BUCKLING OF WEBS FROM LATERAL

COMPRESSIVE FORCES

by

J. J. Shelton

Oklahoma State University
Stillwater, Oklahoma

ABSTRACT

Lateral compressive buckling of a web is often evident in troughs along a tensioned span and in corrugations in a wound roll or in the web wrapping a roller.

The lateral compressive forces are not evident from elementary free body diagrams, but arise from microscopic displacements caused by such phenomena as steering of edge tape elements by deflected rollers and an increasing width as the web passes over a driven roller or is expanded by heat, moisture, or viscoelastic memory.

The theory that buckling of webs is caused by lateral compressive forces is supported by comparisons of wavelengths of troughs and corrugations to predictions by classical buckling theory.

The theory of buckling of webs implies that, for inherent avoidance of harmful wrinkling, rollers should be stiff and as smooth as practical, tensions should be as constant as possible throughout the processing machine, and the web should be as dimensionally stable as possible.

NOMENCLATURE

A	amplitude of a sine wave (one-half the peak-to-peak amplitude)
B	spacing between the bearings which support a roller
E	modulus of elasticity (Young's modulus) of the web
E_R	modulus of elasticity of a roller shell
f	a distributed load, force per unit length
f_i	internal force per unit length in the web
f_μ	force per unit length as limited by lateral slippage
I_R	area moment of inertia of a roller
L	the length of a web span between tangent lines on rollers at each end

R	the radius of a roller or a wound roll
T	total tensile force in a web
t	thickness of a web
W	the width of a web
W_b	the width of the buckled portion of a web
W_t	the width of an unbuckled edge tape element
x,y,z	longitudinal, lateral, and normal rectangular coordinates
δ	the total decrease in lateral width
ε	strain
ε_x	longitudinal strain caused by web tension
ε_y	lateral strain [Note: Except for the specific study of the change in width caused by a change in tension across a roller, ε_y does not (and should not) include the effect of longitudinal strain on lateral strain]
λ	the wavelength of the buckled portion of a web
ν	Poisson's ratio
ρ	weight (gravitational force) per unit volume of a wound roll
σ	stress (force per unit area)
σ_x	longitudinal stress caused by tension
σ_y	lateral stress
$\sigma_y)_{cr}$	the amount of lateral stress which causes buckling

INTRODUCTION

Historical Background

Uniform waves across a tensioned free span or across the surface of a wound roll of a thin web have been a mystery for many decades. Free-span waves have been described as "troughs" or "fluting," while waves across a wound roll have been called "corrugations," "ridges," or "bands." The term "corrugations" is appropriate for either case, in light of their similarity to sheets of old-fashioned corrugated metal and metal pipes which were made by rolling corrugated sheets into pipes, with the corrugations running in the circumferential direction.

The quest for understanding of corrugations has led to many fruitless and sometimes amusing theories. Lanes of varying thickness, "gage bands," have been proposed as a cause, but corrugations of uniform wavelength are seldom traceable to gage variations.

One engineering manager, without checking to find that the wavelengths of corrugations in a wound roll were not the same for different thicknesses of webs and different radii of rolls, speculated that corrugations were the result of gage bands caused by viscoelastic memory of the individual filaments of material which were squeezed through a filter-screen backup plate, and consequently had a backup plate built with a different number of holes.

A similarly futile project was initiated by the author in 1988. After finding that the wavelengths of corrugations were predicted reasonably well by theory of buckling of plates and shells, he speculated that surface tensions of thin films might be large enough to cause enough compression of the body of the film in the absence of external tension in the lateral direction to cause buckling as an Euler column. The project was abandoned in its preliminary stage after it was found that buckling did not occur in a macro-model consisting of a relatively thick rubber sheet which was bonded to an inflated toy balloon on each surface. After the balloons had bonded they were burst and the excess trimmed off, so that the skin of rubber under tension simulated the surface tension of a thin film. The reason that buckling did not occur was that the tensions on the two surfaces produced no moment, whereas the line of action of the force on a structural column does not remain centered within the column.

In another project by engineers for a film manufacturer, an equation for a beam on an elastic foundation seemed to predict the wavelength of corrugations in a free span, but there was no physical explanation of the nature of a loading on the web.

Another theory was that corrugations were really shear buckles caused by the common case, at least for bidirectionally oriented films, of an angle between the principal axes and the machine direction and transverse direction of the film. However, the propensity for wrinkling was not found to be greatly different for such films than for similar films with no angle between the principal axes and the geometric axes. Furthermore, corrugations persist in webs of metal foil and other isotropic materials.

Technical Background

The buckling theory applied in this paper dates from early in the twentieth century. Timoshenko [6] did original work, and reported work by others.

The earliest analysis of buckling of cylindrical shells assumed that internal pressure had no effect on buckling, aside from the obvious additional load-carrying capacity of the pressure multiplied by the internal cross-sectional area of the cylinder. It was found, however, that buckling of an unpressurized cylinder occurred at a load much lower than predicted, while the load carried by the wall of a pressurized cylinder approached that predicted by the original theory, as predicted by Peterson and Dow [2] and experimentally verified by Weingarten, Morgan, and Seide [7]. This outcome was fortunate, because the original equations are very simple, and are applicable to common rolls of web materials and to webs wrapping rollers because the tensile stress in the web has the same effect as hoop stress in a pressurized cylinder.

