STICK-SLIP BEHAVIOR OF PAPER DURING FRICTION TESTING

by

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ABSTRACT

Paper often exhibits oscillatory stick-slip motion during friction testing using a horizontal sled apparatus. The motion consists of a constant-amplitude oscillation in the force required to move the sled, which may occasionally turn into a heavily-damped oscillation. The transition between these two types of behaviour is discrete, not continuous. Depending on the operating conditions, either or both of the above motions can be seen in a single test. A theoretical framework is presented for the analysis of this motion, and it is shown that the distinction between the two classes can be made based on the sled speed and the difference between the static and kinetic coefficients of friction.

It has been found that two different grades of paper may consistently fall into different categories when classified according to the shape of their oscillation. It is suggested that stick-slip behaviour may provide a more reproducible test of the difference between samples than does the traditional friction test which is based only on the measurement of the first peak in the friction curve.

NOMENCLATURE

g = gravitational acceleration
K = kinetic coefficient of friction
k = spring force constant
l = spring length
m = sled mass
Q = dimensionless sled speed
S = static coefficient of friction
t = time

v = sled speed
ϕ = intermediate angle
γ = damping coefficient
ω = oscillator frequency
θ = intermediate angle

INTRODUCTION

The phenomenon of stick-slip motion during a smooth sliding contact is well known in the field of metal-metal friction (1), where it leads to many of the squeals and squeaks that we hear in everyday life. For sliding metal contacts this type of motion is generally undesirable, since it represents erratic and uncontrollable behaviour. In paper-paper contacts this motion has been noted as well during routine friction testing (2-5), but has not been quantitatively analyzed. It is our experience that stick-slip motion may actually be desirable in paper contacts, based simply on the empirical observation that one grade of paper we produce is susceptible to defects such as crepe wrinkles and does not normally show stick-slip motion during friction testing, while the other grade normally does show this behaviour and does not have this particular runnability problem.

Stick-slip motion is characterized by an oscillation in the relative velocity of two surfaces that are sliding over each other. Assume that one surface is being pulled at a constant speed by a spring with a characteristic vibration frequency which is not in any way related to the physical properties of the two surfaces. For metal-metal contact the oscillator frequency would typically be so high that the sound would be audible. For paper-paper contact in a standard friction tester the frequency is much lower, but the oscillation in this case can often be felt by gently placing one's fingertips on the equipment. If the two surfaces are initially at rest, then the oscillator will normally be excited by the initial jerk as sliding begins, and the oscillation will normally die away due to mechanical damping, so that the relative velocity will become constant. However, if we postulate that the static coefficient of friction (COF) is larger than the kinetic, then a situation can arise in which the oscillation does not die away. During the oscillation, it is likely that the relative velocity of the two surfaces will become temporarily zero just as it initially was, and if the static COF is higher than the kinetic, then the two surfaces will stick in such a way as to re-excite the oscillator to the same point that it initially was at, so that the cycle will repeat itself rather than dying away. More generally, it is clear that almost any COF that is a decreasing function of relative velocity will lead to this type of self-sustained oscillation.

For metal-metal contact, because of the importance of the issue, and because of the precision with which metal surfaces can be prepared, the measurement of stick-slip motion has become quite sophisticated (6-8), so that it is typical to see friction measured as a continuous function of velocity, which may or may not include the zero-velocity case. The case of zero velocity, indeed, may be of no practical interest if one is evaluating the merits of a lubricant to be used in a piece of moving equipment. However, for paper, the zero velocity case is of extreme interest, since this is where we wish our paper would always stay, especially in a winder. For this reason, and because the quality of the data we have collected is rather poor, we will use a simple two-parameter model of friction, and describe some of the qualitative features of the data without attempting any numerical analysis. What follows, then, is a phenomenological description of paper friction as opposed to a rigorous theory.

Despite the above disclaimer, we do believe that stick-slip behaviour may lead to new methods of distinguishing between different grades of paper. In our own labs we have become so accustomed to seeing various traces of stick-slip motion on our computer screen that we can almost instantly identify which of our two mills made a certain paper sample simply by the "shape" of the oscillation, with quite a high probability of guessing correctly. The problem is how to turn this from a qualitative judgment into a quantitative measurement tool.

