ON THE DYNAMICS OF WEB TRANSFER

IN AN OPEN DRAW

by

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ABSTRACT

A paper web running in an open draw of a high speed paper machine is subjected to various internal and external forces which stretch it and which can ultimately rupture it. The forces arise from speed differences between various web transfer elements, from pressure loads transmitted through the surrounding air and from the effects of gravity, friction and drag. Since the web travels in a curved path, it is also subjected to centripetal forces, which can reach very high magnitudes.

This study is concerned with the dynamics of paper web in an open draw. The constructed model takes into account external and internal forces acting on the web. There is provision in the model for determining variations in web tension caused by occasional external disturbances.

The sensitivity of web tension to small external disturbances is shown in simulations. The length of the open draw is an important variable in web transfer dynamics. Another essential parameters which must be included are the aerodynamic drag forces, as they contribute to damping the resonance events during web transfer, and therefore to web stability.

NOMENCLATURE

a	distance from the entrance point of the web, m
С	air dissipation coefficient
d	width of the web, m
Е	modulus of elasticity, m^2/s^2
F	force applied to web, N
g	acceleration of gravity, m/s ²
ī	instantenious length of the web, m
1.	length of the open draw (span length), m
m	basis weight of the web, kg/m ²
\mathbf{m}_{0}	basis weight of unstrained web, kg/m ²
Δp	pressure difference, Pa
व	velocity vector, m/s
R	radius of the curvature, m
Т	tension of the web, N/m
$\Delta \mathbf{T}$	tension increase along the web, N/m
T _{tot}	total tension of the web, N/m
u	horizontal velocity, m/s
v	tangential velocity, m/s
W	vertical velocity, m/s
x	horizontal coordinate
Y	Young's modulus, N/m ²
Z	vertical coordinate
3	strain of the web
η	dynamic viscous coefficient, m ² /s
θ	angle of web tangent and horizontal axis, deg.
ν	kinematic viscous coefficient, m ² /s
ρ	density, kg/m ³
σ	stress of the web, N/m ²

 φ angle of velocity vector q and horizontal axis, deg.

Subscripts

а	added mass
- b	friction (air)
đ	drag (air)
-	

- g gravitation
- i air
- p pressure

Superscripts

- ' first derivative with respect to time
- " second derivative with respect to time
- first derivative with respect to coordinate x

1. INTRODUCTION

As speeds of web handling machines increase, the understanding of the web's dynamic behaviour becomes increasingly important. The critical locations in such machines are where the web is unsupported, i.e. where it travels in an open draw. In a drying section of a paper machine, for example, the various external forces acting on such a web cause significant tension changes to occur and these in turn establish how well or how poorly the web will run in the machine. This paper deals with the mathematical modelling of web behaviour in an open draw, based on the well known model for a flexible wire. The model differs from the earlier threadline models (1), (2) in that it takes into account features of mass transportation balance in an open draw, as well as aerodynamic drag forces which have a significant damping effect on the web. Also, unlike the earlier models, the present model deals mainly with web tension and its temporal variations. Although, the general form of our model includes the fluttering phenomena, it's not a issue of this study.

2. MATHEMATICAL MODEL

2.1 Basic Assumptions

In this model, we have assumed the web's material and geometrical properties as well as all forces and the resultant web tension to be constant in the cross-machine direction. We have also assumed that the web has zero bending stiffness, i.e. it is completely flexible and is subjected only to a constant axial tension across its thickness. This assumption is valid only for a very thin web.

If we take an arbitrary point A of the web and assume certain velocity vector \vec{q} (Fig. 1), we can write the following equations:

$$q = \sqrt{u^2 + w^2} \tag{1}$$

$$\frac{w}{u} = \tan \varphi \tag{2}$$

$$\frac{v}{q} = \cos(\varphi - \theta) \tag{3}$$

Using above equations, tangential velocity v can be written:

$$v = u \cos\theta + w \sin\theta \tag{4}$$

Vertical speed w can be written:

$$w = \frac{d z(x,t)}{dt} = \frac{\partial z}{\partial t} + u \frac{\partial z}{\partial x} = \frac{\partial z}{\partial t} + u \tan\theta, \qquad (5)$$

