SHEAR WRINKLING IN ISOLATED SPANS

by

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ABSTRACT

Web wrinkling is a problem which plagues the web process industry. Most webs are quite thin, on the order of 4-150 µm, and become subject to lateral shear during transport either by misaligned rollers or by guide rolls. It is proven herein through analyses verified by experiments, that classical instability theory does apply to this case. The dependence of wrinkles on the available traction between the web and roller surfaces is also modeled and verified experimentally. A verified model is presented which yields the traction as a function of entrained air.

INTRODUCTION

A web is a continuous, flexible strip of material such as paper, plastic film, metal foil, textiles and nonwoven materials. The continuous format is popular as it accommodates high-speed automated manufacturing operations. The web typically undergoes several processes in which value is added by coating or finishing operations prior to being converted to a final product, such as a sheet of paper, a roll of photographic film, a floppy disk, a video cassette, etc. A list of commercial products which use webs or converted webs in some form is virtually endless.

Wrinkling is a problem which pervades web handling industries. Web substrates are typically quite thin and prone to buckle or wrinkle. The effects of wrinkling can range from compromised product quality to down time of a processing operation, leading to less profitability in the manufacturing operation.

Wrinkles can be created by several mechanisms. The web wrinkles which will be discussed in this paper are those that can be affected by roller misalignment, web velocity, web tension, and surface characteristics of the web and rollers which are in contact. Web uniformity will be assumed.

In web machinery the direction in which the web travels is referred to as the *machine direction* and the direction lateral to the web is the *cross machine direction.* A *web* *trough* is defined herein as an out-of-plane deformation in a web span. A *web span* is an unsupported zone of web between rollers. A *web wrinkle* will be defined as an out-of-plane deformation of a web proceeding about a roller. Web troughs often occur in web lines without the occurrence of web wrinkles and may or may not be detrimental to web quality, whereas web wrinkles almost always result in material or quality loss.

The purpose of this publication is to show how web wrinkles can be predicted and ultimately avoided in manufacturing operations. Analytical techniques for predicting the onset of wrinkles in webs will be presented and experimentally verified.

Whether web wrinkles can be supported across a roller depends upon the traction between the web and roller. A new model which predicts this traction is presented and verified.

DISCUSSION

Experimental investigations as early as 1987 demonstrated that two distinct types of wrinkling failures can exist in webs subjected to shear due to roller misalignment [1,2]. Typical results of a wrinkling experiment are shown in Figure 1. The *tram error* which is shown in Figure 1 is defined as the amount of roller misalignment, in radians, required to generate a wrinkle across a roller. The regions below and to the left of these L shaped failure curves are safe havens in which wrinkles do not occur, while the region inside the L shaped curve defines a zone of web stress and tram error that will propagate wrinkles across rollers. The experimental investigations proved that an increase in the web line velocity would result in moving the vertical portion of the L shaped curve to the right, as shown in Figure 1, but that the position of the horizontal portion was unaffected.

The traction between webs and rollers has long been known to be affected by the velocity and tension of the web [3]. The lower portion of the L shaped curve which was independent of velocity was designated as Regime 1, and the vertical portion which was velocity dependent was designated as Regime 2. The web is typically nearly planar up to the point at which wrinkling occurs for wrinkles associated with Regime 1, whereas the web may be either planar, troughed, or have a zone of slackness prior to a Regime 2 wrinkle failure.

ANALYSIS OF REGIME 1 WRINKLES

Regime 1 was studied initially as the planar geometry of the web instilled hope that classic buckling theories generated for plates might be applicable. Web thickness commonly ranges from 4 to 150 μm, hardly a thickness normally associated with a plate but rather with a membrane.

Timoshenko and Gere [4] presented a solution for the buckling stress for a rectangular coupon which allows a condition of tension on two opposing sides and compression on the other two opposing sides, refer to Figure 2. Note that in keeping Timoshenko's sign convention that positive stresses would induce compression into the rectangular coupon. The solution results from implementing the energy method which states that when the work done by forces acting in the middle plane of the plate becomes larger than the strain energy of bending for any shape of out of plane deflection, the plate is unstable and buckling occurs. To find the critical buckling load the work done by the forces is set equal to the strain energy in bending:

$$\Delta T = \Delta V \qquad \{1\}$$

which becomes the following in terms of forces N and out of plane displacement w:

$$-\frac{1}{2}\iint\left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2\right] dx\, dy =$$

$$\frac{D}{2}\iint\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\}dx\,dy \qquad \{2\}$$

The buckled out of plane shapes of the rectangular coupon are assumed to be represented by the equation:

$$w_{mn} = a_{mn}\sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b} \qquad \{3\}$$

which assumes simply supported edges. For a web span simple supports are reasonable where the web approaches a roller, and the condition of support of the edges is unimportant except for points near the edges. In this expression m is the number of half waves in the X direction and n is the number of half waves in the Y direction. Substituting expression {3} into expression {2} yields:

$$N_x\frac{m^2\pi^2}{a^2} + N_y\frac{n^2\pi^2}{b^2} = D\left(\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right)^2 \qquad \{4\}$$

Timoshenko introduced a normalized stress σ_e as:

$$\sigma_e = \frac{\pi^2 D}{a^2 h} \qquad \{5\}$$

Dividing expression {4} by the web thickness, h, and substituting the normalized stress yields:

$$\sigma_x m^2 + \sigma_y n^2\frac{a^2}{b^2} = \sigma_e\left(m^2 + n^2\frac{a^2}{b^2}\right)^2 \qquad \{6\}$$

In applying expression {6} to spans in web lines it will be assumed that the X direction is the machine direction. The stress in the machine direction will be the stress associated with the web line tension. Since there is tension in the machine direction no wrinkles will form perpendicular to the X axis. Thus expression {6} can be reduced as follows by setting m, the number of wrinkle half waves in the X direction, equal to one:

$$\sigma_x + \sigma_y n^2\frac{a^2}{b^2} = \sigma_e\left(1 + n^2\frac{a^2}{b^2}\right)^2 \qquad \{7\}$$

Expression {7} is the basic failure criterion used to indicate if wrinkling has occurred in the web.

