Name: George Wesley Barratt Date of Degree: May 24, 1959

Institution: Oklahoma State University. Location: Stillwater, Okla.

Title of Study: ON THE M.K.S. SYSTEM OF UNITS

Pages in: 34 Candidate for Degree of Master of Science

Major Field: Natural Science

Scope of Study: A system of units is based preferably on a minimum number of independently defined units. Three units are the minimum number in mechanics. For a complete system, one needs a fourth independent unit in defining magnetic and electrical quantities. Great care must be exercised in the choice of basic units and arbitrary constants to ensure a simple and straightforward system that is applicable to all physical phenomena.

Several systems of units have arose throughout the years, including centimeter-gram-second--both electrostatic and electromagnetic, English, and the relatively new meter-kilogram-second system. They are known as C G S-esu and emu, F P S (infrequently), and M K S. The physics student must be keenly aware of the place of each system, and their relationships as concerns the many entities and their defining equations, in the overall set of systems. Various texts and articles were studied in determining the relationship of the M K S system to the older already-defined systems.

Findings and Conclusions: The fundamental units for mechanics are those of mass, length, and time. They are independent units and serve in expressing all other mechanical quantities. With the ampere as the fourth basic unit, we define and discuss the magnetic, electrical, and mechanical entities. With our choice of constants the rationalized M K S A system of units is formulated.

It is evident from working with magnetic and electrical phenomena that the M K S system is very advantageous over the other systems. The older absolute system is still very functional and convenient for atomic and nuclear physical applications. The English system is used by engineers and the general public of English-speaking countries.

An understanding of the many quantities, units, and conversion factors aid manipulations in physics and engineering but an understanding of physical phenomena is a separate endeavor.

Adviser's Approval James H. Zant

ON THE M.K.S. SYSTEM OF UNITS

By

GEORGE WESLEY BARRATT

Bachelor of Science

Northwest Missouri State College

Maryville, Missouri

1954

Submitted to the faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1959

ON THE M.K.S. SYSTEM OF UNITS

Report Approved:

Frank M. Durbin
Report Adviser

James H. Zant

Dean of the Graduate School

ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation to the National Science Foundation and to Dr. James H. Zant, Director, for making this year's study at the Oklahoma State University, possible. Many comments and bits of advise, by Dr. Zant, were very helpful in the organization of the materials and the general procedure involved in the preparation of such a report.

The writer also wishes to express his sincere appreciation for the helpful guidance of Dr. Frank M. Durbin, Professor of Physics, which resulted in the completion of this report.

TABLE OF CONTENTS

Chapter Page

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The purpose of this report is to show the relationship between the relatively new meter-kilogram-second system of units and the centimeter-gram-second, both electrostatic and electromagnetic, and English systems. In order to present the coverage on the M K S system, the author believed it necessary to make many preliminary statements on fundamental units. The foundation equations and defining terms in both esu and emu systems will be discussed, too, where it is felt that such discussion will clarify the subject at hand. The reader is referred to the appropriate appendix for the comparisons of the equivalents, conversion factors, and entities among M K S, emu, and esu systems.

For the beginning student in physics, a rather broad and sometimes confusing list of defining equations, units, and conversion factors must be considered. Some of these must be known for a given subject at hand as well as a satisfactory knowledge of their application. Had it been possible to formulate a simple and orderly system of units, useable in all aspects of science endeavor, at a very early date, we would likely have had a broad metric system that could be applied, handily, to all physical applications. However, some new discoveries made it necessary to systematize an ever-increasing number of quantities.

It has long been recognized that mathematical manipulations utilizing the English system (foot-pound-second) are quite bunglesome. In using this system one is faced with many conversion factors, such as

1728 cubic inches per cubic foot, 144 square inches per square foot, and 5280 feet per mile. In contrast, in the metric system, one can merely move decimal points in changing units. Some of the common prefixes encountered in this operation are centi, milli, and kilo; meaning .01, .001, and 1000, respectively.

Though a complete change to metric units throughout the world would greatly simplify the teaching of science and mathematics, especially on the lower levels, such a change would be a terrific task. Until some better situation is at hand it will be the task of every student, teacher, and layman to have some understanding of our "conglomeration" as his needs require.

A system of units is any set that will allow all physical quantities to be meaningfully expressed, based on a very small number of independent units. The recognized minimum number of such units, in mechanics, is three; the units mass, length, and time. With the choice of meter, kilogram, and second, one has an M K S system. In discussions with electrical and magnetic phenomena one needs to consider a fourth unit. The choice here is an important one for simplicity and straightforwardness in the defining equations of some of the quantities. As one relates the electrical energy to heat energy, he needs to define a unit of heat as well. This report will utilize the meter, the kilogram, the second, and the ampere as its basis. Some authors use such a choice and term it the M K S A system.

Near the middle of the 19th century scientists and engineers working with magnetic and electrical phenomena proved that some of the quantities that arose could be well expressed in terms of distance, mass, and time. Gauss and Weber used the millimeter-milligram-second, to express these entities. The British Association for the Advancement of Science advocated a system based on the meter-gram-second, as the set of basic units. This particular basis, however, led to such a tiny unit for electrical resistance that a more convenient unit such as one 10^7 times as large was recommended. About 1870 a second committee of the same association put forth a system based on the centimeter-gram-second. (CGS). This gave rise to ratios between practical and CGS units of resistance, potential, and capacitance of 10^9, 10^8, and 10^{-9}, respectively. Giorgi in 1904 advocated a practical system--the units of volt, ampere, coulomb, ohm, henry, and the farad--serving with the basic units of length, mass, and time as the meter, kilogram, and second. This lead to the M K S (meter-kilogram-second) system of units. This system was adopted by the International Electrotechnical Commission in 1940. In changing from system to system several constants of various powers of ten are involved. Many of these constants vanish in the M K S system, in contrast to the use of the C G S system.

The units of mass, length, and time are defined and determined without regard for other units or for each other. They are, therefore, called fundamental units. All other units are derived, either directly or indirectly, from the three fundamental units. Chapter II to follow will relate the comparative place of these three entities.

It is the hope of the author that this coverage will serve as a ready reference for a study or review of the M K S system of units.

CHAPTER II

FUNDAMENTAL UNITS OF MASS, LENGTH, AND TIME

The fundamental units for mechanics are those of mass, length, and time. They are independent of one another and serve in expressing all other mechanical units.

