USING FINITE ELEMENT MODEL TO DEFINE HOW WRINKLES FORM IN A SINGLE WEB SPAN WITHOUT MOMENT TRANSFER

H.Gopal & D.M. Kedl

3M Company

St. Paul, MN USA

ABSTRACT

This paper describes a finite element model that successfully predicts the formation of troughs and shear wrinkles in plastic webs for a single span bounded by two idler rollers. For this model, wrinkles are created when the down-stream roller is out of tram with the up-stream roller, and there is no moment transfer across the up-stream roller. Variables considered are the span length web width, caliper, and tension.

NOMENCLATURE

Variables

E	Elastic modulus (Young's Modulus) psi
I	Moment of inertia, in4
L	Span length, inch
W	Span width, inch

DEFINITIONS

For this paper, we will define a *wrinkle* as a permanent out-of-plane deformation in the web going over a roller, and a *trough* as an out-of-plane deformation in the web between rollers. Both *wrinkles and troughs* are formed when the web undergoes a net decrease in width dimension. This will occur when a compressive principal stress is introduced into the cross-web direction by a combination of tension and shear. Wrinkles created under this loading are called *shear wrinkles* (1).

The *span ratio* is the ratio of the span length to width of a web between rollers. In this paper the ratio will be called as the L/W ratio.

The *normal entry rule* is the phenomenon that the web will enter a turning roller normal to the axis of rotation.

Tram is defined as the parallelism between rollers in the web plane.

INTRODUCTION

An ability to predict the wrinkle formation is valuable since wrinkles create defects in processes such as printing, coating, laminating, and winding. Understanding the effects of web line parameters such as width to span ratio, caliper, tension, and material properties upon wrinkle formation is essential for wrinkle control. Though both wrinkles and troughs can be a source of defects, we will consider only the shear wrinkle formation created by misaligned rollers.

BACKGROUND

An analytical model to predict wrinkles was developed using the classical plate buckling theory in conjunction with the beam bending equations (1). This model assumed that at buckling a web will form sine wave shaped troughs uniformly distributed across the free span width of the film (2) as the result of cross-web shearing. The model predictions agree favorably with experimental wrinkle results, but is limited in application. For example, if the web is under a great enough cross-web tension variation, then a portion of the web will go slack, and the buckling criteria will not properly describe either the correct number or the shape of the troughs. The finite element model extends the plate buckling theory to include slack web, and does not limit the trough shape to uniform amplitude sine waves.

Wrinkles are created by compressive stresses in the web. When the down-stream roller is untrammed, the web moves laterally to conform to the normal entry conditions, as shown in Figure 1 below, generating shear in the web between the two rollers. The lateral shear force in combination with the tension in the machine direction generates compressive principal stresses in the web. The wrinkle forms when the compressive stress exceeds the critical buckling stress (1).

If the wrap angle, the friction at the up-stream roller, and the web tension are high, no stresses are transmitted to up-stream spans, and we are left with a moment on the up-stream roller with a zero slope boundary conditions (3). This state of the web is referred to as the *no moment transfer* condition. The scope of this paper is to predict the troughs and wrinkles under this condition.

EXPERIMENTAL WRINKLE GENERATION

To verify the finite element model, it was necessary to create shear wrinkles under controlled conditions. To do this, we used a specially constructed module containing idler rollers, one of which was mounted on a pivoting slide. The web path described in Figure 2 below shows the thread path used to generate shear wrinkles in web passing over an untrammed roller. The rollers in the test section were 3 in. diameter and the distance between bearing of the pivoted roll is 19 inches.

A series of tests were conducted for specific tensions and velocities. The web line was set in motion with the tramable roller at trammed conditions. A micrometer adjustment was used to displace the roller until a wrinkle formed on the roller surface. From the micrometer setting and the roller pivot length, a tram error angle was computed.

Besides wrinkle failure, the shape of the high tension side of the web edge was measured using modified edge guides. The guides were placed as shown in Figure 3. The rollers were coated with a silicon mold release material that provided a friction coefficient between web and roller greater than 2. With friction this high, moment transfer between the test span and the up-stream span was limited. The web deflections at the up-stream span were also measured to assure that there was no moment transfer.

FINITE ELEMENT ANALYSIS

In the finite element analysis two approaches were taken. In the first approach the web was modelled by two dimensional membrane elements in which the elements do not have any capacity to resist compressive stresses. In the second approach the web was modeled using plate elements allowing for the out of plane deformations of the web. The second approach was successful in predicting the deformations and the tram to wrinkle. Both approaches are described in this paper.

