WEB LONGITUDINAL DYNAMICS

by

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ABSTRACT

Tension transients in a moving web can be described and predicted adequately from first principles. The resulting mathematical models are very useful for designing and debugging web conveyance machines and their controls. The basic equations involved will be reviewed. Examples will be presented, including an optimally robust stock roll controller and a balanced inertia float arm. These examples demonstrate the advances which can be accomplished through the application of mathematical modeling.

NOMENCLATURE

A = web cross-sectional area, in^2
c = damping modulus, psi-sec
d = web span damping rate, sec-lbf/in
dal = angular velocity of a float arm, radians/second
ds = web span stretch rate, inches/second
dth = angular velocity, radians/second
E = Young's modulus, psi
gr = gear ratio, motor to stock roll (>1 implies motor faster)
h = web thickness, in
i_m = motor armature current, amps
i_{error} = motor current error, amps
i_{ref} = motor current reference, amps
J_a, J_{arm} = pivot arm rotational inertia (including counter weight and $m*r^2$ of roller, but not gyration of roller), lbf-in-sec^2
J, J_r = roller rotational inertia, lbf-in-sec^2
k = web span spring rate, lbf/in
Kdth = motor back emf constant, volts-sec/radians

Kii = current loop integral gain, volts/amps/sec
Kip = position loop integral gain, 1/sec^2
Kis = speed loop integral gain, amps/radians
Kpi = current loop proportional gain, volts/amp
Kpp = position loop proportional gain, 1/sec
Kps = speed loop proportional gain, amps-sec/radians
Ktqm = motor torque constant, in-lbf/amp
L = span length, in
l_m = motor armature inductance, henries
n = order of polynomial
R, R_r = roller radius, in
R_a, r_{arm}= float arm radius, arm pivot to roller pivot, inches
R_s = stock roll radius, inches
r_m = motor armature resistance, ohms
S, s = web span stretch, in; also, symbolic derivative operator or Laplace transform operator
s_{ref} = motor speed reference, radians/second
t = time, sec
T = web span tension, lbf
Tq_a = torque applied to float arm, inch-lbf
Tq_{motor} = motor torque, in-lbf
trim = position loop trim output, radians/second
V = nip drive velocity, in/sec
v_{line}, v_{lineff} = web speed line reference feedforward, in/sec
voltage = motor armature voltage, volts
w = web width, in
W = suspended weight, lbf
α = float roller pivot arm position, radians
α_{error} = float roller pivot arm position error radians
α_{ref} = float roller pivot arm position reference, radians
α_{sp} = float arm position setpoint, radians
ε = web strain
ρ = weight density, lbf/in^3
τ_{ff} = feedforward torque, in-lbf
τ_{ref} = torque reference, in-lbf
θ_h = vacuum drum angular position, radians
θ_m = motor angular position, radians
θ_s = stock roll angular position, radians
ω_{error} = roller angular velocity error, radians/sec
ω = roller angular velocity, radians/sec
ω_n = natural frequency response, radians/sec
ω_{ref} = roller angular velocity reference, radians/sec

BACKGROUND

A simple web path with active tension control is shown in Figure 1. The basic equations to model the web tensions, roller velocities, float arm position, controller stability, etc. are developed below. This type of model will be referred to as a longitudinal dynamics model.

Roller angular accelerations are based on Newton's second law:

Roller 1:

$$\frac{d}{dt}\omega_1 = \text{input to model} \tag{1}$$

Roller 2:

$$\frac{d}{dt}\omega_2 = \frac{(T_3 - T_2)}{J_2}R_2 \tag{2}$$

Roller 3:

$$\frac{d}{dt}\omega_3 = \frac{(T_4 - T_3)}{J_3}R_3 \tag{3}$$

Roller 4 is driven by a motor that is speed controlled based on the float roller angular position:

$$\frac{d}{dt}\omega_4 = \frac{(T_5 - T_4)R_4 + Tq_{motor}}{J_4} \tag{4}$$

Web stretch rates are based on compatibility equations assuming no slip between web and rollers (see Appendix A [1] for a derivation of these equations and assumptions that apply):

Span 2:

$$\frac{dS_2}{dt} = \left[\frac{R_2 \cdot \omega_2}{1 + \frac{S_2}{L_2}} - \frac{R_1 \cdot \omega_1}{1 + \frac{S_1}{L_1}}\right] \cdot \left(1 + \frac{S_2}{L_2}\right)^2 - r_{arm} \cdot \frac{d}{dt}\alpha \tag{5}$$

Span 3:

$$\frac{dS_3}{dt} = \left[\frac{R_3 \cdot \omega_3}{1 + \frac{S_3}{L_3}} - \frac{R_2 \cdot \omega_2}{1 + \frac{S_2}{L_2}}\right] \cdot \left(1 + \frac{S_3}{L_3}\right)^2 - r_{arm} \cdot \frac{d}{dt}\alpha \tag{6}$$

Span 4:

$$\frac{dS_4}{dt} = \left[\frac{R_4 \cdot \omega_4}{1 + \frac{S_4}{L_4}} - \frac{R_3 \cdot \omega_3}{1 + \frac{S_3}{L_3}}\right] \cdot \left(1 + \frac{S_4}{L_4}\right)^2 \tag{7}$$

Float arm motion, is based on Newton's second law (assuming arm weight to be small compared to suspended weight and ignoring cosine effects):

$$\frac{d^2}{dt^2}\alpha = \frac{\left(T_3 + T_2 - \frac{W}{2}\right)r_{arm}}{J_{arm}} \tag{8}$$

Web tensions are based on the following constitutive laws:

Span 1:

$$T_1 = \text{input to model} \tag{9}$$

Span 2:

$$T_2 = k_2 \cdot S_2 + d_2 \cdot \frac{dS_2}{dt} \tag{10}$$

Span 3:

$$T_3 = k_3 \cdot S_3 + d_3 \cdot \frac{dS_3}{dt} \tag{11}$$

Span 4:

$$T_4 = k_4 \cdot S_4 + d_4 \cdot \frac{dS_4}{dt} \tag{12}$$

Span 5:

$$T_5 = \text{input to model} \tag{13}$$

The spring rates and damping rates are defined as

$$k_i = \frac{w \cdot h \cdot E}{L_i} \tag{14}$$

$$d_i = \frac{w \cdot h \cdot c}{L_i} \tag{15}$$

The web path shown in Figure 1 has active tension control. Three cascaded loops are used to control the armature voltage of the motor connected to roller 4. The position of the float arm is used to trim the speed of the motor. The position control loop logic is

$$\alpha_{error} = \alpha_{ref} - \alpha \tag{16}$$

$$\omega_{ref} = \frac{v_{lineff}}{R_4} + (Kpp \cdot \alpha_{error}) + Kip \cdot \int_0^t (\alpha_{error})dt \tag{17}$$