An in-depth discussion of expression {7} is in order. This expression shows the relationship between the machine direction tensile stress σ_x, the lateral stress σ_y necessary to wrinkle the web, and the number of half waves (n) along the Y axis which will be associated with this wrinkled condition. Further insight may be gained by substituting fixed values of n into expression {7} as follows:

$$\sigma_x + \sigma_y\frac{a^2}{b^2} = \sigma_e\left(1 + \frac{a^2}{b^2}\right)^2 \qquad n=1 \qquad \{8\}$$

$$\sigma_x + 4\sigma_y\frac{a^2}{b^2} = \sigma_e\left(1 + \frac{4a^2}{b^2}\right)^2 \qquad n=2 \qquad \{9\}$$

$$\sigma_x + 9\sigma_y\frac{a^2}{b^2} = \sigma_e\left(1 + \frac{9a^2}{b^2}\right)^2 \qquad n=3 \qquad \{10\}$$

Note that expressions {8}, {9}, and {10} now contain only variables σ_x and σ_y which are plotted in Figure 3 for the case where a=b. Note the locations at which the lines representing n=1 and n=2 intersect. If σ_x is increased to a higher tension level it can be seen that it requires less σ_y stress to wrinkle the web if n becomes two. Since a decrease in σ_y corresponds to a lower energy state, the web would change its wrinkled shape from one half wave to two half waves at this intersection. By properly combining expressions {8} and {9} and then {9} and {10}, σ_y can be removed and the values of σ_x at the intersections can be determined as:

$$\sigma_x = \sigma_e\left(1 - 4\frac{a^4}{b^4}\right) \qquad \text{intersection n=1,2} \qquad \{11\}$$

$$\sigma_x = \sigma_e\left(1 - 36\frac{a^4}{b^4}\right) \qquad \text{intersection n=2,3} \qquad \{12\}$$

Thus for a given value of σ_x the half wave number n can be determined by searching for the bounding σ_x values which correspond to half wave number n=i. This is not complex since expressions {11} and {12}, which are the bounds for σ_x when n=2, can be presented in a generalized expression for n=i as follows:

$$\sigma_e\left[1 - i^2(i-1)^2\frac{a^4}{b^4}\right] > \sigma_x > \sigma_e\left[1 - i^2(i+1)^2\frac{a^4}{b^4}\right] \qquad \{13\}$$

With known web line stress and half wave number, expression {7} is employed to calculate $\sigma_{y_{cr}}$, the compressive lateral stress necessary to wrinkle the web. Solving equation {7} in terms of $\sigma_{y_{cr}}$ yields:

$$\sigma_{y_{cr}} = \frac{b^2}{i^2a^2}\left[\sigma_e\left(1 + \frac{i^2a^2}{b^2}\right)^2 - \sigma_x\right] \qquad \{14\}$$

Expressions {13} and {14} make it possible to calculate the half wave number and the critical stress which is necessary in a cross machine direction to wrinkle the web. What remains to be determined is how these compressive stresses are generated by misalignment of the rollers.

DETERMINING THE SOURCE OF THE COMPRESSIVE STRESS

A web span is shown in Figure 4 which has undergone an end rotation and translation that could be due to an unintentionally misaligned roll or an end pivoted guide roller. If the web is treated as a multispan beam in bending, then shear must be constant in each span and moment will vary at most linearly from one roll to the next. One of the fundamental concepts of web handling derived by Shelton [5,6] is the concept of normal entry of a web to a downstream roller. Thus for each web span the bending moment at the downstream roller is zero, provided there is no slippage across the downstream roll. This implies that immediately upstream of the misaligned roller the condition of stress is a constant tensile σ_x stress (i.e. no bending moment) and a constant shear stress τ on each stress block which might be taken across the web width. The second principal stress, which will always be compressive for the case described, can be calculated from elementary mechanics via the following expression:

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \quad \text{where } \sigma_y = 0 \qquad \{15\}$$

The second principal stress is the maximum compressive stress which the web must withstand. To determine the maximum shear stress which can be applied to the web prior to wrinkling, the critical $\sigma_{y_{cr}}$ stress necessary to initiate wrinkling can be computed from expression {14}, substituted into {15} for σ_2, and rearranged to yield:

$$\tau_{cr} = \sqrt{\sigma_{y_{cr}}^2 - \sigma_x \sigma_{y_{cr}}} \qquad \{16\}$$

To relate this critical shear stress to the web and the degree of misalignment in the web line a deflection analysis of the beam in Figure 4 must be performed. The moment in the beam is:

$$M_{AB} = -F \bullet x - M \qquad \{17\}$$

Invoking Castigliano's second theorem, Shigley [7], and letting the downstream moment M be zero due to the concept of transport of strain, will determine the end rotation of the cantilever beam due to the lateral force F:

$$\theta_A = \frac{\partial U}{\partial M} = \int_L \frac{M_{AB}}{EI} \frac{\partial M_{AB}}{\partial M} dx = \frac{FL^2}{2EI} \qquad \{18\}$$

For thin webs it is common to assume that $\tau = \frac{F}{bh}$ and substituting $I = \frac{hb^3}{12}$ into expression {18} yields:

$$\theta_{cr} = \frac{6\tau_{cr}L^2}{Eb^2} \qquad \{19\}$$

Thus expression {19} yields a critical rotation of the roller at which Regime 1 wrinkling should occur. To predict the critical rotation for a given web stress one must:

- determine i in expression {13} given the web properties and the σ_x stress due to web tension
- substitute i into expression{14} which yields σ_{ycr}
- substitute σ_{ycr} into {16} which yields τ_{cr}
- substitute τ_{cr} into {19} to determine θ_{cr}.

