WRINKLE DEPENDENCY ON WEB ROLLER SLIP

by

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ABSTRACT

The shear forces in a web introduced from an untrammed roller create wrinkles that are affected by web twisting or slipping on the upstream roller. The web span bounded between the untrammed and upstream rollers behaves like a beam when bending in the web plane (2). If the bending stresses exceed the capacity of the friction forces holding the web on the upstream roller, strain will migrate over the roller transferring a portion of the moment to the upstream span. A model is presented that predicts the onset of moment transfer based on equilibrium equations for a beam in bending, and web roller traction.

NOMENCLATURE

A .... Roller with web plane rotation (tram angle) and its entry span

B .... Roller and pre-entry span upstream from A

C .... Roller upstream from B

E .... Elastic modulus of web (N/m^2)

$f_A(M, y)$, $f_B(M, y)$ .... Functions describing the tension in spans A and B (N/m)

F .... Total shear on roller A (N)

I .... Bending inertia of web (m^4)

L_A, L_B .... Lengths of spans A and B (m)

M.................................... Total moment on roller B (N·m)

M_A, M_B .... Moments on roller B from spans A and B (N·m)

M_{lim} .... Maximum possible moment on roller B (N·m)

N.................................... Normal force of web on roller (N)

R .... Total shear on roller B (N)

R_A, R_B............................. Shears on roller B from spans A and B (N)

t...................................... Web caliper (m)

T_A, T_B .... Web tensions in spans A and B (N)

T_{high}, T_{low} .... High and low tensions for belt equation (N)

W.................................... Web width (m)

y_{high}, y_{low}.......................... Cross web location for onset of $\varepsilon_{high}(M,y)$ and $\varepsilon_{low}(M,y)$ (m)

β...................................... Wrap angle of web on roller B (rad)

δ_A, δ_B .... Lateral, cross web displacements at rollers A and B (m)

$\varepsilon_{high}(M, y)$ .... Function describing high tension side moment transfer (N/m)

$\varepsilon_{low}(M, y)$ .... Function describing low tension side moment transfer (N/m)

θ_A.................................... Tram angle and web slope at roller A (rad)

θ_B.................................... Web slope at roller B (rad)

μ...................................... Coefficient of friction between web and roller B

ν...................................... Poisson's ratio for web

τ_{cr}.................................... Critical shear stress to create wrinkles (N/m^2)

DERIVATION OF EQUATIONS

Wrinkles, that form on a roller rotated in the web plane (untrammed), have been studied for a single web span bounded by two rollers (1). This study, has found that wrinkles may also be generated on a roller upstream of the untrammed roller. If the web stress generated by the untrammed roller is high enough to overcome friction on the upstream roller, strain will transfer from the entry span into the pre-entry span of the untrammed roller. This strain transfer can generate stresses which exceed the critical value for the pre-entry span geometry (1). Wrinkles will form on the upstream roller even though it remains in tram. This paper defines the condition for transferring strain and generating wrinkles on the upstream roller.

Strain (or moment) transfer will be derived from the relationship between bending forces acting on the free spans of web adjacent to a roller and frictional forces acting on the web as it passes over the roller. The web span between two rollers has been treated as a simple cantilever beam (1),(2) which this paper extends to include two web spans and three rollers (fig. 1).

Beam Equations

Let roller *C*, (fig 1), act as a foundation for a cantilever beam, representing the web stretched between rollers *C*, *B* and *A*. Let the entry web span to roller *A* be span A, and the pre-entry span be span B. The untrammed roller *A* generates a shear *F*, which bends the web to the tram angle θ_A. Friction, between the web and the upstream roller *B*, provides an intermediate supporting shear *R*, and moment *M*. Equations can be written to express the equilibrium conditions for the web slopes, θ_A and θ_B, and displacements, δ_A and δ_B, at rollers *A* and *B*.

$$\theta_B = \frac{FL_B^2}{2EI} + \frac{FL_A L_B}{EI} - \frac{RL_B^2}{2EI} - \frac{ML_B}{EI} \qquad [1]$$

$$\delta_B = \frac{FL_B^3}{3EI} + \frac{FL_A L_B^2}{2EI} - \frac{RL_B^3}{3EI} - \frac{ML_B^2}{2EI} \qquad [2]$$

$$\theta_A = \frac{F(L_A + L_B)^2}{2EI} - \frac{RL_B^2}{2EI} - \frac{ML_B}{EI} \qquad [3]$$

$$\delta_A = \frac{F(L_A + L_B)^3}{3EI} - \frac{RL_B^3}{3EI} - \frac{RL_B^2 L_A}{2EI} - \frac{ML_B^2}{2EI} - \frac{ML_B L_A}{EI} \qquad [4]$$