Two Dimensional Membrane Element Model

Since webs can resist little compression, the obvious choice for the finite element model was plane stress analysis in which the compressive stiffness of the elements is negligible. A finite element program, ANSYS[1], was used to model the web in one span, as shown below (Figure 4). Element type STIF41 as used in the model was based on three translational degrees of freedom at each node. This 2-D model used only 'X' and 'Y' degrees of freedom, with displacement in the 'Z' direction being suppressed. The element had only in-plane membrane tension stiffness (4), which required evaluating the principal stresses and their directions at every integration point, for each iteration. If both principal stresses at an integration point were positive, i.e. tensile, the web at that point was taut and said to be stiff. If both principal stresses were negative, stiffness contribution was zero. For cases when one of the principal stresses was negative, the integration point was treated as an orthotropic material with no stiffness contribution in the principal, compressive stress direction. This procedure is similar to the methods proposed in (5), (6), and (7). The ANSYS element was validated by redoing the examples in (5) and (8). When the mesh in the model was constructed using triangular elements, it provided a faster convergence than one with rectangular elements.

In the model shown in Figure 4, the nodes at the up-stream roller are constrained in the machine direction, 'X'. At one node the deflection in the cross-web direction, 'Y', was set to zero, to accommodate free contraction from cross-direction tension and Poisson's effect. An artificial compressive stress would be generated, if more than one node on the left support was constrained from 'Y' displacement. The web tension was applied as a uniform pressure to element faces representing the down-stream roller.

From the normal entry rule the rotation angle of the nodes, at the down-stream roller, must be the same as the angle of roller tram error. Also we know that the moment was zero (3), and there was a shear force applied to each node (1).

[1] ANSYS is a registered trade mark for Swanson Analysis Systems Inc.

The program started with a low shear value, which increased with each iteration until the rotation angle at the down-stream nodes at the top edge of the web, (high tension edge) was equal to the tram error angle. For the first model runs, the bottom web edge did not follow the normal entry rule, and indeed, was almost unchanged from the no-load condition. The reason was that the shear force applied at the down-stream roller caused the web to collapse locally in the shear direction. As the web collapsed, it no longer carried shear, and would not displace in the 'Y' direction. To avoid this, for the future runs, the elements immediately next to the down-stream edge were allowed to take compression (Figure 4). This was a reasonable step because that portion of the web would be wrapping the roller, and exhibit a higher stiffness in the lateral direction. With this modification both the top and the bottom edge of the web behaved as though they were in contact with the roller.

Results of 2-D Model Computations

The computations were compared to experimental data in two ways; one, to compare lateral deflections between model and experimental for specific tram error values, and the other to compare model wrinkle failure tram error with experimental values.

Lateral deflections for specific tram error

The deflected shape of the web for the case of 0.00079 inch thick polyester is shown in Figure 5.

In all the analysis, a small total shear force was applied in one-load step, and the equations iterated to achieve an equilibrium condition. The rotation of the web at the down-stream roller was calculated as the difference in the 'X' displacement of the top and the bottom nodes and divided by the web width. The tram error values shown in Figure 5 are the rotation multiplied by the length of the pivoted roller (19 inches). The computed angle from the finite analysis was next compared to the tram error angle. If the difference in angles was negligible, the program was stopped. If not, the shear force was adjusted, and the program rerun. Because there was some tension stiffening from the web tension, the deflected shape was not exactly cubic. The deflected shape matched with the experimentally measured deflections very well, as shown in Figure 5.

Wrinkling failure

To determine the conditions for wrinkling, the shear force was incrementally increased until an unstable equilibrium equation was produced. During the incremental loading the deflected shape of the web is shown in Figure 6 for various tram conditions. The deflected shapes of the web were different from the shapes shown in Figure 5. This showed that in this model the deformed shape of the web is load path dependent. This is a shortcoming of this theory, i.e., the exact shape of the membrane after wrinkling is not definable unless a modification is done to the deformation tensor (Reference 6). Unless the exact deformation is known, the angle of rotation of the web at down-stream roller cannot be predicted. Therefore, this two dimensional membrane element model failed to predict the wrinkles for failure.

