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Statement of Problem: Within the past fifteen years, educators at all levels have become aware of a deficiency in methods of teaching and the content of courses now being taught in mathematics. They feel that a refinement of certain areas of concepts is necessary and that much improvement can be attained. Of these areas, the concept of the "place-value" number system and the concept of "set" are discussed in this paper. Both are important to the "new look" in the field of mathematics.

Method of Procedure: To construct a "place-value" number system, a base must be laid upon which to build. This is done by making tables with a given base and discussing the basic operations of arithmetic, with which the reader is familiar, using this base. The base "7" is used here. Reference is made also to other bases. "Set" theory is introduced by taking an everyday concept of "set" and weaving a link to general logic and showing how it can be used in abstract logic.

Findings and Conclusions: "Place-value" number systems other than base "10" are useful. Base "2" is used in electrically operated machines, and base "12" has been considered as a very usable base. The structure of all those systems, however, is the same and can be learned using any base from which to work. "Set" is becoming more and more the basis for building mathematical concepts. It should be introduced and used in elementary school and developed as higher levels of schooling are attained. Many texts now are being written building large parts of the mathematical structure around "set."

ADVISER'S APPROVAL James H. Zant

TEACHING AIDS FOR THE SECONDARY SCHOOL MATHEMATICS TEACHER

By

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TEACHING AIDS FOR THE SECONDARY SCHOOL

MATHEMATICS TEACHER

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CHAPTER I

INTRODUCTION

This paper is being prepared for the dual purpose of giving the writer a better background in the development of fundamental concepts concerning the number system, symbols, literal numbers, variables, inequalities, and irrational numbers for becoming a better teacher in secondary school mathematics and to give other teachers ideas for consideration in their own planning of classroom activities to meet the current and future needs of the students they teach.

The gap that has appeared between what is being taught and what is needed in secondary mathematics has for some time grown wider and wider. The last decade has shown an easily recognized lag in mathematical instruction. "There is nothing new under the sun," has been believed or at least practiced too long by the mathematics instructor. It takes new knowledge to develop the new things that come out of the laboratories and industries. This new knowledge does not leave out mathematics but, to a large degree, depends upon it. Everett S. Lee, in addressing mathematics teachers, said:

Scientific phenomenon, in one line, is the equation of mathematics...more mathematical knowledge in the solution of an equation results in an expression of variables which allows the phenomena to be resolved into practical use.[1]

The importance of mathematics and the need for keeping it up and ahead of all scientific development should be a code of thinking for all of those involved in the instruction of mathematics. Quite a large number of mathematicians and instructors have done what they could to get the ball rolling in the right direction. In recent years their work has begun to show some influence. Special effort is being made to communicate to those teaching that there is need for taking a long second look. It is becoming apparent to many of these teachers that a new approach to content and method is urgent. These teachers are looking for a way.

Only a few new textbooks have been written to meet the demands of the "new look." Many good articles and papers, however, have been published in the past few years. Each writer contributing a worthwhile part, they have produced much very valuable thinking and material. Much of this material is organized to point in the right direction. Others who can create ideas and have organizational ability should publish their thoughts.

Within the past five years, there also has been considerable activity in organized work. Groups like the School Mathematics Study Group, the Commission of

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[1]Everett S. Lee, "An Engineer Looks at Mathematics," The Mathematics Teacher, XVIII (June, 1950), p. 236.

Mathematics of the College Entrance Examination Board, Summer Institutes for the Study of Mathematics Improvement, and State Committees for the Improvement of Mathematics Instruction have begun to come up with recommendations, many of which quite likely will be used in future instructional work. Many of these groups point toward the development of fundamental concepts that will serve to instil within the students the confidence and poise in mathematics that will insure individual and group creative thinking necessary to bring about the above mentioned scientific phenomena.

If the teacher is to instil confidence and other admirable characteristics in the student, he first must have those traits himself. Certainly he cannot look around and be content with materials and procedures that were thought to be adequate and were popular fifty years ago. The saying, "I will teach as I was taught," has no place in the thinking of today's mathematics instructor. One does not say, "I am content to travel as my grandfather did," or "Farming is like it was when I was a kid." It is difficult to find an adult who is content to drive the automobile bought five years ago. Mathematics does change as it continues to grow, because as mathematics develops, so develops the scientific world.

The refinement and extension of the mathematics now being taught become the job of every practicing teaching. New ideas in high school instruction must come about, and the instructor himself must give birth to or help develop many of those ideas. Research and development by individuals and groups then becomes the task ahead. More fundamental concepts in numbers, symbols, algorisms of arithmetic, variables, and the like must become a greater integral part of the classroom program. Many of these ideas not found in texts may be found in current publications. This paper is an attempt to collect and put into working form some of those ideas and concepts the writer has not used previously in the classroom.

The second chapter will discuss the number system in relation to bases other than ten. The "binary system" then will be introduced showing digital positions, how to write and count, and how to add and multiply. A method for converting a system of base other than ten to base of ten then will be shown. This will be followed by a rather extensive discussion of a system set up to an unusual base of "seven."

Chapter III deals with the number system in terms of sets. The concept of visual sets will lead into real number sets, and this will lead into sets of literal numbers. Consideration will then be given to graphic representations of "greater than or equal to" and "less than or equal to" as they are used in equations of functions.

