WOUND ROLL STRESSES FROM DISPLACEMENT MEASUREMENTS

David R. Roisum, Ph.D
Kimberly-Clark

ABSTRACT

Previous wound roll models required a known Wound-In-Stress boundary condition that is easily obtainable only for the restrictive case of pure centerwinding with measured incoming web tension and caliper. A new boundary condition and model were defined that allows wound roll stresses to be predicted for any winding case from roll diameter and web caliper measurements. These measurements are made during winding using an instrument similar to a density analyzer but extended to include web caliper. These measurements as well as material properties are input into winding models which have been extended to also calculate displacements. The net result is a measurement technique at least an order of magnitude more sensitive that previous methods, as well as fundamental outputs of radial and tangential stresses and strains at any point in the roll and at any time during the winding process.

NOMENCLATURE

b	basis weight	ε_T	tangential strain
c	caliper	μ_R	radial poisson ratio
d	diameter	μ_T	tangential poisson ratio
E_C	core modulus	r	density
E_R	radial modulus	σ_R	radial stress
E_T	tangential modulus	σ_T	tangential stress
l	length	σ_W	wound-in tangential stress
p	pulses	0	subscript for sample at core
ppr	pulses per revolution of encoder	i	subscript for sample i

r	radius	j	subscript for sample j
w	radial displacement	k	subscript for sample k
WIS	wound-in-stress	n	subscript for outer sample
WIT	wound-in-tension	d	drum roller
ε_R	radial strain	r	rewound roll

INTRODUCTION

Analytical winding models have been used since the late 1950's to calculate stresses within wound rolls for a wide variety of materials[1,2]. The reason for modeling as opposed to experimental measurement is that internal roll stresses are often very difficult to measure in practice. Additionally, modeling has the advantage of exploring a variety of conditions in a 'what if' analysis quicker and more economically than running trials on production or pilot winding machines. Typically, the stress output of the winding models has been compared to failure criteria. An example is that the interlayer pressure (radial stress) must be high enough to transmit the required torque for centerwinding and unwinding, yet not so high as to crush or block the material. Similarly, the MD (machine direction) stresses must not be too compressive else buckling and starring might occur, and not so tensile as to yield or break the material.

While conventional winding models have been used successfully by many companies on a wide variety of materials, they have some application limitations. First, many wound roll design questions involve roll geometry such as web length, roll diameter, and roll weight as opposed to just stresses. Secondly, a key input for winding models is the winding tension or stress. Unfortunately, wound-in-tension is easily obtainable only for the case of pure centerwinding under load-cell tension control. This limits application for perhaps most winders that have a nipping roller and/or do not have load cell readouts.

Another approach to wound roll design and analysis is experimental or quality control measurements. While there are at least a dozen methods and devices for wound roll measurement, they all have one or more serious limitations[3]. The hardness measurements such as the Rhometer and Schmidt hammer yield readings on a somewhat arbitrary scale, instead of directly related to the fundamental roll structure variables of stress and strain. Additionally, while they are well suited for CD (cross direction) profiling, diameter based profiling for tuning the roll structure TNT (torque nip & tension) variables is difficult. Some measurements are closely related to wound roll stresses such as the Cameron Gap[4] for measuring wound-in-strain that can be converted to tensions or stresses, and the pull tab for measuring interlayer friction that can be converted to pressure or stress. Unfortunately as mentioned earlier, these stress-like measurements are somewhat difficult and time consuming to make which precludes them from routine and regular use. The last major shortcoming shared by most of the measurement methods is the lack of sensitivity or resolution to roll structure changes.

Of the measurement methods available, the wound roll density analyzer was perhaps the most satisfying because of two major advantages. First, the density analyzer is a computer based data acquisition system that can be easily set up on most any winding machine and can be run in an automatic mode. Because of its automation, it is a cost effective method for obtaining roll structure data with minimal effort. Secondly, the density analyzer was found to be extremely sensitive to wound roll structure, and insensitive to most other variables (except caliper as will be shown later). However, the density analyzer had a serious limitation in that no conversion technique was available between density readings and fundamental wound roll stresses.

Wound Roll Stresses from Mathematical Modeling and Density-like Measurements

Recognizing the strengths and limitations of existing wound roll models and measurements, the author began his PhD thesis project to develop a technique for indirectly measuring stresses in wound rolls. The vision was a marriage of the strengths of the stress outputs from wound roll modeling with the convenience of automated online measurement. Furthermore, if stresses could be reliably measured online, then the winder could be adjusted accordingly to maintain target stresses. Thus, the winder could be run in closed-loop control to the fundamental measure of stress rather than open loop control of torque, nip, and tension.

