COMPUTING WOUND ROLL STRESSES BASED UPON WEB SURFACE CHARACTERISTICS

by

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ABSTRACT

Historically wound roll models have required the input of a radial modulus of elasticity. Heretofore this modulus has been measured using material testing systems which have shown the radial modulus to be dependent upon radial pressure and that it is typically much less than the in-plane moduli of the web. This paper focuses upon a method in which the radial modulus can be predicted based upon surface roughness characteristics. The advantage of this method is that wound roll stresses can be predicted prior to the manufacture of the web. Thus winding strategies can be developed prior to the production of the web. An application of this theory will be presented on polymer webs with experimental verification.

NOMENCLATURE

a_i	radius of circular contact between the ith peak and a flat, nm
A	total area of contact of all peaks which have been compressed with a flat, mm^2
A_i	area of contact between the ith peak and a flat, mm^2
A_o	nominal surface area, mm^2
d	separation between between web surfaces, nm
d_o	separation between web surfaces when subject to little or no pressure, nm
E'	Young's modulus of elasticity for asperities, GPa
$E_{1,2}$	Young's modulus of asperities on surfaces 1 and 2, respectively, GPa
E_r	radial modulus of elasticity of a wound roll, KPa
F	force required to compress all asperities which are contacting, N

F_i force required to compress the ith asperity, N

$F_n(h)$ parabolic cylinder probability expression, defined herein

h standardized surface separation, d/α, dimensionless

h_o standardized separation between web surfaces when subject to little or no pressure, dimensionless

n number of asperities in contact for a given surface separation d

N total number of asperities on a nominal surface area, A_o

P nominal pressure of contact between surfaces, KPa

s normalized peak height

t nominal thickness or caliper of web, μm

z_s summit heights of asperities, nm

$\bar{z}_s$ mean summit height of asperities, nm

α standard deviation of the peak heights on a nominal surface area, A_o, nm

α_{eq} equivalent α for two surfaces in contact, nm

β radius of spherical peak summits, mm

β_{eq} equivalent β for two surfaces in contact, mm

δ asperity deformation, nm

ε nominal strain of asperities deformed between two surfaces and stack strain, dimensionless

$\phi(z_s)$ probability distribution function

$\nu_{1,2}$ Poisson's ratio of asperities on surfaces 1 and 2, respectively, dimensionless

INTRODUCTION

The nonlinear relationship between stress and strain in a stack of web, in a direction normal to the stack, was first documented by Pfeiffer[1] in 1966. Wound roll models which predicted internal stresses became useful and accurate tools for studying the effects of winder and web material parameters after this nonlinear relationship was incorporated. Such models include those which were generated by Pfeiffer [2], Hakiel [3], and Willett and Poesch [4] which allow for nonlinear, anisotropic properties.

The radial modulus of a wound roll is a parameter which encompasses both structural and material nonlinearities. Paper, plastic film, and other webs have asperities upon their surfaces and when the web is wound or stacked asperities from one surface contact asperities upon the next surface. For plastic films and coated papers with low permeability, air may become trapped between the layers in which case the radial modulus becomes a function of the entrained air as well[5,6]. Upon compression, a great deal of strain can occur prior to pressures of significant magnitude being generated. This complicates experimental tests for measuring the radial modulus as it is always difficult to establish the point at which pressure and strain are both zero.

This paper deals specifically for those cases in which entrained air is not a problem. Thus the nonlinearity between stress and strain is assumed to be due to the interaction of asperities from mating web surfaces. Through theories which were previously developed for rough surfaces in contact, relationships for pressure versus strain and for the radial modulus versus pressure will be developed. The accuracy of these relationships will be investigated by comparing them to experimental data obtained via stack tests conducted in a material testing system.

THE THEORY OF ROUGH SURFACES IN CONTACT

Micoparticulates which are often called "fillers" or "slip agents" are added to the resins which are extruded in the forming of polyester, polypropylene, and polyethylene, and other plastic films. These particulates roughen the surface and are added for various reasons which include better web handling properties in web lines and in winders. The following is a condensed rendition of the classical theory of rough surfaces in contact.