In these equations, the web bending inertia I is given by:

$$I = \frac{tW^3}{12} \qquad [5]$$

Equations [1] through [4] may be solved to give expressions for the shears *F* and *R*, and for the moment *M*. Expanding the $(L_A+L_B)^2$ term in [3] and substituting into [1] provides a general expression for θ_A in terms of θ_B.

$$\theta_A = \theta_B + \frac{FL_A^2}{2EI} \qquad [6]$$

Expanding the $(L_A+L_B)^3$ term in [4], substituting into [2], and multiplying by $2/L_B$ leaves

$$\frac{2\delta_B}{L_B} = \frac{2FL_B^2}{3EI} + \frac{FL_A L_B}{EI} - \frac{2RL_B^2}{3EI} - \frac{ML_B}{EI}$$

from which [1] can be subtracted and rearranged to give an expression relating the shears R and F, with additional terms of web displacement δ_B, and angle θ_B, at roller B.

$$R = F - \frac{12EI\delta_B}{L_B^3} + \frac{6EI\theta_B}{L_B^2} \qquad [7]$$

A similar procedure can be used solve for the moment M on roller B, in terms of F, δ_B and θ_B.

$$M = FL_A + \frac{6EI\delta_B}{L_B^2} - \frac{4EI\theta_B}{L_B} \qquad [8]$$

Case I (no moment transfer on roller *B*)

Figure 2 shows the beam with roller A untrammed to some arbitrary angle θ_A, where all of the forces and displacements imposed on the web by the untrammed roller are reconciled in span A. The boundary conditions for this case are $\theta_B = \delta_B = 0$. These are substituted into [6], and solved for the shear force F in terms of the tram angle θ_A.:

$$F = \frac{2EI\theta_A}{L_A^2} \qquad [9]$$

Applying the boundary conditions to [7] and [8] gives R and M in terms of F.

$$R = F \qquad [10]$$

$$M = FL_A \qquad [11]$$

Case II (moment transfer on roller *B*)

In this case, roller B can no longer support the moment M, generated by the shear force F in [11], and the web will twist (Fig. 3a). As long as there is sufficient friction, the web will move laterally to regain normal entry (Fig. 3b). Even though there is strain transfer, and the web is displaced by δ_B, the boundary condition $\theta_B = 0$ still holds. It should be noted that [9] applies for Case II as it did for Case I.

Applying the boundary condition to [7] and [8] provides expressions for R and M on roller B.

$$R = F - \frac{12EI\delta_B}{L_B^3} \tag{12}$$

$$M = FL_A + \frac{6EI\delta_B}{L_B^2} \tag{13}$$

The shear, R, [12] and moment, M, [13] on roller B can be attributed to spans A and B separately by introducing the shears R_A and R_B, and moments M_A and M_B.

$$R = R_A + R_B \quad \text{where} \quad R_A = F \quad \text{and} \quad R_B = -\frac{12EI\delta_B}{L_B^3} \tag{14}$$

$$M = M_A - M_B \quad \text{where} \quad M_A = FL_A \quad \text{and} \quad M_B = -\frac{6EI\delta_B}{L_B^2} \tag{15}$$

From [10] and [11] it is clear that R_A and M_A are appropriate Case I solutions for span A. By analogy with [1], a beam equation can be written for span B in terms of R_B and M_B.

$$\theta_B = -\frac{R_B L_B^2}{2EI} + \frac{M_B L_B}{EI} \tag{16}$$

The Case II boundary condition, $\theta_B = 0$, gives R_B in terms of M_B.

$$R_B = \frac{2M_B}{L_B} \tag{17}$$

Substituting R_B and M_B from [14] and [15] into [17] will show that these are valid expressions for the span B shear and moment.

MOMENT TRANSFER ON A ROLLER

Consider the web wrapping roller B, (Fig. 4) with a wrap angle β, and tensions T_{high} and T_{low} in the spans adjacent to the roller. The maximum tension ratio, that can be obtained for a coefficient of friction μ, is given by the belt equation.

$$\frac{T_{high}}{T_{low}} \le e^{\mu\beta} \tag{18}$$

To simplify the derivation, μ is taken as a representative value and not dependent on local tension.

