MECHANICAL STATES IN WOUND HETEROGENEOUS TAPES BY THE FINITE ELEMENT METHOD

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ABSTRACT

The problem of predicting the mechanical stress and deformation states in wound tapes that result from the winding process addressed by application of the finite element method. The generally heterogeneous tape construction is approximated by continuum finite elements each of which represents many tape plies. Material behavior within the finite element continuum is assumed to be orthotropic elastic. The actual winding process, in which stressed plies are added to an already stressed but partially wound tape, is simulated by sequentially activating layers of finite elements which have initial stress equal to the average winding stress. This model has been implemented in the three-dimensional code JAC3D. Numerical results are presented for the case of a regular two-dimensional circular geometry, for which analytical solutions have been reported in the literature. The favorable comparison between finite element results and analytical results for this example problem validate the finite element approach.

INTRODUCTION

Orthotropic, heterogeneous rolls are fabricated by simultaneously center winding plies of different materials around a common mandrel, or core. Figure 1 shows a capacitor being wound with two plies of aluminum conductor and four plies of mylar dielectric constituting a single wrap. In this construction a common winding tension is applied to all plies as the mandrel turns, so that the stress state in the interior of the partially wound capacitor is continuously altered as new wraps are added. Knowledge of the stress states in completed tapes is necessary to assess reliability of the winding process. For example, insufficient radial compression can result in ply separation, and loss of circumferential tension can result in local ply

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instabilities such as buckling. Additionally, the stress state in the completed tape can be altered in a deleterious manner by electric and magnetic fields, gravity, and thermal gradients imposed during operation of the winding.

Determination of the mechanical states in wound tapes belongs to the general class of accreting body problems discussed by Brown and Goodman [1]. These problems are characterized by the continual addition of stressed material to an already stressed and deformed body. Typical problems include the case of a gravitating sphere which grows to a stable size by acquiring material uniformly through accretion, and a concrete dam which is built up in successive layers. Solutions to accreting body problems are nonlinear in the sense that they are path dependent. In the wound tape problem the final stress state will not be the same if all wraps with initial tension are added simultaneously. Compatibility arguments prevent the individual wraps from fitting together because the final circumferential length of each wrap is not known before it is added to the partially wound tape.

Analytical expressions for the stress states in homogeneous wound tapes have been reported by Harland [2], Altmann [3], and others. Viscoelastic material behavior was considered by Tramposch [4], and heterogeneous ply constructions were reported on by Reuter [5]. Generally these theories have been limited to the geometric and constitutive equations of linear, two-dimensional elasticity. Willett and Poesch [6] introduced the finite difference method as a tool to solve winding problems for nonlinear materials, but only for the case of cylindrical symmetry.

In this paper the finite element method is presented as a method to extend the predictive capability to problems of complex geometry and nonlinear materials. It is also desirable to have the capability to predict winding stresses in noncircular coils, to analyze asymmetric thermal and gravitational loadings, and to study the general problem of interlayer friction on slippage instabilities. The finite element method offers the potential to solve these problems. This paper presents the general methodology for finite element analysis of the tape winding process, and reports comparisons of predicted stresses for a capacitor with cylindrical symmetry with analytic solutions reported in the literature.

THE FINITE ELEMENT APPROACH

Application of the finite element method rests upon the assumption that the discontinuous displacements across individual plies within a representative elementary volume can be approximated by average displacements within a replacement continuum. In practice this means that the characteristic length of the finite element must be large compared to the thickness of the individual plies. Once this assumption is made, the gross behavior of the heterogeneous mixture of plies can then be modelled with a single material constitutive relation. The concept of representative elementary volumes has been previously applied with success to modelling jointed rock media [7].

The finite element method applied to the tape winding problem is illustrated in Figure 2. The mandrel and the final wound geometry are meshed, with the coil winding being discretized into finite element layers. The extent of the discretization, or layering, depends upon the desired resolution of the mechanical states. Each layer contains many plies, and has initial circumferential stress equal to the winding stress, i.e., the winding tension per unit area of winding cross section. At the beginning of the calculation, all finite element layers are inactive in the sense that their nodal generalized coordinates are omitted from the solution process With each load step, an additional layer is added, or accreted, to simulate the winding process The accreting layer is activated and all layers outside of the accreting layer remain inactive. Numerical solutions to the stress and deformation states are obtained for the partially wound tape at each step in the winding process. When the outer, or final, layer is accreted, the resulting stress and deformation state is that for the completely wound tape.

