NON-INTERACTING TENSION CONTROL IN A MULTI-SPAN WEB TRANSPORT SYSTEM

by

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ABSTRACT

A non-interacting tension control algorithm is proposed to reject the disturbance due to the interaction between neighboring processing sections. An 'auxiliary dynamic model' is derived. And an 'auxiliary controller' is designed with the concept of feedback and feedforward control using the auxiliary dynamic model such that it can reject disturbances from the upstream as well as down stream web span. The performance of this controller is compared with that of an existing controller. When the proposed controller is used, the tension in each span is successfully regulated even in a highly interacting multi-span system with less control effort than that of an existing control method.

NOMENCLATURE

A	Cross-sectional area of web
B_{fn}	Rotary friction constant of bearing
E	Modulus of elasticity
J_n	Polar moment of inertia of roll or roller
L_n	Length of web span
R_n	Radius of roll or roller
t_{n0}	Steady-state value of web tension
t_n	Web tension = $t_{n0} + T_n$
T_n	Change in web tension from a steady-state operating value
T_{nref}	Reference tension
u_{n0}	Input value in steady state to a driven motor
u_n	Input to a driven motor = $u_{n0} + U_n$
U_n	Change in input to a driven motor from a steady-state operating value
v_{n0}	Steady-state operating value of web velocity

v_n Web velocity = $v_{n0} + V_n$

V_n Change in web velocity from a steady-state operating value

Subscripts:

0 Steady-state operating condition

n 1,2,3,4 . . .

INTRODUCTION

A web may have to pass through several consecutive processing sections (e.g., cleaning, coating, drying, etc.) in the manufacture of an intermediate or final product. Different web processing sections may require different conditions, e.g., different tension levels. A typical control problem in a multi-span web transport system is maintaining the required longitudinal tension level in each processing section, and at the same time stabilizing the overall web transport system. Mathematical models for a web transport system(1-5,8) and web tension control strategys(6-10) were suggested. Two primary techniques used in the web processing industries for the distributed control of tension are open-loop "draw control"(6) and "progressive set-point coordination" control (open-loop and closed-loop).

In open-loop draw control, tension in a web span is controlled by using the velocities of rollers at the ends of a web span. Control of web tension using draw control requires extremely accurate control of the roller velocities, a requirement which may be very difficult or expensive to achieve. Also, when open-loop draw control is used, a disturbance from an adjacent web span cannot be rejected no matter how accurately the web velocity is controlled. In progressive set-point coordination control, once an input is provided to an upstream driven roller, an input of the same magnitude is automatically provided to each of the downstream driven rollers. It is not a desirable technique for normal operation of a multi-span web transport system. This technique forces the tensions in the downstream web spans to be automatically changed when the tension in an upstream web span is changed. That is, unwanted disturbances are automatically introduced to downstream web spans when there is an input to an upstream processing section. Moreover, the interactions among adjacent web spans and rollers make it much more difficult to maintain the required longitudinal tension level independently in each processing section.

In this paper, a tension control algorithm for a multi-span system is suggested in order to overcome the deficiencies of the open-loop draw control and the progressive set-point coordination control. A non-interacting tension control algorithm is proposed to reject the disturbances due to the interaction between neighboring processing sections caused by the tension transfer phenomena. An auxiliary dynamic model is derived by considering the difference of velocity variations as a state variable. And an auxiliary controller is designed with the concept of feedback and feedforward control using the auxiliary dynamic model such that it can reject disturbances from the upstream as well as downstream web span. The performance of this controller is compared with that of an existing controller through computer simulation. When the proposed controller is used, the tension in each span is successfully regulated even in a highly interacting multi-span system with less control effort than that of an existing control method.

MATHEMATICAL MODEL OF A MULTI-SPAN SYSTEM

Multi-span web transport systems generally can be simplified as shown in Fig. 1. Usually, motors are used to change the tangential velocities of the rollers in order to control the web tension in each processing section. U_n denotes the change in the input to the n-th driven motor, V_n denotes the change in the tangential velocity of the n-th driven roller, and T_n denotes the change in the longitudinal tension in the n-th span.

The system shown in the Fig. 1 can be considered as a set of interconnected subsystems. Each subsystem consists of a web span and a driven roller at the right end of the web span. The n-th subsystem will be called as s_n.