This process can be repeated for various values of σ_x and a curve can be generated which presents the critical roller misalignment required to generate a wrinkle as a function of the machine direction stress in the web line.

In some cases there is not adequate traction at the upstream roller to enforce the zero slope boundary condition and a condition in which moment, and edge deflections, are transmitted upstream into the next span. This condition is known as moment transfer between spans, Young et. al. [8] and Dobbs and Kedl [9], and is not treated in this publication. In some cases the web edges may become slack prior to the occurrence of wrinkling. In such cases expression {18} is incorrect and another expression, such as discussed by Shelton [10], must be used to relate the shear force, F, to the misalignment, θ.

EXPERIMENTAL RESULTS AND DISCUSSION OF THE REGIME 1 WRINKLE CRITERION

Expression {19} was experimentally verified on a web line at 3M Company. A typical experimental procedure was to set the web line tension and velocity and to proceed to slowly misalign the test roll until a wrinkle formed. Misaligning the roller faster than the time constant of the web span (i.e. the web span length divided by the web velocity) can cause dynamic shears which exceed the value of the steady state shear referred to in expressions {17} and {18}. These tests were performed on 20 μm polyester film (E_{md} and E_{cmd} = 4.134 GPa and ν = .35). All tests were performed on webs which were 30.48 cm in width and in a web span 91.44 cm in length. The tests were performed on two different rollers (diameters and surface roughness) as shown in Figures 5 and 6. The results agree very nicely with the theory given by expression {18}.

Recall that in the discussion following the presentation of expressions {8-10} that increased levels of machine direction stress has the effects of (1) increasing the half wave number and (2) increasing the level of stress required in the cross machine direction, σ_{ycr}, to wrinkle the web. Note that this effect shows up clearly in the experimental data and the theoretical results shown in Figures 5 and 6.

Expression {19} was developed using the critical buckling strength of a rectangular coupon of web (expression {14}). The buckling analysis assumed the coupon had simply supported edges, as reflected in the assumed buckled shape given in expression {3}. Thus it might be surmised that expression {19} should calculate the existence of what was described in the Introduction as *web troughs*, out-of-plane buckled deformations in the web span which do not necessarily transgress across rollers to become wrinkles. However, if the web forms troughs, the apparent planform width is narrower than the flattened web width which would exist at a roller, as shown in Figure 7. Thus the web entering the downstream roller is not attaining normal entry until the web on each side of the trough is steered into normal entry, inducing the wrinkle about the downstream roller. Whether this wrinkle can be sustained upon the roller surface is dependent on the available traction, which will be addressed next.

ANALYSIS OF REGIME 2 WRINKLES

Regime 2 wrinkles were empirically proven to be a function of the traction between the web and the roller, over which the wrinkles were forming, in two tests. The first tests, again referring to Figure 1, indicated that the second regime of wrinkling was dependent on velocity. The traction between webs and rollers in web lines is affected by velocity through the height of the air films which are developed. Thus at low web velocities the traction may be as high as the static coefficient of friction between the web and roller surfaces while at high web velocities the surfaces may lose all contact and the traction may decrease to zero. A second set of tests were run to verify that Regime 2 wrinkles were directly dependent on traction. The test roller was covered with a mold release tape product which has such a high affinity for polyester surfaces that apparent traction coefficients greater than one are typical. The results of this test plotted upon the previous data are shown in Figure 5. Note that only Regime 1 behavior was evident in these tests. Both types of tests proved that if the traction could be manipulated by the variation of the air film thickness or by changing the affinity of the roller surface for the web that Regime 2 behavior could be modified or eliminated.

Much higher cross machine direction stresses are required to wrinkle a web in the form of a shell than would be required to wrinkle the same web in a planar attitude, typically orders of magnitude higher. Thus a hypothesis was formed that Regime 2 wrinkles became possible only when there was inadequate traction between the web and roller to sustain a wrinkle which should have formed per the Regime 1 theory. The tests performed with the high coefficient of traction surface supported this hypothesis. For a wrinkle to be transported around the surface of a roller requires that:

- ❑ A trough must form in a span upstream of a roller as predicted per expression {19} and due to the normal entry concept a wrinkle is attempting to form in the web as it passes onto a roller.
- ❑ As the wrinkle proceeds onto a roller σ_y must increase to the value required to buckle a cylindrical shell.
- ❑ If the traction between the web and roller is inadequate to sustain this increased value of σ_y the wrinkle will dissipate upon the roller surface.

Now the hypothesis had to be quantified. Timoshenko and Gere [4] studied the axisymmetric buckling of a cylinder loaded in axial compression and derived:

$$\sigma_{y,cr} = \frac{E}{\sqrt{3\left(1-\nu^2\right)}}\frac{h}{R} \tag{20}$$

Thus if a cylinder is subjected to an axial stress of this magnitude one or more circumferential buckles will form. In the context of a web wrinkle passing over a roller it is obvious that a reaction force is necessary to sustain the wrinkled web upon the roller surface. Compressive internal force within the web can be supported only if there are lateral surface traction forces available between the web and roller which can react the compressive force and maintain equilibrium as shown in Figure 8. The surface traction forces which can be generated are functions of the traction coefficient, the web line tension (which generates a pressure between the web and the roller surface), and the contact area between the web and roller. For a uniform web tension the maximum compressive stress which can be supported by traction, which occurs at the web center, on a per unit circumference basis is:

$$\sigma_{y,max} = \frac{T}{2R}\mu \tag{21}$$

If the maximum lateral compressive stress which can be supported by traction predicted by expression {21} is less than the buckling stress predicted by expression {20}, a wrinkle cannot be sustained in the web upon the roller, Shelton [11]. Wrinkles which try to form as the web enters the roller glide out upon the roller surface since the traction is unable to sustain them.