CHAPTER II

PLACE-VALUE NUMBER SYSTEM

A junior high school teacher is fortunate, indeed, if he has had the experience of a contemporary course in the teaching of arithmetic.[1] This statement was based upon an analysis of textbooks published in the last ten years. Because arithmetic makes up the larger part of the junior high school mathematics program, this would be a very good place to develop the larger concept of arithmetic and a good understanding of the number system. If it cannot be introduced earlier, then selected topics should be presented whenever the opportunity presents itself. Although teachers now entering the teaching field on the elementary level may have had one or more courses in "modern concepts of mathematics," and probably will use much of this material in their teaching, most of it does not yet appear in the grade school textbooks. Currently, unless a deliberate attempt is made to teach topics on the background of the arithmetic number system, it likely will not be done. If it is not taught in elementary school, then it should be introduced to

[1]Myron F. Rosskopf, "Professionalized Subject Matter for Junior High School Mathematics Teachers," The Mathematics Teacher, XLVI (December, 1953), p. 541.

the students not later than the early grades of junior high school. As this is where the students step off from the series of arithmetics to other phases of mathematics, it becomes urgent that every student understands the structure of the natural or "decimal" number system.

The natural number system, which is the system of counting, of United States money, of scientific notation, and of logarithms as they are first introduced, can be learned by everyone to some degree of efficiency. The base of a system, then, is of great importance.

Consideration of bases other than ten would serve as instruction and probably would prove also as a good motivating device. A number system might be developed to any conveniently small base. Because of the extensive use of the base two, or the "binary" system, this and other bases will be discussed in this chapter. Consideration of the development of the place-value number system and basic operations in it using various bases is the dominating feature of this discussion. Also, a general formula by which one may develop any place-value number system will be shown.

The digital positions from right to left in all place-value number systems are for holding the correct count up to a given value. This means that the foremost right digit is used to hold the count up to the value of the base used. Once this base is reached, there must be a store for holding the count, so the next digit to the left keeps count of how many base values there are up to a certain point as again the right digit increases the count by one unit in succession. This procedure is used as the count gets larger and larger; the number of count holding digits to the left increases according to the following formula: With n_i representing any allowable coefficient, from "0" up to one less than the number used as base, b representing base, and n the number of integers in the number value, the formula is: $n_ib^{n-1} + n_ib^{n-2}_{+} \dots + n_ib^{n-n}$. An example of this will appear later.

Starting with the smallest and probably the easiest of the bases to understand, the "binary" system, a short count will be made. The count will be developed by adding successive numbers.

0	1	10	11	100	101	110	111
1	1	1	1	1	1	1	1
1	10	11	100	101	110	111	1000

This represents:

1	2	3	4	5	6	7	8

in the base "ten."

A number in the "binary" system written as 1100101 becomes a very large problem in arrangement and space but may be explained as follows: Using the formula for the place-value system, we will convert the above number to digital values as follows:

$$\begin{array}{ccccccccccccccc} 1 & & 1 & & 0 & & 0 & & 1 & & 0 & & 1 & & \text{means} \\ (1)2^6 & + & (1)2^5 & + & (0)2^4 & + & (0)2^3 & + & (1)2^2 & + & (0)2^1 & + & (1)2^0 & & \\ \text{or } 64 & + & 32 & + & 0 & + & 0 & + & 4 & + & 0 & + & 1 & & \end{array}$$

By adding this last line, the digital values give you 101 in the natural count or the base "ten" count.

The "binary" system is used commercially because of the ease with which machines can be set up to represent the digits. An open and closed circuit represent the values 0 and 1.

Multiplication in the "binary" system is easy and will be illustrated:[2]

$$\begin{array}{r} 10101 \\ \underline{111} \\ 10101 \\ 10101 \\ \underline{10101} \\ 10010011 \end{array} \qquad \text{In the natural number system you get:} \qquad \begin{array}{r} 21 \\ \underline{7} \\ 147 \end{array}$$

The relationship between the natural system and the "binary" system in this problem is shown more fully as follows:

$(1)2^7 + (0)2^6 + (0)2^5 + (1)2^4 + (0)2^3 + (0)2^2 + (1)2^1 + (1)2^0 =$

$128 + 0 + 0 + 16 + 0 + 0 + 2 + 1 = 147$

By working for a short time with the system, an acceptable degree of competency could be acquired by anyone.

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[2]Stanley P. Hughart, Some Topics From Modern Mathematics for Secondary School Science Teachers (Stillwater, 1957), Chapter I.

Table 1 shows the set-up of other number systems as follows:

TABLE 1

PLACE-VALUE NUMBER SYSTEMS OF BASE OTHER THAN 10

Base	n_ib^5	n_ib^4	n_ib^3	n_ib^2	n_ib^1	n_ib^0
2	n_i32	n_i16	n_i8	n_i4	n_i2	n_i1
3	n_i243	n_i81	n_i27	n_i9	n_i3	n_i1
4	n_i1024	n_i256	n_i64	n_i16	n_i4	n_i1
5	n_i3125	n_i625	n_i125	n_i25	n_i5	n_i1

The n_i may equal 0 to b-1 in using this table.

As you can see, the digital values become difficult to hold and handle mentally as the base increases. To illustrate further, the base seven will be developed more completely.

In setting up any system, the addition and multiplication tables should be constructed so that there will be no need to figure each computation before the system has been put to memory.[3]

Tables 1 and 2 will be used to illustrate the associative, commutative, and distributive laws as we know them in the natural number system. Careful inspection will show you how they may be used. If they hold for the base "7" as they

[3]Rosskopf, p. 542.

hold for the base "10," it would be easy to conceive them holding for other bases also.

