

# THE STRESS FIELD IN A WEB DURING SLITTING —OPENING MODE

by

H. Lu<sup>1</sup> and C. Liu<sup>2</sup>

<sup>1</sup>Oklahoma State University

<sup>2</sup>Los Alamos National Laboratory

USA

## ABSTRACT

The dynamic stress field in a web during slitting is presented for the opening mode appropriate for razor slitting in the steady state. The integral equations for the general stress and strain fields in a web near a razor blade are solved for the in-plane stress components for a homogeneous, isotropic and linearly elastic web. A wedge-type razor blade profile is used in the analysis. A Coulomb type of friction relation between the shear and normal traction at the interface between the razor and the web is assumed. Analysis is performed to determine the stress components and the contour of effective stress. It is found that the stress distribution in the web can be separated into two parts. One part depends only on the web speed and the other part depends only on the slitter profile. A parametric study is carried out to determine the dependence of the contour of effective stress on the slitter blade profile, the friction coefficient at the web-slitter interface, the web speed during slitting, as well as the web tension. In the parametric study, it is assumed that the web material properties such as Young's modulus, yield strength and fracture toughness are independent of the strain rate or the web speed. The results on the dependence of the contour of effective stress on the slitting parameters reveal that the size of the permanent deformation zone in the slit edge becomes larger when any one of the slitting parameters among the web tension, the razor wedge-angle and the web speed increases while all the other slitting parameters and web material properties are fixed.

## NOMENCLATURE

$c_l$	Dilatational wave speed in the web
$c_R$	Rayleigh wave speed
$c_s$	Shear wave speed
$E$	Young's modulus of the web
$K_I$	Stress intensity factor in opening mode
$K_{IC}$	Fracture toughness
$u_\alpha$	In-plane displacement components
$v$	Web speed
$\lambda$	Coefficient of friction between the web and the razor slitter
$\mu$	Shear modulus
$\nu$	Poisson's ratio
$\Phi, \Psi$	Displacement potentials
$\sigma$	Normal traction between the web and the slitter
$\sigma_0$	Far field web tensile stress
$\sigma_e$	Effective stress
$\sigma_{\alpha\beta}$	In-plane stress components
$\tau$	Yield stress in shear
$\theta$	Wedge half-angle of a razor blade

## 1 INTRODUCTION

Slitting of a web is a process that converts a web into narrower webs. Web slitting may be considered as a process of a controlled crack propagation in a web material. A crack is initiated by a slitter blade and is propagating in a web under the guidance of a slitter blade. Razor slitting may be considered as an opening mode problem in the framework of fracture mechanics analysis; while shear slitting may be considered as an anti-plane shear mode problem.

Near the crack tip, there exists a high stress zone, where excessive inelastic deformation might occur. Damages, such as micro voids and micro cracks, most likely initiate and coalesce to form visible cracks for some web materials under certain slitting conditions. Part of the damage such as voids and cracks may remain in the wake of the crack, i.e., the slit edges, and produce deterioration in the slit quality, creating defective edges that contain defects such as debris, hairs, slivers and burrs, and possibly creating slit dust. Understanding the stress and strain distribution near the blade, its relation to the slit edge defects and how they affect the slit quality, dust formation, web winding quality following slitting etc. are therefore essential in understanding the slitting process.

Most publications on slitting in the open literature are restricted to qualitative investigations; quantitative investigation is very limited. The identification of appropriate slitting methods relies mostly on experience and trial

and error methods. Among the few quantitative investigations are: Meehan and Burns (1998), which measured the cutting force and determined the isochromatic stress lines in a polycarbonate sheet under a pair of forces aimed at the simulation of shear slitting. Kasuga *et al.* (1977), which investigated the shearing process of ductile materials; Arcona and Dow (1996), which determined a relation between the cutting force and the cutting speed for plastic films; Bollen (1989), which examined the shear cutting of PET film; Bax (1991), which investigated the slitting energy rate and blade forces; Zheng and Wierzbicki (1996), which derived a closed-form solution for the cutting force for a steady-state wedge cutting process.

In this paper, the general representation for the stress and strain fields in a web near a razor blade is used to solve for the in-plane stress components for a homogeneous, isotropic and linearly elastic web under steady-state. Inertia terms in the equations of motions are considered. An ideal wedge-type razor blade profile is used in the analysis. A Coulomb type of friction relation between the shear and normal traction at the interface between the razor and the web is assumed. A parametric study is carried out to determine the dependence of the size of the yield zone on the slitter blade profile, the friction coefficient at the web-slitter interface, the web speed during slitting, as well as the web tension.

