A DYNAMIC MODEL FOR MONITORING AND CONTROL OF A WINDING PROCESS

by

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ABSTRACT

In this paper, a dynamical model for an on-line monitoring and control strategy design is proposed. It describes the dynamic behaviour of in-roll stresses as the roll is being built. The distribution of in-roll stresses is in fact a key issue to tackle the problem of avoidance of in-roll stresses. By acting on boundary variables such as tension and torque, one can drive the in-roll stresses to a non-defect inducing on.

INTRODUCTION

Roll quality is of primary importance for winding processes in paper, printing and film industries. This quality depends on variables such as the winding tension of the web, the speed and acceleration of winding. These variables influence the internal stresses depending on the mechanical properties of the material and the environmental conditions such as moisture and temperature. In this paper, we will concentrate on the centre winding process (Figure 1) leaving the study of other types of winding for future investigations.

In centre winding, the incoming web is added as new layers with a certain tension, which translates to an increase in the internal radial stresses. Therefore, as the roll grows in diameter, the radial pressure builds up for the internal layers. As such, the internal layers are subject to time-varying internal stresses.

In order to link winding variables, several works have focused on the modelling of winding processes [6,1,10,3,9] since the pioneering work of Altmann [1]. These models provide an a priori analysis and a prediction of stresses inside the final wound roll. In hitherto published works, static relations between inner radial and tangential stresses, in-wound tension, core stiffness are given. Nevertheless, parameters such as static and dynamic friction coefficients [2] as well as the diffusion of moisture [7] which involve nonlinear dynamic characteristics are not integrated explicitly in the existing models. For instance, the problem of inter-layer slippage seems to be related to the frictional properties of the material being wound and the relative angular speed of consecutive layers.

In this paper, we shall consider the dynamic modelling problem for on-line stress monitoring and control. Specific solutions pertaining to an axisymmetric geometry with cylindrical material anisothropy will be presented. In winding operations, the layers become consecutively subjected to increasing pressure. Depending on the winding tension profile, i.e., how it is evolving with time as the roll is being built, a model can allow to dynamically adjust the stress profile(s) so that to avoid variations arising from (existing or emerging) roll defects. In fact, controlling the in-roll stresses by acting on boundary variables, such as in-wound tension, rotation torque, is allowed with the model we propose. Moreover, it can help to improve the understanding of roll structure mechanics during winding.

For an effective use, one has to provide on-line information on parameters such as Young's modulus, Poisson's ratio (which are affected by moisture), grammage and web tension. While the web tension and the grammage can be measured, the mechanical properties, such as radial and tangential elasticity moduli, which also depend on the environmental conditions cannot be measured on-line. To overcome this difficulty, an on-line coefficient of restitution measurement is undertaken [11], and measurements from which will be incorporated in the development and validation of the proposed model.

The first section of the paper will discuss the development of the dynamic model and relevant links with previous models. In the second section, a simulation case study is provided to show the use of the model and the design of controllers for winding process control.

DYNAMIC MODELLING FOR A WINDING PROCESS

Consider the following element of material inside the roll and let us describe its dynamics. The element of material is supposed to rotate around the z-axis has the cylindrical coordinates (r, θ, z) in a fixed reference (x,y,z). It is subjected to forces resulting from stresses applied from its environment (Figure 2).

σ_r, σ_θ and σ_z are the radial, tangential and the z-direction stresses respectively. $\tau_{\theta r}$, τ_{rz} and $\tau_{\theta z}$ are the shear stresses. We may now establish the general evolution of this piece of material:

$$\frac{d}{dt}(m(r+\frac{\delta}{2})^2\dot{\theta}) = [\sigma_\theta(\theta+\frac{\alpha}{2}) - \sigma_\theta(\theta-\frac{\alpha}{2})]\delta h(r+\frac{\delta}{2}) +$$
$$[\tau_{\theta r}(r+\delta).(r+\delta)^2 h\alpha - \tau_{\theta r}(r).r^2 h\alpha]$$
$$+ mg(r+\frac{\delta}{2})sin\theta + [\tau_{\theta z}(z+h).r\alpha\delta(r+\frac{\delta}{2}) - \tau_{\theta z}(z).r\alpha\delta(r+\frac{\delta}{2})] \quad (1)$$

