ASPECTS OF TWO DRUM WINDING

by

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ABSTRACT

Two drum winding is not yet described by any model due to the lacking expression for the so-called wound-in-tension. Through dynamic analysis equations for winding velocity and first-drum-tension, which is related to wound-in-tension, are established.

NOMENCLATURE

- a peripheral acceleration of paper roll, m/s^2
- b width of paper, m
- F tangential contact force, N
- h thickness of paper, m
- I moment of inertia, kgm^2
- M applied torque, Nm
- N normal contact force (nip force), N
- r radius, m
- s radius of paper roll, m
- T tensile stress, N/m^{-}2
- t time, s
- v winding velocity (peripheral velocity of paper roll), m/s
- θ wrap angle, deg
- μ coefficient of friction
- ρ density, kg/m^3
- Ω inertia factor, kgm^2/m^2
- ω angular velocity, s^{-1}

Subscripts

1 first drum
2 second drum
c core
f friction outside nip between incoming paper strip and first drum
n incoming paper strip
r rider roller
w web line

INTRODUCTION

Two drum winding is a process often applied when papergrades such as magazine paper and newsprint are wound into rolls. While center winding can be mathematically modeled, models for two drum winding do not exist. The main reason for this is complications due to the driven drums which are nipped against the roll being wound. Through dynamic analysis we will find relations for the two drum winding process. A differential equation for the winding velocity and a partial solution for the first-drum-tension, which is related to wound-in-tension, are the final results. Though these are results and analysis for two drum winding, it should be noted that the same principles may be applied to other kinds of surface winders.

DYNAMIC ANALYSIS

A model of a two drum winder is shown in Fig.1. We assume that the winder is driven by application of torques on the drums. The first drum (i.e. the drum around which the incoming paper is transported) with a radius r_1 and a moment of inertia I_1 is driven by a torque M_1. The second drum with a radius r_2 and a moment of inertia I_2 is driven by a torque M_2. The freely rotating rider roller has a radius r_r and a moment of inertia I_r. Before the winding process is initiated a core with outer radius r_c is resting between the two drums. During winding the radius s of the roll being wound increases from r_c to its final value. The web line stress T_w of the incoming paper acts as a brake on the process. By dividing the configuration into different parts and applying the equations of motion for each part we can find the relation between winding velocity v, the applied torques M_1 and M_2, the web line stress T_w and the rider roller loading N_r. This will also lead to an expression for the web stress T_1 in the outer layer after the first nip, which is an important parameter in wound roll modeling.

Before deriving the equations of motion we have to establish some kinematic relations. We assume incompressible paper roll and rollers (i.e. drums and rider roller) with no slippage between paper roll and rollers. For incompressible rolls we have the following kinematic relations [1]:

$$\dot{s} = \frac{ds}{dt} = \frac{h}{2\pi}\frac{v}{s} \tag{1}$$

where t is time and h is paper thickness, and

$$a = \frac{dv}{dt} = \frac{dv}{ds}\dot{s} = \frac{h}{2\pi}\frac{v}{s}\frac{dv}{ds} \tag{2}$$

where a is acceleration of incoming paper. No slippage between paper roll and rollers indicates equal velocities on all contact surfaces. This implies that

$$\omega_1 r_1 = \omega_2 r_2 = \omega_r r_r = \omega s = v \tag{3}$$

where ω_1, ω_2, ω_r and ω are the angular velocities of the first drum, second drum, rider roller and paper roll respectively. Differentiations of Eqs.(3) give

$$\dot{\omega} = \frac{a}{s} - \frac{\dot{s}}{s}\frac{v}{s} = \frac{h}{2\pi s}\left(\frac{v}{s}\frac{dv}{ds} - \frac{v^2}{s^2}\right) \tag{4}$$

where Eqs.(1) and (2) have been applied, and

$$\dot{\omega}_1 r_1 = \dot{\omega}_2 r_2 = \dot{\omega}_r r_r = a = \frac{h}{2\pi}\frac{v}{s}\frac{dv}{ds} \tag{5}$$

We will also need an expression for the moment of inertia of the paper roll I. If b is the width of the paper and ρ is the paper density, we get

$$I = \frac{\pi}{2}\rho b s^4 + I_c \tag{6}$$

where I_c is a correction due to the core in the middle of the paper roll.