A linearized mathematical model for the n-th subsystem, s_n, can be written as (9):

$$\frac{dT_n}{dt} = -\frac{V_{n0}}{L_n} T_n + \frac{V_{n-1\,0}}{L_n} T_{n-1} + \frac{AE}{L_n}(V_n - V_{n-1}). \tag{1}$$

$$\frac{dV_n}{dt} = -\frac{B_{fn}}{J_n} V_n + \frac{R_n^2}{J_n}(T_{n+1} - T_n) + \frac{R_n}{J_n} K_n U_n. \tag{2}$$

The problem is to design controls U_n, n= 1,...,N, such that these controls together guarantee precise control of tension in each subsystem and the stability of the overall system. N is the total number of subsystems.

The mathematical model for the n-th subsystem given in equations (1) and (2) can be rewritten in a more compact form as shown below :

$$\dot{X}_n = A_n X_n + A_{n\,n-1} X_{n-1} + A_{n\,n+1} X_{n+1} + B_n U_n , \tag{3}$$

$$Y_n = C_n X_n , \tag{4}$$

where

$$X_n = \begin{bmatrix} T_n \\ V_n \end{bmatrix}, \quad B_n = \begin{bmatrix} 0 \\ \frac{R_n}{J_n} K_n \end{bmatrix}, \quad C_n = [\,1 \quad 0\,],$$

$$A_n = \begin{bmatrix} -\frac{V_{n0}}{L_n} & \frac{AE}{L_n} \\ -\frac{R_n^2}{J_n} & -\frac{B_{fn}}{J_n} \end{bmatrix}, \quad A_{n\,n-1} = \begin{bmatrix} \frac{V_{n-1\,0}}{L_n} & -\frac{AE}{L_n} \\ 0 & 0 \end{bmatrix}, \quad A_{n\,n+1} = \begin{bmatrix} 0 & 0 \\ \frac{R_n^2}{J_n} & 0 \end{bmatrix}.$$

A_n and B_n are the system matrix and the input vector, respectively. The matrix $A_{n\,n-1}$ and $A_{n\,n+1}$ are called "interconnection matrices". All pairs (A_n, B_n) are controllable, and all pairs (A_n, C_n) are observable in the above equations.

The problem is to design a set of controllers for the subsystems of distributed system which is called as "local controller". Each local controller includes feedback and feedforward control. Feedforward control may be used to reject some types of disturbances(9). Each local controller has the form of:

$$U_n = -F_n X_n - G_n X_{n+1},$$

where n = 1,..., N.

F_n and G_n are the gains for the feedback and the feedforward control respectively.

The pole assignment technique is quite commonly used for controller design. This technique may be used in the design of a local controller for each subsystem in a multi-span web transport system. It must be assumed that all the states are accessible by either measurement or estimation. Only states and inputs of the corresponding subsystem and adjacent subsystems are used in designing local controllers.

DERIVATION OF AN AUXILIARY DYNAMIC MODEL

Coupling in the equations (1) and (2), which describe the longitudinal dynamics of the web in a multi-span system, complicates the design of a distributed control system. In this section, the equations will be modified such that they are decoupled. The set of modified equations will be called as an "auxiliary dynamic model".

Using equations (1) and (2), the mathematical models for the subsystems s_{n-1} and s_n can be written as:

$$\frac{dT_{n-1}}{dt} = -\frac{v_{n-1\,0}}{L_{n-1}}T_{n-1} + \frac{v_{n-2\,0}}{L_{n-1}}T_{n-2} + \frac{AE}{L_{n-1}}(V_{n-1} - V_{n-2}) \cdot \tag{5}$$

$$\frac{dV_{n-1}}{dt} = -\frac{B_{f\,n-1}}{J_{n-1}}V_{n-1} + \frac{R_{n-1}^2}{J_{n-1}}(T_n - T_{n-1}) + \frac{R_{n-1}}{J_{n-1}}K_{n-1}U_{n-1} \cdot \tag{6}$$

$$\frac{dT_n}{dt} = -\frac{v_{n0}}{L_n}T_n + \frac{v_{n-1\,0}}{L_n}T_{n-1} + \frac{AE}{L_n}(V_n - V_{n-1}) \cdot \tag{7}$$

$$\frac{dV_n}{dt} = -\frac{B_{fn}}{J_n}V_n + \frac{R_n^2}{J_n}(T_{n+1} - T_n) + \frac{R_n}{J_n}K_n U_n \cdot \tag{8}$$

Observing the mathematical model in equation (7) reveals that the velocity difference $(V_n - V_{n-1})$ between the ends of the n-th span can be used as a control variable, instead of the individual velocities. An auxiliary dynamic model may be derived from the mathematical model of the system by introducing a new state variable $(V_{n\,n-1} \equiv V_n - V_{n-1})$.

