THE INFLUENCE OF WEB WARPAGE ON THE LATERAL DYNAMICS OF WEBS

by

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ABSTRACT

The transport of long flexible webs is of great importance in many manufacturing applications, including the processing of paper, textiles, magnetic tapes and photographic film. Beginning with the pioneering work of Shelton and Reid (1,2), analysts have had the ability to model the lateral dynamics of webs, and, thereafter, to apply control algorithms for accurate steering. See, for example, Reid, Shin and Lin (3); Sievers, Balas and von Flotow (4); and Young and Reid (5). Other analysts, such as Soong and Li (6); and Young, Shelton and Fang (7,8), have studied greater web-conveyance systems.

Each of these models assumes that the web is perfectly straight in its stress-free state, which is often a reasonable assumption. However, in some regions of a web, manufacturing flaws and splice misalignments can give the web a stress-free geometry that is warped in the plane of the web. As the imperfection travels between rollers it has an action equivalent to an applied, lateral load that moves with the speed of the web. This causes the imperfect web to deform differently than expected, which, in turn, affects lateral dynamics and control.

In a recent paper, Benson (9) added geometrical imperfections to the web steering analysis, and studied the special case of a splice moving between two rollers. The model utilized the "Timoshenko" beam theory, presented by Shames and Dym (10), to account for the elastic bending and transverse shear deformation of the web. Boundary conditions at the rollers were influenced by the stick/slip studies conducted by Johnson (11) and Smith (12). In the present paper the imperfect web formulation of Benson (9) will be used to examine the case of a sinusoidally warped web. Example results will be presented to show how this imperfection affects the lateral dynamics of the web, and can lead to what Sievers, et al. (4) called "weave regeneration" in downstream web spans.

NOMENCLATURE

A = Area of the web cross-section = hb.
b = Width of the web.
C_i = Constants of integration used in the solution for the web displacement; i = 1,2,3,4.
E = Young's modulus of the isotropic elastic web.
G = Shear modulus of the isotropic elastic web = $E/[2(1+\mu)]$.
h = Thickness of the web.
I = Area moment of inertia of the web cross-section = $hb^3/12$.
k = Factor used in calculating the shear angle = $(10+10\mu)/(12+11\mu)$.
L = Span between the upstream and downstream rollers.
M = Bending moment in the web; varies with x and t.
M_L = Bending moment at the downstream roller; zero if web sticks to roller.
t = Time.
T = Axial tension in the web.
v = Axial velocity of the web.
V = Shear force in the web; varies with x and t.
w = Lateral deflection of the web; varies with x and t.
w_o = Lateral deflection of the web at the upstream roller; varies with t.
w_L = Lateral deflection of the web at the downstream roller; varies with t.
x = Axial coordinate; zero at the upstream roller.
x_o = Offset distance used to position the sinusoidally warped web at time t = 0.
Z = Lateral displacement of the downstream roller; varies with t.

α = Wave number of the sinusoidally warped web.
β = Shear angle in the web; varies with x and t.
γ = Frequently occurring constant used in the solution of the problem.
θ = Angle of the downstream roller; varies with t.
λ = Wavelength of the sinusoidally warped web.
μ = Poisson ratio of the isotropic elastic web.
ϕ = Slope of the web; varies with x and t.
ϕ^* = Slope of the web in its stress-free state; varies with x and t.
ϕ_L = Slope of the web at the downstream roller; varies with t.
Φ^* = Maximum slope of the sinusoidally warped web.
ψ = Bending angle of the web; varies with x and t.
ψ_o = Bending angle of the web at the upstream roller; varies with t.

DEFINITION OF THE PROBLEM

Figure 1 shows the web/roller geometry. An isotropic, elastic web moves with axial velocity v from the Upstream Roller to the Downstream Roller. The span between the rollers is L, the width of the web is b, and the thickness of the web is h (not depicted). The axial coordinate is x, and the lateral deflection for the web is w. The downstream roller may be tilted by an angle θ, and displaced laterally by a distance Z (not depicted). The stress-free web is allowed to have a sinusoidal warpage with maximum slope angle Φ^*, and wavelength λ. The offset distance x_o allows the sine wave to be arbitrarily positioned between the rollers at the start of simulation (see equation {10}). We wish to know how the web will track laterally as a result of the warpage and roller movement.

