# SHEAR IN MULTISPAN WEB SYSTEMS 

by

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#### Abstract

Wrinkling research has been concentrated in past years on understanding web wrinkling in isolated spans in web lines[1]. It is expected that some of the fundamentals which govern single span behavior will also apply to multispan wrinkling problems. For instance it has been found that predicting the shear in the web is necessary to be able to predict shear wrinkles, which are wrinkles due to roller misalignment. Earlier works Young[2] and Dobbs[3] have concentrated on determining shears and moments in moment transfer conditions where the coefficient of traction was assumed constant. It has also been found that knowledge of the traction capacity between webs and rollers allows the calculation a lower bound in web tension below which wrinkling becomes impossible. The focus of this paper is to show how shears in multispan web systems can be predicted. How these shears are impacted by the complications of traction capacities which vary as a function of entrained air and by web edge slackness will be treated.


## INTRODUCTION

In Figure 1 a web is shown crossing three rollers. To simplify the representation both spans have been drawn in one plane when in reality the web changes plane each time a roller is encountered and thus there is some angle of wrap of the web about each roller. There is some frictional engagement between the web and roller surfaces which tend to isolate the shear in each web span. Shelton [4] determined that the moment upstream of the misaligned roller was zero and others such as Lorig[5] had determined earlier the concept of normal entry of a web to a downstream roller. Thus as roller C is misaligned a shear and an associated bending moment develops in span B. In span B the bending moment is maximum as the web exits from roller B and dissipates to zero just prior to entry of roller C.

The maximum moment increases with increasing misalignment until the frictional moment capacity between roller $B$ and the web, denoted $M_{r}$, is exceeded. At this point, bending moment begins to transfer across roller $B$ from span $B$ upstream into span $A$. An added complexity may be that the web may become slack upon one edge and change the relationships that the web shear and moment have as a function of the misalignment of roller C .

## DISCUSSION

## Shear and Moment Expressions for Span_B prior to Moment Transfer or Edge Slackness

Przemieniecki [6] developed stiffness matrices for beams stiffened by tension. In Figure 2 a free body of such a beam is shown. The stiffness matrix for this beam is:
where $E$ is Young's modulus, $I$ is the area moment of inertia, and $T$ is the beam or web tension. The stiffness matrix \{1\} will be used to analyze span $B$. The assumption will be made that $M_{r}$, the moment which can be resisted at roller $B$ due to web/roller traction, is sufficient to react the moment due to roller misalignment. This results in no lateral deflection or rotation of the web at roller $B$. Also known is that the moment in the web at the entry point to roller $C$ is zero. Thus, under these circumstances it is known that $v_{i}, \theta_{\mathrm{i}}$, and $\mathrm{M}_{\mathrm{j}}$ in the equations above are zero. The last equation in the matrix above reduces to:

$$
M_{b j}=0=\left[\frac{-6 E I}{L_{b}^{2}}-\frac{T}{10}\right] v_{b j}+\left[\frac{4 E I}{L_{b}}+\frac{2 T L_{b}}{15}\right] \theta_{b j}
$$

where $\theta_{\mathrm{bj}}$ is the misalignment of roller C in span B , shown in Figure 1 as $\theta$. Thus, the lateral deflection of the web is dependent upon the misalignment per:

$$
v_{\mathrm{bj}}=\left[\frac{\frac{4 \mathrm{EI}}{\mathrm{~L}_{\mathrm{b}}}+\frac{2 T \mathrm{~L}_{\mathrm{b}}}{15}}{\frac{\mathrm{\sigma EI}}{\mathrm{~L}_{\mathrm{b}}^{2}}+\frac{T}{10}}\right] \theta_{\mathrm{bj}}=\frac{4\left(30 \mathrm{EIL}+\mathrm{TL}_{\mathrm{b}}^{3}\right)}{3\left(60 \mathrm{EI}+\mathrm{TL}_{\mathrm{b}}^{2}\right)} \theta
$$