One Rough Surface in Contact with a Rigid Plane

This theory was originally developed by Greenwood and Williamson [7] based upon a rough surface in contact with a rigid, flat plane. In Figure 1 a rendition of a profile trace of a rough surface in contact with a flat plane is shown. For contact problems the summits of the surface asperities are of the upmost importance. If the summit heights are denoted as z_s, having a mean $\bar{z}_s$ and a probability distribution function $\phi(z_s)$, which expresses the probability of finding a summit of height z_s in the interval from z_s to z_s+dz_s. If there are N summits on a nominal surface area A_0, the number of summits in contact at a given separation d is given by:

$$\mathbf{n = N \int_d^\infty \Phi(z_s) dz_s} \qquad \{1\}$$

Greenwood and Williamson made a simplifying assumption that all the asperity summits were spherical and were of constant curvature β. When the summit height of an asperity exceeds the separation it will be compressed. The deformation of the asperity will be:

$$\delta = \mathbf{z_s - d} \qquad \{2\}$$

and the contact surface of an asperity with the rigid flat will be a small circular area of radius a_i. The ith summit will have a contact area which is also a function of the deformation per Hertzian contact theory:

$$\mathbf{A_i} = \pi\ \mathbf{a_i^2} = \pi\beta\delta_i \qquad \{3\}$$

and the total real area of contact is:

$$\mathbf{A = N \int_d^\infty} \pi\beta\ \mathbf{(z_s - d)}\ \phi\mathbf{(z_s)\ dz_s} \qquad \{4\}$$

The force, F_i, required to compress an asperity per Hertzian theory is:

$$\mathbf{F_i} = \frac{\mathbf{4}}{\mathbf{3}} \mathbf{E'}\sqrt{\beta}\,\delta^{3/2} \qquad \{5\}$$

and the force associated with deforming the peaks which are contacting over the surface area A_o is:

$$F = \frac{4}{3}E'\sqrt{\beta}N \int_d^{\infty} (z_s - d)\ \phi(z_s)\ dz_s \qquad \{6\}$$

The peak height z_s can be normalized to a new variable *s* by subtracting the mean peak height, $\bar{z}_s$, and dividing by the standard deviation, a. By developing a new standardized separation h which is equal to d/α expressions {4} and {6} become:

$$A = \pi\ N\ \beta\ \alpha\ F_1(h) \qquad \{7\}$$

and

$$F = \frac{4}{3}E'\sqrt{\beta}N\alpha^{3/2}F_{3/2}(h) \qquad \{8\}$$

where:

$$F_n(h) = \frac{1}{\sqrt{2\pi}}\int_h^{\infty} (s-h)^n e^{-\frac{1}{2}s^2}\ ds \qquad \{9\}$$

for a Gaussian distribution of the peak heights. Greenwood and Williamson showed that the surface height distribution is Gaussian to a very good approximation. If a peak density is defined as:

$$\eta = \frac{N}{A_o} \qquad \{10\}$$

the nominal pressure between the rough surface and the rigid plane will be:

$$P = \frac{F}{A_o} = \frac{4}{3}E'\eta\sqrt{\beta}\alpha^{3/2}F_{3/2}(h) \qquad \{11\}$$

Two Rough Surfaces in Contact

Greenwood and Tripp [8] studied the contact of two rough surfaces. They found that as long as the peak-height distribution was Gaussian that it did not matter whether the asperities were upon one or both surfaces. This allowed the use of Greenwood and Williamson's expressions {1-11} for an "equivalent" rough surface in contact with a flat, rigid plane. Thus an "equivalent" set of surface roughness parameters was required such that two rough surfaces could be modeled as one. The equivalent curvature of the asperities in contact was:

$$\frac{1}{\beta_{eq}} = \frac{1}{\beta_1} + \frac{1}{\beta_2} \qquad \{12\}$$

The equivalent standard deviation of the peak heights is:

$$\alpha_{eq} = \sqrt{\alpha_1^2 + \alpha_2^2} \qquad \{13\}$$

When both sides of the web have identical surface roughness parameters, expressions {12} and {13} become:

$$\alpha_{eq} = \sqrt{2}\ \alpha \ \text{ and } \ \beta_{eq} = \frac{\beta}{2} \qquad \{14\}$$