The relations established above will now be incorporated into the equations of motion for the different parts of the two drum winder. We consider the 5 parts (rider roller, paper roll, first drum, second drum and incoming paper) separately as indicated in Fig.2. The paper roll, rider roller and the two drums have to satisfy the angular momentum equation. If F_r is the tangential contact force between the rider roller and the paper roll, we get the following angular momentum equation for the rider roller:

$$F_r r_r = I_r \dot{\omega}_r = \frac{I_r h}{2\pi r_r}\frac{v}{s}\frac{dv}{ds} \tag{7}$$

The first drum will satisfy the angular momentum equation

$$-(F_f + F_1) r_1 + M_1 = I_1 \dot{\omega}_1 = \frac{I_1 h}{2\pi r_1}\frac{v}{s}\frac{dv}{ds} \tag{8}$$

where F_f is a frictional force between incoming paper and first drum and F_1 is the tangential contact force exerted in the nip between incoming paper and the first drum. For the second drum we find the angular momentum equation

$$-F_2 r_2 + M_2 = I_2 \dot{\omega}_2 = \frac{I_2 h}{2\pi r_2}\frac{v}{s}\frac{dv}{ds} \tag{9}$$

Here F_2 is the tangential contact force between the second drum and the paper roll. The angular momentum equation for the paper roll yields

$$(F_2 - T_1 hb + F_n - F_r)\, s = I\dot{\omega} \tag{10}$$

Here T_1 is the stress in the paper after the nip (i.e. the contact zone) between first drum and paper roll. In addition equilibrium of incoming paper yields:

$$T_1 hb + F_1 + F_f - T_w hb - F_n = 0 \tag{11}$$

where inertia of the incoming paper strip has been neglected.

By combining Eqs.(7)-(11) we obtain the following relation between M_1, M_2, T_w,$\dot{\omega}$, I and v:

$$\frac{M_1}{r_1} + \frac{M_2}{r_2} - T_w hb = \frac{h}{2\pi}\left(\frac{I_1}{r_1^2} + \frac{I_2}{r_2^2} + \frac{I_r}{r_r^2}\right)\frac{v}{s}\frac{dv}{ds} + \frac{I\dot{\omega}}{s} \tag{12}$$

Insertion of Eqs.(4) and (6) for $\dot{\omega}$ and I in Eq.(12) yields the differential equation

$$\frac{M_1}{r_1} + \frac{M_2}{r_2} - T_w hb = \left\{ \frac{h\Omega}{4\pi s} + \frac{1}{8}\rho hbs \right\} \frac{dv^2}{ds} \; - \; \left\{ \frac{1}{4}\rho hb + \frac{I_c h}{2\pi s^4} \right\} v^2 \qquad (13)$$

for the winding velocity v. Note that this differential equation may also be derived from an energy approach [2]. Here

$$\Omega = \frac{I_1}{r_1^2} + \frac{I_2}{r_2^2} + \frac{I_r}{r_r^2} + \frac{I_c}{s^2} \qquad (14)$$

is an inertia factor.

SLIPPAGE

The results above are valid for winding without slippage between paper roll and rollers. However, slippage may occur and this may be accounted for. Considering that we have three nips on the two drum winder, there are 8 possible combinations of slip and no-slip conditions. After the no-slip condition treated above, slippage between the paper roll and the first drum is the most likely condition since the coefficient of friction is relatively low on the first drum. We will now focus on this condition. The same principles are applicable to slippage in another nip or to slippage in several nips.

