LONGITUDINAL DYNAMICS OF A WINDING ZONE

by

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ABSTRACT

The longitudinal dynamics of a moving web in a tension controlled winding zone was modeled and analyzed. The dynamics of the zone included the effects of idler roll inertia, web elasticity, package inertia and feedback control. A computer simulation program was used to solve the system of equations for the natural frequencies and transfer functions. It was found that a typical production type winding zone can have several natural frequencies in the operating range of the winder. The predicted natural frequencies were found to depend significantly on package diameter and feedback gain. As a result, resonant conditions can occur at different times during the winding cycle because the angular velocity of the spindle continually changes. Resonance can also occur with the second and third harmonics of the spindle frequency or any other rotating elements in the winding zone.

NOMENCLATURE

A Web cross sectional area
Bj Bearing friction torque
E Modulus of elasticity
Jj Polar moment of inertia
J0 Total spindle moment of inertia
K Proportional feedback gain
L1 Web span length
M Dancer mass
Rj Roll radius
R0 Package radius
T1 Web span tension
T T1 Winding Tension

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**INTRODUCTION**

The winding of a product on a core is a critical operation in any web manufacturing process. At this stage of the manufacturing cycle, the product has gone through several, value-adding steps and is ready for sale or some additional finishing operation. Thus, problems in winding result in production losses and are usually very costly. There are three major concerns in high speed, precision winding operations: (1) package formation, (2) new roll starts and (3) tension control during winding. These concerns are not necessarily independent since a poor roll start will affect package formation. Similarly, poor tension control will also affect package formation. This paper addresses the problems associated with controlling tension in the winding zone. Besides package quality, inadequate tension control can also be the source of tracking problems, roll slippage, and wrinkle formation in the winding zone.

**PROBLEM DEFINITION**

The two variables which have the most influence on package formation are winding velocity and film tension. Devices such as layon rolls, rider rolls, and taper tension profiling are used to minimize the problems associated with each. Core stiffness, film surface roughness, and film elastic moduli also affect package quality.

It is well known (Ref. 1) that high tension at the beginning of a package can cause the core to buckle. Low tension during winding will cause “shifting” or “telescoping” problems. A period of low tensions followed by a large tension increase creates the “cinching” defect. To avoid these problems and wind good quality packages, it is necessary to maintain uniform winding tension throughout the winding cycle.

**WINDING ZONE DEFINITION**

A winding zone normally consists of a two-spindle (duplex) winder, several transport rolls and a driven roll which feeds web into the zone. The driven roll could be a vacuum roll, S wrap set or pair of nip rolls driven by a motor and speed reduction element. Often the winding zone includes a dancer system with position feedback to control motor speed. In some applications, load cells are used to control tension in place of the dancer (Ref. 2). In this case, the tension response will be different than
when a dancer system is used. Figure 1 shows the different combinations of spindle drives and sensor elements for control. The sensors are used to measure the controlled variables, such as, film tension, dancer position, film speed or motor current (torque). Combinations A, B and C will attempt to provide constant tension winding. System D is constant speed winding and E, F will attempt to provide constant torque winding.

The performance of a winding zone depends, not only on the type of control system, but also the web path, mechanical inertia, geometry, film parameters, and winding velocity. The primary objective of the zone is to provide uniform tension throughout the winding cycle. In some operations, the reference tension is reduced (tapered) as the package builds, thus winding the outer sections of the package at lower tension levels. The basic web transport elements which make up a winding zone are shown in Figure 2. It is important to be able to set the tension (pli) for good winding and maintain that level throughout the winding cycle. Unfortunately, the tension will vary about the set point. Listed below are some of the reasons:

1. Parameters change during winding
   (ie, package inertia, package radius)

2. Mechanical elements are not perfect
   (ie, idler roll runout, package out of roundness)

3. The zone is a dynamic system

4. The control system is not perfect

For a well-performing windup zone, the tension variation should be less than ± 10%. Variations greater than ± 30% are excessive and will generally cause winding problems. Tension variations are caused by excitations which are either transient or periodic in time. An example of transient upset would be a sudden turret move to prepare for a roll start. This paper is concerned with periodic excitations that cause continuous tension variations. Some of the more common excitation sources are: (1) package out of roundness; (2) spindle eccentricity, (3) pull roll speed variation; (4) idler roll eccentricity; (5) motor speed variation; and (6) drive system errors.

