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Scope of Report: This report is an outline of units that can be used for teaching seventh and eighth grade mathematics based on material prepared by the School Mathematics Study Groups for Junior High School at Yale and Maryland universities. The units are listed with aims, objectives, and some of the concepts that should be understood. Some units carry suggested activities.

ADVISER'S APPROVAL James H. Zant

A SUGGESTED OUTLINE OF MATHEMATICAL CONCEPTS
FOR SEVENTH AND EIGHTH GRADES

By

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FOR SEVENTH AND EIGHTH GRADES

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PREFACE

The problem facing every junior high school mathematics teacher is that of revitalizing the content of the curriculum. This report is not an attempt to solve the problem, but it is hoped that the outlines presented will give the student some basic concepts that coincide with the current changes taking place in the field of mathematics.

I would like to acknowledge indebtedness to Dr. James H. Zant for his guidance, suggestions, and loan of material; to Dr. M. L. Keedy for making the material from the University of Maryland Mathematics Project (Junior High School) available; and to my instructors and fellow classmates for the numerous ideas borrowed from them through discussions.

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CHAPTER I

THE PROBLEM

In this age of science there is now, and will be in the future, a greater demand for trained technical personnel. In order to assure this country of sufficient scientific manpower for the future, teachers must in some way impress upon the minds of American youth the importance of technical training.

Mathematics is a basic part of this training. Not only is it necessary in the training of technicians, but a basic knowledge of mathematics is essential in many other fields of endeavor. Begle aptly expresses the need for a sound mathematical background. He says:

> The world of today demands more mathematical knowledge on the part of more people than the world of yesterday, and the world of tomorrow will demand even more. It is therefore important that mathematics be taught in a vital and imaginative way which will make students aware that it is a living, growing subject which plays an increasingly important part in the contemporary world.[1]

The above paragraphs present, informally, some of the problems facing the present school teachers. The writer is chiefly interested in the problem as it concerns the

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[1]E. G. Begle, "The School Mathematics Study Group," The Mathematics Teacher, LI (December, 1958), 616.

teaching of mathematics and the content of the courses taught in the seventh and eighth grades.

Proof that sound mathematical programs are not being executed in the nation's schools is evident in the exigent demands from leading educators and many other sources that changes in the content of curriculum and the manner of presentation be made and that the requirements be increased. The interest shown by the government and many universities to provide special training for the mathematics and science teachers is added proof that there is neglect. Especially, in the opinion of the writer, has the junior high school been neglected.

The writer worked with both junior and senior high school students for six years. During that period, he worked in small schools in two different states. Similar problems were found as follows: (1) lack of interest on the part of the teacher and the pupil; (2) repetition of same principles in different grades; (3) mathematics usually taught by non-majors; and (4) the underestimation of the ability of the junior high school pupil to grasp stronger subjects in mathematics.

The writer's awareness of the need for a revision of the mathematics program in the seventh and eighth grades led to the writing of this report. This report will include an outline of units which may be added to the curriculum in the junior high school. These units and the aims and objectives of each will make up the content of Chapter II. Chapter III will be devoted to grade seven and will contain material suitable for classroom instruction. Chapter IV will be devoted to the eighth grade. Chapter V will be devoted to the summary and suggestions for grade nine. The units and the outlines will be an outgrowth of careful study of reports made by mathematics study groups, books on current mathematical concepts, and articles concerning the revision of the mathematics program in schools.

The writer hopes to gain from this report some ideas, principles, and a clearer understanding of materials which may be presented to the junior high school pupil. He feels that, as a result of this study, he can return to the classroom better equipped for the task of training boys and girls.

CHAPTER II

SUGGESTED UNITS FOR THE SEVENTH AND EIGHTH GRADES

Because of the demands made upon schools by society, there is much public insistence that schools up-date their science and mathematics programs to adapt to the technological facts of everyday living. Quantitative thinking is required constantly in carrying forward everyday enterprises both in and out of school. There is no end to the situations involving the need for mathematical interpretation. If children are to lead economically and socially competent lives, they must be helped to understand number as such and to see its application to daily life. Many educational leaders share this opinion as illustrated by the following statement:

> ...mathematics has a central responsibility in the junior high school curriculum. It is not a collection of detailed skills, suitable for drill method alone. The mathematics curriculum calls for a thoughtful study of problems that effectively use arithmetic, aided by some algebra and geometry of form, size, and position... .[2]

The School Mathematics Study Group, financed by the National Science Foundation, was set up with E. G. Begle of

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[2]Chester Scott, "Mathematical Contributions to the Needs of Youth," The Mathematics Teacher, LI (December, 1958), 612.

Yale University as executive director. As a part of the activity of this group, the committee on grades seven and eight prepared a number of units to be tried out by a group of teachers in these grades during the academic year 1958-59. The topics for the units are:

1. An Introductory Unit on "Why Study Mathematics?"
2. Decimal and Non-Decimal Numeration

2a. Symbols

3. The Natural Numbers and Zero
4. Factoring and Primes

4a. Divisibility (A supplementary unit)

5. Unsigned Rationals
6. Non-Metric Geometry
7. Measurement
8. Informal Geometry I (Angle Relationships)
9. Informal Geometry II (Congruent Triangles, Perpendicular Bisectors, Parallelograms, Theorem of Pythagoras)
10. Approximation
11. Mathematics in Science--The Lever (A Supplementary Unit)
12. Statistics--A Unit on Mathematics in Social Studies
13. Chance
14. Finite Mathematical Systems (A Supplementary Unit).[3]

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[3]Yale University School Mathematics Study Group, Experimental Units for Seventh and Eighth Grades, A Report Prepared by the Committee on Grades 7 and 8 (New Haven, 1958).

Use of material of Maryland's School Mathematics Study Group was made by this committee.

In making an outline for the seventh and eighth grades, these units were considered by the writer, and a list of aims and objectives was formulated.

Aims and Objectives of Units

"Why Study Mathematics?"

The aim of unit one is to give the pupil an appreciation for the importance of mathematics. The objectives are (1) to develop an understanding of what mathematics is as opposed to simple computation and (2) to motivate the pupil by pointing out the need for mathematics.

Decimal and Non-Decimal Numeration

The aim of unit two is to deepen the student's understanding of decimal notation for whole numbers and to delve deeper into the reasons for the mechanical operation of addition and multiplication. The objectives are (1) to become familiar with the number symbols; (2) to learn the definitions of some basic terms of the number system; and (3) to contrast other number systems and symbolisms with the system.