The elastic modulus E' is:

$$\frac{1}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \qquad \{15\}$$

and when the mating surfaces are identical:

$$E' = \frac{E}{2\,(1 - \nu^2)} \qquad \{16\}$$

Substituting expressions {14} and {16} into expression {11} yields:

$$P = \frac{2^{5/4}}{3}\,\frac{E}{1-\nu^2}\,\eta\,\sqrt{\beta}\,\alpha^{3/2}\,F_{3/2}(h) \qquad \{17\}$$

CHARACTERIZATION OF E_r VIA SURFACE ROUGHNESS PARAMETERS

Expression {17} defines the nominal pressure between two surfaces as a function of known material properties and surface roughness parameters and as function of the normalized separation h through the function $F_{3/2}(h)$. The strain in a stack of web is also related to the normalized separation:

$$\varepsilon = \frac{d_0 - d}{t} = \frac{(h_0 - h)\,\sqrt{2}\,\alpha}{t} \qquad \{18\}$$

where d_0 is the separation of the web surfaces when little or no pressure has been applied and t is the web thickness. Expression {18} is an longitudinal strain of the form $\Delta L/L$ where $d_0 - d = \delta = \Delta L$. This equation for strain incorporates an assumption that most if not all the deformation occurs in the contact interface between the mating surfaces. In {18}, h_0 is the normalized separation prior to pressure being applied to the stack. If the peak height distribution is Gaussian in form, the probability is quite low that when h is greater than 3 that any contact exists. Thus expression {18} becomes:

$$\varepsilon = \frac{(3 - h)\,\sqrt{2}\,\alpha}{t} \qquad \{19\}$$

With expressions {17} and {19} pressure and strain can be tabulated as a function of h. The radial modulus, E_r, can be determined by evaluating the slope of the pressure versus strain curve at several locations such that an expression for the radial modulus can be developed as a function of pressure.

RESULTS

This technique was applied to a 23.4 µm polyester film made by ICI(Type S). The surface roughness data was supplied by ICI though the use of their WYKO 3-D surface interferometer. This film has a mean surface roughness of 0.259 µm, the mean radii of the peaks is 0.0385 mm, the density of the asperities is 3480/mm^2, and the standard deviation of the peak heights is 173 nm. The pressure and strain obtained from expressions {17} and {19} are shown with experimental stack test data in Figure 2. At first perusal the discrepancy between the experimental data looks large. Note that the curve has the correct shape but that there seems to be a constant offset in strain between theory and experiments. After reviewing the individual test records from the WYKO interferometer it was found that there were peaks whose heights were greater than three standard deviations and that all but one or two peaks would be included if it was assumed that minimal contact occurred when h, the normalized separation, was greater than 5. Thus equation {19} was modified as:

$$\varepsilon = \frac{(5 - h)\sqrt{2}\,\alpha}{t} \qquad \{20\}$$

The results from the use of expressions {17} and {20} are also shown in Figure 2. The correlation between the theoretical and experimental data is now acceptable. The fallacy in the derivation of expression {19} was that the distribution of peak heights was Gaussian in form, which may not be exact.

Most wound roll models require the radial modulus to be input as some function of pressure and so perhaps the most important matter is how well expressions {17},{19}, and {20} model the radial modulus as a function of pressure and not how well the relationship between pressure and strain is modelled. The radial modulus as a function of pressure as generated by expressions {17},{19}, and {20} and by experimental stack compression tests are shown in Figure 3 and there is now little or no difference between the results of using expressions {19} or {20} to predict the strain. The theoretical modulus is determined by generating tables of pressure and strain data and then using a central finite difference method to estimate the radial modulus at a given pressure. The data shown in Figure 3 was curve fitted using a third order polynomial in radial pressure and using the exponential function first presented by Pfeiffer[9].

