ON THE WEB TENSION DYNAMICS IN AN OPEN DRAW

by

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ABSTRACT

Increasing productivity of paper machines, i.e. speeds, and requirements on decreasing basis weights of paper grades demand a deeper insight of the web tension behavior in the open draws. Runnability of a paper machine is often determined by the web's sensitivity to breaks in the press and dryer sections, where the wet paper web has not yet reached its full strength. Often the measurement of variations in web tension is possible, but determining the factors causing these tension changes is difficult. Mathematical models offer an efficient way to study qualitatively time-dependent physical phenomena in the web.

The aim of this paper is to study web dynamics in an open draw with the help of an advanced mathematical model by including flexural rigidity of the web and geometrically having rolls at both ends of the open draw. The model is based on two-dimensional finite element method (FEM), and both longitudinal and transverse forces have time-dependence in the open draw. FEM implementation of the model makes it possible to study phenomena of the moving web in a detailed manner. Different portions of the total forces affecting the web (inertial and external forces) can be calculated at desired points of the web. As a result, we obtain temporal tension distribution and web deflections. The FEM method offers versatile possibilities to study the responses of different external disturbances at different velocity levels and with different open draw lengths.

The results show that the radius of the roll has a significant effect on the web's behavior in terms of temporal length variation of the open draw. In this case, the open draw has nonconstant boundary conditions, and thus the natural frequencies of the web depend on the length and tension distribution of the open draw. This phenomenon is accentuated especially in cases where the effect of the added mass (air mass) has been included in the model and in cases where the velocity of the web is high. One of the important dynamic phenomena revealed by the model is the rapid changing of web-roll contact points at both ends of the open draw. Due to this, longitudinal and transverse force components undergo rapid changes which could cause web breaks. In the future, the main goal is to make experiments on the model in such a case where the web runs from roll to roll. One of our intentions is also to refine the model by a more realistic material assumption by taking into account the nonlinear behavior of the paper and also expanding the model in a third dimension. With aid of these steps, it is possible to obtain further information of time-dependent stress distributions on the web.

NOMENCLATURE

[B]	kinematic matrix
с	wave velocity. m/s
С	dissipation coefficient
[C]	damping-form matrix, Ns/m
D	flexural stiffness, Nm
d	width of the web, m
E	Young's modulus, N/m ²
F	force, N
g	acceleration of gravity, m/s ²
Ι	cross-sectional property, m ⁴
k	wave number
[K]	stiffness matrix, N/m
m	basis weight of the web, kg/m ²
[M]	mass matrix, kg
L	length, m
[N]	shape function matrix
Δp	pressure difference, Pa
R	radius of the curvature, m
t	time, s
u	horizontal displacement, m
v	tangential velocity, m/s
V	volume, m ³
w	vertical displacement, m
Т	total tension of the web, N/m
х	horizontal coordinate, m
Z	vertical coordinate, m
Θ	(theta) angle of web tangent and horizontal axis, deg.
Φ	(phi) angle of force resultant and horizontal axis, deg.
ρ	(rho) density, kg/m ³
ω	(omega) angular frequency, 1/s

Subscripts

а	air
b	bending
d	drag (air)
е	element
i	time step

1	longitudinal
S	stress (tensile) stiffness
t	transverse
V	volume
v	velocity

superscripts

	1st time derivative
••	2nd time derivative
Т	matrix transpose

1. INTRODUCTION

The speed increase in different web runnability conditions is one of the most demanding tasks in development of a high speed paper machine. This increase aggravates many web handling situations and puts new demands on web strength and quality. Therefore, it is important to know the dynamic behavior of the web. The effect of many physical phenomena changes radically due to velocity increase. This situation is accentuated especially in open draws where the web travels without support. However, the open draws facilitate the control of the web tension. With the model introduced below, it is possible to achieve qualitative information about the dynamic behavior of the open draw in runnability geometry. The effects of different external disturbances and runnability geometries can be studied. The FEM (Finite Element Method) implementation of the model makes it possible to solve this model using many commercial FEM codes. Thus, the limit of the model's versatility is also the limit of the features of the FEM program.