The buckling of plates has received far less analytical attention than cylindrical shells, because the theory has been supported quite well by experiments. However, few experiments have approached the extremely low thickness compared to the other dimensions of a web.

Diagonal corrugations as studied by Gehlbach, Kedl, and Good [1] are caused by buckling in lateral compression. The compressive stress in this case can be understood from a Mohr's circle, in which the small compressive stress results from a lateral shearing stress, such as that caused by a misaligned roller, tapered roller, unbalanced nip force, etc.

Problems Caused by Web Corrugations

In many steps in web processing, corrugations are harmless curiosities, whereas in other cases they result in serious degradation of quality. Plasticity of a web material may cause corrugations to take a permanent set, causing problems in subsequent processes such as printing, or at least problems with appearance. Large corrugations may become creases as they are gripped upon contact with a roller, perhaps leading to rejection of the product.

THEORY OF BUCKLING

Hypotheses for Application of Buckling Theory to Webs

The theory of small-deflection axial buckling of cylindrical shells or lateral buckling of tensioned spans, like the theory of buckling of a column, assumes that the stress after buckling is the same as the stress which caused buckling. This small but finite lateral force, if constraints of the edges were absent as they appear to be, would tend to spread and flatten the web. The hypothesis of this paper is that unbuckled tape elements, not necessarily of uniform width, are steered in a free span by the downstream roller and are gripped by friction in a wound roll.

Figure 1 shows the above hypothesis applied to a span, and Figure 2 is a free body of the forces between the buckled interior and the unbuckled tape elements along the edges. Figure 3 illustrates the cumulative build-up of friction forces on a roller or wound roll from zero at the edge to a level no greater than that limited by the buckling of the web at interior points.

Other Buckling Considerations

Sinusoidal buckling as assumed in the theory of buckling of cylindrical shells cannot occur on a roller if there is no air entrainment. Nonpermeable webs, however, entrain air in accordance with the Knox-Sweeney equation. Corrugations in glossy film under the right conditions of light reflection are often easily visible as it wraps a roller, even at velocities as low as a few hundred feet per minute.

Because the wavelength of corrugations in a winding roll increases with increasing diameter, there must be a gradual mismatching of nesting of waves as the roll builds. For nonpermeable webs, entrained air may fill the voids between the mismatched waves.

Compressive forces which may cause corrugations can result from (1) steering of the edge tapes to perpendicularity to a roller or roll core which is deflected by web tension or, in the case of an upwardly traveling web, by the roller weight, (2) an increase in width caused by the decrease in tension across a powered roller, (3) lateral slippage backward across a braking roller, allowing the wider entering span to buckle to the width of the narrower exiting span, (4) an expansion in width because of a temperature rise or absorption of moisture, (5) compression of the top of a wound roll which is supported at its ends, and (6) an increase in width of a web after it has been wound, because of the decrease in length from viscoelastic memory and the effect of Poisson's ratio.

Just as the internal stresses caused by such prior events as welding and heat treating of a metal part are invisible and generally unknown, the lateral stresses in a flat web are unknown, whether it is on the verge of lateral buckling, at zero lateral stress, or even under lateral tensile stress. Therefore, a source of a given amount of compressive stress or strain may cause buckling under one set of conditions and not under seemingly identical conditions at another time or location.

Shelton [4] derived the relationship between the apparent strain ε_b caused by buckling (ε_b = reduction in width because of buckling per unit of unbuckled width) as a function of the ratio of amplitude and wavelength of the buckled web. Although the precise relationship between the ratio of arc length to wavelength and the ratio of amplitude to wavelength is a complete elliptic integral of the second kind, an approximation which is valid for values of A/λ less than 0.1 is

$$\varepsilon_b = \left(\frac{\pi A}{\lambda}\right)^2 . \qquad (1)$$

Classical analyses of buckling assumed the materials to be isotropic, but the results apparently apply quite well to common anisotropic materials such as paper and oriented plastic film. The following equations for wavelength and critical (buckling) strain are simplified by the assumption that Poisson's ratio is 0.3. Variations in Poisson's ratio between zero and 0.5, the limits for isotropic materials, do not have a major effect on the buckling equations, as the critical strain varies inversely as the square root of $(1-v^2)$ and the wavelength varies inversely as the fourth root of $(1-v^2)$. Except for elastomers and perhaps other soft materials which have a Poisson's ratio of 0.5, continuous materials usually have a Poisson's ratio of about 0.3. Although paper, which is neither isotropic nor continuous, has Poisson's ratios near zero for in-plane to normal (perpendicular to web plane) relationships, the ratios for perpendicular relationships within the plane of the paper are probably closer to 0.3 than to zero. At any rate, because of variations in properties with temperature, coating, moisture, and processing history, values of Poisson's ratio of web materials are rarely published.

Buckling of a Web as a Pressurized Cylindrical Shell

The classical analysis of buckling of a cylindrical shell reported by Timoshenko and Gere [6], for a shell length (web width) great enough for buckling to occur in many waves and for a Poisson's ratio of 0.3, results in the equation for wavelength

$$\lambda = 3.46 \sqrt{RT} \qquad (2)$$

and for critical lateral stress or strain

$$\frac{\sigma_y)_{cr}}{E} = -0.605 \frac{t}{R} . \qquad (3)$$

The minus sign in equation (3) is the conventional indication of compressive stress.