In Figure 4 the radial pressures, computed by a wound roll model similar to that documented by Hakiel, are plotted as a function of normalized radius. The model input is shown in Table 1. Results are shown for the case in which the radial modulus was determined using expressions {17},{19}, and {20} and for the case in which E_r was obtained via experimental stack tests. Overlaid upon this figure are radial pressures which were measured using the pull tab technique in the laboratory[9]. All winding results presented in this paper were for web velocities of 15 m/min on laboratory winders which will accommodate webs which are 15.2 cm in width. Each data point shown is the pressure measured from three experimental winding tests and the error bar shows the standard deviation of the data for a small population. The correlation between the pressures predicted by the models and the data obtained through experiments is excellent, no matter if the radial modulus resulted from stack tests or from the theory derived herein.

This technique was applied to a 50.8 μm polyester film made by ICI(Type S). This film has a mean surface roughness of 0.221 μm, the mean radii of the peaks is 0.0429 mm, the density of the asperities is 3362/mm^2, and the standard deviation of the peak heights is 304 nm. The radial modulus predicted per expressions {17} and {20} and per stack tests are shown in Figure 5. The data shown in Figure 5 was curve fitted and the results are shown with other pertinent winding model data in Table 2. The radial pressures which resulted from use of the winding model are shown in Figure 6 and the correlation is excellent.

Finally the technique was applied to another 23.4 μm polyester film made by ICI(Type 377). This film has a mean roughness of 2.12 μm, the mean radii of the peaks is 0.0783 mm, the density of the asperities is 360/mm^2, and the standard deviation of the peak heights is 1.41 μm. The radial modulus predicted per expressions {17} and {20} and per stack tests are shown in Figure 7. The data shown in Figure 7 was curve fitted and the results are shown with other pertinent winding model data in Table 3. The radial pressures which resulted from use of the winding model are shown in Figure 8 and the correlation is excellent.

CONCLUSIONS

This study has shown that the contact theories of rough surfaces developed by Greenwood and Williamson [7] and Greenwood and Tripp [8] can be used to analytically predict the radial modulus of elasticity for use as a wound roll model input. For the three cases studied the technique developed herein performed admirably. There are exceptions however. Films with low surface roughness (5-10 nm) do not appear to be modelled as well using this technique. It is surmised that it is difficult, if not impossible, to perform stack tests upon very smooth films without air becoming captive in pockets which significantly reduces the apparent modulus as is measured in a stack test. This being the case it would also be difficult, if not impossible, to wind "smooth" films at reasonable velocities without air entrainment being present which would again reduce the modulus and the interlayer pressures within the wound roll. Thus the modelling of winding smooth films cannot be disassociated from aspects of air entrainment unless winding within a vacuum coating operation, for instance.

A logical conclusion which can be drawn from this study is that any web whose surface has a Gaussian distribution of peak heights and whose individual peaks can be modelled using Hertzian contact expressions should be amenable to this technique providing air entrainment is not an issue. For this type of study to be applicable to uncoated paper webs would require at least the development of new contact expressions.

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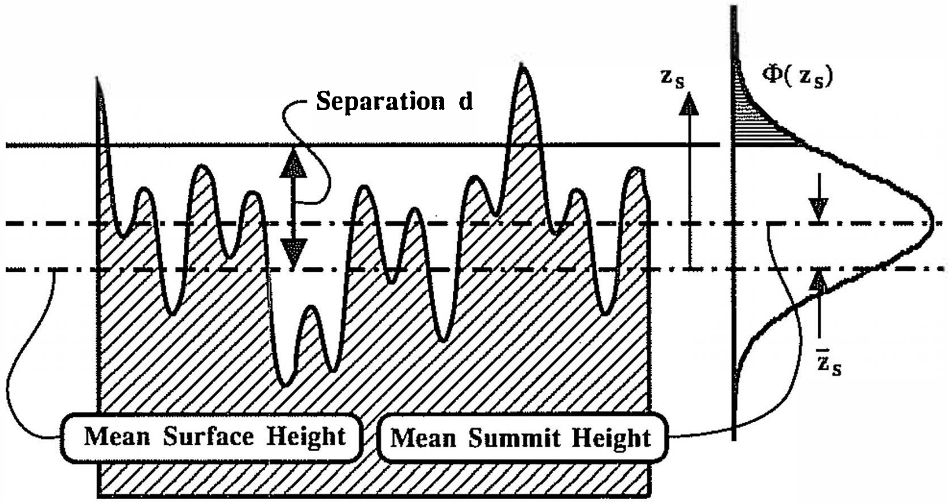

