# DYNAMIC BEHAVIOR OF DANCER SUBSYSTEMS IN WEB TRANSPORT SYSTEMS

Karl N. Reid and Ku-Chin Lin

School of Mechanical and Aerospace Engineering
Oklahoma State University
Stillwater, Oklahoma

## ABSTRACT

A dancer subsystem may be used as a tension measurement or as a disturbance attenuator depending on its design. The design of a dancer subsystem to accomplish a desired result requires the development of a dynamic model for the subsystem. This paper presents the results of a generalized dynamic model of a dancer subsystem. Examples are presented to illustrate the behavior of an unwind-rewind web transport system which incorporates a dancer subsystem, and which has periodic disturbances from an out-of-round unwinding roll.

## NOMENCLATURE

| | |
|---|---|
| $A$ | Cross-sectional area of web |
| $b$ | Width of web |
| $B_f$ | Rotary friction constant of bearing |
| $C_d$ | Damping coefficient |
| $E$ | Modulus of elasticity |
| $h$ | Thickness of web |
| $J$ | Polar moment of inertia of roll or roller |
| $k_s$ | Spring constant |
| $L$ | Length of web span |
| $M$ | Mass of roller |
| $R$ | Radius of roll or roller |
| $s$ | Distance measured along the web |
| $t$ | Web tension |
| $T$ | Change in web tension from a steady-state operating value |

| | |
|---|---|
| $v$ | Roller velocity |
| $V$ | Change in roller velocity from a steady-state operating value |
| $x$ | Displacement of dancer roller |
| $X$ | Change in displacement of dancer roller |
| $\delta$ | Effective translation of a roller |
| $\varepsilon$ | Web strain |
| $\rho$ | Web density |
| $\theta$ | Angle of wrap of web over a roller |
| $\tau$ | Time |
| $\tau_b$ | Brake torque |
| $\tau_m$ | Motor torque |
| $\mu_v$ | Amplitude of sinusoidal disturbances |

Subscripts:

| | |
|---|---|
| 0 | Initial steady state |
| n | 1,2,3, . . . |
| r | Reference signal |

## INTRODUCTION

A dancer subsystem may be used as a tension measurement or as a disturbance attenuator depending on its design. For different applications, a dancer subsystem may be designed with a dancer roller and attachments such as a pneumatic cylinder, a spring, and a damper.

The dynamic behavior of a dancer subsystem is influenced by the following parameters and variables: (i) the diameter, the mass, and the mass moment of inertia of the dancer roller (ii) the angle of wrap of the web over the roller (iii) the parameters of the attachments, and (iv) the forces and torques acting on the roller.

Marhauer (1) has studied a spring-loaded dancer subsystem used for tension measurement. A simplified model was developed but without considering the rotary dynamics of the dancer roller and the interacting dynamics between the roller and web. Marhauer's model is good for understanding the effects of the spring stiffness on the accuracy of the tension measurement. However, the model is not adequate for studying the capability of a dancer subsystem for disturbance attenuation.

Tension disturbances transport over a roller because of the rotation of the roller. For a dancer roller, the disturbances can be attenuated significantly because of the linear translation of the roller. Such counter effects of the rotation and translation of a dancer roller on the incoming web tension disturbances was described by Pfeffer (2). A classical dancer subsystem partially absorbs tension variations. Martin (3) claimed that complete absorption of the disturbances can be achieved if the rotary moment of inertia is precisely balanced with a moment of inertia based upon the translational inertia.

However, the Martin approach does not result in attenuation of disturbances when the dominant frequency of the disturbances is close to the resonant frequency of the web/dancer subsystem (4). To prevent the occurrence of system resonance, the dancer subsystem should be designed so that the subsystem does not have a resonant frequency close to any potential disturbance frequency.

This paper presents a generalized model for a dancer subsystem where the dancer roller moves in the vertical direction. Examples are presented to illustrate the behavior of a web transport system which incorporates a dancer subsystem, and which has periodic disturbances from an out-of-round unwinding roll

## MATHEMATICAL MODEL FOR A DANCER SUBSYSTEM

Figure 1 shows a dancer subsystem which includes a dancer roller, a lead-in roller, a lead-out roller, a pneumatic cylinder, a spring, and a damper. The dancer roller only moves along the central vertical line between the lead-in and lead-out rollers. Assumptions listed below facilitate the derivation of the system model:

1. The displacement of the dancer roller is very small in comparison with the length of the supporting web spans.
2. The change of the wrap angle of the web over the dancer roller due to the displacement of the roller is negligible.
3. No slippage occurs between the dancer roller and the web.
4. The dynamics of the lead-in and lead-out rollers are negligible.
5. The web thickness is very small compared to the radius of the rollers.
6. The cross-sectional area of the web is invariant.
7. The web strain is very small (strain << 1) so that the web can be considered perfectly elastic.
8. The strain is uniformly distributed across the width of web.