The speed control loop logic is

$$\omega_{error} = \omega_{ref} - \omega_4 \tag{18}$$

$$i_{ref} = Kps \cdot \omega_{error} + Kis \cdot \int_0^t (\omega_{error})dt \tag{19}$$

The current control loop logic is

$$i_{error} = i_{ref} - i_m \tag{20}$$

$$voltage = Kpi \cdot i_{error} + Kii \cdot \int_0^t (i_{error})dt \tag{21}$$

A simple model of an armature-controlled D.C. motor [2] is

$$\frac{d}{dt}i_m = \frac{(voltage - Kdth \cdot \omega_4 - r_m \cdot i_m)}{l_m} \tag{22}$$

$$tq_{motor} = Ktqm \cdot i_m \tag{23}$$

The equations above are a mixture of ordinary differential equations and algebraic equations. To solve this set of ordinary differential and algebraic equations it is necessary to transform them into a set of equations that can be solved directly by integration and simple algebraic operations. This is accomplished by expressing the highest order derivative of a variable in terms of its lower order derivatives and other variables. An example of this is the classic model of a mass, spring and dashpot system. If the mass is excited by a force that is a function of time and the mass is connected to ground by a spring and dashpot in parallel, the general form of the equations can be written as

$$\frac{1}{\omega_n} \cdot \frac{d^2 x}{dt} + 2 \cdot \zeta \cdot \frac{dx}{dt} + \omega_n \cdot x = \frac{\omega_n}{K} \cdot F(t) \tag{24}$$

Expressing this equation in terms of the highest order derivative yields

$$\frac{d^2}{dt^2}(x) = \frac{\omega_n^{\ 2}}{K} \cdot F(t) - \left(2 \cdot \zeta \cdot \omega_n \cdot \frac{d}{dt}(x) + \omega_n^2 \cdot x\right) \tag{25}$$

This above equation can then be integrated once for the first derivative of x and integrated twice for x. The output of the integration is referred to as a state variable; thus the number of state variables will equal the number of integration operations. The important state variables for the model of the web path shown in Figure 1 are

Roller angular velocity:

$$\omega_i = \int_0^t \frac{d}{dt}(\omega_i)(dt) + \text{initial conditions} \tag{26}$$

Web span stretch:

$$S_i = \int_0^t \frac{d}{dt}(S_i)(dt) + \text{initial conditions} \tag{27}$$

Float arm angular velocity:

$$\frac{d\alpha}{dt} = \int_0^t \frac{d^2}{dt^2}(\alpha)(dt) + \text{initial conditions} \tag{28}$$

Float arm position:

$$\alpha = \int_0^t \frac{d}{dt}(\alpha)(dt) + \text{initial conditions} \tag{29}$$

Motor current:

$$i_m = \int_0^t \frac{d}{dt}(i_m)(dt) + \text{initial conditions} \tag{30}$$

The easiest way to solve the above set of equations is with a commercially available code.

EXAMPLE PROBLEMS

The longitudinal dynamics of three different web paths will be discussed next. ACSL (Advanced Continuous Simulation Language, by Mitchell & Gauthier Associates) was used to model these examples.

Example 1. Longitudinal dynamic model of the web path shown in Figure 1

A step change in the float arm position reference of 5° will be analyzed. Roller surface speeds, span tensions, and float arm angular position are the outputs of interest. Figure 2 shows the model's response to a float arm disturbance for two different speed loop controller proportional gains. For a lower speed loop proportional gain (Kps = 0.25 amps-sec/radians), the system is unstable (see column 1 of Figure 2). By exercising the model it can shown that the bandwidth of the speed loop (inner loop) is similar to the bandwidth of the position loop (outer loop), thus causing the instability. One obvious solution is to increase the speed loop's proportional gain to make the system stable. For a higher speed loop proportional gain (Kps=1.0 amps-sec/radians), the system is stable, see column 2 of Figure 2.

Active tension control is provided by the float arm in this example. When active tension control is used, equations 5, 6 and 7 can be simplified as shown in appendix A. Using compatibility equations of the form of equation A10 in appendix A, the system response for a value for the higher speed loop proportional gain (Kps = 1.0 amps-sec/radians) is shown in column 3 of Figure 2. There is no difference in the response between compatibility equations of the form of equation A9 or A10 in appendix A when active tension control is used.

The equations developed for the web path shown in Figure 1 only show the essential elements needed for a simulation. This simple model shows the generic methodology used to create a longitudinal dynamic model of any web path. Larger models or additional features such as float roller hysteresis, different constitutive laws (e.g., corrugated webs in dryers), motor gearbox backlash, motor to roller coupling stiffness, models of accumulators, different type of controllers, etc. can easily be added. The modeler's imagination and technical knowledge are the only limitations to what can be modeled with the above approach.

Example 2. Optimally robust stock roll controller:

Synthesis of a control system is best performed on the very simplest description of a system which retains only the dynamics of primary importance. In the case of a stock roll (Figure 3), it is the rotation of the stock roll itself which is of primary concern. The simplest system which retains stock roll motion and contains the control system of interest has the following characteristics:

• ideal motor: torque delivered is simply setpoint torque

• ideal web drive: vacuum drum drive delivers constant web speed

• ideal web: inextensible

These assumptions dramatically simplify the system of equations which describe the system. Once a control system is designed which operates as desired using this simplified system, the full system complete with motor behavior, web stretch, drive shaft flexibility between the stock roll motor and the stock roll, and the like is modeled with the proposed controller. If all these "extra" factors are truly of secondary importance, the control system will function nearly the same in the full model as it does in the simplified model. If the resulting predicted control behavior is acceptable, the system may be implemented and the result observed.