Three Dimensional Plate Element Model

A three dimensional plate element model was made using STRI35 of ABAQUS finite element program. The mesh and constraints are shown in Figure 7.

The program ABAQUS was used because the load could be automatically incremental and iterated to find equilibrium. The triangular rather than rectangular elements were used because of the Hourglass effect associated with the rectangular elements (9). The nodes on the up-stream edge were supported in the 'X' and 'Z' directional displacements. As in the previous model only one node on the up-stream edge was constrained in the 'Y' direction. For this model, gap elements were used for the down-stream edge. The gap element had two nodes; one on the web and the other representing the ground. The element permitted positive Z direction displacements to represent a web lifting off a roller. The gap element did not permit relative X displacements that represented a web in traction with the roller.

The first phase of the solution was to produce a uniform tension field in the machine direction, such as would be in a web wrapped on a turning roller. A uniform tension was applied to the nodes on the down-stream edge, and the 'Y' displacements of the ground nodes set to zero. For this step, the friction in the gap elements was set to zero, allowing the web to "neck down" in the cross web direction to account for the Poisson's effect. It was necessary to apply a small load to the nodes on the right edge in the downward 'Z' direction, in order to keep the web from the rigid body motion. In reality there is downward force when the web is wrapped around the roller.

In the second phase, a shear force was added to simulate a roller with tram error, and the out-of-plane web shape established. For this load step, a 'Y' direction shear force was imposed on all the ground nodes. At the same time, the friction in the gap elements was set to a high number so that there was no 'Y' direction relative motion between the ground nodes and the down-stream edge nodes. Equilibrium was achieved for every incremental loading. Theoretically, displacements could be only in the web plane unless a small out-of-plane perturbation or imperfection was introduce into the model. To simulate a real web, the Z coordinates of several random nodes (usually 8 to 10) were perturbed by one times the web thickness. The shearing load was increased incrementally, in a manner similar to the 2-D model, and displacements found for various tram errors and wrinkle failures.

Results of 3-D Model Computations

As in the 2-D model, the finite element model was tested against experimental results for both matching the web shape for various tram errors, and for wrinkle failure.

Lateral deflections for specific tram error.

The top edge displacement of the web for various tram conditions is plotted in Figure 8. The displacements computed from the finite element analysis compared very well with the experimental results. The deflected shape for the 3-D model is close to the classical beam bending equations with stress stiffening effect, which is dominant for small EI/L ratios. The stress stiffening effect makes the web shape to deviate from the cubic shape of the beam bending.

Figures 9 and 10 show the deformed shape of the web for L/W ratio of 3.0 and 1.5 respectively. The deflected web can be divided into two regions: taut region and trough region. Notice that the beam bending equations with stress stiffening term was closely followed because for a given tram error or angle of rotation, the edge displacement does not depend on the term EI. The deflected shape of a cantilever beam when the end rotation is q due to a shear force is given by 1/3 (x/L)2 (3L-x) q. Where x is the distance from the fixed end. The deflection is independent of the term EI. Therefore since there is small taut region at the top edge, the deflected shape is close to beam bending equation.

Wrinkle failure.

The Z reaction forces at several ground nodes are plotted in Figure 11. Point A in Figure 11 corresponds the tram at which the 'Z' reaction forces in the ground nodes change from the initial reaction force. An instability in the solution occurred at point B and the solution was terminated. Analysis could have been continued further using RIKS algorithm (9) in which lateral shear force would decrease for increased displacement. However, for our purpose the analysis was stopped at B and the tram error identified as the bifurcation point.

Rerunning the model with different random placement and magnitude of the perturbation and the 'Z' forces at the right edge produced a different value for B. However, the value of A was not sensitive to the location and the magnitude of the imperfection. At tram error A, the web has the tendency to lift from the roller. The analysis could not capture the wrinkle formation at the down-stream edge nodes, because the element size in the model was large when compared to a typical height and width of a wrinkle. Point A was taken as the criterion for the wrinkle formation. Although for a static problem B is the bifurcation point and the buckling could occur, for the moving web conditions point A can be treated as the tram to wrinkle.

The results from several experiments are tabulated in Table 1.

The predicted results from the finite element model is in good correlation with the experimental results. The analytical results of (1) did not agree with the experimental results for large and small L/W ratios.