MATHEMATICAL ANALYSIS

The following summary of governing equations for a warped web is taken from Benson (9). Please refer to Figure 1 and the Nomenclature for the definition of variables.

Balance of moments: $$\frac{dM}{dx} = -V \qquad \{1\}$$

Balance of vertical forces: $$\frac{dV}{dx} = -T\frac{d^2w}{dx^2} \qquad \{2\}$$

Moment / curvature relationship: $$\frac{d\psi}{dx} = \frac{d\phi^*}{dx} + \frac{M}{EI} \qquad \{3\}$$

Shear force / angle relationship: $$\beta = \frac{V}{kGA} \qquad \{4\}$$

Slope kinematics: $$\frac{dw}{dx} = \psi + \beta = \phi \qquad \{5\}$$

Upstream boundary conditions: w_o = known function of time. {6}

ψ_o = known function of time. {7}

Downstream boundary conditions: $$\frac{dw_L}{dt} = v(\theta - \phi_L) + \frac{dZ}{dt} \qquad \{8\}$$

$$M_L = 0\,. \qquad \{9\}$$

The warped web is assumed to have a sinusoidal shape in its stress-free configuration:

$$\phi^*(x) = \Phi^* \sin[\alpha(x - vt - x_o)] \qquad \{10\}$$

where the wave number, α , is related to the wave length, λ, by the following formula:

$$\alpha = \frac{2\pi}{\lambda}\,. \qquad \{11\}$$

The solution to these equations is straight-forward. The bending moment, M, shear force, V, web slope, ϕ, and lateral displacement, w, are given by the following equations:

$$M = C_1\sinh(\gamma x) + C_2\cosh(\gamma x) - \frac{EI\gamma^2\alpha\Phi^*}{\alpha^2 + \gamma^2}\cos[\alpha(x - vt - x_o)] \qquad \{12\}$$

$$V = -\gamma C_1\cosh(\gamma x) - \gamma C_2\sinh(\gamma x) - \frac{EI\gamma^2\alpha^2\Phi^*}{\alpha^2 + \gamma^2}\sin[\alpha(x - vt - x_o)] \qquad \{13\}$$

$$\phi = \frac{C_1}{\gamma EI}\left(1-\frac{\gamma^2 EI}{kGA}\right)\cosh(\gamma x) + \frac{C_2}{\gamma EI}\left(1-\frac{\gamma^2 EI}{kGA}\right)\sinh(\gamma x)$$
$$+\frac{\alpha^2 \Phi *}{\alpha^2+\gamma^2}\left(1-\frac{\gamma^2 EI}{kGA}\right)\sin[\alpha(x-vt-x_o)] + C_3 \qquad \{14\}$$

$$w = \frac{C_1}{\gamma^2 EI}\left(1-\frac{\gamma^2 EI}{kGA}\right)\sinh(\gamma x) + \frac{C_2}{\gamma^2 EI}\left(1-\frac{\gamma^2 EI}{kGA}\right)\cosh(\gamma x)$$
$$-\frac{\alpha \Phi *}{\alpha^2+\gamma^2}\left(1-\frac{\gamma^2 EI}{kGA}\right)\cos[\alpha(x-vt-x_o)] + C_3 x + C_4. \qquad \{15\}$$

In the preceding four equations, C_1, C_2, C_3 and C_4 are constants of integration, and the constant, γ, appearing in the hyperbolic-trigonometric functions is given by:

$$\gamma^2 = \frac{T / EI}{1+(T / kGA)}. \qquad \{16\}$$

The constants of integration are determined by the boundary conditions {9}, {7}, {6} and {8}, which, cast in that order, lead to the following 4×4 matrix equation:

