WRINKLING OF WEBS DUE TO TWIST

by

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ABSTRACT

Webs are often required to endure twisting in web process machinery. There is a limit to the degree to which the web can be twisted prior to wrinkling. The objective of this publication is to document a closed form technique that was developed to predict the twist limit of the web.

NOMENCLATURE

a	Half span length (m)
b	Half span width (m)
E	Young's modulus of elasticity (Pa)
R	Radius of rollers (m)
S_1, S_2	Constants that define a parabolic distribution of machine direction stress per Figure 1 (S_1-Pa, S_2-Pa/m^2)
t	Web thickness (m)
T_w	Web Tension (Pa)
u	Web machine direction deformation referenced per Figure 1 (m)
x	Machine direction coordinate per Figure 1
y	Cross machine direction coordinate per Figure 1
α, β, γ	Coefficients in assumed stress functions
ν	Poisson's ratio
σ_{xx}	Machine direction stress (Pa)
σ_{yy}	Cross machine direction stress (Pa)
σ_{xy}	Shear stress (Pa)
ε_{xx}	Machine direction strain (m/m)
Φ	Web twist (rad)

INTRODUCTION

Webs are twisted in process machinery for a variety of reasons. In some cases the web process machine may have outgrown the facility that houses it. To permit additional processing the web is twisted from a horizontal plane to a vertical plane with a set of rollers. After wrapping the vertical roller 90 degrees the web can be twisted back to a horizontal plane and web processing can continue with a new machine direction 90 degrees different than the original machine direction. Webs are often twisted in laminating machines. Various laminae may not be required until other laminae have been processed. Rather than running all the separate laminae the whole length of the process machine over separate sets of rollers, they are often introduced into the process line specifically at the point where they are required. The laminae unwind off of a stand oriented 90 degrees from the machine direction of the process line. The web would be twisted to a vertical plane over the center of the process machine. After wrapping the vertical roller 90 degrees the web would be twisted 90 degrees again, at which point the web will be in a horizontal plane and running in the machine direction of the laminator, ready to be laminated to the other web layers required for that product. Many webs are "scatter wound" by allowing the winder to oscillate laterally during the roll buildup. This is typical for webs with a large variation in thickness across their width. By oscillating the winder the maximum and minimum thickness locations are not allowed to accrue at one cross machine direction location that would result in a corrugated wound roll. When unwound at high speed these "scatter wound" rolls may present a lateral dynamics problem in a subsequent web process line. The lateral oscillations in the unwinding web may be of an amplitude and period that a web guide system can not respond and the lateral oscillations of the web may still exist after entry to the process, which is rarely desirable. To solve this problem the web is twisted to a vertical plane as it is paid off of the unwind stand and then back to a horizontal plane prior to processing, at which point most if not all of the lateral oscillation has been cancelled.

Thus webs are twisted for a host of reasons and as such it is desirable to know to what degree a web can be twisted in a given span prior to wrinkling. As a consequence of the twist of the web, the web edges travel a greater distance than the center of the web between two rollers. A uniform web traveling in a free span between pairs of rollers is assumed to have a uniform machine direction stress across its width, due to web tension. Given that the rollers are well aligned the machine direction stress should be constant along the length of the span as well. Twist however causes the MD stresses to be higher at the web edges than the web center. This variation in MD stress across the width is responsible for inducing cross machine direction (CMD) compressive stresses that can be of sufficient amplitude to wrinkle the web as it approaches a roller. It is possible to model this behavior using the finite element method. The objective of this paper is to develop a closed form solution that can be used by engineers who must design or troubleshoot web machines that will predict at what degree of twist a web will wrinkle.

DISCUSSION

As the web is twisted it will be assumed that it is subjected to a parabolic distribution of MD stress as shown in Figure 1. To develop the stress fields in the x and y directions throughout the membrane the Rayleigh-Ritz method will be employed[1]. An Airys stress function of the form:

$$\phi = \frac{S_1 y^2}{2} + \frac{S_2 y^4}{12} + \left(x^2 - a^2\right)^2 \left(y^2 - b^2\right)^2 \left(\alpha + \beta x^2 + \gamma y^2\right) \qquad \{1\}$$

is chosen where α, β, and γ are unknown coefficients that must be determined. This form was selected as it satisfies the parabolic force boundary condition. By premultiplying the unknown terms by the $\left(x^2 - a^2\right)^2 \left(y^2 - b^2\right)^2$ term it is assured that the unknowns yield no normal or shear stresses on the boundaries. The stresses are:

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} = S_1 + S_2 y^2 + 2\left(x^2 - a^2\right)^2 \begin{bmatrix} -2b^2\alpha + 6y^2\alpha - 2b^2 x^2\beta + \\ 6x^2 y^2\beta + b^4\gamma - 12b^2 y^2\gamma + 15y^4\gamma \end{bmatrix} \qquad \{2\}$$

$$\sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} = 2\left(y^2 - b^2\right)^2 \left[a^4\beta - 2a^2\left(\alpha + 6x^2\beta + y^2\gamma\right) + 3x^2\left(2\alpha + 5x^2\beta + 2y^2\gamma\right)\right] \qquad \{3\}$$

$$\sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = -8xy\left(y^2 - b^2\right)\left(x^2 - a^2\right)\left(2\alpha - a^2\beta + 3x^2\beta - b^2\gamma + 3y^2\gamma\right) \qquad \{4\}$$