Since $v_{i}$ and $\theta_{j}$ are zero and $v_{j}$ is now known in terms of $\theta_{j}, F_{y i}, M_{i}$, and $F_{y j}$ can be determined in terms of $\theta$ from the stiffness matrix \{1\} as:

$$
\begin{gather*}
\mathrm{F}_{\mathrm{byi}}=\mathrm{V}_{\mathrm{b}}=-\frac{240(\mathrm{EI})^{2}+104 \mathrm{EIL}_{\mathrm{b}}^{2} \mathrm{~T}+3 \mathrm{~L}_{\mathrm{b}}^{4} \mathrm{~T}^{2}}{2 \mathrm{~L}_{\mathrm{b}}^{2}\left(60 \mathrm{EI}+\mathrm{L}_{\mathrm{b}}^{2} \mathrm{~T}\right)} \theta \\
\mathrm{M}_{\mathrm{bi}}=-\mathrm{M}=-\left[\frac{2 \mathrm{EI}}{\mathrm{~L}_{\mathrm{b}}}+\frac{\mathrm{T}_{\mathrm{b}}}{6}\right] \theta
\end{gather*}
$$

$$
\begin{equation*}
F_{b y j}=-V_{b}=\frac{240(E I)^{2}+104 E_{L} L_{b}^{2} T+3 L_{b}^{4} T^{2}}{2 L_{b}^{2}\left(60 E I+L_{b}^{2} T\right)} \theta \tag{6}
\end{equation*}
$$

Thus the maximum moment, shear, and lateral deflections are known in span B. These equations remain valid until:

- the moment $\mathrm{M}_{\mathrm{bj}}$ exceeds $\mathrm{M}_{\mathrm{r}}$, at which point moment transfer begins
- a slack edge occurs.


## Moment Transfer between Spans B and A

An algorithm must be developed for $M_{r}$ such that the occurrence of moment transfer can be predicted. In Figure 3 a web in contact with a roller is shown. The cylindrical area of contact between the web and the roller has been flattened out for illustrative purposes. The pressure P which exists between the web and roller due to web tension is calculated using equilibrium. If the web slips and rotates on the roller about its own centroidal axis, the moment $\mathrm{M}_{\mathrm{r}}$ is:

$$
M_{\Gamma}=\frac{\mu T \beta W}{4}
$$

When $\mathrm{M}_{\mathrm{bi}}$ from \{5\} exceeds $\mathrm{M}_{\mathrm{r}}$ moment transfer will begin. Expression \{7] is valid for conditions in which the coefficient of traction is constant. It is known that traction is affected by entrained air which is a function of web tension, among other variables. Prior to moment transfer the web stress is uniform on the span A side of roller B and linearly varying due to the bending moment $\mathrm{M}_{\mathrm{bi}}$ on the span B side. Thus as the entrained air film thickness becomes significant the validity of expression \{7\} comes into question. Knox and Sweeney[8] proved that foil air bearing expressions were applicable to webs passing over rollers. Their expression for the air film thickness was:

$$
\begin{equation*}
h_{0}=0.65 R\left[\frac{12 \eta V}{T}\right]^{\frac{2}{3}} \tag{8}
\end{equation*}
$$

where $\eta$ is the dynamic viscosity of air, V is the web velocity, T is the web tension in units of load per unit width, and $R$ is the radius of the roller in question. Research has shown[9] that the following set of algorithms work quite well in determining $\mu_{t}$, the traction coefficient which has been diminished by entrained air to be:

$$
\begin{gather*}
\mu_{\mathrm{t}}=\mu_{\mathrm{st}} \quad \mathbf{h}_{\mathrm{o}} \leq \mathbf{R}_{\mathbf{q}} \\
\mu_{\mathrm{t}}=-\frac{\mu_{\mathrm{st}}}{2 \mathbf{R}_{\mathbf{q}}} \mathbf{h}_{\mathbf{0}}+\frac{3}{2} \mu_{\mathrm{st}} \quad \mathbf{R}_{\mathrm{q}} \leq \mathbf{h}_{\mathbf{0}} \leq 3 \mathbf{R}_{\mathbf{q}}  \tag{9}\\
\mu_{\mathrm{t}}=\mathbf{0} \quad \mathbf{h}_{0} \geq \mathbf{3} \mathbf{R}_{\mathbf{q}}
\end{gather*}
$$

where $\mathrm{R}_{\mathrm{q}}$ is the combined r.m.s. roughness of the web and roller surfaces in contact and is defined as:

$$
\mathbf{R}_{\mathbf{q}}=\sqrt{\mathbf{R}_{\mathbf{q}, \text { roller }}^{\mathbf{2}}+\mathbf{R}_{\mathbf{q}, \text { web }}^{2}}
$$