$$\begin{bmatrix} \sinh(\gamma L) & \cosh(\gamma L) & 0 & 0 \\ \frac{1}{\gamma EI} & 0 & 1 & 0 \\ 0 & \frac{1}{\gamma^2 EI}-\frac{1}{kGA} & 0 & 1 \\ \left(\frac{1}{\gamma^2 EI}-\frac{1}{kGA}\right)\sinh(\gamma L) & \left(\frac{1}{\gamma^2 EI}-\frac{1}{kGA}\right)\cosh(\gamma L) & L & 1 \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{EI\gamma^2 \alpha \Phi *}{\alpha^2+\gamma^2}\cos[\alpha(L-vt-x_o)] \\ \psi_o - \frac{\alpha^2 \Phi *}{\alpha^2+\gamma^2}\sin[\alpha(-vt-x_o)] \\ w_o + \frac{\alpha \Phi *}{\alpha^2+\gamma^2}\left(1-\frac{\gamma^2 EI}{kGA}\right)\cos[\alpha(-vt-x_o)] \\ w_L + \frac{\alpha \Phi *}{\alpha^2+\gamma^2}\left(1-\frac{\gamma^2 EI}{kGA}\right)\cos[\alpha(L-vt-x_o)] \end{Bmatrix} \qquad \{17\}$$

This linear system is easily solved for C_1, C_2, C_3 and C_4, and then, through back-substitution, all other variables of interest can be determined.

EXAMPLE RESULTS AND DISCUSSION

Table 1 provides a set of web/roller specifications that will be used to illustrate results for a sinusoidally warped web with a "short" wavelength of 3.048 meters. This is equal to the roller span. The warped web has a maximum slope angle of 0.1 degrees, and the offset is zero. To focus strictly on the influence of web warpage on lateral tracking, it is assumed that the downstream roller does not pivot or slide, and that the web is fed in perfectly at the upstream roller with no lateral displacement or bending angle:

$$\theta(t) = 0, \quad Z(t) = 0, \quad w_o(t) = 0, \quad \psi_o(t) = 0 \qquad \{18\}$$

Figure 2 shows the resulting deflection at the downstream roller, $w_L(t)$. Five wavelengths are tracked, and it is seen that the web very quickly reaches a steady-state oscillation of about 1.2 millimeters in amplitude.

Figure 3 shows how this web deforms between the upstream and downstream rollers. A single steady-state oscillation is broken down into ten equally spaced segments. As it takes 0.2 seconds for the oscillation to repeat, each curve is separated by 0.02 seconds. The perfect entry of the web at the downstream roller is seen in the figure; there is no lateral deflection or bending angle. Nevertheless, at the downstream roller, the ± 1.2 mm oscillation, first seen in Figure 2, is evident. It is believed that this is an illustration of what Sievers, et al. (4) called "weave regeneration." Even if the web is perfectly controlled at one roller, by the next downstream roller web imperfections can reestablish a lateral oscillation.

The web can, of course, be forced to maintain a zero deflection at the downstream roller, through either a tilting or sliding correction maintained by the roller. In place of equations {18}, either:

$$\theta(t) = \phi_L(t), \quad Z(t) = 0, \quad w_o(t) = 0, \quad \psi_o(t) = 0 \qquad \{19\}$$

for tilting correction, or:

$$\theta(t) = 0, \quad \frac{dZ}{dt} = v\phi_L(t), \quad w_o(t) = 0, \quad \psi_o(t) = 0 \qquad \{20\}$$

for sliding correction, will force $w_L(t) = 0$ in equation {8}. Either choice leads to the same web-span deflections, which are depicted in Figure 4. As in Figure 3, ten equally spaced deflections are shown for a single steady-state oscillation. Although zero deflection is maintained at the downstream roller, it is a short-lived "victory." As depicted in Figure 3, weave regeneration can be expected to occur at the *next* roller downstream. Further, this simple form of control has only caused the deflection to be zero at the downstream roller. Nothing has been done to drive the bending angle to zero, although that might be achieved through a combination of roller tilt and slide. The graphs for $\theta(t)$ and $Z(t)$ used to generate the results of Figure 4 are shown in Figure 5. Not surprisingly, the corrective motions have the same steady-state periodicity as the warped web.

Table 2 gives specifications for a "long" wavelength example. Half of a cosine arc – concave down – is attached to straight segments on either side. The half-wavelength is 30.5 meters, which is ten times the roller span. The warped web bends from –1 degree to +1 degree for a total rotation of two degrees. The offset of 16.8 meters positions the start of the bend between the rollers when the simulation begins. Equations {18} are again used to set conditions at the upstream and downstream rollers. Figure 6 shows how this web with a long, slow bend tracks on the downstream roller. The bent web segment is in the roller span for 2.2 seconds, and the web reaches a maximum excursion of 2.74 mm at 1.23 seconds. At 2.2 seconds the web is nearly back to zero deflection at the downstream roller – but not completely so – and it then asymptotes to zero by the usual lateral dynamics for flat webs. It is thus seen how an isolated imperfection in a web will induce an isolated lateral displacement as the warpage approaches, and passes over a roller.