Figure 1. - Contact of a Rough Surface with a "Smooth" Flat

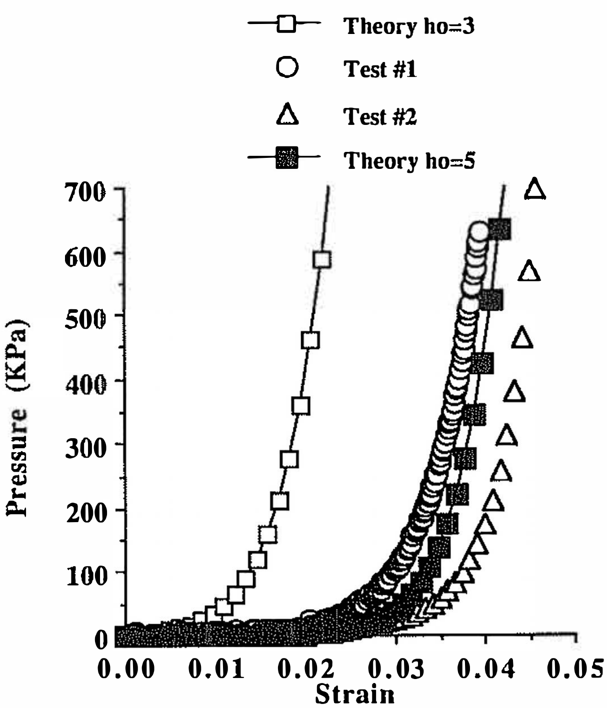

Figure 2. - Normal Pressure vs. Strain for ICI Type S PET 23.4 um Film

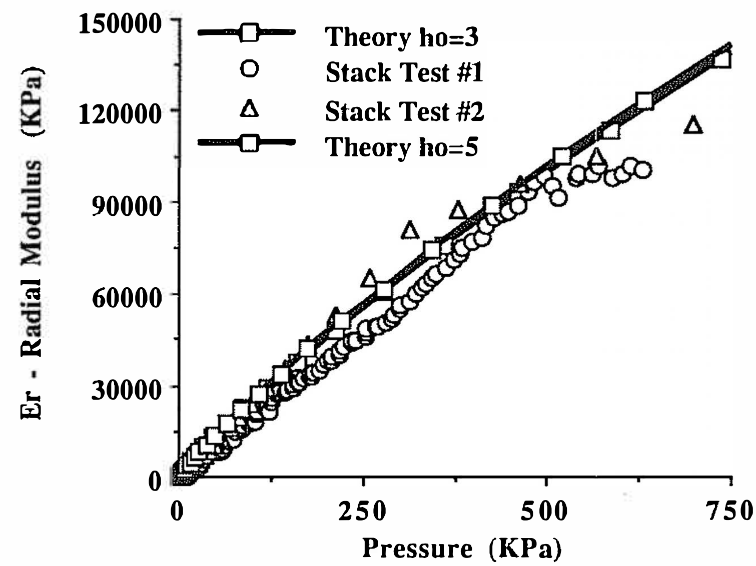

Figure 3. - Normal Modulus vs. Pressure for ICI Type S 23.4 um PET film

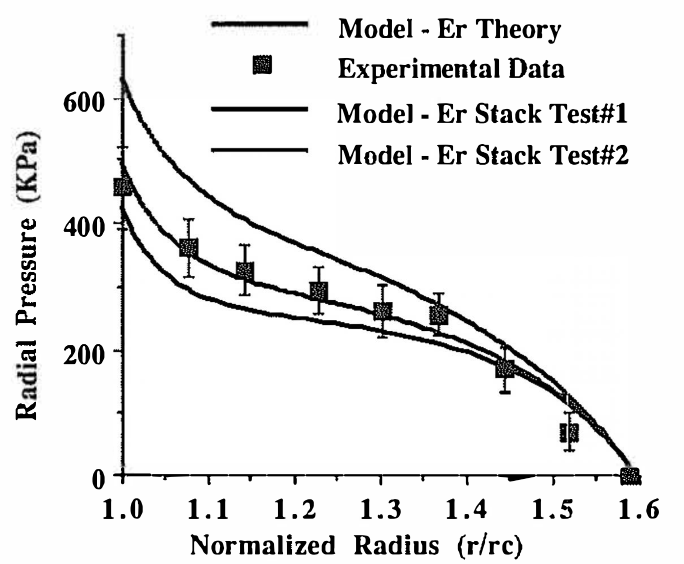