These stresses were then used to form the total complimentary potential using the plane stress expression:

$$U' = \frac{t}{2E} \int_{-b}^{b} \int_{-a}^{a} \left[\sigma_{xx}^2 + \sigma_{yy}^2 - 2\nu\sigma_{xx}\sigma_{yy} + 2(1+\nu)\sigma_{xy}^2\right] dx\, dy \qquad \{5\}$$

or

$$U' = \frac{t}{2E} \begin{bmatrix} 4ab S_1^2 + \frac{8}{3}ab^3 S_1 S_2 + \frac{32768\alpha^2\left(a^9 b^5 + a^5 b^9\right)}{1575} + \frac{32768 a^9 b^9\left(\beta^2 + \gamma^2\right)}{3675} + \\[2mm] \frac{131072\alpha^2 a^7 b^7}{11025} + \frac{1024 S_2\left(\beta a^7 b^5 + \gamma a^5 b^7\right)}{1575} + \frac{65536\left(\alpha\beta a^{11} b^5 + \alpha\gamma a^5 b^{11}\right)}{17325} + \\[2mm] \frac{65536\left(\alpha\beta a^7 b^9 + \alpha\gamma a^9 b^7\right)}{11025} + \frac{32768\left(\alpha^2 a^{13} b^5 + \gamma^2 a^5 b^{13}\right)}{75075} + \frac{4}{5}ab^5 S_2^2 + \\[2mm] \frac{131072\left(\beta^2 a^{11} b^7 + \gamma^2 a^7 b^{11}\right)}{121275} + \frac{1024}{225}a^5 b^5 \alpha S_2 + \frac{65536\beta\gamma\left(a^{11} b^7 + a^7 b^{11}\right)}{121275} \end{bmatrix}$$

where t is the web thickness and E is Young's modulus of elasticity. Then the principle of minimum total complementary potential is invoked and expression {5} is minimized with respect to each of the three unknown coefficients α, β, and γ:

$$\frac{\partial U'}{\partial \alpha} = \frac{t}{2E} \begin{bmatrix} \frac{1024 a^5 b^5 S_2}{225} + \frac{65536\alpha\left(a^9 b^5 + a^5 b^9\right)}{1575} + \frac{262144\alpha a^7 b^7}{11025} + \\[2mm] \frac{65536\left(a^{11} b^5\beta + a^5 b^{11}\gamma\right)}{17325} + \frac{65536\left(a^7 b^9\beta + a^9 b^7\gamma\right)}{11025} \end{bmatrix} = 0 \qquad \{6\}$$

$$\frac{\partial U'}{\partial \beta} = \frac{t}{2E} \begin{bmatrix} \frac{1024 a^7 b^5 S_2}{225} + 65536\left(\frac{\alpha a^{11} b^5}{17325} + \frac{\alpha a^7 b^9}{11025} + \frac{\beta a^{13} b^5}{75075} + \frac{\beta a^9 b^9}{3675}\right) + \\[2mm] \frac{262144 a^{11} b^7\beta}{121275} + \frac{65536\gamma\left(a^{11} b^7 + a^7 b^{11}\right)}{121275} \end{bmatrix} = 0 \quad \{7\}$$

$$\frac{\partial U'}{\partial \gamma} = \frac{t}{2E}\left[\frac{1024a^5b^7S_2}{1575} + 65536\left(\frac{\alpha a^5 b^{11}}{17325} + \frac{\alpha a^9 b^7}{11025} + \frac{\gamma a^9 b^9}{3675} + \frac{\gamma a^5 b^{13}}{75075}\right) + \frac{262144\gamma a^7 b^{11}}{121275} + \frac{65536\beta(a^{11}b^7 + a^7 b^{11})}{121275}\right] = 0 \quad \{8\}$$

Expressions {6},{7}, and {8} can now be solved for the three unknowns:

$$\alpha = -\frac{77S_2}{64}\left[\frac{1430(a^8 + b^8) + 9477(a^6 b^2 + a^2 b^6) + 74219 a^4 b^4}{\psi}\right] \quad \{9\}$$

$$\beta = -\frac{1001S_2}{64}\left[\frac{715a^6 + 1235a^4 b^2 + 170a^2 b^4 + 22 b^6}{\psi}\right] \quad \{10\}$$

$$\gamma = -\frac{1001S_2}{64}\left[\frac{22a^6 + 170a^4 b^2 + 1235a^2 b^4 + 715 b^6}{\psi}\right] \quad \{11\}$$

where ψ is:

$$\psi = 25025(a^{12} + b^{12}) + 129740(a^{10} b^2 + a^2 b^{10}) + 911998(a^8 b^4 + a^4 b^8) + 726044 a^6 b^6 \quad \{12\}$$

Thus with expressions for α, β, and γ the stresses σ_{xx}, σ_{yy}, and σ_{xy} are now known in terms of closed form expressions upon substitution of α, β, and γ into {2-4}. The MD deformation of the web is also needed such that the web twist can be related to the stresses. The MD strain is developed using the biaxial constitutive expression:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{1}{E}\left[\sigma_{xx} - \nu \sigma_{yy}\right] \quad \{13\}$$

Integration of {13} yields an expression too lengthy to present in this context. Also it should be noted that integration of {13} involves integrating a partial derivative and as such one must assume that the solution for u may involve some functions of y alone that could be grouped as f(y). After integrating {13} and substituting x=0 into the result it is found to be equal to zero for all y values. Through symmetry of structure and load and a fortunate choice for the origin of the x-y axes in Figure 1 it is known that the u deformation must be zero for all y locations and thus f(y) must be equal to zero. At specific locations the expressions for the MD deformation, u, are reasonably compact. The deformation of the end of the span where x becomes a is:

$$u_{|x=a} = \frac{a}{105E}\left[105(S_1 + S_2 y^2) - 16a^4\left(\begin{array}{c}\alpha(14 b^2 - 42 y^2) + \beta a^2(2 b^2 - 6 y^2) \\ + \gamma(84 b^2 y^2 - 7 b^4 - 105 y^4)\end{array}\right)\right] \quad \{14\}$$