The first insight of how the new stress measurement system might be constructed is to observe that as the wound-in-tension increases, the finished diameter for a given amount of a certain material decreases. In other words, the harder the wind, the smaller the roll. This relationship also applies incrementally such that the rate of roll diameter increase, all else being equal, decreases with increasing wound-in-tension. The next insight required is that the density analyzer does not directly measure density, rather it measures the rate of increase in roll diameter for a fixed number of wound roll revolutions. The final insight is that wound rolls are determined systems. Thus, the behavior or state of any roll is uniquely determined by material properties and physical laws which are closely described by newer nonlinear winding models.

The wound roll model running inside the stress measurement system contains the same elements as conventional wound rolls, as will be described later. However, instead of merely calculating internal stress distributions; strains and then displacements are also calculated. It is the calculated displacement of the outer diameter that is compared with the measured increase in diameter using the density-like system. The data acquisition system also contains the same elements as the conventional density analyzer, namely encoders which feed into a computer for measuring footage and diameter. However, the density analyzer measurements are supplemented by online caliper measurements. The marriage of modeling and measurement is done by noting that the determined winding system can have only one unique value of wound-in-stress that will produce a change in diameter corresponding to what was measured by the density-like analyzer. Side benefits from the stress measurement system are that in the process of determining what the unique wound-in-stress is for a given sample, all stresses, strains and displacements are calculated.

Conventional Wound Roll Models

All consistent winding models are composed of the following ingredients or their equivalents:

Equilibrium
$$r\frac{d\sigma_R}{dr} + \sigma_R - \sigma_T = 0 \tag{1}$$

Strain-Displacement
$$\varepsilon_R = \left(\frac{dw}{dr}\,dr/dr\right) = \frac{dw}{dr} \tag{2a}$$

$$\varepsilon_T = \frac{w\,d\theta}{r\,d\theta} = \frac{w}{r} \tag{2b}$$

Stress-Strain
$$\varepsilon_R = \frac{\sigma_R}{E_R} - \mu_T\frac{\sigma_T}{E_T} \tag{3a}$$

$$\varepsilon_T = \frac{\sigma_T}{E_T} - \mu_R\frac{\sigma_R}{E_R} \tag{3b}$$

These ingredients can be combined into a second order differential equation with two boundary conditions such as:

Winding Equation
$$r^2\frac{d^2\sigma_R}{dr^2} + A\,r\frac{d\sigma_R}{dr} + B\,\sigma_R = 0 \tag{4}$$

where
$$A = 3 \text{ and } B = 1 - \frac{E_T}{E_R}$$

Core B.C.
$$E_c = \left.\frac{\sigma_R}{w/r}\right|_{r=r_0} \tag{5}$$

Outside B.C.
$$\sigma_R = \frac{-\,WIT}{r_n} \quad @\ r = r_n \tag{6}$$

The first winding models[1,2] were limited to linear isotropic materials ($E_R = E_T$). However, even isotropic materials display anisotropic behavior in a stack. Thus, though these models provided invaluable insight into winding behavior, they could not be used for most applications. The next stage in the evolution[5,6] was to accommodate anisotropic behavior ($E_R \neq E_T$). Though these linear anisotropic models came closer to describing many real applications, the radial or stack modulus was often a function of applied stress. Thus, a nonlinear anisotropic model[7] was developed that serves as the core of this work as well as that of many others ($E_R = f(\sigma_R)$). Finally, more complex material behavior such as stress relaxation[8,9], hygrothermal strains[10] and other phenomenon has also been successfully modeled. Other differences between the models are the choice of mathematical solution technique for the winding equations which ranges from numerical integration, to series expansion, to finite difference.

The articles cited are but just a few of the published works on wound roll modeling. With so many models available, choosing which one(s) are appropriate for a given application has at times been confusing. However, a first cut can be made by a consistency check between the models for identical sets of inputs[11]. In other words, we expect an identical output of stresses for an identical input, subject only to numerical accuracy which should leave at least the first few digits intact. It is satisfying to note that several of the authors of those models that survived the consistency check, also took the care to independently verify the stress results through measurement of wound rolls. Of those models which survive the consistency check, the appropriate for any application is the simplest which adequately describes the particular situation being modeled. This author has found Hakiel's nonlinear anisotropic model[7] to be satisfactory for most situations.