2-1. Mass. The mass of a body is a measure of the amount of matter of which it is composed. Mass might also be defined as the inertia possessed by a given body, with inertia as the property of matter causing it to remain at rest or in uniform motion.

The choice of the mass unit was arbitrary but in the beginning it was based on some common or well-known unit. The metric system has as its mass unit, the gram. It is defined as the mass of a cubic centimeter of pure water at maximum density(near 4^0 centigrade). As should be readily understood, the choice of basic units should allow each to be defined possessing a definite uniqueness. Such is not the case for the gram. In a recent discovery it was proved that heavy hydrogen and heavy water exist; making the cubic centimeter of "pure" water slightly variable. One gram is generally accepted as the mass of one cubic centimeter of pure water, however, and is equivalent to 1/1000 of a kilogram. The gram is the unit of mass in the C G S system of units.

The kilogram is the basic unit of mass in the M K S system of units. The standard kilogram is the mass of a certain metal block, kept at the International Bureau of Weights and Measurements in Sevre, France. One kilogram is equivalent to 1000 grams.

For the English gravitational system of units, the unit of mass is the pound. The origin of this unit is based on 7000 grains of wheat as taken from the average wheat stem. Because of the non-scientific aspect of the "average" wheat grain, the pound is now based on a standard platinum-iridium cylinder placed in London, England. Several countries of the world have copies of this standard. We recognize 2.204622 pounds as being equivalent to one kilogram.

The English engineering system uses the slug (w/g), as the unit of mass. When force units are discussed, the slug will be more thoroughly discussed.

2-2. Length. The second basic unit is that of length. In the metric system, the meter is the fundamental unit. The meter was formulated to equal 10^{-7} of the earth's quadrant. The following problem will illustrate an approximation of this:

$\frac{25000 \text{ miles}}{4 \text{ quadrants}} = 6250$ miles/quadrant. (1610 meters = 1 mile) and 6250 times 1610 = 10^7. Thus one meter = $1/10^7$ of the earth's quadrant.

A platinum-iridium bar is now maintained in Sevre, France. It contains two marks that show the one standard meter-separation at 15 degrees centigrade. One meter is equivalent to 100 centimeters or 1000 millimeters.

In 1892-93 A. B. Michelson showed that a standard of length could be replaced by reference to the measurement of wave lengths of light. In 1927 the 7th General (international) Conference on Weights and Measures adopted provisionally a supplementary definition of the meter in terms of wave length of light. According to this definition the relation for red cadmium light waves under specified conditions of temperature, pressure and humidity is 1 meter = 1,553,164.13 wave lengths.

The accuracy of this value is usually stated as being about 1 part in 10,000,000.

The yard is the fundamental unit of length in the English system. A bar in England shows a distance between two lines at 0 degrees as the standard yard. In the United States we recognize the yard as 3600/3937 of the standard meter.

There are, of course, many other common distance units derived from the meter and yard. Some of these subdivisions are large, some small, but all seem to expedite the handling of various measurements with more convenient units. Some examples include the centimeter, millimeter, and the kilometer, in the metric system, which equal .01 meters, .001 meters, and 1000 meters, respectively. The English system employs the inch, foot, and mile, as 1/36 yard, 1/3 yard, and 1760 yards, respectively.

2-3. ~~Time~~. One of our near precise units is that of time. Our basic time unit, the second, is based on the motion of the earth. A day is defined as the interval of time between successive meridian passages of the sun. The length of the day varies throughout the year due to the elliptical orbit of the earth about the sun at a variable speed. The average over the year is called the mean solar day. This is divided into 24 hours, 1440 minutes, or 86,400 seconds. Fortunately, the second is basic to all systems of units.

CHAPTER III

NON-ELECTRIC AND NON-MAGNETIC UNITS

3-1. General Comments. The metric system of units is used throughout the world in scientific study. The English speaking courtries, such as Great Britain and the United States, use the English system, however, for engineering and general public applications. Because of this dual system it is necessary to know many conversion factors and obscure relationships.

Concerning advantages and disadvantages of the systems, many controversial ideas have sprung up. For the metric system one could first note the convenience of the decimal point in subdividing measurements. There seems to be a correlation between measures of length, mass, and volume. This has made it possible to build up somewhat of an absolute structure based on the meter, the gram, and the liter. One liter is one thousand cubic centimeters. These relationships are very convenient for simple chemical applications. Proposals have been made to include time, temperature, intensity of light, and some of the electrical entities in such an absolute system. The following diagrams should illustrate one advantage of the metric system over the English system of units.

The prefixes such as centi, milli, and kilo, may be assigned to the meter, gram, and liter, which make the use of subdivisions very convenient. Some authors point out the potential confusion of having a liquid, dry, and imperial quart. Also confusing are such terms as the imperial gallon, British gallon, and standard gallon. One encounters such conversions as, one pound of butter is equivalent to 16 ounces Avoirdupois or 7000 grains or 454 grams, while one pound of gold equals 12 ounces Troy or 5760 grains 373 grams. These points, thus far, seem to be against the English system.

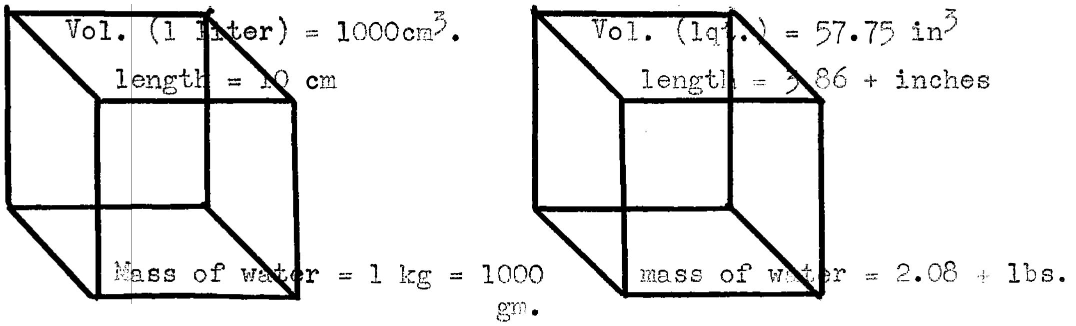

Fig. 1. Relations Among the Units of Mass, length and Volume.

There are some so-called familiar equable quantities in the English system too. Some examples that are common follow:

one carat equals 1/5 grams or 200 milligrams.

one nickel equals about 5 grams.

one dime is one millimeter thick (or thin) and equals 2.5 grams.

one non-scientific drop of water equals 1/20 cubic centimeters or 50 milligrams.

one teaspoonful equals 4 cubic centimeters.

one fluid ounce equals 30 cubic centimeters.

one cupful equals 8 fluid ounces.

