MODELING WEB TRACTION ON ROLLERS

by

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ABSTRACT

When web tension changes between entry and exit on a roller, the tension change occurs in a slip zone in the last part of the wrap, whose size is given by the well-known Capstan or Belt equation. A method for modelling moving film over rollers has been developed using the Finite Element analysis program ABAQUS. This has been used to reproduce the analytical results for the angular extent of the slip zone, and the relative movement on the roller surface of the web as it extends or contracts.

Normally, the web has a non-zero Poisson's Ratio, and therefore a tension change is accompanied by changes in stress and strain in the transverse (width) direction. As a consequence, the relative motion of the web has a transverse component in the slip zone. ABAQUS has been used to model this, as an analytic solution is no longer possible. The slip zone is larger at the edges of the web, and transverse movement changes continuously from zero in the centre of the web to a value similar to that in the machine direction at the edge. The effect of varying web modulus, entry and exit tensions, and friction coefficient has been studied.

More commonly, machines use sequences of rollers and it is desirable to know how web tension varies from one span to the next. If the roller surface speeds differ by only a fraction of a percent, large tension differences can be set up resulting in slip on some of the rollers. A scheme for obtaining a self-consistent solution for the tensions has been developed. An elementary system of rollers driven at set speeds with controlled unwind and rewind tensions has been modelled analytically at steady state. Common drive strategies increase roller speed through the machine, but the simulation shows this can lead to slippage on many rollers and tensions greatly exceeding the unwind and rewind values midway through the roller sequence.

NOMENCLATURE

e	Base of natural logarithms, 2.718
E	Young's modulus of web
FEA	Finite Element Analysis
f	Maximum tension ratio, calculated using equation 6
f_i	Maximum tension ratio across the ith roller
i	Index denoting roll number: 1,2,3 etc.
MD	Machine Direction
R	Roller radius
T	Web tension (per unit width)
T_{entry}	Tension at the entry to a roller
T_{exit}	Tension at the exit from a roller
T_i	Tension at the entry to the ith roller (and the exit from roller i-1)
TD	Transverse direction
t	Web thickness
v	Roller surface velocity
v_i	Surface velocity of the ith roller
x	Relative displacement of points on the film and roller after contact
Δ	Tension change across a roller without total slip (equation 5)
μ	Effective coefficient of friction
μ_i	Coefficient of friction on the ith roller
σ	Poisson's ratio of the web
θ	Angular co-ordinate on roller with origin at the web entry
ϕ	Wrap angle on roller
ϕ_i	Wrap angle on the ith roller

INTRODUCTION

In many manufacturing processes involving webs, the loading applied and the interaction with the contacting rollers are of interest to the designer and the operator. Incorrect tensions in the web can lead to production problems. Driving the rollers in a web process line is used as a way of adjusting the web tension. This is sometimes done using tension sensing and closed loop control, but more often it is achieved purposely or inadvertently by controlling roller surface speed through the angular velocity and diameter, resulting in open loop control of tension. If the rollers apply sufficient traction to the web, the speed differences between rollers produce strain changes and hence tension variations. These are simply calculated for elastic webs. (The cases of significant inelasticity or viscoelasticity will not be considered further).

However, the traction available is limited, especially at high speeds owing to aerodynamic lubrication effects. The tension change possible across a single roller is limited by the effective coefficient of friction and the angle of wrap. A simple analysis of this results in the well-known "Capstan" or "Belt" formula, which is briefly reviewed below.

The Capstan Formula assumes that all relative movement of web on the roller occurs in the machine direction (MD). However, real webs have width and a non-zero Poisson's Ratio, so relative motion will have a component in the transverse direction (TD) also. Part of the traction will act to oppose this component of the motion, leaving less available to support an MD tension change. This is expected to reduce the maximum tension change which can be achieved across a roller. This problem can only be solved numerically: a Finite Element solution method is outlined later.

Many web processes involve transport over a sequence of rollers. In some cases, the rollers may be driven at a given speed ratio, either on purpose or as a result of diameter differences and the same angular velocity. For example, slitter-rewinders used for polyester film use a controlled tension unwind, several driven rollers whose surface speeds are identical or progressively increasing, followed by a torque-controlled rewind. It would be desirable to calculate the web tension through the machine as a result of a specified set of roller speeds and given offwind and rewind tensions. This is straightforward only if the web is elastic and the limiting tension ratio is not reached on any of the rollers. The final section of the paper describes an analytic method for tackling this problem when slip is more widespread. An idealised process is modelled as an example. As yet, no validation data have been obtained but an experimental test of the theory is clearly desirable.