The total deformation of the center of the web is:

$$u_0 = 2u_{|x=a,y=0} = \frac{2}{E}\left[aS_1 - \frac{16}{15}\left(2\alpha a^5 b^2 + 2\beta \frac{a^7 b^2}{7} - \gamma a^5 b^4\right)\right] \quad \{15\}$$

The total deformation of the edge of the web is:

$$u_b = 2u_{|x=a,y=b} = \frac{2}{E}\left[aS_1 + a b^2 S_2 + \frac{64}{15}\left(\alpha a^5 b^2 + \beta \frac{a^7 b^2}{7} + \gamma a^5 b^4\right)\right] \quad \{16\}$$

It should be noted that the variables S_1 and S_2 are not independent in a web line. A fundamental web line operating parameter is the web tension, T_w, and the parabolic stress profile assumed to exist at the panel ends must be in equilibrium with the web tension per:

$$T_w = \int_{-b}^{b} \left(S_1 + S_2 y^2 \right) t \, dy = 2bt \left(S_1 + \frac{S_2 b^2}{3} \right) \qquad \{17\}$$

In Figure 2 a web subject to a twist ϕ is shown. The edge of the web is required to travel further than the web center due to the twist, an amount that will be equal to the difference between u_b and u_o. Assuming that the web at the edge must travel a distance $2a + (u_b - u_o)$ a simple model was developed to relate the web deformations to the angle of twist ϕ:

$$\phi = \frac{\sqrt{(u_b - u_o)^2 + 4a(u_b - u_o)}}{b} \qquad \{18\}$$

This model could be much more complex. As a roller twists the angle of wrap of the web increases about the rollers on one side and decreases on the other side, in fact the free span length will be longer on one edge of the web with respect to the other side. Expression {18} has been found to be adequate to model the behavior as will be shown in the experimental results.

As later will be shown in an example, the maximum compressive CMD stress occurs at the web center immediately prior to the roller (i.e. x=a, y=0). Substitution of this location and expressions {9-11} into {3} yields:

$$\sigma_{cmd,max} = -\frac{77}{8} S_2 a^2 b^4 \left[\frac{10725 a^8 + 25532 a^6 b^2 + 76429 a^4 b^4 + 25532 a^2 b^6 + 1430 b^8}{\psi} \right]$$

$$\{19\}$$

If the CMD stress exceeds the critical value required to buckle the cylindrical shell of web upon the roller surface wrinkling will occur. Timoshenko [2] gives the critical value for a cylindrical sector as [2]:

$$\sigma_{cr} = \frac{E t}{R \sqrt{3(1 - v^2)}} \qquad \{20\}$$

where R is the radius of the roller.

Thus it is apparent from expressions {19} and {20} that given the web span, width, thickness, web modulus of elasticity and Poisson's ratio, and the roller radius that a critical value of S_2, that will wrinkle the web, can be determined. With known web tension (Tw) S_1 can be extracted from expression {17}. The web MD deformations at web center and the edge can then be determined from {15} and {16}. The web twist associated with a critical level of CMD stress that will wrinkle the web can now be determined from expression {18}.

AN EXAMPLE

Assume a polyester web, 17.8 μm thick, that is 15.2 cm wide spans two rollers 12.7 cm apart. It is also given that Young's Modulus is 4.13 GPa, that Poisson's ratio is 0.3, that the rollers are 7.37 cm in diameter, and that the web has 40 Newtons of web tension. The quest is to determine the amplitude of twist the web can be subjected to prior to wrinkling.

From expression {20} the critical web CMD stress that will induce a wrinkle is found to be 1.17 MPa. S_2 is found to be 57.33 KPa/cm^2 from expression {19} and S_1 is found to be 13.65 MPa from expression {17}. The MD and CMD stress profiles for the web are now known from substitution of S_2 into expressions {9-11} that are then substituted into {2} and {3}. The stress profiles have been evaluated at discrete points and plots thereof

are shown in Figures 4 and 5. These plots are not needed to determine the twist but are shown to promote an understanding of how the stresses can vary throughout the web span. Finally S_2 is substituted into {15} and {16} which yield:

$$u_o = 0.424 \text{ mm}$$

$$u_b = 0.505 \text{ mm}$$

Substitution of u_o and u_b into expression {18} yields that the critical twist ϕ is 0.059 radians or 3.4 degrees.

EXPERIMENTAL VERIFICATION

An experimental apparatus, as shown in Figure 6, was constructed and inserted into a web line. The apparatus allowed the span length (2a) to be fixed at various values. A typical experiment would consist of establishing a desired tension level in the web and then slowly increasing the web twist (ϕ) until a wrinkle formed upon the roller. These experiments were conducted several times for various web widths (2b), span lengths (2a), web thickness (t), and web tensions (T_w).