SUMMARY

The lateral dynamics of a sinusoidally warped web has been modeled using a linear elastic beam theory that allows the web to have a non-flat, stress-free shape. It has been shown that even under ideal entering conditions at an upstream roller, warpage in the web will induce weaving at a downstream roller. A useful extension of this work would be to imbed this single-span model in a complete web-path analysis. This would allow for more interesting conditions to be applied at the "upstream" rollers, and would permit a more detailed examination of weave regeneration in multiple roller systems carrying warped webs.

ACKNOWLEDGMENTS

Funding from the Eastman Kodak Company in support of the Mechanics of Flexible Structures Project at Penn State University is gratefully acknowledged. The author thanks Kevin Cole, Eric Mockensturm, Pratima Singh, Ken Stack and Bob Walton for many helpful discussions about this work.

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Specification		**Metric**		**English**	
Web Length,	$L =$	3.048	m	120.0	in
Web Width	$b =$	0.305	m	12.0	in
Web Thickness	$h =$	0.178	mm	0.007	in
Young's Modulus	$E =$	3.448	GPa	500,000	psi
Poisson Ratio	$\mu =$	0.45		0.45	
Tension/Width	$T/b =$	350.0	N/m	2.00	lb/in
Axial Velocity	$v =$	15.24	m/s	600.0	in/s
Maximum Slope	$\Phi^* =$	0.00175	rad	0.1	deg
Warpage Wavelength	$\lambda =$	3.048	m	120.0	in
Warpage Offset	$x_o =$	0.0	m	0.0	in

Table 1: Nominal Specifications for Short Wavelength Examples

Specification		**Metric**		**English**	
Web Length,	$L =$	3.048	m	120.0	in
Web Width	$b =$	0.305	m	12.0	in
Web Thickness	$h =$	0.178	mm	0.007	in
Young's Modulus	$E =$	3.448	GPa	500,000	psi
Poisson Ratio	$\mu =$	0.45		0.45	
Tension/Width	$T/b =$	350.0	N/m	2.00	lb/in
Axial Velocity	$v =$	15.24	m/s	600.0	in/s
Maximum Slope	$\Phi^* =$	0.0175	rad	1.0	deg
Warpage Wavelength	$\lambda =$	61.0	m	2400.0	in
Warpage Offset	$x_o =$	16.8	m	660.0	in