REVIEW OF CAPSTAN FORMULA

When a web passes over a roller, the behaviour can be approximated to that of a narrow belt. The web approaches the roller at known values of speed and tension. In steady state, the difference between exit and entry tensions T_{exit} and T_{entry} balances the braking or driving torque (including bearing friction) applied to the roller. However, friction limits the maximum tension ratio which can be realised. For an elastic or rigid web passing over a roller with angle of wrap ϕ radians and obeying Coulomb's law of friction with coefficient of friction μ, an analytic solution is possible for the maximum tension ratio (1). Consideration of a small element $d\theta$ of the wrap angle in mechanical equilibrium with change in tension T balanced by frictional force leads to limits on the rate of tension change with angle:

$$-\mu T \leq \frac{dT}{d\theta} \leq \mu T \tag{1}$$

Integrating the limiting cases over the whole of the wrap gives:

$$e^{-\mu\phi} \leq \frac{T_{exit}}{T_{entry}} \leq e^{\mu\phi} \tag{2}$$

In the limiting conditions, the web is slipping over the whole of the wrap angle, and hence the roller speed is indeterminate. In practice, braking or driving torque will be dependent on roller speed , so a steady state will be reached with the roller either stationary or at some speed lower than the web speed in the case of braking, or at the maximum drive speed or a speed between this and the web speed in the case of driving. Of course, the web to roller coefficient of friction may itself be speed-dependent (2).

For an imposed tension ratio within the limits of equation 2, the web moves at the roller speed over the first part of the angle of wrap (the "stick zone") with constant tension, and then changes smoothly in tension over the last part of the angle of wrap (the "slip zone") until it reaches the exit tension at the point of leaving the roller. It can be argued logically that this arrangement must exist in the steady state. Any tension profile in the web between the limits of equation 1 tends to be transferred along the roller surface at the roller speed. Hence motion alone tends to result in the entry tension propagating through the wrap angle. However, this results in a discontinuity at the roller exit point. This is resolved by the existence of the slip zone leading up to the roll exit, where one of the two limiting rates of tension change given in equation 1 applies. Note that the term "stick" does not imply any adhesion, merely the absence of relative tangential motion between web and roller.

Within the stick zone, the belt tension remains constant; beyond this, the tension at any point can be calculated as:

$$T = T_{entry} e^{\mu(\theta - \theta_{stick})} \tag{3}$$

Here the case of braking is considered, and the stick zone extends for an angle θ_{stick}.

In the slip zone, the tension and hence the web strain are changing, resulting in a displacement of each point on the web relative to the point on the roller where it makes contact on entry. A simple analysis for the case of braking gives this displacement as:

$$x(\theta) = \frac{T_{entry}(1-\sigma^2)R}{Et}\left(e^{\mu(\theta-\theta_{stick})} - 1 - \mu(\theta - \theta_{stick})\right) \tag{4}$$

In equation 4, R is the roller radius, t the web thickness, E the Young's modulus of the web and σ its Poisson's Ratio. The $(1-\sigma^2)$ factor appears because constant width is assumed.

When the limiting cases of constant width or zero transverse stress in the web cannot be assumed, the problem cannot be solved analytically. Hence, a numerical approach based on a commercially available Finite Element Analysis (FEA) program has been used to deal with the added complexity.

FINITE ELEMENT MODEL

The FEA model for the problem shown in figure 1 has been developed using ABAQUS/Standard, version 5.3, for all pre-processing and analysis. First order, finite thickness shell elements, S4RF, have been used to represent the web, a rigid body represents the roller while IRS4 contact elements are used to describe the contact condition. In addition, a DASHPOT1 element connects the roller to ground introducing a difference between entry and exit tensions. The material of the web is assumed to be isotropic, linear elastic and classical Coulomb linear friction behaviour is implemented.

The intention of the model is to calculate the steady state condition of the web at which stage the stress field in the web and the contact condition will have stabilised. To achieve this, the STATIC STEP procedure is used and a length of web is run over the roller to allow steady state conditions to be established. However, an analysis step is first used to set up the problem. In this step, the web, initially flat, is laid on to the roller and a uniform tension is applied to the web at the trailing end. This approach simplifies the mesh generation and allows for a quicker mathematical solution. The set up step is carried out with no friction so that a uniformly tensioned web is in contact with the roller at the start of the second step. Now the friction coefficient is altered to the desired value and the leading edge is moved using boundary conditions at the desired line speed while the tensile force remains active on the 'trailing' end. A representation of the steps applied during the analysis is shown in figure 2.