EXPERIMENTAL

Friction tests were done with a standard TMI horizontal sled tester, with a load cell to measure the force required to move the sled, and with an adjustable sled speed. The coupling between the load cell and the sled is one-way, so that the sled can be pulled but not pushed. For the tests shown here the sled weight was 2331 gm, sled size was 5 in. by 5 in., giving an applied pressure of 1.4 kPa. The instrument has an internal microprocessor which provides a readout of static and kinetic COF, but the sample rate used in the instrument is not sufficiently fast to capture the peak height properly, so we monitor the analog output from the load cell directly, using an A/D converter and a personal computer. The computer measures the output from the load cell 350 times per second, which is more than adequate, since the typical oscillation frequency is about 20 Hz. The analog signal is calibrated by hanging a known weight from the load cell. The COF is obtained using the first peak as the static COF, and the average of about the next 10 cycles as the kinetic COF.

There are a number of practical problems associated with this type of analysis. The analog signal sometimes has sharp electrical spikes in it which have to be removed as they would lead to a false peak. Secondly, the first peak observed is very often not representative of the following peaks, which are seen due to stick-slip motion. Thirdly, the average value of the signal tends to drift downwards with time, so that the kinetic COF is not at all constant. However, these are outside the scope of the present discussion, which will concentrate only on the form of the oscillation.

Figure 1 shows experimental results obtained using the above method. Samples from our Thorold and Baie Comeau mills were tested side by side, with 5 tests done on each mill, at 3 different sled speeds. The aim was to try to distinguish between the two mills, since they have very different fibre furnish, and different runnability characteristics.

The kinetic COF's for the two mills are clearly different, but we do not normally wish to do 5 replicate tests just to detect a difference that should be quite pronounced. The static COF's, on the other hand, have so much experimental error that it is not clear whether the difference is significant.

For our present purposes the main point of interest is that the difference between the static and kinetic COF's appears to increase as the sled speed increases. This will be discussed below, where it will be shown that these samples are in fact very different in a qualitative sense.

STICK-SLIP MOTION WITH NO DAMPING

Figures 2 and 3 show the raw analog signals obtained from the tester at a sled speed of 12 in./min for 3 replicate tests of Thorold and Baie Comeau papers. The vertical scale is not calibrated and the graphs have been deliberately offset vertically so that the 3 samples can be distinguished more clearly. Note that the Thorold samples always show a damped oscillation with a half-life of about 5 cycles. The Baie Comeau samples show an oscillation that is constant-amplitude for an indefinitely long time. In some cases this oscillation will persist in Baie Comeau samples even on repeated testing of the same sample as many as 5 times. The difference in shape of these two sets of curves is so persistent and so systematic that we would like to exploit it to distinguish between samples in a quantitative way.

The oscillations have the following observed characteristics :

1 - The frequency appears to be sample-independent, although it does depend on the sled speed. At sled speeds of 3, 6, and 12 in./min we obtain oscillation frequencies of roughly 14, 17, and 20 Hz, respectively.

2 - The amplitude of the oscillation is sample-dependent and is also speed-dependent, as shown in Fig. 1, where we have plotted essentially the peak height and average value of the oscillation.

3 - The oscillation is not entirely sinusoidal in nature, as is shown by Fig. 4. This figure shows the first derivative of the force with respect to time, i.e. sample speed relative to the load cell, for 3 Thorold samples at different sled speeds. The peaks in the velocity curve are chopped off, especially at the lowest sled speed, as will be explained below.

4 - The presence or absence of the constant-amplitude oscillation is very much speed-dependent, since at sled speeds of 6 or 3 in./min all the samples tested above invariably had a constant-amplitude oscillation, so that it was no longer possible to qualitatively distinguish between Thorold and Baie Comeau samples at these speeds.