Thus, as a first attempt to model how $\mathrm{M}_{\mathrm{r}}$ is effected by entrained air, the traction coefficient $\mu_{t}$ as given per expressions \{9\} will replace $\mu$ in expression [7]. It is evident that $h_{0}$ will vary as a function of CMD location on the downstream side of roller $B$. This is due to localized variation in the web tension which results from the bending moment
which was generated by the misalignment of roller C . This is a complexity which will be studied later if required.

Shelton [4] defined the condition in which there is inadequate traction to react the moment $\mathrm{M}_{\mathrm{bi}}$ but adequate traction to react the shear forces that spans A and B present upon roller B as circumferential slippage. Even after moment begins transferring into span A the web is attempting to gain normal entry to roller B. The stiffness matrix from expression \{1\} can now be used to develop relationships for span $A$ if the $i$ and $j$ locations now become the locations of rollers A and B . $\mathrm{v}_{\mathrm{ai}}, \theta_{\mathrm{ai}}$, and $\theta_{\mathrm{aj}}$ are all zero assuming no slippage at roller A and that normal entry is satisfied at roller B, and that the moment at the downstream end of span $A$ is:

$$
\begin{align*}
& \mathrm{M}_{\mathrm{aj}}=\mathbf{0} \quad \text { when } \mathrm{M}_{\mathrm{bi}}<\left|\mathrm{M}_{\mathrm{r}}\right| \\
& M_{a j}=-\left[M_{b i}-M_{r}\right] \quad \text { when } M_{b i}>M_{r} \text { and } M_{b i}(+) \\
& M_{a j}=-\left[M_{b i}+M_{r}\right] \quad \text { when }\left|M_{b i}\right|>M_{r} \text { and } M_{b i}(-)
\end{align*}
$$

The sign changes and conditionals in \{11\} are necessary to keep track of the sign conventions laid out in Figure 2 as expression \{1\} is applied to span A. The only nonzero degree of freedom is the lateral deformation $\mathrm{v}_{\mathrm{aj}}$. From $\{1\}$ the lateral deformation will be:

$$
v_{\mathrm{aj}}=-\frac{\mathrm{M}_{\mathrm{aj}}}{\frac{6 \mathrm{EI}}{L_{\mathrm{u}}^{2}}+\frac{T}{10}}
$$

With $v_{a j}$ determined using $\{11,12\}$, expression $\{1\}$ may now be used to compute the end forces and shears as:

$$
\begin{gather*}
\mathrm{F}_{\mathrm{ayi}}=\mathrm{V}_{\mathrm{a}}=-\left[\frac{12 E I}{\mathbf{L}_{\mathrm{a}}^{3}}+\frac{6 \mathrm{~T}}{5 \mathrm{~L}_{\mathrm{a}}}\right] \mathrm{V}_{\mathrm{aj}} \\
\mathrm{M}_{\mathrm{ai}}=-\mathrm{M}=-\mathrm{M}_{\mathrm{aj}}=-\left[\frac{6 E I}{\mathrm{~L}_{\mathrm{a}}^{2}}+\frac{\mathrm{T}}{10}\right] \mathrm{V}_{\mathrm{aj}} \\
\mathrm{~F}_{\mathrm{ayj}}=-\mathrm{V}_{\mathrm{a}}=\left[\frac{12 \mathrm{EI}}{\mathrm{~L}_{\mathrm{a}}^{3}}+\frac{6 \mathrm{~T}}{5 \mathrm{~L}_{\mathrm{a}}}\right] \mathrm{v}_{\mathrm{aj}}
\end{gather*}
$$

After moment transfer begins $v_{b j}$ as given by expression \{3\} is still valid in a relative sense as the change in lateral deformation between rollers B and C. However, if the deformations in span $B$ are desired in an absolute sense the deformation per expression \{3\} must be added to the deformation of span A, per expression \{12\}:

$$
v_{b j, m t}=v_{b j}[3]+v_{a j[12]}
$$

Expressions $\{4\},\{5\}$, and $\{6\}$ are still valid under moment transfer conditions.