Test results are shown in Figure 7 for a polyester (PET- polyethylene terephthalate) film 23.4 μm thick and 15.2 cm wide for four span lengths. These tests were conducted at 3M Company. Note that the experiments show that higher twist angles are required to initiate wrinkling at low nominal web stress when compared to the tests run at higher web stresses for a given span length. The nominal or average web stress is the web tension divided by the cross sectional area of the web. Also note that with increased nominal web stress the twist required to initiate a wrinkle appears to approach a horizontal asymptote. The theory presented herein predicts the twist required to wrinkle the web is independent of length. Note the agreement between the twist predicted by the theory compared to the test results at high nominal web stress is quite good. The theory presented herein does not account for the friction between the web and the rollers. With low friction the roller behaves as a spreading device and the analysis of this behavior was thoroughly treated in an earlier publication [3].

In Figure 8, test results and theory are shown for four span lengths for a 17.84 μm thick PET web that was 15.2 cm wide. The experimental data presented in this and the remaining figures were collected at OSU. These tests were conducted with a roller covering that was coated with Dow 236, a silicone based product that has a high friction coefficient when in contact with PET. Note that the twist required to initiate a wrinkle for a given span length is more constant with respect to the web stress than was shown in Figure 7. There is a small but notable positive slope of the data for a given span length that was evident for all the cases in which the high coefficient covering was used. Note that the theory estimates the wrinkles nicely in this case as well.

Results from tests and theory for a 6.6 μm thick PEN (polyethylene naphthalate) film 15.2 cm wide is shown in Figure 9 for two span lengths. These tests were run with the high coefficient of friction covering. Again the theory agreed with the experiments well.

In Figure 10, test results and theory are shown for two span lengths for a 50.8 μm thick polyethylene (PE) web that was 15.2 cm wide. These results are particularly interesting in that the tests were conducted with and without the high coefficient of friction covering on the twist roller. Note at low nominal web stress how the bare aluminum roller surface with lower friction coefficient allows much greater angle of twist to induce a wrinkle than when the roller is covered with the high coefficient covering. At

higher web stress the test results converge for a given span length. The theory predicted the occurrence of the wrinkles quite well.

In Figure 11, the test results shown in Figure 8 for the 17.8 μm web are shown again. Overlaid in this case are curves that show the degree of twist required to reduce the MD stress at the center of the web where it contacts a roller (S_1) to zero. This is the amount of twist required to generate slackness at the web center. The twist amplitude was calculated by setting S_1 equal to zero and substituting given values of web tension, web width, and thickness into expression {17}. This yielded a value for S_2 that could be substituted into {15} and {16} to obtain u_0 and u_b, which in turn were substituted into {18} to obtain the twist associated with a zero value of S_1. Note that wrinkles were generated prior to a slack web condition in every case.

Finally in Figures 12 and 13, the results of some parametric studies that were performed using the model such that the effects of span ratio (L/W or 2a/(2b)) and web thickness on the twist required to generate a wrinkle could be shown. The critical twist is very sensitive to the span ratio, per Figure 12, which is fortunate since it is the variable an engineer is most likely to be able to alter. The critical twist is less sensitive to variations in the web thickness than span ratio, comparing Figures 12 and 13. Although the web thickness is usually determined by the needs of the product, the results show that it is much easier to eliminate wrinkles due to twist by increasing the span ratio rather than through an increase in web thickness.

CONCLUSIONS

A closed form solution for predicting wrinkling due to web twist has been presented. Test results for four polymer webs have been presented providing experimental verification of the theory.

REFERENCES

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2. Timoshenko, S.P., "Theory of Elastic Stability," ," McGraw-Hill, 1936, pp. 467-470
3. Good, J.K, Kedl, D.M., and Shelton, J.J., "Shear Wrinkling in Isolated Spans," Proceedings of the Fourth International Conference on Web Handling, Oklahoma State University, Stillwater, Oklahoma, June 1-4, 1997.
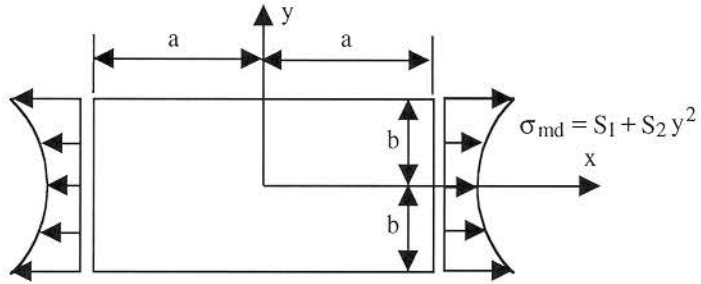
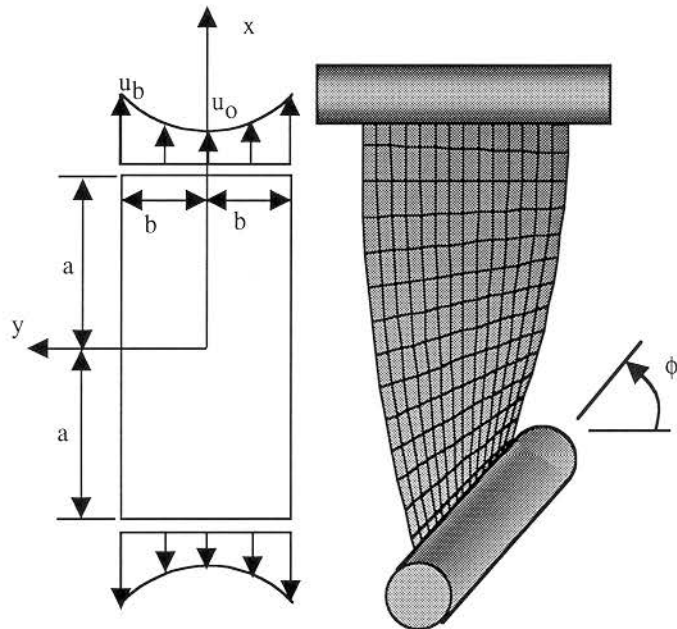**Figure 1 – A Web Span Subject to a Parabolic Distribution of Stress**