## 2 GENERAL SOLUTION

A general representation for the dynamic steady-state stress and strain distribution in a homogeneous, isotropic and linearly elastic web under razor slitting has been derived based on the dynamic fracture mechanics derived (Liu, Lu and Huang, 1999). The solutions are summarized in this section.

Consider a moving web being slit by a razor slitter, as shown in Figure 1. In the web plane, a Cartesian frame  $(X_1, X_2)$  is established and it moves with the web at a speed of  $v$ . Another Cartesian frame  $(x_1, x_2)$  is also established as shown in Figure 1, it is fixed with its origin located at the crack tip in the web. The two coordinate systems  $(X_1, X_2)$  and  $(x_1, x_2)$  have a simple translation relation, i.e.,

$$x_1 = X_1 - vt, \quad x_2 = X_2.$$

Here we have assumed that the two coordinate systems coincide at the moment of time  $t = 0$ .

### 2.1 Deformation Field Surrounding the Slitter

In the analysis of this two-dimensional planar deformation problem, we may utilize two displacement potentials,  $\Phi(X_1, X_2, t)$  and  $\Psi(X_1, X_2, t)$ , in the Cartesian system  $(X_1, X_2)$ . With respect to the undeformed field, the two

non-zero in-plane displacement components can be expressed through

$$u_\alpha = \frac{\partial \Phi(X_1, X_2, t)}{\partial X_\alpha} + e_{\alpha\beta} \frac{\partial \Psi(X_1, X_2, t)}{\partial X_\beta}, \quad \alpha = 1, 2, \quad (2.1)$$

where the summation convention has been used here.  $e_{\alpha\beta}$  is the two-dimensional alternator defined by

$$e_{12} = -e_{21} = 1, \quad e_{11} = e_{22} = 0.$$

The stress components in a homogeneous, isotropic, and linearly elastic material that we consider, can be expressed through the displacement potentials as

$$\left. \begin{aligned} \sigma_{11} &= \mu \left\{ \frac{c_l^2}{c_s^2} \cdot \frac{\partial^2 \Phi(X_1, X_2, t)}{\partial X_\alpha \partial X_\alpha} - 2 \frac{\partial^2 \Phi(X_1, X_2, t)}{\partial X_2^2} + 2 \frac{\partial^2 \Psi(X_1, X_2, t)}{\partial X_1 \partial X_2} \right\} \\ \sigma_{22} &= \mu \left\{ \frac{c_l^2}{c_s^2} \cdot \frac{\partial^2 \Phi(X_1, X_2, t)}{\partial X_\alpha \partial X_\alpha} - 2 \frac{\partial^2 \Phi(X_1, X_2, t)}{\partial X_1^2} - 2 \frac{\partial^2 \Psi(X_1, X_2, t)}{\partial X_1 \partial X_2} \right\} \\ \sigma_{12} &= \mu \left\{ 2 \frac{\partial^2 \Phi(X_1, X_2, t)}{\partial X_1 \partial X_2} + \frac{\partial^2 \Psi(X_1, X_2, t)}{\partial X_2^2} - \frac{\partial^2 \Psi(X_1, X_2, t)}{\partial X_1^2} \right\} \end{aligned} \right\}, \quad (2.2)$$

where  $c_l$  and  $c_s$  are, respectively, the dilatational and shear wave speeds of the material. They can be expressed, in terms of the shear modulus  $\mu$ , Poisson's ratio  $\nu$ , and the mass density  $\rho$ , as

$$c_l = \left( \frac{\kappa + 1}{\kappa - 1} \cdot \frac{\mu}{\rho} \right)^{1/2}, \quad c_s = \left( \frac{\mu}{\rho} \right)^{1/2}, \quad (2.3)$$

where  $\kappa = 3 - 4\nu$  for plane strain deformation and  $\kappa = (3 - \nu)/(1 + \nu)$  for plane stress deformation, respectively.

Suppose that the slitting process has reached a steady state. Introduce two parameters  $\alpha_l$  and  $\alpha_s$  defined by

$$\alpha_l = \left( 1 - \frac{v^2}{c_l^2} \right)^{1/2}, \quad \alpha_s = \left( 1 - \frac{v^2}{c_s^2} \right)^{1/2},$$

and two complex variables defined by

$$z_l = x_1 + i \alpha_l x_2, \quad z_s = x_1 + i \alpha_s x_2,$$

where  $i = \sqrt{-1}$ . Then the general solutions for the displacement and stress components in the absence of body force density can be represented by (Freund, 1990)