Some writers are of the opinion that the real material difference in the United States and British units, relative to weight and volume, is that of the weight of a gallon. Britain used 10 pounds as the weight equivalent for one gallon of water and the United States used 8 1/3 pounds.

---

[1] Harvey A. Neville, "Educating the Public in the Case of the Metric System," Journal of Chemical Education, July, 1925, pp. 596-597.

for the same volume. This allows 100 pounds of water to match 10 gallons or 12 gallons, in Britain and the United States, respectively.

There seems to be a difference of opinion as to the origin of the systems. Some credit the authorship of the decimal (metric) system to James Watt, English scientist and engineer. Others claim the metric system was French in origin. For the English system, most are of the opinion that it just grew, with much of its foundation based on works of the Egyptions, Greeks, and Romans.

All national changes in standards have been toward the adoption of the metric system. One might wonder if the people of Great Britain and the United States would be very appreciative of an established metric system in their respective countries.

S. C. Lind[2] presented six reasons for the adoption of the metric system; they follow:

1. To conform with a decimal system.
2. To conform with Universal scientific use.
3. To have the advantage of the simple relationships between distance, volume, and weight units.
4. To promote our relations with Latin American countries, all of which use metric units.
5. To conform with the usage of most civilized countries.
6. To show that we as a nation are not frozen in a pattern of conservatism and that we have the force and initiative to adopt a change long overdue that would put us in step with world progress.

The Congress of the United States voted in 1866 for the adoption of the fundamental standards of our units to be English.

While it is quite true that many more mathematical manipulations are required in a complete application of the English subdivisions in units, it remains a large task to organize a switch from English to Metric units. Granted that it would be nice to rid ourselves of such

---

[2] S. C. Lind, "Letter to Editor," Journal of Chemical Education, XX (1943), 593.

numbers as 1728, 30 1/4, 5 1/2, 27, and countless others, it remains a challenge for us to acquaint ourselves with numerous relationships and to be able to apply them to their immediate needs.

It is quite interesting to read of the opinions of many writers on such controversial subjects as systems of units, but further discussion of ideas would be beyond the need of this writing. We have presented some of the factors of the broad systems of units that confront writers of text-books today.

We will present some of the more important quantities using the C G S, English, and M K S systems of units, in the remainder of this chapter. Electrical and magnetic units will be handled separately in Chapter IV.

3-2. Velocity. Velocity refers to change of distance per unit of time in a given direction. Another similar term is that of speed[3]. Examples of velocity might include 10 miles per hour south and 15 centimeters per second bearing 10° north of west. The defining equation for velocity is $V = K\ L/T$ where V is the velocity, L the length, T the time, and K the constant of proportionality, taken as unity and without dimensions.

Since velocity or speed has dimensions of length over time, the equation is applicable to any system of units. In the English system we find such expressions as miles/second, feet/second, and miles/hour.

In the C G S system, centimeter/second is the most common unit. The M K S unit for velocity is meter/second.

---

[3] Speed refers to change of distance per unit of time with no indicated direction while velocity does indicate direction. Thus, velocity is a vector quantity.

3-3. Acceleration. The acceleration of a body may be defined as the rate of change of its velocity. As noted above the unit for velocity is a rate of change of distance, which determines the unit for acceleration is:

$A = K (V_1-V_2)/T$ with $k = 1$, T the time in seconds, and $(V_1 - V_2)$ showing the change in velocity. The unit of velocity as cm/sec would allow the unit of acceleration in the C G S system to be cm/sec/sec or cm/sec^2.

The above equation can be applied in any system of units. In the English system common examples of acceleration are ft/sec^2, miles/hr/sec, and miles/sec/sec.

In the M K S system the most common unit for acceleration is meters/sec/sec.

The acceleration of gravity, denoted by g, for freefalling bodies is approximately 980 cm/sec/sec, 32 ft/sec/sec, and 9.8 m/sec/sec for the C G S, English, and M K S systems, respectively.

3-4. Force. The word force is usually accepted to mean a push or a pull. It may be applied to prevent or produce motion. The gram of force is the basic unit of force in the metric gravitational system. It may be defined as the earth's pull on an object having a mass of one gram. A careful consideration of meaning of terms, namely, mass and weight (force), is suggested here. Mass and weight, often used in correctly are two different quantities. Mass is a measure of the amount of matter a given body possesses while weight is a measure of the gravitational pull on a given body. Weight is varable, depending upon the location of the given body. A body losses weight as it leaves the surface of the earth, outward in space or downward toward its center, such as in a well or mine. Some authors use gram-force and grammass or gram alone, for mass, in distinguishing between these two quantities.

The rate of change of momentum of a body is proportional to the force acting upon it, and takes place in the direction in which the force acts. This is Newton's second law of motion. F = K M A, where K is unity and dimensionless. M A denotes the change of momentum since A is the rate of change of velocity and momentum is the product of mass and velocity.

The dyne is the unit of force in the C G S system of units. It is a force just large enough to give an acceleration of 1 cm/sec/sec to a body that has a mass of one gram. The dyne is too small for much practical use. One gram of force is equal to about 980 dynes.

Force in the English system is usually expressed in pounds. One pound-force equals the weight of an object with a mass of one pound. Some engineers prefer to use gravitational units of force throughout their calculations. This necessitates the use of the factor 1/g in the equation, F = (w/g)A, if conventional units of mass are used. One can retain gravitational units of force and the simple form of the law if he uses a new unit of mass. If one pound-force gives a mass of 1 pound an acceleration of 32 ft/sec/sec, then 1 pound-force will give a mass of 32 pounds an acceleration of 1 ft/sec/sec. When masses are expressed in multiples of 32 pounds and force in pounds, the equation F = M A is valid. This has brought forth a unit, the slug, which is equivalent to a mass of 32 pounds. A slug is that mass which acquires an acceleration of 1 ft/sec/sec under the application of 1 lbf.

The poundal is a unit in the English system similar in nature to the dyne in the C G S system. The poundal is a unit of force found by F = M A, when the mass is in pounds and the acceleration in ft/sec/sec.

Hence, any time the equation F = M A is used, or one based on it, one must use care in using the dyne and the poundal as the force units, the gram or pound as the unit of mass, and the cm/sec/sec or ft/sec/sec for the acceleration units.