Wound Roll Model Extensions

The conventional wound roll models calculate the radial (σ_R) and tangential (σ_T) stress distributions at any radial position in the roll as a function of material properties, geometry, core stiffness and the wound-in-tension profile. Yet, it is a simple extension to calculate radial strains from the stress distribution from equation (3a). The radial strain at any layer is related to the displacements of its inner and outer boundaries as[11]

Strain-displacement
$$\varepsilon_{R,i} = \frac{dw}{dr} = \frac{w_i - w_{i-1}}{h} \qquad (7)$$

Summing the radial displacements over all wraps gives the displacement of the current outer surface as

Displacement of Outside
$$w_k = w_0 + h\sum_{i=1}^{k} \varepsilon_{R,i} \qquad (8)$$

where the displacement of the core, w_0, is given as

Displacement of Core
$$w_0 = \frac{r_0 \sigma_{R0}}{E_c} \qquad (9)$$

Combining equations (3a) with (7-9) yields

Displacement-stress
$$w_k = \frac{r_0 \sigma_{R0}}{E_c} + h\sum_{i=1}^{k} \left(\frac{\sigma_{R,i}}{E_R} - \mu_T \frac{\sigma_{T,i}}{E_T}\right) \qquad (10)$$

Equation (10) is a very powerful tool for wound roll design that can be used to calculate diametral geometries, and can be used in at least two distinct manners. The first is to compute the finished roll size based on final accumulated radial and tangential stresses. Conceptually it works like this. If the roll were wound under no tension, the finished diameter could be easily predicted from the free web length, the unstressed stack ply height (height of a stack under no load divided by the number of plies), and the core diameter. This problem is simply a matter of equivalent cross-sectional areas. To calculate the real stressed roll diameter, the change in diameter is calculated from (10) where the stresses are the accumulated values calculated by the conventional winding models. Further calculations of unstressed and final density as well as roll weight can be obtained from the basis weight of the material.

The other application of the extended model can be used to infer winding stresses. Now instead of using accumulated stresses, incremental stresses contributed by winding a given number of revolutions of the roll are used. This application begins by noting that the pressure added just under the outer layer caused by winding under tension causes the layers to compress. As seen in Figure 1, the incremental compression is greatest at the outside and rapidly decreases with depth. Subjectively, the rapidly decreasing compaction with depth is due to the self supporting nature of the rings. Quantitatively, this decrease is directly due to the incremental radial stress profile which also decreases rapidly with depth as described by winding physics.

This behavior is described by a boundary condition that is different (but consistent) than the conventional radial stress under the outer layer caused by wound-in-tension and equation (6). The new boundary condition is easiest to understand if the effects of caliper addition and roll deformation are separated into two steps. Figure 2 shows a general diagram of a roll with j wraps, upon which will be added the kth wrap. Focusing on the kth wrap, if it is first added in an unstressed condition, as in the center diagram, the roll would then assume a diameter of $rk = rj + ck$. However, as wrap k is added under a wound in tension, a pressure is developed between wrap j and wrap k. As a consequence, the thickness of wrap k is reduced from ck to $ck(1 + \varepsilon_R,k)$. Similarly, the incremental interlayer pressure increases will cause all wraps under wrap j to decrease in thickness, thus radius rj will decrease to rj' and so on. The core, which is the foundation of the wind, will also decrease slightly in radius in response to increasing radial compression. Thus, all layers of the roll as well as the core will experience an incremental decrease in thickness due to the incremental increase in interlayer compression as quantified by winding models and illustrated in Figure 1.

This new boundary condition can be expressed as

Displacement $$w_k = (r_k - r_j) - \sum_{n=1}^{N} c_{k,n} \qquad (11)$$

The first term of this equation is the *measured* increase in radius, while the second term is the *expected* increase in radius due to the addition of a given number of wraps with an unstressed caliper c. The difference between the measured and expected increase in radius is due to the compression of the layers under incremental pressure. The utility of this new boundary condition is that radii can be measured with a density analyzer, and caliper can be measured with gaging sensors.

Using this new boundary condition, along with an extended winding model, allows calculation of all stresses, strain and displacement field. As seen in Figure 3, the conventional model inputs a *known* wound-in-stress boundary condition (as well as other required parameters) and outputs stress profiles. While the stress measurement system also has imbedded within it the conventional model, there are extensions on each end. On the input end, the displacement boundary condition is calculated using (10) and the radii and caliper measured with the density-like system. Another difference on the input end is that an assumed wound-in-stress is input to the conventional model. On the output end, the differential strains and then outer roll differential displacements are computed. Finally, the roll differential displacements for the assumed wound-in-stress condition is compared with that measured by the density-like system. The ratio of the assumed and measured displacements are used in a linear correction of all calculated variables. This sequence is repeated for every sample gathered by the data acquisition system.