$$u_1(x_1, x_2) = \operatorname{Re}\{F'(z_l) + \alpha_s G'(z_s)\}, \quad u_2(x_1, x_2) = -\operatorname{Im}\{\alpha_l F'(z_l) + G'(z_s)\}, \quad (2.4)$$

and

$$\left. \begin{aligned} \sigma_{11}(x_1, x_2) &= \mu \operatorname{Re}\{(1 + 2\alpha_l^2 - \alpha_s^2)F''(z_l) + 2\alpha_s G''(z_s)\} \\ \sigma_{22}(x_1, x_2) &= -\mu \operatorname{Re}\{(1 + \alpha_s^2)F''(z_l) + 2\alpha_s G''(z_s)\} \\ \sigma_{12}(x_1, x_2) &= -\mu \operatorname{Im}\{2\alpha_l F''(z_l) + (1 + \alpha_s^2)G''(z_s)\} \end{aligned} \right\}, \quad (2.5)$$

where functions  $F(z_l)$  and  $G(z_s)$  are analytic everywhere in the complex  $z_l$ - or  $z_s$ -planes except along the nonpositive real axis  $-\infty < x_1 \leq 0$  occupied by the crack, the prime (') represents the derivative with respect to the corresponding complex argument. The complete deformation field will be determined if the two analytic functions  $F(z_l)$  and  $G(z_s)$  can be obtained.

Now, consider that the crack surface is subjected to a distributed normal traction  $\sigma(x_1)$ . The associated friction traction is therefore, given by  $\lambda\sigma(x_1)$ , where  $\lambda$  is the friction coefficient between the web and the slit. Behind the crack tip, the crack opening displacement at any position  $x_1$  is given by

$$\delta_2(x_1) = u_2(x_1, 0^+) - u_2(x_1, 0^-), \quad -\infty < x_1 \leq 0. \quad (2.6)$$

The two analytic functions,  $F(z_l)$  and  $G(z_s)$ , can be expressed as

$$\left. \begin{aligned} F''(z_l) &= \frac{1 + \alpha_s^2}{\pi D(v)} \int_{-\infty}^0 \frac{\sigma(s)}{\mu} \cdot \frac{\sqrt{-s}}{\sqrt{z_l}(z_l - s)} ds - \frac{2\lambda\alpha_s}{\pi D(v)} \int_{-\infty}^0 \frac{\sigma(s)}{\mu} \cdot \frac{ds}{z_l - s} \\ &\quad + \frac{1}{2(\alpha_l^2 - \alpha_s^2)} \cdot \frac{\sigma_0}{\mu} \\ G''(z_s) &= -\frac{2\alpha_l}{\pi D(v)} \int_{-\infty}^0 \frac{\sigma(s)}{\mu} \cdot \frac{\sqrt{-s}}{\sqrt{z_s}(z_s - s)} ds + \frac{\lambda(1 + \alpha_s^2)}{\pi D(v)} \int_{-\infty}^0 \frac{\sigma(s)}{\mu} \cdot \frac{ds}{z_s - s} \\ &\quad - \frac{1 + \alpha_s^2}{4\alpha_s(\alpha_l^2 - \alpha_s^2)} \cdot \frac{\sigma_0}{\mu} \end{aligned} \right\}. \quad (2.7)$$

By combining (2.5) and (2.7), the stress field surrounding the slit can be calculated and therefore, the deformation field can be determined. Also, by using the superposition scheme, the dynamic stress intensity factor at the crack tip,  $K_I$ , can be obtained through

$$K_I = \sqrt{\frac{2}{\pi}} \int_{-\infty}^0 \frac{\sigma(s)}{\sqrt{-s}} ds. \quad (2.8)$$

## 2.2 Interaction Between Moving Web and Slitter blade

By using the fact that at the crack tip,  $\delta_2(0) = 0$ , the crack opening displacement can be obtained as

$$\delta_2(x_1) = \frac{2\alpha_l(1 - \alpha_s^2)}{\pi D(v)} \int_{-\infty}^0 \frac{\sigma(s)}{\mu} \ln \left| \frac{\sqrt{-s} + \sqrt{-x_1}}{\sqrt{-s} - \sqrt{-x_1}} \right| ds, \quad -\infty < x_1 \leq 0. \quad (2.9)$$

Note that in the above expression, the normal opening displacement  $\delta_2(x_1)$  depends only on the normal traction applied on the crack surface or web edge,  $\sigma(x_1)$ . The shearing traction associated with the friction coefficient  $\lambda$ , and the remote tensile stress  $\sigma_0$ , do not enter the expression for the crack opening displacement.