Applying a force balance on the dancer roller gives:

$$M_n \ddot{x}_n = M_n g_c + f_a - f_s - f_d - (t_{n-1} + t_n) \sin(\theta_n/2) \tag{1}$$

where the spring force, $f_s$, is proportional to the displacement of the dancer roller from its neutral position,

$$f_s = k_s (x_n - x_{n0}) \tag{2}$$

the damping force, $f_d$, is proportional to the translational velocity of the dancer roller,

$$f_d = C_d \dot{x}_n \tag{3}$$

and the force, $f_a$, generated by the pneumatic cylinder is invariant.

Applying a torque balance on the dancer roller gives:

$$J_n \dot{v}_n = - B_{fn} v_n + R_n^2 (t_n - t_{n-1}) . \tag{4}$$

## MATHEMATICAL MODEL FOR A WEB SPAN

Figure 2 shows a schematic diagram of a web span. If it is assumed that no slippage occurs between the rollers and web, the law of conservation of mass can be written as:

$$\frac{d}{d\tau}\int_0^{L_n} \rho_n A_n \, ds = \rho_{n-1} A_{n-1} v_n - \rho_n A_n v_{n+1} \ . \tag{5}$$

If the web is assumed perfectly elastic, Hooke's law can be written as:

$$[t_n]_v = A_n E_n [\varepsilon_n]_v \ , \tag{6}$$

where $[t_n]_v$ and $[\varepsilon_n]_v$ denote the tension and strain in the web respectively, and which are created due to the velocity difference at the ends of the span as shown in Figure 3(a).

Through considering an infinitesimal element in the web span of length ds, Equations (5) and (6) can be combined to give the following nonlinear differential equation (5):

$$L_n [\dot{t}_n]_v = v_n [t_{n-1}]_v - v_{n+1} [t_n]_v + E_n A_n (v_{n+1} - v_n) \tag{7}$$

Web tension also is influenced by the translation of a roller as shown in Figure 3(b). The change of web tension is proportional to the effective translation, $\delta_n$, in the direction of the web span:

$$[t_n]_d = E_n A_n \delta_n / L_n \tag{8}$$

where $[t_n]_d$ denotes the web tension which is created due to the roller translation.

As shown in Figure 4, the effective translation of a vertical-displacement dancer roller is

$$\delta_n = X_n \sin(\theta_n/2) \tag{9}$$

From assumption 7,

$$t_n = [t_n]_v + [t_n]_d \tag{10}$$

Combining Equations (7) through (10) gives:

$$\begin{aligned} L_n \dot{t}_n = {} & v_{n+1} E_n A_n \left(1 - \frac{t_n}{E_n A_n} + \frac{X_n}{L_n}\sin(\theta_n/2)\right) - \\ & v_n E_n A_n \left(1 - \frac{t_{n-1}}{E_{n-1} A_{n-1}} + \frac{X_n}{L_{n-1}}\sin(\theta_n/2)\right) + \dot{X}_n \sin(\theta_n/2) \end{aligned} \tag{11}$$

Equations (1) and (11) are coupled because of the consideration of the roller effective translation that describes the interaction between the dancer subsystem and web.

## EXAMPLES

Consider the unwind-rewind web transport system shown in Figure 5. The system dynamic equations are as follows:

$$J_1 \dot{v}_1 = -B_{f1} v_1 + R_1^2 t_1 - R_1 \tau_b \tag{12}$$

$$L_1 \dot{t}_1 = v_2 E_1 A_1 \left(1 - \frac{t_1}{E_1 A_1} + \frac{X_2}{L_1}\sin(\theta_2/2)\right) - v_1 E_1 A_1 \left(1 + \frac{X_2}{L_1}\sin(\theta_2/2)\right) + \dot{X}_2 \sin(\theta_2/2) \tag{13}$$

$$J_2 \dot{v}_2 = -B_{f2} v_2 + R_2^2 (t_2 - t_1) \tag{14}$$

$$M_2 \ddot{X}_2 = -C_d \dot{X}_2 - k_s X_2 - (T_1 + T_2)\sin(\theta_2/2) \tag{15}$$