where z(x,t) is the vertical deflection of the web. Using equations 4 and 5 velocity components u and w are:

$$u = v \cos\theta - \frac{\partial z}{\partial t} \sin\theta \cos\theta$$
(6)

$$w = v \sin\theta + \frac{\partial z}{\partial t} \cos^2\theta \tag{7}$$

By differentiation (Eq. 7) we obtain:

$$\frac{dw}{dt} = \frac{\partial^2 z}{\partial t^2} + 2u \frac{\partial^2 z}{\partial x \partial t} + u^2 \frac{\partial^2 z}{\partial x^2} + \frac{du}{dt} \frac{\partial z}{\partial x}$$
(8)

Since $\dot{z}(x,t)$ is the tangent of function z (Eq. 5), we can write:

$$\frac{dw}{dt}\cos\theta - \frac{du}{dt}\sin\theta = \cos\theta \left(\frac{\partial^2 z}{\partial t^2} + 2u \frac{\partial^2 z}{\partial x \partial t} + u^2 \frac{\partial^2 z}{\partial x^2}\right)$$
(9)

It can be seen that velocity differentiation yields the usual inertial, coriolis and centripetal terms.

2.2 The Model

<u>2.2.1 Equation of motion</u>. Let us take an infinitesimal element, $dl = Rd\theta$, of the moving web and assume the following force components (Fig. 2):

$$F_{p} = \Delta pR \ d\theta \qquad \text{External pressure}$$

$$F_{g} = mgR \ d\theta \qquad \text{Gravitation}$$

$$F_{b} = \frac{1}{2}C_{b}\rho_{i}v^{2}R \ d\theta \qquad \text{Friction caused by air}$$

$$F_{d} = \frac{1}{2}C_{d}\rho_{i}w^{2}R \ d\theta \qquad \text{Drag caused by air}$$

Using Newtonian mechanics we obtain differential equations for both T and dT:

$$T = \Delta pR + mgR \cos\theta + mR \left(\frac{dw}{dt} \cos\theta - \frac{du}{dt} \sin\theta\right) - \frac{1}{2}C_d\rho_t R (w^2 \cos^2\theta - u^2 \sin^2\theta)$$
(10)

$$dT = \frac{1}{2}C_{b}\rho_{i}v^{2}R \ d\theta + mgR \sin\theta \ d\theta + mgR \sin\theta$$

If we substitute Eq. 9 in Eq. 10 and Eqs. 6 and 7 in both 10 and 11 we obtain the following partial differential equations:

$$T = \Delta pR + mgR \cos\theta + mR \cos\theta \left(\frac{\partial^2 z}{\partial t^2} + 2u \frac{\partial^2 z}{\partial x \partial t} + u^2 \frac{\partial^2 z}{\partial x^2}\right) - \frac{1}{2}C_d\rho_i v^2 R\left(\left(\frac{\partial z}{\partial t}\right)^2 \cos^2\theta \left(1 - 2\sin^2\theta\right) + 2v \frac{\partial z}{\partial t}\sin\theta \cos^2\theta\right)$$
(12)

$$dT = \frac{1}{2}C_{b}\rho_{i}v^{2}R \ d\theta + m\left(R\frac{dv}{dt} - u\frac{\partial z}{\partial t} + \left(\frac{\partial^{2}z}{\partial x\partial t} / \frac{\partial^{2}z}{\partial x^{2}}\right)\right) d\theta + mgR \sin\theta \ d\theta - \frac{1}{2}C_{d}\rho_{i}R \sin\theta \ \cos\theta \left(v^{2} - \left(\frac{\partial z}{\partial t}\right)^{2}\cos^{2}\theta\right) d\theta$$
(13)

Total tension in the web $T_{\mbox{\tiny tot}}$ is:

$$T_{tot} = T + \oint dT \tag{14}$$

Above integral is calculated along the instantaneous trajectory of the web.