Distributed Web Stress on the Roller

For the case of no moment transfer in Figure. 4(a), functions $f_A(y, M_0)$ and $f_B(y, M_0)$ may be constructed to describe the tension at any cross-web location y, with an imposed moment, M_0. The following integrals must hold for these functions, where T_A and T_B are the average tensions for spans A and B.

$$T_A = \int_{\frac{-W}{2}}^{\frac{W}{2}} f_A(y, M_A) \cdot dy \quad \text{and} \quad M_A = \int_{\frac{-W}{2}}^{\frac{W}{2}} f_A(y, M_A) y \cdot dy \qquad [19]$$

$$T_B = \int_{\frac{-W}{2}}^{\frac{W}{2}} f_B(y, M_A) \cdot dy \quad \text{and} \quad M_B = 0 = \int_{\frac{-W}{2}}^{\frac{W}{2}} f_B(y, M_A) y \cdot dy \qquad [20]$$

The following expressions for $f_A(y, M_0)$ and $f_B(y, M_0)$ satisfy [19] and [20].

$$f_A(y, M_0) = \frac{T_A}{W} + \frac{12yM_0}{W^3} \quad \text{and} \quad f_B(y, M_0) = \frac{T_B}{W} \qquad [21]$$

The onset of moment transfer occurs when, at any position y, the tension ratio exceeds the belt equation [18]. For this derivation, the web is treated as individual ribbons which can transfer strain independent of one another. Moment transfer can occur either on the high or low tension edge in span A which leads to the following two solutions for y.

$$\frac{f_A(y_{high}, M_0)}{f_B(y_{high}, M_0)} = e^{\mu\beta} \quad \text{yielding} \quad y_{high} = \frac{W^2}{12M_0}\left(T_B \cdot e^{\mu\beta} - T_A\right) \qquad [22]$$

$$\frac{f_B(y_{low}, M_0)}{f_A(y_{low}, M_0)} = e^{\mu\beta} \quad \text{yielding} \quad y_{low} = \frac{W^2}{12M_0}\left(T_B \cdot e^{-\mu\beta} - T_A\right) \qquad [23]$$

Limits are imposed by [18] such that $\frac{-W}{2} \le y_{low} \le 0$ and $0 \le y_{high} \le \frac{W}{2}$.

Following the onset of moment transfer in Figure 4(b), tension will decrease on the high tension side of the roller and increase on the low tension side. Let the function
$\varepsilon_{high}(y, M_0)$ represent the strain transfer across the roller in the region y_{high} to $+W/2$. The following form of the belt equation [18] holds in this region.

$$\frac{f_A(y, M_0) - \varepsilon_{high}(y, M_0)}{f_B(y, M_0) + \varepsilon_{high}(y, M_0)} = e^{\mu\beta}$$

$$\varepsilon_{high}(y, M_0) = \frac{f_A(y, M_0) - f_B(y, M_0) \cdot e^{\mu\beta}}{e^{\mu\beta} + 1} \qquad [24]$$

Moment transfer in this region can be integrated from $\varepsilon_{high}(y, M_0)$.

$$M_{high} = \int_{y_{high}}^{\frac{W}{2}} \varepsilon_{high}(y, M_0) y \cdot dy = \frac{\frac{W}{8}\left(T_A - T_B \cdot e^{\mu\beta}\right) + M_0\left(\frac{1}{2} + 2\left(\frac{y_{high}}{W}\right)^3\right)}{e^{\mu\beta} + 1} \qquad [25]$$

A similar function $\varepsilon_{low}(y, M_0)$ can be constructed for $-W/2$ to y_{low} with a sign change indicating that strain transfer is from span B to span A in this region.

$$M_{low} = -\int_{\frac{-W}{2}}^{y_{low}} \varepsilon_{low}(y, M_0) y \cdot dy = \frac{\frac{W}{8}\left(T_B - T_A \cdot e^{\mu\beta}\right) + M_0 e^{\mu\beta}\left(\frac{1}{2} - 2\left(\frac{y_{low}}{W}\right)^3\right)}{e^{\mu\beta} + 1} \qquad [26]$$

The moment remaining in span A is reduced by the sum of [25] and [26] which is the moment transferred into span B.

$$M_A = M_0 - (M_{high} + M_{low}) \qquad M_B = M_{high} + M_{low} \qquad [27]$$

Since y_{high}, y_{low}, M_{high} and M_{low} depend on the imposed moment M_0, it is necessary to iterate [22] through [27] to find an M_0 so that the expression for M_A in [27] matches the one in [15].