Before we consider slippage between the first drum and the paper roll, we define *nip-paper* as the part of the incoming paper which is situated in the nip (i.e contact zone), and *wrap-paper* as the remaining part of the incoming paper (see Fig.3).For slippage to have an effect on the established equations, it can be shown that slippage has to be present both between the first drum and the wrap-paper and between the first drum and the nip-paper. It is also possible for the wrap-paper to slip on the first drum without the nip-paper slipping. However, this has no effect on the above equations. The tangential contact forces F_1 and F_f are given by

$$F_1 \leq \mu_1 N_1 \qquad (15)$$

$$F_f \leq \pm T_w hb \left(1 - e^{\mp \mu_1 \theta}\right) \qquad (16)$$

where equality represents slip conditions, the upper part of the $\pm$ or $\mp$ signs represents relative angular velocity of the first drum with respect to the incoming paper in counterclockwise direction and the lower part represents relative angular velocity in the clockwise direction. Here N_1 is the normal contact force between the first drum and the incomming paper strip, μ_1 is the coefficient of friction between paper and first drum and θ is the wrap angle of the incoming paper strip (see Fig.3). The normal contact force N_1 is calculated from straight forward trigonometric relations including the effect of the rider roller loading N_r and the weight of the paper roll. We insert Eqs.(15) and (16) in the first drum equation, Eq.(8), and get the following criterion for no slippage with a driving torque:

$$M_1 \leq I_1 \dot{\omega}_1 \pm \mu_1 N_1 r_1 + T_w hbr_1 \left(1 - e^{\mp \mu_1 \theta}\right) \qquad (17)$$

and this criterion for no slippage with a braking torque:

$$M_1 \geq I_1 \dot{\omega}_1 \mp \mu_1 N_1 r_1 + T_w hbr_1 \left(1 - e^{\pm \mu_1 \theta}\right) \qquad (18)$$

If these criteria are not satisfied, we have slippage (i.e. slip between first drum and wrap-paper and slip between first drum and nip-paper).

In a dynamic analysis with slippage between first drum and paper roll the surface velocities of first drum and paper roll are not equal. We are thus unable to express the angular acceleration of the first drum in Eq.(8) in terms of the winding velocity. This is fortunately insignificant since we may use Eqs.(15) and (16) instead of Eq.(8). For slippage between first drum and paper roll we apply Eqs.(7) and (9)-(11) for rider roller, second drum, paper roll and incoming paper strip respectively, and Eqs.(15) and (16). This yields the following differential equation for the winding velocity when slippage is present at the first drum:

$$\frac{M_2}{r_2} \pm \mu_1 N_1 - T_w hbe^{\mp\mu_1\theta} = \left\{\frac{\Omega_1 hb}{4\pi s} + \frac{1}{8}\rho hbs\right\}\frac{dv^2}{ds^2} - \left\{\frac{1}{4}\rho hb + \frac{I_c h}{2\pi s^4}\right\}v^2 \quad (19)$$

Here

$$\Omega_1 = \Omega - \frac{I_1}{r_1^2} \quad (20)$$

is a reduced inertia factor. Compared with the no-slip equation, Eq.(13), we see that for slippage between paper roll and first drum the applied torque M_1 and the inertia of the first drum do not affect the winding velocity. Instead the nip force (i.e. normal contact force) N_1 and the wrap angle θ affect the winding velocities. Whether the upper part or the lower part of the $\pm$ or $\mp$ signs apply is decided by the direction of the relative velocity between the first drum and the incoming paper. This is equivalent to the opposite direction of the tangential contact forces $F_1 + F_f$ which is given by Eqs.(7) and (9)-(11)

$$F_1 + F_f = T_w hb - \frac{M_2}{r_2} + \left\{\frac{\Omega_1 hb}{4\pi s} + \frac{1}{8}\rho hbs\right\}\frac{dv^2}{ds^2} - \left\{\frac{1}{4}\rho hb + \frac{I_c h}{2\pi s^4}\right\}v^2 \quad (21)$$

If $F_1 + F_f$ is positive we have a relative velocity in the counterclockwise direction, and the upper part of the $\pm$ or $\mp$ signs should be applied. If $F_1 + F_f$ is negative we have a relative velocity in the clockwise direction, and the lower part of the $\pm$ or $\mp$ signs should be applied.

Similarily it is possible to find differential equations for the winding velocity with other slippage conditions.