Because the values of stress and strain in equation (3) are so small, specialized laboratory techniques would be required for measurement, which has not been attempted. Figure (4) shows good verification of equation (2) from commercial wound rolls, as well as from the pattern on two idlers which handled a given thickness of film.

Buckling of a Tensioned Free Web Span

The analysis of buckling of a rectangular plate with tension in the x (machine) direction, compression in the y (lateral) direction, and simple supports at all edges in *Theory of Elastic Stability* by Timoshenko and Gere [6] can be greatly simplified for common web conditions if the web is wide enough for it to buckle in many waves and if Poisson's ratio is assumed to be 0.3. The simplification assumes that the longitudinal strain, (σ_x/E), is much larger than $(t/L)^2$ and $(t/L)^3 (E/\sigma_x)^{0.5}$, conditions which are usually met in common large process lines used in the manufacture of thin films.

The simplified equation for wavelength is

$$\lambda = 1.95 \sqrt{LT}\,(E/\sigma_x)^{1/4}\,, \tag{4}$$

and for critical lateral stress or strain the equation is

$$\frac{\sigma_y)_{cr}}{E} = -1.90\,\frac{t}{L}\sqrt{\frac{\sigma_x}{E}}\,. \tag{5}$$

Again, verification of equation (5) has not been attempted. In fact, observation of web behavior gives reason to suspect that the actual buckling stress of a web span is much lower than that predicted by equation (5). Another (perhaps related) discrepancy between theory and observation is that the theory assumes a sinusoidal profile in edge view along a buckled span, whereas actual webs in long spans appear to have a constant amplitude along most of the length of the span.

Equation (4), nevertheless, is reasonably well supported by data plotted in Figure 5. Corrugations in adjacent spans under approximately the same tension can be observed to have wavelengths related to the span length. Two such pairs of data are plotted in Figure 5 along with unrelated data points.

Lateral Slippage on a Roller or Winding Roll

Whether or not the web wrapping a cylinder is buckled (corrugated), it may creep outwardly to relieve some of the lateral strain, possibly preventing wrinkling at the roller where slippage occurs or at subsequent rollers. The band of creep at each edge (assuming symmetry) may be narrow, or it may extend as far as the center of the web, thereby relieving lateral strain across the entire width of the web.

The maximum lateral force per unit length within the web which can be sustained by frictional contact with the web on the roller is equal to the pressure of the web on the roller multiplied by the coefficient of friction and the distance from the edge, as shown in Figure 3:

$$f_\mu = \frac{T}{WR}\,\mu y\,. \tag{6}$$

The internal force per unit length of the web is

$$f_i = \sigma_y t\,, \tag{7}$$

where σ_y is a function of y, the distance from the edge. The lateral stress σ_y is likely to be nearly constant over the portion of the web width which is not slipping laterally, but must taper to zero at the edge. Lateral slippage limits f_i to be no greater than f_μ, and buckling limits σ_y to be no greater than $\sigma_y)_{cr}$. The width W_t of the cylindrical (unbuckled) tape elements can be found by combining equations (3), (6), and (7), with W_t substituted for y and $\sigma_y)_{cr}$ for σ_y:

$$W_t = 0.605\,\frac{t}{\mu\varepsilon_x}\,. \tag{8}$$

The level of lateral load at a roller cannot be maintained as the web passes from the roller to a free span, unless the span is extremely short. A comparison of equations (3) and (5) shows that the buckling stress of the web as a tensioned free span is usually much less than as a cylindrical shell. An unbuckled web may therefore become buckled as it passes into the free span, and is likely to have corrugations with a much larger period and amplitude than at the roller.

Lateral Compression Caused by Change of Tension Across a Roller

A longitudinal tensile stress σ_x in a web causes a strain ε_x equal to σ_x/E in the longitudinal direction and a strain ε_y equal to $-\upsilon\varepsilon_x$ in the transverse direction, with the minus sign denoting a decrease in width.

In its pure definition as a property of isotropic elastic materials, Poisson's ratio can vary from zero to 1/2. Poisson's ratio is sometimes extended to apply to y-direction strains caused by x-direction stresses for viscoelastic and plastic behavior. Because of the dependence of elastic, viscoelastic, and plastic properties of "plastic" films on many variables, including time and history, Poisson's ratio is seldom published.

Plastic webs which are relatively stable and stiff, such as processed cellophane or oriented styrene, polypropylene, and polyester, probably have values of Poisson's ratio somewhat lower than 0.5. Tests on Mylar by Weingarten, Morgan, and Seide [7] found a ratio of 0.3, while tests on nine varieties of celluloid in Germany before 1930 found ratios varying from 0.36 to 0.5, with all but one variety below 0.42.

A representative longitudinal strain of plastic film as it is being processed in web form is 0.001, although the strain is sometimes less and sometimes several times greater. A typical change in width caused by longitudinal tension and Poisson's ratio therefore is of the order of 0.05 percent.