$$L_2 \dot{t}_2 = v_3 E_2 A_2 \left(1 - \frac{t_2}{E_2 A_2} + \frac{X_2}{L_2}\sin(\theta_2/2)\right) - v_2 E_2 A_2 \left(1 - \frac{t_1}{E_1 A_1} + \frac{X_2}{L_1}\sin(\theta_2/2)\right) + \dot{X}_2 \sin(\theta_2/2) \tag{16}$$

$$J_3 \dot{v}_3 = -B_{f3} v_3 - R_3^2 t_2 + R_3 \tau_m \tag{17}$$

The unwinding roll is resisted by a brake torque, $\tau_b$, which determines the average tension level in the system. The rewinding roll is driven by a motor and the speed of the motor is controlled by a feedback controller. It is assumed that the dynamics of this speed control system are negligible compared to the dominant dynamics of the system.

First, consider a free-dancer subsystem (i.e., $k_s = C_d = 0$) with an inertia-compensated dancer roller. An inertia-compensated roller has the following relationship between its mass and mass moment of inertia (4):

$$J_2 = M_2 R_2^2 \tag{18}$$

Now, suppose the dancer subsystem is subject to periodic disturbances from an out-of-round unwinding roll that are equivalent to sinusoidal disturbances on the roll velocity, i.e.,

$$v_1 = v_{10} + \mu_v \sin\left(\frac{v_{10}\,\tau}{R_1}\right), \tag{19}$$

where $\mu_v$ denotes the amplitude of the sinusoidal disturbances.

The system equations were solved in the time domain based on the initial conditions and the parameter values given in Table 1. Also, the same system without the dancer subsystem was considered for comparison. The system tension responses to the disturbances from an out-of-round unwinding roll are plotted in Figures 6(a), 6(b), and 6(c) for the cases without the dancer subsystem, with a classical dancer subsystem, and with an inertia-compensated dancer subsystem, respectively. The results show that without a dancer roller web slackness or buckling may occur if the operating tension level at the unwinding section is low. Web slackness can be prevented by incorporating a dancer into the system. As shown in Figures 6(b) and 6(c), the magnitudes of the tension changes have been reduced by incorporating a dancer subsystem into the web transport system. The inertia-compensated dancer subsystem produces similar results as those with the classical dancer subsystem; the former subsystem does not completely absorb the disturbances as claimed by Martin (3).

The resonant frequency of the web/dancer subsystem for the web transport system studied is

$$\text{Resonant Frequency} = \frac{1}{2\pi}\sqrt{\frac{\sin(\theta_2/2)}{M_2}\left(\frac{E_1A_1}{L_1} + \frac{E_2A_2}{L_2}\right)} \approx 7.7 \text{ Hz} \tag{20}$$

When $R_1 = 75.9$ cm (30 in), the frequency of the disturbances is 3.2 Hz. This frequency increases as the processing continues and the unwinding roll radius decreases. When $R_1 = 38.0$ cm (15 in.), Figure 7 shows that the disturbances with frequency 6.4 Hz are amplified by the inertia-compensated dancer subsystem.

Consider the system in Figure 5 subjected to a step change in the rewinding roll reference velocity from an initial steady-state condition, $v_{3r0}$. The same system conditions and parameter values given in Table 1 were used for simulating the system tension responses for a step change of $V_{3r} = 0.05$ m/sec (10 fpm).

Figure 8(a) shows the tension responses for the cases with an inertia-compensated dancer and without a dancer subsystem. The change in the tension in the web spans during the transient is substantially greater for the system without the dancer than the system with the dancer. That is, the dancer subsystem for this case acts as a low-pass filter and not as a suitable tension measurement device.

A spring-loaded dancer normally responds to tension variations faster than a free dancer does. As the stiffness of the spring increases, the influence of the dancer on the system dynamics decreases. Figure 8(b) shows the step responses for the system with high spring stiffness ($k_s >> 1$, such as a load cell). The responses are very close to those for the system without a dancer. That is, for this case the dancer subsystem acts as a high-pass filter and serves as a suitable tension measurement device.

## CONCLUSIONS

This paper presents a generalized model which includes the dynamics of a dancer subsystem and its interacting dynamics with the web. The model can be used to study the dynamic behavior of a dancer subsystem when it is used for disturbance attenuation or for tension measurement.