2. MATHEMATICAL MODEL

2.1 Tension model

The foundation of this model is in previous paper considering web transfer dynamics in an open draw (1). In this paper, the web tension T was expressed in the following form:

$$T = \Delta pR + mgR + mR\cos\Theta\left(\frac{\partial^2 w}{\partial t^2} + 2\frac{d u}{d t}\frac{\partial^2 w}{\partial x \partial t} + \left(\frac{d u}{d t}\right)^2\frac{\partial^2 w}{\partial x^2}\right) - \frac{1}{2}C_d \rho_a R\left(\left(\frac{d w}{d t}\right)^2\cos^2\Theta - \left(\frac{d u}{d t}\right)^2\sin^2\Theta\right)$$
(1)

In above equation velocities $\frac{d u}{d t}$ and $\frac{d w}{d t}$ can be written:

$$\frac{d u}{d t} = v \cos \Theta - \frac{\partial w}{\partial t} \sin \Theta \cos \Theta \quad , \quad \frac{d w}{d t} = v \sin \Theta + \frac{\partial w}{\partial t} \cos^2 \Theta \tag{2}$$

Local radius of the web curvature and trigonometric functions can be expressed in the following form:

$$R = \frac{\left(1 + \left(\frac{\partial w}{\partial x}\right)^2\right)^{\frac{3}{2}}}{\frac{\partial^2 w}{\partial x^2}}, \cos \Theta = \frac{1}{\sqrt{1 + \left(\frac{\partial w}{\partial x}\right)^2}}, \sin \Theta = \frac{\frac{\partial w}{\partial x}}{\sqrt{1 + \left(\frac{\partial w}{\partial x}\right)^2}}$$
(3)

If we substitute eqs. 2 and 3 in eq. 1 we get full nonlinear equation for transverse vibration in case of ideally flexible moving web:

$$\frac{\mathrm{T}\frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x}^{2}}}{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}} = (\Delta \mathrm{p} + \mathrm{mg})\sqrt{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}} + \mathrm{m}\frac{\partial^{2} \mathrm{w}}{\partial \mathrm{t}^{2}} + \frac{2\mathrm{m}\left(\frac{\partial \mathrm{w}}{\partial \mathrm{t}^{2}}\right)^{2}}{\sqrt{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}} + \frac{\partial \mathrm{w}}{\partial \mathrm{t}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}}}{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}}\right) \frac{\partial^{2} \mathrm{w}}{\partial \mathrm{x} \partial \mathrm{t}} + \frac{\mathrm{m}\left(\frac{\partial \mathrm{w}}{\partial \mathrm{t}^{2}}\right)^{2}}{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}} + \frac{2\mathrm{w}\frac{\partial \mathrm{w}}{\partial \mathrm{t}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}}}{\left(1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}\right)^{\frac{2}{2}}} + \frac{\mathrm{m}\left(\frac{\partial \mathrm{w}}{\partial \mathrm{t}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}}\right)^{2}}{\left(1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}\right)^{\frac{2}{2}}} - \frac{1}{2}\mathrm{C}_{\mathrm{d}} \mathrm{\rho}_{\mathrm{a}} \left(\frac{\left(\frac{\partial \mathrm{w}}{\partial \mathrm{t}}\right)^{2}}{\sqrt{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}}} \left(1-\frac{2\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)}{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}}\right) + \frac{2\mathrm{w}\frac{\partial \mathrm{w}}{\partial \mathrm{t}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}}}{1+\left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)^{2}}\right)$$
(4)

If we assume that the web undergoes only small deflections, the rotation angles are small and, therefore, we can write the following:

$$\frac{\partial w}{\partial x} = \tan \Theta = 0 \tag{5}$$

Now the eq. 4 reduces to familiar form:

$$\Delta p + mg + m\frac{\partial^2 w}{\partial t^2} + 2mv\frac{\partial^2 w}{\partial x \partial t} + (mv^2 - T)\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}C_d \rho_a \frac{\partial w}{\partial t} \left| \frac{\partial w}{\partial t} \right| = 0 \quad (6)$$

It can be observed that in its full nonlinear form, the behavior of the moving web is very complicated. If the web undergoes large deflections, the transverse velocity of the web (dw/dt) dominates the behavior while the effect of the web's normal longitudinal velocity is smaller. It is also important to notice that the aerodynamic drag term (4th row in eq. 4) is independent on longitudinal velocity in this small deflection approximation.