NUMERICAL EXAMPLE

With a forward difference method we solve Eq.(13) for winding of 45g/m^2 newsprint roll wound on a two drum winder at the paper mill at Skogn in Norway with properties as listed in Table 1. Other inputs to the calculation are specific torques M_1/b and M_2/b, web line tension $T_w h$ and rider roller loading N_r/b given by the controll program which is installed at the winder. Plots for a chosen winding process is illustrated in Figs.4 and 5. Note that it is the electric motor currents and not the the specific torques of the drums that are given by the controll program. Applied torques are related to the currents and may be calculated from the currents if loss factors such as bearing resistance is accounted for [2]. Note also that though rider roller loading is not present as a parameter in Eq.(13) we need it to check for slippage with Eqs.(17) and (18). If slippage is present Eq.(19) replaces Eq.(13). For the process to be analyzed no slippage occured.

Calculations result in winding velocity as function of peripheral radius (i.e. outer radius) of paper roll as displayed in Fig.6. Compared with measured values of the winding velocity we see a slight overestimate in the winding velocity found from theory. This is consistent with the fact that the system friction have been neglected

[1]. The discrepancy between theory and experiments are, however, acceptable, and for the first part of the process where friction is insignificant compared with inertia, there is convincing consistency between theory and observation.

The theory presented above provides an improved understanding of the two drum winding process. In addition to enabling calculations of the winding velocity, the theory may be applied to other aspects. For instance is it possible to calculate the loss in acceleration due to increased inertia if drums are strenghtened to resist bending. The comparission between calculated and observed winding velocity serves as a verification of the theory. It is important to know that the theory is supported by such observations, before we apply it on the more debatable aspects of wound-in-tension.

WOUND-IN-TENSION

The most important input to wound roll models is the wound-in-tension. Wound-in-tension is traditionally defined as the tension in the outer layer of the roll. For two drum winding, or surface winding in general, it is a fact that the tension in the outer layer varies with tangential position [3]. Thus a more specific definition of the wound-in-tension is required. If we define first-drum-tension as the tension T_1 (see Figs.2 and 7) in the outer layer of the roll after the nip between the paper roll and the first drum, rider-roller-tension as the tension after the rider roller nip and second-drum-tension as the tension after the second drum, we can argue that we have at least three candidates for the wound-in-tension. By imposing equilibrium conditions on the outer layer we can argue that the rider-roller-tension is the sum of the first-drum-tension, a contribution from the rider roller nip and a contribution from interlayer slippage between first drum nip and rider roller nip. The second-drum-tension is a sum of the first-drum-tension, contributions from rider roller nip and second drum nip and a contribution from interlayer slippage.

rider-roller-tension	=	first-drum-tension	+	rider roller nip	+	interlayer slippage		
second-drum-tension	=	rider-roller-tension	+	second drum nip	+	interlayer slippage		
	=	first-drum-tension	+	rider roller nip	+	second drum nip	+	interlayer slippage

The first-drum-tension is therefore a contributor to the tension in all tangential positions in the outer layer, and thus we may argue that the first-drum-tension is related to the wound-in-tension.

From the equations established above, Eqs.(8), (10), (15) and (16), we find that the first-drum-tension T_1 is

$$T_1 = T_w + \frac{F_n}{hb} - \frac{M_1}{hbr_1} + \frac{I_1}{2\pi b r_1^2}\frac{v}{s}\frac{dv}{ds} \tag{22}$$

when no slippage is present, and

$$T_1 = T_w e^{\mp\mu_1\theta} + \frac{F_n}{hb} \mp \frac{\mu_1 N_1}{hb} \tag{23}$$

when there is slippage between the paper roll and the first drum. Note that the term including the tangential contact force between paper roll and incoming paper,

F_n (see Fig.2), is recognised as nip-induced tension. We see that the first-drum-tension increases with increasing web line tension and nip-induced tension. When no slippage is present we find (see Eq.(22)) that first-drum-tension decreases with applied torque on the first drum. This is all consistent with empirical observations of the wound-in-tension which is related to the first-drum-tension.

Apparently it looks as if formulas for the first-drum-tension T_1 have been derived, and that we are close to having formulas for the wound-in-tension. In reality this is not true since F_n is an unknown quantity. However, we know the maximium and minimum values of F_n. We have

$$0 \leq F_n \leq \mu N_n \tag{24}$$

where μ is the paper-to-paper coefficient of friction and N_n is the normal contact force between the paper roll and the incoming paper. By applying these limits on F_n it can be shown that F_n is a significant quantity which can not be neglected [1]. This is physically reasonable since nip-induced tension from practical experiences is found to be highly significant for the wound-in-tension of two drum winding.