ORTHOTROPIC MATERIAL MODEL

An orthotropic, elastic material law is employed to model the behavior of a finite element composed of many heterogeneous plies. In three-dimensional cylindrical coordinates, these standard relations are,

$$
\begin{aligned}
e_{rr} &= \frac{\sigma_{rr}}{E_{rr}} - \frac{\nu_{r\theta}\sigma_{\theta\theta}}{E_{\theta\theta}} - \frac{\nu_{zr}\sigma_{zz}}{E_{zz}} \\
e_{\theta\theta} &= -\frac{\nu_{r\theta}\sigma_{rr}}{E_{rr}} + \frac{\sigma_{\theta\theta}}{E_{\theta\theta}} - \frac{\nu_{z\theta}\sigma_{zz}}{E_{zz}} \\
e_{zz} &= -\frac{\nu_{rz}\sigma_{rr}}{E_{rr}} - \frac{\nu_{\theta z}\sigma_{\theta\theta}}{E_{\theta\theta}} + \frac{\sigma_{zz}}{E_{zz}} \\
\gamma_{r\theta} &= \frac{\tau_{r\theta}}{G_{r\theta}} \\
\gamma_{\theta z} &= \frac{\tau_{\theta z}}{G_{\theta z}} \\
\gamma_{zr} &= \frac{\tau_{zr}}{G_{zr}}
\end{aligned}
\qquad (1)
$$

EXAMPLE: CYLINDRICALLY SYMMETRIC CAPACITOR

As an example of the finite element approach to tape winding processes, consider the problem of a cylindrically symmetric capacitor. Analytical solutions for this example have been reported by Reuter [5]. Each capacitor winding is composed of four plies of mylar dielectric and two plies of aluminum conductor, as illustrated in Figure 1. This six-ply construction is wound onto a Lexan mandrel. Material and geometric properties for the mylar plies are,

$$
\begin{aligned}
E_{rr} &= 1.72 \times 10^{2} \text{ MPa} \\
E_{\theta\theta} &= 4.41 \times 10^{3} \text{ MPa} \\
\nu_{r\theta} &= 0.017 \\
t &= 1.02 \times 10^{-2} \text{ mm}
\end{aligned}
\qquad (2)
$$

for the aluminum plies,

$$
\begin{aligned}
E_{rr} &= 6.89 \times 10^{4} \text{ MPa} \\
E_{\theta\theta} &= 6.89 \times 10^{4} \text{ MPa} \\
\nu_{r\theta} &= 0.33 \\
t &= 5.59 \times 10^{-3} \text{ mm}
\end{aligned}
\qquad (3)
$$

and for the Lexan mandrel,

$$\begin{aligned} E_{rr} &= 6.89 \times 10^3 \text{ MPa} \\ E_{\theta\theta} &= 6.89 \times 10^3 \text{ MPa} \\ \nu_{r\theta} &= 0.40 \\ R &= 3.96 \text{ mm} \end{aligned} \tag{4}$$

where t is the thickness of an individual ply, and R is the radius of the solid mandrel. This design of capacitor has 50.8 mm wide ply materials, 2.4 N of tension which is constant throughout the winding process, and 200 windings. This results in a total of 1200 plies in the completely wound capacitor.

Equivalent, orthotropic material properties for the six-ply construction were derived by Reuter. These are,

$$\begin{aligned} \overline{E}_{rr} &= 2.20 \times 10^2 \text{ MPa} \\ \overline{E}_{\theta\theta} &= 1.83 \times 10^4 \text{ MPa} \\ \overline{\nu}_{r\theta} &= 0.085 \end{aligned} \tag{5}$$

The finite element calculations were performed with the three-dimensional code JAC3D [8]. The two-dimensional geometry was modelled in the three-dimensional code by applying appropriate symmetry conditions. The completed capacitor windings were divided into 30 concentric finite element layers, which are shown in Figure 2. Each finite element layer therefore modelled the behavior of 40 plies.