If the width of the web is much smaller than its length, the added mass (air mass) coefficient is $m_a = \pi p_a d/4$ (4). Now eq. 6 can be written:

$$\Delta p + mg + (m + m_{a})\frac{\partial^{2} w}{\partial t^{2}} + 2(m + m_{a})v\frac{\partial^{2} w}{\partial x \partial t} + ((m + m_{a})v^{2} - T)\frac{\partial^{2} w}{\partial x^{2}} + \frac{1}{2}C_{d}\rho_{a}\frac{\partial w}{\partial t}\left|\frac{\partial w}{\partial t}\right| = 0$$
(7)

2.2 Finite Element Approximation of the Model

When we solve time-dependent FEM approximation, the matrix equation always has the following form (5):

$$[\mathbf{M}]\left\{\frac{\mathrm{d}^{2} z}{\mathrm{d} t^{2}}\right\} + [\mathbf{C}]\left\{\frac{\mathrm{d} z}{\mathrm{d} t}\right\} + [\mathbf{K}(z)]\{z\} = \{\mathbf{F}(t)\}$$
(8)

Here, variable z denotes all necessary time-dependent variables (degrees of freedom).

In normal linear cases, the stiffness matrix [K] is not dependent on displacements $\{z\}$ but, in our case, the nonlinear tension and web-roll contact behavior demand the use of displacement-dependent stiffness matrix. Due to the method's universality we could simultaneously solve both longitudinal (2) and transverse dynamical behaviour. The equations for these cases are:

Longitudal:

$$\rho \frac{\partial^2 u}{\partial t^2} + 2\rho v \frac{\partial^2 u}{\partial x \partial t} + (\rho v^2 - E) \frac{\partial^2 u}{\partial x^2} = 0$$

(9)

Transverse:

$$(m+m_{a})\frac{\partial^{2} w}{\partial t^{2}} + 2(m+m_{a}) v \frac{\partial^{2} w}{\partial x \partial t} + ((m+m_{a}) v^{2} - T) \frac{\partial^{2} w}{\partial x^{2}} + D \frac{\partial^{4} w}{\partial x^{4}} + \frac{1}{2} C_{d} \rho_{a} \frac{\partial w}{\partial t} \left| \frac{\partial w}{\partial t} \right| = f(x,t) \sqrt{1 + \left(\frac{\partial w}{\partial x}\right)^{2}}$$

Each equation term above can be transformed to matrix form. Mass, gyroscopic (3) and damping matrices can be written respectively:

Mass matrix:

$$\begin{bmatrix} \mathbf{M} \end{bmatrix} \left\{ \frac{\mathrm{d}^{2} \mathbf{z}}{\mathrm{d} t^{2}} \right\} = \left(\rho \int_{\mathbf{V}} \begin{bmatrix} \mathbf{N} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \mathbf{N} \end{bmatrix} \mathrm{d} \mathbf{V} \right) \left\{ \frac{\mathrm{d}^{2} \mathbf{z}}{\mathrm{d} t^{2}} \right\} \iff (\mathbf{m} + \mathbf{m}_{a}) \frac{\partial^{2} \mathbf{w}}{\partial t^{2}}, \ \rho \frac{\partial^{2} \mathbf{u}}{\partial t^{2}}$$
(10)

Gyroscopic and

and damping matrix:
$$[C] \left\{ \frac{dz}{dt} \right\} = \left([C]_{v} + [C]_{d} \right) \left\{ \frac{dz}{dt} \right\} \Leftarrow$$

$$2v(m+m_{a}) \frac{\partial^{2} w}{\partial x \partial t} , 2\rho v \frac{\partial^{2} u}{\partial x \partial t} , \frac{1}{2}C_{d} \rho_{a} \left| \frac{\partial w}{\partial t} \right| \frac{\partial w}{\partial t}$$

$$(11)$$

Tensile stiffness and bending stiffness matrices as well as the nonlinear portion of the force vector (tangent stiffness matrix) are included in full stiffness matrix. With tangent stiffness matrix it is possible to update the web's force balance between the time steps and obtain time-dependent tension and system frequency behavior:

Stiffness matrix:
$$[K(z)]\{z\} = ([K(z)]_s + [K(z)]_b)\{z\} \iff$$

 $((m + m_a)v^2 - T)\frac{\partial^2 w}{\partial x^2}$, $(\rho v^2 - E)\frac{\partial^2 u}{\partial x^2}$, $f(x, t)\sqrt{1 + (\frac{\partial z}{\partial x})^2}$, $D\frac{\partial^4 w}{\partial x^4}$ (12)