There is a limit to the moment that can be obtained by twisting the web on roller B. This can be seen by imposing a very large moment M_0, in [22] and [23] in which case y_{low} and y_{high} become zero. Using [15], [27], [25] and [26] gives a limiting moment, M_{lim}.

$$M_{lim} = M_A - M_B = M_0 - 2(M_{high} + M_{low})$$

$$M_{lim} = M_0 - \frac{\frac{W}{4}\left(T_A - T_B \cdot e^{\mu\beta}\right) + M_0}{e^{\mu\beta} + 1} - \frac{\frac{W}{4}\left(T_B - T_A \cdot e^{\mu\beta}\right) + M_0 e^{\mu\beta}}{e^{\mu\beta} + 1}$$

$$M_{lim} = \frac{W(T_A + T_B)}{4} \cdot \frac{\left(e^{\mu\beta} - 1\right)}{\left(e^{\mu\beta} + 1\right)} \qquad [28]$$

EXPERIMENTAL RESULTS

Figure 5 shows a schematic diagram of the experimental setup used to verify the theory. The tram angle θ_A, was adjusted by mounting roller A on adjustable slides such that the rotation was always about the center of the web, and in the plane of span A. The forces F, R, A_O, A_M,, B_O, and B_M were measured by force transducers mounted on the roller shafts. The shear forces, F and R, were measured directly, but the moments, M_A and M_B had to be computed from A_O, A_M, B_O, and B_M. Web displacements, δ_A and δ_B, were measured by edge sensors placed in span A, close to rollers A and B. By keeping the wrap angle on rollers A and B at 90°, the tension distribution in the input and output spans did not interact with each other, and the transducers measured forces in a single span.

The theoretical computations were based entirely on θ_A, span dimensions, and web physical properties. One additional variable, the web-roller friction coefficient, was needed to make the theoretical computations. This was done as a special test measuring the tension drop across roller B as a function of speed and tension(5). The computed forces, moments, and displacements were done according to, and in the order shown in Table 1.

All force, moment and displacement data are plotted against roller A tram angle, θ_A. The web used for verification of the theory was a 0.037 mm thick, 254 mm wide, polyester film with a modulus of 4.137 GPa. The friction coefficient was measured from tension velocity tests. A constant coefficient of 0.25 was used for the 133 N tension at a velocity of 0.254 m/s.

Additional wrinkle experiments were run using the same web and velocity, but different tensions. These data were used to determine the wrinkle failure criteria as a function of the tensile stress in the web.

Displacement Verification

Figure 6 compares the experimental displacements with those computed from theory. For Case I, δ_B was zero. At around 0.0028 radians tram angle, θ_A,, there was sufficient moment in span A to transfer into span B. The negative web displacement, δ_B, returned the web to normal entry on roller B in agreement with theoretical prediction. Figure 3(b) shows the final web shape for Case II. The poorer fit at larger tram angles indicates an overestimation of R from the model.

Shear Force Verification

Figure 7 compares the experimental shear forces on rollers A and B with theory. According to [10], F and R are equal throughout Case I. In Case II, R becomes greater than F for a negative web displacement, δ_B as predicted by [12]. The transition from Case I to Case II was about 0.0028 radians tram angle which agrees with the transition observed for the web displacement above. In the derivation of the beam equations [1] through [4], all of the deflecting force on the beam was attributed to the shear force, F, on roller A. As shown in Figures 2 and 3, a small component of the web tension, T, produces a deflection not accounted for in the derivation. The experimental shear was found to be lower than that predicted by theory. A better fit between experimental and theory can be obtained by applying a tension compensation factor to the shear forces.(3) The theoretical shear force, F, in figure 7 was corrected for tension by multiplying by a factor of $(1\text{-}TL_A^2/2EI)$.