Thus a means of determining whether wrinkles can be sustained upon a roller has been determined in terms of expressions {20} and {21}. The difficulty of applying these expressions lies in the ability to determine the coefficient of friction in expression {21} which required the development of a predictive model.

DEVELOPMENT OF A TRACTION ALGORITHM

An air film layer due to hydrodynamic lubrication was first shown to exist between moving webs and idler rollers by Knox and Sweeney [3]. Knox and Sweeney verified the following relationship:

$$h_0 = 0.65R\left[\frac{12\mu V}{T}\right]^{\frac{2}{3}} \tag{22}$$

where R is the radius of the idler roller, μ is the dynamic viscosity of air ($3.077*10^{-7}$ N-min/m^2 @ 27°C), V is the web velocity (m/min), and T is the web tension (N/m). This air film layer causes less asperity contact between webs and rollers and decreases the normal forces of asperity contact, due to the pressure of the lubricating air film partially supporting the web. The air film thickness can become so large that no contact will exist between the web and roller, at which point the traction can decrease nearly to zero, as provided by the viscosity of air. The reduction of the coefficient of friction due to entrained air films has been documented empirically for some time, Daly [12] and Ducotey and Good [13]. No models existed, however, predicting how the friction coefficient varies as a function of web line operating parameters, such as tension and velocity, and web and roller surface characteristics. Empirical data was obtained by applying a braking torque to a roller which was driven by the web, an idler, and measuring the tensions upstream and downstream from that roller, T_1 and T_2, respectively. With a known angle of wrap, ϕ, the friction coefficient can be determined using expressions developed initially for band-type clutches or brakes, Shigley [7]:

$$\mu = \frac{\log_e\left(\frac{T_2}{T_1}\right)}{\phi} \qquad \{23\}$$

It is assumed that the braking torque is sufficient to cause slippage throughout the entire angle of wrap. Empirical traction data collected for five roller/web surface combinations are shown in Figure 8. After collecting much of this type of data it was found that the friction coefficient retains its static value until the air film thickness surpasses the equivalent root mean square roughness, Rq, of the surfaces in contact. For surfaces with Gaussian distributions of peak heights this roughness is:

$$R_q = \sqrt{R_{q,roller}^2 + R_{q,web}^2} \qquad \{24\}$$

For Gaussian surfaces no contact would be predicted if the roller and web surface achieved a mean surface separation of 6 Rq, at which point the friction should be minimal and due only to the viscosity of air. Since approximately half of the space between the roller and web is occupied by roller and web surface asperities, an air film thickness of 3 Rq is all that is required to prevent asperity contact between the web and roller. Thus, based upon empirical evidence and assumptions of Gaussian distributions of asperity heights on the roller and web surfaces, the following algorithm was developed for traction:

$$\begin{aligned} &\mu_t = \mu_{st} \quad h_o \le R_q \\ &\mu_t = -\frac{\mu_{st}}{2R_q}h_o + \frac{3}{2}\mu_{st} \quad R_q \le h_o \le 3R_q \qquad \{25\} \\ &\mu_t = 0 \quad h_o \ge 3R_q \end{aligned}$$

This algorithm has been proven for surface-ground rollers in contact with web surfaces. The efficacy of the algorithm is demonstrated in Figure 9. Thus an algorithm for traction has been developed which is dependent upon roller and web surface roughness, roller radius, the static coefficient of friction μ_{st}, and web tension and velocity through expression {22} for h_o, the air film thickness.

EXPERIMENTAL VERIFICATION OF THE LOWER BOUND FOR REGIME 2 WRINKLES

With the ability to predict the traction available between the web and roller from expression {25} , equations {20} and {21} can be equated and the minimum tension required to sustain a wrinkle can be calculated at a given web velocity. These tensions when converted to web stress correspond quite nicely to the asymptotes in the experimental data as shown in Figures 10-14 for both 20 and 23 μm thickness webs. These figures show how well the theory is verified on five rollers with roughness covering the gamut of roughness seen in industry.

CONCLUSIONS

Many interesting things have been discovered in the research which led to this publication. One item of intrigue was the applicability of classic buckling equations developed for plates by legendary engineers such as Timoshenko to thin webs which would usually be assumed to be membranes due to their minute thickness. Results of this work prove that there is no lower bound on thickness where bending effects are truly negligible; thus, use of an expression such as {1} is invaluable in predicting the onset of wrinkling in the thinnest of webs. Another interesting feature of this work is the changing nature of the web traveling through a machine. During passage through free spans the web may have little buckling strength, whereas the same web passing over a roller can have significant buckling strength. A final component of this work, which has many implications within web handling besides wrinkling, was the development of a simple, robust, algorithm {25} to estimate the traction between webs and rollers as a function of the air film thickness which can be calculated using expression {22}.

The results of this work can be employed to predict Regime 1 shear wrinkles and to compute a minimum level of machine direction tension required to propagate any wrinkle across a roller which can be used to locate the Regime 2 asymptote. The mechanism that causes Regime 2 wrinkles is still unknown. It is obvious that a misalignment in excess of that required to generate a Regime 1 wrinkle is required to generate a Regime 2 wrinkle. The web/roller traction reduction due to air entrainment is not only affecting the ability of a wrinkle to transport around a roller but also the ability for the misaligned roller to enforce the normal entry condition, and thereby the shear predicted by expression{18}, upon the entering web.