Force vector:

 $\{F\} = f(x,t)$

Furthermore, the components of the damping and stiffness matrices are:

Gyroscopic inertia matrix:
$$[C]_{v} = mv \int_{V} \left([N]^{T} \frac{d[N]}{dx} - \frac{d[N]^{T}}{dx} [N] \right) dV$$
Drag – damping matrix:
$$[C]_{d} = \frac{1}{2} C_{d} \rho_{a} \int_{V} \left([B]^{T} [B] \right) dV \left| \frac{\partial w}{\partial t} \right|$$
Stress stiffness matrix:
$$[K]_{s} = (m v^{2} - T) \int_{V} \left(\frac{d[N]^{T}}{dx} \frac{d[N]}{dx} \right) dV$$
Bending stiffness matrix:
$$[K]_{b} = \int_{V} \left([B]^{T} [E] [B] \right) dV$$
(13)

Kinematic matrices: $[B]_{t} = \frac{d[N]}{dx}$ (longitudinal) $[B]_{t} = \frac{d^{2}[N]}{dx^{2}}$ (transverse)

In FEM procedure, we normally define transformation matrices and with the help of these, the local element matrices are calculated and, finally, the global mass, damping and stiffness matrices are formulated. It has to be noted that, in structural problems, all matrices are normally positive-definite. However, gyroscopic inertia matrix is in its basic position (i.e. element longitudinal axis is parallel with x-axis in cartesian coordinate system) semipositive-definite but in all other positions negative-definite.

Two-dimensional (6 d.o.f.) gyroscopic inertia matrix with zero diagonal terms and skew-symmetric form is presented as follows:

$$m d v \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & L_{e}/5 & 0 & 1 & -L_{e}/5 \\ 0 & -L_{e}/5 & 0 & 0 & L_{e}/5 & -L_{e}^{2}/30 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -L_{e}/5 & 0 & 0 & L_{e}/5 \\ 0 & L_{e}/5 & L_{e}^{2}/30 & 0 & -L_{e}/5 & 0 \end{bmatrix}$$
(14)

The effect of the web-roll contact behavior cannot be expressed in explicit form but it can be calculated between time steps using the same iterative procedure as used in determining web tension behavior. The main principle of the contact procedure can be seen in fig. 1. This procedure is used between time steps and it demands an iterative solution method since the use of the pseudoforces also affects to the displacements of the other nodes in the web model. The intensity of the pseudoforces is linearly dependent on the spring constant (Young's modulus) of roll surface and thus also on the amount of penetration (6).

2.3 Model Solution

The development of mass, damping and stiffness matrices obeys normal FEMprocedure. All different assumptions of shape functions can be used. For this reason, the matrix formulation expressed in eqs. 10 and 13 is also applicable in three-dimensional cases.

The solution of the model is divide into two parts: displacement solution of the steadystate system and solution of the non-steady state (time-dependent) system. Details of the solution procedure go separate ways in different FEM codes but the main principle remains the same:

> - Firstly, nonlinear tensile-stiffened contact problem is solved in order to find out the initial displacements. The usual iterative solution method is Newton

Rhapson.

 Secondly, the time-dependent displacements caused by desired disturbance are solved by using some explicit or implicit time-stepping method.

- Between each step, the nonlinear analysis is repeated to determine the force balance of the system.

To find out the tension behavior on the basis of solved displacements, nodal force components have to be calculated. This computation takes place at the end of every time step as follows:

$$\left\{f\right\}_{i} = \left[K(z)\right]_{i} \left\{z\right\}_{i} + \left[C\right] \left\{z\right\}_{i} + \left[M\right] \left\{z\right\}_{i}^{**}$$
(15)

Tension can be calculated from nodal force components using the principle expressed in fig. 2. By using force components (F_x and F_y), the force resultant F_r can be calculated, and when the angles Θ and Φ are known, the tangent-directional tension T can easily be determined.

3. DEFINITION OF EXAMPLES

As an example we solve a model presented in fig. 3. In this case we study the problem at two velocity levels, v = 0 m/s ja v = 15 m/s. The length of the open draw is 0.9 m and primary web tension was T = 300 N/m. The radii of both rolls are 0.9 m. Basis weight of the web is assumed to be 50 g/m^2 and the thickness of the web is $70 \mu\text{m}$.