Consider the situation shown in Figure 2. Here the slitter blade has a finite length. The distance between the crack tip and the tip of the blade is  $q$  and the distance between the crack tip and the end of the blade is  $b$ . The profile of the slitting blade is characterized by the function  $\delta(x_1)$ . We assume that the slitter blade and the moving web keep contact in the region of  $-b \leq x_1 \leq -a$  where  $a \geq q$ . Therefore, when the slitter blade is perfectly aligned with the web moving direction, within the contact region, we have  $\delta_2(x_1) = \delta(x_1)$ . By denoting  $\eta_1 = -x_1/b$  and  $\eta = -s/b$ , from (2.9), one can write

$$\int_{\alpha}^1 \overset{*}{\sigma}(\eta) \ln \left| \frac{\sqrt{\eta} + \sqrt{\eta_1}}{\sqrt{\eta} - \sqrt{\eta_1}} \right| d\eta = \frac{\pi D(v)}{2\alpha_l(1 - \alpha_s^2)} \overset{*}{\delta}(\eta_1), \quad \alpha \leq \eta_1 \leq 1, \quad (2.10)$$

where we have defined the following,

$$\overset{*}{\sigma}(\eta_1) = \frac{\sigma(x_1)}{\mu}, \quad \overset{*}{\delta}(\eta_1) = \frac{\delta(x_1)}{b}, \quad \alpha = \frac{a}{b}.$$

One can observe from (2.10) that the interacting traction between the moving web and the slitter blade,  $\overset{*}{\sigma}(\eta_1)$ , can be expressed in the form of

$$\overset{*}{\sigma}(\eta_1) = \frac{D(v)}{2\alpha_l(1 - \alpha_s^2)} \Sigma(\eta_1), \quad \alpha \leq \eta_1 \leq 1, \quad (2.11)$$

which indicates that  $\overset{*}{\sigma}(\eta_1)$  is composed of two parts. The first one is a function of the web moving speed,  $D(v)/2\alpha_l(1 - \alpha_s^2)$ , which intrinsically also depends on the Poisson's ratio,  $\nu$ . The second part, denoted by  $\Sigma(\eta_1)$ , does not depend on the web speed, it is only a function of the blade profile. We may refer to the first part as the speed factor and the second part as the shape factor.

Here we have assumed that the deformation is plane stress. One can see that the speed factor is a monotonic function of the web moving velocity. It starts from the value of  $2/(\kappa + 1)$  at  $v = 0$  and decreases to zero when  $v = c_R$ , where  $c_R$  is the Rayleigh wave speed of the material.

The shape factor, or  $\Sigma(\eta_1)$ , can be expressed as

$$\Sigma(\eta_1) = -\frac{1}{\pi\sqrt{1-\eta_1}} \int_0^1 \frac{\sqrt{1-\eta} \delta'(\eta)}{\eta - \eta_1} d\eta, \quad 0 \leq \eta_1 \leq 1. \quad (2.12)$$

One can show that as  $\eta_1 \rightarrow 0^-$ , the above function  $\Sigma(\eta_1)$  is unbounded. However, from (2.8), we have observed that the normal traction  $\sigma(x_1)$  has to be bounded as  $x_1 \rightarrow 0^-$ . Such observation indicates that in (2.15) we must have  $\alpha \neq 0$ .

To study the properties of the shape factor,  $\Sigma(\eta_1)$ , we consider an ideal shape, a wedge profile, for the slitter blade. The wedge profile can be described as

$$\delta(x_1) = 2|x_1 + q| \tan \theta, \quad -b \leq x_1 \leq -q, \quad (2.13)$$

where  $2\theta$  is the angle of the wedge. In nondimensional form, the wedge shape in (2.13) can be rewritten as

$$\delta(\eta_1) = 2(\eta_1 - \gamma) \tan \theta, \quad \gamma \leq \eta_1 \leq 1, \quad (2.14)$$

where  $\gamma = q/b$  and  $\gamma < \alpha$ . The shape factor,  $\Sigma(\eta_1)$ , is expressed for slitter blade with wedge slit as

$$\Sigma(\eta_1) = -\frac{1}{\pi} \sqrt{\frac{\eta_1 - \alpha}{\eta_1(1 - \eta_1)}} \int_\alpha^1 \sqrt{\frac{\eta(1 - \eta)}{\eta - \alpha}} \cdot \frac{\delta'(\eta)}{\eta - \eta_1} d\eta, \quad \alpha \leq \eta_1 \leq 1. \quad (2.15)$$

To maintain a non-negative interacting traction between the moving web and the slitter blade cannot be negative, there exists a lower bound of  $\alpha_0$ , which can be obtained from the following condition (for details, see Liu, Lu and Huang, 1999)

$$\left. \frac{d\Sigma(\eta_1)}{d\eta_1} \right|_{\eta_1=\alpha_0} = 0, \quad (2.16)$$

for any given slitter blade profile.