If a high tension is desired in one span and a lower tension is desired in the next span (for example, a high tension for better control during coating and a low tension through an oven), an increase in width occurs as the web passes over the powered roller. The web, however, cannot suddenly increase in width, and must either compress laterally or form corrugations as shown in Figure 1, as it enters the span beyond the powered roller. If the longitudinal strain were 0.002 upstream from the roller and 0.001 downstream, for example, and if Poisson's ratio were 0.3, the resulting lateral strain would be 0.0003. This value of lateral strain is greater than the buckling strain of most wide webs of thin film. If the residual lateral stress in the downstream span were already high enough to cause the web to be on the verge of buckling, the additional apparent strain of 0.0003 would cause corrugations with an amplitude of 0.55 percent of their period from equation (1). As shown in Figure 6, corrugations may lead to wrinkling at the next roller.

Wrinkles may occur because of a tension increase across a roller, as shown in Figure 7. The wrinkles would then occur at the braked roller or accelerating idler roller because the web cannot make an instant transition in width. The narrower downstream web therefore may cause buckling of the wider upstream web. Isolation of the upstream span from the downstream span, as by a nip, would prevent this type of wrinkling.

Lateral Compression Caused by an Increase in Temperature or Moisture

Most plastic webs have a high coefficient of thermal expansion, in some cases higher than 0.0001 per degree F, and paper usually expands significantly as it absorbs moisture. Heating occurs on a heating roller or in a free span in many processes involving plastic webs, such as drying and corona or flame surface treatment. An example of a sudden addition of moisture to paper is a sizing process.

If the lateral expansion of the web occurs at a roller, the behavior in the downstream span is the same as if the expansion in width had occurred because of a decrease in tension across the roller, as analyzed in the previous article. The risk of wrinkling is again at the next roller.

Lateral Compression Caused by Viscoelastic Memory

One of the more nebulous sources of dimensional changes in a plastic web is viscoelastic memory. Quantification is difficult because production conditions can seldom be duplicated in a laboratory, and samples cannot be taken from intermediate points in a production machine and instantly tested for short-term changes in dimensions. The effects of viscoelastic memory are particularly significant in films which have been oriented.

The goals of machine-direction orientation are to reduce the thickness and/or to improve the mechanical properties. Neckdown (a reduction of width) accompanies the reduction of thickness. When the tension is reduced after drawing, the length decreases and the width increases because of viscoelastic memory. This increasing width could cause troughing and wrinkling of a thin, wide web which is oriented only in the longitudinal direction similarly to the increases in width for other reasons as previously discussed. However, if transverse orientation follows the longitudinal orientation, the web is relatively thick and narrow and is therefore unlikely to wrinkle before the transverse orientation.

During transverse orientation, the web is obviously constrained laterally by the tenter clips, but it is also constrained longitudinally by tension into and out of the tenter. After the web has left the tenter, there is seldom significant constraint against lateral shrinkage; in fact, the primary conclusion of this study is that the web is subjected to lateral compressive loading at most rollers and free spans. The shrinkage caused by memory of the transverse orientation is beneficial toward prevention of corrugating and wrinkling, but it may be masked by other phenomena, such as the width change caused by a tension change across a roller.

The continuous stress in the longitudinal direction, however, delays the longitudinal shrinkage, especially if a high stress is required between the tenter and the winder. Longitudinal shrinkage in the wound roll is then accompanied by an increase in width. Because the edges of the outer layers of the wound roll are constrained by frictional contact with the inner layers, the cylindrical shell comprising the outer layer(s) buckles into corrugations. This viscoelastic source of compressive stress is believed to be the principal cause of corrugations in wound rolls of freshly oriented thin plastic films.

Lateral Compression Caused by Bending of a Wound Roll

A wound roll is often supported at the ends of the core. This article is a study of the possibility that such support could cause corrugations in the top of the outer layers in the roll, as shown in Figure 8.

It is assumed that the entire roll deflects as a beam, with no interlayer slippage. Justification for this assumption is (1) a roll cannot be satisfactorily wound if interlayer friction is very low, and (2) the horizontal shearing stress approaches zero as the upper surface, where the corrugations would occur, is approached.

The stiffness of the core is neglected and the roll is approximated as a homogeneous solid. Even if a metal core is used, its stiffness (EI) is likely to be small in comparison to the stiffness of a roll of material with a common ratio of outside diameter to core diameter of 3.0 or greater.

The uniformly distributed load f on the wound roll as a beam is its weight per unit length:

$$f = \pi R^2 \rho \, . \tag{9}$$

The maximum stress for the uniformly loaded solid cylinder which has simple supports at its ends is:

$$\sigma_{max} = \frac{fW^2}{2\pi R^3} \, . \tag{10}$$

The stress would be a tensile stress at the bottom center and a compressive stress at the top center of the roll, where buckling would first occur.

Combining equations (9) and (10) to find the maximum stress caused by bending of the roll:

$$\sigma_{max} = \frac{W^2 \rho}{2R} \, . \tag{11}$$

Equation (3) applies to a cylindrical shell which is axially loaded, not in a bending mode as in this article; however, Timoshenko [6] (page 483) states that this equation may be used as a "satisfactory approximation" which is "on the safe side," meaning that the buckling stress predicted by the "much more complicated" bending analysis would be higher. If equation (3) is therefore substituted into equation (11) and successively solved for thickness and width, the results are that buckling caused by simple support of a wound roll (with zero initial lateral stress) would occur if

$$t < 0.83 \, \frac{\rho W^2}{E} \tag{12}$$

or if

$$W > 1.10 \sqrt{\frac{Et}{\rho}} \, . \tag{13}$$

The radius of the roll is not a variable in the above two equations, but they are based on the assumption that the core diameter is a small fraction of the outside diameter.