In fig. 3, the model is in the steady-state. A disturbing force of 50 N with a duration of 0.05 s is directed at distance of 0.35 m from the beginning of the unsupported web. The duration of the simulation is 0.25 seconds. Correspondingly, points A, B and C present the starting, disturbance and ending points of the web model in steady-state situation. Web-roll contact situation are assumed to be frictionless.

4. EXAMINATION OF THE RESULTS AND DISCUSSION

In the stationary case (0 m/s), both sides around the target point deflect in symmetrical form at the beginning of the disturbance. Even if the effect of the deviating force ends, web will continue its vibrational movement due to normal inertial forces. In this stationary case, the vibration exhibits quite normal string mode. In fig. 4, deflection state at t = 0.1 seconds from the beginning of the simulation is presented. Due to the non-constant boundary conditions, the point of maximum deflection moves sideways depending on highest or lowest deflection situation.

Important feature of the web model behavior is related to tension changes as a result of longitudinal and transverse force components acting in a periodical manner in the open draw. Fig. 6 presents the tension behavior in points A, B and C (from up to down) of the web model in the stationary case. All these figures include periodically repeating tension peaks which are caused by traveling transverse force distribution. In this case, the distribution travel speed can be approximated as $c \approx \sqrt{T/(m + m_n)}$ (ref. to appendix) and

thus the time between tension peaks is approximately twice as long as force distribution's travel time in the other direction. However, it has to be noted that the travel time is dependent on the tension distribution state and on the length of the open draw and therefore the intervals between tension peaks are not exactly equally spaced.

Tension peaks are caused by high-frequency vibration modes between web and roll surface. According to the principal expressed in fig. 3 longitudinal and transverse force components are connected with tension and thus traveling transverse force distribution entering the other end of the open draw reflects back. In the reflecting region, the web is forced to vibrate with connecting roll and then the kinetic energy of the vibrating web is concentrated on a small and closed area and therefore causing tension peaks described earlier. In this case, the web model has not internal friction or viscoelastic material behavior and so the results are only qualitative. Also, in normal cases

At velocity level v = 15 m/s, the behavior of the web is somewhat different. Gyroscopic effects take place by causing phase shifts between separate points of the web and thus we obtain a fluttering-type deflection during simulation. In fig. 5, deflection state of the web model at t = 0.1 seconds from the beginning of simulation is presented.

In fig. 7 is again tension behaviour in points A, B and C of the web at velocity level 15 m/s. The same periodical manner can be also examined in these cases but due to unequal wave velocities in opposite directions and centripetal velocity affecting on the web, the interval between tension peaks is somewhat longer than in the previous example.

Due to fast entry of the transverse force distribution to right-side end of the open draw, rapidly changing web-roll contact forces can be observed almost immediately. Reflection of the force distribution (and also wave form) demands the wave to travel in an opposite direction compared to the web's velocity. Therefore, the transverse force distribution lingers longer time in web-roll contact region causing more tension peaks to web. Again, it has to be noted that the wave velocity is dependent on web tension and, therefore, analytically calculated value of c is only approximate.

It also has to be noted that normal web-roll friction feature can change tension behaviour described earlier causing local buckling or wrinkling behavior in the web.

5. SUMMARY

In previous cases, we have only discussed few features concerning this model. Preliminary examples above demonstrate that possibilities to utilize this kind of runnability model are very extensive. For example, different kinds of time-dependent disturbing forces or force distributions can easily be installed in the model. The nature of disturbances can be longitudinal or transverse and the shape of these forces is dependent on the versatility of the finite element method program.

These cases show that disturbances affecting the web in the open draw cause remarkable tension changes at both ends of the open draw. In these regions, the rapid web-roll contact dynamics causes tension changes through the coupling of the force components. Nonlinear tension behavior also makes it possible to study the frequency behavior of the model. Velocity effects can be obtained by using gyroscopic inertia matrix and thus it is possible to use normal stationary FE-models in problems where these effects are important.

Because the matrix equations are based on volume integrals, this modeling can be done in three-dimensional form using available commercial finite element programs.

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Fig. 1 Nonlinear web-roll contact situation. Surface effect is described by using pseudoforces F_{px} and F_{py} in contact area. Forces are non-existent until web (a node in web's FE-model) tries to penetrate roll surface.