To verify the numerical model, a simulation of the constant width case was carried out by restricting the width of the web to one element and constraining both edges. When the behaviour in the TD was of interest, a wider mesh was used. One edge was constrained to represent a symmetry condition along the web centre line, while the second edge was unconstrained.

Model Verification

The constant width model has been verified against the analytical expressions (equations 3 and 4) using the following parameters:

$$
\begin{gathered}
R = 100 \text{ mm} \\
t = 0.1 \text{ mm} \\
E = 2000 \text{ MPa} \\
\nu = 0.45 \\
\mu = 0.3 \\
T_{entry} = 12.0 \text{ N/mm} \\
T_{exit} = 14.38 \text{ N/mm} \\
\phi \ = 90 \text{ deg.}
\end{gathered}
$$

The plot of web tension as a function of angular position on the roller is shown in figure 3 including analytical results (equation 3) and numerical ones; agreement is excellent. In figure 4, the amount of relative slip is plotted against angular position. Once more, agreement between the analytical solution (equation 4) and the numerical model is excellent.

Three Dimensional Model

The full three dimensional model is shown in figure 5. The web width is 1000 mm of which only one half is represented, taking advantage of symmetry; other parameters remain the same as those used in the previous section. The main interest in this problem is the influence of the transverse forces and movement of the web on the MD conditions.

Contours of the MD tension in the web which has exited the roller are shown in figure 6. It is clear that the free edge disrupts the uniformity of the MD tension across the width of the web as it exits the roller, resulting in higher tension at the edge.

The extent of the slip zone, where the web moves relative to the roller, is indicated by a changing web tension. Figure 5 shows that the slip zone is larger at the edge of the web than in the centre. At the centre, it is close to the value predicted using equation 3. Furthermore, slip takes place in the TD to an extent similar, near the edge, to that in the MD. At the centre of the web, transverse slip is nil. The slip paths at positions across the web are illustrated in figure 7. The amount of MD slip is similar to that predicted by equation 4 at the centre, but over twice as large at the edge.

Benefit from the model is increased if its use can indicate ways of reducing the magnitude of slip. To this end, a parametric study was carried out where each of four parameters was varied in turn with the other three held constant. The parameters were: the web elastic modulus, the web to roller coefficient of friction, the entry tension, and the entry to exit tension difference. The effect of the changes on the amount of slip at the edge of the web in the MD and TD, as well as the stick zone angle there, is recorded in table 1. Because each parameter was varied within an arbitrarily selected range, it is not possible to assess the relative effect of the variation of the parameters. However, the qualitative influence of each parameter is indicated. For the purpose of reducing slip, high values of web stiffness, coefficient of friction and entry tension are desirable; a low difference in tension across the roller is also beneficial. The results also show that the slip in the TD and MD changes by a similar amount when the problem parameters are varied.

Examination of the development of the stress fields in the web throughout the solution progress indicated that, within the time allowed, a steady state solution had not yet been reached. One approach to overcome this difficulty would be to run more web through the model until the distance of the leading end from the roller became much larger than the web width. Another approach would be to add a second roller which would act to constrain the width of the web at a fixed position after it has left the first roller, as shown in figure 8.

ROLLER SEQUENCES

In order to calculate the web tensions through a sequence of rollers when significant slip occurs, the web speed and the extent of slip on each roller must be determined. One possible approach is to use the numerical methods mentioned in the previous section, extending the model of figure 8 to more complex situations. However, considerable computer power is required to run models containing enough elements for a sufficient time to enable a steady state to be reached. Nevertheless, this approach has the merit of producing an accurate solution.

As an alternative, an analytic approach may be adopted where the MD tensions only are considered. If complete slip occurs on any roller, this will be slightly inaccurate as the tension ratio across that roller will be smaller than the limits given by equation 2, since part of the traction available must resist TD rather than MD movement. In addition, the MD tension depends not only on the MD strain but also on the state of TD strain. Both of these effects are assumed negligible in the following discussion.

An idealised steady state web process will be considered in the following subsections: the entry and exit tensions, and the speeds of one or more rolls in between are fixed.

One driven roller

The geometry is shown in figure 1: the only difference is that the roller speed, not the web speed, is fixed. The permissible tension ratio achievable is given by equation 2. The incoming web speed matches the roller speed. Tension ratios outside the limits are not possible in steady-state operation.