Figure 4. - Radial Pressure for ICI Type S 23.4 um PET Film Centerwound at 3.45 MPa

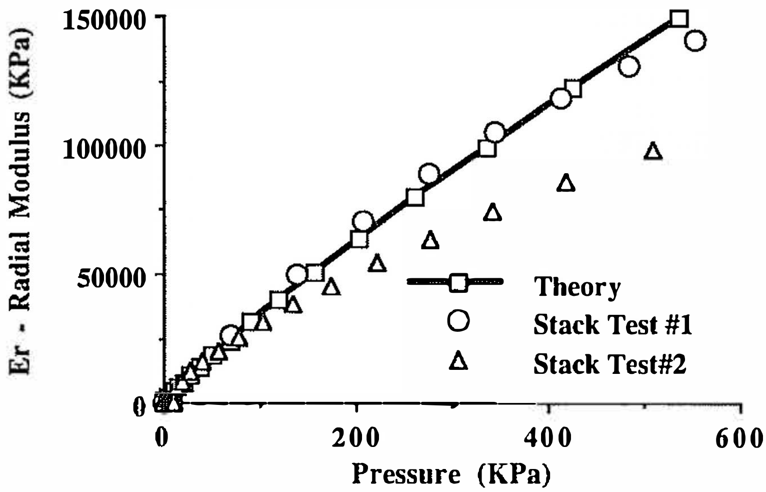

Figure 5. - Normal Modulus vs. Pressure for ICI Type S 50.8 um PET Film

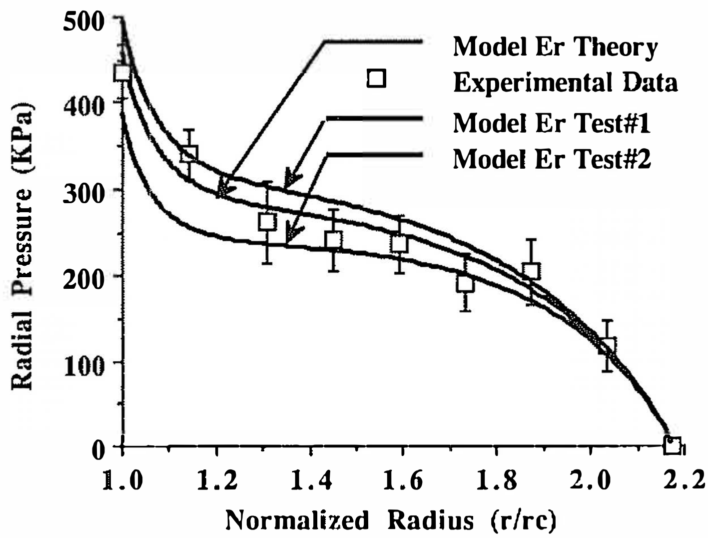

Figure 6. - Radial Pressure for ICI Type S 50.8 um PET Film Centerwound at 2.41 MPa