5 - When the oscillation decays, it does so at a rate which is sample-independent. Furthermore, the decay curve cannot be explained in terms of a gradual increase in a damping coefficient as a function of time. The transition from a constant-amplitude oscillation to a damped oscillation is instantaneous and discrete.

The classical explanation of stick-slip behaviour, for the case where there is no damping of the oscillator, has been given in Ref(1). The sample is initially at rest. As we start to pull on the sled the force rises linearly to a point determined by the static COF. The spring pulling the sled is extended and the sample starts to slip at this point. The slip motion is sinusoidal and is governed by the kinetic COF. There is a certain amount of overshoot in the extension of the spring before it actually starts to contract, due to the fact that it is necessary to match the slope of a straight line to the slope of a sine wave. Before a full oscillation is completed, the relative velocity between the two surfaces will be zero again, after the spring has been fully compressed and is starting to expand again. At this point, the two surfaces will stick together, either because the coupling is one-way or because the static COF is higher than the kinetic COF. The force on the load cell will then rise linearly until it regains its original position determined by the static COF.

Re-expressed in terms of velocity, dl/dt, this means that the velocity curve is also sinusoidal during the slip portion of the movement, but during the stick portion it has a peak height given by v, the sled speed, above which it cannot rise. This is why the peaks are truncated in Fig. 4, especially at low speeds.

As shown in more detail below, the oscillation has the following properties, if the damping factor, γ, is zero:

Average force : $$\frac{kl(\text{average})}{mg} = K \tag{1}$$

Maximum force : $$\frac{kl(\text{maximum})}{mg} = K + (S - K)(1 + Q^2)^{1/2} \tag{2}$$

Sticking time : $$\omega t(\text{stick}) = 2/Q \tag{3}$$

Slipping time : $$\omega t(\text{slip}) = 2\pi - 2\tan^{-1}(1/Q) \tag{4}$$

where $$Q = \frac{\omega v}{g(S - K)} \tag{5}$$

The kinetic COF will therefore be unaffected by the oscillation, while the static COF will be over-estimated at high speeds, Q. The total period of the oscillation will depend on sled speed, and can become arbitrarily large at low speeds.

Table 1 shows some typical theoretical predictions of how the apparent static COF and apparent frequency are affected by sled speed. These results are qualitatively consistent with the measured oscillation frequencies in observation 1 above. They are also consistent with the speed dependence of the static COF, shown in Fig. 1. The dependence of the kinetic COF on sled speed in Fig. 1 is not understood, but is probably due to the downward drift in the average signal versus time, which is not considered in this theory.

These equations adequately deal with observations 1, 2, and 3 above, but they do not address points 4 and 5, which are the only two observations that offer us any hope of qualitatively distinguishing between samples without a great deal of very messy numerical computation. To be plausible, even a phenomenological description must be able to explain the discrete nature of the transition from stick-slip behaviour, which is by definition constant-amplitude, to non-stick-slip motion which is heavily damped. For example, it is simply not good enough to postulate that the oscillations die away due to some time-dependent phenomenon that causes the static COF to drop during the course of the test. This could easily explain the observed results, but it is not physically reasonable. However, it is reasonable to suppose that there may be a threshold value for a static COF and/or a sled speed such that above the threshold we always have constant-amplitude oscillations and below the threshold we never have them. The introduction of damping into the oscillator allows us to define such a threshold.

STICK-SLIP MOTION WITH DAMPING

The reason why damping changes the character of the theory so much is that it is now no longer inevitable that the sample will stick after one oscillation. With no damping, one can say unequivocally that, if the sample stuck once, then it will always continue to do so. For this to happen, the oscillator must go through more than half a cycle of its motion, from slightly before a full extension to slightly after a full compression, and must still have enough energy to accelerate the sample so that its speed matches (or attempts to overshoot) the sled speed. If the oscillator is damped then this will not always be true. Furthermore, if the sample fails to stick during the first oscillation, then it will clearly fail on all successive tries. We will then see a damped oscillation with no sticking, and our experimental observation is that the damping will be characteristic of the equipment, not the paper sample.