The auxiliary dynamic model can be derived as follows.

Let

$$V_{n\,n-1} = V_n - V_{n-1}, \tag{9}$$

where n =1,...,N.

Using equation (9) in equation (7) yields:

$$\frac{dT_n}{dt} = -\frac{v_{n0}}{L_n}T_n + \frac{v_{n-1\,0}}{L_n}T_{n-1} + \frac{AE}{L_n}V_{n\,n-1} \cdot \tag{10}$$

The tension in the n-th span depends only on the velocity difference ($V_{n\,n-1}$) instead of the absolute variations of each tangential velocity (V_n and V_{n-1}) of the rollers in the n-th subsystem. Substituting equation (9) into equation (8) yields:

$$\frac{d(V_{n-1} + V_{n\,n-1})}{dt} = -\frac{B_{fn}}{J_n}(V_{n-1} + V_{n\,n-1}) + \frac{R_n^2}{J_n}(T_{n+1} - T_n) + \frac{R_n}{J_n}K_n U_n \,. \tag{11}$$

Subtracting equation (6) from (11) gives:

$$\begin{aligned}\frac{dV_{n\,n-1}}{dt} = &-\frac{B_{fn}}{J_n}V_{n\,n-1} - \left(\frac{B_{fn}}{J_n} - \frac{B_{f\,n-1}}{J_{n-1}}\right)V_{n-1} \\ &+ \left[\frac{R_n^2}{J_n}T_{n+1} - \left(\frac{R_n^2}{J_n} + \frac{R_{n-1}^2}{J_{n-1}}\right)T_n + \frac{R_{n-1}^2}{J_{n-1}}T_{n-1}\right] \\ &+ \frac{R_n}{J_n}K_n U_n - \frac{R_{n-1}}{J_{n-1}}K_{n-1} U_{n-1} \,. \end{aligned} \tag{12}$$

In equation (12), let the input difference ($U_{n\,n-1}$) between the n-th and (n-1)-th subsystem as:

$$U_{nn-1} = \frac{R_n}{J_n}K_n U_n - \frac{R_{n-1}}{J_{n-1}}K_{n-1} U_{n-1} \,. \tag{13}$$

Substituting equation (13) in equation (12) gives:

$$\begin{aligned}\frac{dV_{n\,n-1}}{dt} = &-\frac{B_{fn}}{J_n}V_{n\,n-1} - \left(\frac{B_{fn}}{J_n} - \frac{B_{f\,n-1}}{J_{n-1}}\right)V_{n-1} \\ &+ \left[\frac{R_n^2}{J_n}T_{n+1} - \left(\frac{R_n^2}{J_n} + \frac{R_{n-1}^2}{J_{n-1}}\right)T_n + \frac{R_{n-1}^2}{J_{n-1}}T_{n-1}\right] + U_{n\,n-1} \,. \end{aligned} \tag{14}$$

Equations (10) and (14) constitute the auxiliary dynamic model for the n-th subsystem. The input difference $U_{n\,n-1}$ in equation (13) is defined as the "auxiliary control".

DESIGN OF LOCAL CONTROLLERS USING AUXILIARY DYNAMIC MODEL

In this section, a procedure for designing each local controller in the distributed control system is developed. The advantage of using the auxiliary dynamic model in the design of the distributed control system will be illustrated using a numerical example in the next section.

To simplify the problem, it is assumed that $J_n = J$, $R_n = R$, $B_{fn} = B_f$, and $K_n = 1$ for n=1,...,N in equation (14). Then, the auxiliary dynamic model for the n-th subsystem s_n, represented by equations (10) and (14), can be rewritten in a compact form as:

$$\dot{X}_n = A_n X_n + A_{n\,n-1} X_{n-1} + A_{n\,n+1} X_{n+1} + B_n U_{n\,n-1}, \tag{15}$$

$$Y_n = C_n X_n \ , \tag{16}$$

where

$$X_n = \begin{bmatrix} T_n \\ V_{n\,n-1} \end{bmatrix} , \quad B_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_n = [\ 1 \quad 0\ ] ,$$

$$A_n = \begin{bmatrix} -\dfrac{v_{n0}}{L_n} & \dfrac{AE}{L_n} \\ -2\dfrac{R^2}{J} & -\dfrac{B_f}{J} \end{bmatrix}, \ A_{n\,n-1} = \begin{bmatrix} \dfrac{v_{n-1\,0}}{L_n} & 0 \\ \dfrac{R^2}{J} & 0 \end{bmatrix}, \ A_{n\,n+1} = \begin{bmatrix} 0 & 0 \\ \dfrac{R^2}{J} & 0 \end{bmatrix}.$$