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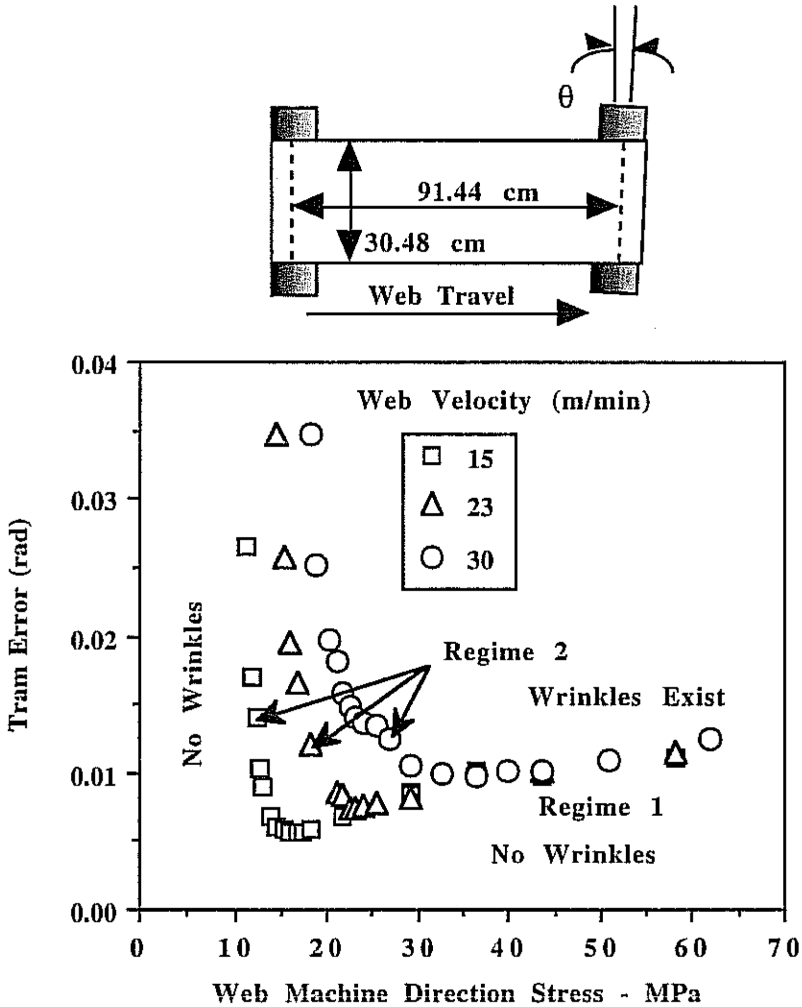

Figure 1 - Experimental Results Exhibiting Regime 1 and Regime 2 Wrinkles for 20 μm PET film - 8.89 cm O.D. roller, 0.46 μm rms surface

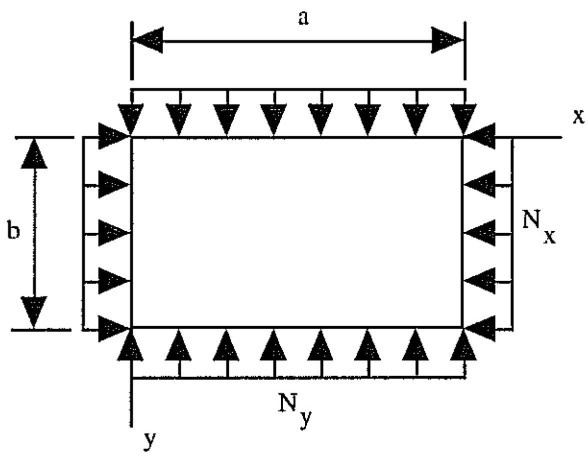

Figure 2 - A Web Panel Subject to Edge Loads

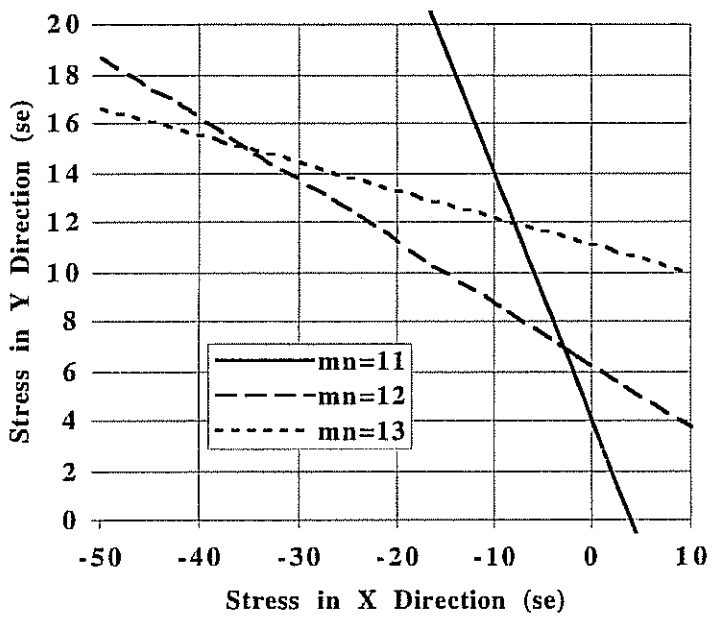

Figure 3 - Y Direction Stress Required for Buckling as a Function of X Direction Stress and Wave Number