Two driven rollers

This configuration, shown in figure 9, will be discussed in some detail as the same principles govern the operation of many-roller systems. The ith roller has speed v_i, entry tension T_i and exit tension T_{i+1}. The parameter Δ is defined by:

$$\Delta = Et\frac{v_2 - v_1}{v_1} \tag{5}$$

and is the tension difference T_2-T_1 which is obtained across roller 1 provided that total slip does not occur on either roller. Strains are assumed to be small. Each roller can sustain a maximum tension ratio f_i given by:

$$f_i = e^{\mu_i \phi_i} \tag{6}$$

The first hypothesis is that total slip occurs on neither roller and so the incoming web speed is equal to v_1 and the tension T_2 equal to $T_1+\Delta$. If the two tension ratios satisfy equation 2, then this hypothesis is correct.

For example, if $f_1=f_2=2$ and $T_1=T_2=100$ N/m, a value of Δ of 50 N/m results in acceptable tension ratios. However, if Δ is equal to 200 N/m, neither roller can sustain the tension ratio of 3 and T_2 is limited to 200 N/m.

If, in the last case, the value of f_1 is increased to 4, the tension ratio of 3 is only possible on roller 1. The hypothesis is again invalid as this tension ratio is not sustainable across the second roller. Total slip must therefore be occurring across roller 2, setting T_2 equal to $T_3 f_2$, 200 N/m. This revised tension still gives an acceptable tension ratio across roller 1, so the final solution has been reached. The incoming web speed is equal to the first roller speed.

However, if the values of f_i are interchanged so that $f_1 = 2$ and $f_2 = 4$, the tension ratio cannot be sustained over roller 1. As a result, total slip must be occurring, and therefore T_2 is equal to $T_1 f_1$, 200 N/m. Roller 2 still has a sustainable tension ratio, and now the stick zone is located in the first part of the wrap on roller 2. Hence the web speed coming onto roller 2 under a tension of 200 N/m matches the speed of roller 2.

General Principles

These results are general, and can be set in the form of an algorithm to apply to a system containing many rollers.

1. Assume the incoming web speed matches the first roller speed.

2. Calculate the entry tensions assuming total slip does not occur on any roller.

3. Adjust tensions so that equation 2 is satisfied on all rollers. If this is possible, then the correct solution has been reached.

4. If not, total slip must be occurring on one or more rollers at the start of the sequence. Adjust the incoming web speed so that roller and web speed are matched on the earliest roll in the sequence which results in equation 2 being satisfied on all rollers.

A solution will not be reached if the ratio of first roller entry tension to last roller exit tension is too large (greater than the product of the f_i's) or too small (less than the reciprocal of the product).

Example: 5 driven rollers

The above procedure has been used to analyse a simple 5-roller configuration shown in figure 10. Each roller is assumed to have $f=2$, and the offwind and rewind tensions both equal to 100 N/m. The values of Δ are calculated from the roll speed increments and a web modulus of 5 GPa and thickness of 20 microns. A speed differential of 0.1% then corresponds to a Δ of 100 N/m.

The tension profiles calculated for several constant speed increments are shown in figure 11. For a small increment of 0.02%, the tension increase follows the speed increase and there is a stick zone on each roller. As the increment goes up, slip starts to occur at the end of the sequence. At a speed increment of 0.05%, slip occurs on roller 5, and at 0.1%, on rollers 4 and 5 with roller 1 on the point of slipping. At an increment of 0.2%, slip also occurs on rollers 1 and 2, so roller 3 is the only one with a stick zone. In the last 2 cases, the tension reaches a value of 400 N/m, much greater than the start and end values.

If the objective of driving rollers with a constant speed increment is to provide a similar incremental variation of tension, this is only achieved if the speed increments are small, which requires tight tolerances on diameter.

CONCLUSIONS

The ABAQUS FEA software has been used to model the movement of a web on a roller taking account of the interaction of the two in all directions. Results obtained from a restricted model have been verified against an analytical solution. Changes in web conditions over time have been considered and the model has been used to carry out a parametric study which indicates ways of reducing the relative slip between the web and the roller surface.

A systematic procedure for estimating web speeds, tensions and the occurrence of slip in a sequence of rollers, neglecting the width direction, has been developed. Idealised systems demonstrate that the web can frequently be in the total slip condition, and the tension rise midway through a roller sequence can be substantial.

REFERENCES

1. Johnson, K. A., "Contact Mechanics", Cambridge University Press, 1985.

2. Ducotey, K. S., " Traction between Webs and Rollers in Web Handling Applications", Ph.D. dissertation , Oklahoma State University, 1993.