In the MKS system the newton is the unit of force which may be defined as that force which acting upon a mass of one kilogram will produce an acceleration of 1 m/sec^2. From this we see the relationship between the dyne and newton as: 1 newton is equivalent to 10^5 dynes.

3-5. Work. In the physical sense, work is done on an object when a force is applied causing motion in the same direction that the force was applied. One may consider a unit of work as any unit of force times any distance unit. The amount of work done is directly proportional to the force applied and to the distance moved, in the direction of the force. For example, a force of one pound applied through a distance of one foot, would do one foot-pound of work. The defining equation for work is, W = K F S where F is the force, S the distance, K the constant of proportionality, and W the unit of work. From the equation (F = M A), we can see that the dimensions of work are $M L^2 T^{-2}$. In the C G S system the unit of work is the erg, defined as a dyne-centimeter with the dimensions of $gm\text{-}cm^2/sec^2$. It follows that dynes of force may be converted to grams by dividing by the value of g in cm/sec^2.

The most common unit for expressing work in the English system is the foot-pound. One foot-pound is the amount of work done when a force of one pound produces motion of one foot in the direction of the applied force.

In the M K S system of units work is normally expressed in joules. The joule may be defined as 1 nt-m with dimentions of $kg\text{-}m^2/sec^2$. It can be shown that 1 joule = 10^7 ergs.

3-6 Power. Power is the rate of doing work. It can be expressed as the work done per unit time.

P = W/T. Since work is the product of force and distance, we have power as force-distance /time.

In the CGS system power is expressed in ergs/second.

In the MKS system power is expressed in joules/second. We also have the term watt which is a joule/sec.

In the English system of units we have power expressed in foot-pounds/second and horsepower. One horsepower is defined arbitrarily as 550 ft-lb/sec.

With proper utilization of units and dimensions one can prove that 1 horsepower = 746 watts = .746 kilowatts. Also, one watt is equivalent to 10^7 ergs/sec.

3-7. Momentum and Impulse. Two other quantities associated with motion are momentum and impulse. Momentum is defined as the product of the mass of the body and its velocity. One of the laws of dynamics, comparable to the law of conservation of mass and energy, is that of the conservation of momentum. Many problems in physics, classical and modern, deal with this law that states that the momentum of an isolated system of particles remains constant. Momentum = mass X velocity.

A less important term is that of impulse. Impulse may be defined as the product of the force and time for a given application. It is reasonable to expect that for a given body, the greater the momentum the more difficult it would be to overcome its motion. Newton's second law states that F T = M (V - V') or (impulse = change of momentum).

The units of impulse and momentum depends upon the units of force, time, mass, and velocity in the various systems of units. Some examples for impulse are: dyne-sec, lb-sec, and nt-sec. For units of momentum we have: gm-cm/sec, lb-ft/sec, and kg-m/sec.

3-8. Pressure. Pressure may be defined as the force per unit of area. P = F/A. This equation shows at once the units of pressure to be force units per squared units of distance. Some examples of units of pressure are: gm/cm^2, lbs/in^2, lbs/ft^2, $tons/yd^2$, nt/m^2, $dynes/cm^2$, and inches of mercury.

3-9. Specific Gravity and Density. The specific gravity of a substance may be defined as the ratio of the densities of two substances where one is the standard. Water is used as the standard for liquids and solids and air at standard conditions for gases. Density is the mass per unit of volume in any system.

C G S density = specific gravity X 1 gm/cm^3.

M K S density = specific gravity X 1000 kg/m^3.

English density = specific gravity X 62.43 lb/ft^3.

3-10. Angular Motion. A body is capable of two different types of motion; translatory and rotary. Thus far we have dealt with linear or translatory motion. Although there is almost enough marked similarity between quantities, formulas, and units, to allow one to quickly transfer his application from linear to circular motion, we will include a short discussion of angular motion.

First we define the radian which is the most common unit for expressing angular measure in contrast to the degree in simple geometry and trigonometry. The radian may be defined diagramatically as θ (theta) = $\frac{s}{r}$; the radian being the angle such that the length of the arc s subtending it is equal to the radius r of the arc AB.

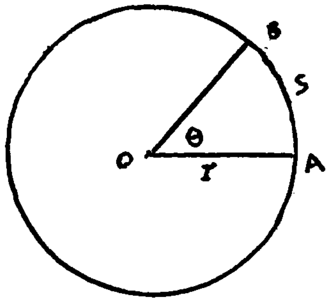

Figure 2. Relation of linear to rotary motion.

We measure angular velocity ω (omega) in radians per second and angular acceleration α (alpha) in radians per second per second. Thus we have:

a. $S = r\theta$ where S is length of arc, r the radius, and θ the angular distance.

b. $V = r\omega$ where V is linear velocity, r radius, and ω the angular velocity.

c. $A = r\alpha$ where A is linear acceleration, r radius, and α the angular acceleration.

3-11. Rotational Inertia. $I = M r^2$ defines rotational inertia of a particle of mass M at a distance r from the axis of rotation. Since I is a property of a body that determines its resistance to angular acceleration, one should readily expect that it is highly variable for different shapes, masses, and bodies with mass unusually distributed. The choice of axis is very important in problems of this topic, too. Some common units for rotational inertia are gm-cm^2, lb-ft^2, and kg-m^2.

3-12. Torque. $T = I\alpha$ Torque is the product of the rotational inertia I and the angular acceleration α. Torque is sometimes defined as the moment of force denoting the ability of the given force to produce motion in a circular pattern. Torque is then the product of force applied and the perpendicular distance from the axis of rotation to the line of action. Separate meanings for torque and work are based on different directions in the application of the force. The units of torque are dyne-centimeter, newton-meter, and pound-foot.

3-13. Angular Momentum. Ang. Mom. = $I\,\omega$. This equation shows the angular momentum of a rotating body is measured by the product of the moment of inertia and the angular velocity. One might also relate the change in angular momentum to the product of time and torque similar to article 3-7 for linear motion. Units for angular momentum are determined by $I\,\omega$.

CHAPTER IV

ELECTRICAL AND MAGNETIC UNITS IN THE M K S SYSTEM

4-1. Introduction. One of the main reasons a need arose for such a system as the M K S system was to make it more convenient to manipulate quantities in problems of a magnetic and electrical nature. Many of the problems of the physicist and engineer are those of organizing variable quantities toward a solution which may involve several different units. An understanding of the place each unit occupies in each system and a knowledge of the conversion factors and unique constants is basic.