The block diagram for a closed-loop subsystem shown in Fig. 2 can be obtained using equations (9), (10), and (13) through (16). The block diagram shows the structures of a closed-loop subsystem which use the auxiliary dynamic model in the design of local controller. The purpose is to design local controls U_n, n=1,..N, such that Y_n , n=1,..,N can be controlled at the desired level within given performance specifications and such that the overall system is stable.

Auxiliary controls $U_{n\,n-1}$, n= 1,...,N, which were defined in equation (13) may be designed using the auxiliary dynamic model represented by equations (10) and (14) and the specifications for each subsystem. The control for each subsystem is determined from equation (13) as follows:

$$U_n = \frac{J_n}{R_n K_n}\left(U_{n\,n-1} + \frac{R_{n-1}}{J_{n-1}} K_{n-1} U_{n-1}\right). \tag{17}$$

Feedforward control may be used in the design of a local controller to reject some types of disturbances. Each local controller incorporates both feedback and feedforward control. Let the auxiliary control be:

$$U_{n\,n-1} = H_n(U_{nc} - F_n X_n - G_n X_{n+1}), \tag{18}$$

where $U_{n\,n-1}$ is the auxiliary control for the n-th subsystem
U_{nc} is the reference input for n-th subsystem
F_n is the feedback gain vector for the n-th subsystem
G_n is the feedforward gain vector for the n-th subsystem
H_n is the overall gain for the n_th local controller.

Substituting equation (18) into equation (15) gives:

$$\dot{X}_n = \overline{A}_n X_n + A_{n\,n-1} X_{n-1} + \overline{A}_{n\,n+1} X_{n+1} + B_n H_n U_{nc}, \tag{19}$$

where

$$\overline{A}_n = A_n - B_n H_n F_n ,$$
$$\overline{A}_{n\,n+1} = A_{n\,n+1} - B_n H_n G_n .$$

$A_{n\,n+1}$ in equation (19) can be factored by B_n and H_n as:

$$A_{n\,n+1} = B_n H_n G_n \,. \tag{20}$$

By using equation (19), G_n can be selected such that

$$\overline{A}_{n\,n+1} = 0 \,. \tag{21}$$

H_n and F_n can be selected by the pole placement technique such that the closed-loop local subsystem meets a desired performance specification.

EXAMPLES FOR THE DESIGN OF LOCAL CONTROLLERS

To illustrate advantages of using the auxiliary dynamic model and the auxiliary control, simple numerical examples for the design of a distributed control system were solved for a three-span web transport system. The object of the control is to regulate the tension in each web span in the initial operating value even with disturbances from neighboring spans of the system. Two cases are considered.

Case 1: Using the auxiliary dynamic model.
Case 2: Using the original mathematical model.

Consider the three-span system as shown in Fig. 3. The control U_1 is considered as the master speed control.

Assume that desired closed-loop transfer functions for subsystems s_n, n=2,3,4, were given as:

$$Y_n = \frac{100}{s^2 + 14\,s + 100}\; U_{nc} \,, \quad \text{for } n = 2, 3, 4 \,. \tag{22}$$

And $U_{2c} = T_{2ref} = 0.0$, and $U_{3c} = T_{3ref} = 0.0$, and $U_{4c} = T_{4ref} = 0.0$.

Case 1: Using the auxiliary dynamic model and control.
The auxiliary dynamic model for the three-span system shown in Figure 3 is:

$$\dot{X}_n = A_n X_n + A_{n\,n-1} X_{n-1} + A_{n\,n+1} X_{n+1} + B_n U_{n\,n-1} \tag{23}$$

$$Y_n = C_n X_n \,, \; n = 2, 3, 4 \,, \tag{24}$$

where

$$X_n = \begin{bmatrix} T_n \\ V_{n\,n-1} \end{bmatrix}, \quad B_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_n = [\,1 \;\; 0\,]$$

$$A_n = \begin{bmatrix} -\frac{v_{n0}}{L_n} & \frac{AE}{L_n} \\ -2\frac{R^2}{J} & -\frac{B_f}{J} \end{bmatrix}, \; A_{n\,n-1} = \begin{bmatrix} \frac{v_{n-1\,0}}{L_n} & 0 \\ \frac{R^2}{J} & 0 \end{bmatrix}, \; A_{n\,n+1} = \begin{bmatrix} 0 & 0 \\ \frac{R^2}{J} & 0 \end{bmatrix}$$

and n = 2, 3, 4.