FIGURES

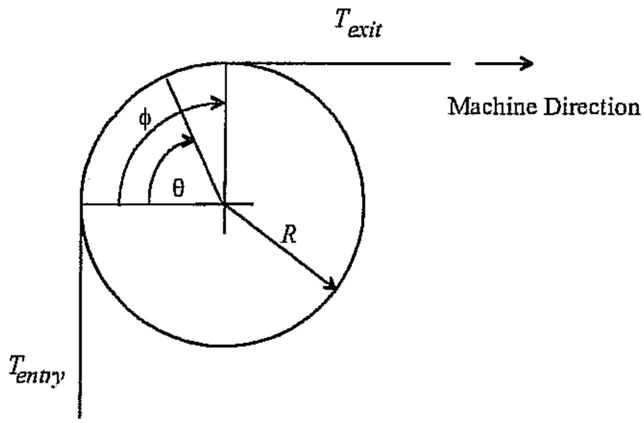

Fig. 1 Web on roller geometry

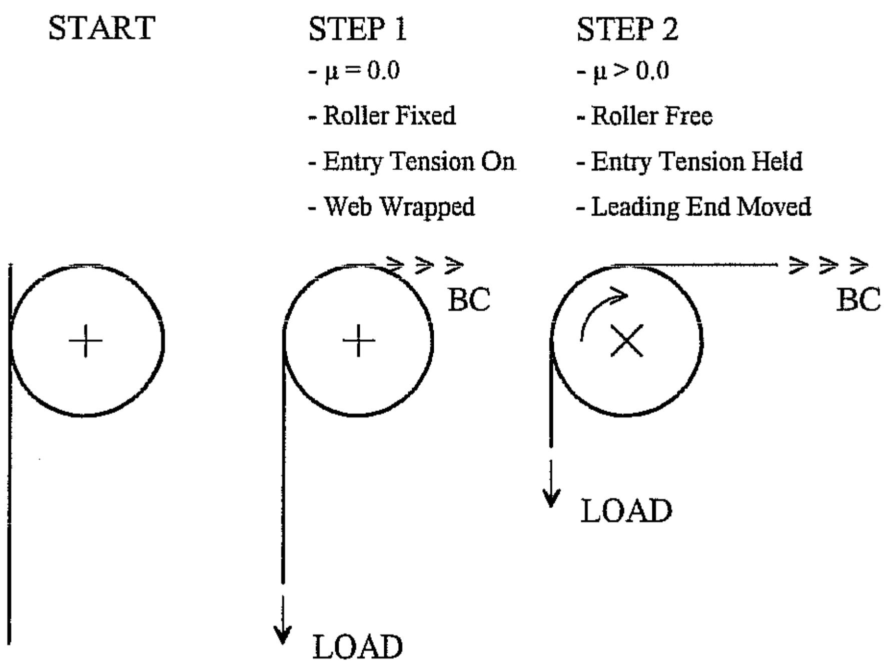

Fig. 2 Numerical simulation steps.

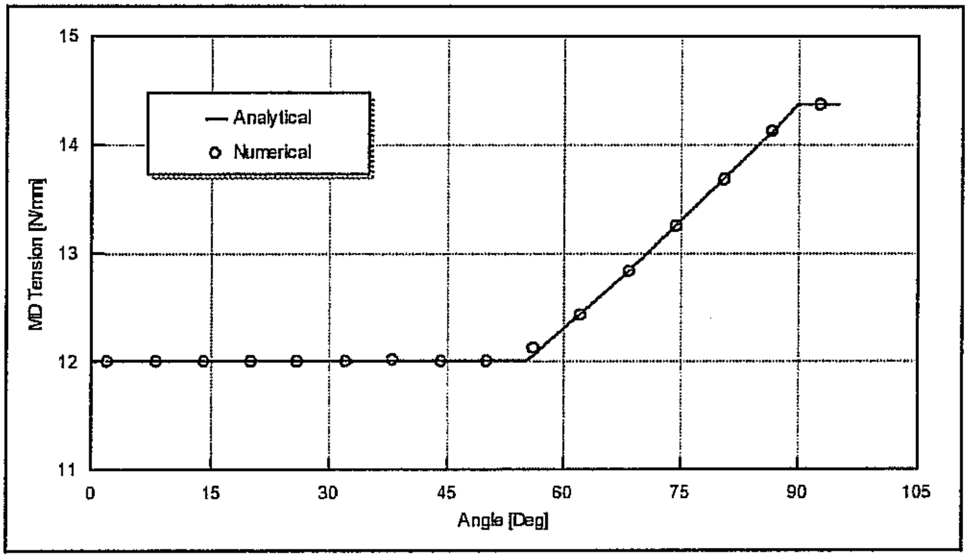

Fig. 3 Analytical and numerical calculations of MD tension vs angle compared.