The International Electrotechnical Commission, meeting in Scheveningen, Brussels, in June, 1935, adopted the meter, kilogram, and second as the basic units of length, mass, and time. This action become effective in January, 1940.

To discuss electrical and magnetic units one needs to start by defining a basic unit such as the ampere, and proceed from there. Any one of three effects--heating, magnetic, electrolytic, could be used in defining a unit of current (ampere). The International Committee on Weights and Measurements adopted the magnetic effect.

The following topics will discuss the important electrical and magnetic quantities and their M K S units. Appendixes A, B, and C show a complete comparison of C G S (esu and emu) units, definitions, conversion factors, and constants, to similar quantities in the M K S system of units. We will, however, express some of the basic quantities in C G S as well as M K S units where such discussion will clarify the topic at hand.

4-2. Current. We may define the ampere (amp) as that constant current which, if present in each of a pair of parallel wires of unlimited length and one meter apart, in a vacuum, will cause each wire to experience a force of 2×10^{-7} newtons for each 1 meter of length of wire. If the currents are equal and oppositely directed the force will be one of repulsion, and one of attraction if the currents are in the same direction.

The dependence upon the medium is very important in electrical applications, especially in motors and generators. The medium is represented in physical equations by the product $(\mu \mu_0)$, read (mu and mu sub zero). μ_0 denotes the effect in free space and μ expresses a ratio relating the effectiveness of one given medium to another and has no units. Thus we have $\mu = 1$ for free space. (μ necessarily greater than μ_0). By name μ_0 and μ are permeability of free space and relative permeability, respectively. This points up our previous discussion of choice of basic units, namely the choice of meter, kilogram, second, and ampere as fundamental MKSA units.

From the definitions and discussion above we may write; $F = k \mu \mu_0 I_1 I_2 L/d$ where k is defined to be 1/2(pi) and $\mu_0 = 4 \text{ pi} \times 10^{-7}$; I_1 and I_2 refer to the currents, L the length of conductor, d the distance between conductors, and F the force. This equation in dimension form is: $F = \mu_0$ ampere ampere meter/meter allowing units for μ_0 to be newtons/ampere^2. With proper substitution in the above equation we have:

$F = 2 \times 10^{-7} I_1 I_2 L/d$; which completely defines the ampere. The unit for current is abbreviated as amp or just a, ma for milliamperes and μa for microamperes. The electrostatic unit of current is rarely used. The electromagnetic unit for current is the abampere which is equivalent to 10 amperes.

4-3. Quantity of Electricity. Following the above equation; if the current in one wire is kept constant, the current in the other is directly related or proportional to the force between them. It follows that the product of the current in a wire and the time will give the quantity of electricity that passes a point on the wire in that given time. We may define the unit of electrical charge or quantity of electricity, the coulomb, as the quantity which passes a point on a conductor in one second when the given conductor carries a steady current of one ampere. Thus, we have; 1 coulomb is equivalent to one ampere-second. The statcoulomb ($1/3 \times 10^{-9}$ coulombs) is the esu for charge.

To correlate this discussion with some modern theory we know that each electron, (a) has the same negative charge as all others and (b) has a charge of 1.60×10^{-19} coulombs. Therefore, 1 coulomb = the charge on 6.25×10^{18} electrons. We may think of an ampere as being equivalent to the motion of 6.25×10^{18} electrons/second past a given point.

4-4. Potential Difference. The potential difference between two points in a circuit may be defined as the quotient obtained by dividing the amount of energy W expended in moving a given amount of electricity Q from one point to the other, by this quantity Q.

$V = W/Q$. The volt is the potential difference which exists between these two points when one joule of energy must be expended in moving one coulomb of electricity from one point to the other. The esu and emu for potential difference are the statvolt (300 volts) and the abvolt (10^{-8} volt), respectively.

4-5. Electrical Power. From the above equation and the appropriate definitions; $V = W/Q = W/T \div Q/T = P/I$. One can see the simple relationship that exists between potential difference, charge, energy, and power. Then $W = VQ$ (joules= volts X coulombs) and $P = VI$ (watts = volts X amperes). Since 1 joule/second is 1 watt, one can see that power will come out in watts when I is in amperes and V in volts. 1 Kilowatt = 1000 watts.

4-6. Resistance. The electrical resistance between two points on a conductor is found by dividing the potential difference between these points by the current in the conductor. $R = V/I$. If V is in volts and I in amperes, R will be in ohms, the M K S unit for resistance. The esu and emu for resistance are the statohm (9×10^{11} ohms) and the abohm (10^{-9} ohms), respectively.

The resistance of most metals is almost constant if physical conditions, mainly temperature, are constant. Ohm found this to be true whether the current was small or large. Equation $R = V/I$ is known as the algebraic form of Ohm's Law.

4-7. Mechanical Equivalent. If the element of a circuit between two points consists of resistance only, the total energy, $W = V\,Q$, becomes heat. We are assuming here that no electromotive force (emf) is present. Emf is the rise in potential produced by such things as batteries and generators. To relate heat and mechanical units, most authors use the letter J. $W = J\,H$ or $H = V\,Q/J$. Experiment has shown that the value of J is 4.185 joules/calorie or 4185 joules per large Calorie (C). Other useful relations are: $W = V\,Q = V\,I\,T = I^2\,R\,T = V^2\,T/R$.

4-8. Resistivity. Experiment has shown that $R = \rho\,L/A$ where L and A are the length and cross-section area of a given conductor and ρ (rho) is a constant depending upon the material of the conductor. The dimensions of ρ may be shown to be ohm-meters by ρ = R A/L = ohm-meters^2/meters = ohm-m. Rho is a variable number for the many conductors but constant for any one conductor at a given temperature. The engineering field has a similar constant measured in ohms-cir-mils/foot.

4-9 Magnetic Fields. The repulsive and attractive forces, discussed in defining the ampere were magnetic in nature. Like poles repel and unlike poles attract. The immediate space about a magnet is known as a magnetic field. The direction of the field is the direction in which a north pole of a compass points, when placed in the field. The strength of the field, at a certain point, is proportional to the force on a magnetic pole placed at this point. Otherwise stated, the strength of the field is proportional to the torque on a magnet held at right angles to the direction of the field at the point in question. The M K S unit for magnetic field strength is the newton/weber or more frequently ampere-turn/meter. The emu of magnetic field strength or intensity (oersted) is equivalent to 1000/4(pi) ampere-turns/meter.