## Effects of Slack Edges in Spans A and B

At some point the bending moments in span B and/or span A will become great enough that the maximum compressive bending stress will exactly cancel the nominal tensile stress due to web tension and a slack edge will occur. Depending upon the relative lengths of the spans and moment transfer conditions span A or span B may witness a slack edge first, as shown in Figure 4.

This work will concentrate on the case in which the edge slackness first appears in span B since results of experiments showed this to be the case for the web and web line
parameters chosen. Also note that if the web edge is slack in span $B$ at roller $B$ does not necessarily dictate slackness in span $A$ at roller $B$. The web achieves a finite shell stiffness as it is wrapped about roller B which can isolate the slackness between the spans. Shelton's previous research [7] in slack edge behavior included an assumption that the web in span $B$ still achieved uniformity in stress prior to entry of roller $C$.
Observations of slack edges in thin webs, refer to Figure 5, show clearly that the web slackness exists as a subtriangular region within the web span. Note as well that troughs exist within the web span which parallel the slack edge. It is assumed herein that the uniform stress condition may be achieved only if the web thickness is adequate to prevent buckling at the web edge due to CMD compressive stresses which are generated as the web transfers from the slack region to the taut, uniform stress, region. Thus the decision was made to remain modeling the slack edge as a triangular region of slackness.

In Figure 6 a web encountering a misaligned roller is shown. The shear force required to drive the web to normal entry is shown as well neglecting shear due to web tension which can be obtained from \{6\} setting the web tension to zero. The bending strain in the lower edge of the web nearest the upstream roller per Euler is:

$$
\varepsilon_{\text {bending }}=\frac{\mathrm{My}}{\mathrm{EI}}=\frac{2 \mathrm{EI} \theta / \mathrm{L}_{\mathrm{b}}^{2} * \mathrm{~L}_{\mathrm{b}}^{* \mathrm{~W} / 2}}{\mathrm{EI}}=\frac{\mathrm{W}}{\mathrm{~L}_{\mathrm{b}}} \theta
$$

If the roller was not misaligned a uniform tensile elongating strain would occur in the web due to the web line tension:

$$
\varepsilon_{\text {web tension }}=\frac{T}{\mathrm{AE}}=\frac{\mathrm{T}}{\mathrm{WhE}}
$$

Web slackness will appear whenever the misalignment $\theta$ generates a bending strain which is larger than the uniform tensile strain. The onset of slackness can be determined by equating $\{17\}$ and $\{18\}$ and solving for $\theta$ :

$$
\theta_{c r}=\frac{\mathbf{T}_{\mathbf{b}}}{\mathbf{W}^{2} \mathbf{h E}}=\frac{\sigma_{x}}{E} \frac{\mathbf{L}_{\mathbf{b}}}{\mathbf{W}}
$$

Also of importance is what width of web remains taught at the upstream roller ( $\mathrm{W}_{\mathrm{O}}$ ) when $\theta>\theta_{\mathrm{Cr}}$ ? Refer to Figure 5 and note that the slack edge contributes no structural stiffness to the web. Assume that the taught edge of the web can be modeled as a tapered beam, as shown in Figure 7, in bending and that the beam width as a function of $x$ can be represented as:

$$
\mathbf{W}(\mathbf{x})=\mathbf{W}_{\mathbf{0}}+\left(\frac{\mathbf{W}-\mathbf{W}_{\mathbf{0}}}{\mathbf{L}}\right) \approx \mathbf{x}
$$