Moment Verification

Figure 8 compares the experimental moments measured from the load cells with those derived from theory. For Case I, M_A increased linearly with θ_A as predicted by [9] and [11], while M_B was zero. After the transition to Case II, at θ_A of about 0.0025 radians, M_B started to increase according to [27]. The Case II transition agreed favorably with both displacement and shear data. The lack of fit may come from either the experimental transducers or the theoretical model. The transducers, made especially for the experiment, were found to be subject to hysteresis and interactions with one another. A special calibration was made to minimize this interdependency, and the final accuracy for measuring force was better than 15%.

The model for computing moment transfer was based on a monoaxial stress field. The web width was divided into ribbons where the belt equation strain transfer for any ribbon acted independent of its neighbor. In reality, the web stresses were in a biaxial stress field over the roller, where interactions would retard moment transfer. The model predicted more moment than was measured, indicating some error in the simplified approach.

Wrinkle Verification

Figure 9 shows data for wrinkles in span B. During the tests, there were no wrinkles observed in span A. The critical shear τ_{cr}, for wrinkling on an untrammed roller, has been derived (1)(4) and verified experimentally. The upper curve in Figure 9, labeled "Wrinkle failure from tension and compression" and computed using this theory, shows a poor fit to the experimental data.

In reference (6), Timoshenko develops buckling criteria for thin plates in bending as well as tension and compression. The tension/compression models have been successfully applied to wrinkle creation.(1)(4) The stress field in span B includes a bending moment, M_b , that does not exist in span A. The curve marked "Wrinkle failure from bending only" uses a simplified expression for buckling from pure bending as an alternate wrinkle criteria which predicts failure well below the experimental data. The tension/compression and pure bending failure curves bracket the experimental data. A good theoretical prediction may be possible from combining tension stiffening with the pure bending failure.

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ILLUSTRATIONS

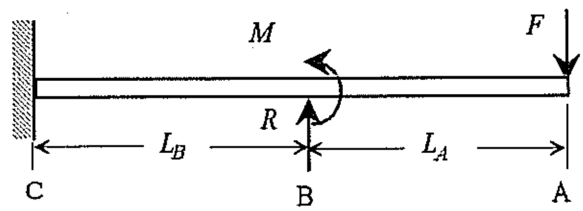

Figure 1.- Two span, three roller web line represented as a beam.

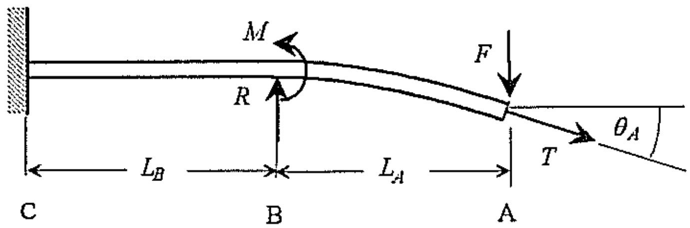

Figure 2.- Case I bending in Span A only.

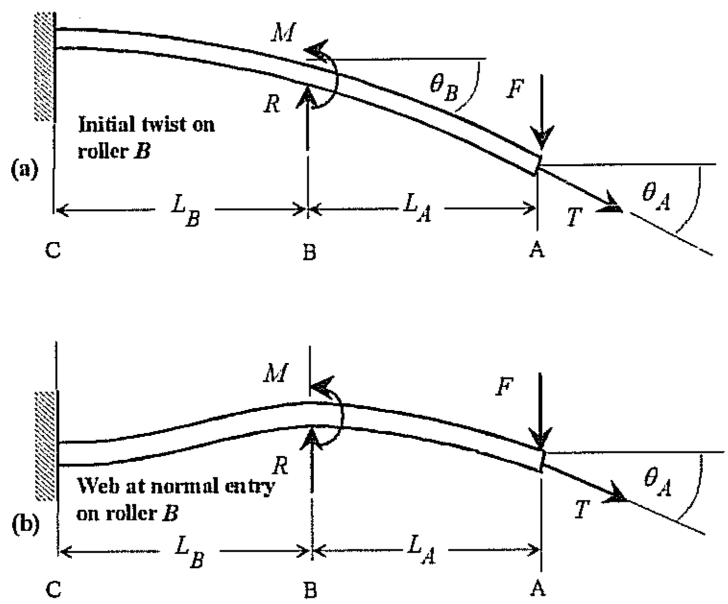

Figure 3.- Case II bending with moment transfer.