An inertia-compensated dancer subsystem is useful for disturbance attenuation providing the disturbance frequency is well below or well above the dominant natural frequency of the system. The manufacturer's claim of complete rejection of disturbances is overstated!

In the design of a dancer subsystem, the possible occurrence of system resonance should be considered. The system model can be used to select appropriate system parameter values or web processing speed to avoid the potential of operation near resonance.

## REFERENCES


1. Marhauer, H. H., "The Dynamics of Web Tension Measurement", ISA Transactions, Vol. 5, No. 3, 1966, pp. 242-247.
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3. Martin, J. R., "Tension Regulation Apparatus", United States Patent, May 2, 1972.
4. Sayed, A. A. M. E., "Critical Analysis of Portable Web-Tension Measuring Equipment", Doctoral Thesis, Loughborough University of Technology, September, 1976.
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Table 1 System Conditions and Parameter Values for Simulation

| |
|---|
| $\tau_b = 98.3$ N-m (70 lbf-ft) |
| $v_{10} = v_{20} = v_{30} = 15.2$ m/sec (3000 fpm) |
| $t_{10} = t_{20} = 5.2$ N/cm (2.8 pli) |
| $L_1 = L_2 = 0.9$ m (3 ft) |
| $E_1 = E_2 = 2.4*10^9$ N/m$^2$ (350,000 psi) |
| $b_1 = b_2 = 25.3$ cm (10 in.) |
| $h_1 = h_2 = 2.53*10^{-3}$ cm (1 mil) |
| $B_{f1} = B_{f2} = B_{f3} = 0$ |
| $C_d = 0$ |
| $R_1 = 75.9$ cm (30 in.) |
| $R_2 = 7.6$ cm (3 in.) |
| $R_3 = 15.2$ cm (6 in.) |
| $M_2 = 15.2$ N-sec$^2$/m (1 slug) |
| $J_1 = 175.5$ N-m-sec$^2$(125 lbf-ft-sec$^2$) |
| $J_2 = 0.088$ N-m-sec$^2$(0.063 lbf-ft-sec$^2$) |
| $J_3 = 14.0$ N-m-sec$^2$(10 lbf-ft-sec$^2$) |
| $\theta_2 = 180^o$ |
| $\mu_v = 0.152$ m/sec (30 fpm) |