where $m = \rho r\alpha\delta h$ is the mass of the piece of material and ρ is its density (in Kg/m^3); $r\alpha\delta h$ is the volume of the element. The notation $\sigma_i(j)$ (resp. $\tau_l(k)$) indicates the evaluation of σ_i at j (resp. of τ_l at k). The above equation leads to:

$$\frac{d}{dt}(\rho r^2\dot{\theta}) = \frac{\partial\sigma_\theta}{\partial\theta} + \frac{1}{r}\frac{(\partial r^2\tau_{\theta r})}{\partial r} + \rho g r sin\theta + \frac{\partial\tau_{\theta z}}{\partial z}r \quad (2)$$

Now, consider the momentum equation for the radial displacement:

$$\frac{d}{dt}(m\dot{u}) = [\sigma_r(r+\delta).(r+\delta)h\alpha) - \sigma_r(r).r\alpha h] + [\tau_{rz}(z+h).r\alpha\delta - \tau_{rz}(z).r\alpha\delta]$$
$$+ [\tau_{\theta r}(\theta+\frac{\alpha}{2}).\delta h - \tau_{\theta r}(\theta-\frac{\alpha}{2}).\alpha\delta] - [\sigma_\theta(\theta+\frac{\alpha}{2}) + \sigma_\theta(\theta-\frac{\alpha}{2})]\frac{\alpha}{2}h\delta - mgcos\theta \quad (3)$$

where u is the radial displacement of the piece of material and r its current radial position. This leads to:

$$\rho\ddot{u} = \frac{\partial\sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) + \frac{\partial\tau_{rz}}{\partial z} + \frac{1}{r}\frac{\partial\tau_{\theta r}}{\partial\theta} - \rho g cos\theta \quad (4)$$

The third equation is the expression of the momentum on the CD-axis:

$$\frac{d}{dt}(m\dot{w}) = [\sigma_z(z+h).r\delta\alpha - \sigma_z(z).r\alpha\alpha] + [\tau_{rz}(r+\delta).(r+\delta)\alpha h - \tau_{rz}(r).r\alpha h]$$
$$+ [\tau_{\theta z}(\theta+\frac{\alpha}{2}).\delta h - \tau_{\theta z}(\theta-\frac{\alpha}{2}).\delta h] \quad (5)$$

This equation clearly leads to:

$$\rho\ddot{w} = \frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta} \quad (6)$$

w is the CD displacement of the piece of material. This later equation will not be used in this paper.

A first step for using this model is to understand how it can be related to the dynamic behaviour of a layer. For this purpose, the stresses acting on the layer have been depicted in the following figure.

As shown in Figure 3, tangential stresses at the edges of the layer are considered to act on different radii thus allowing to tighten each layer on the roll. Moreover, owing to the fact that the thickness is small, each layer can be considered to be a concentric cylinder. It is also assumed to be of uniform density and thickness. This allows to write the rotational motion of the layer from the integration of equation (2):

$$\frac{d}{dt}(J_i\omega_i) = t_i(r_i + \frac{t_i}{2})H\sigma_{\theta i} - \sigma_{\theta i+1}t_{i+1}H(r_{i+1} + \frac{t_{i+1}}{2}) + 2\pi H(r_i^2\tau_{r\theta_i} - r_{i+1}^2\tau_{r\theta_{i+1}}) \tag{7}$$

where ω_i is the angular speed of the layer. This implies that ω_i is assumed to be uniform for each layer. J_i is the inertia of the layer: $J_i = \int_\theta \int_r \int_z \rho r^3 dr dz d\theta$.

$$\int_{V_i} \frac{\partial \sigma_\theta}{\partial \theta} dV_i = t_i(r_i + \frac{t_i}{2})H\sigma_{\theta i} - \sigma_{\theta i+1}Ht_{i+1}(r_{i+1} + \frac{t_{i+1}}{2}) \tag{8}$$

where H is the width of the paper in the cross direction, t_i is the thickness of the layer i. $\sigma_{\theta i}$ is the mean value of σ_θ on layer i.