SUMMARY

A finite element model is described to predict the wrinkle formation in the web span between two rollers. The model of one span was made using triangular plate elements of ABAQUS finite element program. A nonlinear large displacement analysis of the model was carried out for the applied tension and the lateral shear force generated due to the untrammed conditions. A failure criterion was established regarding wrinkles that form when the nodes on the untrammed roller lift from the roller surface. The model predicted the deflected shape of the web and the tram to cause the wrinkle that matched very well with the experimentally measured values.

ACKNOWLEDGMENTS

The authors would like to thank Dr J. Keith Good of Oklahoma State University for the valuable assistance during the progress of this work, and the 3M company for providing research funds for deriving and verifying the models.

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Span (L) inches	Tenison (T) lbs.	Width (W) inches	L/W	Expr. Tram mils	FE Tram mils	Ref. 1 mils
36	6	6.0	5	325	304	174
18	6	3.0	5	140-150	135	123
18	6	3.0	10	185-200	190	207
18	12	1.5	10	57	56	31
18	12	1.5	20	85	97	53

Table 1. Experimental versus theoretical tram for wrinkle formation

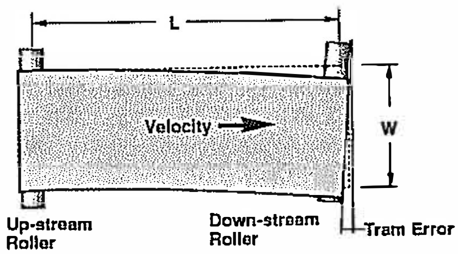

Figure 1. Lateral deflection of the web

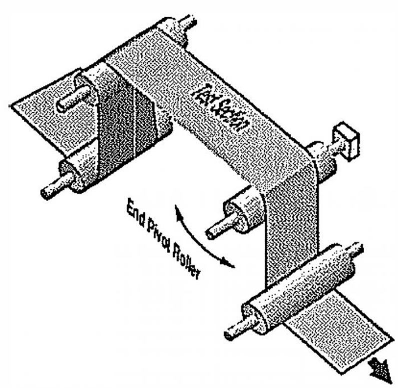

Figure 2. Wrinkle test section

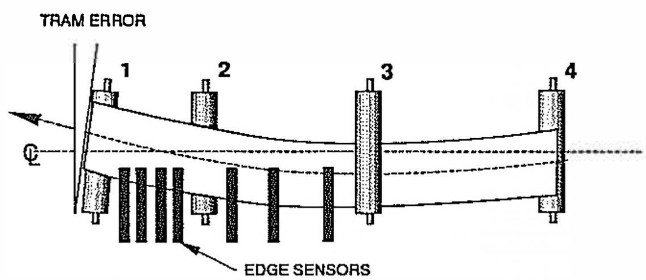

Figure 3. Edge sensor location

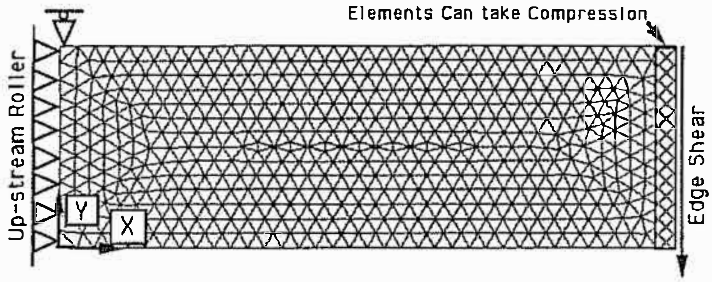

Figure 4. Two dimensional membrane element model

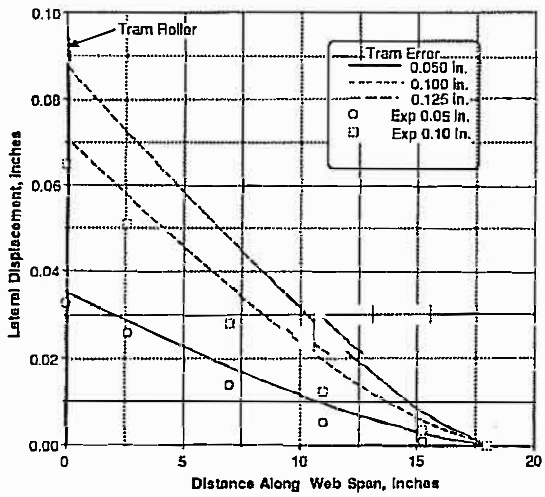

Figure 5. Two dimensional model results for ten lbs. tension

The curve is for conditions of:

1. Width .... 6 inches
2. Span length................................. 18 inches
3. Elastic modulus........................... 680,000 psi
4. Poisson's ratio 0.34
5. Web velocity .... 50 ft./min.