TABLE 2

ADDITION TABLE

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	10
2	2	3	4	5	6	10	11
3	3	4	5	6	10	11	12
4	4	5	6	10	11	12	13
5	5	6	10	11	12	13	14
6	6	10	11	12	13	14	15

TABLE 3

MULTIPLICATION TABLE

X	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	11	13	15
3	0	3	6	12	15	21	24
4	0	4	11	15	22	26	33
5	0	5	13	21	26	34	42
6	0	6	15	24	33	42	51

Using the tables the associative law of addition states:

$$6 + (5 + 4) = 6 + 12 = 21 \text{ and}$$

$$(6 + 5) + 4 = 14 + 4 = 21.$$

The distributative law gives:

$$2(3 + 2) = 2(5) = 13 \text{ and}$$

$$2(3) + 2(2) = 6 + 4 = 13.$$

The commutative law holds in the same way. Now, we consider "carrying" in an addition problem. To add 64 and 25 write:

$64 = 60 + 4 = 6$ sevens $+ 4$ units
$\underline{25 = 20 + 5 = 2 \text{ sevens} + 5 \text{ units}}$

Adding we get,

$110 + 12 = (1)7^2 + (1)7^1$ and $(1)7^1 + (2)7^0$

This gives $(1)7^2 + (2)7^1 + (2)7^0$ or

1 forty-nine + 2 sevens + 2 units.

Use was made of the addition table to secure the partial sums 4 + 5 and 60 + 20. The exercise can be done mentally as follows: 5 and 4 gives 1 seven plus 2 units; write down the 2 and carry the 1; 6 sevens plus 2 sevens plus the 1 seven that was carried gives 7 sevens plus 2 sevens, written as 12 sevens; therefore, the sum is 122. With a little practice one can become quite adept in addition in this system.[4]

Next, we consider a multiplication exercise. You will find the procedure for the base "7" is the same as for the base "10."

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[4]Ibid., p. 543.

$$\begin{array}{r} 325 \\ \underline{64} \\ 1636 \\ \underline{2622} \\ 31156 \end{array}$$

The answer to this problem by the formula above is:

$(3)7^4 + (1)7^3 + (1)7^2 + (5)7^1 + (6)7^0$.

Describing the development of the problem in words, it sounds like this: 4 times 5 gives 2 sevens plus 6 units; put down the 6 and carry the 2; 4 times 2 gives 1 seven^2 plus 1 seven; now add the carried sevens and get 3 sevens; put down the 3 and carry the 1 seven^2; 4 times 3 gives one seven^3 plus 5 seven^2; now add the carried sevens^2, and you get 16. The first partial product is 1636, which is: $(1)7^3 + (6)7^2 + (3)7^1 + (6)7^0$. Now, multiplying the top row by 6; 6 times 5 gives 4 sevens and 2 units; put down the 2 and carry the 4 sevens; 6 times 2 gives 1 seven^2 plus 5 sevens; adding the carried 4 sevens 1 more seven^2 and 2 sevens; put down the 2 and carry the 2 seven^2; 6 times 3 gives 2 seven^3 and 4 seven^2; adding the carried 2 seven^2 gives 26. The second partial product is 2622. This is placed below the first partial product in the same manner as when dealing in natural numbers. If there were more numbers to be multiplied, the problem is the same but longer. The second partial product is: $(2)7^4 + (6)7^3 + (2)7^2 + (2)7^1$. The answer is found by adding the partial products by the method shown above.

Subtraction of the smaller number from the larger can be done by the process of "borrowing." Take the example:[5]

$$\begin{array}{r} 612 \\ \underline{35} \\ 544 \end{array}$$

This can be explained in the same manner as it is in base "ten" also. Five will not subtract from 2, so a seven is taken from the second digital position and added to the 2; this gives 12 from which you subtract 5; this is 5 from 2 more than seven which gives 4; when the seven was taken from the second position that left 0 sevens from which to subtract 3; take a seven^2 from the third position and add it to the 0 sevens. This gives 7 sevens from which you subtract 3; this leaves 4; then with nothing to subtract from the remaining 5 seven^3, you bring it down. The answer is 544.

Doing several exercises in this manner will enable the student to acquire both the meaning and method of subtraction. Mastering subtraction in this system will help one to understand the theory of the natural number system.

Long division is rather difficult to learn even in the familiar decimal system. We shall see if it is harded in another system. A problem will be worked and then explained.

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[5] Ibid.

```
       3
      30
     200
23 )6024
    4600
    1124
    1020
     104
     102
       2
```

In the division $6024 \div 23$, first estimate how many groups of 23 will be contained in 6024. One can figure that there are 200. Hence, the number 200 is placed in the quotient position. Multiplying back and subtracting, the difference is 1124. The divisor 23 divides into 1124, 30 times; multiplying back and subtracting, the difference is 104; dividing again by 23 you get 3 with a 2 remainder. With a little practice, again, division can be streamlined, the 0's can be eliminated, and the quotient may be written as in the base "ten."[6]

Development of this system into the introductions of positive and negative numbers and fractions might be done, but having carried the base "7" system this far, one can see the development might well be the same as the base "ten." One now can see and appreciate the decimal or natural number system and the power and efficiency of the place-value number system in general.

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[6] Ibid., p. 544.

CHAPTER III

SET AND THE LITERAL NUMBER SYMBOL

The concept of set, and the relation it has with the understanding of the number system in general, the literal number symbol, and equation formation has been demonstrated to be very useful. The lack of the development of this concept in the past has left an important method of teaching some very basic and important areas to methods that are either harder to understand or are not being taught at all. It becomes an important area, then, in bringing teaching methods up to date.