Through the above discussions, we have shown that the interacting traction between the moving web and the slitter blade is the product of two

factors, the speed factor and the shape factor. We also showed that the parameter  $\alpha$  has to be in the range of

$$\alpha_0 \leq \alpha < 1, \quad (2.17)$$

in order for the interacting traction to be positive. Nevertheless, the constant  $\alpha$  still remains as a free parameter and needs to be determined. Recall that the dynamic stress intensity factor at the crack tip,  $K_I$ , is given in (2.8). Also note that during the slitting process, the stress intensity factor,  $K_I$ , has to be equal to the fracture toughness of the web material, which is denoted by  $K_{IC}(v)$ . Here the notation has suggested that the fracture toughness of the web material,  $K_{IC}$ , also depends on the web speed  $v$ . In terms of the shape factor,  $\Sigma(\eta_1)$ ,  $K_{IC}(v)$  can be rewritten as

$$K_{IC}(v) = \mu \sqrt{\frac{2b}{\pi}} \cdot \frac{D(v)}{2\alpha_1(1-\alpha_s^2)} \int_{\alpha}^1 \frac{\Sigma(\eta)}{\sqrt{\eta}} d\eta. \quad (2.18)$$

Using the requirement (2.18), the parameter  $\alpha$  is therefore determined. From (2.18), one can see that  $\alpha$  will depend on the material's fracture toughness, web speed, and the shape of the shape of the slitter blade.

### 3 RESULTS AND DISCUSSIONS

In this section, some general characteristics of the dynamic steady-state stress field in the web surrounding the blade are studied, and the dependence of the contours of the effective stress on some slitting parameters, such as web tension, web speed, razor blade wedge angle, as well as friction coefficient between the blade and the web, are investigated.

#### 3.1 Dynamic Stress Field

Figures 3, 4 and 5 are the contour plots of the three components of the dynamic steady-state stress field surrounding the a wedge-type razor blade. The parameters used in this simulation are:  $v/c_s = 0.1$ ,  $\nu = 0.3$ ,  $\theta = 5.0^\circ$ ,  $\lambda = 0.2$ . We assume that the fracture toughness of the web material is a constant and  $K_{IC}/\mu\sqrt{b} = 0.075$ , and  $b$  is the distance from the crack tip to end point of the contact region as shown in Figure 2. The web tension is assumed to be  $\sigma_0/\mu = 0.008$ . The length of the contact region can be determined and from (2.18), we have  $\alpha = 0.3913$ .

In Figure 3, we observe that ahead of the crack tip and in a wedge-shape region, the stress component  $\sigma_{11}$  is positive. The positiveness of the  $\sigma_{11}$  component ahead of the crack has some significant implications in the quality control of web slitting. Since the so-called  $T$ -stress (the second term next to the singular term in the asymptotic expansion near the crack tip) in mode-I type of deformation, is determined by the distribution of the  $\sigma_{11}$



component ahead of the crack tip, the value of  $T$ -stress will be positive as well. According to the analysis by Cotterell and Rice (1980), the level of  $T$ -stress determines the stability of the crack. For large positive  $T$ -stress, the crack is stable and will remain its orientation. For lower or even negative  $T$ -stress, the crack will become unstable and tends to kink out of its original direction or bifurcate into several branches. Therefore, if the  $T$ -stress value is below certain value and the crack becomes unstable, we might see many micro or small cracks along the cutting edge of the web. Thus the quality of the final product would be poor. From the dynamic fracture mechanic analysis, the  $\sigma_{11}$  component ahead of the crack tip is determined by the remote tension  $\sigma_0$  and the web speed  $v$ . As a result, the web slitting quality will be improved by adjusting these two parameters to maintain a high value of the  $T$ -stress. This will eliminate the tendency that the crack loses its stability during slitting. Meanwhile, from Figures 3 and 4, we see that near the contact region between the moving web and the slitter, i.e., from  $x_1 = -0.3913b$  to  $x_1 = -b$ , both  $\sigma_{11}$  and  $\sigma_{22}$  are negative. One can deduce that the two principal stresses will also be negative near the contact region. This situation might trigger the web to buckle and this will also lead to poor slit edge quality.