Web tension term T_{tot} contains only the constant axial tension since the model ignores the web's bending stiffness. If we assume that the material obeys Hooke's law, we can write:

$$\sigma = Y \epsilon \tag{15}$$

However, if the behaviour of the material is viscoelastic we may obtain linear Kelvin-Voight material model:

$$\sigma = Y \epsilon + \eta \epsilon' \tag{16}$$

Tension T_{tot} for linear material is:

$$T_{tot} = m E \epsilon \tag{17}$$

Tension T_{tot} for viscoelastic material is:

$$T_{tot} = m \left(E \,\epsilon \,+\, \nu \,\epsilon' \right) \tag{18}$$

<u>2.2.2 Continuity equation.</u> The usual theoretical consideration dealing with a moving web assumes no temporal changes in mass transportation or in velocity. In practice however, there are always time-dependent disturbances (pressure, velocity, etc.) causing tension variations in the web. Therefore it is necessary to derive a continuity equation which defines mass transportation balance in the open draw.

If the web is at rest in an open draw, it is subjected to a constant tension T and according to Newton's first law of force balance, such a purely axially tensioned web is subjected to a constant strain. However, when it is in motion, material exchange causes a strain nonlinearity to a moving web in an open draw (3), which empirically can be represented by the following equation:

$$\epsilon(a) = \epsilon_{out} \left(\frac{a}{l}\right)^n \tag{19}$$

Fig. 3 shows plots of above function when 0 < n < 1. Using above equation, the average strain can be calculated:

$$\overline{\epsilon} = \frac{\epsilon_{out}}{n+1}$$
(20)

A disturbance at web exit causes a longitudinal stress wave motion towards the web entrance with velocity \sqrt{E} . During the interval the stress wave moves over the open draw, the length of the arriving new material is v*1/ \sqrt{E} . A good approximation for n is v/ \sqrt{E} , which is small in all practical cases. Modifying the Eq. 20 to form:

$$\overline{\epsilon} \approx \epsilon_{aut}$$
 (21)

The total mass of the web in the open draw is $(m_0 \text{ constant})$:

$$M = \frac{m_0 l}{1 + \overline{\epsilon}} \tag{22}$$

By differentiation we get an equation of mass flow rate in an open draw:

$$\frac{dM}{dt} = \frac{m_0 l'}{1 + \overline{\epsilon}} - \frac{m_0 l \epsilon'}{(1 + \overline{\epsilon})^2}$$
(23)

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This flow rate has to be equal to the difference between the incoming and outgoing flow rates:

$$\frac{dM}{dt} = m_0 v_1 - \frac{m_0 v_2}{1 + \epsilon_{out}}$$
(24)

Simplifying, we get the continuity equation:

$$l' - \frac{\epsilon' l}{1 + \epsilon} = (1 + \epsilon)v_1 - v_2$$
⁽²⁵⁾

3. WEB PATH AND AIR LOADING

Equations 17, 18 and 25 contain the necessary information to define the behaviour of the web. However, the solution of equations above is analytically impossible.

As a good approximation for the unknown vertical deflection function z(x,t) is an arc with time-dependent radius R(t) (Fig. 4) satisfying the necessary boundary conditions:

$$z(x,t) = \sqrt{R(t)^2 - s^2} - \sqrt{R(t)^2 - x^2}$$
(26)

Through this we can rebuild equations 17,18 and 25 to more manageable forms.

Other approximation functions could be second-order polynomial or hyperbolic (chain) functions but substitution of these in Eqs. 11 and 12 would lead to more complicated mathematics without improving the results. However the approximation of z(x,t) as an arc has the following restrictions:

- The upper limit of the radius R(t) represents a situation when the web is straight. The radius is then infinite and function z(x,t) does not have a finite value.

- The various forces acting on the web can produce sheet flutter. In this model, however, the definition of the arc excludes such possibility.