$$\int_{V_i} \frac{1}{r}\frac{(\partial r^2 \tau_{\theta r})}{\partial r} dV_i = 2\pi H(r_i^2\tau_{r\theta_i} - r_{i+1}^2\tau_{r\theta_{i+1}}) \tag{9}$$

$\tau_{r\theta_{i+1}}$ and $\tau_{r\theta_i}$ coincide with the friction-induced stresses at the inter-layer contact. A first approximation of these stresses can be made using classical friction theory, although recent development on friction modelling exists [5]. Let us recall that friction is the tangential reaction force between two surfaces in contact. It is well-known that when there is no relative motion between both surfaces, the friction force (Coulomb) is proportional to the normal force exerted where the coefficient of proportionality is the static coefficient of friction. When a relative motion appears, the friction phenomenon is said to be viscous. The friction force is then proportional to the relative velocity. Therefore, we will consider $\tau_{r\theta_j} = f_s\sigma_{r_j}$. The shear stress is neglected compared to the inter-layer friction induced stress. Finally, we consider:

$$\int_{V_i} (\rho g r sin\theta) dV_i = 0 \tag{10}$$

and

$$\int_{V_i} \frac{\partial \tau_{\theta z}}{\partial z} dV_i = 0 \tag{11}$$

An interesting feature of this model is that the inter-layer friction can be captured in both static and kinetic ways. Indeed, friction is an important parameter in winding processes. It has been shown to be influenced by moisture (see, for instance, [2]), roughness and air film (mostly in non-porous materials such as polymer films). In the case of paper, the composition (fillers, fibres, polymers) can also influence the frictional properties.

At the core interface i.e. for the first layer, we have:

$$\frac{d}{dt}(J_1\omega_1) = -H\sigma_{\theta 2}t_2(r_2 + \frac{t_2}{2}) + 2\pi H(r_2^2\tau_{r\theta_2} - r_1^2\tau_{r\theta_1}) \tag{12}$$

$\sigma_{\theta 1} = 0$ since the first edge of the first layer is free. This means that the winding of the first layer is obtained by the material-to-core friction. For the outer layer k, equation (2) becomes:

$$\frac{d}{dt}(J_k\omega_k) = t_{k-1}(r_{k-1} + \frac{t_{k-1}}{2})H\sigma_{\theta,k-1} - T(r_k + \frac{t_k}{2}) - 2\pi H r_{k-1}^2 \tau_{r\theta_{k-1}} \quad (13)$$

where $\tau_{r\theta_k} = 0$ because the outer surface is free.

$$H\sigma_{\theta,k}t_k = T$$

the outer layer tension of the paper. Also ω_k the outer layer winding speed has to track the speed of the incoming paper. Finally the driving motor has the following dynamics:

$$\frac{d}{dt}(J\omega) = (r_1 T_m - r_1^2 \tau_{r\theta_1})2\pi H \quad (14)$$

where J is the (core + motor rotor) inertia and ω is the speed of the motor; T_m is the torque of the motor.

The implementation of the above dynamic equations implies that they are computed for each layer each revolution of the roll. Therefore, from a control viewpoint, this may be too demanding to implement. On the other hand, the above equations can be used for monitoring some internal phenomena such as inter-layer slippage which is said to be the origin of winding defects (see e.g. [10]).

The aim of winding is presumably to make compact rolls by adding layers to existing ones; the continuity of stresses can be expected unless discontinuities such as breaks (as, for instance, in web breaks and restarting of winding), and nonuniform angular speed due to inter-layer slippage appear. Set $J_r = \sum_j J_j$ the total inertia of the roll being wound.

$$\frac{d}{dt}(J_r\omega_r) = 2\pi H r_1^2 \tau_{r\theta_1} - T(r_k + \frac{t_k}{2}) + \frac{d}{dt}(\sum_{j=1}^{N} J_j(\omega_r - \omega_j)) \quad (15)$$