### 3.2 Effect of Slitting Parameters on the Contour of Effective Stress

We next change some slitting parameters and investigate how these parameters affect the contour of effective stress. The effective stress  $\sigma_e$  is defined by

$$\sigma_e = \left( \frac{5}{6}\sigma_{11}^2 + \frac{4}{3}\sigma_{11}\sigma_{22} + \frac{5}{6}\sigma_{22}^2 + 3\sigma_{12}^2 \right)^{1/2}. \quad (3.1)$$

In writing the expression in (3.1), we have assumed that the deformation is plane stress.

In Figure 6, two different web speeds are considered and they are  $v/c_s = 0.1$  and  $v/c_s = 0.3$ , respectively. Other parameters are the same as those shown in Figure 3. By noting that the von Mises yield criterion can be expressed as

$$\sigma_e \leq \tau, \quad (3.2)$$

where  $\tau$  is the yield stress in pure shear, the contour plots shown in Figure 6 indicate the effect of the web speed on the size of the yield zone near the slitter blade. There are two singular points in the stress field. One is the crack tip and the other is the end of contact region. From Figure 6, we can see that although the stress concentration at the tip of the crack will cause plastic deformation, the majority of the yielding occurring in the area surrounding the slitting blade is dominated by the contact condition, especially near the

end of the contact region. Also, we can see that a higher web speed will enlarge the yielding area surrounding the slitter blade.

In Figure 7 two different wedge-angles are used, the other slitting parameters are the same as in Figure 3. We can see that the size of the yield zone increases as the wedge angle increases. A smaller wedge angle may lead to higher slit-edge quality. Figure 7 indicates the effect of tension, again except the tension the other slitting parameters are the same as those used in Figure 3. We observe that a higher tension tends to increase the size of the yield zone.

#### 4 SUMMARY

A stress analysis for the slitting process of a thin web material is presented in this study based on the dynamic fracture mechanics. The purpose here is to identify the key parameters that will contribute to the quality control during the web slitting process. The instability of the crack tip, web buckling in the region near the slitter blade, and the plastic yielding surrounding blade are believed to be the major factors that affect the slit-edge quality of a web. A parametric study indicates that the size of the yield zone in a web during razor slitting is affected by the web speed, the slitter wedge angle and web tension.

#### ACKNOWLEDGMENTS

H. Lu would like to acknowledge the support from 3M Non-Tenured Faculty Grant, the support under grant No.9697-1 from the NSF IUC/RC research center-Web Handling Research Center-at Oklahoma State University (OSU), sponsored by NSF and twenty-one companies that include 3M, Eastman Kodak, ALCOA, AET Films, Fife, Mobil Chemical, Polaroid, Rockwell Automation and Xerox, and the support from NSF under grant No. 9872350. The work of C. Liu was supported financially by the US Department of Energy, Defense Programs, Technology Transfer Initiative Program.

#### REFERENCES

- Arcona, C. and Dow, T.A. (1996), "The role of knife sharpness in the slitting of plastic films," *Journal of Materials Science*, **31**, pp. 1327-1334.
- Bax, A.J. (1991), "Characterization of razor slitting in thin sheets and membranes," M.S. thesis, University of Tulsa.
- Bollen, D., Deneir, J., Aernoudt, E. and Muylle, W. (1989), "Shear cutting of PET film," *Journal of Materials Science*, **24**, pp. 2957-2966.
- Cotterell, B. and Rice, J.R. (1980), "Slightly curved or kinked cracks," *In-*

*ternational Journal of Fracture*, **15**, pp.155–169.

Freund, L.B. (1990), *Dynamic Fracture Mechanics*. Cambridge University Press, New York.

Kasuga, Y., Tsutsumi, S. and Mori, T. (1977), “On the shearing process of ductile sheet metals (2nd report Theoretical analysis of the ‘Scissors Type’ shear),” *Bulletin of the JSME*, **20**(148), pp. 1336–1343.

Liu, C., Lu, H. and Huang, Y. (1999), “Dynamic Steady-State Stress Field in a Web During Slitting”, To be submitted to *Journal of Applied Mechanics*

Meehan, R.R. and Burns, S.J. (1996), “Mechanics of slitting and cutting webs,” *Experimental Mechanics*, **38**(2), pp.103–109.

Meehan, R.R. and Burns, S.J. (1998), “Mechanics of slitting and cutting webs,” *Engineering Mechanics*, **38**(2), pp.103–109.

Muskhelishvili, N.I. (1953), *Some Basic Problems in the Mathematical Theory of Elasticity* (translated by J.R.M. Radok). Noordhoff, Groningen, Holland.

Zheng, Z.M. and Wierzbicki, T. (1996) “A theoretical study of steady-state wedge cutting through metal plates,” *International Journal of Fracture*, **78**, pp.45–66.