"Set" is not a new idea in our thinking, as almost everything that one talks about or thinks about is classified or placed into a set of one classification or another. Taking a somewhat limited area for establishing the concept of "set," one might think of a set of dishes. This would include the plates, cups, saucers, butter plates, serving bowls, soup bowls, sugar bowl and creamer, and perhaps extra items. When one sets the table for a meal, the way the table is set and the kind of dishes used depend upon the kind of meal to be served. The set of breakfast dishes placed upon the table depends upon whether bacon and eggs with coffee are being served or whether cereal and coffee are being served. In the first instance, one might put plates, cups, saucers, and bread plates on the table. This is a set which might be thought of as a "subset" of the larger set of dishes. If cereal is being served, then cereal bowls instead of plates would be placed on the table. Bread plates might be needed in the latter case where they were not in the former. Cups and saucers and sugar bowl and creamer would still be needed. This is another "subset" of the set of dishes. An inclusion graph, Figure 1, will help show the relationship between the set and "subset."[1]

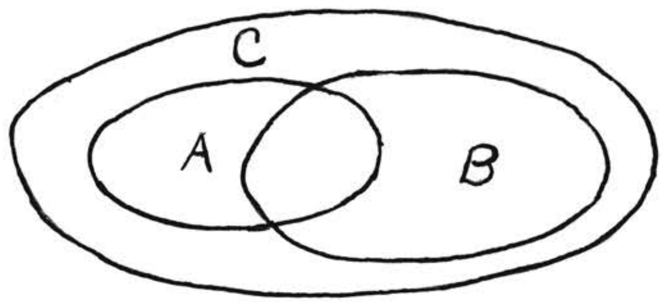

Figure 1

The larger enclosure C represents the full set of dishes; the one to the left and inside represents the A breakfast of bacon and eggs; and the one on the right represents the B breakfast of cereal. The two inside "subsets" of the set overlap because they contain some common articles

[1]Moses Richardson, Fundamentals of Mathematics (New York, 1958), pp. 8-18.

of dishes. The idea presented in showing the overlapping "subsets" and the inclusion of both within the set is a common way of thinking in everyday life. The graph will be used later to show ways to approach more complicated ideas.

"Set" and Logic

This is no attempt to go deeply into logic but only to show that the concept of "set" is used to an advantage when dealing with logic. In plane geometry one of the main objectives is to teach the student an approach to logical thinking. In this area one might get a problem something like this:

Hypothesis. (a) All freshmen are undergraduates.

(b) All undergraduates are geniuses.

Conclusion. All freshmen are geniuses.

Here the hypothesis implies the conclusion, since it is clear that if the hypothesis is true, then the conclusion must be true. This may or may not be actually true, but the conclusion that is drawn is the only one possible. This can be illustrated as shown in Figure 2.[2]

[2] Ibid.

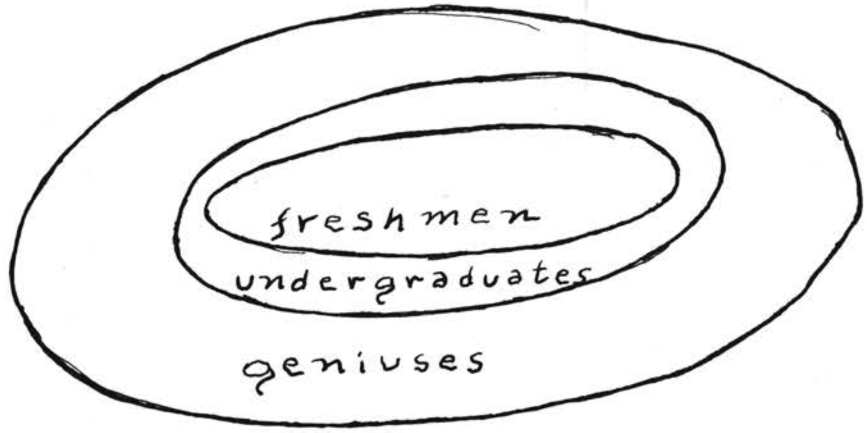

Figure 2

A second simple illustration will be shown in Figure 3. See if there is a difference in the kind of solution that can be drawn from it.

Hypothesis. (a) All freshmen are human.

(b) All undergraduates are human.

Conclusion. All freshmen are undergraduates.

Disregarding the possibility that this is a poor problem, how can you show whether the conclusion is a valid one? This can be shown by using the illustrative "set" diagram as follows:[3]

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[3]Ibid.