The Density Analyzer and Extension

Web density, along with caliper and basis weight, are properties that have been monitored for quality control in the paper industry for many decades. Additionally, it was observed that *wound roll* density increased with high wound-in-tensions. This is a result of higher interlayer pressures which cause ZD compressive strains. In 1980, the first practical measurement method for roll density became available with the invention of a computerized roll density analyzer[12]. In addition to its automated ease of use, the density analyzer has been shown to be quite sensitive to winder changes of torque, nip, tension, speed, acceleration and splices[13].

The conventional density analyzer usually consists of two incremental rotary encoders, pulse counters, and a microcomputer as seen in Figure 4. The encoder connected to a drum or roller traveling at web speed yields footage and speed measurements. The ratio of the roller encoder and wound roll encoder allows calculation of current diameter of the winder and/or unwind roll. From these encoder pulse counts over a sample number of revolutions, the following can be calculated

Length

$$l_i = \frac{\pi \, d_d \, p_{d,i}}{ppr_d} \tag{12}$$

Diameter

$$d_{r,i} = d_d \, \frac{p_{d,i}}{p_{r,i}} \, \frac{ppr_r}{ppr_d} \tag{13}$$

Density

$$\rho = \frac{2 \, b \, n^2 \, ppr_d}{d_d \left(p_{d,i} - p_{d,i-1} \right)} \tag{14}$$

While the density analyzer is sensitive to changes in wound-in-tension, it has also been found to have an undesirable cross-sensitivity to caliper[14]. The problem stems from an assumption of constant basis weight in the conventional derivation, which is somewhat idealistic for real webs. However, equation (14) can be rederived to allow for basis weight variation, but expressed in terms of measured caliper as

Density, $$\rho = \frac{2\,\rho_0\, c\, n^2\, ppr_d}{d_d\,(p_{d,i} - p_{d,i-1})} \quad \text{(Caliper Compensated)} \qquad (15)$$

The next problem, is how to measure caliper online and accurately. Though many gaging systems exist, finding one suitable for a web production machine, and yet extremely accurate, is difficult. After reviewing several technologies and options, the most promising seemed to be to adapt a noncontacting gage made by Schaevitz. As seen in Figure 5, the gage is a pneumoservo valve nozzle which floats above the surface at a constant height. The position of the valve, measured by a LVDT to an accuracy better than 0.00001", is directly related to the web thickness. Though there are many complications and subtleties with this approach, the resulting improvement on density using caliper compensation is dramatic. As seen in Figure 6, in addition to the caliper measurement that in itself is very useful, increased the resolution of the density analyzer about tenfold over the conventional uncompensated density[11].

Stress Measurement System

We can now assemble the winding model with its extension to strains and displacements, and the density analyzer with its extension to caliper measurement, into a stress measurement system. The quality of the system can then be evaluated by comparing its outputs to independent measurements of stress. Figure 7 shows an example result of the comparison of the stress measurement with wound-in-stress measured by a Beloit Wound-in-Tension-Wound-off-Tension pilot winder. As seen here, the results are mixed. While the values are similar, they can differ by more than a comfortable engineering design margin. The reason for the similarity is that winding behavior can be described, modeled and solved mathematically. The reasons for the difference is due both to measurement as well as numerical accuracy. First, the precision required for the input measurements can be around 0.001" on the diameter, and 0.0001" on caliper for typical web materials. This is difficult to obtain with present technology. Second, the calculations as presently formulated can be lengthy, over several hours on a fast PC, which is ample time for numerical roundoff errors to accumulate.

Though the results of this prototype are neither overwhelmingly successful nor suitable for immediate application, it should not be surprising due to the tremendous scope and complexity of integrating this type of modeling and measurement. Yet, that the integration of differential stress modeling and diameter measurement has been achieved is an evolution in winding. Also, we should expect that the system can be improved as it is not contingent on any particular winding model, solution technique, diameter or caliper measurement. Indeed, the contribution is more one of philosophy than of detail, that is that winding models can be posed in many different ways than the conventional tension to stress approach.