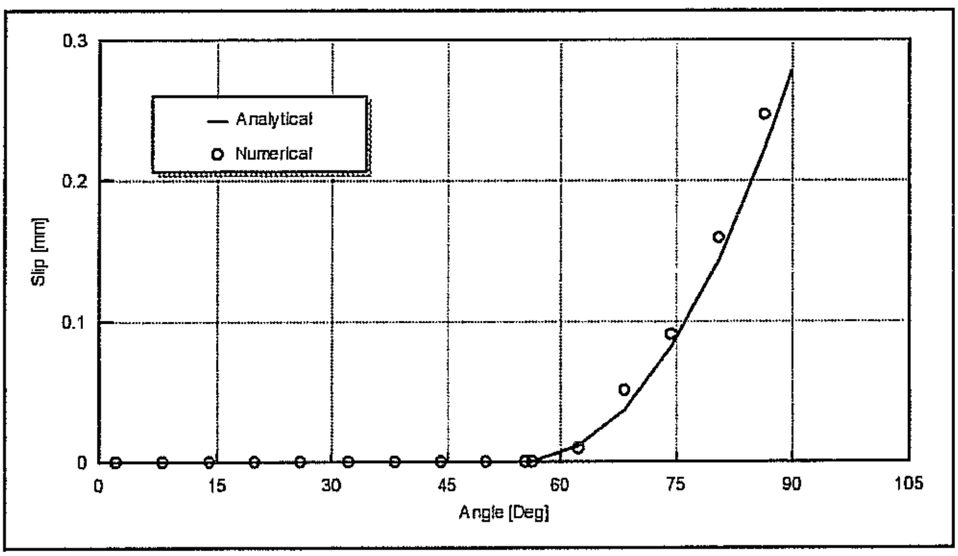

Fig. 4 Analytical and numerical calculations of slip vs angle compared.

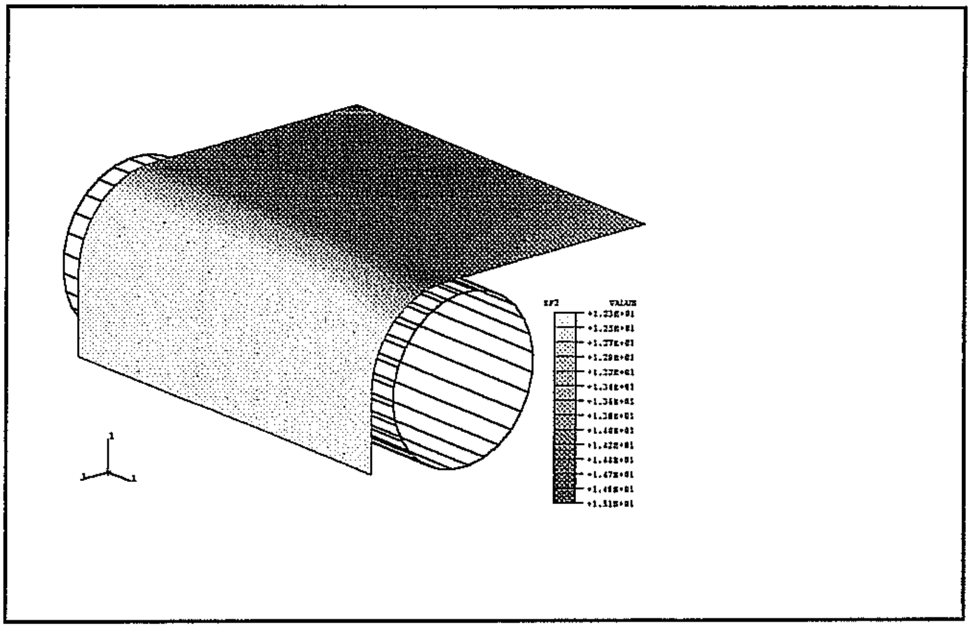

Fig. 5 Three-dimensional model with contours of MD tension.