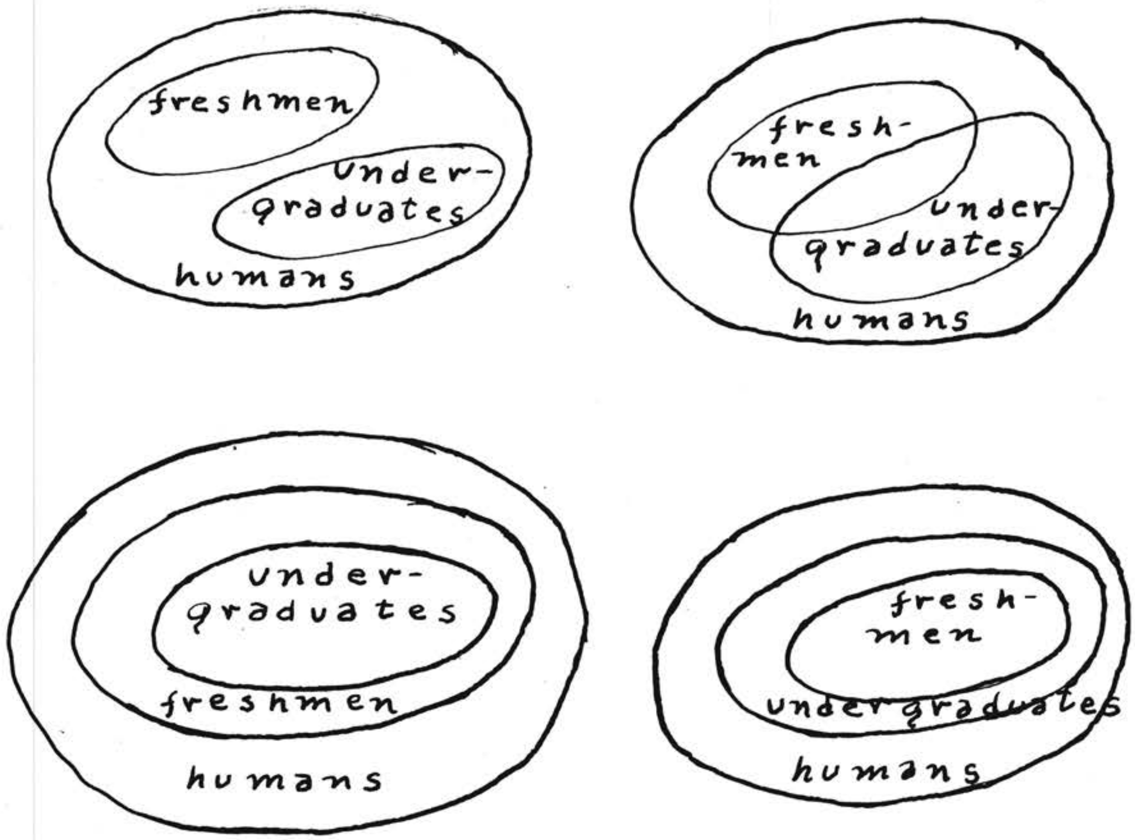

Figure 3

Still another problem may be considered. In this one there may be difficulty in showing whether there is a real connection between the parts of the hypothesis in order to draw a valid conclusion.

Hypothesis. (a) All timid creatures are bunnies.

(b) Some timid creatures are dumb.

(c) Some bunnies are dumb.

Conclusions. (a) Some bunnies are dumb.

(b) Some freshmen are bunnies.

(c) Some freshmen are dumb bunnies.

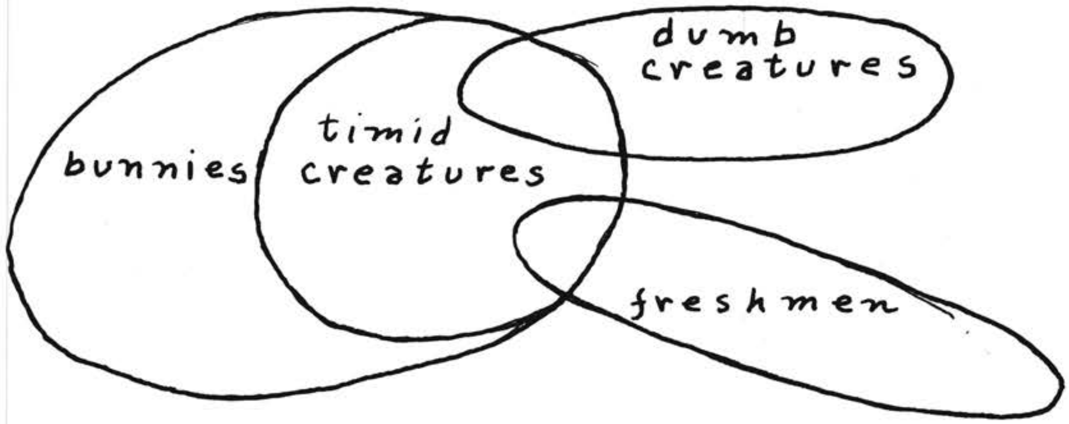

Figure 4

Notice that the illustrative "set" diagram, Figure 4, shows the overlapping of some of the "sets" and the complete enclosure of the one "set" which makes it a "subset" of another set.[4]

"Set" is not used to its fullest unless it can be applied to the abstract. The student who has reached algebra and literal numbers concepts in the early secondary grades will find this useful. If instead of using named objects or things, letters represent the particular "set" or "subset," the procedure is much the same. Consider the first example above, and let it read thus:

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[4]Ibid.

Hypothesis. (a) All x's are y's.

(b) All y's are z's.

Conclusion. All x's are z's.

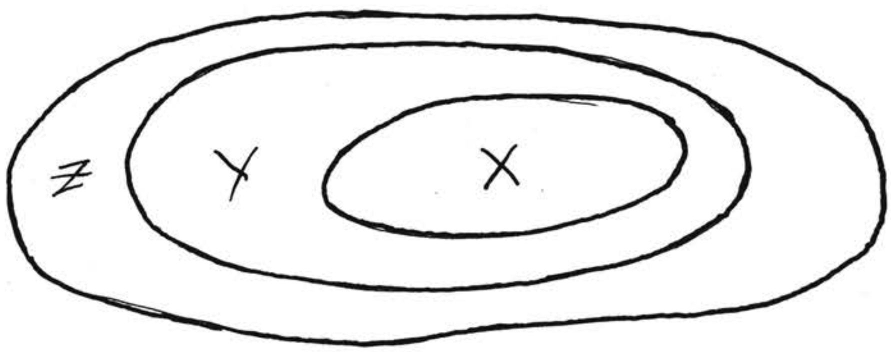

Figure 5

Note that the "set" diagram, Figure 5, is the same as that above and can be explained in the same manner.[5]

Moving rather quickly now into the general area of symbolic language, we must consider enough of it to establish a concept of how "set" applies. In this discussion the normal group of new symbols connected with explaining and using a new idea will not be introduced. The symbols generally used in algebra will be utilized when possible, and word descriptions will be used otherwise.

You are familiar with the general usage as applied to integers. Let us consider the following statements in regard to the "set" of all integers: (a) the product of an integer and zero is zero and (b) the square of an integer is less than twenty-five. Notice that the first statement is

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[5]Ibid.

true for each and every integer of the "set" of integers, while the second is true for only a part of the full "set." Again, this last group is thought of as a "subset" of the "set" of integers. This leads to the realization that it becomes necessary to know whether one is considering the "set" or a "subset" when seeking the truth of a problem in abstract.