The Natural Numbers and Zero

To develop in the pupil an appreciation for the number system and to lead him to an understanding of its basic operations are the aims of this unit. The objectives are (1) to recognize the advantage in one-to-one correspondence; (2) to learn some of the properties of the natural number and zero; (3) to learn the principles that govern the basic operations of natural numbers; and (4) to give practice in the use of the symbols "is less than" and "is greater than."

Factoring and Primes

The aims of unit four are to review the fundamental skills of arithmetic and to give some inductive and informal deductive reasoning skill. The objectives are (1) to identify prime and composite numbers; (2) to recognize quickly odd and even numbers by inspecting their decimal numerals; and (3) to identify factors of a natural number and a product.

Unsigned Rationals

The aim of unit five is to develop in the pupil an understanding of fractional notation and why it is needed. The objectives are (1) to learn some of the properties of rational numbers; (2) to learn the relationship between rationals expressed as integers and rationals expressed as decimals; and (3) to learn the principles that govern the fundamental operations on rational numbers.

Non-Metric Geometry

In unit six, the aim is to develop in the student some important geometric properties without involving the idea of measurement. The objectives are (1) to understand the concepts of a line, a set, an element, a plane, and space; (2) to be able to recognize line segments and their end points and skew lines; (3) to be able to identify half-lines, rays, end points of rays, and simple closed curves in a place; and (4) to be able to define and use words and terms that describe non-metric properties in geometry.

Measurement

To develop within the student the ability to recognize the different systems of measurement and their uses is the aim of the seventh unit. The objectives are (1) to learn the basic units of the two widely used systems; (2) to learn the comparison and conversion principles; and (3) to learn the units that are standard the world over.

Informal Geometry I

The aim of unit eight is to learn the properties of geometric figures by experimenting, measuring, and formulating conclusions by inductive reasoning. The objectives are (1) to introduce certain geometric concepts and relations; (2) to give the pupil experience in verification of experimental results and informal deductive argument on the basis of previously stated principles; and (3) to develop an awareness of the occurrence in the environment of illustrations of points, lines, planes, and three dimensional regions bounded by points, lines, and planes.

Informal Geometry II

In unit nine, the aim is to learn the construction of geometrical figures using the concepts of points and lines to improve upon inductive and deductive reasoning. The objectives are (1) to learn the kinds of angles and their relationship; (2) to learn the kinds of lines and their relationship; (3) to learn the principles of congruency; and (4) to study the various theorems and their application.

Approximation

To develop in the pupil the idea that the measurement of a single thing is an approximation and not exact is the aim of the tenth unit. The objectives are (1) to recognize the difference between the process of counting separate objects and measuring a single object; (2) to learn the difference between the terms "precision" and "accuracy"; and (3) to learn the principles involving error and significant digits.

Mathematics in Science

The aim of unit eleven is to help the pupil to become aware of the importance of mathematics in science and of the necessity of mathematical data to carry out experiments. The objectives are (1) to give the pupil experience in collecting mathematical data in an experiment in science; (2) to give the pupil experience in the typical inductive method of science; (3) to help the pupil to see the importance of the quantitative aspect of the physical experiment;

and (4) to give the pupil skill in mathematical application to science.

Statistics

To help the pupil to become aware of the varied uses of mathematics in the social sciences is the aim of unit twelve. The objectives are (1) to help the student develop the elementary concepts of statistics; (2) to help the student to develop an appreciation of mathematical applications in the social studies; (3) to show that mathematical interpretation of statistics is very important and necessary for certain governmental agencies to make predictions; and (4) to show how important mathematical interpretation of statistics is in business.

Chance

In unit thirteen, the aim is to introduce some of the elementary properties of probability. The objectives are (1) to help the student understand the use and importance of probability to science, industry, and business and (2) to introduce to the student the concept of classifying counting, ratio, estimation, and the meaning and measure of chance.

Finite Mathematical Systems

To help the student to achieve some appreciation of the nature of mathematical systems is the aim of unit fourteen. The objectives are (1) to increase understanding of commutative, associative, and distributive properties of numbers; (2) to develop an understanding of closure, identity element, and inverse element concept; and (3) to increase understanding of the inverse operations of division and subtraction.

CHAPTER III

MATHEMATICS FOR THE SEVENTH GRADE

> Interest in the improvement of the teaching of mathematics is more widespread than at any other time in recent years. The explosive growth of the subject, recent interest in its foundations, and new and spectacular applications to fields hitherto unaffected by mathematics have implications for all levels of instruction--from the kindergarten to the graduate school... .[4]

Ideas which employ the use of abstractions and generalizations have taken precedence over once important subject matter included in the curriculum because they are more in line with newer mathematical concepts. The following outline, based on a study of the units prepared by the University of Maryland Mathematics Projects, will adhere closely to the foregoing statements.

Decimal and Non-Decimal Numeration

Systems of Numeration

The purpose of teaching non-decimal systems of numeration is not to produce facility in calculating with them. The modern decimal system is wholly adequate. It is so familiar, however, that its structure and the concepts

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[4]National Council of Teachers of Mathematics, As We See It (Washington, 1958), p. 3.

involved in its algorithms are too easily overlooked. As a study of a foreign language aids in the understanding of the mother tongue, so a study of less familiar numeration aids in understanding the familiar. A look at a different symbolism for numbers can bring out the fact that the symbols used are not themselves numbers--a distinction which is important to make. It is apparent that, when one confines this arithmetic activity to the use of the decimal system, he is apt to think of arithmetic as consisting of manipulation of decimal symbols rather than as a study of numbers themselves and their properties. The idea that numbers have properties independent of any system of symbolism must be brought out. Emphasis also should be placed upon the importance of systematic position.

It is essential that the student understands these facts:

1. Numbers may be named in different ways;
2. A numeral with several digits refers to a sum, each addend of which may be expressed as a product of factors (not using the word "factor" at this time);
3. The number property of a collection can be thought of in groups other than ten;
4. The smaller the base, the fewer the symbols but the longer the numeral to name a number; and
5. The larger the base, the greater the number of symbols but the shorter the numeral to name the number.

As an activity, one should provide students with a collection of things to count, such as sticks, matches, and paper squares. One may have them take fifteen of the objects and then ask questions concerning groupings to tell the number of objects without saying "fifteen." One should explain that writing "fifteen" means one ten and five ones; this is grouping by tens. The teacher should have the students group objects on other bases and should point out the positions and their relations to the various bases.

Symbols

The understanding and correct use of the symbols used in mathematics are essential to the person engaged in mathematical activity; they are the tools of mathematics. Precise use of language is mandatory in the mathematical processes. A common error in using symbols is that of confusing them with the thing they represent.