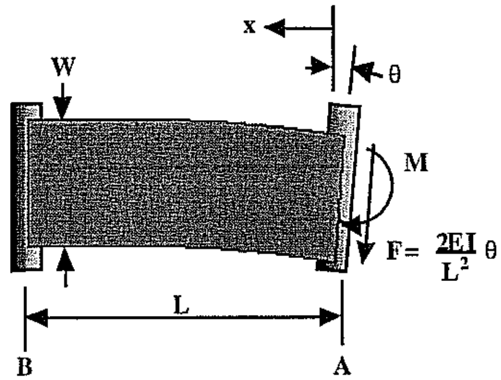

Figure 4 - Lateral Force Generated by a Misaligned Roller

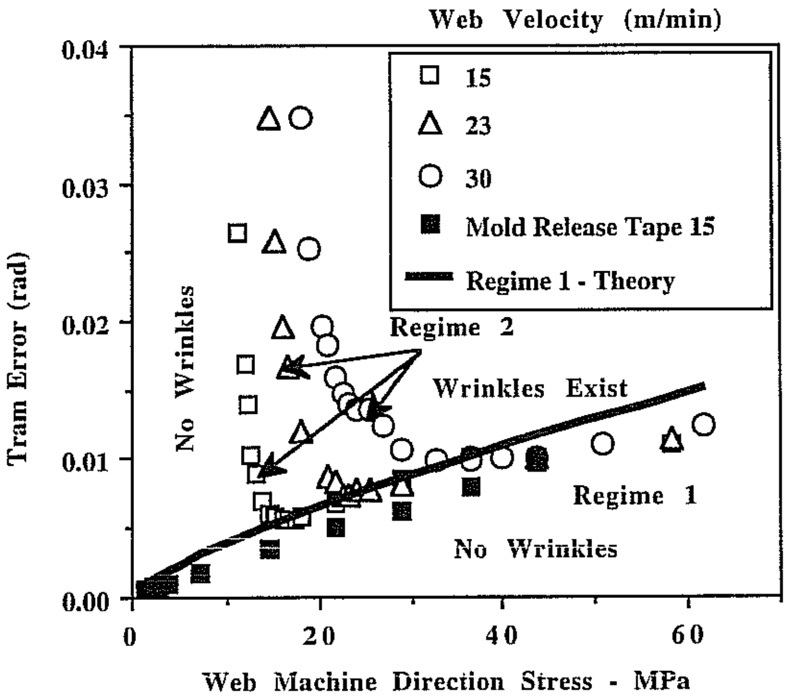

Figure 5 - Results of Regime 1 Theory and Tests with High Traction Overlaid upon the Data of Figure 1 - 8.89 cm O.D. roller, 0.46 μm rms surface

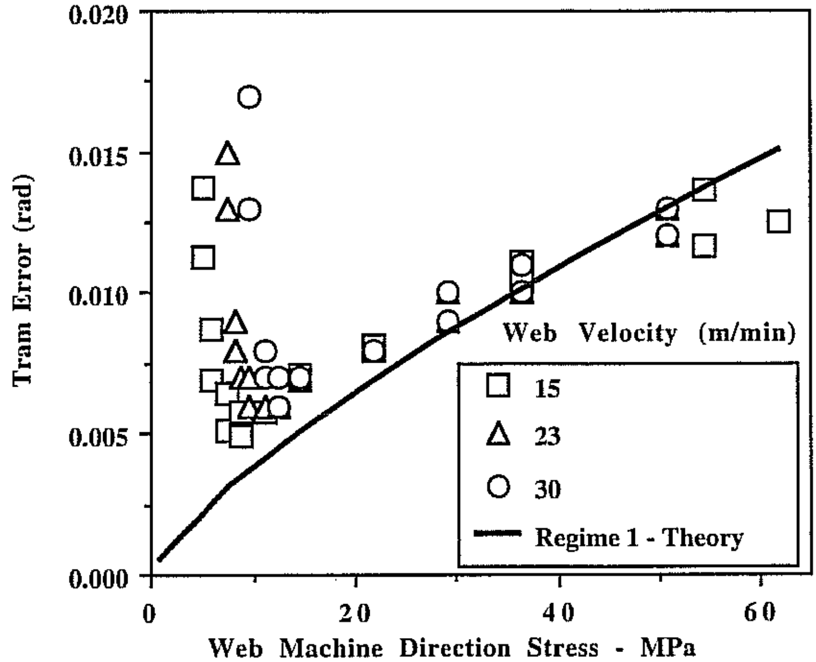

Figure 6 - Results of Regime 1 Theory Compared with Test Results on a 7.3 cm O.D. roller, 0.64 μm rms surface

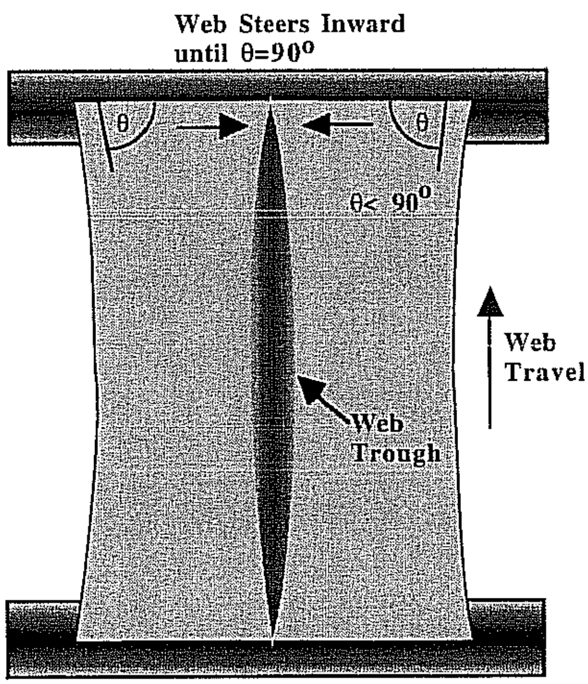

Figure 7 - Troughs resulting in Wrinkles

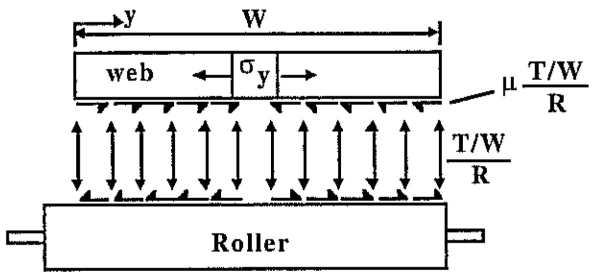

Figure 8 - Free Body Diagram of the web on a roller showing the requirement of traction to sustain a lateral compressive stress, σ_y.