Fig. 2 Linkage between calculated nodal force components $(F_x \text{ and } F_y)$ and web tension (T).



Fig.3 Example model. Web was assumed to be narrow compared to its length. Length of the open draw was 0.9 m.



Fig. 4 Web deflection at velocity v = 0 m/s at time t = 0.1 s from the beginning of simulation.



Fig. 5 Web deflection at velocity v = 15 m/s at time t = 0.1 s from the beginning of simulation.









Figs. 6 a, b and c. Tension behavior of stationary (0 m/s) web respectively in points A, B and C in fig 3.







Figs 7 a, b and c. Tension behavior at 15 m/s velocity respectively in points A, B and C in fig 3.

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APPENDIX

Nonlinear flutter equation can be achieved from eq. 1 by taking inertial, coriolis and centrifugal terms in their nonlinear form and adding the portion of air mass (slender web assumption):

$$\mathbf{T} = (\mathbf{m} + \mathbf{m}_{a}) \mathbf{R} \cos \Theta \left(\frac{\partial^{2} \mathbf{w}}{\partial t^{2}} + 2 \frac{\mathrm{d} \mathbf{u}}{\mathrm{d} t} \frac{\partial^{2} \mathbf{w}}{\partial \mathbf{x} \partial t} + \left(\frac{\mathrm{d} \mathbf{u}}{\mathrm{d} t} \right)^{2} \frac{\partial^{2} \mathbf{w}}{\partial \mathbf{x}^{2}} \right)$$
(16)

If we assume that the solution of above equation is arbitrary function, we obtain the following:

$$w(x,t) = f(\omega t - k x)$$
(17)

By substituting d u/d t from eq. 2 to eq. 16 and constructing the derivatives the simplification will result:

$$T = (m + m_{\mu}) \left(v - \frac{\omega}{k} \cos \Theta \right)^2$$
(18)

Furthermore, $(\omega/k) = v + c$ and if we assume that web only undergoes small deflections, we finally obtain the following equation for wave velocity:

$$c = \sqrt{\frac{T}{m + m_a}}$$
(19)

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Question - Looking at your force balance there, I'm not sure I understand it, but the one thing I'm looking for is the adhesion of paper to the surface. What paper will adhere either to the pressed roll or to a drier surface. Is that incorporated into your model? Does it take into account that the paper is sticking?

Answer - You mean the friction behavior of that model?

Question - Adhesion.

Answer - No, it is not included in this model.

Question - That seems an important boundary condition because that's going to determine your dynamics.

Answer - That is true. It is a limitation of this solving method. Adhesive differences often leads to numerical problems and relates very closely to stability analysis of these webs and this model is not capable yet of performing these kinds of things. And so in this model there is no adhesive behavior included.

Question - I have another question. You have a number of material parameters that are in this model., for instance, Young's modulus. What paper is very viscoelastic. And Young's Modulus depends on the strain rate. Where do you get these parameters?

Answer - Yes, it is another thing not included in this model. For example, the height of the extension peaks is closely related to the viscoelastic behavior. In this case, we have only generalized general linearly elastic material law. This is also the feature not included in this model. The main thing is only to study the basic behavior in the qualitative sense in this model.

Question - Have you validated this model on a commercial paper machine or pilot paper machine?

Answer - We haven't done the tension measurements, but we have used high speed cameras where we have photographed the behavior with these kinds of disturbances effecting the web and in this case it is exactly the same behavior.

Question - What kinds of disturbance have you taken into account on your system-internal or external?

Answer - In this case, I have only used an example. These two problems are identical. In this case, I have only used...what do you mean?

Question - What kinds of disturbances?

Answer - It is only a force peak. I tried to be very simple, because if I included many disturbances, the behavior is very complicated.

Comment - Thank you.

Question - I have a simple question. Your tensions in Newton's per meter . I assume we need the width to figure out how that 50 Newton's of force is proportional to the Newton's in tension. What is the width of the web in the model?

Answer - This is a very narrow web. This is only a thread line model.

Question - But to get the response of the 50 Newton's into the web and the tension offsetting that, you have Newton's per meter . So I need to know what is the Newton's per meter loading? Your units don't seem to jive. You've got Newton's loading into the web, but the tension is described in Newton's per meter, so I need the width of the web to determine how that's offsetting the Newton force.

Answer - If I remember correctly, the width of the web is 0.6 meters.

Thank you.