We therefore consider the following system :

$$m\frac{d^2l}{dt^2} = -kl - m\gamma\frac{dl}{dt} + mgK \tag{6}$$

where K is the only paper-related parameter. Define an oscillator frequency as

$$\omega = (k/m - \gamma^2/4)^{1/2} \tag{7}$$

and a dimensionless sled speed as

$$Q = \frac{kv}{m\omega g(S - K)} \tag{8}$$

Note that this Q is slightly more general than the one defined in Eq(5), due to γ in Eq(7). Also define some intermediate angles which help to simplify the algebra :

$$\tan\theta = Q + \gamma/2\omega \tag{9}$$

$$\tan\phi = \gamma/2\omega \tag{10}$$

Then the solution to Eq(6) is given by

$$\frac{kl(t)}{mg(S - K)} = \frac{K}{S - K} + \frac{\exp(-\gamma t/2)\cos(\omega t - \theta)}{\cos\theta} \tag{11}$$

while the velocity is given by

$$\frac{k\,dl(t)/dt}{\omega mg(S - K)} = -\frac{\exp(-\gamma t/2)\sin(\omega t - \theta + \phi)}{\cos\theta\cos\phi} \tag{12}$$

At t = 0 this solution satisfies the conditions

$$kl(0) = mgS \tag{13}$$

$$dl(0)/dt = v \tag{14}$$

where Eq(14) states that the sample is sticking at t = 0, and Eq(13) states that the force is given by the static COF, while Eq(6) states that the slipping motion is governed by the kinetic COF.

There are two times that are of particular interest in a slip cycle, namely the time, t_1, of maximum force:

$$\omega t_1 = \theta - \phi \tag{15}$$

and the time, t_2, of maximum slipping speed (minimum dl/dt):

$$\omega t_2 = \omega t_1 + \pi/2 - \phi. \tag{16}$$

Most of these parameters are not physically observable. However, there are two relevant physical measurements that can easily be made. The first is the amplitude of the stick-slip motion, where amplitude is arbitrarily defined as half the range. We find

$$\frac{k(l(max) - l(min))}{2mg(S - K)} = \frac{\exp(-\gamma t_1/2)(1 + \exp(-\gamma\pi/2\omega))\cos\phi}{2\cos\theta} \tag{17}$$

This factor is really an "amplitude amplification ratio" which measures how much larger the apparent (S - K) is than the true (S - K). Equation (2) above is an example of the use of this factor for the special case of no damping. The second measurement is the ratio of the maximum slipping speed to the sticking speed. This is given by

$$-\frac{dl(t_2)/dt}{v} = \frac{\exp(-\gamma t_2/2)\cos\phi}{\sin(\theta - \phi)} \tag{18}$$

It is now technically feasible to analyze the stick-slip motion to extract true values of the static and kinetic COF, even in the presence of constant-amplitude oscillations. One would begin by noting that the damped oscillation takes about 5 cycles to decay to half amplitude. This leads to the result

$$\gamma/2\omega = 0.022 \tag{19}$$

or, alternatively, $\phi = 1.26$ degrees. Then one would use the velocity curve in Fig 4 to determine the left hand side of Eq (18). Since t_2 is a unique function of θ and ϕ, we could extract θ from Eq (18). With this information we can calculate the right hand side of Eq (17) to determine the correction factor with which to convert an apparent (S - K) to a true (S - K).

While this is feasible, it is not necessarily advisable. A much more promising line of inquiry is to investigate the possibility of a threshold speed for stick-slip motion. Towards the end of the first cycle, the maximum positive velocity of the oscillator occurs at t_4, where

$$\omega t_4 = \omega t_2 + \pi. \tag{20}$$

For the sample to be on the verge of not sticking, it is necessary that

$$dl(t_4)/dt = v. \tag{21}$$

This can be re-expressed as

$$\exp(\gamma t_4/2) = \frac{\cos\phi}{\sin(\theta - \phi)} \tag{22}$$

which follows immediately from inspection of Eq (18) above.