$$\sigma_{md} = S_1 + S_2\, y^2$$

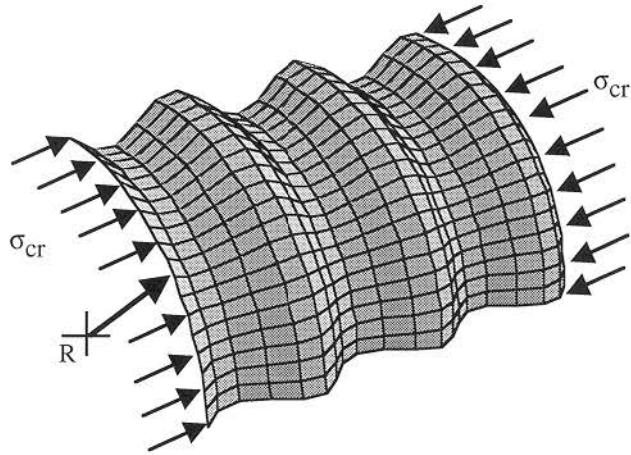**Figure 2 – A Web Span Subject to a Twist ϕ**

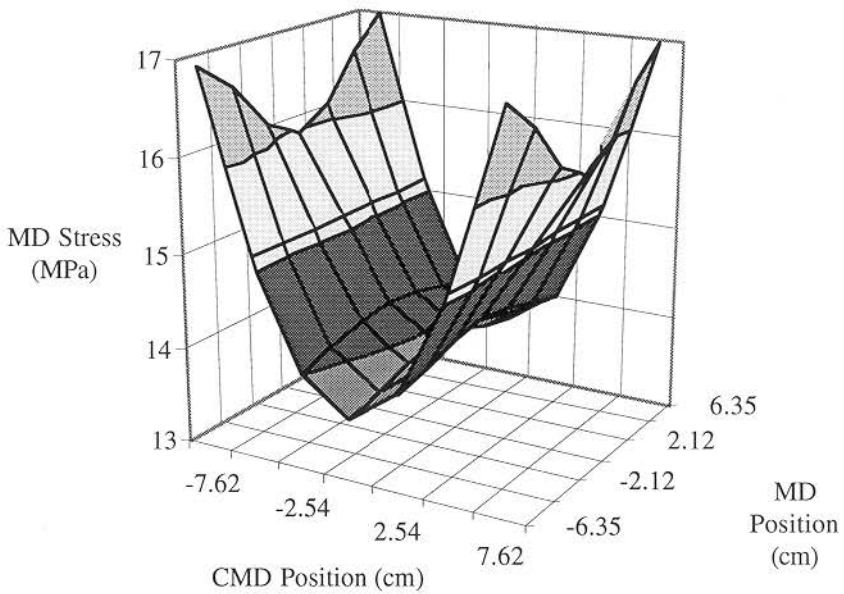**Figure 3 – A Sector of Web Wrapping a Roller that has Failed due to Cylindrical Instability**