The operating conditions and parameter values used for the design are given in Table 1.

When X_{n-1} and X_{n+1} are assumed zero, the closed-loop transfer function for the n-th subsystem can be obtained using equations (19) through (21) as:

$$Y_n = \frac{a_0}{s^2 + a_1 s + a_0} U_{nc},\ n = 2, 3, 4, \tag{25}$$

where

$$a_0 = \frac{v_{n0}}{L_n}\left(\frac{B_f}{J} + H_n f_n^2\right) + \frac{AE}{L_n}\left(2\frac{R^2}{J} + H_n f_n^1\right),$$

$$a_1 = \frac{v_{n0}}{L_n} + \frac{B_f}{J} + H_n f_n^2, \text{ and } F_n = [\, f_n^1 \;\; f_n^2 \,].$$

Equating equations (22) and (25) allows us to select H_n and F_n. And then, by using equation (21), G_n can be selected as:

$$G_n = \frac{1}{H_n}\begin{bmatrix} \frac{R^2}{J} & 0 \end{bmatrix},\ n=2, 3, 4. \tag{26}$$

The gains for auxiliary controls are:

$$H_n = 0.02381,\ n = 2,3,4. \tag{27}$$
$$F_n = [\ -1.0661 \quad 517.8496\ ],\ n = 2, 3, 4. \tag{28}$$

With H_n, F_n and G_n from equations (26), (27) and (28), the auxiliary control can be obtained by using equation (18) as :

$$U_{n\,n-1} = H_n(U_{nc} - F_nX_n - G_nX_{n+1}),\ n = 2, 3, 4.$$

With the auxiliary control $U_{n\,n-1}$, the local control can be obtained as follows using equation (17).

$$U_n = \frac{J_n}{R_n}\left(U_{n\,n-1} + \frac{R_{n-1}}{J_{n-1}} U_{n-1}\right),\ n= 2, 3, 4.$$

It was assumed that $K_n = 1.0$, n= 2, 3, 4. The performance of the controller based on the auxiliary dynamic model were determined for a step change in v_1 (i.e., $V_1(0^-) = 0$, $V_1(0^+) = 1.0$ for this example).

Case 2: Using the original model.
The original mathematical model for the system shown in Fig. 3 is(9):

$$\dot{X}_n = A_nX_n + A_{n\,n-1}X_{n-1} + A_{n\,n+1}X_{n+1} + B_nU_n \tag{29}$$

$$Y_n = C_nX_n,\ n = 2, 3, 4, \tag{30}$$

where

$$X_n = \begin{bmatrix} T_n \\ V_n \end{bmatrix}, \quad B_n = \begin{bmatrix} 0 \\ \frac{R}{J} \end{bmatrix}, \quad C_n = [\,1 \quad 0\,],$$

$$A_n = \begin{bmatrix} -\frac{v_{n0}}{L_n} & \frac{AE}{L_n} \\ -\frac{R^2}{J} & -\frac{B_f}{J} \end{bmatrix}, \quad A_{n\,n-1} = \begin{bmatrix} \frac{v_{n-1\,0}}{L_n} & -\frac{AE}{L_n} \\ 0 & 0 \end{bmatrix}, \quad A_{n\,n+1} = \begin{bmatrix} 0 & 0 \\ \frac{R^2}{J} & 0 \end{bmatrix}.$$

With original mathematical model represented by equations (29) and (30), the control for the n-th subsystem can be given as:

$$U_n = H_n(U_{nc} - F_nX_n - G_nX_{n+1}) \,. \tag{31}$$

The controller represented by equation (31) will be called "original control".
Following similar procedures as those of case 1,

$$G_n = \frac{1}{H_n}[\,R \quad 0\,] \;, n = 2, 3, 4 \,. \tag{32}$$

And

$$H_n = 0.4475 \;, n = 2, 3, 4 \,, \tag{33}$$

$$F_n = [\;-0.138 \qquad 517.8496\;] \,, n = 2, 3, 4 \,. \tag{34}$$

The performance of the controller based on the original mathematical model was determined for a step change in v_1 (i.e., $V_1(0^-) = 0$, $V_1(0^+) = 1.0$ for this example).