The term $\frac{d}{dt}(\sum_{j=1}^{N} J_j(\omega_r - \omega_j))$ describes the slippage of each individual layer. In the case where no slippage appears, this term vanishes. We refer to this equation as the global dynamic equation of the roll. In addition to these equations, a mass balance of the wound material can be used:

$$\frac{d}{dt}(M_r) = \rho V t_k H \quad (16)$$

$$= GVH \quad (17)$$

where M_r is the actual mass of the roll, V is the speed of the incoming web, $G = \rho t$ is the grammage of the material, t is the thickness of the web and H is the width of the roll. This last equation expresses the growth of the roll and can be used to calculate its actual radius using the fact that $M_r = \pi(r^2 - r_c^2)H\rho$, r_c

is the radius of the core and r is the actual radius of the core. Note that when this radius is measured (see [11]), it can be used directly in the computation.

Let us now consider the well-known elasticity equations. In the axisymetric case, the strain-stress equation is given by:

$$\epsilon_r = \frac{\sigma_r}{E_r} - \mu_\theta \frac{\sigma_\theta}{E_\theta} \tag{18}$$

$$\epsilon_\theta = \frac{\sigma_\theta}{E_\theta} - \mu_r \frac{\sigma_r}{E_r} \tag{19}$$

where E_r and E_θ denote the young modulus in the r and θ directions. The radial modulus, when dealing with a stack of sheets, has been shown to depend on the radial pressure. In [6], the author has propose a linear variation with internal pressure. In [3],a polynomial dependence has been suggested while in [9], a modified Pfeiffer relation is proposed. This matter is not discussed in the present paper.

μ_r, μ_θ are the poisson ratios. $\epsilon_r, \epsilon_\theta$ are the corresponding strains. Moreover the displacement u is related to the strains as follows: In the cylindrical coordinates, the strains are given by:

$$\epsilon_r = \frac{\partial u}{\partial r}; \quad \epsilon_\theta = \frac{u}{r} \tag{20}$$

For the actual outer layer, $u = 0, \dot{u} = 0$. Therefore, the boundary conditions on the displacement can be found from:

$$-\sigma_{r_{k-1}} = \frac{t_{k-1}}{r_{k-1}} \sigma_{\theta_{k-1}} \tag{21}$$

which is given in [3]. $\sigma_{\theta_{k-1}} = \frac{T}{t_{k-1}H}$ where T is the actual applied winding tension. The other necessary boundary condition is that pressure is continuously applied to the core.

In [4], the author has established that the effective tension of winding T_e is defined by:

$$T_e = T - \rho V^2 \tag{22}$$

where V is the line velocity of the web. Indeed, when dealing with the control of the centre-winding process, equation (15) will allow the control of the rotational speed so that the external layer follows the line speed. The torque is then calculated to realise this goal according to a predetermined effective tension profile. The optimal profile takes into account the prediction of the internal pressure so that defects such as wrinkles and slippage may be avoided. To do so, a sensor is needed to report the actual evolution of the stresses through, for example, the continuous measurement of the coefficient of restitution of rolls during winding [11].

SIMULATIONS RESULTS

The following simulations have been performed to show the evolution of the stresses as layers are being added. The first simulation shows the evolution of radial stress as the roll is being wound, i.e., at different time intervals, for different radii (Figure 4). The simulations have been conducted using MATLAB.

For a constant hoop stress, the speed is controlled in order to follow the web line speed. For this purpose, a simple PI controller is used. Figure 5 shows the curves of the angular speed (in radian) versus time (in seconds). The reference angular speed corresponds to the line speed divided by the actual outer radius.

Conclusion and future works

In this paper, a dynamic model for the control of winding processes is given. It consists of combining an internal dynamic model which describes the internal stresses and another describing the global evolution of the roll. The latter is used for control purposes, whereas the former is used for monitoring. A preliminary simulation study has been conducted whereby the evolution of the stress distribution inside the roll is shown while control actions are undertaken.

Future works include the determination of an optimal tension profile so that the stress distribution can follow a "non-defect inducing path". Also, as CD tension is known to be non-constant, investigations will be undertaken to incorporate this fact.