Using the unit load method to derive the lateral force $F$ required to enforce an entry angle $\theta$ at the adjustable roll yields:

$$
\theta=\int_{L} \frac{M^{*} m(\theta)}{E I} d x=\int_{L} \frac{F^{*}\left(x-L_{b}\right)^{*}(-1)}{E h W(x)^{3} / 12}=\frac{6 F L_{b}^{2}}{E h W W_{0}^{2}}
$$

or more conveniently:

$$
F=\frac{E h W W_{0}^{2}}{6 L_{b}^{2}} \theta
$$

The bending strain in the lower edge of the tapered beam at $x=0$ is:

$$
\varepsilon_{\text {bending }}=\frac{M y}{E I}=\frac{\left[\frac{E h W W_{0}^{2}}{6 L_{b}^{2}} \theta L_{b}\right] * \frac{W_{0}}{2}}{E\left[\frac{h W_{0}^{3}}{12}\right]}=\frac{W}{L_{b}} \theta
$$

which is equal to the bending strain calculated for the full web width in expression \{17\}. Now the assumption is made that the web was initially tapered and that prior to the misalignment the uniform stress at the upstream web was:

$$
\varepsilon_{\text {web tension }}=\frac{T}{A E}=\frac{T}{W_{0} h E}
$$

Equating expressions \{23\} and \{24\} yields a slack edge criteria which can be solved to yield the critical misalignment required to cause the tapered beam to become slack:

$$
\begin{equation*}
\theta=\frac{\mathbf{T}_{\mathbf{b}}}{\mathbf{W} \mathbf{W}_{\mathbf{0}} \mathbf{h E}} \tag{25}
\end{equation*}
$$

which can be rearranged to yield the following expression for the reduced width Wo:

$$
\mathbf{W}_{\mathbf{o}}=\frac{\mathbf{T} \mathbf{L}_{\mathbf{b}}}{\mathrm{W} \theta \mathbf{h} \mathbf{E}}
$$

Note this expression should only be used when $\theta>\theta_{c r}$ in which case $W_{0}<W$. This relationship was verified in the laboratory by viewing the reflection of a projected plane of laser light at a low angle of incidence on the web just downstream of the upstream roller, the results of which are shown in Figure 8. The lateral deflection of the web in span $A$ at roller $B, v_{a j}$, is calculated using expression \{12\} as long as span A remains taut. The lateral deflection of the web as it passes through span $B$ under slack edge conditions is computed using the unit load method.

$$
\begin{gather*}
v=\int_{L} \frac{M^{*} m(v)}{E I} d x=\int_{L} \frac{F^{*}\left(x-L_{b}\right)^{2}}{E h W(x)^{3} / 12} d x \\
=6 L_{b}^{3} F-\frac{W^{2}-4 W W_{0}+3 W_{0}^{2}+2 W_{0}^{2} \ln \left[\frac{W}{W_{0}}\right]}{E h\left[W-W_{0}\right]^{3} W_{0}^{2}}
\end{gather*}
$$

If equation $\{27\}$ is solved for F and equated to $\{22\}$ the following relationship between v and $\theta$ can be generated:

$$
v=L_{b} W \frac{W^{2}-4 W W_{0}+3 W_{0}^{2}+2 W_{0}^{2} \ln \left[\frac{W}{W_{0}}\right]_{\theta}}{\left[W-W_{0}\right]^{3}}
$$

Note [28] is valid only for a slack edge condition in span B. The total lateral deflection of the web at roller $C$ for the slack edge condition in span $B$ is therefore:

$$
v_{b j}=v_{\mathrm{aj}}+L_{b} W \frac{W^{2}-4 W W_{0}+3 W_{0}^{2}+2 W_{0}^{2} \ln \left[\frac{W}{W_{0}}\right]_{\theta}}{\left[W-W_{0}\right]^{3}}
$$

Now with expressions for Wo and for the deformation of the web the forces on rollers $B$ and $C$ due to the deformation of span $B$ can be derived. As shown in Figure 9 it is assumed that the web tensile stress is triangular in distribution. This has been proven by
inserting arrays of force sensitive resistors between the web and roller during slack edge conditions. The lateral force expressions are composed of two terms, the first being expression $\{22\}$ and the second term is due to the resultant tension not being normal to the downstream roller. The lateral force on roller $C$ due to span $B$ is:

$$
\mathbf{F}_{\mathrm{b} y \mathrm{i}}=\frac{\mathbf{E h W} \mathbf{W}_{0}^{2}}{\mathbf{6} \mathbf{L}_{\mathrm{b}}^{2}} \theta+\mathbf{T} \sin \left[a \tan \left(\frac{v+\frac{W-\mathbf{W}_{0}}{3}}{\mathbf{L}_{b}}\right)-\theta\right] \text { for } \theta>\theta_{\mathrm{cr}}
$$