Before leaving this section of the unit, the student should be familiar with these facts:

1. A symbol is a means of expressing an idea;
2. Symbols have meaning by agreement;
3. A symbol may have different meanings in different situations;
4. A symbol is not the thing it represents;
5. The same idea may be expressed by different symbols;

6. A symbol for a number is called a numeral;
7. A definition in mathematics is often an agreement about the meaning of a symbol;
8. A sentence using "=" for its verb is reversible; and
9. A numeral with dimensions attached is a symbol which is not a numeral.

Activities could include discussions which bring out the definition of symbols. The teacher should have students list symbols they know, and he should be certain to have some names included. He should ask for some words as symbols and present to them "$<$," "$>$," "$=$," "$\neq$," and "()" with explanations as to their meaning and use. Then he should show how the use of quotation marks around symbols indicates that the symbol itself is being referred to.

The Natural Numbers and Zero

Properties of Natural Numbers

The natural numbers are the ones used for counting beginning with the number one. The natural numbers are important because they were the first numbers invented. When other kinds of numbers, rational, irrational, and imaginary, were invented, they were given as many of the properties of natural numbers as possible. This made the new numbers more useful and easier to handle. Therefore, a good understanding of the basic properties of natural numbers is essential to the real understanding of more recently invented numbers.

The essential properties of natural numbers are:

1. There are two operations, addition (+) and multiplication (·), for which closure exists. Given any two numbers, a and b, $a + b = c$ where c is another natural number. Similarly, $a \cdot b = d$ where d is another natural number. Subtraction (the inverse of addition) and division (the inverse of multiplication) are not always possible within the realm of the natural numbers; the set is not closed with respect to these operations;
2. The operations (+) and (·) are commutative for any two natural numbers. Given natural numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$. Subtraction and division are not commutative;
3. Both addition and multiplication are associative. Given natural numbers, a, b, and c, $(a + b) + c = a + (b + c)$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. Subtraction and division are not associative;
4. Multiplication is distributive over addition. This property is essential to arithmetic computation as it is done today.
5. There is a natural number, 1, having the property that $1 \cdot a = a$. This number is the identity element for the operation of multiplication.

6. The natural numbers are an ordered set with a first number but no last number; every member of the set has an immediate successor.

These properties are fundamental, and it should be emphasized that they do not depend on any system of numeration. Understanding of the concepts, (1) the natural numbers are not sufficient for measurement; (2) subtraction is the opposite of addition; (3) division is the opposite of multiplication; and (4) number is an abstract concept (an idea rather than something which can be seen or felt), is essential in this unit.

The teacher should have students count sets of things in the room. Then he should ask if numbers are used in counting. It should be explained that man invented numbers for counting, and the importance of this idea should be stressed. One should review the difference between numbers and numerals, and one should bring in the idea that counting is learned by use of sets of things. One then should provide students with objects for counters and have them find 4 + 3 by making a pile of four on the left and three on the right and count them. He should have them write 4 + 3 = 7. Then he should repeat this process in reverse. It should be emphasized that this is the commutative property of addition.

The Number One

The first natural number, one, is worthy of study because of its properties which are useful in making applications. "Fractional numerals" and "decimal numerals" should be introduced here, as their introduction is made easier while discussing the properties of one. The properties of one are (1) that one is the first natural number (the starting number used in counting); (2) adding one to a natural number gives the next natural number (the "successor" to the number); (3) subtracting one from a natural number gives the "predecessor"; (4) one has no "predecessor"; (5) the product of one and any number is the same number (the identity property for multiplication)--given a number, n, $n \cdot 1 = n$ (applies to all numbers, not just natural numbers); and (6) in any of the numbers, natural, real, or complex, $n/n = 1$ and $n/1 = n$ for any number n $(n \neq 0)$.

The teacher should introduce the fraction after stating that one is sometimes used to represent all of something (the whole), and a fraction is not a whole number. The division and multiplication of fractions and decimals is studied at this point.

As an activity, the teacher can list the properties of one on the board and discuss them briefly. He can ask: "If a number does not have the factor 2 and it is divided by 2, what is the remainder?" Then he can ask: "If you take an even or odd number and add one, what do you get?" He should explain the relationship of one and the division of decimal numerals. Any number of activities may be used to clarify in the minds of the students the properties of one and the definition of terms used in this section of the unit.

The Number Zero

The invention of zero was a great thing in the development of the mathematical system. The erroneous belief that zero is not a number stems from the fact that zero usually is defined as nothing. Actually, zero is a symbol used to represent the absence of a quantity. Since numbers are created by definition, zero is a number with certain special properties: (1) It is the number which should be used to answer the question "how many" (meaning--"not any"); (2) it is the number obtained by subtracting any number from itself; (3) zero is less than any natural number; (4) the sum of zero and any number is that number (zero is the identity for addition); (5) if any factor of a product is zero, the product is zero; (6) no non-zero number may be divided by zero, and zero cannot be divided by zero; and (7) when zero is divided by any other number, the result is zero.

As an activity, the teacher can ask: "Suppose someone asks you how many horses you have, and you do not have any? What number would you use to answer?" He can explain that taking the set of natural numbers and putting zero with them, the new set would be called the set of "whole numbers." He can draw a line on the board representing the natural numbers and ask where zero would come. He should bring out that zero is less than any natural number and point out that this scale is like that of a ruler with zero at the beginning. Students should illustrate by making paper scales with zero at the left end.

The System of Integers Under Addition

Man invented and used numbers years before the invention and use of negative numbers. This interval probably was due to an assumption that numbers had a pre-existence and were discovered rather than man-made. The idea that man invented numbers and number systems is of major importance.

The approach to constructing the system of integers is arithmetic rather than geometric. The system is restricted to the operation of addition and its opposite, subtraction, in order to simplify the procedure. Concentration is on commutativity and associativity rather than on jumps along a line. After the properties of this mathematical system are familiar, the idea of vector interpretation is brought in, and several applications based on the vector idea are considered.

This section should give an understanding of the following:

1. The system of whole numbers is not closed under subtraction--zero is the only whole number which has an inverse for addition;

2. A system of "integers" using only addition can be invented such that every member will have an inverse for addition--then subtraction always will be possible;
3. A numeral for a positive integer has two parts--a positive sign and a numeral for a natural number;
4. A number for a negative integer has two parts--a negative sign and a numeral for a natural number;
5. A "+3" is read "positive three"--the "+" does not mean addition, and a "-3" is read "negative three"--the "-" does not mean subtract;
6. The inverses for all positive integers are the negative integers;
7. The positive integers behave like natural numbers in that they have commutative and associative properties for addition;
8. The integer zero is the identity for addition;
9. Addition can be defined in any way except that previous agreements must not be violated;
10. Subtraction is the opposite of addition--subtraction of one integer from another is the same thing as adding the inverse; and
11. The integers can be ordered along a line.