4-10. Magnetic Induction. The force which a conductor experiences while carrying a current I, in a field H, might at first appear to be dependent only upon the current, length of conductor, and the strength of the field. The space immediately about the conductor has a considerable effect, too. A combination term, including these factors, is B, the magnetic induction. $F = BIL\sin\theta$ defines B in nt/amp-m, when F is in nt., I in amp., and L in meters. This equation holds only for uniform fields. A calculus equation of $dF = BI\,dL\sin\theta$ handles non-uniform applications. Another unit for magnetic induction to be discussed later is the weber/meter^2. The emu (gauss) is equivalent to 10^{-4} weber/meter^2.

4-11. Permeability. $B = \mu H$. This relates the magnetic induction B and the magnetic field strength H at any point. The constant μ has the value 4 pi X 10^{-7} weber/amp-m or nt/amp^2. This particular value of is represented by μ_0 and is referred to as the permeability of free space. The value of this constant arises from the definition of the ampere. Its value in most gases is approximately equal to that in free space. For most materials the ratio of B to H may be constant and equal to μ_0 but for such metals as iron, nickel , and cobalt, and some of the alloys, the permeability may be several hundred times that of free space.

4-12. Magnetic Flux. We previously mentioned lines of force in dealing with magnetic fields. This use of lines to represent B diagramatically makes one think next of the magnetic flux density. The magnetic flux ϕ may be defined as the product B A, where A is the area perpendicular to the magnetic induction B, assumed uniform over the area. Again, calculus must be employed to compute ϕ if B is not uniform. To express ϕ in terms of lines of induction; it is the total number of lines through A, while B is the lines per unit area. The unit of magnetic flux ϕ is the weber, since the unit of B is weber/meter^2.

Another formula of interest at this point is based on work of Faraday and Lenz. In $V = -N\, d\phi \,/\, dt$, N is the number of turns in a coil, $d\phi$ the change in flux, dt the change in time, and V the potential difference.

We now have B, H, and μ established in relation to each other. We shall now proceed to discuss other magnetic properties of substances. For some of these it is more convenient to use other terms, such as pole strength, intensity of magnetization, and magnetic moment.

4-13. Magnetic Moment. The symbol M is called the magnetic moment of a magnet and is proportional to the length of the magnet and its magnetization strength. If a bar magnet is situated in a magnetic field it tries to line itself up in the direction of the field. The torque τ it experiences depends upon the magnitude of the field and on the sine of the angle it makes with the given field. $\tau = M\ H \sin\theta$. One may make the assumption that the ends of the magnet have the concentrated strength of the entire magnet, and then consider them as the magnetic poles. The torque is then $F\ L \sin\theta$, where L is the distance between poles or the length of the magnet itself. Thus we have two things equal to the torque. Equating these we have: $M\ H \sin\theta = F\ L \sin\theta$ and we see that $F = HM/L$ or $F = HP$, where P replaces M/L. P under these conditions is called the magnetic pole strength. This unit of strength is the weber. To summarize somewhat, the force per unit pole is the magnetic field strength and the product of the pole strength and the length of the magnet is the magnetic moment. It may be noted further that the torque on a magnet and the force on a pole are dependent upon the value of H, while the torque on a coil with a current and force on a conductor with a current are proportional to B and thereby to μ H.

4-14. Intensity of Magnetization. One more term, besides the pole strength P and the magnetic moment M, is necessary to assign the strength or magnetization of a given piece of iron. This term is that of the intensity of magnetization. Intensity is defined as the moment of the magnet per unit volume or strength per unit surface, where the surface or area in question is perpendicular to the direction of the magnetization. Int = M/vol or PL/AL = P/A. The unit of Int. is weber/meter^2.

4-15. Attraction and Repulsion of Magnets. Coulomb made many quantitative studies of the forces between like and unlike magnetic poles. His work brought forth the relation: $F = PP'/4\ \text{pi}\ \mu\ r^2$. This shows that the force between magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them. μ may be taken as μ_0 or as our previously discussed permeability constant of free space.

A careful study of the relationships among P, ϕ, B, and H, will show the unit pole as having the same dimensions as the unit of magnetic flux, the weber. One additional statement made for exactness is that B and H apply only to the space exterior to the magnet itself.

4-16. *Coulomb's Law.* We have discussed the basic magnatic units and the units that apply to current electricity. We now turn our discussion to that of stationary charges. Under the topic electrostatics come such terms as electric field, electric flux, capacitance, and dielectrics.

We will note a great similarity between the laws to attraction and repulsion for magnetic poles and electric charges. For electric charges at points in a uniform space of infinite magnitude, the force is given by: $F = qq'/4 \text{ pi } ee_0r^2$. The letters q and q' refer to the two charges, r the distance between them, e is the dielectric constant for the medium in which we find the field, and e_0 is a calculated or experimental constant. After the unit of electrical capacitance, the farad, is discussed we will be in a better position to give a complete discussion of dielectrics.

4-17. *Electric Field Strength.* $E = F/q$. This equation defines the electric field strength. Electric field strength indicates the amount of force a given field is capable of exerting on a unit charge placed in the field. The equation relates q the charge in coulombs placed in the field, F the force in newtons acting on q, and E the electric field strength in newtons/coulomb. The direction of the field is arbitrarily taken as the direction of the force on a positive charge placed in the field.

An electric field exists between the conductors of a condenser. If the field is uniform, its intensity equals V/d. V is the potential difference in volts and d is the distance between the plates (conductors) in meters. Therefore, another unit for electric field strength is the volt/meter.

4-18. Electric Induction. In $D = e\ E$, e is the dielectric constant. It is interesting to note the similarity between this equation and $B = \mu H$. If $D = eE$ is combined with the equation for field due to a point charge, we have $D = e\ E = q/4\ pi\ r^2$ and $B = \mu\ H = P/4pi\ r^2$.

Electric flux is represented by lines, just as magnetic flux. Just as B, the magnetic induction, is the number of magnetic lines per unit area, D, the electric induction, is the number of electric lines per unit area. We think of one line coming from each unit of charge, so we may use the coulomb as a unit of flux as well as charge.

In considering parallel charged plates, if large and with small separating distance, we find uniform flux densities if the electric field is uniform. Regions near the edges of such plates are not included in the uniform field. The charge per unit area is q/A, where A is the surface area of one plate and q the charge on that plate. From the above discussion we see that q/A is also the flux density or electric induction D. Therefore, $D = q/A$ or eE. The unit for electrical induction is coulomb/meter^2.