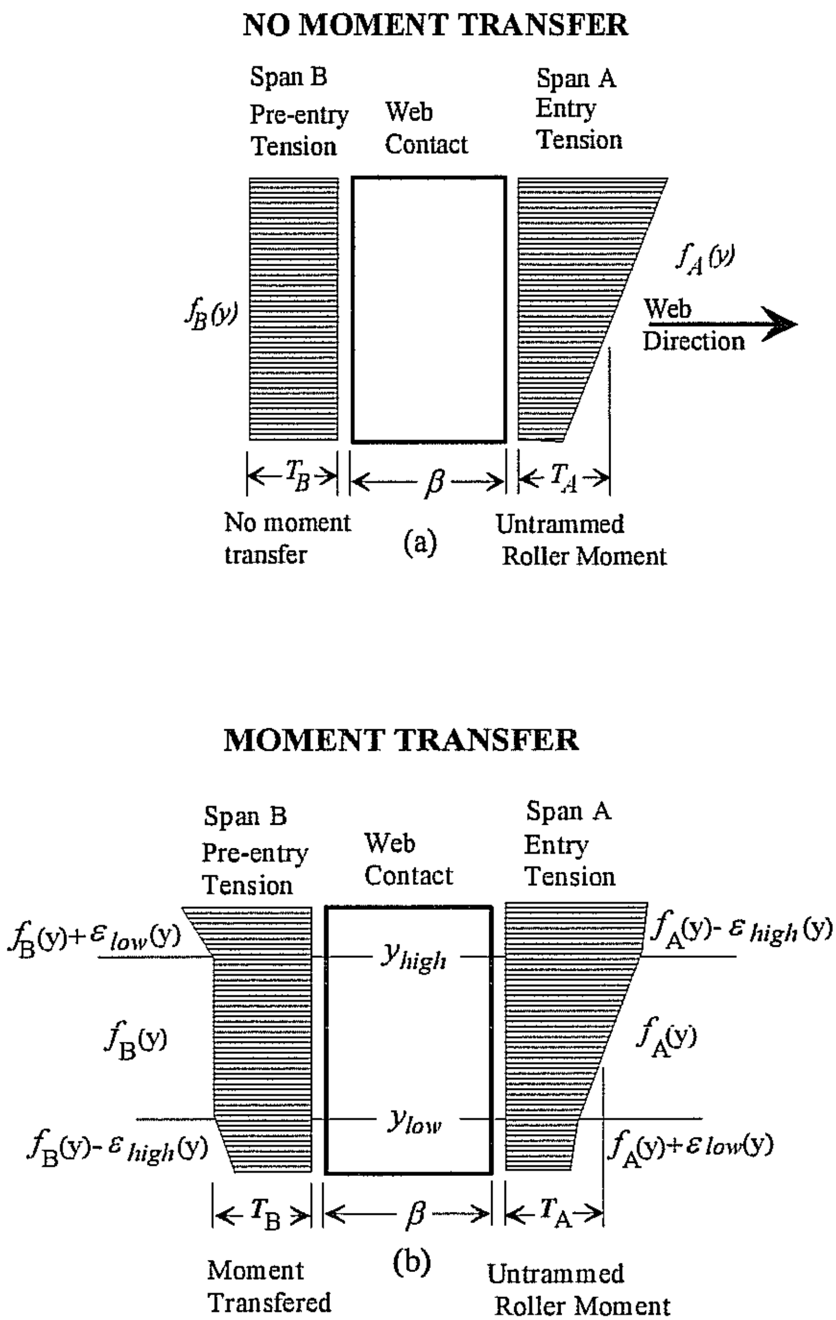

Figure 4.-Distributed tensions on roller *B* from web spans *A* and *B*.