Radial and circumferential stresses are compared with Reuter's analytical solutions in Figures 3 and 4, respectively. Analytical solutions have been obtained for cases in which the completed capacitor winding has been divided into 10, 20 and 200 layers. The analytical solution for 200 layers is the correct, or convergent, solution for this example because 200 turns of the mandrel was required to produce the completed geometry. It can be seen that the the finite element stresses, obtained for 30 layers, fall between the analytically computed stresses obtained for 20 and 200 layers. However, more resolution is needed to model the circumferential stresses in those windings adjacent to the mandrel. Mechanical states at these locations are primarily governed by the initial windings, and only minimally altered by subsequent windings. The nonlinear distribution of stresses in partially wound tapes is similar to those in the completed tape. The finite element model used for this example has difficulty modelling the nonlinear distributions in the first few windings because of the assumption of linear displacement fields within the continuum elements. This problem exists down to the level of a single winding. More study is needed to arrive at appropriate discretization of these initial windings, and the effect of the discretization on mechanical states in the completed tape.

CONCLUSIONS AND FUTURE WORK

It has been shown that the finite element method can be applied to computing process winding stresses in heterogeneous tape constructions. More work is needed to assess the layering resolution required to predict circumferential stresses in windings adjacent to the mandrel.

The finite element method is a potentially valuable tool for studying noncircular tapes, and the effects of asymmetric temperature and gravitational loadings. Neither of these problems can be attacked by the two-dimensional analytical formulations.

Inter-ply friction can potentially be modelled by incorporating a Mohr-Coulomb failure criteria in the orthotropic material constitutive model. By this method, initiation of slip resulting from asymmetric loading, for example, may be addressed. Post-slip behavior would be difficult to model because large geometric deformations are not easily or accurately established through constitutive relationships.

The extension of the finite element method to slip problems is not straightforward if the slip instability results from the spiral winding. Continuum finite element modelling of spiral windings may not be possible because of the extremely large number of elements which would be needed to model the entire coil. Slip instabilities which initiate at a point would be difficult to model within the continuum assumption of representative elementary volumes. Conceptually, spiral windings can be modelled by orthotropic, truss-like elements which are linked together and maintain compatibility during the winding process. The feasibility of this approach will be investigated in our future work.

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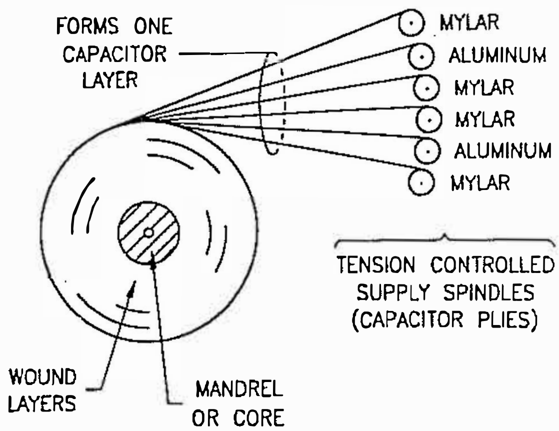