The above simplifying assumptions lead to the following set of equations:

From Newton's second law:

$$\frac{d^2}{dt^2}\theta_s = \frac{\tau_{ref} + R_s \cdot T_w}{J_s} \tag{31}$$

From the inextensible web, we see that the float arm velocity is determined by the vacuum drive and stock roll velocities:

$$\frac{d}{dt}\alpha = \frac{(R_s \cdot \dot{\theta}_s - (R_h \cdot \dot{\theta}_h))}{2 \cdot R_a} \tag{32}$$

The speed loop controller gives

$$\tau_{ref} = K_{ps} \cdot \varepsilon_s + \tau_{ff} \tag{33}$$

$$\varepsilon_s = s_{ref} - \dot{\theta}_m \tag{34}$$

$$\theta_m = \theta_s \cdot gr \tag{35}$$

$$s_{ref} = \frac{V_{line}}{R_s} + trim \tag{36}$$

$$trim = K_{pp}(\varepsilon_p + K_{ip} \cdot \int \varepsilon_p) \tag{37}$$

$$\varepsilon_p = \alpha_{sp} - \alpha \tag{38}$$

Expressing the time-derivative operator as an algebraic variable "s", these equations may be rewritten into a format suitable for the Macsyma algebra and calculus symbolic manipulator:

```
/*variable dictionary:
   al = float arm position, radians
   alsp = float arm position setpoint, radians
   errp = position loop error, radians
   errs = speed loop error, rad/sec
   gr = gear ratio, stock roll motor to stock roll
   js = stock roll inertia, including motor, rollers, float arm, in-#-sec^2
   kip = position loop integral gain, 1/sec^2
   kpp = position loop proportional gain, 1/sec
   kps = speed loop proportional gain, in-#-sec/rad
```

```
   ra = float arm radius (pivot to pivot distance), inches
   rh = vacuum drum radius, inches
   rs = stock roll radius, inches
   s = derivative operator (d/dt), 1/sec
   sref = speed loop reference, rad/sec at motor
   thh = vacuum drum position, radians
   thm = motor speed, rad/sec
   ths = stock roll position, radians
   tqff = motor torque feedforward, in-#
   tqref = motor torque reference, in-#
   trim = speed trim from position loop, rad/sec
   tw = web tension, #
   vline = line speed reference, in/sec
*/

/*float arm position as a function of stock roll and vacuum drum position*/
al:(rs*ths-rh*thh)/2/ra;
/*position error*/
errp:alsp-al;
/*speed trim*/
trim:kpp*(errp+kip*errp/s);
/*speed reference*/
sref:vline/rs+trim;
/*motor speed*/
thm:ths*gr;
/*speed error*/
errs:sref-s*thm;
/*torque reference*/
tqref:kps*errs+tqff;
/*stock roll dynamics*/
ths=(tqref*gr+rs*tw)/js/s^2;
/*solve for stock roll position*/
solve(%,ths);
```

The result of running Macsyma on this program is a complicated expression which forms the basis of a set of transfer functions:

```
                                                 2
(d10) [ths = (2 gr kps ra s vline + 2 ra rs  s tw + 2 gr ra rs s tqff +

 (gr kpp kps rh rs s + gr kip kpp kps rh rs) thh + 2 alsp gr kpp kps ra rs s

                                                   3       2             2
 + 2 alsp gr kip kpp kps ra rs)/(2 js ra rs s  + 2 gr  kps ra rs s

                 2                       2
 + gr kpp kps rs  s + gr kip kpp kps rs )]
```

It is the denominator of this expression which affects control behavior. We would like to control the position of the roots of this cubic polynomial in "s" to be at a particular spot, namely as far left of the imaginary axis as is possible, consistent with the response capabilities and requirements of the stock roll system. For a given polynomial, this occurs when the polynomial is of the form

$$\left(\frac{s}{\omega_n}+1\right)^n \qquad (39)$$

Expanded, this polynomial is of the form

$$\left(\frac{s}{\omega_n}\right)^3 + A_2 \cdot \left(\frac{s}{\omega_n}\right)^2 + A_1 \cdot \frac{s}{\omega_n} + 1 \qquad \text{where } A_2{=}A_1{=}3 \tag{40}$$

The denominator of our transfer function above may be converted to this form and the individual coefficients obtained using some additional Macsyma code:

```
(c11) /*grab denominator -- denominator determines control behavior*/
denom(rhs(d10));
                3        2            2                      2                    2
(d11) 2 js ra rs s  + 2 gr  kps ra rs s  + gr kpp kps rs  s + gr kip kpp kps rs

(c12) /*place into canonical form, with units coefficient 1*/
ans:expand(%/ratcoef(%,s,0));
                                  3                 2
                       2 js ra s         2 gr ra s      s
(d12)              ----------------- + ---------- + --- + 1
                   gr kip kpp kps rs   kip kpp rs     kip

(c13) /*now compare to the canonical form (s/wn+1)^3*/
q3:ratcoef(ans,s,3)=1/wn^3;
                                   2 js ra          1
(d13)                         ----------------- = ---
                              gr kip kpp kps rs     3
                                                  wn

(c14) q2:ratcoef(ans,s,2)=a2/wn^2;
                                   2 gr ra     a2
(d14)                            ---------- = ---
                                 kip kpp rs     2
                                              wn

(c15) q1:ratcoef(ans,s,1)=a1/wn;
                                       1    a1
(d15)                                 --- = --
                                      kip   wn

(c16) /*and solve for the control gains*/
solve([q1,q2,q3],[kpp,kip,kps]);
/packages/macsyma2/SunOS5/library1/algsys.so being loaded.
/packages/macsyma2/SunOS5/library1/grobner.so being loaded.
                      2 a1 gr ra wn        wn        a2 js wn
(d16)          [[kpp = -------------, kip = --, kps = --------]]
                          a2 rs             a1           2
                                                       gr
```

Solving for the three gains (shown immediately above) gives expressions for the gains in terms of the natural frequency, system parameters, and the A_i coefficients. Then, for a given machine and a desired natural frequency, A_1 and A_2 may be set to 3, and the gains needed to obtain this behavior are obtained.

These control gains may now be put back into a "full-blown" model of the control system. Such a model includes factors such as motor behavior, web stretch, and stock roll shaft flexibility. The model is implemented in the ACSL simulator (Advanced Continuous Simulation Language), but is not shown due to space considerations.

In this model, the resulting control behavior may be observed for deviation from the desired behavior and for robustness problems. If none are found, the scheme may be implemented, and the results of the implementation compared to the desired behavior. The results of one such implementation are shown in Figure 4.