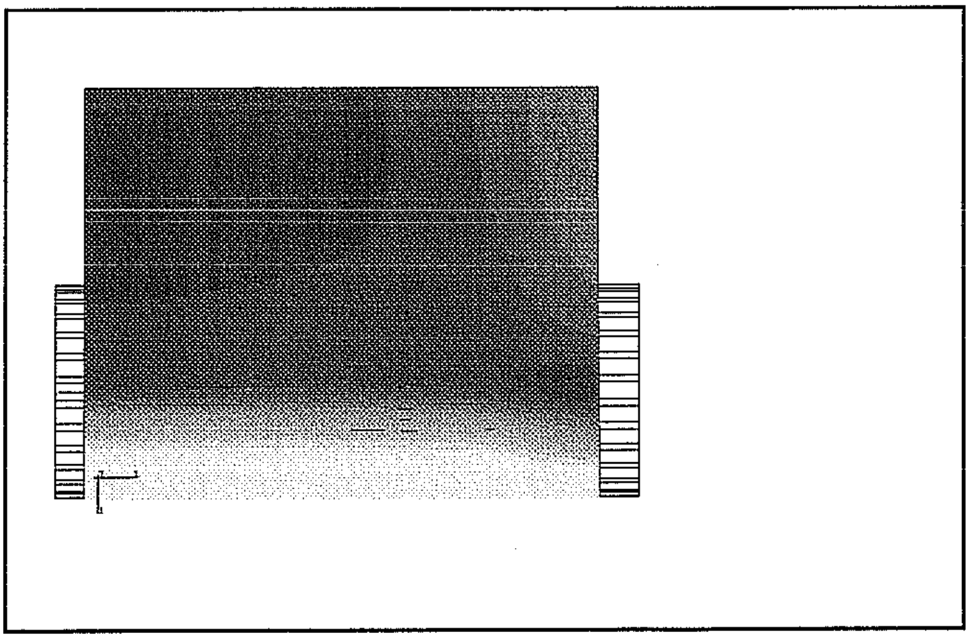

Fig. 6 Top view of MD tension contours, legend as in Figure 5.

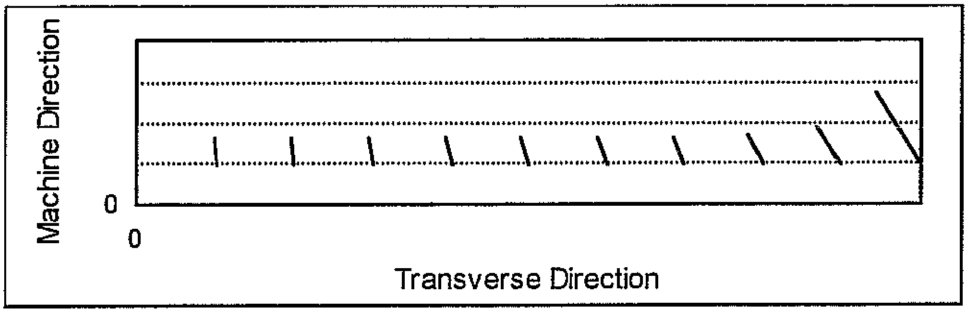

Fig. 7 Schematic of slip paths at positions across web (centre line at left).