Acknowledgement

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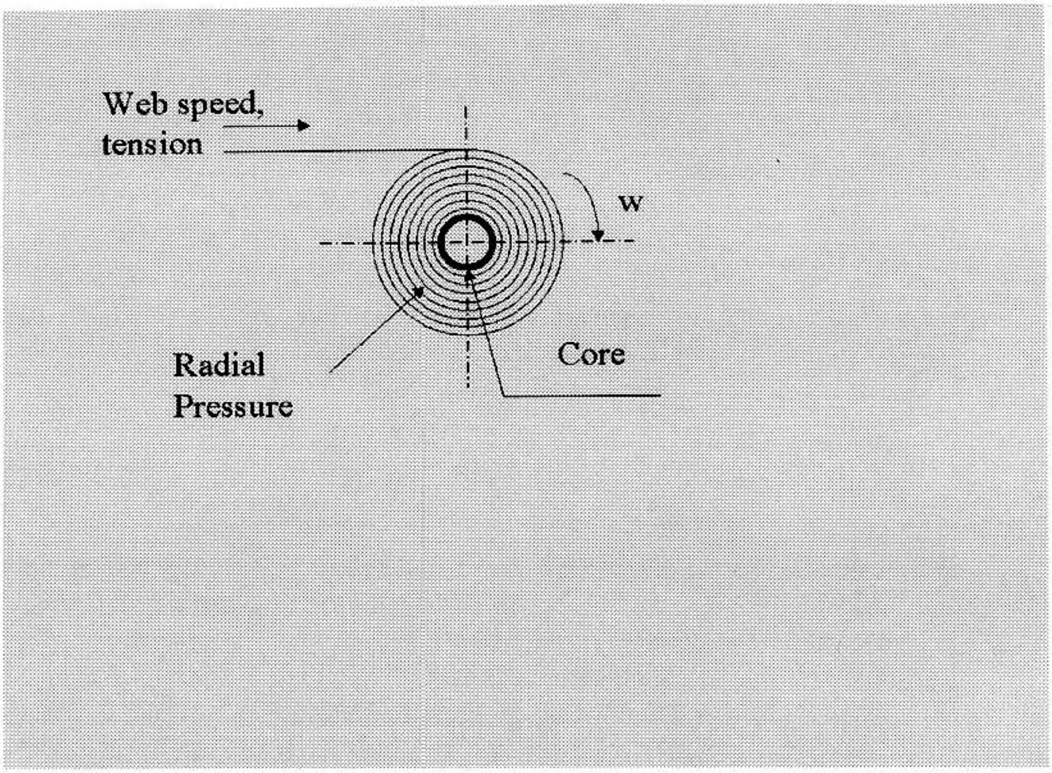