Figure 1. Tape Winding Process for Six-Ply Construction

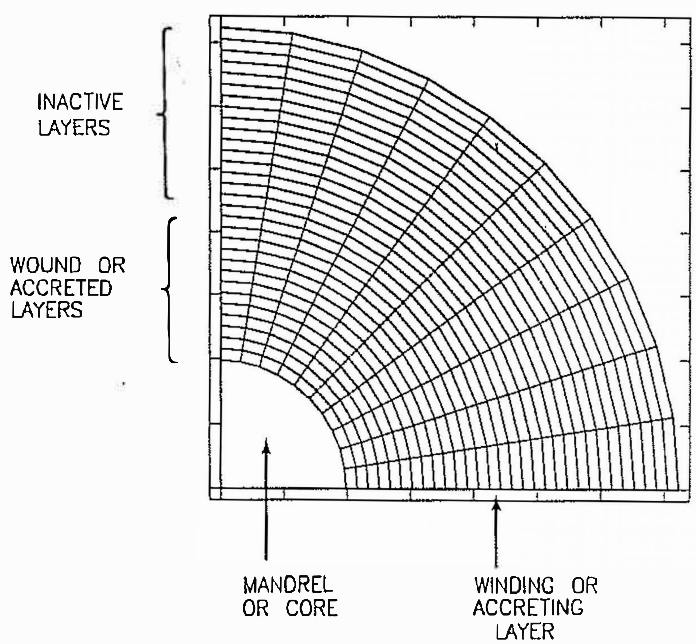

Figure 2. Finite Element Approach to Tape Winding

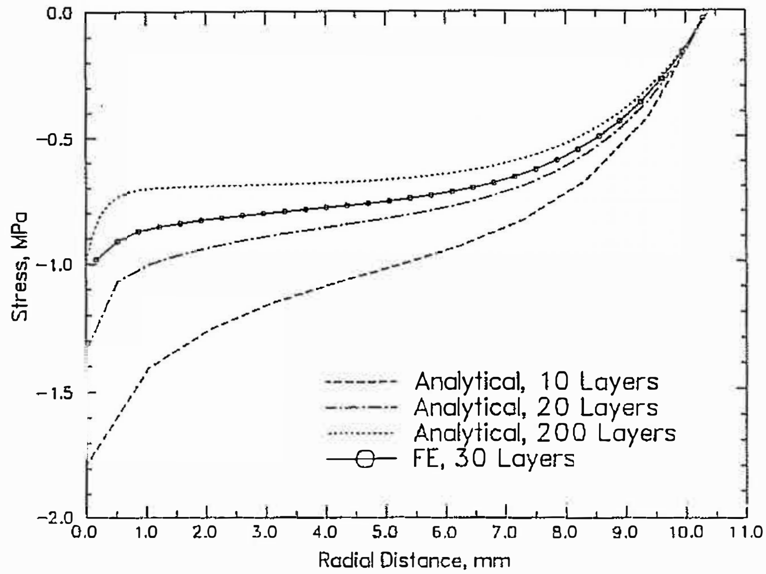

Figure 3. Radial Stresses in Capacitor Winding

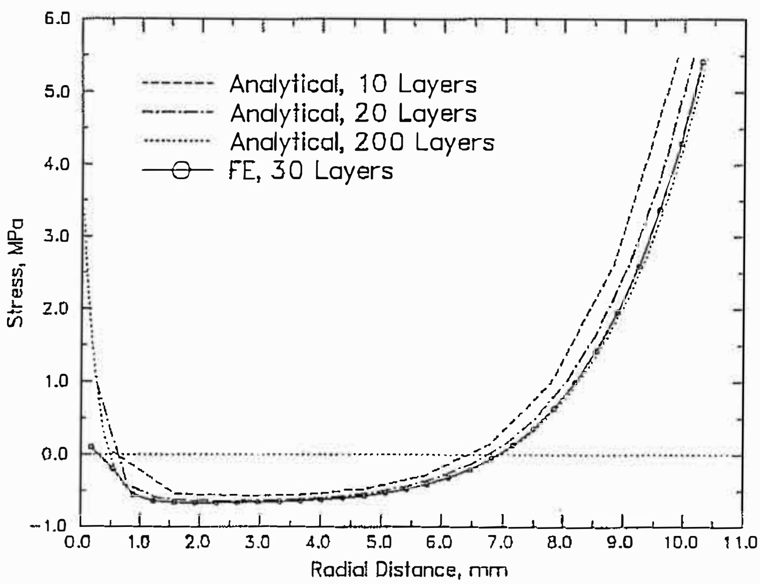

Figure 4. Circumferential Stresses in Capacitor Winding

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Did you ever try a lower Poisson ratio for the mandrel? (You have ν = 0.40) This might clear up, or explain, the "problem" you have at the core area at the start of winding, where there is a light blue ring.
Dave Pfeiffer, McGill

In an earlier talk this morning it was stated that depending upon what material properties you use for the core you can reduce or eliminate that gradient on the first few windings. What I did was I assumed the same material properties that were used earlier. The gradients were present so I didn't change the Poission's ratio. I think in the earlier formulation if the Poission's ratios were changed or some other properties, you could make that gradient do what you wanted.

In your calculation E_r is constant. Could you have made E_r as a function of e_r?
Gopal, 3M

It's fairly easily done when you run explicit finite element codes. All you have to do is remember the last e_r and reset a new modulus. I think thats the mechanics that's involved.

Why didn't you use 1° sector angle in the model rather than 90°?
Gopal, 3M

We have a Cray XMP. For a problem of this small size, it's easier to model the 90° sector.

The last layer is not where it was in the original stress free condition so it means that each new element going on is something slightly different than what your original grid was defined as, and this error is an accumulating error that propagates out to your model. How did you accomplish that?
Gopal, 3MM

You're correct, although I don't know if I would consider that an error In actuality when you're winding the core (the coil) you're seeing this contraction. If you were to take all these windings and lay them in a circle they would be much larger than if you had wound that many layers. That geometry is coming in anyway and in the finite element code the elements just follow it in.

You're adding a certain delta radius in your new element then without caring what happened before?
Gopal, 3M

All the elements outside the winding layer are following the inner layerings. I'm inducing a geometry change but no strain or stress.