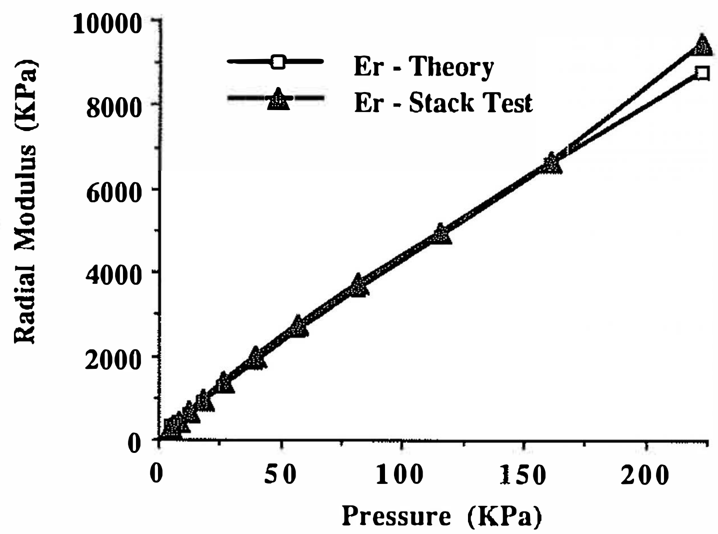

Figure 7. - Normal Modulus vs. Pressure for ICI Type 377 PET Film

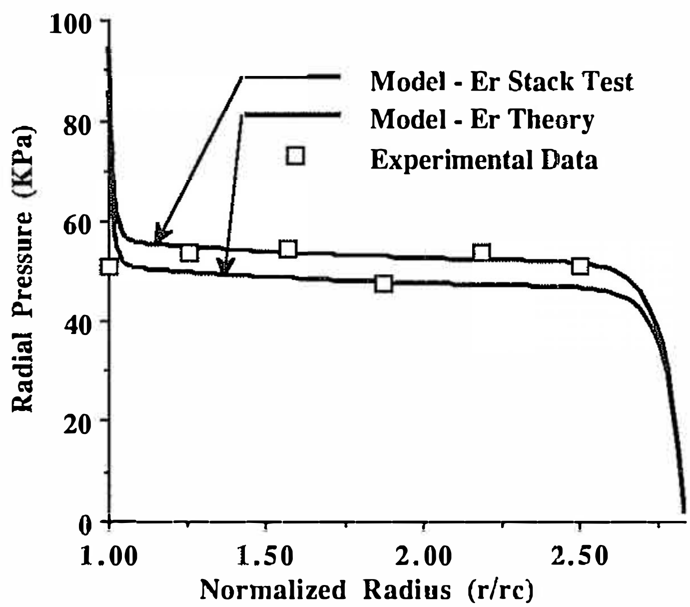

Figure 8. - Radial Pressure for Centerwinding ICI Type 377 PET Film at 2.76 MPa

Outer radius of Wound Roll (cm)	7.12		
Outer radius of Core - r_c (cm)	4.45		
Core Stiffness (GPa)	33.1		
Winding Tension - T_w (MPa)	3.45		
E_θ (GPa)	4.152		
$\nu_{r\theta}$	.01		
	$E_r = C_1\sigma_r + C_2\sigma_r^2 + C_3\sigma_r^3$		
E_r	C_1	C_2	C_3
23.4 µm - Stack Test #1	247.9	.0307	-2.230E-04
23.4 µm - Stack Test #2	171.0	.1541	-2.58E-04
23.4 µm - Theory $h_o=3\alpha$	266.1	-.1946	1.159E-04
23.4 µm - Theory $h_o=5\alpha$	270.0	-.1924	1.11E-04
	$E_r = K_2(\sigma_r + K_1)$		
E_r	K_1	K_2	
23.4 µm - Stack Test #1	.0238	223.9	
23.4 µm - Stack Test #2	.5181	179.1	
23.4 µm - Theory $h_o=3\alpha$	43.89	181.8	
23.4 µm - Theory $h_o=5\alpha$	.0775	215.3	