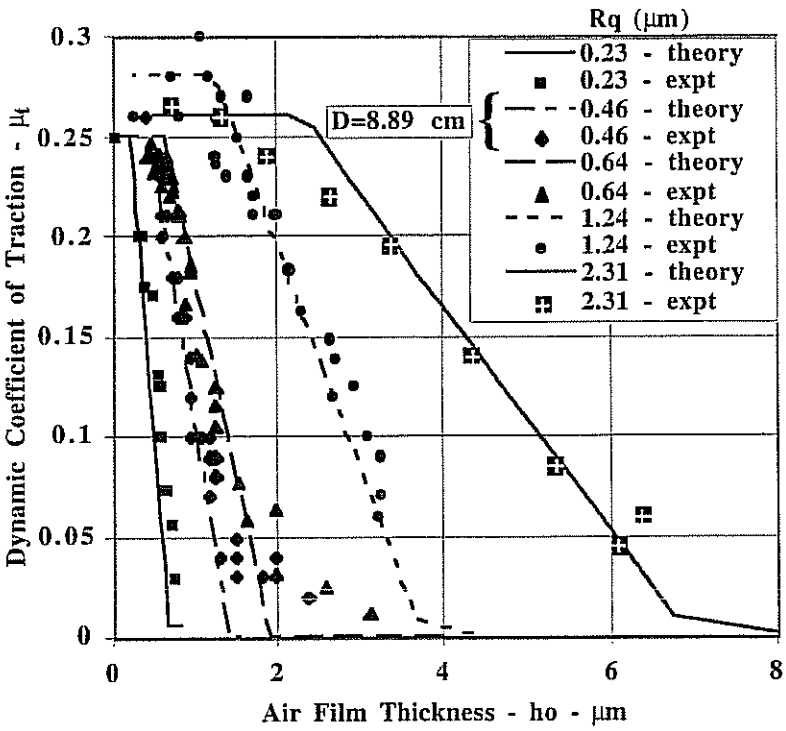

Figure 9 - Verification of Traction Model (Roller Diameters are 7.37 cm except as noted)

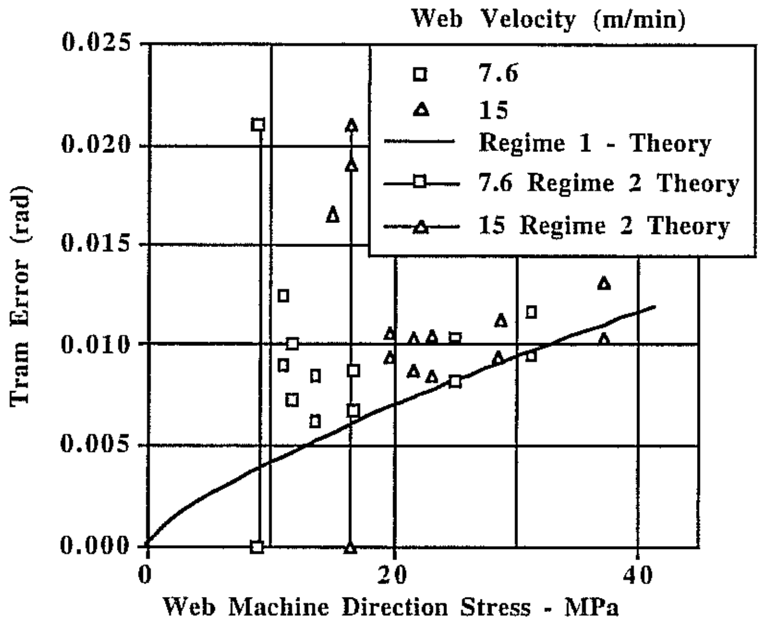

Figure 10 - Results of Theory on 23 μm PET on a 7.3 cm Diameter Roller with an Rq of 0.23 μm

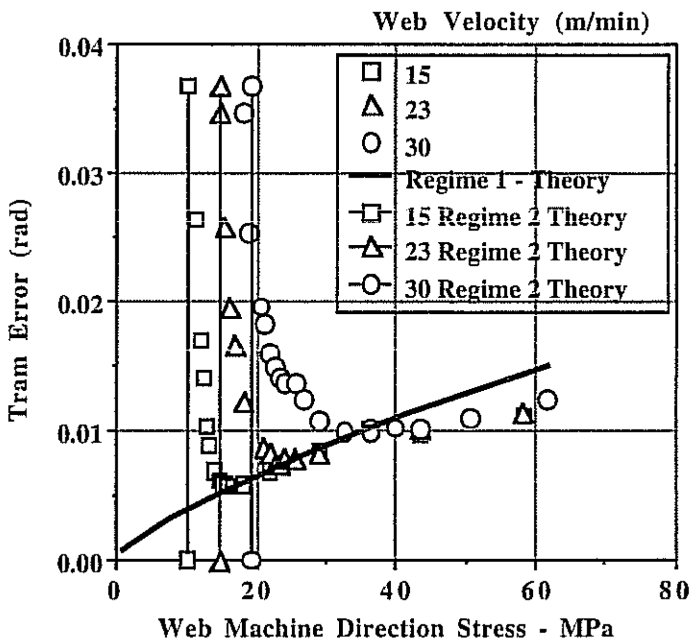

Figure 11 - Results of Theory on 20 μm PET on a 8.89 cm Diameter Roller with an Rq of 0.46 μm