For $\theta<\theta_{\text {cr }}$ the shear equation given in expression \{6\} should be used. The force at roller $B$ end of $\operatorname{span} B$ is quite similar:

$$
\mathbf{F}_{\mathbf{b y i}}=\frac{E h W W_{0}^{2}}{6 L_{b}^{2}} \theta+\mathbf{T} \sin \left[\operatorname{atan}\left(\frac{v+\frac{W-W_{0}}{3}}{L_{b}}\right)\right] \text { for } \theta>\theta_{\mathbf{c r}}
$$

Note that $v$ and Wo in \{30\} and \{31\} are determined from expressions \{28\} and \{26\}, respectively, both being functions of $\theta$.

A value of the bending moment is needed at roller $B$ to determine if moment is being transferred from span B into span A per expression \{8\}. This moment must be computed about the center of the beam, regardless of the slackness, as both $\mathrm{M}_{\mathrm{r}}$ from expression \{7\} and $M_{a j}$ in \{8\} are assumed to act at the beam center. Since the tension resultant will always be located $\mathrm{Wo} / 3$ from the upper edge of the web beam shown in Figure 9 the moment will be:

$$
\mathbf{M}_{\mathrm{bi}}=-\mathbf{T}\left[\frac{\mathbf{W}}{2}-\frac{\mathbf{W}_{0}}{3}\right] \operatorname{sign}[\theta]
$$

and thus a positive $\theta \mathrm{j}$ as defined in Figure 2 yields a negative $\mathrm{M}_{\mathrm{bi}}$. Results from expression \{32\} would be used in \{11\} to evaluate $M_{a j}$. Then as before $M_{a j}$ would be used to determine $v_{a j}$ in $\{12\}$ and $F_{a y i}$ and $F_{a y j}$ in $\{13\}$ and $\{15\}$.

## EXPERIMENTAL VERIFICATION

Experiments were performed at 3M Company's E.S.\&T laboratories. The first tests were run on $35.6 \mu \mathrm{~m}$ polyester web which was 15.24 cm wide and Young's modulus of 3.8 GPa . The experimental rig is shown in Figure 10. Lateral reaction forces are measured for rollers A and B . Web lateral displacements are measured prior to roller $A$, just after leaving roller $B$ and just prior to roller $C$ as shown.

The traction for this web against the roller B is shown in Figure 11, the theoretical results are per expression [9]. At 15.2 and $45.7 \mathrm{~m} / \mathrm{min}$ the traction is . 26 but at 91.4 $\mathrm{m} / \mathrm{min}$ it decreases to about 0.18 . The reaction forces at rollers $A$ and $B$ were measured at $15.2,45.7$, and $91.4 \mathrm{~m} / \mathrm{min}$ at a web tension of 66.7 N and are plotted in Figure 12 along with the results of the theory. Under conditions in which a slack edge is not present and moment transfer does not occur, expressions $\{4\}$ and $\{6\}$ would be used to predict the reaction at roller B and the force C due to the misalignment of roller C :

$$
\begin{align*}
& F=F_{\text {byj }}\{6\} \\
& R_{B}=-F_{\text {byi }}\{4\}
\end{align*}
$$

and when moment transfer conditions begin:

$$
\begin{align*}
& F=F_{\text {byi }}\{6\} \\
& R_{B}=-F_{\text {byi }}\{4\}-F_{\text {ayj }}\{15\} \\
& R_{C}=-F_{\text {ayi }}\{13\}
\end{align*}
$$

and finally after slack edge conditions in span $B$ begin:

$$
\begin{align*}
& F=F_{\text {byj }}\{30\} \\
& R_{B}=-F_{\text {byi }}\{31\}-F_{\text {ayj }}\{15\}  \tag{35}\\
& R_{C}=-F_{\text {ayi }}\{13\}
\end{align*}
$$