This scheme has the advantage of permitting control gains to be automatically computed for product changes such as product width or density. The method eliminates field tuning of stock roll behavior and generates control behavior which does not ring and does not vary with stock roll radius. Mathematical modeling of machine control systems coupled with control system synthesis procedure can lead to dramatically improved results. For this machine, the control computer vendor recommended a float arm capacity of four feet; the machine was actually constructed with nine inches of float arm capacity, which was a large cost and space savings.

Example 3. Balanced inertia float arm

A balanced-inertia float arm is a regular float arm, but with flanges added to the roller in such a fashion that

$$J_r = J_a \cdot \left(\frac{R_r}{R_a}\right)^2 \qquad (41)$$

Applies only to float arms with 180° degree roller wrap and roller translation nominally parallel to web spans

The equations which define the essence of this system are 42 through 46:

$$T_{in} = K_{in} \cdot s_{in} + D_{in} \cdot ds_{in} \qquad (42)$$

$$T_{out} = K_{out} \cdot s_{out} + D_{out} \cdot ds_{out} \qquad (43)$$

for the "in" and the "out" web spans

$$ddth = \frac{R_r \cdot (T_{out} - T_{in})}{J_r} \qquad (44)$$

$$ddal = \frac{R_a \cdot (T_{in} + T_{out}) - Tq_a}{J_a} \qquad (45)$$

$$\mathrm{ds}_{\mathrm{in}} = R_r \cdot dth - V_{in} - R_a \cdot dal \qquad (45)$$

$$ds_{out} = V_{out} - R_r \cdot dth - R_a \cdot dal \qquad (46)$$

As before, these equations are converted to the frequency domain with the Laplace transform by replacing the time derivative d/dt with the algebraic variable "s". The equations are represented in Macsyma as

```
/*roller acceleration*/
q1:jr*s^2*th=(tout-tin)*rr;
/*arm acceleration*/
q2:ja*s^2*al=ra*(tout+tin)-tqa;
/*incoming span stretch rate*/
q3:s*sin=rr*s*th-vin-ra*s*al;
/*outgoing span stretch rate*/
q4:s*sout=vout-rr*s*th-ra*s*al;
/*incoming span stretch rate*/
q5:tin=kin*sin+din*s*sin;
/*outgoing span tension*/
q6:tout=kout*sout+dout*s*sout;
```

These algebraic equations are solved for a judicious choice of variables with

```
solve([q1,q2,q3,q4,q5,q6],[tout,tin,th,al,sin,sout])$
```

Skipping over the voluminous algebra involved and removing the components of the result which are attributable to things other than incoming speed variation (float arm torque variation and outlet speed variation), the following expression for the outlet tension is obtained:

```
                          2          2
(d11) tout = - (ja rr  - jr ra ) s (din s + kin) (dout s + kout) vin

          4              2  3              2  3                2  3               2  3
/(ja jr s  + dout ja rr   s  + din ja rr   s  + dout jr ra   s  + din jr ra   s

                     2    2  2               2  2              2  2               2  2
 + 4 din dout ra   rr   s  + ja kout rr   s  + ja kin rr   s  + jr kout ra   s

                2  2                   2    2                    2    2
 + jr kin ra   s  + 4 din kout ra   rr   s + 4 dout kin ra   rr   s

                     2    2
 + 4 kin kout ra   rr  )
```

The thing to note above is the term $(ja*rr^2\text{-}jr*ra^2)$ which is a factor of the entire expression. By setting this expression to zero, incoming speed variation does not generate outgoing tension variation. This result was demonstrated in the laboratory on equipment shown schematically in Figure 6. Results of the laboratory demonstrations and model comparisons are shown in Figures 7 through 11.

The model may be exercised to determine the sensitivity to flange radius, as shown in Figure 12. With a validated simulation of the process, "experiments" such as this sensitivity demonstration may be performed in a few minutes at a desk rather than spending days fabricating flanges for laboratory experiments.

REFERENCES

[1] Kee-Hyun Shin, "Distributed Control of Tension in Multi-Span Web Transport Systems." Ph.D. Thesis, Web Handling Research Center at Oklahoma State University, 1991, pp. 20-24.

[2] Katsuhiko Ogata, Modern Control Engineering, 3rd ed. Prentice Hall, 1997, pp. 143-144.

Appendix A- Derivation of Web Stretch Rates

This derivation of web stretch rates follows closely a derivation by Dr. Kee-Hyun Shin [1]. Figure A1 shows web entering a nip drive at x_1 and leaving a nip drive at x_2. Using a control volume just to the left of both nips and assuming no slippage between web and roller, the law of conservation of mass can be written as follows:

$$\frac{d}{dt}\left(\int_{x_1}^{x_2}(\rho_2\cdot A_2\cdot dx)\right) = \rho_1\cdot A_1\cdot V_1-\rho_2\cdot A_2\cdot V_2 \tag{A1}$$

Figure A2 shows an infinitesimal element of unstretched web. Mass must be conserved when this element is stretched, thus

$$\frac{\rho_i\cdot w_i\cdot h_i}{\rho\cdot w\cdot h} = \frac{dx}{dx_i} = \frac{\rho_i\cdot A_i}{\rho\cdot A} = \frac{dx}{(1+\varepsilon_i)\cdot dx} \tag{A2}$$

Note, the letter "i" is an index referring to the number of the web span (e.g. ρ_2, is the density of the web in span 2 after it is stretched). Simplifying equation A2 further yields:

$$\rho_i\cdot A_i = \frac{\rho\cdot A}{1+\varepsilon_i} \tag{A3}$$

Substituting equation A3 into equation A1 yields

$$\frac{d}{dt}\left(\int_{x_1}^{x_2}\frac{\rho\cdot A}{(1+\varepsilon_2)}dx\right) = \frac{\rho\cdot A\cdot V_1}{1+\varepsilon_1}-\frac{\rho\cdot A\cdot V_2}{1+\varepsilon_2} \tag{A4}$$

Assuming that the strain state is uniform within span 2, equation A3 can be written as

$$\frac{d}{dt}\left(\frac{L_2}{1+\varepsilon_2}\right) = \frac{V_1}{1+\varepsilon_1}-\frac{V_2}{1+\varepsilon_2} \tag{A5}$$

From here the derivation differs from Shin's in that no simplifying assumptions are made in order to solve the above equations. Taking the derivative of the left hand side of equation A5 yields equation A6:

$$L_2\cdot\frac{d}{dt}(\varepsilon_2) = \left[\frac{V_2}{1+\varepsilon_2}-\frac{V_1}{1+\varepsilon_1}\right]\cdot(1+\varepsilon_2)^2 \tag{A6}$$