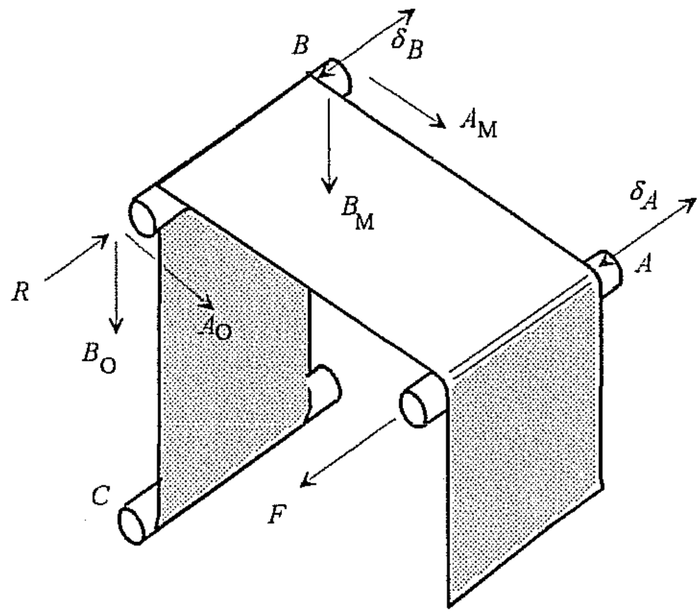

Figure 5.- Experimental force and displacement measurements.

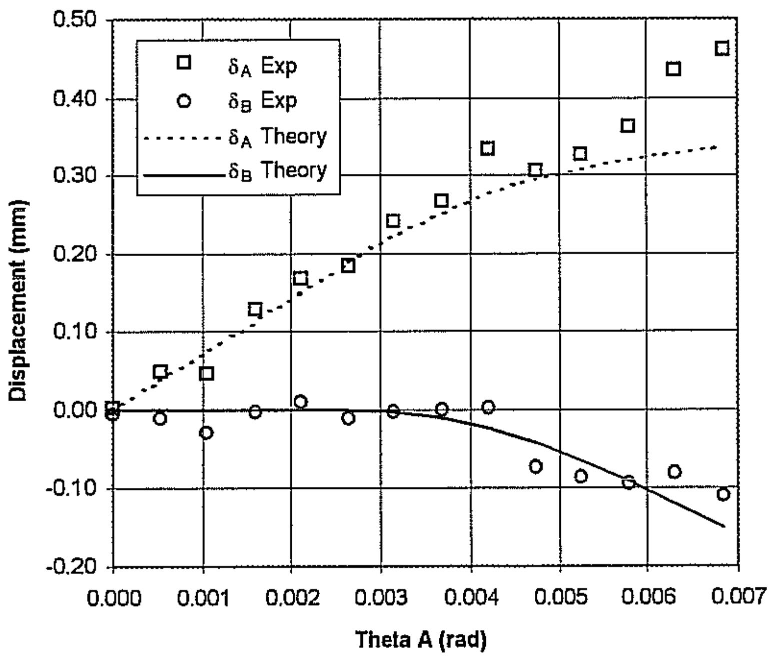

Figure 6.- Experimental and theoretical web displacement as a function of tram angle.

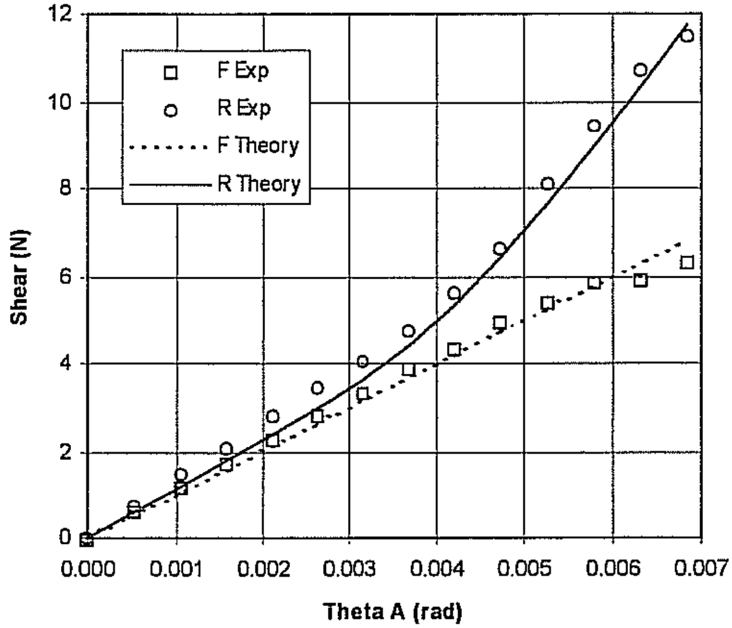

Figure 7.- Experimental and theoretical shear force on rollers A and B as a function of tram angle. The theoretical force, F, was corrected for web tension.