As an activity the teacher can ask: "In the system of natural numbers, what is 10 - 4, 5 - 3, and 7 - 7?" He should bring out that this subtraction (7 - 7) cannot be done in the system of natural numbers, or "7 - 7" is not a name for any number of the natural system. He should explain that a new system called whole numbers was invented so that subtraction like this could be done.

Factoring and Primes

This unit is a brief introduction to number theory. Number theory is that branch of mathematics concerned with properties of natural numbers--properties which are independent of numeration. A number is a prime whether it is named in base ten, base n, or in any other manner, and the study of primes is fundamental in the theory of numbers. The applications of the fundamental theorem of arithmetic (the "unique factorization property") are many. Two other important ideas introduced in this unit are inductive and deductive methods of proof.

This unit should give an understanding of the following things:

1. The process of finding factor is called "factoring."
2. In a product of two or more numbers, each of these numbers is a factor of the product.
3. Every number has the factor one; every number is a product of itself and one.
4. A natural number is called a "prime" if it has exactly two different factors, itself and one.

5. A natural number is called "composite" if it has more than two factors other than itself and one.
6. A number is called "even" if it has the factor two. Otherwise, it is falled "odd."
7. A product is even if one or more of its factors is even.
8. The sum of two odd numbers is even; the sum of two even numbers is even; the sum of an odd and an even number is odd.
9. Every composite can be factored as a product of primes in only one way except for order of the factors. This is the unique factorization property for natural numbers.
10. Given two or more numbers, any number which is a factor of each of them is called a "common factor."
11. If one number is a factor of another, the second is called a "multiple" of the first.
12. Given any two numbers, any number which is a multiple of each of them is called a "common multiple" of the two numbers.

It should be pointed out that just a few illustrations do not prove a general principle. Statements must be made, and reasons given for them, in such a way as to apply to all possible situations in question. Induction must be planned for and encouraged. Clear distinction must be made between induction and deduction. The principle of greatest common factor and least common multiple should be emphasized.

The teacher should ask students for the sum of the two primes, three and five; for sums of several pairs of primes, always making sure that two is not included as one of the primes; and have them write the sums. He should ask what they notice about the numbers which are the sums of the two primes and hope for the conclusion that they are all even (or at least composite). He can then have them try to give a deductive argument. Finally, he should show an example like two plus three. He should explain that no matter how many examples are given (unless all possible are given) proof is not substantiated.

Some of the ideas used in this unit should come from the students. This is important, as student participation develops creative activity. The role of the teacher is to encourage the free flow of ideas and moderate discussions.

The Non-Negative Rational Numbers

The Number System of Ordinary Arithmetic

In this section of the unit, the system of rational numbers is introduced as "arithmetic" numbers rather than rational. The definition of rational number given here is a fractional number whose numerator and denominator are whole numbers rather than a set of equivalent ordered pairs of whole numbers. These are called "arithmetic" numbers. These "arithmetic" numbers include all of the numbers familiar to the seventh grader. The system of real numbers should not be considered here. The teacher should point out that the need for numbers other than the natural or whole numbers arose in connection with measurement and that the invention of the fraction served as a necessary convenience in carrying out division.

Clear definitions with examples must be given for terms encountered, such as numerator, denominator, fractional numeral, decimal numeral, reciprocal, discount, and commission. Emphasis should be given to the properties of arithmetic numbers: (1) multiplication and addition are associative; (2) two arithmetic numbers with the same denominator have the same name; (3) these numbers must have the same name to add them; (4) multiplication and addition of fractions is commutative; and (5) the distributive property is also true for these numbers.

An understanding of the following is necessary:

1. Decide on the properties arithmetic numbers should have to convey the ideas they represent.
2. Inverses for multiplication are called "reciprocals" of each other.
3. To divide an arithmetic number by another gives the same results as multiplying by the reciprocal of the second.
4. The arithmetic numbers have order.
5. Decimal numerals for arithmetic numbers either end or repeat.

6. A fractional numeral for a number which is named by a repeating decimal numeral may be found in a systematic way.

An activity for this section may consist of writing some pairs of numerals on the board such as 6, 2 and 9, 3. The teacher then should ask: "For each pair, by what would you multiply the second to get the first?" Then he should ask similar questions for pairs, such as 2, 5 and 3, 7. He should bring out that whole numbers as answers cannot be given for the latter cases. To find these answers, one must divide the first number by the second. The teacher should include activites to show the relationship between fractions, decimals, and per cents. This is somewhat of a transition section, and in the end note can be made of the similarity between the properties of arithmetic numbers and natural numbers.

<u>System of Rational Numbers</u>

In this system of numbers, definitions are given to the operations of addition and multiplication so that the properties of commutativity, associativity, and distributivity are preserved. Making these definitions of such that the operations have the desired properties introduces a new name to this system--"Field." The set of elements in this system is infinite.

A clear understanding should be given to the following:

1. The system of ordinary arithmetic is not closed to subtraction. Zero is the only number having an additive inverse.
2. Invent the system of rational numbers such that every member will have an inverse for addition. Subtraction will then always be possible.
3. The positive rationals behave like the arithmetic numbers.
4. The proper reading and writing of numerals for rational numbers is necessary as well as the identification of positive and negative rationals.
5. Addition is defined so that it will have as many familiar properties as possible.
6. Addition is commutative and associative; subtraction is the opposite of addition; and subtraction is the same as adding to the inverse.
7. Multiplication for rational numbers is defined so that it has the properties of closure, commutativity, associativity, and distributivity of multiplication over addition.
8. Division of rational numbers is always possible except by zero.
9. A positive one is the identity for multiplication for rational numbers.
10. The rational numbers have order; the order has properties; there is no largest or smallest; given

a rational number there is no "next" one; and the order is dense.

As an activity, the teacher should ask: "In the number system of ordinary arithmetic, what is 1/3 - 2/3?" He should bring out that this subtraction, and many like it, cannot be done in this system. He should ask: "Which of the numbers have an inverse for multiplication?" and "For addition?" Bring out that only zero has inverse. He should explain and relate the invention of the system of rational numbers for the purpose of solving examples like the one suggested.

Non-Metric Geometry

Usually, geometric concepts which do not involve notions of measurement or distance are taught as needed. The non-metric concepts are basic and in many ways more fundamental than metric notions. Teaching these concepts leads the student to the discovery of basic material and introduces sets and set intersections which are fundamental for the study of geometry as well as for the study of mathematics in general.