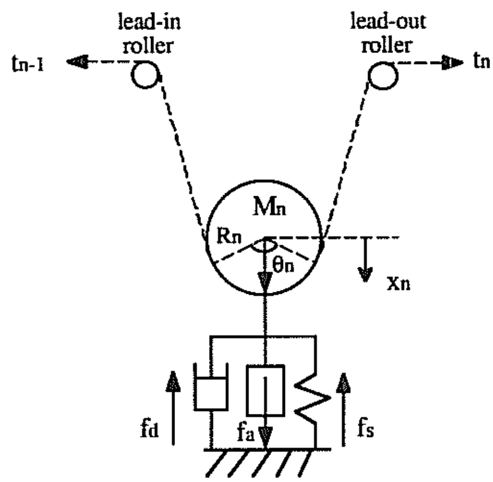


Fig. 1 Schematic of a Dancer Subsystem

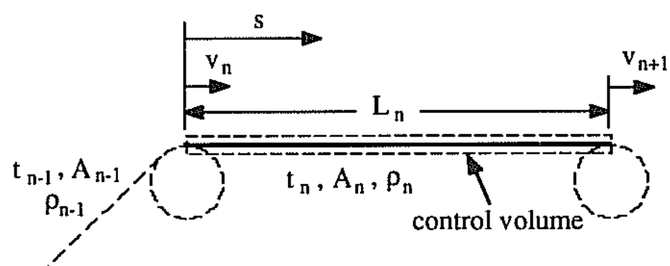


Fig. 2 Schematic of a Web Span

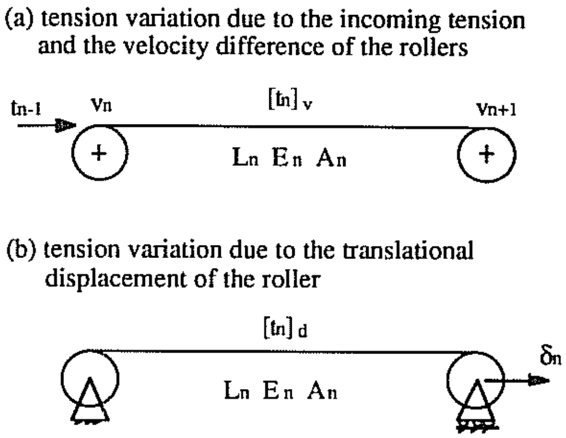

Fig. 3 Dominant Factors Which Influence Web Tension

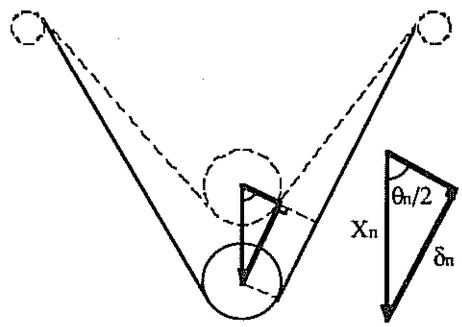

Fig. 4 Geometry of the Displacement of a Dancer Roller

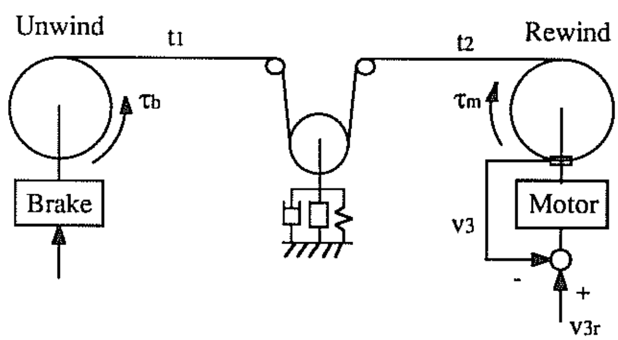

Fig. 5 Unwind-Rewind Web Transport System

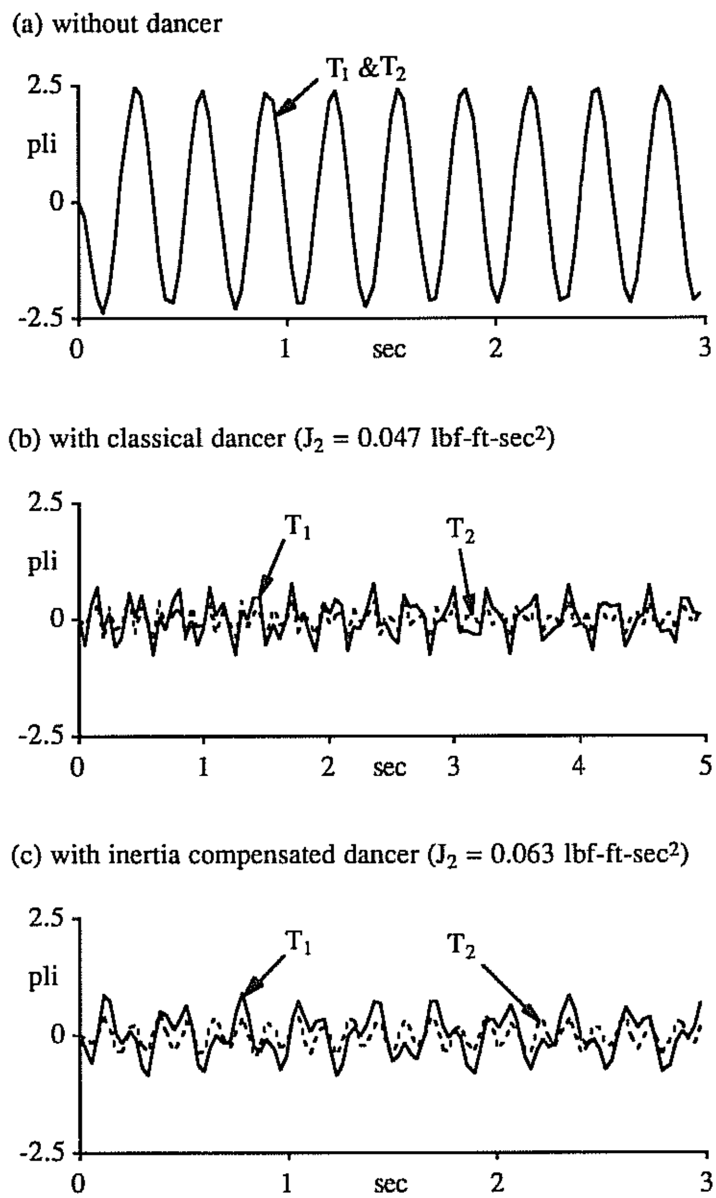


Fig. 6 Tension Variations due to the Sinusoidal Disturbances, $R_1 = 75.9$ cm (30 in.)