The solutions for the examples are given in Fig. 4 -7. In Fig. 4, tension variations T_2, T_3, and T_4 due to a step change in velocity, v_1, of the system shown in Fig. 3 are compared for two cases (tension control with the auxiliary and the original control). When the original control is used, tensions are not regulated properly with big steady-state errors. But, with the auxiliary control, the tensions are regulated properly since the velocity variation in the upstream is reflected in the auxiliary control design as shown in Fig. 5. It is shown in Fig. 6 and 7 that the total control efforts is less in the system using the auxiliary dynamic model and control than that using original mathematical model and control.

When the auxiliary dynamic model is used in the controller design, control signals (U_2, U_3) are generated such that they induce necessary velocity difference between rollers just enough for the required tension variations in the web span.

CONCLUSIONS

An auxiliary dynamic model is derived by considering the difference of velocity variation as a state variable. And a non-interacting tension controller is designed with the concept of feedback and feedforward control using the auxiliary dynamic model such that it can reject the disturbances from the upstream as well as down stream web span. The performance of this controller is compared with that of an existing controller through computer simulation. When the proposed controller is used, the tension in each span is successfully regulated even in a highly interacting multi-span system with less control effort than that of an existing control method.

One disadvantage of using velocity difference as a control variable for tension control is that the magnitude of velocity difference is very small, and so measurement must be very accurate.

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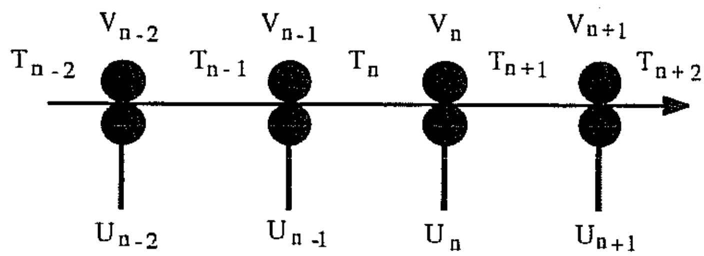
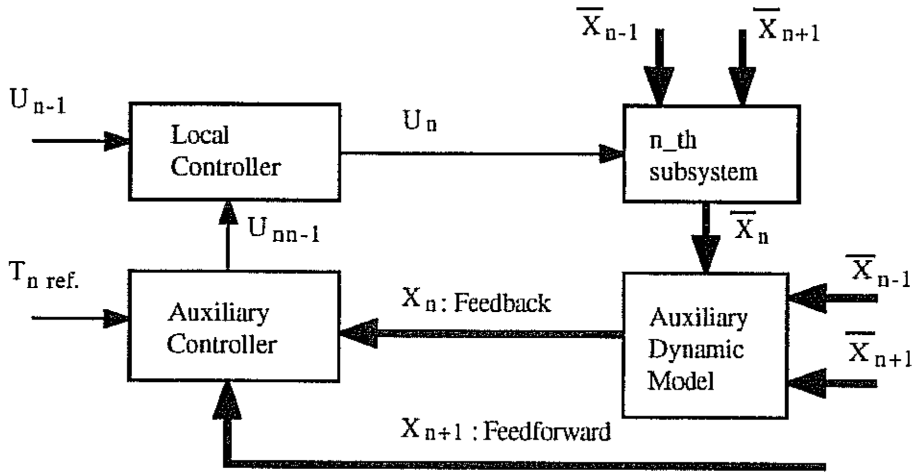


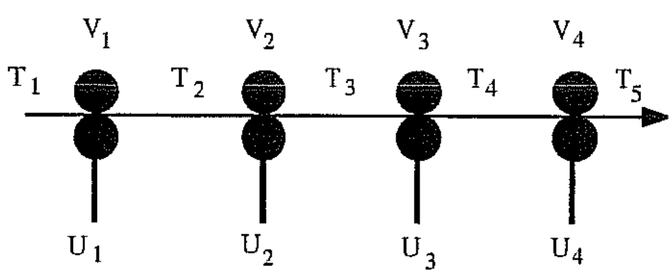
$t_n = t_{n0} + T_n, \quad u_n = u_{n0} + U_n, \quad v_n = v_{n0} + V_n$