Lateral Compression Caused by Roller Deflection

The theory of lateral behavior of a web as developed by Shelton [3] is based on beam theory, which does not consider strain or buckling in the lateral direction. This theory, however, could be used to predict the behavior of planar tapes along each edge of a corrugated web if they were of constant width. If the width of the planar edge tapes vary along the span, an extension of the theory of lateral behavior was shown by Shelton [5] to involve a nonlinear differential equation, and analysis becomes unwieldy. The simpler first-order theory (not considering width variation or curvature of a tape element) will be applied in this article for a study of lateral compression.

The deflection of a roller caused by the tension in the web causes a lateral compressive stress in the web, as lanes of the web attempt to become aligned perpendicularly to the roller which they are approaching. Similarly, the deflection caused by the weight of the roller causes a lateral compressive stress if the entering span has an upward component, but spreads the web or reduces the lateral compression if the entering span has a downward component.

If the deflection of a roller is symmetrical about its centerline, the lateral condition of the web is affected only by the deflection within the width of the web, not by deflection of the portion of the roller outside the edges or by the deflection of the shaft(s). For analysis of lateral web behavior, it is therefore convenient to have the origin of the coordinates at the center of the roller, moving with the deflection of the roller, as shown in Figure 9. Although the bearing spacing B is sometimes less than the width of the web when the shaft is stationary, it is here assumed that B is equal to or greater than W.

The deflection is:

$$z = \frac{fW^4}{8E_RI_R}\left[\left(\frac{B}{W} - \frac{1}{2}\right)\left(\frac{y}{W}\right)^2 - \frac{1}{3}\left(\frac{y}{W}\right)^4\right]. \tag{14}$$

The deflection of the edge of the web relative to the center is

$$z_{W/2} = \frac{fW^4}{32E_RI_R}\left(\frac{B}{W} - \frac{7}{12}\right). \tag{15}$$

The slope of the deflection curve at the edge of the web is:

$$\left.\frac{dz}{dy}\right)_{W/2} = \frac{fW^3}{8E_RI_R}\left(\frac{B}{W} - \frac{2}{3}\right). \tag{16}$$

An example in which all of the roller deflection is in the plane of the entering span and is therefore fully effective (undesirably) in gathering the web is a 180 degree wrap, for which f = 2T/W. The lateral change in width for the hypothetical assumption of straight edges perpendicular to the deflected roller is the roller slope multiplied by 2L, and the average strain is the change in width divided by the width. The strain caused by roller deflection therefore could approach

$$\varepsilon_{max} = \frac{TWL}{2E_R\,I_R}\left(\frac{B}{W} - \frac{2}{3}\right). \tag{17}$$

By substitution of example parameters into equation (17) and comparison to an evaluation of equation (5), one can readily find conditions for which the strain caused by roller deflection greatly exceeds the buckling strain, even if the lateral compressive stress is zero as the web enters the span.

ACKNOWLEDGEMENTS

Appreciation is extended to John Slavens, former Plant Manager of Mobil Chemical Company, Films Division, Shawnee, Oklahoma, and to the many Mobil employees who gave assistance and cooperation in gathering of data. Data points for corrugations in rolls and a web wrapping a roller were also furnished by Doug Kedl, Tim Walker, and Deborah K. LeMire of 3M Company, St. Paul, Minnesota.

The support by the Web Handling Research Center, Oklahoma State University, Stillwater, Oklahoma, and its sponsors, is appreciated.

REFERENCES

1. Gehlbach, L.S., Kedl, D.M., and Good, J.K., "Predicting Shear Wrinkles in Web Spans," TAPPI Journal, August, 1989.

2. Peterson, J.P. and Dow, M.B., "Structural Behavior of Pressurized Ring-Stiffened, Thin-Wall Cylinders Subjected to Axial Compression," NASA Technical Note D-506.

3. Shelton, J.J., "Lateral Dynamics of a Moving Web," Ph.D. Dissertation, Oklahoma State University, July, 1968.

4. Shelton, J.J., "Machine Direction Troughs in Web Spans and Corrugations in Wound Rolls," Report to Web Handling Research Center, Oklahoma State University, Stillwater, Oklahoma, August, 1991.

5. Shelton, J.J., "An Initially Straight Moving Web with a Slack Edge," ASME Winter Annual Meeting, November, 1992.

6. Timoshenko, S.P. and Gere, J.M., "Theory of Elastic Stability," McGraw-Hill Book Company, New York, Second Edition, 1961.

7. Weingarten, V.I., Morgan, E.J., and Seide, P., "Elastic Stability of Thin-Walled Cylindrical and Conical Shells Under Combined Internal Pressure and Axial Compression," AIAA Journal, Vol. 3, No. 6, June, 1965.