In addition to the integration of modeling and measurement, many perhaps more useful tools have been developed. First, winding models have been tested and verified for consistency. Second, solution acceleration techniques were discovered which can speed calculations by up to a 100 times. Third, an option for inexpensive online caliper measurement has been developed that exceeds the accuracy of those typically used in quality control lab testing. Fourth, the caliper corrected density or alternatively a new radial compression measurement has improved wound roll structure sensitivity by 10 times over conventional density, making it the most sensitive yet available. Finally, the thesis is the most complete reference and treatment on wound roll modeling, measurement and mechanics in the public domain.

Looking Forward

While conventional models have served for stress based design criteria, their potential for a much wider range of applications is just being realized. This is a result of extending the models to strain, displacement, geometry and weight. With this larger list of variables to work with, many new relationships can be established. This work has posed and solved a geometry to stress relationship, which is essentially solving an extension of conventional models in the reverse direction.

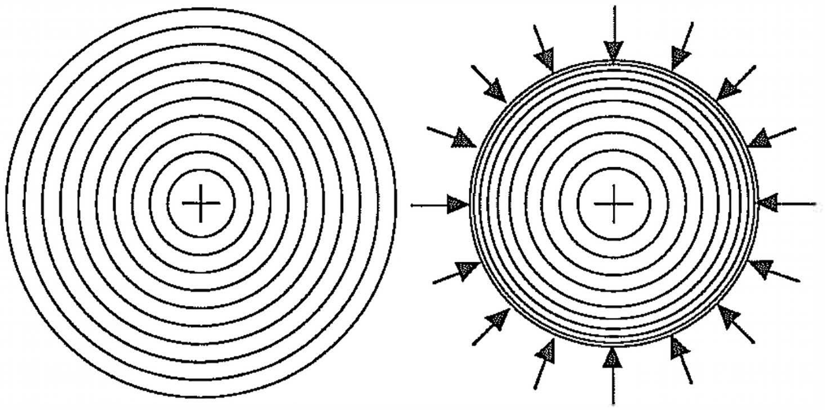
Yet, there are many other relationships to be explored. For example, winding tensions can be reverse engineered from a density or Rhometer hardness vs winding tension as seen in Figure 8. While the curve must be established by measuring hardness and tension for a given material, perhaps on a lab winder, it can later be applied to determine winding tension for any winder running that material by measuring Rhometer hardness, perhaps in a production environment. We know that this relationship between Rhometer and constant winding tension is unique and determined for any material because winding physics and material properties determine the outcome. This is just one of many arrangements of determined dependency that can be made. This broader view of wound roll design is perhaps the greatest contribution of this work.

BIBLIOGRAPHY

1. Gutterman, R. P., "Theoretical and Practical Studies of Magnetic Tape Winding Tensions and of Environmental Roll Stability," US Contract No. DA-18-119-SC-42, 1959.
2. Catlow, M. G. and Walls, G. W., "A Study of Stress Distribution in Pirns," J. of Textile Institute Part 3, pp. T410-429, 1962.
3. Roisum, D. R., "How to Measure Roll Quality," Tappi J., vol. 71, no. 10, October 1988.
4. TAPPI, "Routine Control Method RC317," (Cameron Test for Determining residual Strain in Paper), Tappi J., vol. 46, no. 12, pp. 123A, December 1963.
5. Altmann, H. C., "Formulas for Computing the Stresses in Center-Wound Rolls," Tappi J., vol. 51, no. 4, pp. 176-179, April 1968.
6. Yagoda, H. P., "Generalized Formulas for Stresses in Wound Rolls," Tappi J., vol. 64, no. 2, pp. 91-93, February 1981.
7. Hakiel, Z., "Nonlinear Model for Wound Roll Stress," Tappi J., vol. 70, no. 5, pp. 113-117, May 1987.
8. Tramposch, H., "Anisotropic Relaxation of Internal Forces in a Wound Reel of Magnetic Tape," J. of Applied Mechanics, Transactions of the ASME, vol.

34, no. 4, no. 89, pp. 888-894, 1967.