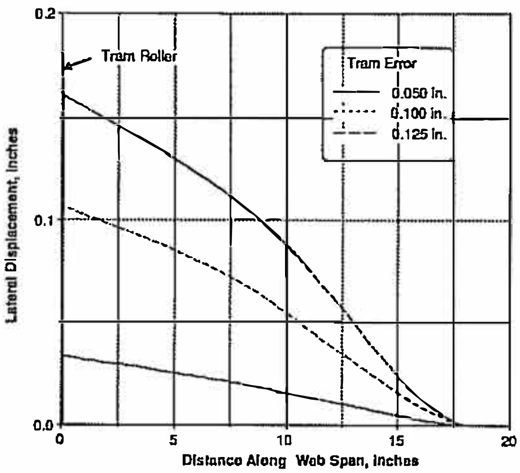

Figure 6. Two dimensional model, incremental loading

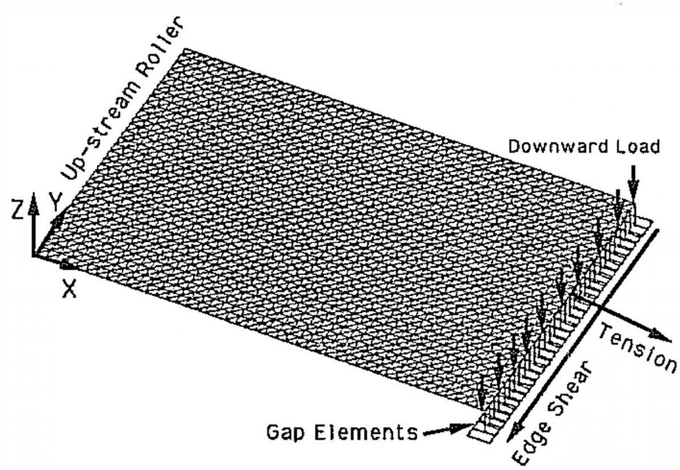

Figure 7. Three dimensional plate element model

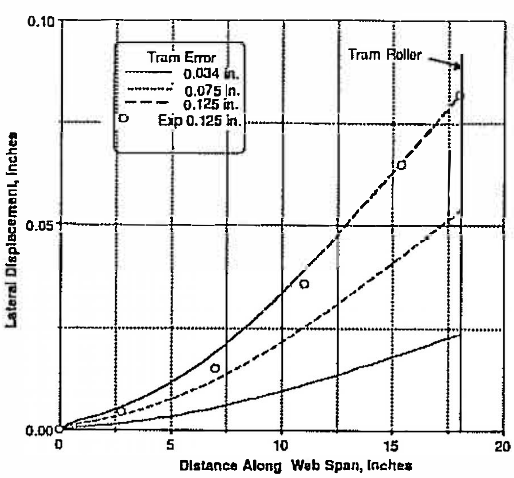

Figure 8. Lateral web deflection from the three dimensional model

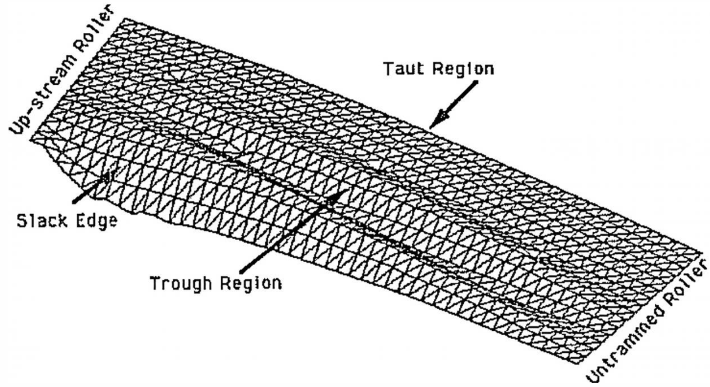

Figure 9. Deformed web shape for L/W= 3.0

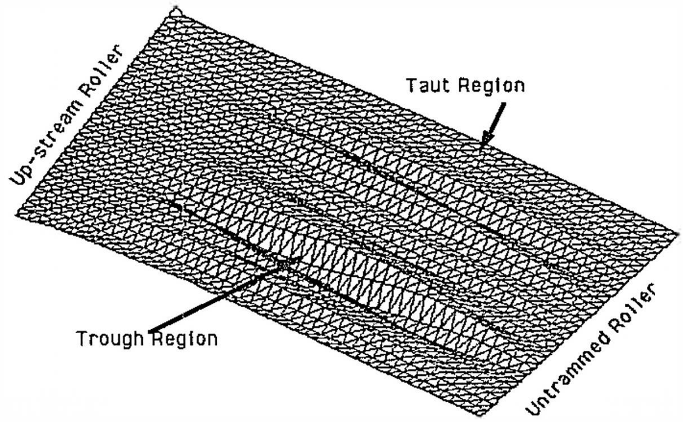

Figure 10. Deformed web shape for L/W = 1.5

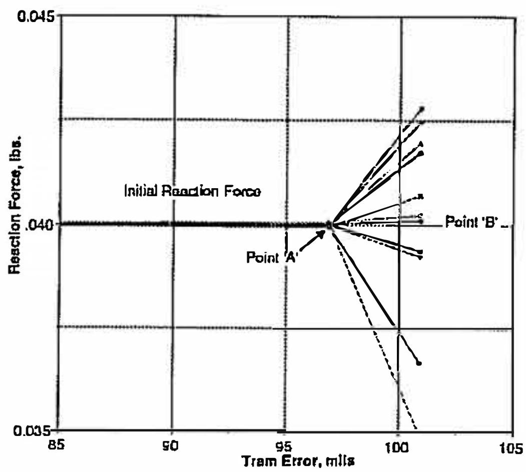

Figure 11. Ground node reaction forces

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H. Gopal

Is it possible to have shear wrinkles for zero tram error for an orthotropic web under uniform longitudinal strain when the material symmetry axes are not coincident with the web longitudinal and width axes?
Keith Bennett, Weyerhauser Paper Co.

We haven't really done anything on the orthotropic web. Keith has looked at how the orthotropic materials will form the influence on the formation of the wrinkle. I don't see any reason why I can not take this model and then just change few commands with an orthotropic material properties and then see. I really can not answer whether it's going to form the wrinkle at zero tram. "Keith do you want to add any thing to that? Any experience? (KG) "No." Maybe we can do it later.

Rolls diameter and wrap angle also play role in wrinkle formation: The bigger diameter and or wrap angle, the less wrinkles formed and on contrary. You didn't mention neither diameter nor wrap angle. Could you please comment on this?
Mark Kleiman, Polaroid Corp.

I did not because I was looking at the wrinkle formation in regime 1. If you go into regime 2 both tension, wrap angle and the diameter can play a role. Anything that gets lifted on the roller can be spread depending on the wrap angle and the tension you have. I think they can play an important role in regime 2 and I haven't looked into regime 2. The scope is just regime 1 wrinkle formation . Once the wrinkle forms it is going to be there. That's the whole idea in this paper.