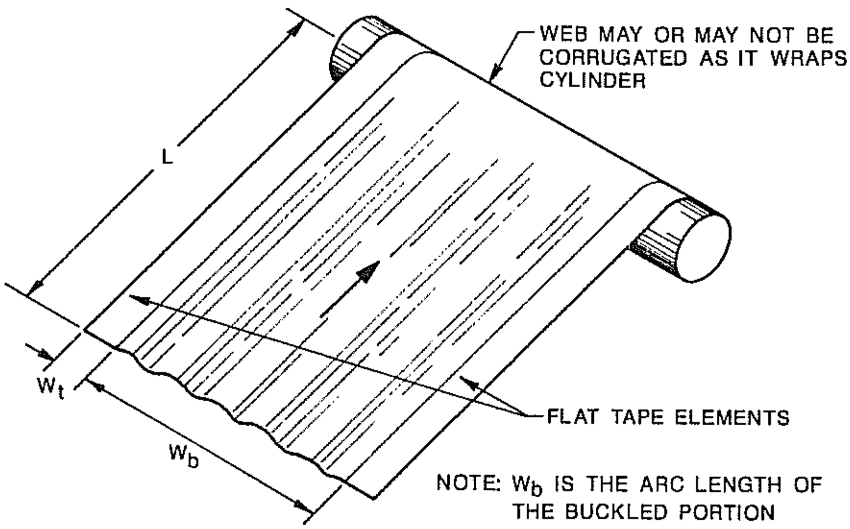


Fig. 1 Steering of flat tape elements at edges with buckled center portion

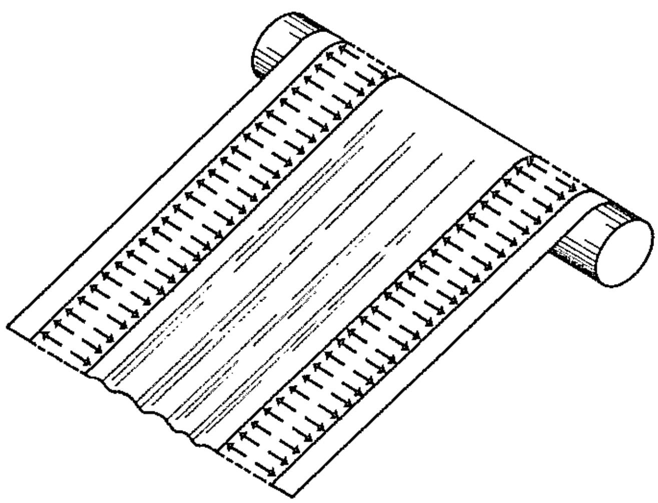
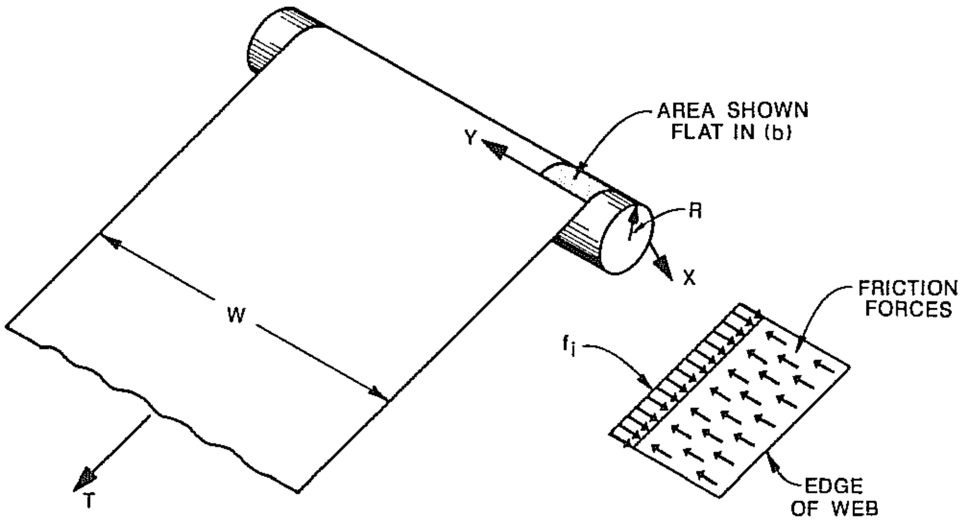


Fig. 2 Lateral forces on tape elements and buckled center portion



a) Web in contact with roller   b) Lateral force on edge tape

Fig. 3 Internal force limited by friction

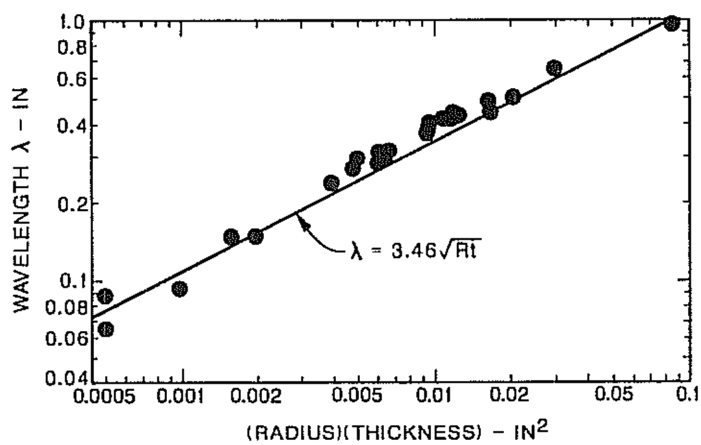


Fig. 4 Experimental data compared to equation (2)