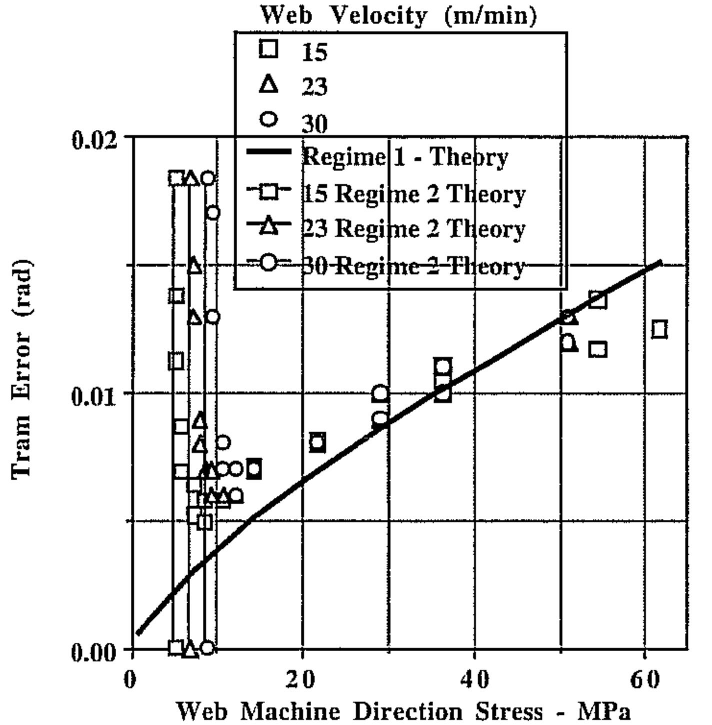

Figure 12 - Results of Theory on 20 µm PET on a 7.3 cm Diameter Roller with an Rq of 0.64 µm

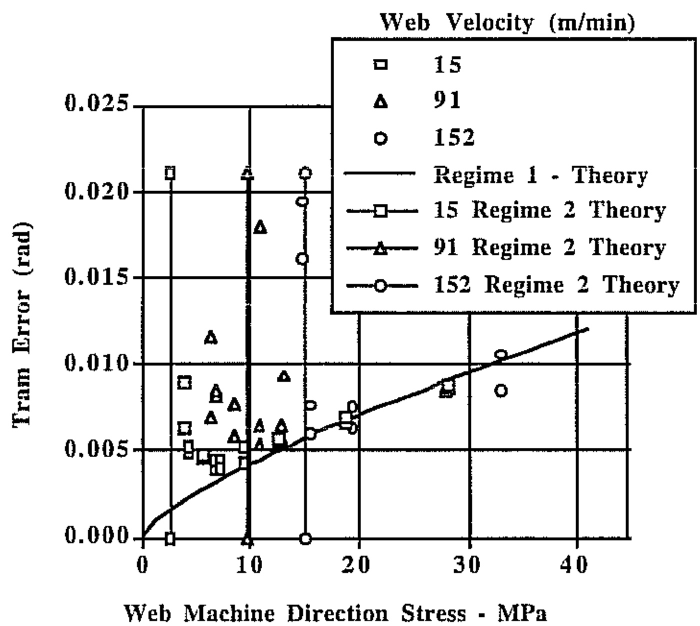

Figure 13 - Results of Theory on 23 µm PET on a 7.3 cm Diameter Roller with an Rq of 1.24 µm

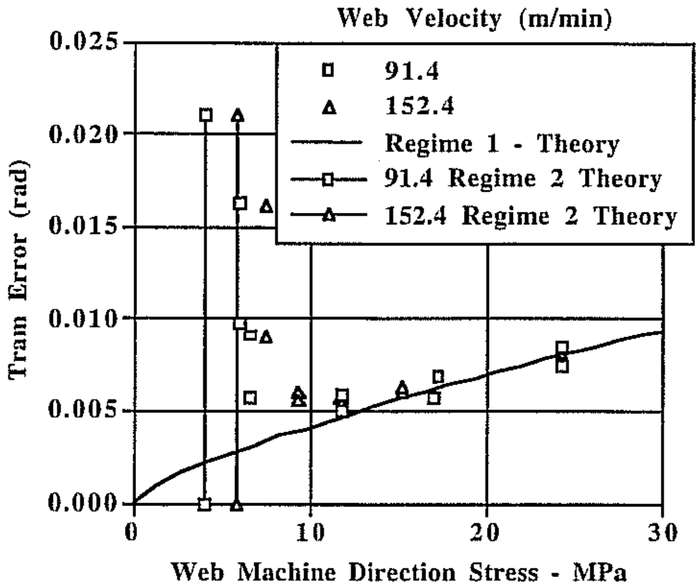

Figure 14 - Results of Theory on 23 μm PET on a 7.3 cm Diameter Roller with an Rq of 2.31 μm

Question -First, the use of conditions for buckling instability when the web is over a roller. I think the case commonly quoted is for a thin shell. I think that if the web is forced to conform to a rigid roller that the buckling stress is going to be a lot higher and no longer able to reduce its large radius of curvature. So that well may be a point that makes wrinkles fall on rolls than you might think.
Second point is that on polyester films at least we do see significantly surface asperity which is greater than 3 times the rms roughness for example some 12 micron film might have an rms roughness of 1 microns and yet there are definitely some areas where the height of the surface is greater than 3 microns from the line.

Answer - I agree totally. I made the supposition of the traction equation based totally upon Guassian surfaces and if truly Guassian I think they work. Now if you look at nominal peak height distribution for a lot of polyester webs, I agree they don't look really Guassian. Sometimes they slope off a little bit and look more like a chi square or some other distribution. But also note in many of the cases I'm showing you here, the roller roughness is quite a bit greater than the web roughness.

Question - It looks real beneficial but what about instances where the gauge variation in thickness of films where it goes across effects distribution?

Answer - There's not a lot you can say in terms of the regime 1 calculation. That is specifically for shear wrinkling of webs with reasonably uniform thickness. Certainly having nonuniform thickness is probably going to cause the regime1 failure to occur at a lesser tram error. But what you can say is I believe that these same equations that I have presented to you for regime 2 will still apply quite nicely for the case where we generate compressive stresses due to that non-uniform thickness. We see a lot of cases where our web lines look terrible, there are troughs all over the place but somehow we make it over rollers. The troughs we are seeing in many cases are not due to misalignment of rollers, but due to something that happened while we were tinting the film. Maybe the head box wasn't throwing all the paper pulp out in the same direction so we had longer and shorter path length or paper length. We see a lot of troughs out in web spans for a number of reasons, but that regime 2 asymtote equation is quite universal in application.