Please note that the experiments the reactions at rollers A and B were measured and the force F due to the misalignment of roller C was inferred experimentally by adding the reactions at rollers $A$ and $B$ and that $F$ was calculated either from expression [4\} or \{31]. The agreement seems to be quite good. You will note a few rogue data points at a tram error of .011 radians. These data points were taken at a tram error of negative .011 radians to check the uniformity of the web(i.e. web camber) and the experimental apparatus. It should be noted that for $\mathrm{F}, \mathrm{R}_{\mathrm{B}}$, and $\mathrm{R}_{\mathrm{C}}$ the same shear levels were obtained whether the tram error be positive or negative.

The lateral deformations measured at rollers $\mathrm{A}, \mathrm{B}$, and C were also compared to theory. The theory is represented by expression \{3\} when neither moment transfer or slack edge conditions are present, expressions \{12\} and \{16\} when moment transfer begins, and \{12\} and \{29\} when both moment transfer and slack edge conditions exist. Please note that the experimental edge measurements are not made exactly when the web exits roller B or when the web first contacts roller C due to the width of the detectors as shown in Figure 10 whereas expressions $\{3,12,16,29\}$ predict the lateral deformations at the tangent points were the web either contacts or leaves a roller.

Next the web tension was doubled to a value of 133.4 Newtons and the web shear and the edge deflections which resulted are shown in Figure 14. The higher web line tension results in a larger $\mathrm{M}_{\mathrm{r}}$ such that moment transfer does not occur until the misalignment of roller C is roughly three times higher than it was for the case in which the web tension was 66.7 Newtons. The final data points were taken at .022 radians at which point wrinkles were precipitating on roller B and explains the deviation between theory and experiment for that data.

The next set of experiments were run on polypropylene which had a modulus of 1.38 GPa. At $15.2 \mathrm{~m} / \mathrm{min}$ and both web tensions studied the coefficient of friction between the web and the aluminum roller with a $1.19 \mu \mathrm{~m}$ (rms) surface roughness was 0.36 . The results for the low tension case are shown in Figure 15 and are quite interesting in that good correlation to theory is shown but note the critical tram errors which indicate the beginning of moment transfer and slack edge behavior are extremely close to one another. In Figure 16 the results of the case in which 133.4 Newtons of web tension was applied is shown. Again the theory correlates well to experiments.

Finally as an additional test of the slack edge expressions developed for span B, \{30\} and \{31\}, a comparison is made against experimental data taken by Shelton. This data is shown in Figure 17 for a polystyrene web with the span ratios and tensions indicated.

## SUMMARY

The results prove that the expressions developed herein for web shear and deformation that develop as functions of roller misalignment, moment transfer, and web edge slackness are reasonable. There have been some simplifications made which may not apply to every web. The robustness of expression \{7\} is questionable under slack edge conditions and in situations where the assumption of an average traction coefficient, which is dependent on the average air film height, is not valid. Edge slackness in span A has been neglected herein. Figure 18 shows a picture of span A on the foreground, note that there is no cross section of the web which is completely taut.

It is clear from all of the force and restraint data shown for polyester and polypropylene that once moment transfer begins that the reaction at roller B now has to react the shear from both spans A and B. In Figure 14, in which the final data point was taken when a wrinkle appeared at roller $B$, the shear in span $A$ was about 6.1 N . When added to about a 4.4 N shear in span B this sums to a total reaction of 10.6 N at roller B . It is very common when wrinkling occurs due to roll misalignment that the wrinkle will form on the roller upstream from the misaligned roll.