This unit should give a clear understanding of the following:

1. Use "line" to mean straight line, and it is unlimited in extent.

2. A line is a certain set of points; in a set, the things are called "elements of the set"; and a set without an element is called the "empty set."
3. For any two sets there is a set which is their intersection (the set whose elements are in both sets).
4. A line segment is determined by two points on a line called "end points" of the segments.
5. A line segment can be named in more than one way.
6. Between any two points on a line, there is a third point.
7. A point on a line separates the line into three sets--to half-lines and a set containing just the given point. If the point is put with a half-line, a set of points called a "ray" is formed.
8. A line lies on a point as well as a point lies on a line, and there is a "set of lines on a point" and a one-to-one correspondence between lines on a point and points on a line.
9. A plane is an idealization--a flat surface, unlimited in extent, and without thickness.
10. A triangle is a set of points determined by three points and consists of the three points together with the line segments joining them.
11. "Curve" is a general word for certain sets of points.

12. A line is a particular kind of curve.
13. A "closed curve" is one which can be traced so that a pencil comes back to the original point.
14. A "simple" closed curve is one which separates a plane into three sets--namely, an outside, an inside, and the curve or boundary.
15. A line on a plane separates the plane into three sets--two half-planes and the line itself.
16. Reference can be made to a plane on a line as well as a line on a plane and a set of lines on a plane.
17. The intersection of two planes is either a line or the empty set; two lines in different planes, whose intersections are the empty set, are called "skew lines."

The teacher should explain to students that drawing a sketch, or picture for a line, is to draw a picture of only a part of a line--a line segment. He should ask them to think of something that represents line segments. Bring out by these examples that a line segment could be extended to get a line.

Measurement

Much of the material in this unit is traditional, but the introduction of the metric system is given more than ordinary importance. Mass and weights are distinguished in this unit since it is felt that they are important for understanding some significant physical phenomena in this last half of the twentieth century. The concept of a conversion factor is introduced at this time. It is important to note that the word "equivalent" rather than "equal" is used in this unit. This reserves the use of the word "equal" for cases in which its more special meaning (two names for the same thing) applies best. Toward the end, time and temperature scales are introduced. These are different classes of measure scales.

The importance of metric prefixes should be emphasized, and it is necessary for the student to understand that:

1. A system of measure is one in which there are various units for different kinds of measure.
2. Standardization of systems of measures is essential.
3. The British-American System of measure is the most widely used.
4. Mass and weight are two different concepts; weight refers to the pull of the earth on an object, and mass refers to the amount of matter in an object.
5. The units of mass and weights usually have the same name.
6. The metric system is used widely by scientists and is in general use in most parts of the world. It is necessary, therefore, to be able to convert between American and metric units.

7. Measures of time are standard the world over; time measure is related to angle measure in rotation of the earth, revolving of the earth in orbit, and rotation of clock hands.

8. Measurement of temperature depends on calibrating an instrument. There is an arbitrary zero and an arbitrary unit.

As an activity, the teacher should discuss and review the concept of a measure and develop the concept of a system of measures. He should have students do research on early systems of measures. He should discuss these early systems in class to bring out how the need for standardization became gradually stronger. He should show that the units picked for measures are chosen arbitrarily. He should give complete explanation of the British-American System of Measures and the metric system.

Informal Geometry

Plane Figures I

A plane figure, in this section, is defined as any set of points on a plane. This definition, chosen because it is inclusive and general, is much broader than Euclid's notion of a figure. Certain class of figures, or sets of points, merit more attention than others. Angles, lines, segments, polygons, and interiors of simple closed curves are some of the important classes and will be studied in this section.

Much of the material is traditional except for the new point of view. Here the non-metric concepts are considered further, and metric ideas are not mentioned until progress along non-metric lines becomes difficult. Non-metric notions are concerned with the number of sides of a polygon, the number of vertices, and the number of diagonals of a polygon.

Emphasis should be given to the terms Nominal Scales, Relation Scales, Ordinal Scales, Interval Scales, and Ratio Scales, and an explanation should be made of their differences in relation to measures and measurement. The students should learn and understand the definitions for plane, figure, quadrilateral, vertex, diagonal, concave, unit, sub-unit, perimeter, and degree. They should be able to understand, define, and use parallel, to measure, measure, intersecting, altitude, and triangle (right, isosceles, and equilateral). It is also important that they be able to recognize simple polygons, an ordering among one, two, and three dimensional figures, perpendicular lines, and adjacent, obtuse, acute, and vertical angles. Additional skills should be acquired in induction and informal deduction. Skills in the use of the protractor and in measuring and sketching angles should be learned.

As an activity, the teacher should explain the definition of a plane figure. He should review the definitions of closed and simple closed curves. He also should infer that several kinds of plane figures have been learned previously (lines, half-lines, rays, and triangles). He should give explanation for and have a student sketch an angle, a closed curve made up of line segments, and some simple polygons.

Plane Figures II

This section is a continuation of the preceding one. The study of sets of points is extended, and various classes of quadrilaterals are to be studied using the concepts of sets. Construction of geometric figures is introduced here, and the language problem associated with measures should be emphasized. Distinction between a radius of a circle and the length of a radius must be made. The determination of areas using the concept of a sweeping line segment also is introduced. Volume, or three dimensional measure, is given here for completeness.

Complete, clear definitions should be learned for these terms: (1) interior and exterior angles, (2) parallelogram, (3) rhombus, (4) transversal, (5) rectangle, (6) circle, (7) radius, (8) diameter, (9) circumference, (10) area, (11) volume, and (12) cube.

Students should have an understanding of the following:

1. When there are two lines and a third intersecting, the third one is called a transversal.
2. The sum of the measures of the angles on a simple quadrilateral is three-hundred sixty degrees.
3. Dividing the circumference of a circle by the measure of its diameter will give a number which is the same for all circles.

4. The symbol "π" is used as a numeral for this number and is approximately 3.14.
5. A number sentence can be written which gives the relation between the area of the interiors of a rectangle ($A = l \cdot W$), a parallelogram ($A = b \cdot a$), a triangle ($A = \frac{1}{2} b \cdot a$), and a circle ($A = \pi \cdot r^2$).
6. A number sentence can be written for circles ($C/D = \pi$ or $C/2 \cdot r = \pi$); the same thing is said when multiplying by inverse ($C = \pi \cdot D$ and $C = 2\pi \cdot r$).

The teacher should draw a figure on the board showing a transversal and explain the ideal of a transversal. He should point out that there are several angles on this figure and see if students can name some of them. He also should explain that these angles are referred to as "interior" and "exterior." He should see if students can figure out which are interior and which are exterior. He should bring out that the ones opening toward the inside are the interior angles.