At steady state equation A6 reduces to the draft tension control law

$$\varepsilon_2 = \frac{(1+\varepsilon_1)\cdot V_2-V_1}{V_1} \tag{A7}$$

Using the relationship

$$S_i = \varepsilon_i\cdot L_i \tag{A8}$$

Equation A6 can also be written in terms of web stretch:

$$\frac{dS_2}{dt} = \left[\frac{V_2}{1+\frac{S_2}{L_2}}-\frac{V_1}{1+\frac{S_1}{L_1}}\right]\cdot\left(1+\frac{S_2}{L_2}\right)^2 \tag{A9}$$

When draft tension control is used equation A6 or A9 must be used. When active tension control is used (float roller or load cell feedback), equation A6 can be simplified further under the assumption that strains are much less than unity:

$$\frac{d}{dt}L_2\cdot\varepsilon_2 = \frac{dS_2}{dt} = V_2-V_1 \tag{A10}$$

Obviously equation A10 cannot be used for draft tension control.

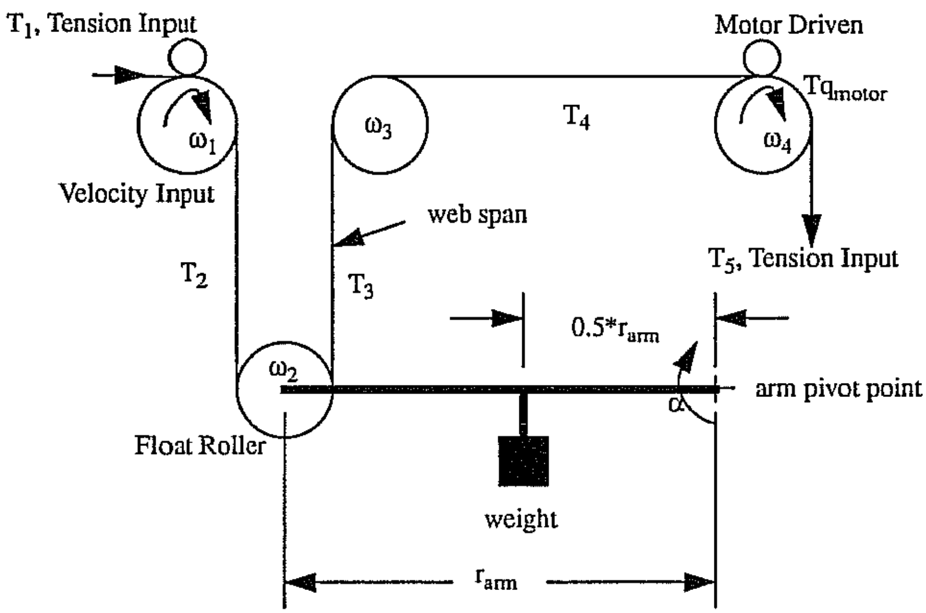

Figure 1- Web path with active tension control

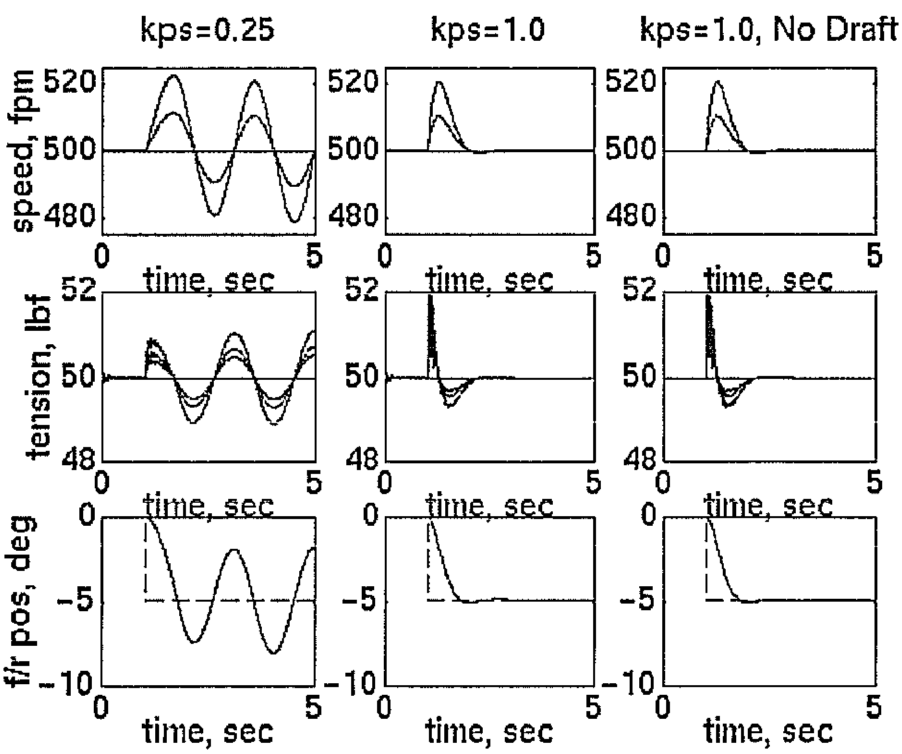

Figure 2-Web path model results

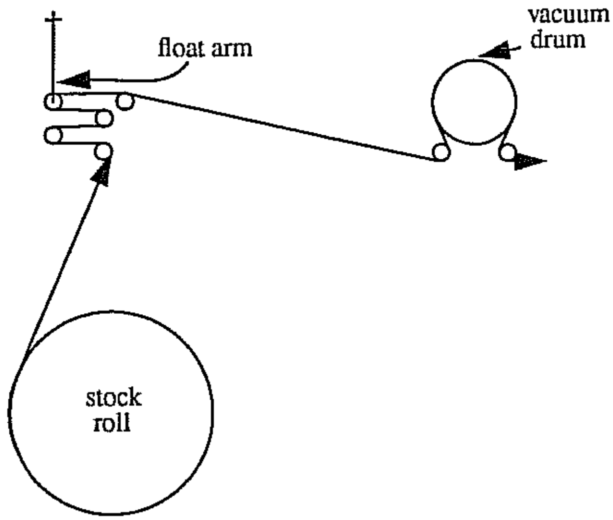

Figure 3: Stock roll system

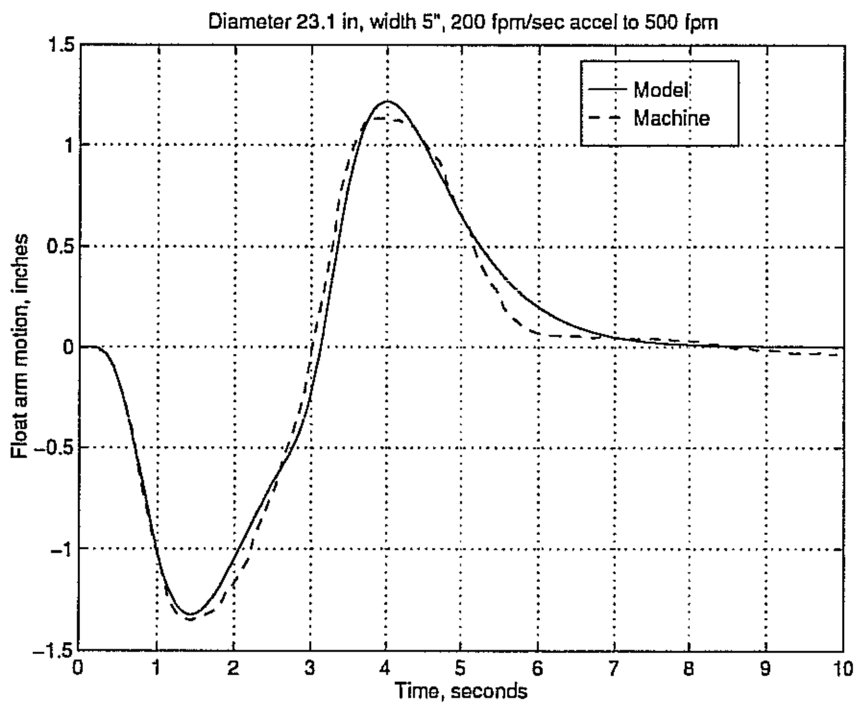

Figure 4: Float arm behavior at stock roll start, model and machine

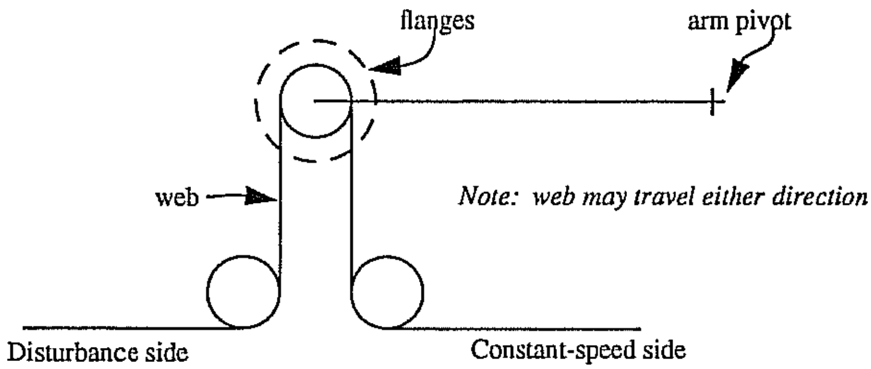