So far, the interaction of the web with the surrounding air has been neglected in this analysis. Many previous studies have shown, that air loading has a significant effect on the dynamics of the web in an open draw. Air contribution can be included it's simplest form by using potential theory, i.e. the mass of the interacting air can be added to the mass of the web. Assuming web width to length to be small, Eq. 12 can be rewritten to include the effect of air:

$$T = \Delta pR + mgR \cos\theta + (m+m_a)R \cos\theta \left(\frac{\partial^2 z}{\partial t^2} + 2u \frac{\partial^2 z}{\partial x \partial t} + u^2 \frac{\partial^2 z}{\partial t^2}\right)$$
(12')
$$- \frac{1}{2}C_d\rho_t R \left(\left(\frac{\partial z}{\partial t}\right)^2 \cos^2\theta (1 - 2 \sin^2\theta) + 2v \frac{\partial z}{\partial t} \sin\theta \cos^2\theta\right)$$

Where $m_a = \pi \rho_i d/4$.

4. EQUATIONS FOR THE OPEN DRAW

4.1 Non-Steady State Solution

Substituting Eq. 26 in Eqs. 12 and 13 we obtain equations for both T and dT which have the following form:

$$T = T(R''(t), R'(t), R(t))$$
(27)

$$dT = dT(R'(t), R(t)) \tag{28}$$

.....

It can be shown that for an arc the tension increase ΔT is:

$$\Delta T = \int_{-\theta_0}^{\theta_0} dT \tag{29}$$

By integration we get:

$$\Delta T = C_b \rho_i v^2 l + \tilde{m} (v l)' \tag{30}$$

Eq. 12' can be used in various ways. It can either be used to solve local tensions within different points of the web, or it can be integrated (Eq. 31) to give the web's average tension \overline{T} (Fig. 4).

$$\overline{T} = \frac{1}{2\theta_0} \int_{-\theta_0}^{\theta_0} T \, d\theta \tag{31}$$

By integration we get:

$$\overline{T} = \Delta pR + mgR - \frac{2\overline{m}R(R')^2 s^3}{l (R^2 - s^2)^{\frac{3}{2}}} + \overline{m}RR'' \left(\frac{l_0}{l \sqrt{1 - \frac{s^2}{R^2}}} - 1\right) + \frac{1}{l \sqrt{1 - \frac{s^2}{R^2}}} + \overline{m}(RR')^2 \left(\frac{1}{2(R^2 - s^2)} - \frac{l_0}{2lR\sqrt{R^2 - s^2}}\right) - \frac{1}{2lR\sqrt{R^2 - s^2}} - \frac{1}{2lR\sqrt{R^2 - s^2}} + \frac{1}{2lR} + \frac$$

Combining equations 17, 18, 25, 26 and 32 the mathematical model for the open draw is:

$$T_{tot} = \Delta pR + mgR + \tilde{m}v^{2} - \frac{\bar{m}R(R')^{2}l_{0}^{3}}{4l\left(R^{2} - \frac{l_{0}}{4}\right)^{\frac{3}{2}}} + C_{b}\rho_{i}v^{2}l + \tilde{m}(vl)' + \frac{\tilde{m}RR''}{4l\left(R^{2} - \frac{l_{0}}{4}\right)^{\frac{3}{2}}} - \frac{1}{2lR\sqrt{R^{2} - \frac{l_{0}}{4}}} - \frac{1}{2lR\sqrt{R^{2} - \frac{l_{0}^{2}}{4}}} - \frac{1}{2l\sqrt{R^{2} - \frac{l_{0}^{2}}{4}}} - \frac{1}{2l\sqrt{R$$

$$\tilde{m} = m + m_a = \frac{m_0}{1 + \epsilon} + \frac{\pi \rho_i d}{4}$$
(34)

$$T_{tot} = mE\epsilon \quad \forall \quad T_{tot} = m (E\epsilon + \nu\epsilon')$$
 (35)

$$l' - \frac{\epsilon' l}{1 + \epsilon} = (1 + \epsilon)v_1 - v_2$$
(36)

$$l' = R' \left(\frac{l}{R} - \frac{l_0}{\sqrt{R^2 - \frac{l_0^2}{4}}} \right)$$
(37)

$$l = 2 R \arcsin\left(\frac{l_0}{2R}\right) \tag{38}$$

4.2 Steady State Solution

In steady state there is no time dependency in Eqs. 33 - 38, which simplifies the equations to:

$$T_{tot} = \Delta pR + mgR + \bar{m}v^2 + C_b \rho_i v^2 l \tag{39}$$

$$T_{tot} = mE\epsilon \tag{40}$$

$$\epsilon = \frac{\nu_2 - \nu_1}{\nu_1} \tag{41}$$

$$l = 2 R \arcsin\left(\frac{l_0}{2R}\right) \tag{42}$$

Thus, as shown in Eq. 39, tension in the web is the sum of the external pressure, gravitational, centripetal and air friction forces.