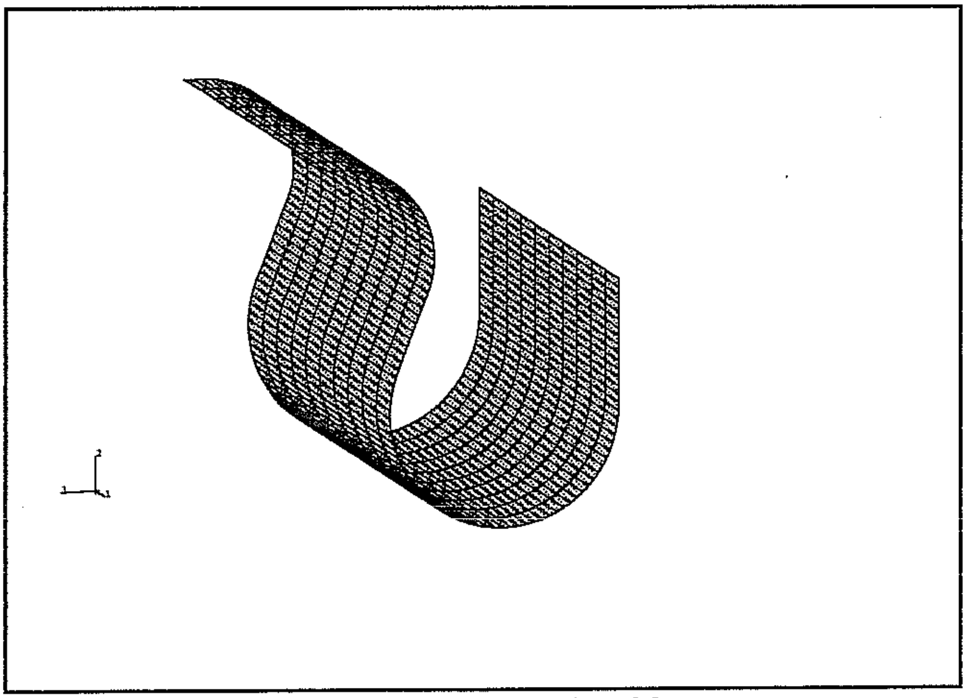

Fig. 8 Web in 2-roll FEA model.

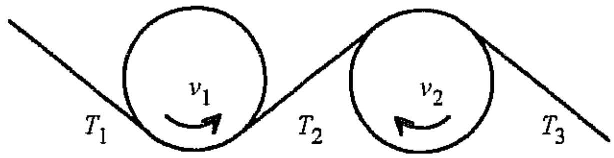

Fig. 9 Configuration of 2-roll model

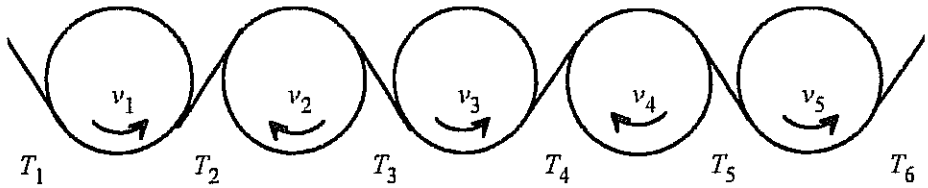

Fig. 10 Configuration of 5-roll model

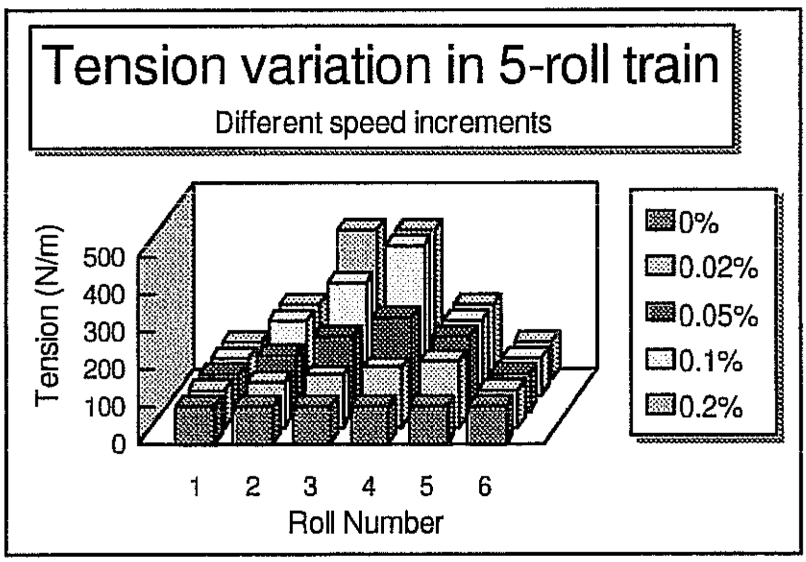

Fig. 11 Tension variation through 5-roll model

TABLE

		MD Slip [mm]	TD Slip [mm]	θ_{stick} [deg]
Web Stiffness (MPa)	1000	2.09	1.52	50
	2000	0.91	0.62	47
	3000	0.56	0.37	52
Friction Coefficient	0.15	1.41	0.93	14
	0.3	0.91	0.62	47
	0.6	0.63	0.45	65
Entry Tension (N/mm)	7	1.16	0.76	29
	12	0.91	0.62	47
	21.5	0.69	0.49	64
Tension Difference (N/mm)	1.28	0.36	0.26	56
	2.4	0.91	0.62	47
	5.12	2.73	1.79	12

Table 1. - Effect of problem parameters on contact conditions at the edge of the web.

Zahlan, N.; Jones, D.P.
Modeling Web Traction on Rollers
6/19/95 Session 3 3:15 - 3:40 p.m.

Question - In your equation 5 which defines Delta it seems that you are making the common slight errors using the change in velocity relative to the first roller speed, no to the velocity an unstrained web would have.

Answer - Yes, you are correct, but the error is very small.

Thank you.