It is important to recognize that this condition will not normally be satisfied. It is satisfied only for one special value of Q, the dimensionless sled speed. All the parameters in Eq (22) are unique functions of θ and ϕ. If we have an independent measurement of γ (or ϕ), then Eq (22) allows us to calculate θ (or Q). We can then predict theoretically that above a certain sled speed the stick-slip motion will spontaneously decay, while at lower sled speeds it will not.

Figure 5 shows a theoretical simulation of the shape of such a threshold oscillation. We have chosen $\phi = 1.26$ deg as above. Using Eq (22), this leads us to $\theta = 62.94$ deg, and the threshold value of $Q = 1.94$. We have arbitrarily used $K = 0.3$, $S = 0.35$. In the simulation we have arbitrarily allowed the oscillator to stick three times in a row, where the stick portion of the motion is drawn as a thicker line than the slip portion. On the fourth cycle we chose to let the oscillator slip continuously. Note that this does not require a discontinuous change in any of the physical properties of the system. Since we are on the threshold, it is purely a matter of chance as to which will occur.

The overall shape of Fig 5 is perfectly consistent with any of the experimental observations we have made above, and it explains the apparently discontinuous nature of the transition from constant-amplitude oscillations to heavily damped oscillations.

It also contains one other minor new feature of interest, namely asymmetry in the vertical direction. That is, the average value of the force is now slightly higher than the kinetic COF. This can be seen by inspecting the stick portion of the motion, which begins at slightly below the kinetic COF and ends at precisely the static COF. If no damping were present, this motion would have been perfectly symmetric about the kinetic COF.

FUTURE WORK

We are now in a position to speculate as to why Baie Comeau samples almost invariably oscillate for indefinitely long periods of time, while Thorold samples never oscillate at a sled speed of 12 in./min. It appears that the threshold speed for Baie Comeau samples is above 12 in./min, while for Thorold it is below this value. The threshold speed, of course, is proportional to (S - K), so we are really making a statement about COF when we say this.

This suggests the following possibility for future work: since our equipment is not designed to run at a variable sled speed, it might be worthwhile to design a piece of equipment to run at continuously variable speed. For example, if the sample were tested at a constant acceleration rate, rather than a constant speed, then one might theoretically predict that the amplitude of the stick-slip motion would grow with time initially due to the amplification factor in Eq (17), and then would spontaneously die away as we passed through the threshold sled speed. This would give us a measurement of the threshold speed, which in turn would allow us to calculate (S - K).

There are a number of practical reasons why this approach would be preferable to present testing methods. First of all, it represents a direct measurement of (S - K), rather than an indirect one by measuring S and K separately. This would improve the precision by about a factor of 5 or so. Secondly, the effects of long-term downward drifting in the signal would be automatically removed, since the amplitude of the osillation in Fig 3 appears to be independent of the fact that the signal is drifting. Thirdly, we would no longer have to rely on the initial transient behaviour of the signal in order to extract our information. It is quite possible that the abnormalities seen in the very first "stick" peak are actually more indicative of mechanical maladjustments in the equipment than they are of any paper-related properties, and it would be desirable to begin data acquisition only after this transient behaviour was gone.

CONCLUSION

It is quite feasible to quantitatively analyze the stick-slip oscillations and extract all the information that is needed to apply the standard two-parameter model of paper friction. Correction factors can be determined for the effects of sled speed and damping of the oscillator. Whether this is advisable or not is another question entirely. It may be more fruitful to explore some of the more qualitative features of the oscillation, such as their tendency to be strongly velocity-dependent and to disappear under certain conditions.