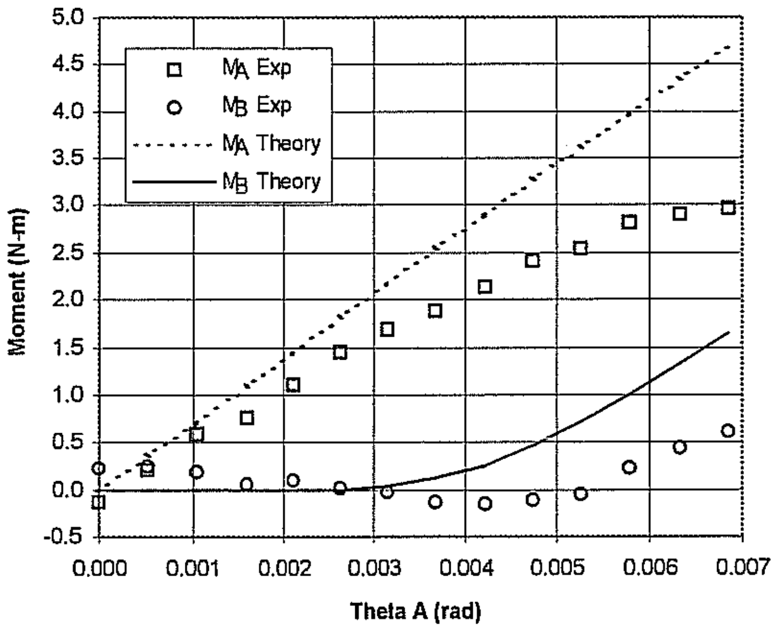

Figure 8.- Experimental and theoretical moments at roller B as a function of tram angle.

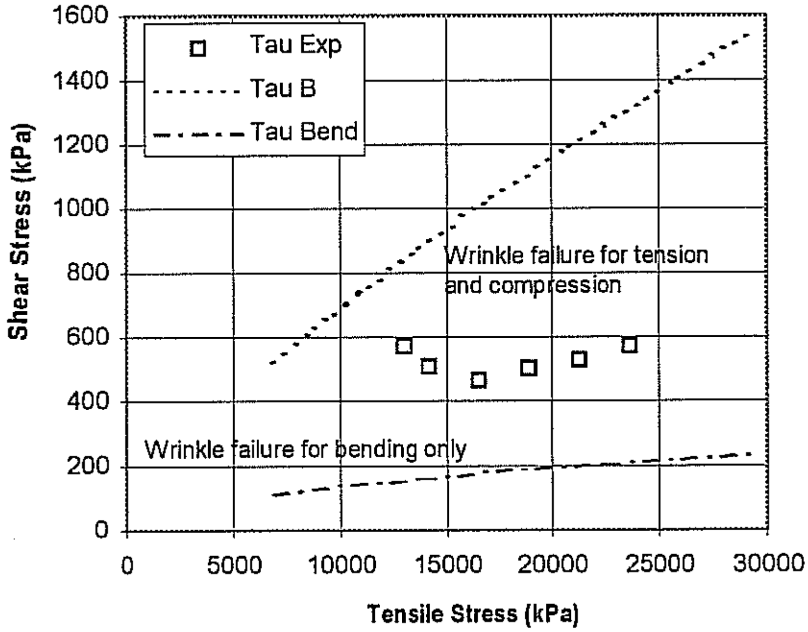

Figure 9.- Shear stress to create wrinkles in span *B* as a function of web tensile stress.

VARIABLE	DETERMINATION
F	Direct from [9]
M_A	Direct from [15]
M_B	Iterating [22] through [27]
M	From [15], M_A and M_B
R_B	From [17] and M_B
R, R_A	From [14] and R_B
δ_B	From [2], F, R, and M
δ_A	From [4], F, R, and M

Table 1.- Computation process for finding theoretical variables.

Dobbs, J.N.; Kedl, D.M.
Wrinkle Dependency on Web Roller Slip
6/21/95 Session 5 9:00 - 9:25 a.m.

Question - In your beam equations you neglect the tension?

Answer - We're not putting the tension in the beam equation as such. The deflections are very small. In the data we mention we collect the shear force F for tension component. But other than that we neglect that. Obviously, John on his Ph.D. thesis did a far more rigorous attack on that problem. But we've neglected it in the actual beam equations and we only correct our shear force F for tension component.

Answer - Its a simplification.

Thank you.