Estimating

This unit leads up to the topic of Approximations, Approximate Calculations, and Mensuration. It is not easy to establish the fact that measurements are approximations, and there is hope that this unit will prepare the student to accept and understand this fact. The idea that certain quantities are known only within limits should be stressed, and the idea of a rounded number is given a more significant interpretation. The fact that guessing can be done intelligently will be a step toward maturity for the student.

Definitions for estimate, upper number, lower number, and rounded number should be made clear to the students. When the unit is completed, the student should understand these facts:

1. There are some sets of things in the physical world which have an exact number, but it would be difficult to determine.
2. There are sets of things which have an exact number, but it is impossible to determine.
3. There are other things that do not have an exact number.
4. In all of these cases, it is not necessary or helpful to know the number exactly even if it does exist.
5. In any of these cases, the number can be estimated.
6. To estimate a number, or make an estimate of it, try to make the most intelligent guess possible.
7. When estimating, find an upper number (one known to be too large) and a lower number (one known to be too small).
8. Symbols are written to describe estimates.
9. Better estimates are gotten by measuring.

10. When calculating, use upper and lower numbers.
11. Estimating is important when working with numbers.
12. Experimenting is sometimes helpful in making estimates.
13. Numerical estimating is useful when estimating physical quantities.

As an activity, the teacher should prepare a jar full of beans or a similar arrangement to use in illustrating estimating. He should ask the students to guess the number of beans in a jar, write the estimate down, and hand them in. The teacher should tabulate guesses on the board to show the great variability. He should explain what has been done in ordinary guessing, give steps used in estimating, and bring out the advantages of using this method.

CHAPTER IV

MATHEMATICS FOR THE EIGHTH GRADE

An investigation of the contents of traditional textbooks for the seventh and eighth grades verifies the opinion that one is somewhat the repetition of the other. For years the eighth grade has been a review of seventh grade work with but little addition of more or new concepts. For a student to remain interested in a subject, he must have some fresh ideas and concepts constantly challenging him. The student entering the eighth grade is at an age when his quest for learning is at a high potential. To keep this enthusiasm at its peak, a program must be provided with new and different material.

The basis for an outline for the eighth grade grew out of this idea. It is the wish of the writer that the outline, as presented here, will furnish new ideas and concepts which will maintain the student's interest.

"Why Study Mathematics?"

The major purpose of this unit is to implant in the student's mind the idea that mathematics is a great deal more than learning to calculate and computate numbers using formulas and equations. A student reaches mathematical maturity when he realizes that mathematics involves inductive and deductive thinking along with logical reasoning to prove and verify ideas and ideals related to all walks of life.

This unit should give an understanding that:

1. To the mathematician, mathematics is an art just as music or painting is to the musician or painter.
2. Mathematics and mathematical principles and concepts were invented and developed by man.
3. Mathematical principles are used to explain physical science concepts.
4. The use of mathematics is constant in everyday living.
5. Inductive and deductive reasoning is necessary in developing ideas and ideals.

Deductive reasoning may be reviewed by taking the total number of students in the classroom (at least thirteen or more) and proving that there are at least two students whose birthdays are within the same month. As an additional example, one can use eight marbles, all of which are the same color, shape, and size but one which differs in weight. Using a balance scale, one can find the heavy marble in only two weighings. Emphasize that the inductive method of learning consists of observing and experimenting, and the deductive method is logical reasoning.

The Integers

Non-Negative Integers

The non-negative integers are the counting numbers and zero. These counting numbers are known as the natural numbers. The development of mathematics begins with these numbers, and it is important that the nature and properties of them be understood.

Emphasis should be placed on these facts:

1. The notation for naming natural numbers and zero uses the ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
2. The position of each numeral in a number is essential in determining its value.
3. Natural numbers under the operations of addition (+) and multiplication (·) have the properties of closure, associativity, commutativity, and distributivity.
4. One is the multiplicative identity, and zero is the additive identity.
5. The natural numbers are discretely ordered.
6. Subtraction and division, the inverses of addition and multiplication, respectively, are not always possible within the realm of natural numbers.
7. Subtraction and division do not have the properties of closure, associativity, and commutativity.

8. Other bases and systems have the same general properties as the decimal system.

The principles for the cancellation law, maxima and minima, sets, and one-to-one correspondence should be given. Practice in the use of the symbols, "$<$" (is less than), "$>$" (is greater than), "$=$" (is equal to), and "$\neq$" (is not equal to) is necessary. The definitions for ordinal and cardinal numbers should be learned.

The properties of natural numbers may be learned by using objects that students can count. One should have them group the objects so that the properties can be reviewed.

Negative Integers

Negative means less than zero; positive means more than zero. Zero is the representative for the empty set, the answer to the question "not any." This implies that a negative integer is the inverse of a positive integer. After using the natural or positive numbers for many years, man found that they were not sufficient to allow unrestricted subtraction. To lift this restriction, the negative numbers were invented. These were given as many of the same properties as the natural numbers as possible.

The student should have knowledge of the following:

1. The negative numbers are the inverse of positive numbers.
2. Negative numbers have the property of commutativity and associativity.

3. Inverses have the same related numbers but different signs.
4. The integers can be ordered along a line.
5. Subtraction is the opposite of addition.
6. An integer is a collection of pairs of natural numbers.

The unit on integers should afford challenging activities for the student and should give a foundation for the principles involved in mathematical computation.

Factoring and Primes

This unit could be called an introduction to the analysis of integers. A factor is one of two or more numbers used to find a product. A prime number is one which has no other factors besides itself and one. These definitions, together with the fundamental theorem of arithmetic, give some of the basic principles of the theory of numbers.

Things to be understood are as follows:

1. The process of finding factors is called "factoring."
2. In a product of two or more numbers, each number is a factor of the product.
3. A number is even if it is divisible by the number two; if not, it is odd.
4. A number is said to be "prime" if it has no other factors besides itself and one.

5. Numbers that are not prime numbers are called composite numbers.

6. A number (positive integer) is a product of primes in essentially one way only (this is the fundamental theorem of arithmetic).

7. The prime factors of numbers may be used to find their greatest common divisor and least common multiple.

Discussions on even and odd numbers and their sums and on the "sieve of Eratostenes" should be included here. Activities involving the inductive and deductive method of proofs for certain properties associated with primes would be appropriate for this unit. This unit should give the students confidence and assurance that their ideas can play an important role in the study of mathematics.

Congruences

Two integers a and b are said to be congruent modulo an integer m if and only if there exists an integer k such that a - b = km.[5] This definition by Weiss in the text Higher Algebra merely says that m divides a-b. This unit is included to introduce the student to congruence and Euclidean Algorithm. A complete analysis of the definition

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[5]Marie J. Weiss, Higher Algebra (New York, 1949), p. 17.

of congruence should be given along with the properties involved with congruences.