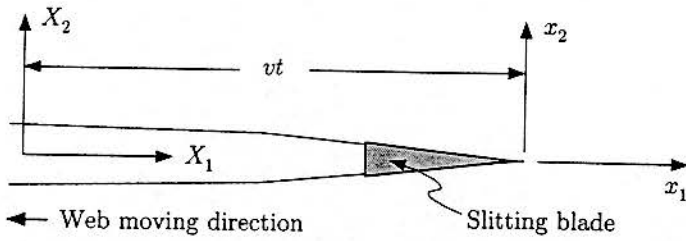


Figure 1: A schematic diagram of the razor slitting process

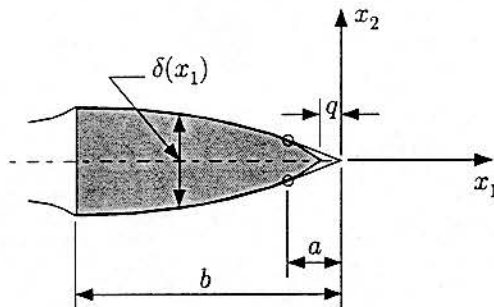


Figure 2: Geometric profile of the slitter and contact condition.



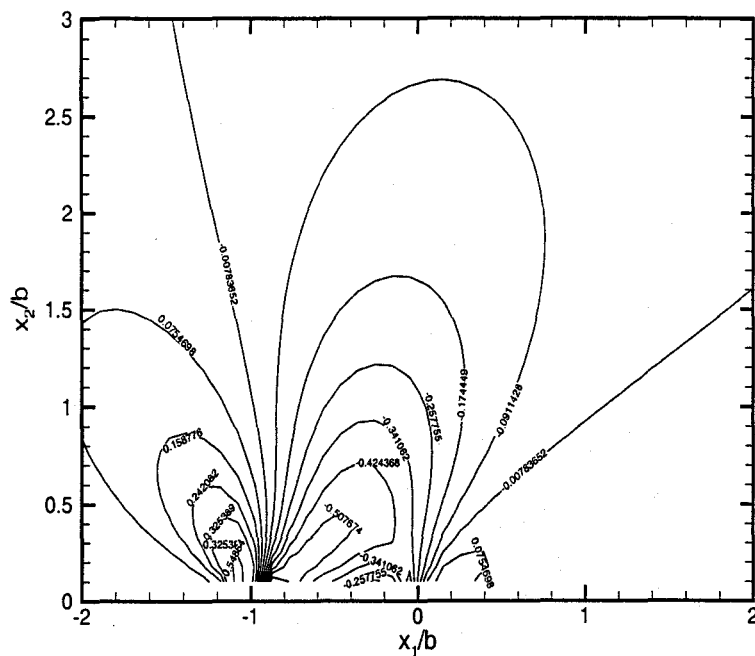


Figure 5: Contour plot of  $\sigma_{12}$ .

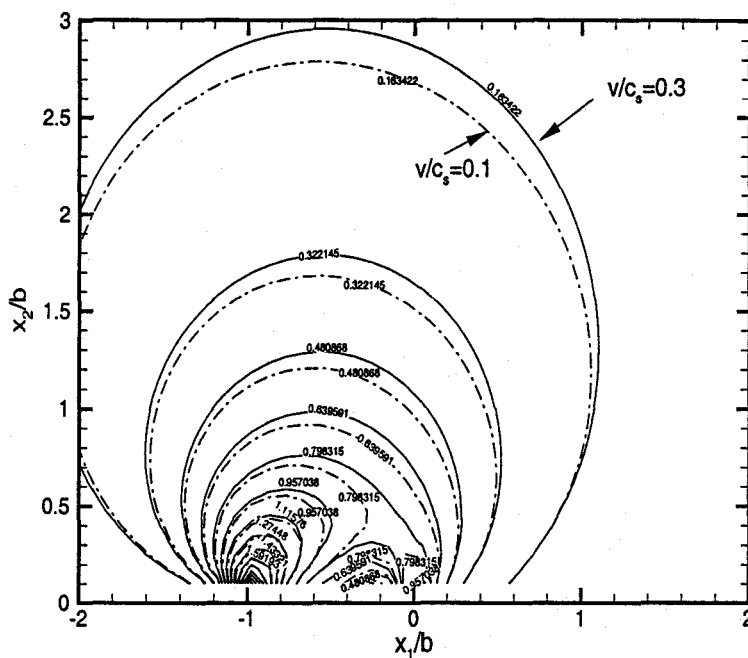


Figure 6: Contour plots of  $\sigma_e$ , for (a)  $v/c_s = 0.1$  and (b)  $v/c_s = 0.3$ .