Figure 4 – Machine Direction σ_{xx} Stresses for a 17.8 μm PET Film span 12.7 cm Long and 15.2 cm Wide due to a Twist of 3.56 Degrees.
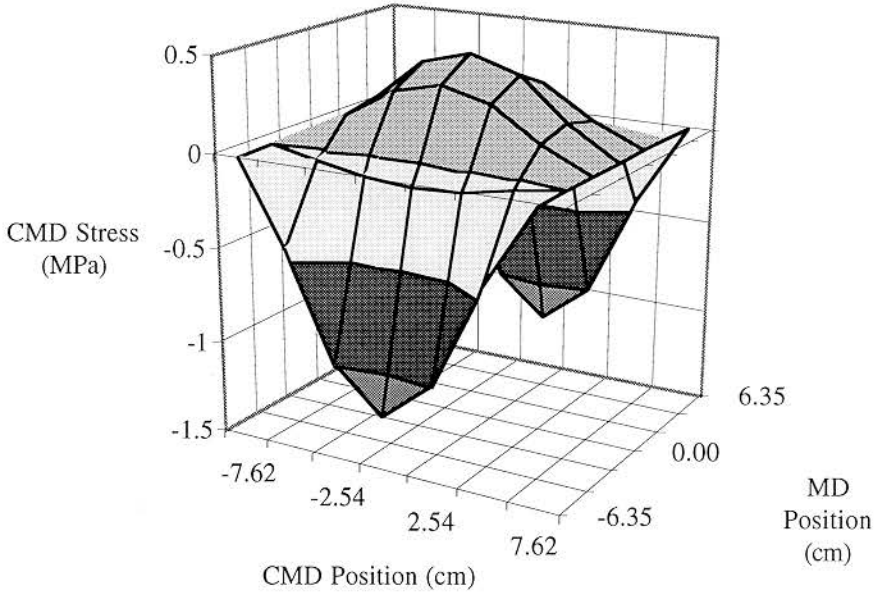
Figure 5 – Cross Machine Direction σ_{yy} Stresses for a 17.8 μm PET Film span 12.7 cm Long and 15.2 cm Wide due to a Twist of 3.56 Degrees. The compressive stress at the center (-1.17 MPa) is just sufficient to wrinkle the web when it encounters a 7.37 cm diameter roller per expression {19}.
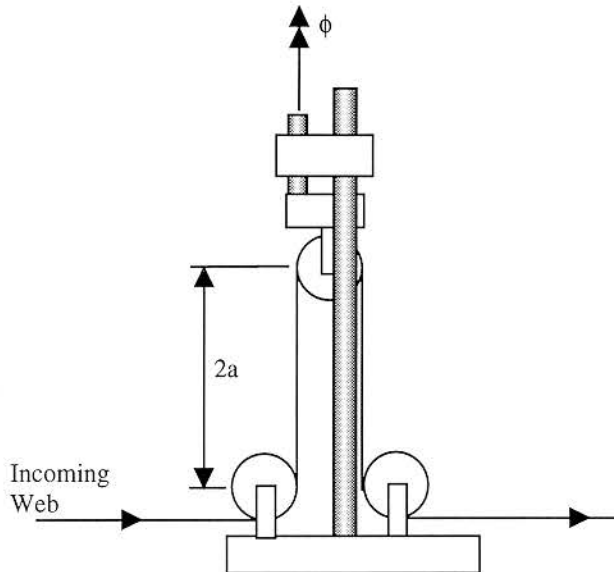
Figure 6 – Experimental Apparatus
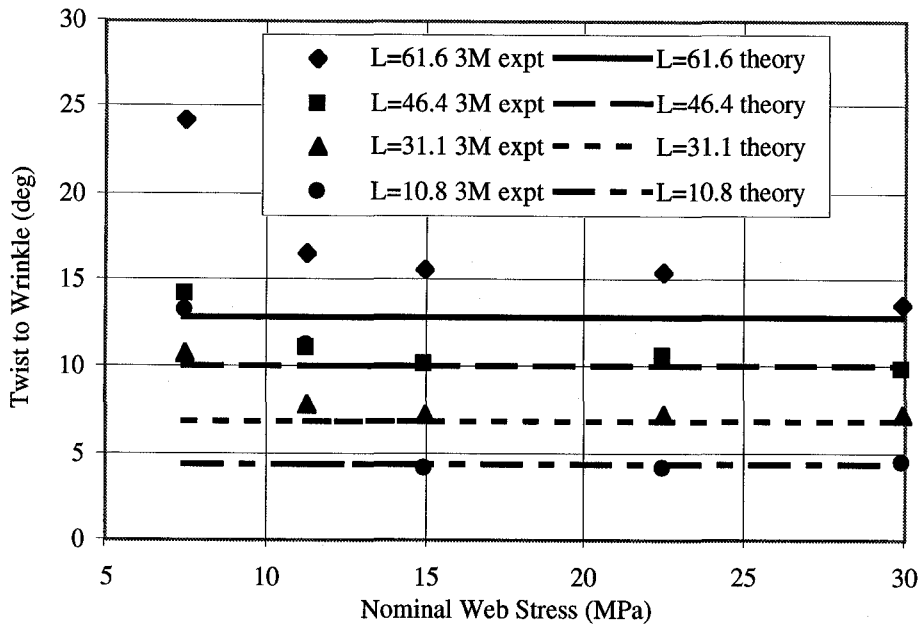
Figure 7 – Twist required to wrinkle a 23.4 μm PET film 15.2 cm wide of Span Lengths (L(cm)) per the Legend. Tests Conducted with no covering on the Twisting Roller.(E=4.13 GPa, v=.3, R=3.68 cm)
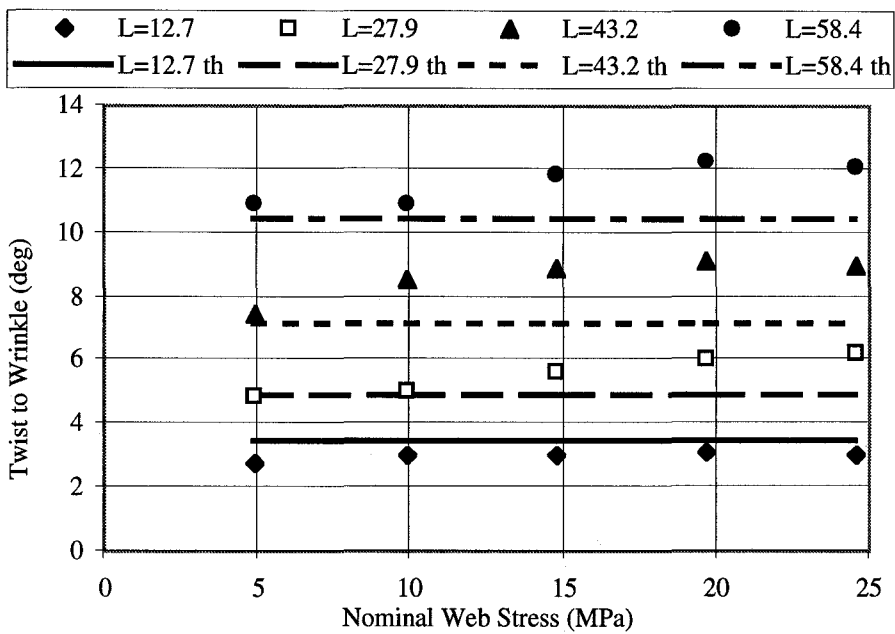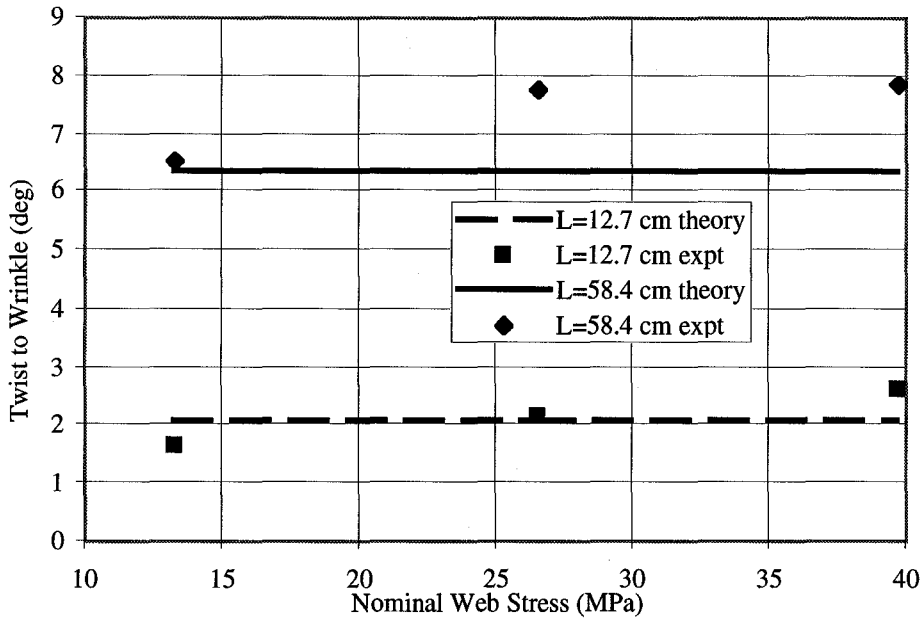