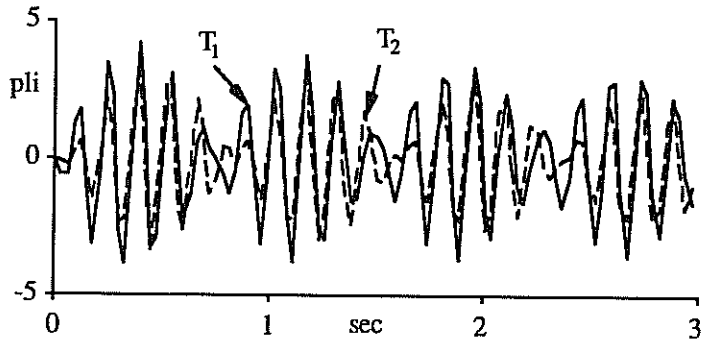


Fig. 7 Tension Variations in the System with Inertia-Compensated Dancer due to the Sinusoidal Disturbances, $R_1 = 38.0$ cm (15 in.)

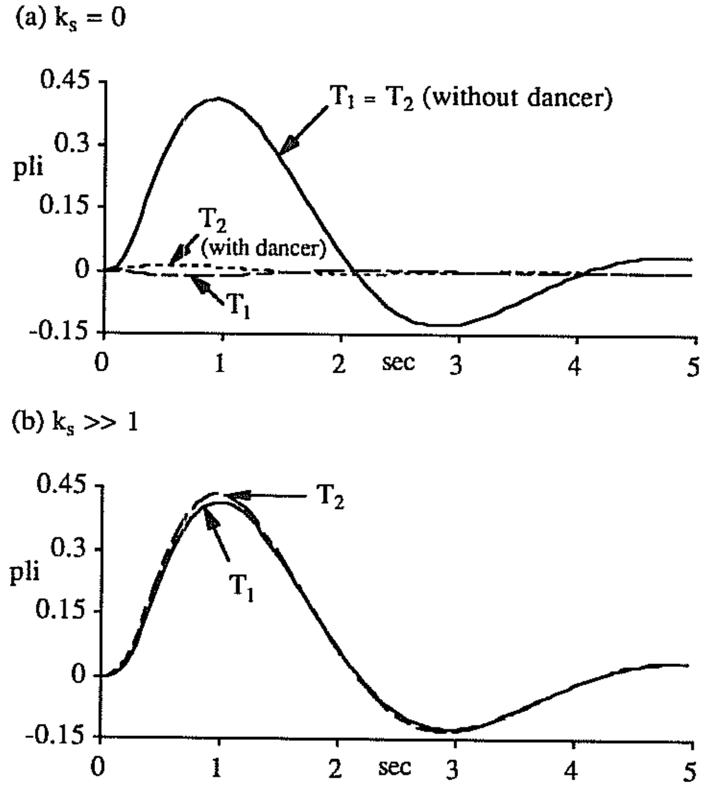


Fig. 8 Tension Responses to the Step Change of $V_{3r} = 0.05$ m/sec (10 fpm)

**QUESTIONS AND ANSWERS**

Q. In addition to a dancer subsystem, a driven roller also can attenuate system tension disturbances. What's the difference between a dancer subsystem and driven roller on the attenuation of tension disturbances?

A. Yes, you are right. A driven roller or an idle roller does attenuate tension disturbances somewhat. But, driven and idle rollers have no translational displacement. Such displacement is a dominant factor for disturbance attenuation. As shown in the paper, a dancer subsystem does significantly (although not completely) attenuate tension disturbances if the translational inertia of the dancer roller is designed in good balance with its rotary inertia.

Q. Is the dancer subsystem considered in this paper the same as that considered in the Martin's inertia compensation? What's the difference between the Martin's mathematical equations and yours?

A. In the Martin's derivation of inertia compensation, only the rotary and translational dynamics of the dancer roller are considered. Since the rotation and translation of a dancer roller have the opposite effects on the web tensions, the effects can be canceled if the moment of inertia of the dancer roller is equal to its mass times radius square. That is the basic concept of inertia compensation. However, if web dynamics and the interaction between the dancer roller and web are considered, the concept of inertia compensation is no longer adequate, especially when the dominant frequency of tension disturbances is close to the system natural frequency.