4.3 Numerical Procedure

Equations 33 - 38 form a nonlinear second-degree differential system of equations which cannot be solved analytically. Through substitution this system can be expanded to first-degree system of equations which can be solved using standard numerical methods. Steady-state solution is used as an initial condition for non-steady state solution.

5. APPLICATION OF THE MODEL

5.1 The Paper Making Process

In the paper making process the web moves continuously from head-box to reeler. As it dries in the machine, the web is subjected to varying tensions. To ensure good runnability in the machine the web is progressively stretched by means of speed differences between the paper machine driven groups. In this process, the web is therefore pre-tensioned at each individual draw. In an ideal situation the paper machine runs for long periods with no breaks and web tension changes are at a minimum. In practice, such stability is unusual and the process is more or less subjected to various sudden disturbances which can have a large effect on web runnability. These can be caused by variations in basis weight, uncontrolled air flows, or simply by a felt crimp. Even small amplitudes in disturbances can effect high enough tension variations to cause a web brake.

In the following calculations of web tensions some parameters have been chosen to be constant: length of the open draw (0,46 m), entrance velocity v_1 (20 m/s), exit velocity v_2 (20,05 m/s, corresponding speed difference 0,25 %), pressure difference across the web (150 Pa), basis weight of the paper (80 g/m²) and the modulus of elasticity (300 000 m²/s²). The pre-tension is estimated to be 400 N/m. These values are considered to be representative of commercial operation.

5.2 Steady State

Substituting the above values in Eqs 39-42 gives the total increase in tension in the open draw as 60 N/m and the midpoint displacement as 16.8 mm.

As an example, Tab. 1 shows relative percentages of individual force components calculated for three different velocities (15, 20, 25 m/s). The increase in tension is maintained at 60 N/m for each velocity. As can be seen, at higher speeds the centripetal force becomes the dominant component.

5.3 Non-Steady State

<u>5.3.1 Pressure Pulse.</u> A pulse disturbance in external pressure difference (150 Pa) with duration of 0.1 s and absolute value of 50 Pa (Fig. 5a) causes a synchronous displacement having a similar shape as the disturbance (Fig. 5b). The sudden drop in tension to below average, shown in Fig 5c, is due to the fact that web velocity at exit does not adjust sufficiently fast to compensate for the extra web length.

In the case that the pressure difference across the web in a steady state is twice as much as above (300 Pa), the basic displacement is also two times higher than before, but the basic web tension is the same (Fig. 6). Here pressure pulse with the same absolute value and duration as in Fig. 5a causes a smaller displacement and also smaller tension increase because of the longer web path.

A shortened pressure pulse of 0.05 s (Fig. 7a) sets up oscillations in the web approaching resonance (Fig. 7b and 7c), which may increase tension in the web to beyond its rupture strength.

5.3.2 Pressure Jump. A sudden jump in pressure (rise time 0.1 s) causes temporary tension and permanent displacement increase (Fig. 8b and 8c). The web path becomes permanently longer and assumes a new, steady state path. At first the tension increases, then gets its original level. No matter how much the pressure increases, after the jump web tension always gets its basic level.

At shorter rise time (0.01 s), the displacement adjusts to the same level as before, but tension peak is higher (Fig. 9c). The rise time of the pulse approaches the web's natural frequency (32 Hz) and the solution starts to resonate.

<u>5.3.3 The velocity pulse.</u> A sudden change (2cm/s \approx 0.1%) in exit velocity causes a temporary decrease in displacement and an increase in tension (Fig. 10b and 10c). The web path shortens thereby shortening the displacement. We can see much more sensitive response to the web tension in the case of velocity disturbance compared to pressure disturbance.