Figure 8 - Twist required to wrinkle a 17.8 μm PET film 15.2 cm wide of Span Lengths (L(cm)) per the Legend. Tests Conducted with Dow 236 covering on the Twisting Roller. (E=4.13 GPa, v=.3, R=3.68 cm)

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Figure 9 - Twist required to wrinkle a 6.6 μm PEN film 15.2 cm wide of Span Lengths (L(cm)) per the Legend. Tests Conducted with Dow 236 covering on the Twisting Roller. (E=8.87 GPa, ν=.3, R=3.68 cm)
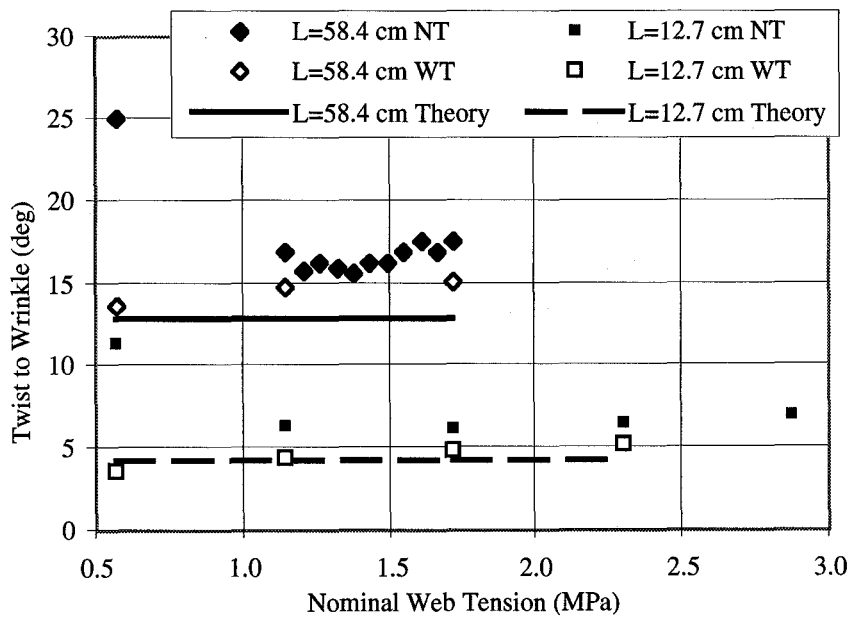