The properties of congruence are as follows:

1. Reflexive: $a \equiv a \pmod{m}$ read a is congruent to a modulo m.
2. Symmetric: If $a \equiv b \pmod{m}$, then $b \equiv a \pmod{m}$.
3. Transitive: If $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$ then $a \equiv c \pmod{m}$.

The student should become familiar with the Euclidean Algorithm and the principles of casting out nines (residue of a modulo). The teacher should include activities which will involve these principles.

The Rational and Real Numbers

Rational Numbers

Addition, subtraction, multiplication, and division are terms in arithmetic known as the four fundamentals. These terms along with their assigned properties are the tools used for computation. The rational numbers were invented to solve certain problems which grew out of the fundamental called division. The integers permit subtraction, but they do not always allow division; that is, when one integer is divided by another, the remainder is not always zero. This problem of division not being a closed operation with the integers brought about the invention of numbers known as rationals. The term "fraction" is familiar to the eighth grader, and a review of its interpretation will make the understanding of rationals easier.

A fraction is defined as a part of the whole of something and is written as 1/2, 2/3, for example. The 2/3 may be interpreted as: (1) two parts of a whole which has been broken into three equal parts; (2) one of the three equal parts of two things; (3) a quotient of two numbers ($2 \div 3$); and (4) a comparison or a ratio (2 : 3). The interpretations three and four describe the rational numbers. One author's definition of the rational numbers is as follows:

> If a,b are whole numbers of which b is not zero, the ratio of a to b is a/b (the result of dividing a by b). A rational number is defined as the ratio of two whole numbers. The class of all whole numbers is a subclass of all rational numbers, as is seen by restricting the divisor b to be 1.[6]

From this, the relationship between fractions and rational numbers is seen readily. In this unit emphasis should be given to the following:

1. The system of rational numbers is called a "field."
2. Rationals have the properties of closure, commutativity, and associativity for addition and multiplication.
3. Rationals have the property of an additive identity and an additive inverse.
4. There is a multiplicative identity and a multiplicative inverse (restricted) for rationals.

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[6] E. T. Bell, Mathematics (New York, 1951), p. 46.

5. Rationals have the property of distributivity.
6. Division of rational numbers is always possible, except by zero.
7. The rational numbers have order, and the order is dense (there is no largest or smallest).
8. Rational numbers may be expressed as repeating decimals. These properties should be analyzed, and examples involving proofs for them should be worked by the student.

Real Numbers

The invention of rational numbers proved that the existing numbers were not sufficient to meet the needs. So, from these rational numbers an ordered field called real numbers was constructed. These include all of the rational and irrational (any number that is not the ratio of any pair of whole numbers) numbers. Any number which can be expressed as a decimal is known as a real number. All numbers previously discussed are included in this category--natural, integers, fractions, decimal fractions, rational plus the irrationals. Example of irrational: Consider the problem of finding the square root of a rational number a/b where $(a/b)^2 = 2$ then $\frac{a^2}{b^2} = 2$ and $a/b = \sqrt{2}$, and it can be shown that there is no rational number whose square is 2 that is $a/b \cdot a/b = 2$. The term $\sqrt{2}$ is then called a real number.

The important concepts here for the student to grasp are those of realizing that real numbers were invented to meet the needs of certain problem solving; and with certain principles involving odd and even numbers, proof can be constructed for irrational numbers.

Activities for this unit should consist of examples that will give the student practice in the construction of rational and real numbers and proof of some of the properties of the rationals.

Geometry

Non-Metric Geometry

The material which makes up the contents of this section could be called a vocabulary study of terms used in geometry. However, basic concepts will be developed also. The idea of sets and set intersection is a basic part of this section (a fundamental idea for geometry as well as for the study of mathematics in general). Studying this section before metric geometry makes it easier to understand, and its concepts become more meaningful.

The student should be able to define and analyze the following words: (1) point, (2) line, (3) space, (4) plane, (5) intersection, (6) ray, (7) set, (8) segment, (9) vertex, (10) quadrilateral, (11) elements, (12) plane figures, (13) Desargues' configuration, (14) curves, and (15) end points.

Emphasis should be given to the following concepts:

1. The word "line" means a straight line.
2. A line is a set of points.
3. For any two sets there is a set which is their intersection.
4. An intersection may be the empty set.
5. A line segment is determined by two points on a line; these points are called end points.
6. Lines are on a point, and planes are on a line.
7. There is a one-to-one correspondence between lines on a point and points on a line.
8. Between any two points on a line there is a third point.

A discussion on the kinds of lines and the types of curves should be included. As an activity, the teacher should discuss triangles and their definitions. He should sketch some figures on the board, bring out their differences, and explain the connection of shapes and names of curves.

Metric Geometry

Geometry is defined as that branch of mathematics which investigates the relations, properties, and measurement of solids, surfaces, lines, and angles. Lines, angles, and plane surfaces will be studied in this portion of the unit. An introduction to plane geometry is the objective of this unit, and emphasis should be given to the following:

1. A plane figure is a set of points on a plane.
2. A polygon is a plane figure which is a closed curve consisting entirely of line segments.
3. Polygons are named according to the number of sides.
4. An angle is a set of points on two rays with a common end point, and the unit of measure for an angle is called the degree.

Discussions should involve the relationship in angles and figures formed by parallel lines and transversals. Geometric symbols should be learned and used. An analysis of formulas for area, circumference, perimeter, and volume is necessary. The idea of converse principle is essential in discussing the congruency of figures. The teacher should introduce the Pythagorean Theorem at this time. Before completing the unit, a brief discussion on vectors is recommended.

As an activity, the teacher can sketch different size circles on the board. He should ask: "Do the larger ones have bigger circumferences?" He should bring out that the ratio of the circumference to the diameter is the same in any circle. Other activities should give the student practice in the use of the protractor.

Approximations

The process of evaluation is employed by most people. The number of objects in a set and the amount of a certain substance in a given space or area are expressions of evaluations. To find the number of objects in a set, the set of natural numbers is placed in a one-to-one correspondence with the set of objects. To find amounts it was necessary to invent a form of comparison (unit of measure). Observation of a balance, a ruler, or a graduated cylinder will show that these units are not exact but approximate. Approximations are a part of everyday life, and it is essential to learn something of approximate numbers.

When this unit is completed, the student should have an understanding of the following:

1. Numbers used for measurement are called denominate numbers.
2. The metric system is used by scientists and a number of nations of the world.
3. The United States uses the English system.
4. Precision and accuracy are different terms based on apparent and relative errors.
5. Computing with approximate numbers involves significant digits.
6. The irrational numbers are approximate. Discussions should include a history of measurement, and

the importance of denominate numbers and their standardizations should be stressed.