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Figure 1-A Two Span Web System


Figure 2 - Beam Element showing ( + ) Sign Convention for Loads and Deformations


Figure 3-Contact between Webs and Rollers


Figure 4-A Two Span Web System Subject to Moment Transfer and Slack Edges


Figure 5-A 30.5 cm wide, $20 \mu \mathrm{~m}$ thick, PET web in a 91.4 cm long span under slack edge conditions


Figure 6 - A Single Span Web Subjected to a Shear Force F


Figure 7-A Single Web Span with Edge Slackness


Figure 8 - Verification of the Taut Width Expression \{23\}


Figure 9 - Web Tension Forces Acting upon a Slack Edge Web


Figure 10 - Experimental Apparatus


Figure 11 - Traction between a $35.6 \mu \mathrm{~m}$ thick PET web ( $\mathrm{Rq}=152 \mathrm{~nm}$ ) and a 7.37 cm diameter aluminum roller ( $\mathrm{Rq}=1.19 \mu \mathrm{~m}$ )

| 国 | RB (15.2) | A RA (15.2) $\mathrm{F}^{(15.2)}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ | RB (45.7) | $\Delta$ | RA (45.7) | 0 | F (45.7) |
| 回 | RB (91.4) | $\Delta$ | RA (91.4) | 0 | F (91.4) |
| F (15.2-91.4) - RA(15.2 |  |  |  |  |  |



Figure 12 - Forces or Reactions at Rollers for a 15.2 cm wide, $35.6 \mu \mathrm{~m}$ thick, PET web with a Web Tension of 66.7 Newtons.


Figure 13 - Lateral Deformations for a 15.2 cm wide, $35.6 \mu \mathrm{~m}$ thick, PET web with a Web Tension of 66.7 Newtons.


Figure 14 - Forces, Reactions, and Edge for a 15.2 cm wide, $35.6 \mu \mathrm{~m}$ thick, PET web with a Web Tension of 133.4 Newtons at a Web Velocity of $15.2 \mathrm{~m} / \mathrm{min}$.


Figure 15 - Forces, Reactions, and Edge Deformations for a 15.2 cm wide, 82.6 mm thick, Polypropylene with a Web Tension of 29.8 N at a Web Velocity of $15.2 \mathrm{~m} / \mathrm{min}$.


Figure 16 - Forces, Reactions, and Edge Deformations for a 15.2 cm wide, 82.6 mm thick, Polypropylene with a Web Tension of 133 N at a Web Velocity of 15.2 and $91.4 \mathrm{~m} / \mathrm{min}$.


Figure 17 - Comparison of results of expressions $\{4,31\}$ with Shelton's data[4]


Figure 18 - Span $A$ showing slackness in the foreground as the web leaves Roller $C$ (not shown) on the left moving to slackness on the right at Roller $B$

Question - Is the traction algorithm only for a small wrap angle?
Answer - No that's for any angle less than 360 degrees, for any wrap angle.
Question - Do you think there is any major side leakage with any wrap angle?
Answer - It is a factor of speed. What time does the air layer spend in the air layer zone? If you had a very large air layer coming in than I would say yes it is a function of that wrap angle. Your seeing the results of a traction algorithm that was in its infancy about $11 / 2$ years ago. If you start off with a very tiny layer it decays slower. The time wise algorithm's you saw presented by Robert Taylor and Butous show that it is a function of 1 over width squared it doesn't decay fast.

Question - We seem to have a fast decay of traction with respect to roughness of the surface roller, does that have an effect on air leakage or side leakage?

Answer - It's possible, its hard for me to answer that question right now.
Question - The faster you go the less time you go the less time you are on a roller, the less the leakage. With aluminum or with a metal with a thicker material you have more anaclastic edge that lifts it more and keeps it off the roller, and touches the tip which might be why the bottom curve comes out to non zero.

Answer - That traction equation I showed is applicable for a lot of things, it has been used for telescoping on round rolls, it has a wide range of use and it works.

Question - In terms of predicting when the web will wrinkle. When you can tell if the equation are valid and not valid? Have you tried applying the coupon method that I believe you and Doug have used to both web spans.

Answer - The expressions are valid to the point to where the web does wrinkle. Where we saw the web coming down in the video there was some complexity to the upstream span. You don't have the nice uniform stress ahead the roller. We haven't proceeded quite that far. Thats the next step.

Question - I's it a thick plate theory or thin beam theory?
Answer - Its a classic beam theory, and no shear effects are taken into account. We didn't think that was needed at the time because John Shelton had already proved that those effects were minimal in his thesis, Its a beam because it has tension in it. Short beams are practical and the major problem is that tension makes it a beam.