Figure 1: Centre winding

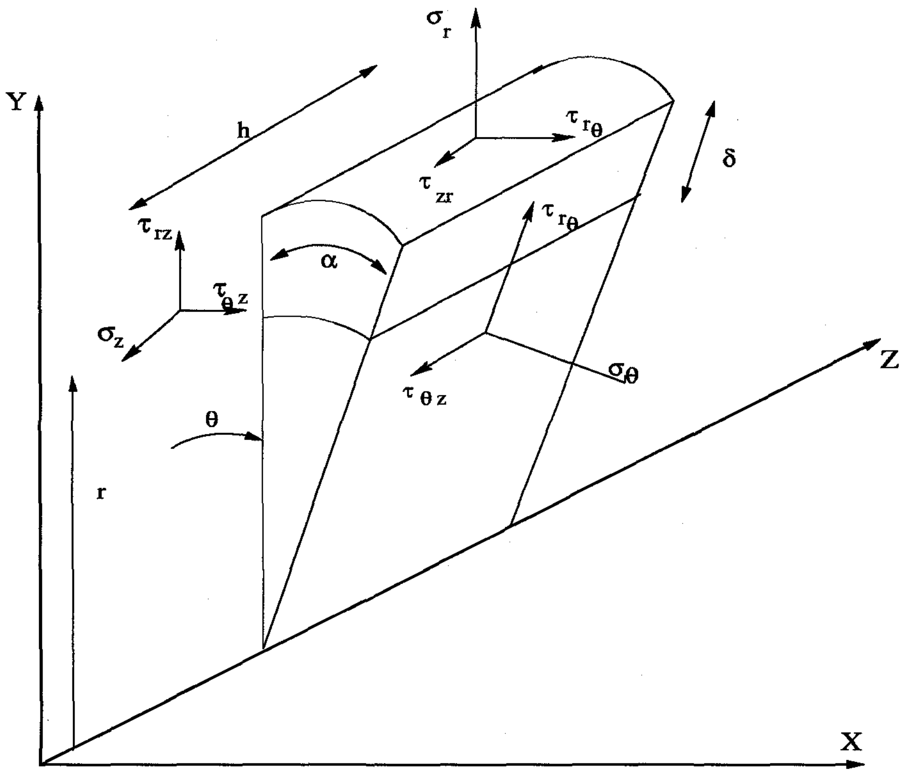

Figure 2: Element of material

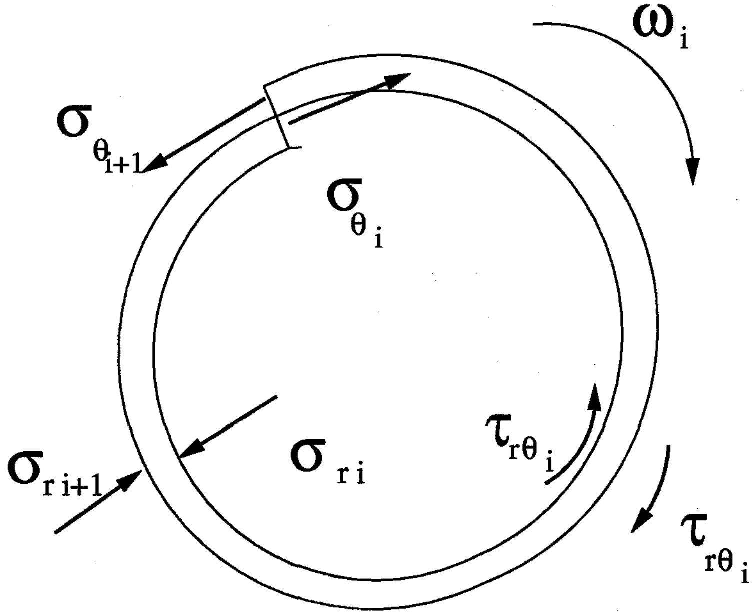

Figure 3: Stresses on a layer

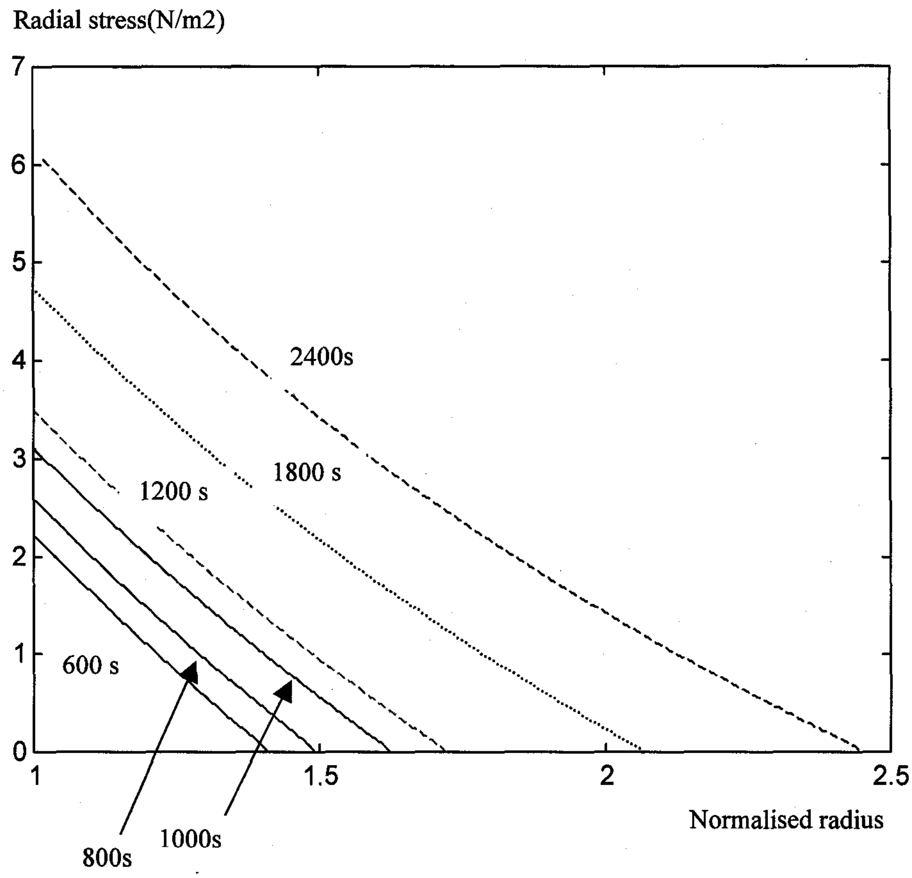

Figure 4: Stress distribution change with time

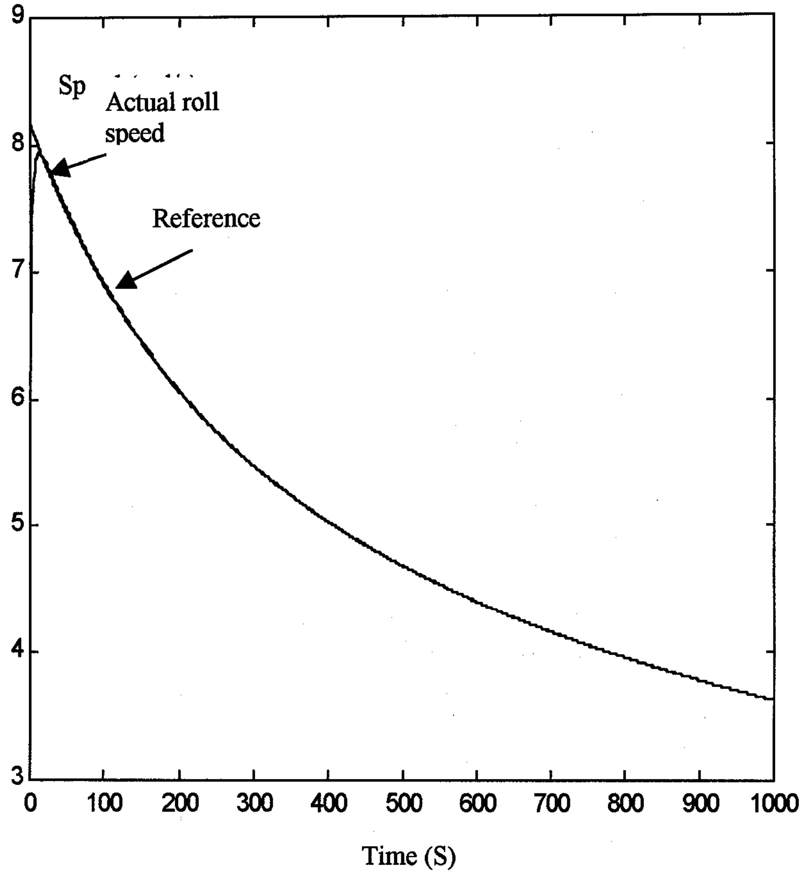

Figure 5: Speed Control of the roll

P. Kabore, H. Wang, H. Jaafar and W. Hamad
A Dynamic Model for Monitoring and Control of a Winding Process
6/8/99 Session 2 1:45 - 2:10 p.m.

Question - Doug Goodenneq, University Bouhome
You have accounted for slippage in your model, but at the moment you are modeling center winding. Do you expect a lot of slippage in center winding or is it that you are already prepared for the future when you analyzed as in surface winding?

Answer – H. Jaafar, UMIST
We are looking at future work. We started using center winding because it's simpler to understand, rather than starting with something more complex. Slippage at this stage is not very critical.