Table 2: Nominal Specifications for Long Wavelength Example

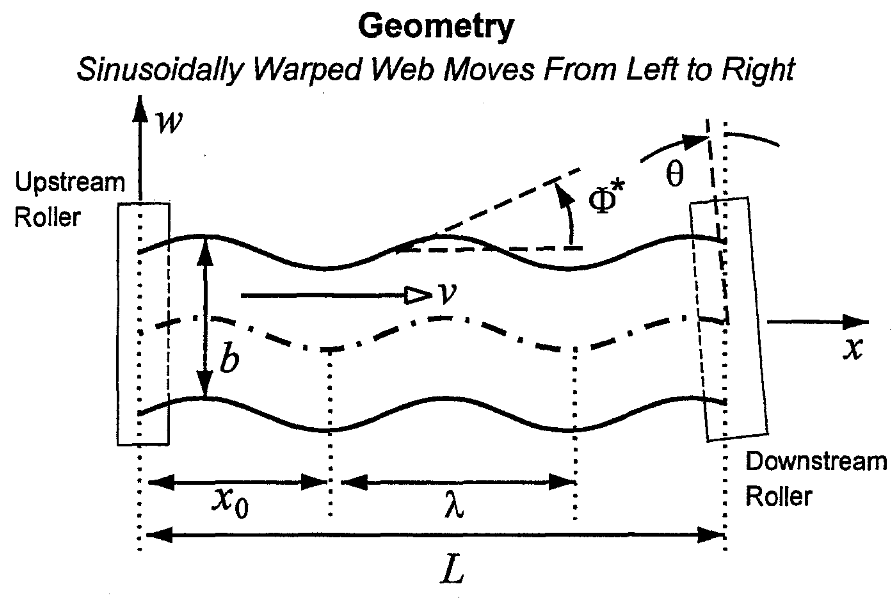

Figure 1

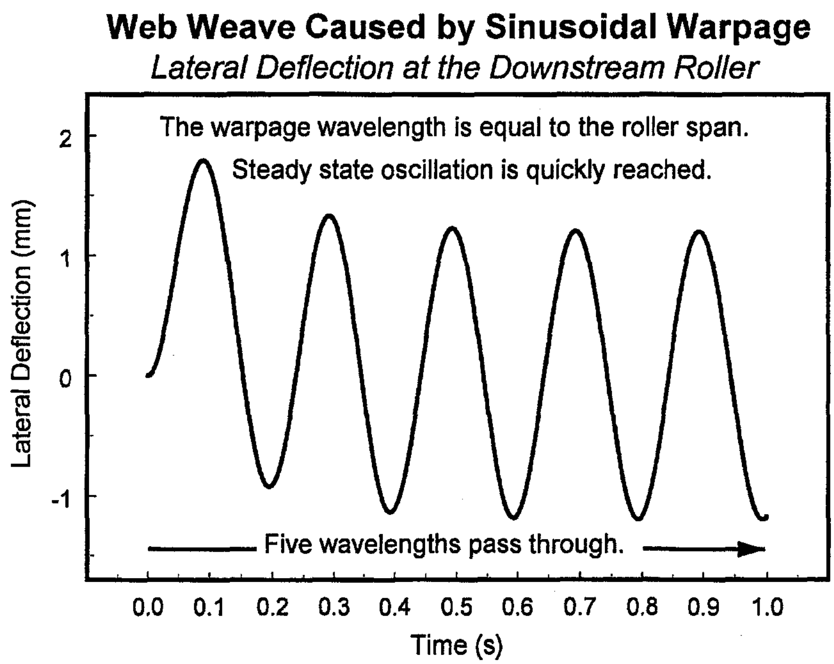

Figure 2

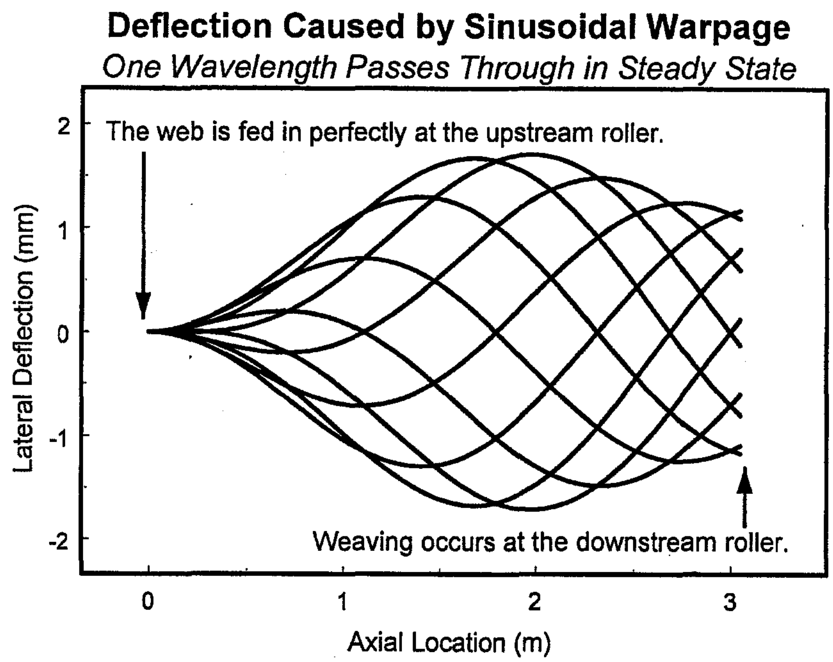

Figure 3

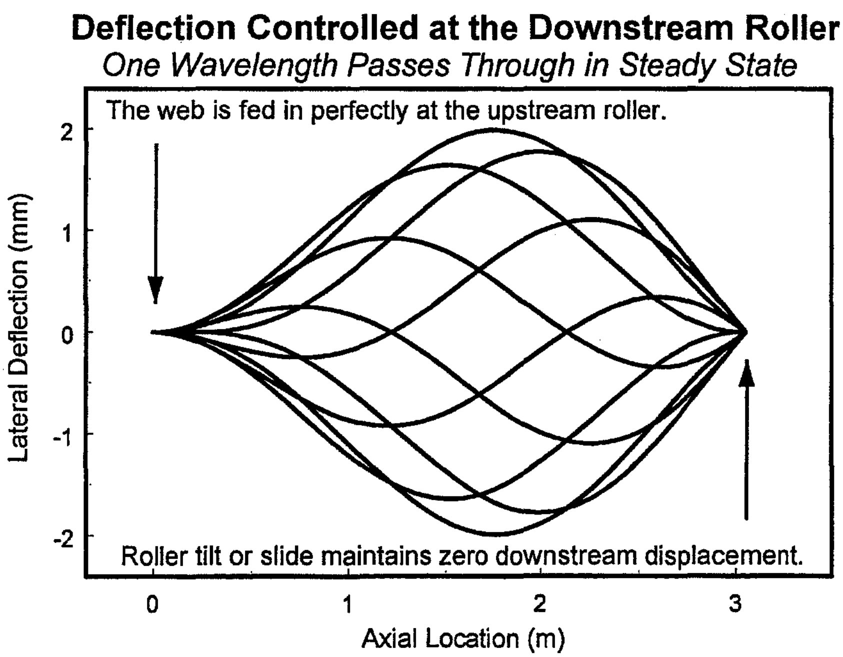

Figure 4

Sliding or Tilting Movement at the Downstream Roller

Web Deflection at Downstream Roller is Held to Zero

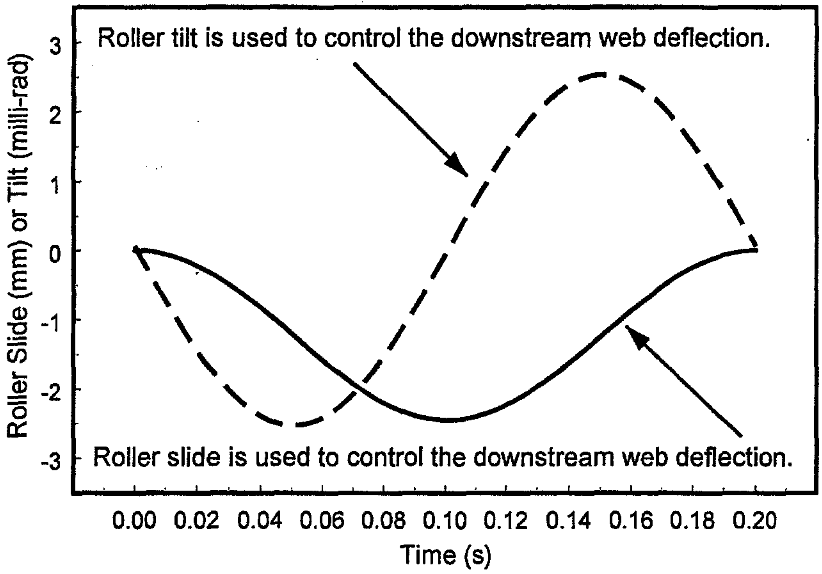

Figure 5

Deflection Caused by a Long Bend in the Web

Lateral Deflection at the Downstream Roller

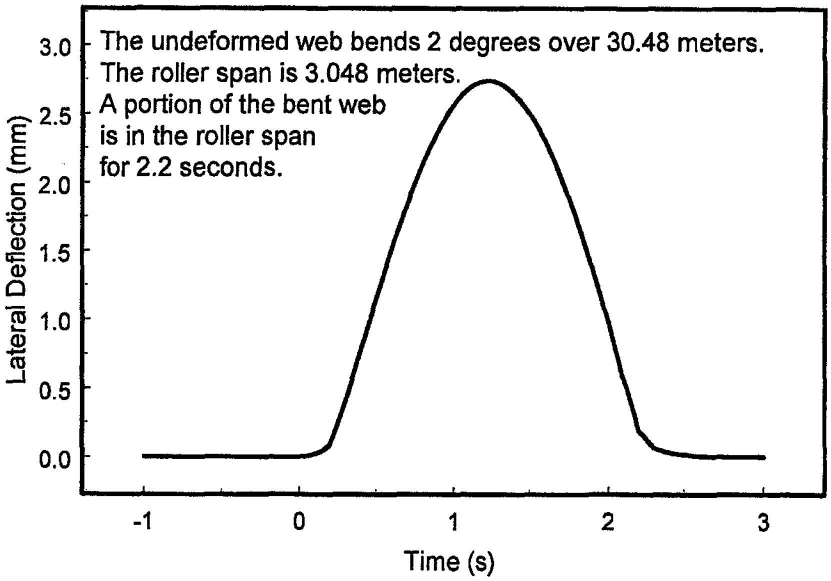

Figure 6

R. Benson

The Influence of Web Warpage on the Lateral Dynamics of Webs

6/9/99 Session 4 9:25 - 9:50 a.m.

Question - Peter Werner, Rockwell Automation

I notice in your examples, that you have wavelengths that are either equal to your span or much greater than your span. Did you do any investigations with the wavelength considerably shorter than the span?

Answer – R. Benson, Penn State University

I have, but it didn't seem to give me anything that was very different. A short wave length moved just as quickly to steady state oscillation. So I didn't see anything there. One thing that occurred to me was that conceivably you can see something like a resonance. This is a quasi-static analysis, I want to make clear. But there is a time history because of what's happening on the boundaries. So I was curious at what we might see. However, everything I looked at, just seemed to be quasi-static, so the wavelength seemed to have very predictable implications.

Question:

There was no averaging to having the wavelengths significantly shorter than the roll span.

Answer – R. Benson, Penn State University

I might have missed your point on averaging.

Questions:

The influence of the sheet, averaging the fact that you have multiple periods between the rolls.

Answer – R. Benson, Penn State University

I didn't see any.

Questions:

In Mr. Swanson's paper he saw that in his experiments that in short spans there wasn't a tracking issue to begin with.

Answer – R. Benson, Penn State University

To get a good comparing between his paper and mine I would have to examine a constant bend and the closest I came was in my long wave length example. When it comes to the top of the arch approximately a constant radius curve at the peak of my cosine curve. Ron and I discussed this yesterday, and I think we agree on the shape of the web deflection when a constant radius bend is between the rollers. I am going to study his paper carefully after the reference and try to work out a constant radius solution.

Question – David Pfeiffer, JDP Innovations Inc.

How does a web know where its own centroid is really? Because you could image a perfectly straight true web or newsprint that was accurately trimmed and someone draws a sine wave to web and trims it with scissors to match the sine wave so that web original centroid was straight. But what happens in that situation?

Answer – R. Benson, Penn State University
I agree the web doesn't know where its centroid is, that's why I argue on that stick condition applies everywhere. If its going to stick one place the web doesn't know not to stick somewhere else. If I cut out boundaries, here is what happens. You have to be careful, this is a beam theory in which we're averaging through the thickness. You put a curvature in. If you pull on a perfectly straight web with uniform tension, it would just stretch. But if I have these curved edges your actually bending influences and see curvature change. There would be no curvature change in the former case. By changing the geometry of the beam, the resultant loads have been affected. I'd like to think about this some more, and give you a more concise answer later.

Comment:
There is another example seen frequently in newspaper and print winding, in which slitting is done after oscillating unwind, which is to get rid of the trim by taking a triangle trim, wide sometimes, narrow sometimes, by moving the unwind stands by slitting. The interesting thing is it shows up on the windup in the derivative or slope of the oscillation. So you get stair-step offset every time the direction of the unwind and travel reverses. It shows up as derivative on the windup side.

Question - Bob Lucas, Beloit Corporation
A practical issue would be a very poorly slit web that has a lot of run-out in the slitter band. So you are creating a scallop slit edge and then you expose that to an edge guide without trying to straighten that edge out. So this is something that happens in day to day mill applications; so then we have to get into some dynamics because you have the mass of paper that asks to go back and forth.

Answer – R. Benson, Penn State University
Some of the machines I saw yesterday are moving so fast, that I think dynamic influences are going to be pretty important.

Comment:
As far as a practical application to what you have in your presentation, its a shame you didn't expand more in your paper on the spliced edge. This is a practical problem that deals with offsets in the mill and it would be nice to have a chance to get a better insight on that.

Question – R. Benson, Penn State University
Is it a displacement offset or an angle offset that you are talking about?

Answer:
An angle offset, but both can happen.