MODELING THE WINDING ZONE

The purpose in modeling a winding zone is to develop an understanding of how tensions and speeds can vary throughout the winding cycle. A system of equations will be developed from engineering fundamentals (Ref 3, 4, 5). Separate equations must be written for each idler roll and each web span.
Equations of Motion

The dynamics of the zone are described by a set of first order differential equations. The velocity of each idler roll is determined by the tension difference across the roll and the bearing friction torque:

\[
\frac{J_i}{R_i} \frac{dV_i}{dt} = R_i(T_i - T_{i-1}) - B_i
\]

(1)

\[i = 1, 2, ..., n, \quad j = 1, 2, ..., m\]

Equation 1 assumes that there is no slippage between the web and the roll. Similarly, the tension in each web span can be described by a differential equation developed from a mass flow balance:

\[
\frac{dT_j}{dt} = \frac{T_j + EA}{I_j} \left\{ V_j - V_{j-1} \left( \frac{T_j}{I_{j-1}} + \frac{EA}{T_{j-1}} \right) \right\}
\]

(2)

The form of equation (2) is nonlinear and assumes that the distance between rolls \(i\) and \(i-1\) does not change.

The winding package is mounted on a spindle which is driven by torque from a motor. Since the inertia increases with time,

\[
\frac{d}{dt} \left( J_0 \dot{\theta}_0 \right) = T_q - R_0 T_1
\]

(3)

where \(J_o\) has a fixed portion and a variable component which increases as the web is being wound. Generally the zone is under tension control using either a loadcell for tension or a dancer for position feedback. In either case, the motor torque could be expressed as,

\[T_q = f(T_j X)\]

(4)

where function \(f\) depends on the type of feedback control. Equation (4) assumes that the motor is in torque control and that motor dynamics are faster than the web dynamics and can be neglected.

When a dancer roll is used in the zone, the equation of motion can be determined by summing forces on the moving mass. In general terms:

\[M \ddot{x} = \Sigma \text{forces}\]

(5)

Solution Method

The model for a multi-span, web transport zone (Ref. 6) is represented by \(N\) first order differential equations. The \(N\) variables are roll velocities, span tensions and
those associated with the feedback element. Since the purpose of this paper is to
explore the frequency response (Ref. 7) of the winding zone, an in-house computer
program is normally used to generate the solutions. The program is specific to web
system dynamics and computes the system natural frequencies and transfer functions.
Like the WTS program (Ref. 8), this program first computes the steady state solution.
The differential equations are then linearized about the steady state. Finally, for a
specified sinusoidal excitation, the equations are solved for the small variations from
steady state. Since package radius and inertia change with time, the solution had to be
repeated at several different radii. This assumes that the rate of change for these two
parameters is small compared to the rate of change due to the excitation.

CASE STUDY

The analysis technique described in the above section was applied to the
winding zone shown in Figure 3. The zone is part of a web line which is used to test
web handling equipment, study phenomenon such as wrinkling, and develop improved
on-line measurement techniques. In the past, this line has experienced significant
variations in winding tension at certain line speeds. These often lead to package
formation problems. In the configuration shown, the zone consists of a vacuum roll,
11 idler rolls and a center winder with an 8 inch diameter core. Line speed is set by the
vacuum roll which runs at a controlled speed. Winding tension is controlled by a
loadcell which provides tension feedback to the windup motor drive. The control
system is PC based and has several algorithm options. For this analysis, a simple
proportional control model will be used.

Parameters

There are numerous parameters needed to generate a solution to the winding
zone model. Web parameters such as elastic modulus, thickness and width along with
the location of each idler roll are required. For each idler roll, values are needed for
radius, moment of inertia, wrap angle and coefficient of friction (web to roll). Since
the winding package changes radius and mass, information is needed on how the
moment of inertia changes with package radius. This moment of inertia must also
include the mass of the spindle and the other rotating elements in the winding drive.
Numerical values used in this case study are listed below:

- Number of idler rolls: 11
- Web width: 56"
- Web thickness: 0.004"
- Elastic modulus: 500,000 psi
- Average winding tension: 2 pli
- Winding radius: 4" to 12"
- Winding inertia: 3764 to 97443 lbin²
- Line speeds: 200, 400, 600 FPM
- Feedback gains: 1, 10, 100
Natural Frequencies

The computer solution predicted 12 natural frequencies for the winding zone. Four of the natural frequencies were below 20 Hz. Figure 4 shows how the four natural frequencies varied with package radius. The first frequency (lowest) decreased as the package wound. The second and third frequencies were close together and also decreased as the radius increased. The fourth frequency >15 Hz increased as the radius increased. The curves in Figure 4 are for a feedback gain of \( K = 10 \). The effect of gain on the first natural frequency is shown in Figure 5. As the gain was increased, the first natural frequency increased. At \( K = 100 \), the first natural frequency remained nearly constant with package radius at slightly above 6 Hz. Thus, increasing feedback gain is a way to reduce the effect of package inertia.