The symbols of arithmetic and algebra have been established for the purpose of describing and using certain members of certain sets. The symbols like "10," (-4," "15%," and "π," of course, stand for specified numbers, though in some cases there might conceivably be a set or subset containing only one number. Letter symbols like "K," "x," and "p" can be used to represent non-specified numbers of sets of numbers or, for that matter, sets of objects. When letters are used in this way, they are called "literal numbers."[6] These two types of numbers will be used to discuss certain relationships which follow.

If x, y, and z are members of the set of real numbers, we may consider the following statements:[7]

1. $x + y = y + x$	4. $(xy)z = x(yz)$
2. $x + 1 \geq 7$	5. $xy = yx$
3. $x + 1 = x$	6. $x + y = x + y + 1$

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[6]Max Beberman and Bruce E. Meserve, "The Concept of a Literal Number Symbol," The Mathematics Teacher, XLVIII (April, 1955), p. 198.

[7]Ibid., p. 199.

It should be noticed that some of the above statements are true for every member of the set of real numbers, and the other statements are true for only a subset of the set of real numbers. In the first of the statements, no matter what value is given x and y the statement is true. In the second statement, notice that x cannot be less than 6 and the statement be true. The other statements may be checked for truth for all values and for limited values. In doing exercises such as these, one may develop an understanding that the literal number symbol represents unspecified numbers of a set and that further knowledge of the given statement in symbolic form is needed to determine the set member value.

In the statement $x + y = y + x$ it is easy to show the relationship by some graphic means. Though the size of x and y are not known, one can see in Figure 6 that the sum of the first plus the second is the same as the sum of the second plus the first.

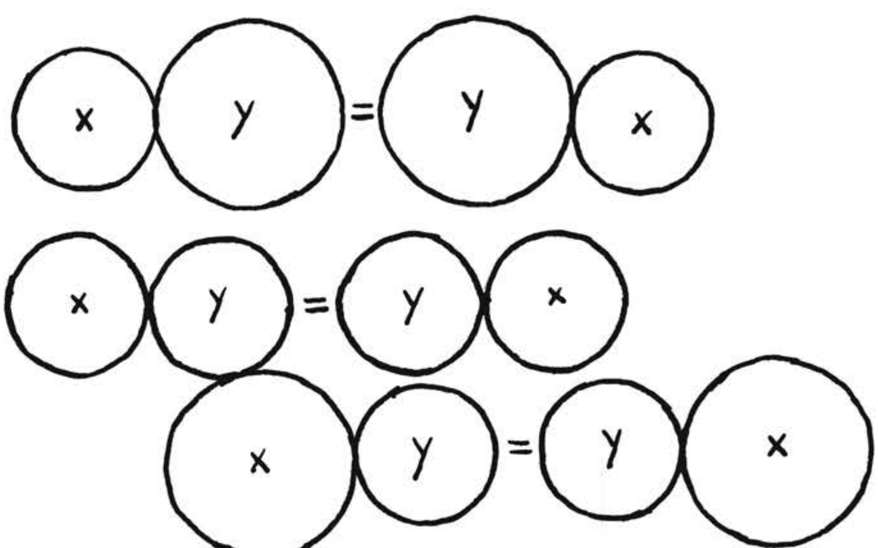

Figure 6

The consideration is not whether x is larger than y, y larger than x, or whether they are the same size, but whether the sum of the two in opposite order is the same. However, x is the same, and y is the same on both sides of the equal sign.

Now consider the second equation, $x + 1 \geq 7$, shown in Figure 7.

$$6 + 1 \geq 7 \qquad\qquad >6 + 1 \geq 7$$

Figure 7

Here, the size of the x is limited in that it must be greater than the lower limit of 6 units. Where in the previous formula the two sides of the equation may be exchanged, in the second the symbol prevents this from happening.

If the two sides of the second equation did change sides, then $7 \geq x + 1$ takes on a completely different meaning. The x now is a different subset of the real number system. It must now be less than 6 instead of greater than 6.

In dealing with the set of negative real numbers, these two statements might be considered: (1) The square of a negative number is equal to the square of the positive of that number, $(-2)^2 = (2)^2$, and (2) the square of a number is seven less than five, $(x)^2 = 5 - 7$. It is seen easily that, in dealing with the square of any member of the real number system, the limited area of answers forms a subset. This subset is 1, 4, 9, 16,... . There are, of course, no negative numbers that can fit into this subset.

The graphic representation of all literal number equations such as those above may be done by extending, only a little, the concept already understood in graphing on the rectangular coordinate system. By starting with the number line, Figure 8, one can grasp easily the idea that subsets of the set of real numbers may be treated with the straight line, the curved line, as well as the closed curved line.

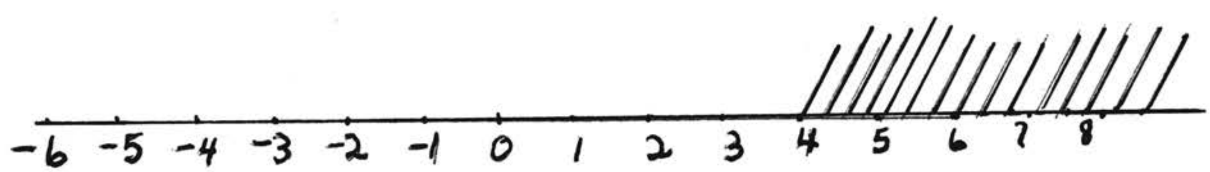

Figure 8

Consider the number line in representing all real numbers greater than 4. Then the space to the right of four would be shown to represent that subset. The same area of the line would include all numbers such that $x^2 > 4$, this being another subset. Other examples to be considered will be listed later for the interest of the reader.