4-19. Capacitance. The electrical capacitance of a body may be defined by $C = q/V$ where q is the charge that must be assigned to a body to alter its potential by the amount V. If the charge is measured in coulombs and the potential in volts, the unit of capacitance is the farad. The farad, being a relatively large unit for much use, has two rather important subdivisions. They are the microfarad (uf) and the micromicrofarad (uuf). One uf = 10^{-6}f and uuf = 10^{-12}f. The capacitance of a body depends not only upon its shape and size but upon the surrounding conductors. The esu for capacitance, the statfarad, is equal to 1/9 X 10^{-11} farad.

4-20. Dielectric and Relative Dielectric Constants. We have used the letter e in several instances and have compared it with μ the permeability. The letter e denotes the dielectric constant for the medium in which we find the electric field. The dielectric constants for some substances such as glass and oil are much greater than the dielectric constant for a vacuum or air. The ratio of the dielectric constant of any substance to that of a vacuum is called the relative dielectric constant. Some authors use specific inductive capacity or this term. The letter k is equivalent to the quotient of e/e_0. e_0 has a value of 1/4 pi X 9 X 10^9 farads/meter. The value of e_0 may be calculated from Coulomb's equation, but more accurately from $c = 1/(\mu_0 e_0)^{1/2}$, where c is the velocity of light. This calculation using c = 3 X 10^8 m/sec and μ_0 = 4 pi X 10^{-7} nt/amp^2 yields the value of e_0 as 1/4 pe X 10^9 coul^2/nt-m^2 or farad/m.

4-21. Inductance. Most of the preceding discussion of magnetic and electrical phenomena dealt with steady currents and in a few instances with stationary charges. We will now consider the effects of variable currents. In ordinary circuits such as battery-switch-electromagnet, one finds that the current does not reach its maximum immediately, but acts as though it had a stubbornness to move. If an ordinary filament bulb is put into the circuit in place of the electromagnet, with equal resistance, the current starts much more quickly, and there is much less arc when the switch is opened. The important property in existence here is known as inductance.

We previously discussed the induced emf in coils of wire accompanying a change of flux within the coil. Current in a wire also produces a magnetic flux and a change in current naturally produces a corresponding change in flux. It follows that an emf is produced with these changes. The emf is in such a direction as to be opposite to the producing action. Starting current is opposed by an emf and a stopping current is opposed by an emf. As was discussed before an emf only exists while there is a change in effect; here it is the current that is changing. The emf here is that of selfinduction.

The amount of emf of self-induction depends upon the nature of the circuit considered and the rate of change of current. The emf equation is: $E = -L\, dI/dt$, where L is the inductance in henrys, E the potential difference in volts, I the current in amperes, and t the time in seconds. One can see that if a coil has a back emf of 1 volt with a current change rate of 1 ampere/second, one henry exists. One henry is equivalent to 10^9 emu of inductance.

Often circuits contain pairs of coils. One coil has an emf produced based upon the rate of change of current in a second coil and the second has a change of current which in turn changes the flux; thus mutual induction is formed by the interaction of currents in two coils.

CHAPTER V

SUMMARY

In this report on the meter-kilogram-second system of units, we have attempted to define and relate the important entities with their appropriate defining equations. For the non-electrical and non-magnetic quantities we have shown the centimeter-gram-second, English (foot-pound-second), as well as the MKS units. We kept the electrical and magnetic quantities separate in chapter IV. These quantities were defined in the MKS system with the appropriate CGS unit alongside as needed for clarification. A complete listing of comparisons among the systems can be found in appendixes A, B, and C.

We realize that the main need of such a practical system as the MKS arose because of the advantages in manipulating quantities that were magnetic and electrical in nature.

There are two versions of the MKS system being used currently; one is called the rationalized system, the other the unrationalized. This report has made use of the rationalized system.

The rationalized system allows one to define quantities and express equations without following the classical development of physical ideas. by choosing $k = 1/2(\text{pi})$ and $\mu_o = 4 \text{ pi} \times 10^{-7}$ in the equation

$F = k \mu \mu_o I_1 I_2 L/d$ with $\mu = 1$ for a vacuum,

with no units, we may write the dimension form as

Newtons = μ_o (amperes) (amperes) (meters)/meters. Hence μ_o has the dimensions of newtons per square ampere or henrys per meter. μ_o

relates the effectiveness of one medium to another. Upon substitution the above equation becomes $F = 2 \times 10^{-7} \times I_1 I_2 L/d$ and we use this equation to define the ampere. In summary, we have the meter, kilogram, second, and ampere, as the basic entities forming the rationalized MKSA system of units.

We do not want to suggest that the two absolute systems, CGS electrostatic and CGS electromagnetic, be classed as obsolete, but that the MKS system seems to have an advantage over these systems in expressing electrical and magnetic quantities. Little can be said for MKS units over CGS units in mechanical applications, but the newer MKS system makes use of the practical units in electrical and magnetic applications, to a greater advantage than the others. The important difference between the absolute system and the MKS system in computations is the factor 4 pi that appears in the MKS form of Coulomb's Law.

The study of particle physics, atomic and nuclear, is enhanced by the continued use of absolute units. There is no great use of permeability and permittivity (dielectric constant), in that atoms consist of particles in free space.

It is important to keep in mind that e_o is determined by purely electrical experiments and the quantity μ_o is arbitrarily defined.

The English system of units is currently being used by the engineers of the United Kingdom and the United States and the general public of the English-speaking countries. Because of its haphazard growth having brought forth unrelated subdivisions and the inconvenience of no decimal factors, this system is much more difficult to use in computations.

Systems of units are essential, in that physical theories may be improved upon by exact measurements and mathematical relationships to illustrate exact quantitative and qualitative descriptions as nearly as possible. Systems of units stand alone and serve as meaningful expressions of interrelated quantities. The conversions from one system of units to another and from larger to smaller units, etc, must be understood but is not directly related to an understanding of physical phenomenon.

It is the wish of the author that this report serve as a simple and direct review booklet for those interested in comparing systems of units and seeing the place that the M K S A system occupies.

BIBLIOGRAPHY

American Association for Physics Teachers Committee, "On Electrical and Magnetic Units." *American Journal of Physics,* VI, CXLIV (1938), p. 33.

Bingham, Eugene C., "Helping the Metric Campaign," *Journal of Chemical Education,* II (January,(1925), p. 62.

Durbin, Frank M., *Introduction to Physics,* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1955.

Jauncey, G. E. M., and A. S. Langsdorf, *M K S Units and Dimensions,* New York: The Macmillan Company, 1940.

