ABSTRACT

A web break model, based on fracture mechanics, was used to investigate unknown coater web breaks. Runnability was defined as $L_\alpha$, the length between breaks. The model includes the size distribution of defects (holes and light spots, for example), web strength and web tension. Defects, such as holes, were most detrimental to $L_\alpha$.

Tenacity and tensile were used interchangeably, due to their high correlation on these grades. Tenacity is a simple, precise, valid fracture toughness test that is easy to use. Strength correlated strongly with web breaks. A 10% increase in tensile related to a 26% increase in $L_\alpha$.

NOMENCLATURE

- $a = \text{flaw size, in.}$
- $a_p = \text{flaw size defined under } N_{a_p}, \text{ in.}$
- $K = \text{stress intensity factor, pli} \sqrt{\text{in.}}$
- $Ko = \text{tenacity, pli} \sqrt{\text{in.}}$
- $L_\alpha = \text{length between web breaks, million lineal feet}$
- $N_{a_p} = \text{distribution parameter, number of defects larger than } a_p$
- $N_{tot} = \text{total number of defects of a given type per million lineal feet}$
- $p_{a_p} = \text{probability that the flaw size is less than } a_p$
- $P_{a_p} = \text{probability that the flaw size exceeds } a_p$
- $\alpha = \text{distribution parameter}$
- $\beta = \text{geometry factor}$
- $\lambda = \text{distribution parameter, in.}$
- $\sigma = \text{failure tension, pli}$
INTRODUCTION

Web breaks influence coater productivity and mill profitability. Two studies were conducted to exploit the predictive capability of fracture mechanics, including the tenacity test. The first study was an eight-month effort covering 847 logs. The second was a focused 4-month study covering 1,633 logs with a total of 370 million lineal feet and 418 relevant breaks.

We developed a probabilistic web break model based on fracture mechanics, which included flaws, web strength and web tension. Flaws included light spots and holes. Sheet strengths were tested to identify the best one for resistance to web breaks. Some of these "unknown" breaks can now be explained.

TENACITY

Paper can fail at stresses below its tensile stress when a flaw causes a severe stress concentration. The stress intensity factor, \( K \), is a measure of the stress at a flaw. At fracture, this critical value of \( K \), is denoted \( K_c \). At failure:

\[
K_c = K = \beta \sigma \sqrt{a}
\]

(1)

where \( \sigma \) is the failure tension (in psi or N/mm), \( a \) is the initial flaw size (in inches or mm's), and \( \beta \) is a dimensionless geometry factor. Any shape and size sample can be used, provided \( \beta \) is known. Failure tension can be predicted at different flaw sizes. The critical flaw size can be predicted, too.

\[
\sigma = \frac{K_c}{\beta \sqrt{a}}
\]

(2)

\[
a = \frac{K_c^2}{(\sigma \beta^2 \pi)}
\]

(3)

Tenacity was developed for paper and paperboard from extensive projects by Battelle Columbus Laboratories and Fracture Research, Inc. Figure 1 shows the dimensions of the test specimen (5" wide by 8" between the grips of a tensile tester, 1½" central slit). A 0.2"/minute elongation rate is used. Paper samples tend to wrinkle during the test. These wrinkles are suppressed by two plastic anti-buckling guides as shown in Figure 1. There is enough clearance between them to allow the sample to slide freely. A narrower sample (4" x 8" with a 1" inch central slit), without the guides, can be used for paperboard.

The geometry factor, \( \beta \), for a center crack specimen, as used in our paper tenacity test is:

\[
\beta = \sqrt{\sec \left( \frac{\pi a}{w} \right)} \times \sqrt{\sec \left( \frac{0.75 \pi}{5} \right)} \times 1.0594, \text{ then}
\]

(4)

\[K_c \approx 0.3252 \times \text{(Failure Load in pounds) psi} \sqrt{\text{in}}\]

(5)

For a standard specimen, one may simply compare the fracture loads, which is often not possible with \( J \), fracture toughness.
**Figure 2** shows the predictive capability of tenacity. The data points show tests with different initial flaw sizes for two sample widths. A tenacity curve is calculated using Eq(2), based on one data point. Additional information on the tenacity test is available (1,2,3). This approach was applied successfully to other published data (4,5,6,7).

Actual defects were also tested. Patches produced a strength nearly equal to unflawed samples. Larger holes or ones with ragged edges were most detrimental. "Light" spots were not.

The coefficient of variation (standard deviation/mean) of tenacity was about 5% for most reel averages. This is slightly lower (better) than tensile, indicating excellent consistency. The larger the size of the tenacity specimen, the better the consistency. Tenacity and tensile correlated well with each other for similar grades, but will not correlate for diverse grades.

**RUNNABILITY MODEL**

**Flaws and runnability**

Web breaks are caused by defects that need not be edge tears. Holes and thin spots have jagged edges and may act as crack-like defects. At a given tension, a web breaks when a defect of critical size is present. The average length between breaks, \( L_a \), will be equal to the average distance between defects of critical size. The probability of such a defect occurring depends upon the flaw distribution (meaning the statistical population of defect sizes, not their locations).

The flaw distribution is a Weibul distribution. The probability that a flaw has a size less than \( a_p \) is \( p_{1-p} \), and is given by Eq(6). (See Table 1). For the present application it is more useful to work with \( P_{a_p} = 1 - p_{a_p} \), given by Eq(7), which is the probability that the flaw size exceeds \( a_p \). For a sufficiently large number of observations, for which we take all the flaws occurring in one million feet, the probability is \( P_{a_p} = N_{a_p}/N_{tot} \) where \( N_{a_p} \) is the number of defects larger than \( a_p \), and \( N_{tot} \) the total number of defects of a certain type. Thus \( N_{a_p} \) can be expressed as in Eq(8).