Does your 3D finite element model predict the numbers and spacing of troughs, and are these predictions in agreement with observation?
Dilwyn Jones, ICI

It is definitely in agreement with the observation, but is not really a representative measure of the height or the width of the troughs. Probably we didn't need to, or we didn't have good technique to measure the height of each trough. I think we are in agreement with what we observed on the web in single span.

Did you consider modeling the right-hand side roller and wrapping the web over the roller instead of cutting the web at the roller and imposing boundary loads?
Bob Thomas, Sandia

I did do some work on wrapping the web. That's a good question, but the thing we all need to realize here is that web handling or with the web the problem is not exactly like a structural problem where you know the displacement and you analyze it. The reason is that sometimes you know the kinematic boundary conditions. The slope has got to be certain value to make a normal entry. I think you will end up with some problems in terms of when you rotate the roller you also have to move it with the web because it's turning. There are some problems with making the full model with the rollers. It's a good question.

You mention the tram error in mils 50, 75, 100 mils but you didn't tell us exactly about the face of that roller where that measurement took place. So was the roller exactly the width of the web, because you did give us the web width and if not how can we relate the tram in mils to the tram in angle error.
D. Pfeiffer, McGill University

Probably I didn't mention that in my presentation, but the roller web is 19 inches. The tram error or the tram error divided by 19 inches gives an idea of the angle of rotation at the downstream roller.