Since both natural frequencies and spindle speed vary during winding, it is possible that they will cross at some time (radius) during winding. This condition is referred to as resonance.

Figure 6 shows the first natural frequency \( (K = 1) \) as the package radius grows from 4 to 12". Spindle speed (Hz) is also plotted for three line speeds. At 400 FPM, the spindle frequency and natural frequency are near each other when the package starts at \( R=4" \). As the package winds, the spindle slows down faster than the natural frequency changes. But, when winding at 600 FPM the spindle passes through the natural frequency at \( R=7" \). This would be a resonant condition and large tension variations would occur at about 2.75 Hz. One solution to this problem would be to increase the feedback gain in the control system and raise the natural frequency. This strategy is shown in Figure 7. Although the spindle frequency does not pass through the natural frequency curve, the second harmonic \( (2X) \) would match at \( R=8.5" \). The tension variations would occur at a frequency of 4.7 Hz. This would be a less severe resonance, but the tension variations would increase, nevertheless.

Transfer Functions

A transfer function is defined as the ratio of a system variable to an excitation variable. It represents the response of a measurable variable to a unit sinusoidal excitation and is plotted as a function of frequency. In web transport systems, for example, a transfer function would show the amount of tension variation that would result from a 1 FPM variation in vacuum roll speed at a given frequency. The natural frequencies discussed in the previous section appear as peaks in the transfer functions and represent conditions of large variations in either tension or roll velocity.

In a winding zone analysis, the two key transfer functions are for the spindle (package) velocity and the tension in the span entering the package. For this discussion, the excitation was assumed to be the vacuum roll velocity variation. However, regardless of the excitation source, the natural frequencies will remain the same.

Figures 8 and 9 contain the transfer functions for winding tension and velocity at the start of the winding cycle. A gain of \( K=10 \) was chosen to illustrate the response functions to an excitation from the vacuum roll. The frequency range of interest is 0.1 to 20 Hz. The tension variable is film stress which is tension divided
by the cross sectional area of the film. The excite function is the angular velocity of the vacuum roll. Thus the units for the amplitude are psi/(rad/sec). The units for the velocity transfer function amplitude are [(rad/sec)/((rad/sec)] or dimensionless. The plots also show the phase angle between the excitation and response variable.

Figure 8 shows that for low frequency excitations the amplitude of the winding tension is low. As the frequency is increased, the amplitude increases and reaches a peak at the first natural frequency. The other peak is the fourth natural frequency. For this package diameter and gain, the second and third natural frequencies were very close together (see Figure 4) and have a canceling effect. Thus, the transfer function does not show a peak. At low frequencies, the amplitude ratio for package velocity (Figure 9) remains constant. In this case, large variations in package velocity result when the excitation frequency approaches the first and fourth natural frequency.

As the package winds, the transfer functions gradually change. Figures 10 and 11 show the result near the end of the wind cycle at R=12". Although the shapes are similar to the earlier plots, the first natural frequency has moved to a lower value. The second and third natural frequencies have moved apart and now appear as distinct peaks in the response curves. It should also be noted that the amplitude ratio has shifted downward as a result of the increased inertia.

CONCLUSION

A necessary requirement to winding high quality packages is good control of the winding tension. Because of the changing parameters of the winding zone and the numerous sources of excitation, the dynamic tension variations often become a problem. A dynamic model of a typical zone was used to explore the characteristics of the winding zone. It is not uncommon for a winding zone to have three or four natural frequencies below 20 Hz. They generally decrease as the package builds but not necessarily. The natural frequencies are also affected by the feedback gain of the tension control system. This effect can be used to tune the drive to improve tension control.

Resonant conditions can occur with any rotating element in the zone. Higher harmonics can also pass through the natural frequency and create significant tension variations. The package rotation is a common source of resonance because it decreases as the package builds. An existing line was used to illustrate this phenomena and identify ways to avoid the problem.

In summary, the transfer functions for winding tension and spindle velocity are a good way to quantify the dynamic characteristics of a winding zone. Natural frequencies and amplitude ratio can be used to compare different designs or evaluate the effect of changing parameters, such as web thickness.