Figure 5: Typical balanced-inertia float arm

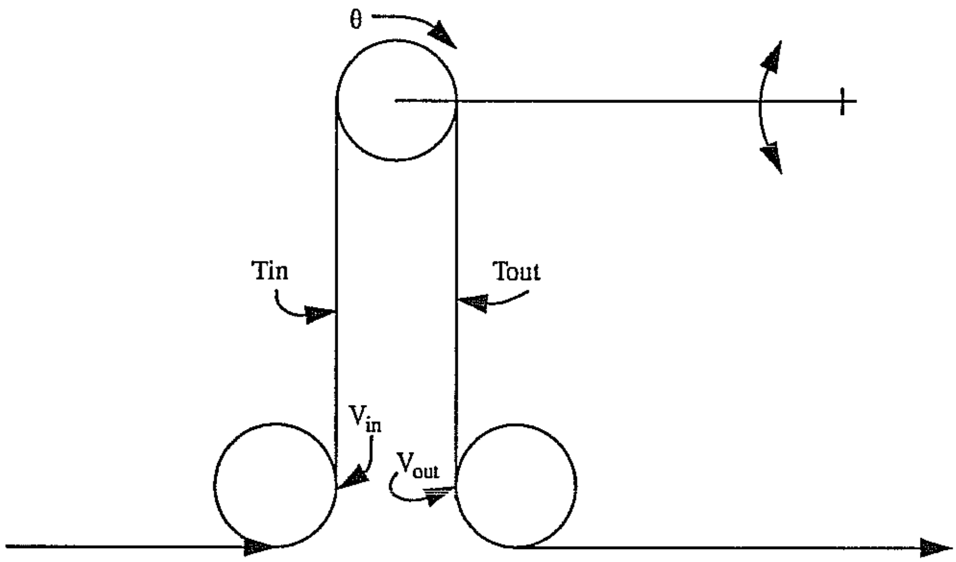

Figure 6: Generic float arm with inlet and outlet web spans

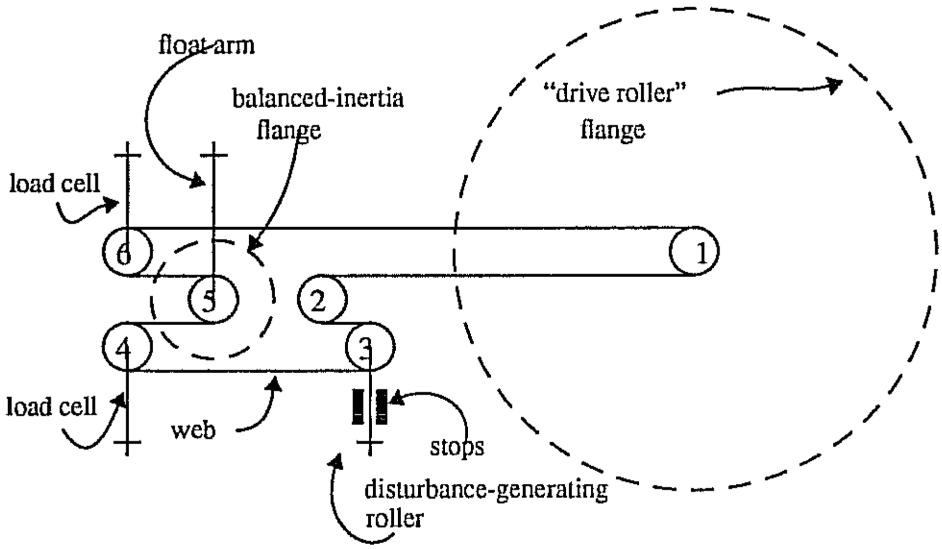

Figure 7: Schematic layout of laboratory equipment

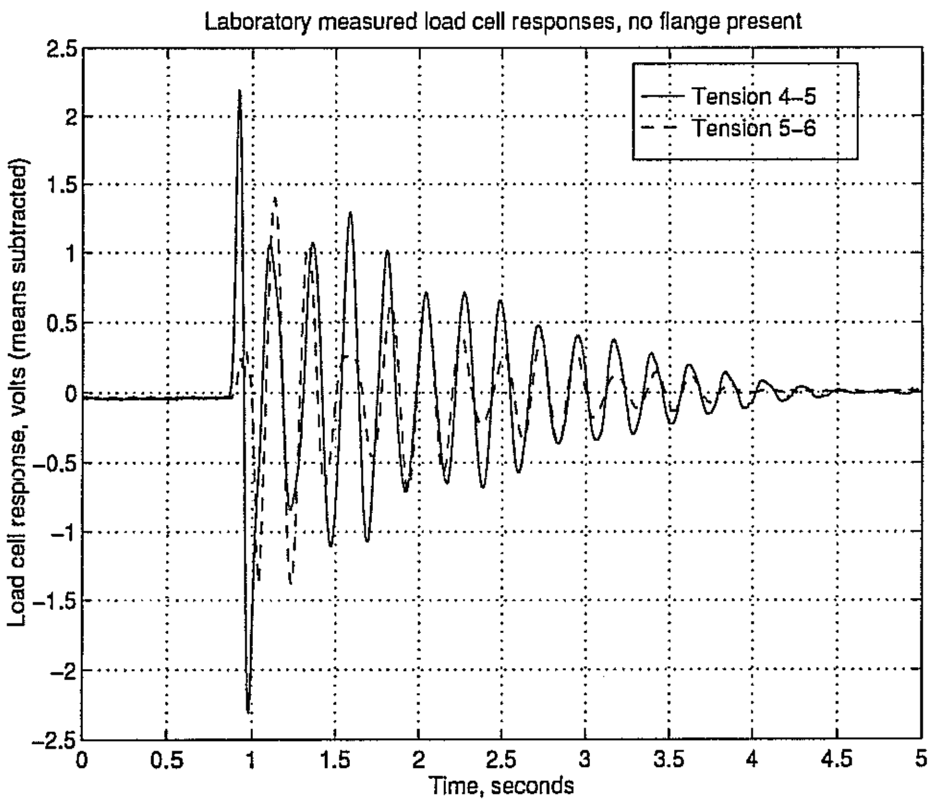

Figure 8: **Laboratory measurement of load cell response with no flange present on the float arm (typical float arm, not a balanced inertia float arm)**

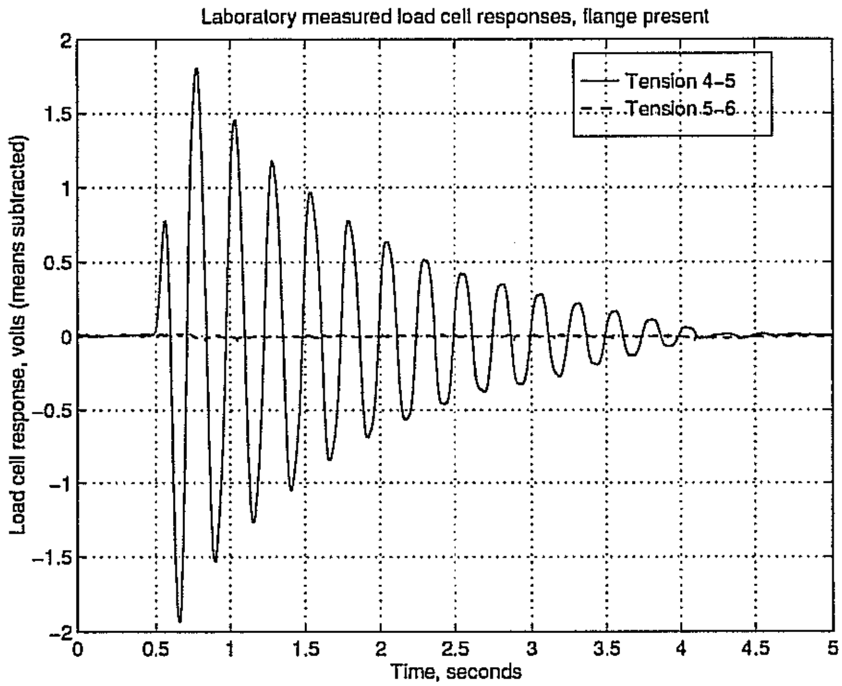

Figure 9: Laboratory measurement of load cell response with the balanced inertia flange present