It is reasonable to assume that the unknown tangential contact force F_n is a function of nip forces, applied torque on the first drum, web line tension and material parameters. Further research should focus on finding an expression for F_n. Since the nonlinear behavior of the paper roll limits the possibility for analytical solutions, finite element methods or experiments are required.

CONCLUSION

Theoretical equations for winding velocity and first-drum-tension for two drum winding have been derived from a dynamic analysis. Calculations on winding velocity show consistency with observations. A slight overestimate can be explained by the system friction which are not included in the calculations. The expressions found for first-drum-tension show that first-drum-tension increases with web line stress and nip induced tension. This is consistent with knowledge from empirical observations on the wound-in-tension which is related to the first-drum-tension

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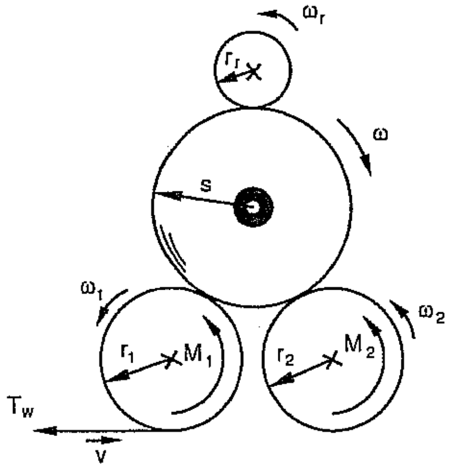

Figure 1: Model of windup section of two drum winder.

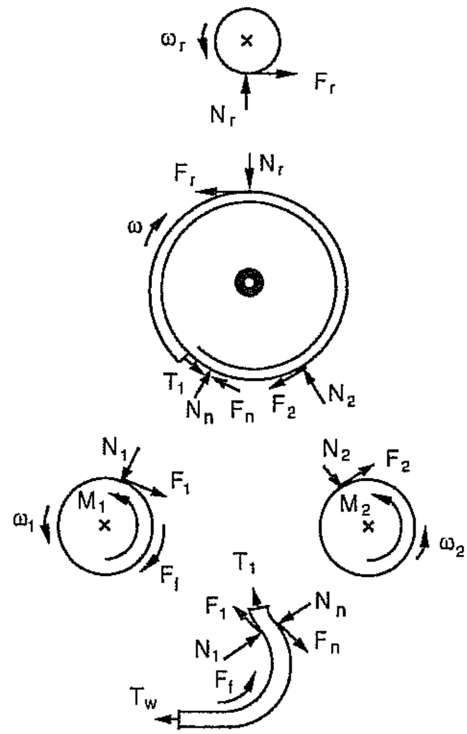

Figure 2: Two drum winder split into different parts.

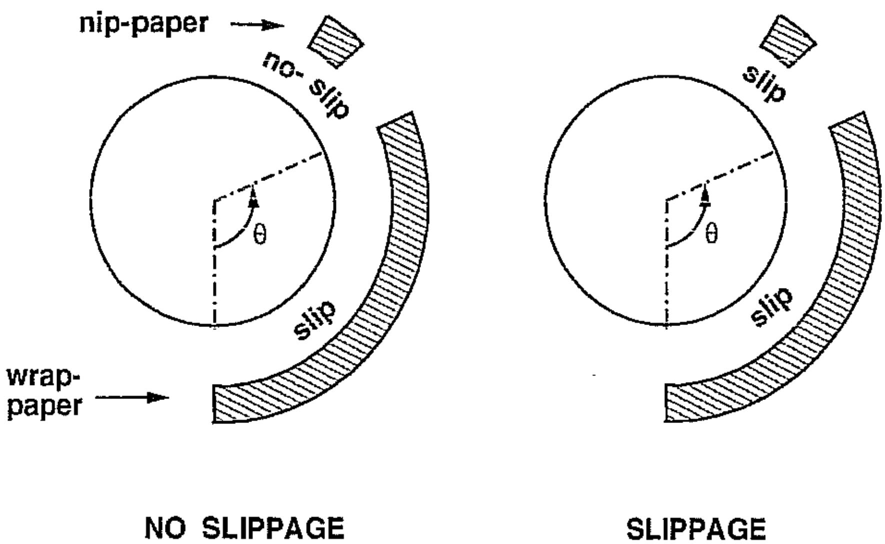

Figure 3: Illustration of slippage conditions between incoming paper and first drum.