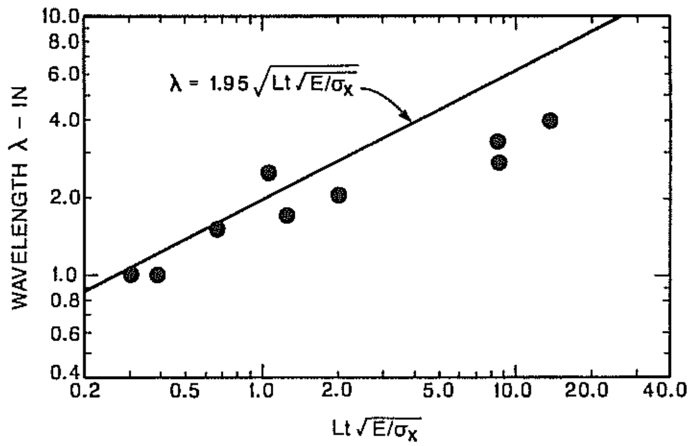
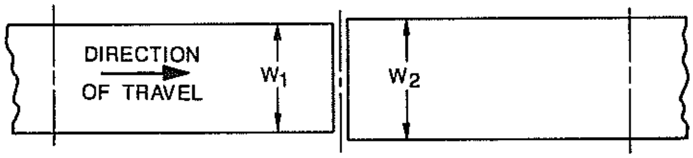
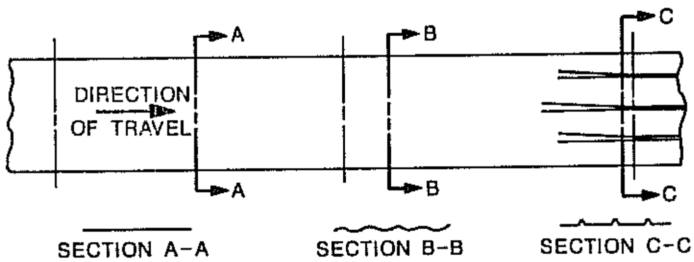


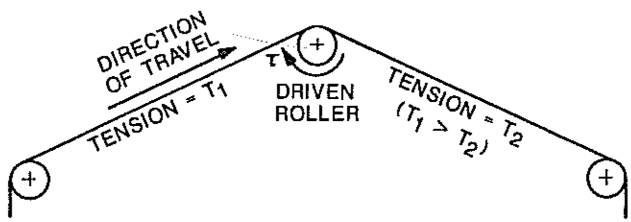
Fig. 5 Experimental data compared to equation (4)



a) Width change if web is cut and laid flat under operation tension

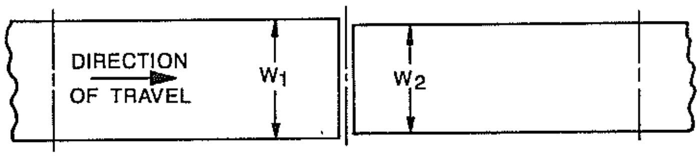


b) Corrugating and wrinkling because of constraint of the wide span by the narrow span

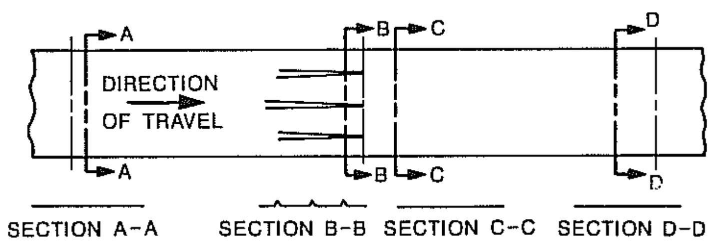


c) Side view of web with decreasing tension

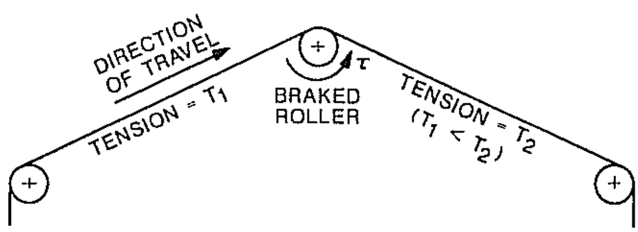
Fig. 6 Wrinkle formation caused by decrease in tension



a) Width change if web is cut and laid flat under operating tension



b) Wrinkling caused by imperfect isolation of narrow downstream span from wide upstream span



c) Side view of web with increasing tension

Fig. 7 Wrinkle formation caused by increase in tension without isolation of upstream span from downstream span