Anyone having had elementary algebra has worked with limited sets or subsets in the form of graphing the straight line and higher degree equations. The description of such graphic lines is described as the "locus of all points that satisfy the given conditions of the equation." These lines and equations when used together will produce some interesting results. The only considerations given here will be those dealing with the explanation of inequalities. The subsets represented on the coordinate graph represented by inequalities are of importance in understanding the purpose of inequalities. Above we considered the points on the real number line to the right of 4. This was a limiting statement. Now, let us consider the equations representing all points so that $x > 4$ and $y > 2$. Note, in Figure 9, the plotting of the lines on the graph are drawn for the equations $x = 4$ and $y = 2$ but that the further interpretation gives a graph in which the shaded area represents the solution.

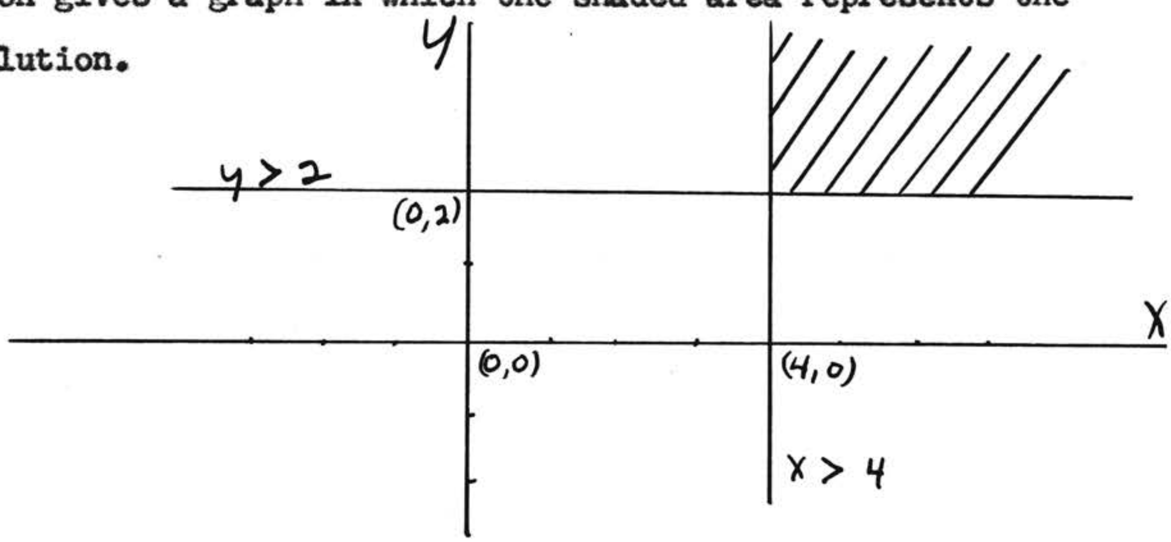

Figure 9

All points "greater than" $x = 4$ will lie to the right of a vertical line drawn through the point $x = 4$, and all points "greater than" $y = 2$ are above the horizontal line drawn through the point $y = 2$. The points that meet both requirements will lie in the area as shown.

Another pair of equations forming a system are $x + 3y > 6$ and $y < x - 2$ (see Figure 10)[8]. Using these, there is a set of points that satisfies both of the inequalities. The part that is common to both equations is again shaded. The plotting of the equations $x + 3y > 6$ and $y < x - 2$ gives the area to be set off to fit the requirements.

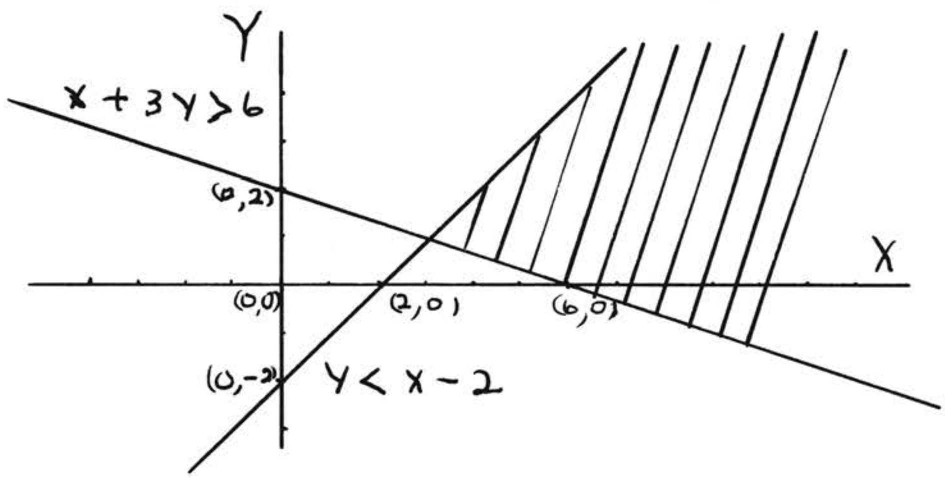

Figure 10

You are familiar with the graphing of the circle coming from the equation $x^2 + y^2 = 1$ or the variations of the ellipse. If the equal sign is replaced with the "greater than" or the "less than" sign, then the concept is extended

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[8]Richardson, pp. 259-261.

to a much wider use. Figure 11 shows the equations $x^2 + y^2 > 1$ and $x^2 + y^2 < 1$ together with the equation $x^2 + y^2 = 1$ and gives three sets of points that added together will describe the whole plane. This concept is much more useful than the singular idea of using only the equal values.