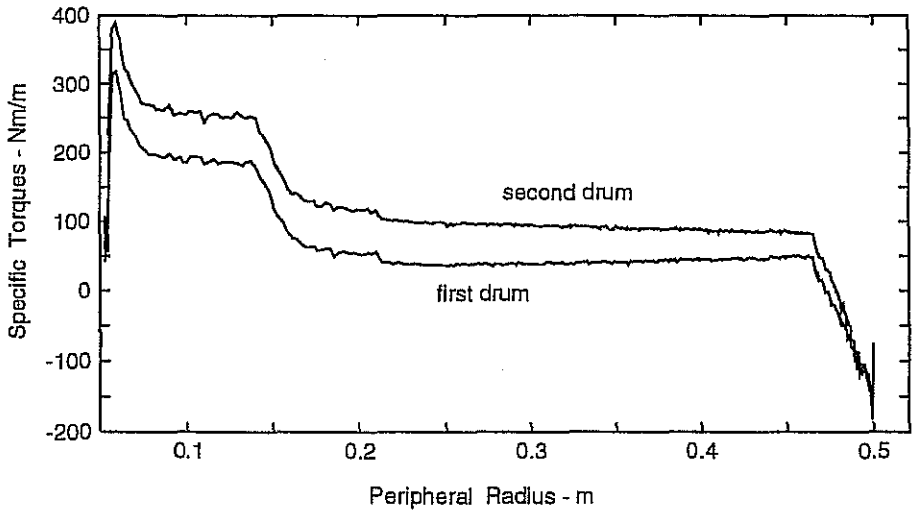

Figure 4: Specific torques as functions of peripheral radius.

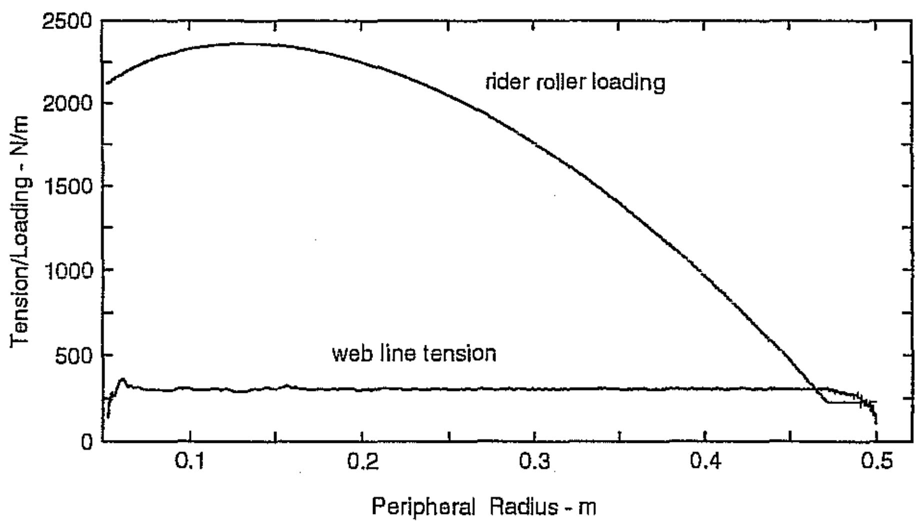

Figure 5: Web line tension and rider roller loading as functions of peripheral radius.