Lind, S.C., "Letter to Editor," *Journal of Chemical Education,* XXI (December, 1943), p. 623.

Marshall, John Stewart, and Elton Roy Pounder, *Physics,* New York: The Macmillan Company, 1957.

Michener, W. H., "A Brief Table of M K S Units," *American Journal of Physics,* VIII (October, 1940) p. 318.

Michener, W. H., *Physics,* New York: John Wiley & Sons, Inc., 1947.

Milligan, Robert A., and John Mills, *Electricity, Sound, and Light,* Boston: Ginn & Company, 1908.

Neville, Harvey A., "Educating the Public in the Use of the Metric System," *Journal of Chemical Education,* II (July, 1925) p. 593.

APPENDIX A

METER-KILOGRAM-SECOND UNITS[1]

SYMBOL	QUANTITY	DEFINITION	UNIT	EQUIVALENT
L	Length	-	meter	100 cm; 39.37 in.
M	mass	-	kg.	1000gm; 2.2+ lbs.
F	force	$F = M A$	newton	10^5 dynes
W	work	$W = \int F\cos\theta dL$	joule	10^7 ergs
P	power	$P = dW/dt$	watt	10^7 ergs/second
τ	torque	$\tau = FL \sin\theta$	nt-meter	10^7 dyne-cm
I	electrical current	see 4--2	ampere	.1emu;3.10^9 esu
Q	electrical charge	$Q=\int I\ dt$	coulomb	.1emu;3.10^9esu
V	potential difference	$V=W/Q = P/I$	volt	10^8emu;1/300esu
R	resistance	$R = V/I$	ohm	10^9emu;$1/9.10^{11}$ esu.
E	electric field strength	$E=F/Q=-dV/dr$	nt/coul. volt/m	10^6emu;$1/3.10^4$
C	capacitance	$C = Q / V$	farad	9.10^{11}esu
e	dielectric constant	$F=QQ'/4pier^2$	farad/m	$4pi\cdot 9\cdot 10^9$esu
k	relative diel. constant	$k = e/e_o$	-	free space,k=1
$\mathcal{F}$	magnetomotive force	$\mathcal{F} = N I$	amp-turn	.4pi gilberts
L'	inductance	$V=-L'dI/dt$	henry	10^9 emu
H	magnetic field strength	$H=\frac{\int I\sin\theta dl}{4pir^2}$	amp/m nt/weber	$4pi\cdot 10^{-3}$oersted
M'	magnetic mom.	$M H \sin\theta=\tau$	weber-m	$10^{10}/4pi$ emu
P'	pole strength	$P = M /L$	weber	$10^8/4pi$ emu
ø	magnetic flux	$V=-N d\phi/dt$	weber	10^8 maxwells
B	mag.induction or flux dens.	$B = \phi/A$ or $dF=BI\sin\theta dL$	weber/m^2	10^4 gauss
μ	permeability	$\mu=B / H$	web/amp-m	$10^7/4pi$ emu
μ_r	rel.perm.	$\mu_r = \mu/\mu_o$	-	free space; μ_r=1
Int.	intensity of magnetization	Int=M/V=P/A	web/m^2	$10^4/4pi$ emu
D	electric induction	D=eE	coul/m^2	$12pi\cdot 10^5$esu
e_o				$1/4pi\cdot 9\cdot 10^9$f/m
μ_o				$4pi/10^7$ web/a-m

---

[1] W. H. Michener, Physics, (New York-1947) p. 636.

APPENDIX B

THE ESU AND THEIR RELATIONS TO CORRESPONDING MKS UNITS[2]

quantity	name	conversion factor 1 unit of column 2 equals
charge	statcoulomb	$1/3 \times 10^{-9}$ coulombs
current	statampere	$1/3 \times 10^{-9}$ amperes
pot.diff.	statvolt	300 volts
elec.field strength	statvolt per cm.	30,000 volts per meter
capacitance	statfarad	$1/9 \times 10^{-11}$ farad ($\doteq$ 1 uuf)
resistance	statohm	9×10^{11} ohms

APPENDIX C

THE EMU AND THEIR RELATIONS TO CORRESPONDING MKS UNITS[3]

quantity	name	conversion factor 1 unit of column 2 equals
pole str.	abpole	$4\,\text{pi} \times 10^{-8}$ webers
mag.field strength	oersted	1000/4 pi amp/meter
flux dens.	gauss	10^{-4} weber/meter/meter
flux	maxwells	10^{-8} webers
current	abampere	10 ampere
pot. diff.	abvolt	10^{-8} volts
resistance	abohm	10^{-9} ohms
inductance	abhenry	10^{-9} henrys

From the appendixes we have the following pair of equations:

$$\frac{1\text{abamp}}{1\text{statamp}} = \frac{1\text{ statvolt}}{1\text{ abvolt}} = \sqrt{\frac{1\text{ statohm}}{1\text{ abohm}}} = \sqrt{\frac{1\text{ abfarad}}{1\text{ statfarad}}} = 3 \times 10^{10}\text{ cm/sec}$$

(vel. of light)

$$1/\sqrt{e_o \mu_o} \doteq \left(\frac{10^{-9}}{36\text{ pi}} \times 4\text{ pi} \times 10^{-7}\right)^{-\frac{1}{2}} = (9 \times 10^{16})^{\frac{1}{2}} = 3 \times 10^{8}\text{ m/sec}$$

(velocity of light)

---

[2] J. S. Marshall, and E. R. Pounder, Physics (New York, 1957)p.774.

[3] Ibid p. 775-776.

VITA

George Wesley Barratt

Candidate for the Degree of

Master of Science

Report: ON THE M.K.S. SYSTEM OF UNITS

Major Field: Natural Science

Biographical:

Personal Data: Born in Mount Moriah, Missouri, March 25, 1926, the son of James T. and Lillian Maude Barratt.

Education: Attended grade school in Mount Moriah and Cainsville, Missouri; graduated from Cainsville High School in 1943; received the Bachelor of Science Degree in Education, with a mathematics major, from the Northwest Missouri State College, Maryville, Missouri, in May, 1954; completed requirements for the Master of Science degree in May, 1959.

Professional experience: Served in the United States Naval Reserve (active) from March, 1944, to February, 1946. Taught in high schools for five years; past member of Kappa Delta Pi in Maryville, Missouri; member of the National Education Association. Attended Oklahoma State University, Stillwater, Oklahoma, as a member of the National Science Foundation--Academic Year Institute in 1958-59.