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STATIC COF	0.35		
KINETIC COF	0.30		
GRAVITY	32 ft/sec^2		
OSCILLATOR FREQUENCY	20 Hz		
SLED SPEED (in./min)	Q	apparent STATIC COF	apparent FREQUENCY (Hz)
3	0.327	0.353	12.7
6	0.654	0.360	17.1
12	1.309	0.382	19.3

Table 1 - Friction Parameters as a Function of Sled Speed (No Damping)

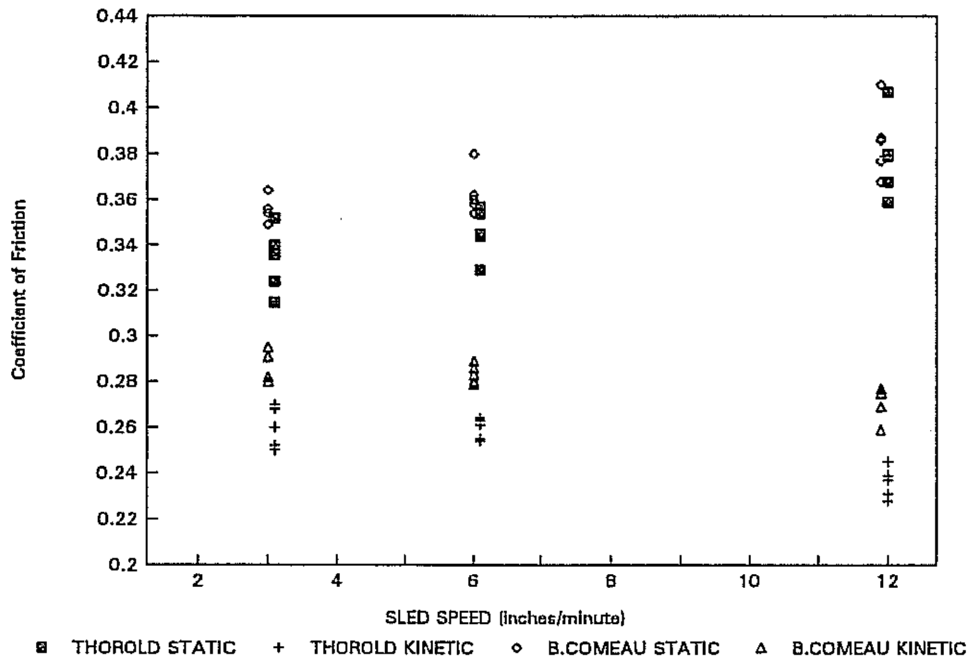

Fig. 1 - Coefficient of Friction versus Sled Speed

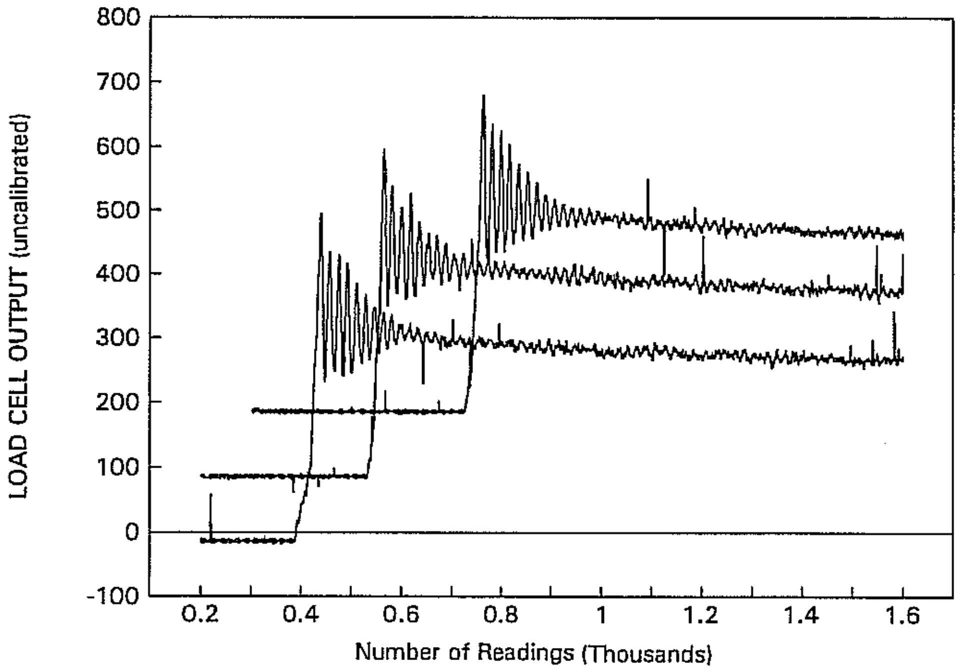

Fig. 2 - Three Thorold Samples : Sled Speed = 12 in./min

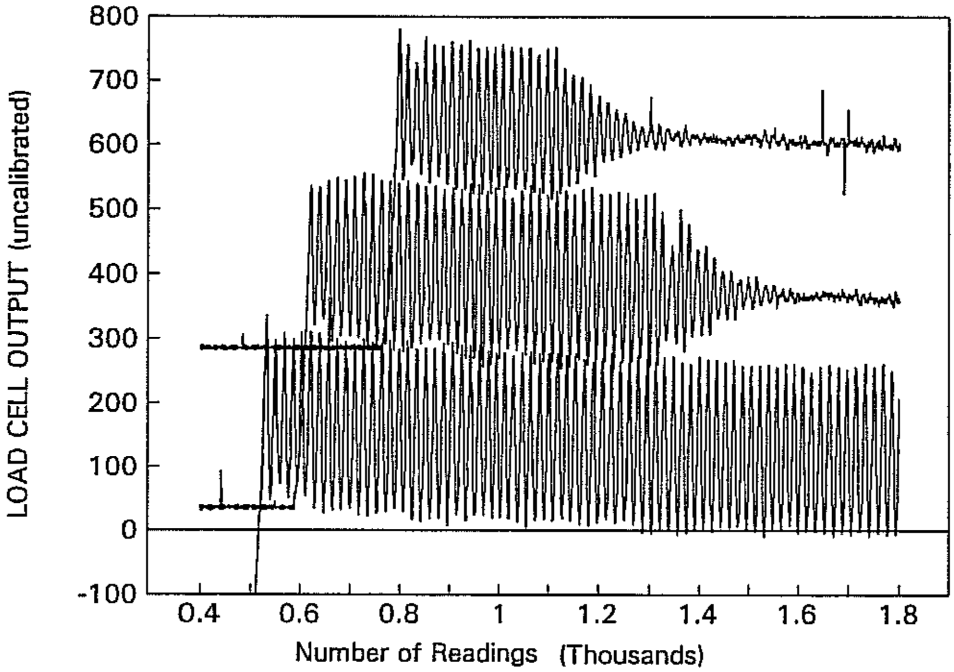

Fig. 3 - Three B.Comeau Samples : Sled Speed = 12 in./min

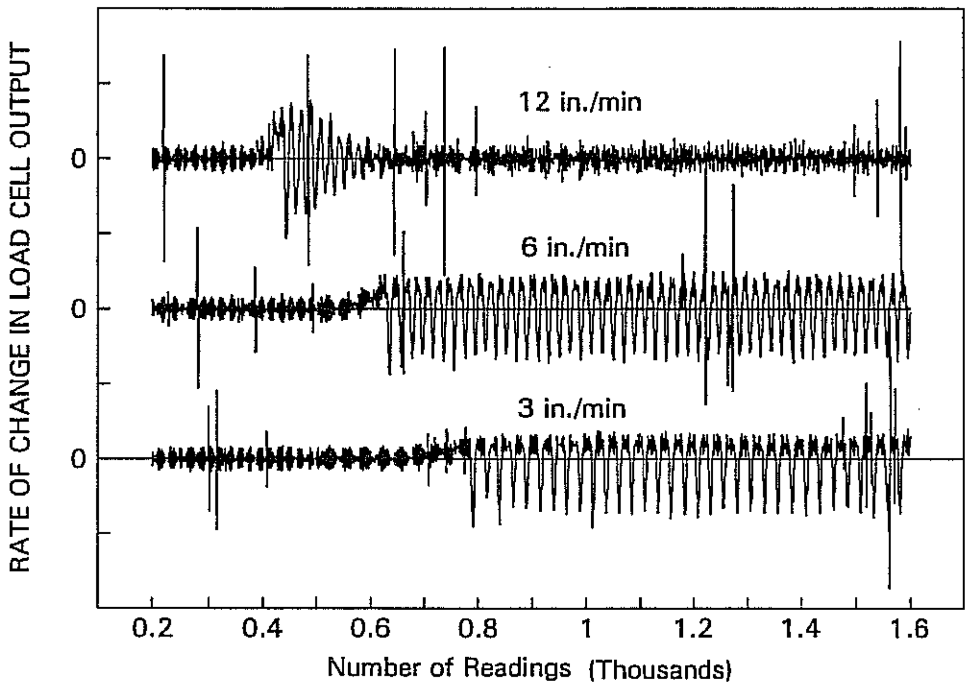

Fig. 4 - Three Thorold Samples at Different Sled Speeds

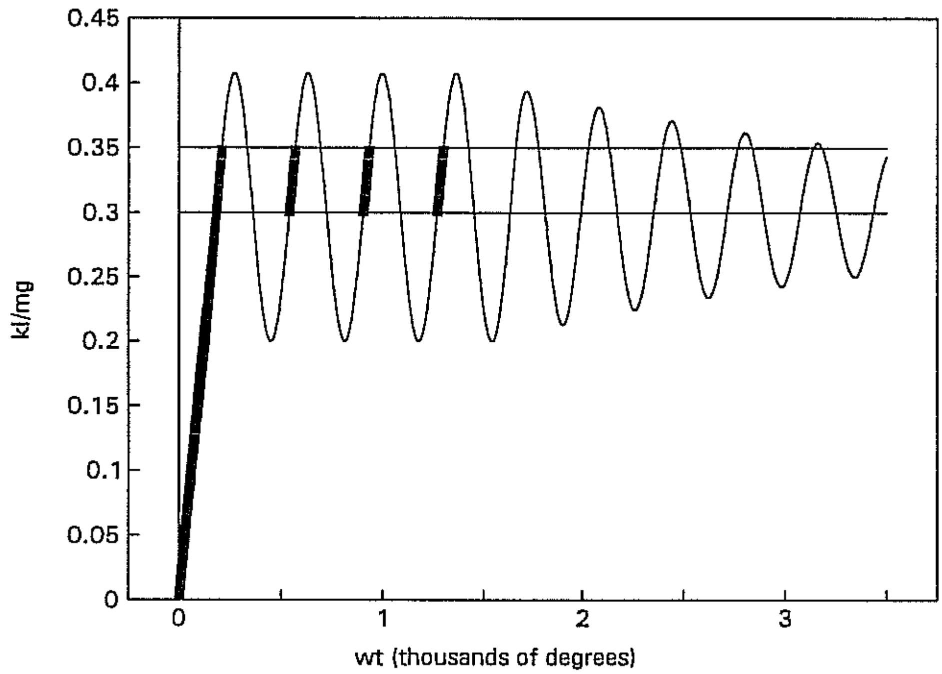

FIG. 5 - Theoretical Calculation of Stick-Slip Function
For the case of a THRESHOLD OSCILLATION

Penner, A.P.
Stick-Slip Behavior of Paper During Friction Testing
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Question - In order for a mechanical oscillation of that sort to take place, there has to be some energy stored and released and some momentum change in sled so it would be kind of an artifact of that particular tester to be drawn with a steel cable or band or drawn with a nylon cord. Could you say any more about how the tester is supposed to be hooked up?

Answer - Well, it's a TMI Tester. What you're doing, is setting the oscillator when you give it the initial jerk to speed up the sled. You're exciting the oscillator by a certain amount. That first linear line is setting the oscillator, then after it starts to slip it's on its own. But the initial energy comes from the initial jerk. This will certainly be very much machine dependent. If you change the mechanical coupling, you could get a much higher frequency. And I expect that under those conditions, you might not see the oscillations at all. We've gone to the other extreme, deliberately using a very lose coupling like, a rubber band and then you would see the oscillations become slow and gradual and become much more pronounced than they are now. The strength or rigidity of the coupling is entirely up to choice. The point is, do we want to encourage this behavior or don't you? I guess what I'm suggesting is that we should try to encourage it because it's quite interesting.

Thank you.