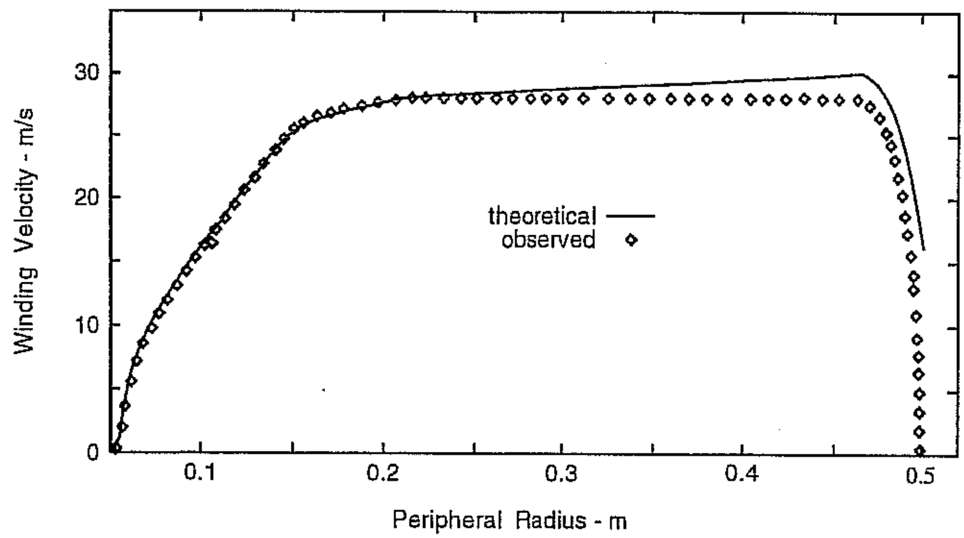

Figure 6: Winding velocity as function of peripheral radius.

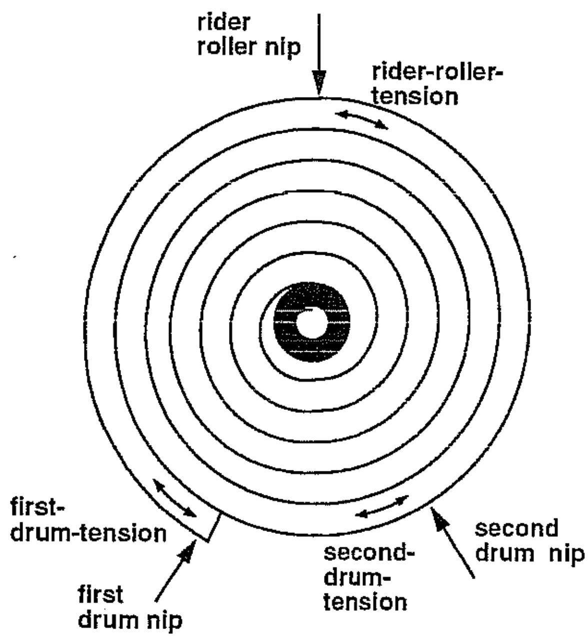

Figure 7: Definitions of first-drum-tension, rider-roller-tension and second-drum-tension.

Moments of inertia/paper width	
First drum ($kg\, m^2/m$)	167.0
Second drum ($kg\, m^2/m$)	167.0
Rider roll ($kg\, m^2/m$)	2.5
Core correction ($kg\, m^2/m$)	-0.0036
Radii	
First drum (m)	0.425
Second drum (m)	0.425
Rider roll (m)	0.09
Core (m)	0.055
Horizontal gap between drums	
Gap (m)	0.02
Paper	
thickness (μm)	74.75
density (kg/m^3)	602.0
coeff.friction	0.2

Table 1: Winder and paper properties

Question - Is this analysis specifically focusing on the acceleration and deceleration of the winder?

Answer - No, this is for the entire winding process.

Question - Why is there so much emphasis on the inertia of the drums and the winding rolls when they have little to do with the dynamics of the system and the steady state speed?

Answer - Inertia only affects acceleration and deceleration.

Question - How does inertia have so much influence during fixed steady state speed?

Answer - It doesn't.

Question - The slide which had the winding velocity on it where you compared the experimental measurement to theory. Near the end of the line - the line approximated nearly a couple of meters per second between experiment and theory at a peripheral radius of about 4 or something like that. That error in velocity looks like a small error in velocity but could be a tremendous error in web strain. It could be in the area of 10%.

Answer: I agree.