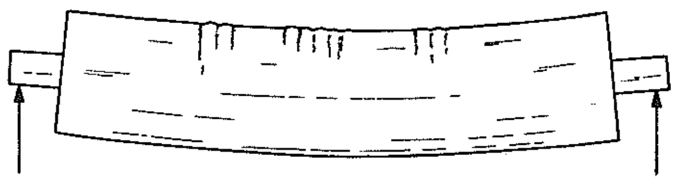


Fig. 8 Corrugations at top of simply supported roll

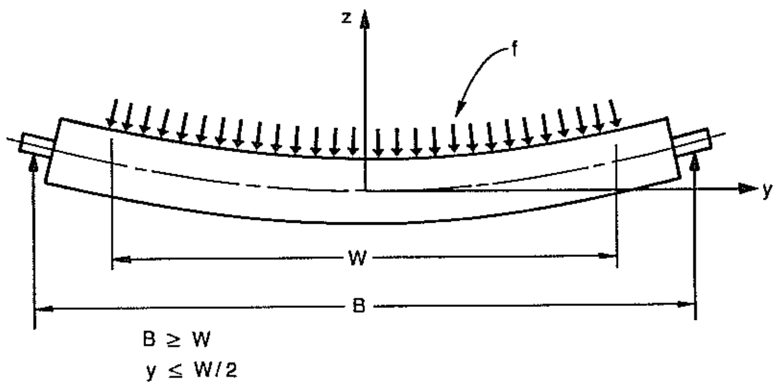


Fig. 9 Roller deflection caused by a centered web with uniform tension

QUESTIONS AND ANSWERS

Q. The narrowing of the web caused by deflection of a roller must be very small. Can this small deflection commonly cause corrugations?

A. Yes, the lateral strain which can cause corrugations in a span may be $(10)^{-5}$ or even less in practical cases if the web is thin. Both the tension of the web and the weight of the roller contribute to narrowing of the web in a span in which the web is traveling upward, whereas the deflection caused by weight in a downward span causes spreading, or at least reduces the lateral compression. The paper industry has fewer problems with corrugations than the film industry because the rollers are usually very stiff, and the web is thicker than a thin plastic film.

Q. What was the width of the web in the slides which showed corrugations?

A. Approximately 120 inches. However, corrugations can be seen in spans of webs on WHRC lab lines while handling widths of about two feet, and edge tapes commonly appear to be two or three inches wide.

Q. Can corrugations occur in narrow webs?

A. Yes, particularly if the web is very thin, such as 0.0005 inch.

Q. Can't corrugations be caused by imperfections in the web?

A. Yes, but I devised this theory of buckling between edge-tape elements because of observations of persistent, uniform corrugations in webs which were homogeneous and isotropic, and which appeared to be flat in an untensioned condition. Corrugations appear even in free spans of metal strip and foil.

Imperfections which can cause or modify corrugations include local variations in flatness (causing puckers), a variation of locked-in stress through the thickness of the web (causing edge curl) and permanent set of corrugations from a previous condition.

The anisotropy of bidirectionally oriented films has not been found to significantly affect corrugations. In fact, most of the data points in the graphs in this paper were from bidirectionally oriented film.

Q. Do grooving patterns on rollers affect corrugations and wrinkling?

A. Yes, but the effects have not been quantified. Generally, grooving or roughening of rollers because of air entrapment when handling thin webs should be done only to the extent required to achieve the necessary friction between the web and the roller. Grooves which spiral outwardly from the center give an illusion of spreading and sometimes eliminate creases at this roller by allowing a place for harmless buckling of the excess width to occur; however, the web was not spread by the grooves, so that wrinkling may occur at the next roller. Grooving or roughening of a roller only in a band near its center, with the rest of the roller smooth to allow the web to slide outward, is worth considering.

Q. Is the friction which locks in corrugations of the stick-slip type?

A. Undoubtedly, stick-slip friction sometimes occurs, but friction of plastic against itself or against another material is often more nearly viscous than stick-slip, especially with entrained air as a lubricant.

Q. Can a concave roller spread a web and prevent corrugations?

A. Yes, corrugations may be prevented on a concave roller if the entering span is sufficiently long, but corrugations may reappear downstream and in the winding roll, as previously explained. The edge tapes are steered outwardly by the end portions of the concave roller, which appear to each edge tape as a conical roller. The edge tapes must form S shapes in the entering span as they become wider and perhaps merge as they approach the roller. Analysis of a constant-width web which is steered by a conical roller readily shows that the entering span must be long for significant steering to be achieved, but quantification of the spreading of the variable-width tapes would be difficult.

Q. Can you predict the amplitude of the corrugations?

A. Yes, if the relationship between the flat, relaxed width of the web and the width under the operating constraints is known. Equation (1) then shows that, for a small wave amplitude compared to the wavelength and for a known web thickness and radius, the change in apparent width compared to the unbuckled width varies with the square of the amplitude of the corrugations. Conversely, the amplitude varies with the square root of the change in the apparent width compared to the actual width when the web was on the verge of buckling.

Q. Is this an empirical relationship between the amplitude of the waves and the wavelength?

A. Knowledge of the lateral stress required for buckling to occur does not predict the amplitude of the waves, just as small-displacement column-buckling analysis does not predict the amount of buckling. The problem with trying to apply equation (1) is that the true relaxed width at the exact conditions of operation, including stress history, is seldom known. However, if the amplitude of the waves is measured, equation (1) gives the relationship between the unstressed width when the web is on the verge of buckling compared to the buckled width, regardless of the cause of buckling. If A/l is greater than 0.1 or so, accuracy requires that a series expression of an elliptic integral be evaluated.