Fig. 1 A Multi-Span Web Transport System



$$\overline{X}_n = \begin{bmatrix} T_n \\ V_n \end{bmatrix} \quad \overline{X}_{n+1} = \begin{bmatrix} T_{n+1} \\ V_{n+1} \end{bmatrix} \quad X_n = \begin{bmatrix} T_n \\ V_{n\,n-1} \end{bmatrix} \quad X_{n+1} = \begin{bmatrix} T_{n+1} \\ V_{n+1\,n} \end{bmatrix}$$

Fig. 2 Block Diagram for a Closed-Loop Subsystem



$t_n = t_{n0} + T_n, \quad u_n = u_{n0} + U_n, \quad v_n = v_{n0} + V_n$

Fig. 3 A Three-Span Web Transport System

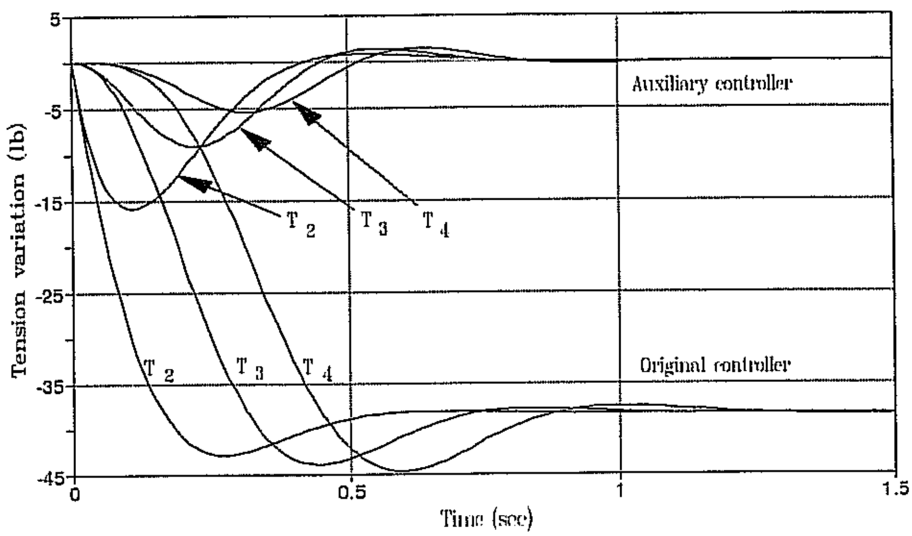


Fig. 4 Tension Variations: with Auxiliary and Original Contol

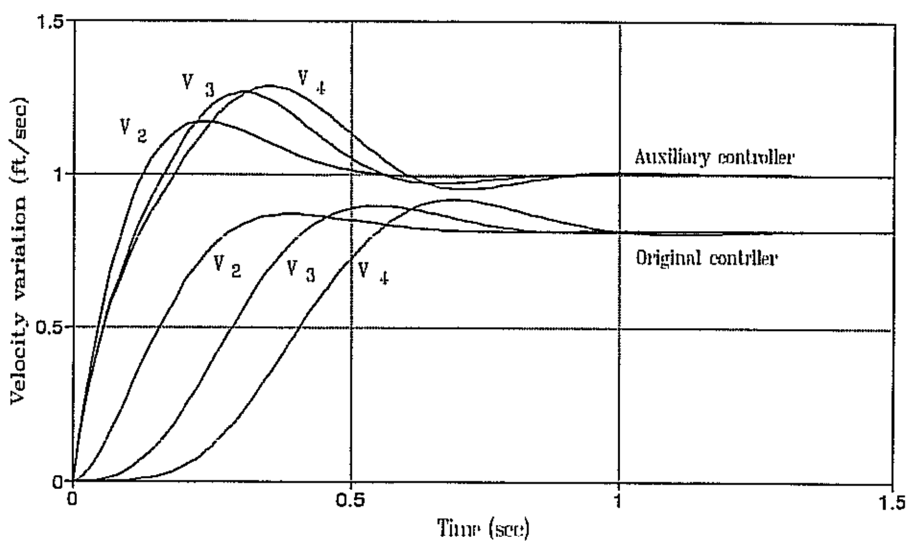


Fig. 5 Velocity Variation: with Auxiliary and Original Control

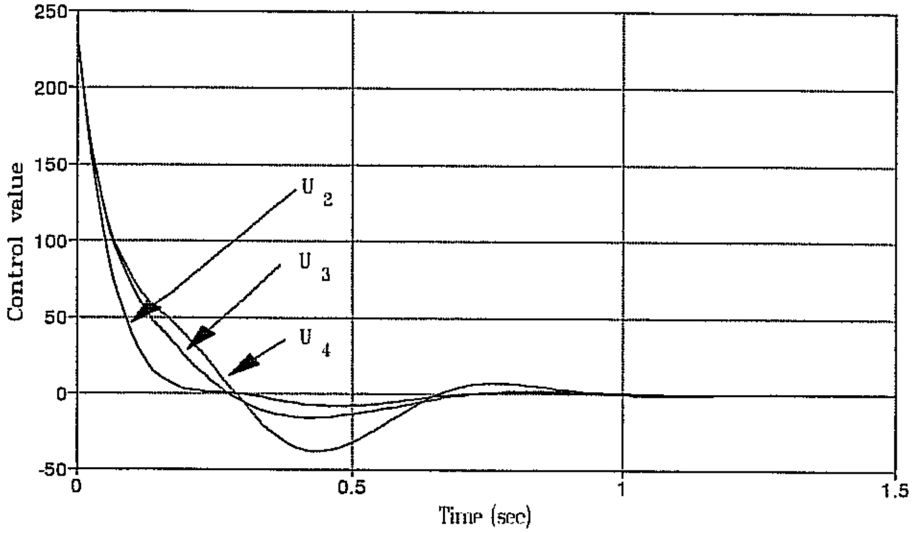


Fig. 6 Input Values: with Auxiliary Control

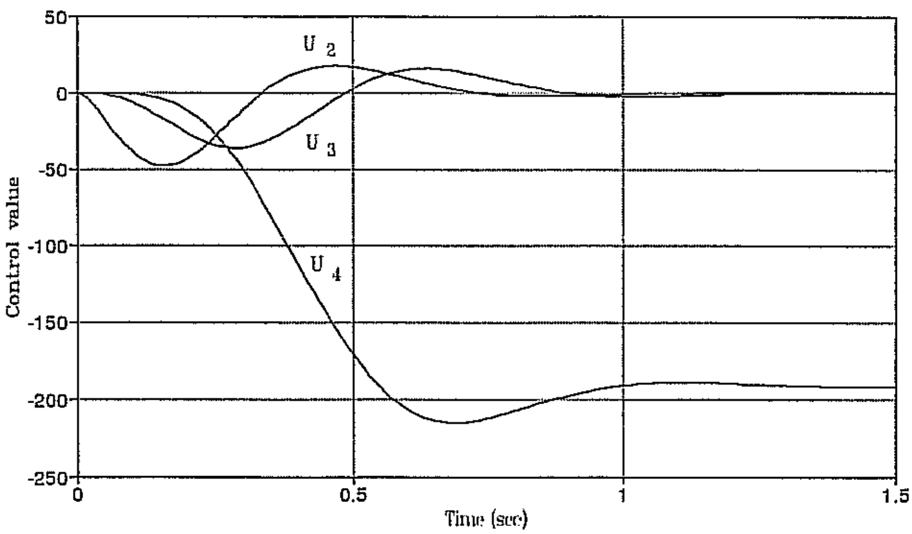


Fig. 7 Input Values: with Original Control

Table 1. Parameter Values for Simulation

parameter	value
A	0.12 in^2
E	350000 lbf/in^2
J_n	94 lbf in sec^2
L_n	120 in
v_n	1000 ft/min
B_f	0.025857
R	5 in

Shin, K.H.; Reid, K.N.; Kwon, S.O.
Non-Interacting Tension Control n a Multi-Span Web Transport System
6/20/95 Session 4 1:10 - 1:35 p.m.

Question - In the auxiliary model, is the difference of the speeds in the output of the system? Is it a model of all neighboring system or the next one?

Answer - The system input is the input to the motor that drives the roller. But, eventually the speed difference can be used to control the tension as shown in the equation. The purpose is to generate the control U_n to drive the roller. This part can be considered as a controller as a whole. Its a little bit complicated but it gives you more and better way to control some kind of disturbances and reduce the size of order as very important.

Question - You had a steady-state error if you use only the lower control without auxiliary control. What is the reason for the steady-state error? Did you use an integrator in your reference or what?

Answer - You might be able to reduce the steady-state error by using another approach. But it may need to increase the order of the system to account the interaction which I don't like. In the proposed method, the disturbance due to the change in the velocity of the master speed drive can be rejected when the auxiliary controller was used without increasing the order of the system.

Question - I don't think so because the coupling value are like disturbances. And if you use state space control, you don't have a steady state error, if you measure velocity or tension.

Answer - You may need more control effort to remove the steady-state error even though you measure it.

Thank you.