REFERENCES


8. Shin, K., “A Computer Based Analysis Program for Multi-Span Web Transport Systems”, WHRC Project 8600-10, Oklahoma State University, Stillwater, OK.
### Fig. 1 Different types of windup control systems

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Spindle Actuator</th>
<th>Motor Drive</th>
<th>Motor Clutch</th>
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<td>Tachometer</td>
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<td>Motor current</td>
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### Fig. 2 Elements of a typical winding zone
Fig. 3 Winding zone web path for case study
Fig. 4 Effect of package radius on natural frequencies for feedback gain $K=10$

Fig. 5 Effect of feedback gain on the first natural frequency
Fig. 6 Resonant conditions with the first natural frequency and spindle rotation, K=1

Fig. 7 Resonant condition with second harmonic of spindle rotation, K=10
Fig. 8 Transfer function for winding tension with $R=4''$ and $K=10$, amplitude=$\text{psi}/(\text{rad/sec})$.

Fig. 9 Transfer function for spindle velocity with $R=4''$ and $K=10$, amplitude=$\text{(rad/sec)}/\text{(rad/sec)}$. 

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Fig. 10 Transfer function for winding tension with $R=12''$ and $K=10$, amplitude=$\text{psi/}(\text{rad/sec})$.

Fig. 11 Transfer function for winding tension with $R=12''$ and $K=10$, amplitude=$\text{(rad/sec)}/(\text{rad/sec})$. 

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Question - These are mechanical resonance's, right? This is not to do with band width or instability from the controller or frequent mechanical resonance's, right? From a drive supplier perspective, it's good to see customers doing this analysis before they get the drive systems in so we have an idea if there's going to be a drive problem before we get there.

Answer - Thank you.

Question - I'd like to suggest two other sources of resonance. I assume this is a turret-winder with ?

Answer - No, it's a center winder.

Question - Oh, okay, because a lot of people would transfer the operations using ..

Answer - No, I'm sorry, it's a turret.

Question - Sometimes you have a problem with the mechanical stiffness of the turret arms so that the whole arm bounces up and down so this is a rotational frequency of the turret arm stiffness and the mass of the package hung on the end of the cantilever. That's one frequency problem. The other is, sometimes people use a timing belt to couple from the first shaft out to the spindle, so the timing belt is fine if it's accurately manufactured and not stretch during installation and that's in cases where the timing belt is stretched which gives you a funny resonance because it doesn't occur every spindle revolution. It's every 1.8 or something, depending on the peaking of the timing belt. That can be another confuser with these resonance problems.

Answer - The timing belt is a source of excitation but also an elastic element. I assume it's rigid from the roll back to the motor, so there's no mechanical resonance there. You say mechanical, but it's web-roll resonant frequency. It's a stretching of the web because the model for the web span is the basic equation for the stretch web between two spans.

Question - We have a good analysis of the winding system. Did you make investigations to avoid the disadvantage of the frequencies in your control system.

Answer - Yes, I have. Unfortunately, the paper wasn't long enough to include the experimental work in there. Certainly, we've changed the gain. Making changes in the controls to reduce the resonance. I only used proportional feedback. Using an integral control with that will also, you a change and can provide some relief there.

Question - And what kind of control did you use and was it cascaded?

Answer - Straight PI and there some proprietary control algorithm in the drive to compensate for it.

Question - And my second question. I remember on your pictures, you had some zones with high and low pressure in your alignment and if you're winding up, these changes will be frozen on the winder and if you unwind these changes will come out and cause
some slip zones on the surface. Did you consider these problems? These slip zones will be on the surface and cause some damages. Did you consider this?

Answer - My intent was not to consider the modeling of the package stresses. Modeling always assumes that there is constant winding tension. That's not been the case. In any web line I've ever looked at, I've seen variations, some as much as 70 or 80% peak to peak at times. There is no uniform tension and I consider uniform less than 10% peak to peak. That's the best I've ever seen because there are so many exhortations.

Question - Operating velocity? The frequency may depend on the operating velocity so there might be a way to avoid the critical frequency. Did you try several winding velocities in this experiment?

Answer - That's the production approach. Often what I do is define a line speed zone or operating region to stay out of. You either wind less than that or a little higher, but stay away from that frequency, and that's the easiest and often the solution we use for a quick and easy solution. It's only when we get resistance that we have to make this product at that speed and we have no choice. Then we have to resort to some other changes, like control changes.

Thank you.