Figure 10 - Twist required to wrinkle a 50.8 μm PE film 15.2 cm wide of Span Lengths (L(cm)) per the Legend. Some Tests were Conducted with Dow 236 covering (WT) and some with no covering (NT) on the Twisting Roller.
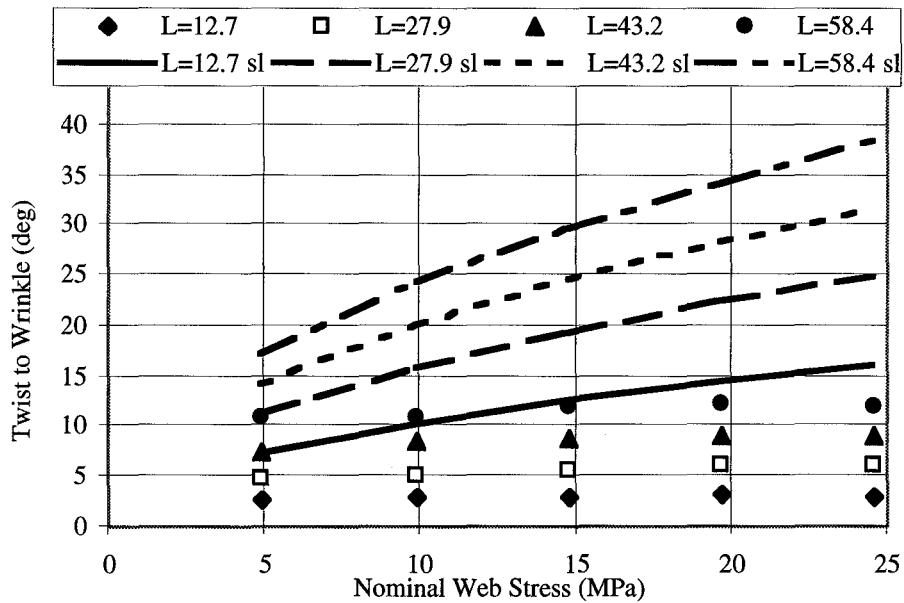
Figure 11 – Twist required to wrinkle a 17.8 μm PET film 15.2 cm wide of Span Lengths (L(cm)) per the Legend. The lines indicate the amount of twist required to induce slackness at the web center. Tests were conducted with Dow 236 covering on the twisting roller. (E=4.13 GPa, ν=.3, R=3.68 cm)
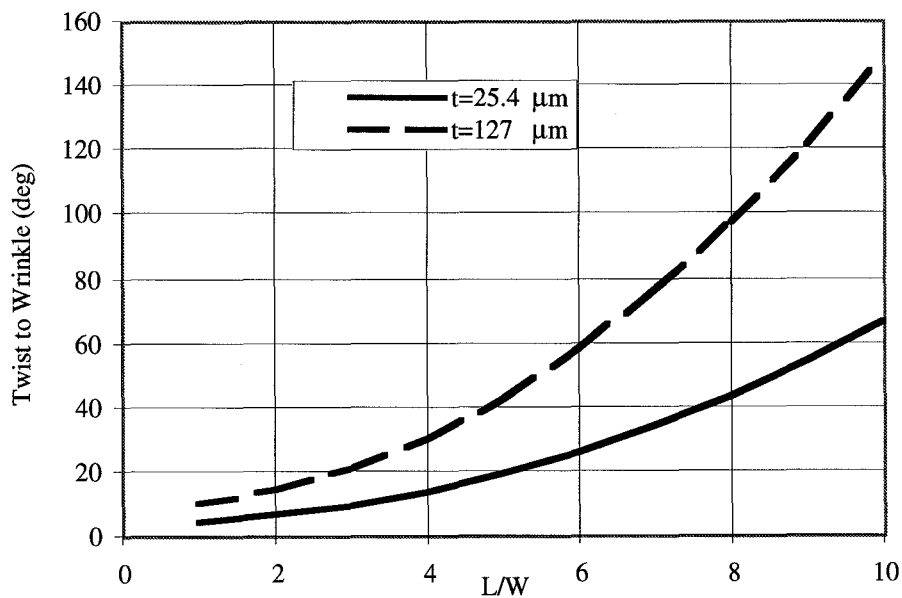

Figure 12 - Effect of Span Ratio (L/W) on the Twist Required to Wrinkle a Web (E=4.13 GPa, ν=0.3, D=7.37 cm)
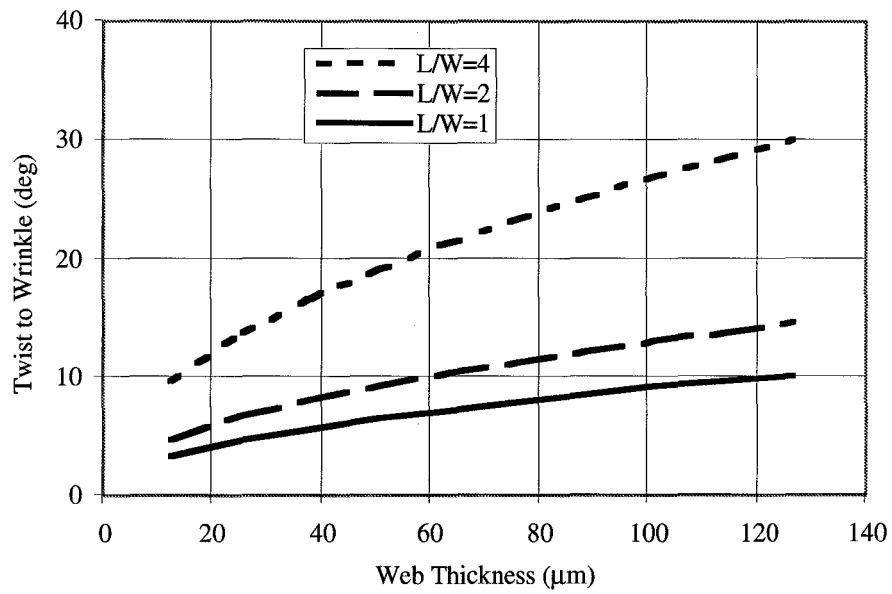
**Figure 13 – Effect of Web Thickness on the Twist Required to Wrinkle a Web
(E=4.13 GPa, ν=0.3, D=7.37 cm)**

J. K. Good And P. Straughan
Wrinkling of Webs Due To Twist
6/9/99   Session 4   11:05 - 11:30 a.m.

Question - Mark O'Neil, Procter and Gamble
I was wondering if this is enforcing that the edges of the web are following a helical path or is it allowing them to take a straight-line path through the rollers?

Answer – Keith Good, Oklahoma State University
Actually, the theory has no real idea that the web is deforming in a 3-D space. The stress theory that you saw me was for a flat panel. The assumption would be that that edge is reasonably straight. And you are right in reality, I think it's a second order effect but, the web does take sort of a helix path but that didn't seem to be enough of an issue to effect my experimental results. There were several assumptions that went into the building of the theory and the question of what are good assumptions and what are bad assumptions are provided by the experimental results. The experiments proved that the assumptions were reasonable.