The distribution function contains three parameters, namely \( N_{tot} \), \( \lambda \), and \( a \). The value of these parameters can be obtained from measurements. For example, a flaw detector can count the number of flaws falling within certain size windows. From these measurements, the distribution parameters can be calculated as shown in Figure 2, for holes in two grades of paper. The lines drawn through the measured data points in Figure 3 represent Eq(8).

Given a certain web tension, \( \sigma \), and a certain type of flaw with geometry factor \( \beta \), the critical flaw size \( a_p \) follows from the tenacity as in Eq(9). All flaws larger than this will cause a break. This number is given by Eq(10). Substitution of Eq(9) in Eq(10) gives the number causing breaks by Eq(11). This being the number of breaks per million lineal feet. Its inverse is \( L_a \), the length between breaks as shown in Eq(12). When more than one type of defect can cause breaks, as is usually the case, the number of breaks due to all types of defects must be added first by Eq(11), and the total inverted to get the length between breaks.
Model Predictions

Predictions made by this model are compared with mill data in Figure 4. They are not perfect, due to limitations of the flaw detector. Not all distribution parameters of all flaw types could be obtained and some had to be estimated. The model permits the calculation of the relative effects of parameters such as tenacity and total number of defects, as shown in Figure 5.

Eq (12) shows web tension is important. The length between breaks depends as strongly upon tension as it does upon tenacity (Figure 5). If the tension is not known, it and the geometry factor can be lumped into one unknown parameter, the value of which can be backed out of actual data for the length between breaks. A predictive equation is obtained by this calibration of the equation.

Surges in tension (and speed) are not as important as one might think. The combined probability of a surge occurring at a certain location, and that of a defect of sufficient size just passing that location simultaneously, is extremely small. However, if stress and geometry factor are lumped and backed out of the data, as discussed above, any effect of surges will be accounted for automatically.

1. Equations for the Runnability Model

\[ P_{ap} = 1 - e^{-\left(\sigma_p/\lambda\right)^s} \]  
\[ P_{ap} = 1 - P_{ap} = e^{-\left(\sigma_p/\lambda\right)^s} \]  
\[ N_{ap}/N_{tot} = P_{ap} \quad \text{or} \quad N_{ap} = N_{tot} e^{-\left(\sigma_p/\lambda\right)^s} \]  
\[ a_e = K_e^2/(\pi \beta^2 \sigma^2) \]  
\[ N_{ae} = N_{tot} e^{-\left(\sigma_e/\lambda\right)^s} \]  
\[ N_{ae} = N_{tot} e^{-\left(\sigma_e/\lambda\right)^s} \]  
\[ L_b = \left[ e^{\left(\sigma_e^2/(\pi \beta^2 \sigma^2)\right)^s} \right]/N_{tot} \]

RUNNABILITY STUDY

Runnability was defined as \( L_b \), the length between breaks. It is calculated from the lineal footage and the number of relevant breaks. A 20 log minimum, with at least 4 breaks, was often used. Relevant breaks should depend upon fracture mechanics (flaws, strength and stress). These included "unknowns" (by location including unwind, 1st coater head, 1st dryer section, 2nd coater head, 2nd dryer section, reel and unknown) and defect related breaks (holes and rawstock cracks).
Tenacity was not tested in the second study, to reduce the amount of testing, since it correlated with tensile for these grades (No. 3 and No. 4 Medium, blade coated). The strong correlation between tensile strength and web breaks is shown in Figure 6. A 10% increase in tensile related to an increase of $L_a$ by 26%, which is similar to the prediction in Figure 5. $L_a$ also correlated negatively with the amount of holes. Holes, when combined with tensile strength, improved the correlation ($R^2 = 0.92$). See Figure 7 for the raw data for holes. The web break model would have been more useful with better tension data. Assuming all defects have the same shape (same $\beta$) was another limitation.

**CONCLUSIONS**

A fracture mechanics web break model was developed with $L_a$ as the best measurement of runnability. $L_a$, the length between breaks, is not easily calculated.

Flaws were more important than web strength. Holes were detrimental flaws, while light spots and patches were not. Since defects cannot be entirely avoided, increasing web strength is one way to reduce breaks.

Tenacity and tenacity correlated well for specific grades. Tensile strength correlated strongly with web breaks. A 10% increase in tensile related to an increase of $L_a$ by 26%.

Tenacity is a material property that provides a measure of the strength of webs containing flaws, and thus of runnability. It can be used to predict the length between breaks, based on the flaw content of the web. Tenacity is precise and easy to use. It predicts strength (failure load) at other crack sizes easily with a calculator or spreadsheet.

**REFERENCES**

Figure 1. Paper Tenacity Test
Figure 2. Tenacity Curves & Residual Strength

Figure 3. Size Distribution of Holes
Figure 4. Prediction of Runnability

Figure 5. Effect of Tenacity and Total Number of Defects on Lb
Figure 6. Length Between Breaks vs. Tensile Strength

Figure 7. Length Between Breaks vs. Holes
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Question - General question, why did you choose to use a controlled flaw in the middle of your web as opposed to, I would have anticipated that you would have worked with a controlled edge crack which is very often a place to propagate a web failure?

Answer - Our web break model allows for any size and location of defects. This study excluded edge cracks because we didn't have a way to measure them. Their inclusion would have improved the results.

Answer - The tenacity test does have a central crack. This fixed geometry was selected as the best way to easily determine fracture toughness ($k_c$). This fundamental material property is needed for the web break model. Other geometry's, with appropriate geometry factors ($b$), can be used.

Thank you.