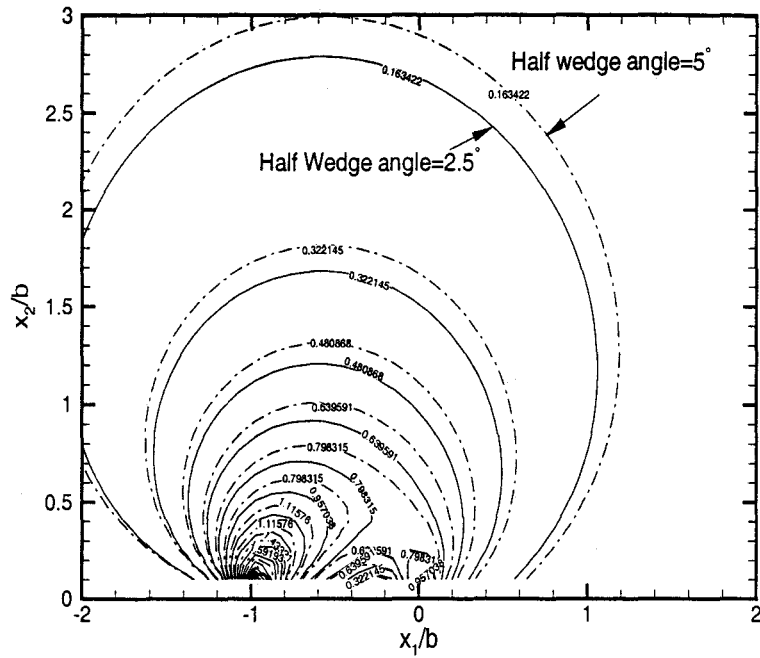


Figure 7: Contour plots of  $\sigma_e$  for (a)  $\theta = 2.5^\circ$  and (b)  $\theta = 5^\circ$ .

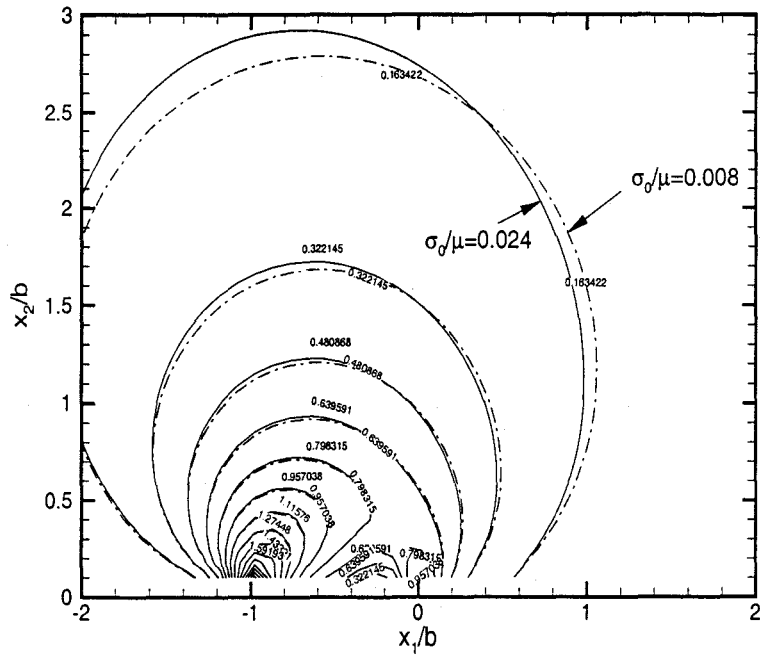


Figure 8: Contour plots of  $\sigma_e$  for (a)  $\sigma_0/\mu = 0.008$  and (b)  $\sigma_0/\mu = 0.024$

H. Lu and C. Liu

*The Stress Field in a Web During Slitting: Opening Mode*

6/8/99

Session 2

10:15 - 10:40 a.m.

Question - Dave Roisum, Finishing Technologies, Inc.

In many cases of slitting the crack runs ahead of the blade in cases you can actually see it with your eye. However we can infer that in other cases the web must be in contact with the front of the blade, otherwise you wouldn't have wear. The picture you show would give a self sharpening situation and we know in many cases these blades will dull themselves on the leading edge so there must be contact. Do you have any comment on that?

Answer - Hongbing Lu, Oklahoma State University

Actually I do believe sometimes that there is contact between the slitter and the web. Sometimes if the web material is brittle and does not buckle in the slitting area, that may be a situation where you do not have contact.