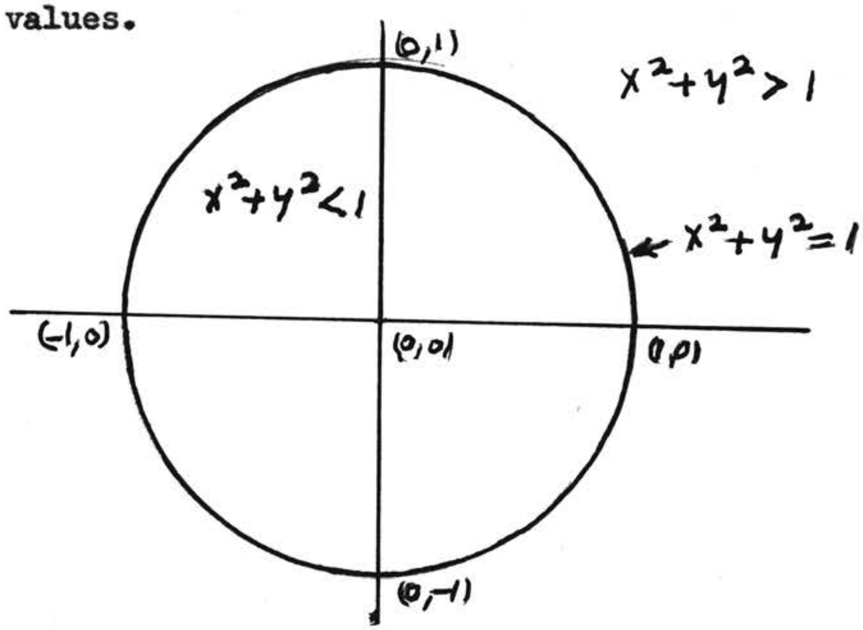

Figure 11

Below is a list of problems that can be used to establish permanency for future use.[9, 10]

[9]Beberman and Meserve, p. 201.

[10]Richardson, p. 261.

1. $x + 1 = 3$
2. $x \nless -3$
3. $x - 1 > 3$
4. $3 < x < 5$
5. $3 > x < 5$
6. $x = x$
7. $x + y \geq 7$
8. $\begin{cases} x^2 + y^2 \geq 4 \\ x + y \geq 2 \end{cases}$
9. $\begin{cases} x + y \leq 7 \\ x - y \geq 1 \end{cases}$
10. $\begin{cases} y \geq x^2 \\ y \leq x \end{cases}$

The number scale and the coordinates can be used to accept, reject, or limit the values that must be given a literal number and, therefore, place it in its proper subset in the number system.

The graphic representation may be used to simplify the reasoning in more complicated equations as well. Higher mathematics and statistics as well as general economic areas have accepted set methods to illustrate the point to be made.

CHAPTER IV

CONCLUSION

The understanding of the "place-value" number system is a step toward the understanding of the structure of the decimal system with which the current population of the United States and of most of the world is familiar. In the discussion of this system, the writer attempts to define the "place-value" number system and to show that there can be a great variation in the bases on which such a system may be built.

The base "two" or the "binary" system permits a structure of only two units. The whole system representing the count of integers as we know them can be represented by the two symbols "1" and "0." The operations which can be permitted in the decimal system may be used in this system. Although the representation of a large number in this system is space consuming and would be cumbersome for the everyday student of arithmetic, the system is used commercially because the "1" and the "0" can be represented by open and closed electrical circuits. This can be done very rapidly, and the working of a problem of many hours or days with a pencil may be done in a few seconds or minutes.

The structure of the "place-value" number system is the same whether the base to be used is two, seven, ten, or twelve. With the drill necessary to understand the structure to the base "seven," for instance, a student will become familiar enough with the general structure of the system so that the decimal system becomes relatively simple.

"Set," and some simple interpretations of its use, can also become a useful tool with which the teacher and student alike should become familiar. The very small corner of the area of "set" is hurried through to give a glimpse of what can be done by one who has built a background in "set" theory. The last few years have brought out to higher level educators the possibilities of its use and the need for presenting it to those who pursue the advanced areas of mathematics and logic.

This does not mean that one who has but limited background in "set" cannot find some simple applications which will help him explain many areas in number relations, both real and literal, applications in logic as studied in geometry, or in other lower level educational situations.

"Set" may be explained by everyday illustrations using "things" which are everyday observances of everyone. The dishes used in serving the various kinds of breakfasts one might eat may be used to represent the "set" and the "subset" and may be used to show that there is more than one subset of a set and that subsets may overlap each other.

Sets and subsets may be shown graphically by circular type drawings, overlapping or separate, according to the picture to be represented. One of the better ways to help formulate a logical conclusion is to use such graphs.

Logic may be brought through the stages of complication by starting with simple everyday occurrences and continuing step by step into the abstract.

The elementary algebra problem often can be simplified by the use of circular graphs as they are used in other logical situations. Extensions of concepts may be introduced easily and taught. For instance, graphing of the linear and quadratic equations on the rectangular coordinate plane is familiar to all algebra students as the "locus of all points that satisfy the given conditions of the equation," and "the points on this locus must satisfy the conditions of the equation." The introduction of the graphing of inequalities on the secondary school level may be shown easily by the same basic method extended but a little.

The algebra student is familiar with the graph that represents the circle equation $x^2 + y^2 = 4$, but how many are familiar with the equation written as an inequality and the graph of that equation? The graph of the equation $x^2 + y^2 > 4$ is the set of all the points outside of the graph line of the circle.

A "set" of concepts on "set" learned by the teacher of any grade level would be highly profitable. The more one uses this concept, the easier it is to use again and, therefore, it can become a very useful tool in the teaching profession.

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