96.36	93.09	90.18	87.64	85.45	83.64	82.18	81.09	80.36	80.00	(A)
1/5	-1/4	940.0	80.00	83.64	86.91	89.82	92.36	94.55	96.36	97.82	98.91	99.64	100.00	(B)
1/6	-1/5	1140.0	100.00	103.64	106.91	109.82	112.36	114.55	116.36	117.82	118.91	119.64	120.00	(B)

Winder Direction

Unwinder Direction

Stationary Rollers



John J. Shelton
Dynamics of a Web Accumulator
6/8/99 Session 2 12:45 - 1:10 a.m.

Question –Albert Forrest, DuPont
Have you experienced accumulator problems in a plant environment?

Answer – John Shelton, Oklahoma State University
I have little personal experience with accumulators, but have heard of rollers continuing to turn after the web has stopped, and an extreme case of the web hanging in free loops with no contact with the lower rollers. Further, an accumulator is a mass/spring system with the rollers acting as masses and the web spans as springs, but such dynamic behavior was not analyzed in this paper. The constant acceleration analyzed in this paper should be helpful in design of new accumulators and in troubleshooting of existing accumulators.

Question – Brian Boulter, Rockwell Automation
We have studied accumulators in the lab, but have complaints from plants, particularly when huge rollers with radii of 1.0 to 3.0 feet are used with cables positioning the carriage.

Comment – Bruce Feiertag, OSU
I haven't seen a large accumulator in the steel industry which doesn't have a 50 percent variation in tension when it operates.

Answer – John Shelton, Oklahoma State University
In addition to the web tension problems caused by the acceleration of large rollers and by imperfect control of the velocity or force of the cables, lateral problems are caused by tilting of the carriage because of off-center or cambered webs acting on the springy cables.

Question – Mike Madaras, Goodyear
Please summarize optimum roller design for accumulators.

Answer – John Shelton, Oklahoma State University
A low mass of the shell of the roller is desirable for maximizing the rate of acceleration/deceleration while avoiding slippage and maintaining an acceptable tension difference across the accumulator. The low mass should be achieved with a thin wall and large diameter, so that stiffness in bending is also achieved. A high web-to-roller coefficient of friction is also desirable for avoidance of slippage.