Table 1. - Winding Properties Required to Model ICI Type S 23.4 µm PET Film

Outer radius of Wound Roll (cm)	9.78		
Outer radius of Core - r_c (cm)	4.45		
Core Stiffness (GPa)	33.1		
Winding Tension - T_w (MPa)	2.41		
E_θ (GPa)	4.152		
$\nu_{r\theta}$	.01		
	$E_r = C_1\sigma_r + C_2\sigma_r^2 + C_3\sigma_r^3$		
E_r	C_1	C_2	C_3
50.8 µm - Stack Test #1	333.1	-2.951	.01397
50.8 µm - Stack Test #2	405	-2.228	-2.58E-04
50.8 µm - Theory $h_o=3\alpha$	266.1	-.1946	4.328E-03
	$E_r = K_2(\sigma_r + K_1)$		
E_r	K_1	K_2	
50.8 µm - Theory $h_o=3\alpha$	.01077	291.7	

Table 2. - Winding Properties Required to Model ICI Type S 50.8 µmPET Film

Outer radius of Wound Roll (cm)	12.15		
Outer radius of Core - r_c (cm)	4.30		
Core Stiffness (GPa)	25.6		
Winding Tension - T_w (MPa)	2.76		
E_θ (GPa)	4.502		
$\nu_{r\theta}$	.01		
	$E_r = C_1\sigma_r + C_2\sigma_r^2 + C_3\sigma_r^3$		
E_r (KPa)	C_1	C_2	C_3
23.4 μm - Stack Test #1	56.16	-.1691	4.815E-04
23.4 μm - Theory $h_o=3\alpha$	50.11	-.0864	1.738E-04
	$E_r = K_2(\sigma_r + K_1)$		
E_r (KPa)	K_1	K_2	
50.8 μm - Theory $h_o=3\alpha$	10.90	37.48	

Table 3. - Winding Properties Required to Model ICI Type 377 23.4 μm PET Film

QUESTIONS AND ANSWERS

Q. How do you measure wound roll pressures?

A. We use two different techniques here. Given the choice, when the pressure is low enough, we use pull tabs, and the pull tab will be a piece of brass shim stock, which is folded over into an envelope and there will be a piece of steel shim stock inside of that, which seems to give a more controlled coefficient of friction than just inserting a tab between the sheets, if you will, of the wound roll. If the pressures are too high you can't physically pull the tabs out of the roll. In such a case, we use force sensitive resistors. There's a discussion of that for the paper that I wrote for the first IWEB conference.

Q. Would you comment upon how stack height influences this?

A. The height of the stack controls the number of interfaces, if you will, that are in the problem. Per the theory, you're only looking at one interface, right? The pressure and strain in that interface. The pressure and stack that we do, we follow a standard that was set by Dave Pfeiffer many years ago, not too many years ago, excuse me, Dave. And the stack is nominally 1 inch, 2.54 centimeters thick. And so, in our stack tests, if you will, the number of interfaces are changing because what we hold in common is the whole pile depth, being 2.5 centimeters.

Q. R/R_c in your experiments is never greater than 3, why?

A. I'll give you a reason. The reason R/R_c does not ever get very big is, number 1 my winder is not very big, and you'll see it this afternoon in the laboratory. The other reason is, to get good results here, you have to wind at pretty low speeds, because if you're center winding, air entrainment becomes a problem at very low speeds. Now, you'll see me present a paper a little later this morning where we begin to encompass those effects of air entrainment, where the speeds become reasonable. But, my students, they don't seem to be very, well, it takes a long time for these rolls to wind. I'll just say that.

Q. Is the difference between E_r and E_T significant?

A. Well, mathematically, I can say that I think the best way to approach this is to think about the inputs to a winding model. And I'll tell you that in terms of sensitivity, the greatest impact on wound roll pressure is whatever the wound-on-tension is on the outer layer. There's two second most important parameters, and one is the in plane modulus, and the other is what you call the Z direction or the radial modulus. Now that comes in as a ratio, if you will, in Hakiel's model of E_t/E_r, and so both parameters are equally important in determining what your wound roll pressures will be, but I'd like to say that even though films are quite a bit stiffer than paper, that ratio of E_t/E_r can easily be 100 to 200.