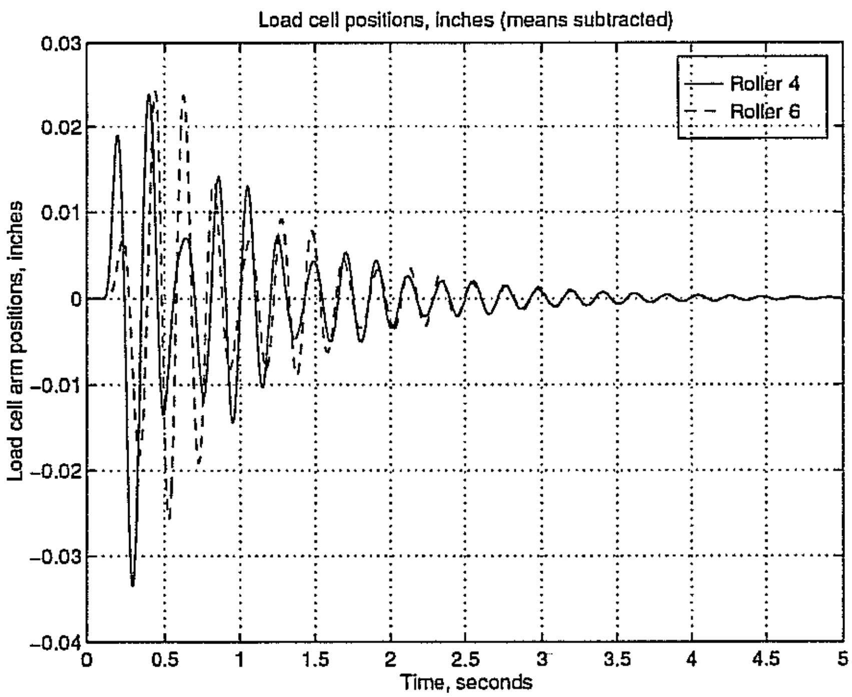

Figure 10: Model prediction of load cell positions with no flange present

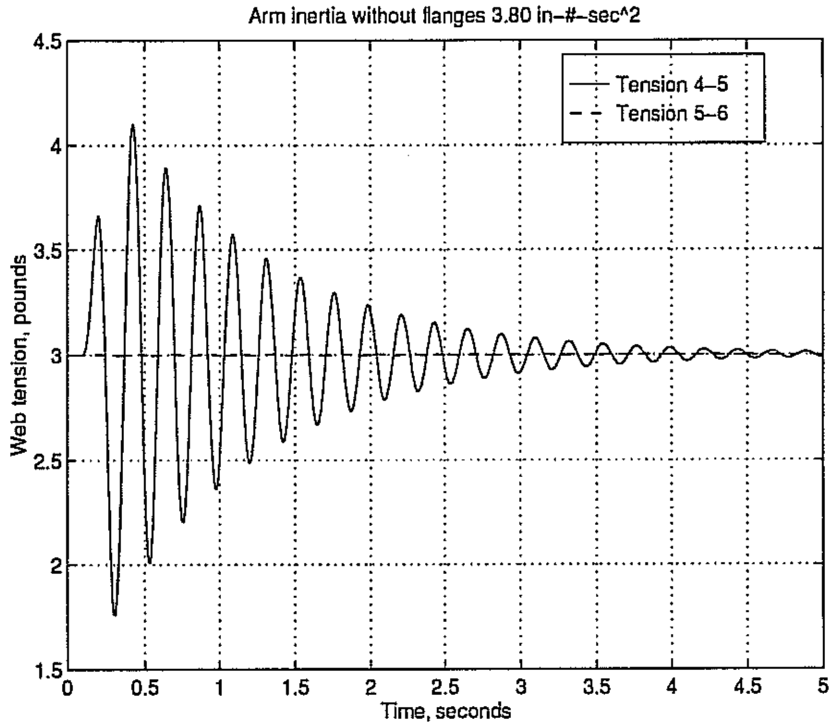

Figure 11: Modeled web tension before and after the balanced-inertia float arm

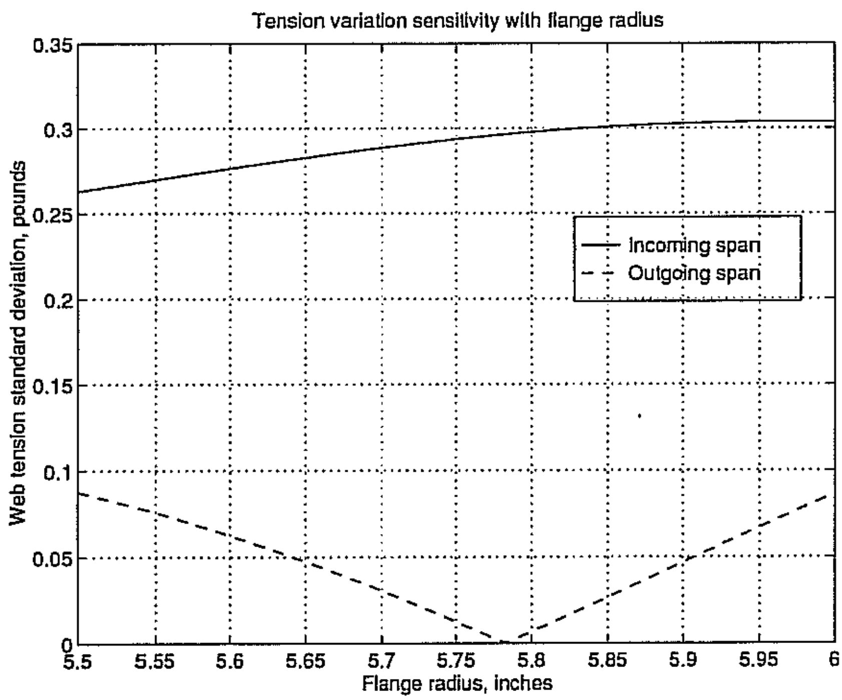

Figure 12: Tension variation sensitivity: the effect of flange radius

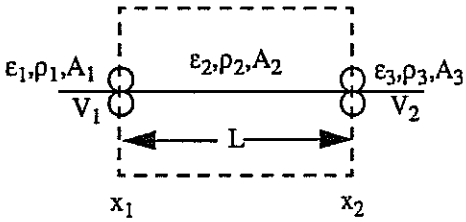

Figure A1

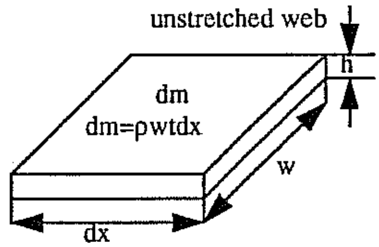

Figure A2

Question - In equation #10 what does 'D' represent?

Answer - It is the viscous damping coefficient of the material. Some form of damping is needed or oscillations would continue indefinitely throughout the controller loops. The value of "D" was obtained experimentally by suspending a weight from a mechanical ground by the web. The weight was tapped, and both the modulus and the damping of the web were observed in the resulting decaying oscillation.

Question - Why did you use a dancer roller in example #1 instead of a load cell?

Answer - Either could have been used, this example was for illustrative purposes only. Generally a load cell requires very high controller band widths with stiff web spans to achieve good tension control. Also, a stock controller with a dancer roller is much more forgiving of out-of-roundness defects in the stock roll, and is the method of choice if web damage is to be minimized.

Question - Was there earlier work involving the balanced-intertia float arm?

Answer - Yes. Lance Stryker of Eastman Kodak, reviewed the technique years ago. Also, John Martin of Martin Automatic holds a patent (expired) on the technique.