For an activity, the teacher can get a dozen balls and a container of water. He should have each student count the balls and observe that each should get the same number. This answers the question, "How many?," and is exact. Using different cups, he should let several students find the number of cupfuls of water in the container. This answers the question "How much?" It is not exact because the cups may be of different sizes; therefore, it is an approximation.

Mathematics in Science

To give in detail the relationship between mathematics and the sciences would take several volumes. E. T. Bell gives an idea of the vast connection in his quotation of C. F. Gauss.

> Mathematics is Queen of the Sciences and arithmetic the Queen of Mathematics. She often condescends to render service to astronomy and other natural sciences, but under all circumstances the first place is her due.[7]
>
> -- C. F. Gauss

In this unit the basic principles that underlie scientific notation and experiment are studied. This would include the introduction of powers and exponents and an

[7] Bell, p. 1.

explanation of the advantage in using exponents when writing large numbers. Some of the principles involved in the study of this unit are:

1. The four steps of scientific experimenting:
   A. Observation and collection of data
   B. The formulation of a hypothesis
   C. Predictions that can be made from these hypotheses
   D. The testing of a reasonable number of cases to verify or nullify the predictions made.
2. If the experiment is verified, it is called a theory or principle.
3. The quantitative aspects of physical experiments depend on the collection of mathematical data.
4. Predictions usually are made with the use of mathematical formulas or equations.
5. Proofs of scientific predictions are found by applying mathematical principles to the collected and observed data.
6. Graphs are used to show scientific information.

Emphasis should be given to the steps and principles that are necessary in doing calculation with scientific notations. For an activity, the teacher can write several large numbers on the board and explain how these can be written using exponents.

Statistics

This unit deals with the elementary concepts of statistics. The part mathematics plays in securing and developing material used by government agencies, insurance companies, and businesses of all kinds to make predictions is not usually known. The idea that statistical information can be presented in such way as to prove or disprove a point is an essential principle that the student should realize. Some of the terms to be analyzed and understood are as follows: (1) arithmetic mean, (2) median, (3) mode, (4) range, (5) average deviation, (6) data, and (7) average.

The student should learn that certain statistics provide better descriptions than others of specific situations. This unit should give him an appreciation of the use of mathematics in the social studies and of the important part mathematics plays in predictions that affect our everyday lives.

For an activity, the teacher should have students collect news items containing references to averages. The teacher should discuss these and check them for the kinds of averages used. He should bring out that the kind of average used is important for interpretation of information presented.

Chance

This unit is an introduction to probability and logic. Statements are used to show the difference between conjunctions, disjunctions, implications, and negations. The use of true and false statements for finding a truth value is given. Emphasis should be given to the following:

1. The probability or chance is the ratio of the number of favorable cases or true statements to the total number of cases. This may be written symbolically as C = T/S C = chance, T = true statement, and S = the sum of the true and false statements.
2. A letter or symbol may be used to represent a statement or a numerical value or both.
3. Substitution for variables always must come from some specific set.
4. Symbols are called variables.
5. Some sentences containing variables are statements, and some are not.
6. The truth value of a statement is different for each connective, conjunction, disjunction, and implication.
7. The solution of equations can be attacked with the use of number sentences.
8. Inferences are drawn from the various possibilities that a specific case presents.

For an activity, the teacher should have the students check the calendar for the prediction of weather several days ahead and make observation of the number of predictions that come true. They should work out the probability and use statements to build a truth table using examples of all the connectives.

Finite Mathematical Systems

The comparison of similar objects for the purpose of learning more about either of them is not a new idea. The biologist learns by observing one animal and using the collected information to make inferences about others. The structure of most physical objects has some similarity, and comparison aids in learning about all of them. Mathematical application is used in making physical comparisons, but it is thought of as an abstraction itself. The fact that all systems of mathematics are thought of as abstract reveals that there is a similarity between them. This unit gives the student an idea of the sameness of finite mathematical systems or the kinds of algebras. He should get a clear picture of the framework in which finite mathematics is carried out.

When the unit is completed, the student should have an understanding of the following:

1. Any mathematical system consists of a set of elements and an equal or equivalence relation between pairs of elements.
2. There are one or more operations on a pair of elements of the set.
3. When the operation on a pair of elements of a set produces a third element of the same set, the operation is called a binary operation.
4. A singularly operation involves just one element of the set.
5. The binary operations of addition and multiplication possess the properties of closure and associativity.
6. The set considered will have an identity element and an inverse element for each element in the set.
7. Addition may be of another kind besides the familiar kind.
8. In modular arithmetic, the properties may be checked by studying tables.
9. Modular addition is associative and commutative.
10. The members of a mathematical system may not be numbers.
11. Modular arithmetics are mathematical systems with two operations.
12. Modular arithmetics have the distributive property.

For an activity, the teacher should have students make a table for the base ten and one for another base. He should let them compare the bases by doing the binary operations of addition and multiplication. He should allow the students to make up their own symbols and systems and compare them.

CHAPTER V

SUMMARY

The problem of what units should constitute a mathematics course for the seventh and eighth grades to meet the demand of the current change now taking place in mathematics was the chief concern of the writer in preparing this report.

The writer has not tried to solve this problem of revising the junior high school curriculum. This can be done only through the cooperation of many groups working together to improve the entire mathematics program as a whole. He has, however, tried to make a small contribution toward a solution.

This contribution, though limited in scope, is aimed at all students in the seventh and eighth grades--not just the gifted. It is meant to reach that pupil who will pursue a scientific career as well as the student whose use of mathematics will be confined to affairs of everyday life.

While organizing these units, the writer had certain objectives in mind which he feels will sum up the entire purpose of this report:

1. To provide material which is fundamental yet challenging enough to arouse interest to the point of possible pursuit of further mathematical study.
2. To develop in each student an appreciation for mathematics as a useful art.
3. To show the close relationship between mathematics and the other sciences.
4. To instil in the student the fundamental ideas of mathematical language.
5. To provide material that will help the student to attain accuracy in reasoning and computation.
6. To instil in the student the idea that mathematics constantly affords opportunities to search out truth and to seek new applications for use in daily living.
7. To provide material that will give the student a desire for developing new skills in mathematics.

It is the desire of the writer that the units as presented in these outlines, together with competent teaching, will accomplish these objectives. The writer feels that the outlines can be used as guides for teaching seventh and eighth grade mathematics. These outlines were not intended to be complete, but they are blueprints which can be used with other material in teaching these grades.

No outline is given for the ninth grade, but certain units are recommended. These probably could include advanced topics in algebra and introductory units in numerical trigonometry and demonstrative geometry.

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