In fig. 11 one can see the effects of open draw length and velocity pulse duration on web tension. When the open draw is shorter than 0,01m, the pulse duration has no significant effect on tension. Beyond this length, the shorter the pulse duration, the lower the web tension. Tension variations are high for long durations or short draw lengths.

5.4. Viscoelastic Behaviour

In all cases above we have dealt with a linearly elastic material. If we solve the case described in Fig.9 (sharp pressure jump) using a viscoelastic material model described in Eq. 18, we obtain the results shown in Fig. 12. If we compare results in Fig. 9 with Fig. 12 we see that damping is much faster in case of the viscoelastic material.

6. SUMMARY

A sophisticated model has been constructed to study the behaviour of a web as it travels in an open draw. Its main objective is to define variations in web tension in non-steady state when the web is exposed to external disturbances. The model is based on Newtonian mechanics combined with elastic and viscoelastic material theories.

In steady state the relative importance of main tension components at different velocities is calculated. Centripetal force becomes the dominating force when speeds increase.

In non-steady state sufficiently small disturbance duration times, corresponding to the first natural frequency of the web, may create oscillatory tension behaviour. The dampening of this oscillation is controlled by dissipative forces, such as friction and drag caused by surrounding air as well as by possible viscoelastic properties of the web.

It is generally accepted that short open draws improve the web stability thereby reducing web breaks. However, for velocity disturbances the model predicts the exact opposite. As open draw length decreases, tension variations also increase effecting higher probability for web breaks.



Fig. 1. Velocity coordinate system.



Fig.2 Infinitesimal web element and external forces.



Fig.3 Strain curves according to Eq. 19 when 0<n<1.



Fig. 4. The web geometry in the open draw.



Fig. 5 a, b and c. A disturbance pulse in the external pressure. Duration of the pulse is 0.1 s and the absolute disturbance is 50 Pa.



Fig. 6 a, b and c. A disturbance pulse in the external pressure. Duration of the pulse is 0.1 s and the absolute disturbance is 50 Pa (steady state 300 Pa).



Fig. 7 a, b and c. A disturbance pulse in the external pressure. Duration of the pulse is 0.05 s and the absolute disturbance is 50 Pa.



Fig. 8 a, b and c. An increase in the external pressure. Duration of the jump is 0.1 s and the absolute value of the increase is 50 Pa.



Fig. 9 a, b and c. An increase in the external pressure. Duration of the jump is 0.01 s and the absolute value of the increase is 50 Pa.



Fig. 10 a, b and c. A pulse disturbance in exit velocity v_2 . Duration of the pulse is 0.1 s and the absolute disturbance is 2 cm/s.



Fig. 11. Web tension amplitude as a function of draw length with different exit velocity pulse durations.



Fig. 12 a, b and c. An increase in the external pressure. Duration of the jump is 0.01 s and the absolute value of the increase is 50 Pa. Material of the web is viscoelastic and the kinematic viscous coefficient $v = 200 \text{ m}^2/\text{s}$.

Component / web velo- city	15 m/s %	20 m/s %	25 m/s %
Pressure difference	68,3	44,0	12,8
Gravity	0,3	0,3	0,0
Centrifugal	30,2	53,5	83,7
Air friction	1,2	2,2	3,5

Tab. 1. The relative percentage of individual force components at different velocity levels

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QUESTIONS AND ANSWERS

- Q. It is normally accepted that a shorter open draw improves runnability and web stability. If we look at picture number 11, it shows the opposite. How do you explain this?
- A. In this picture, the web is affected by the same absolute velocity disturbance. In case of short open draw, less web has to stand the stress due to the change in velocity and the increase in tension per web length becomes high. Now we have to remember, that the sheet fluttering is not included in the model. With sheet fluttering the situation might be totally different.
- Q. Are you saying that more stabile and accurate drives are required?
- A. I will give you Mr. Taskinen to answer this question.
- Taskinen: In principle, yes, but the accuracy of speed control in each case depends on the process, the viscoelastic properties of the web in process allows some inaccuracy.