9. Lin, J. Y. and Westmann, R. A., "Viscoelastic Winding Mechanics," J. of Applied Mechanics, vol. 56, no. 12, pp. 821-827, December 1989.
10. Roisum, D. R., "Hygroscopic Diffusion in a Paper Roll - Revisited," Beloit Research internal report, D87-016, March 2, 1987.
11. Roisum, D. R., "The Measurement of Web Stresses During Roll Winding," PhD thesis, Oklahoma State University, May 1990.
12. Eriksson, L. G. et al, "Measurement of Paper Roll Density During Winding," Tappi J., vol. 66, no. 1, pp. 63-66, January 1983.
13. Odell, M. H. et al, "Control of the Winding Process by Roll Density Measurement," Appita, vol. 38, no. 5, pp. 359-369, September 1985.
14. Baum, S. A., and Roisum, D. R., "The Effect of Caliper on Wound Roll Density," to be published in the Tappi J..
15. Pfeiffer, J. D., "Nip Forces and Their Effect on Wound-in Tension," Tappi J., vol. 60, no. 2, pp. 115-117, February 1977.
16. Pfeiffer, J. D., "Wound-off Tension Measurement in Paper Rolls," Tappi J., vol. 64, no. 4, pp. 106-108, March 1977.



Before
Outer Radial Stress BC Addition

After
Outer Radial Stress BC Addition

Note: This schematic is for the thickness resulting from *incremental* radial stresses. The actual thickness profile will follow the *accumulated* radial stress distribution where the compaction will be near uniform throughout except that it tapers to zero at the outer boundary.

Figure 1. - INCREMENTAL LAYER COMPACTION

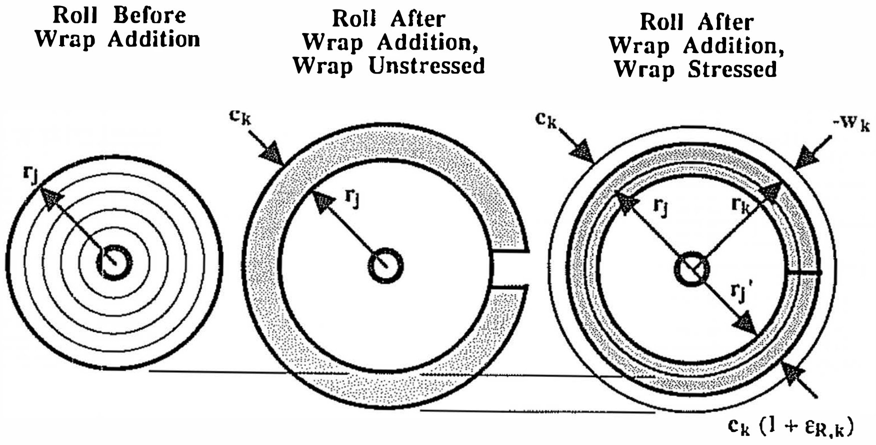


Figure 2. - RADIAL DISPLACEMENTS

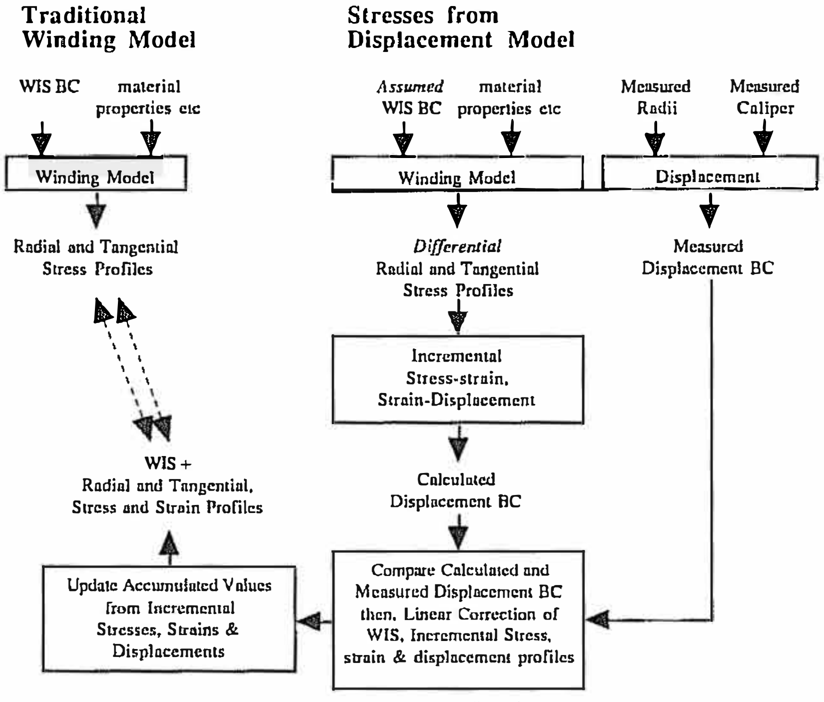


Figure 3. - MODEL COMPARISON

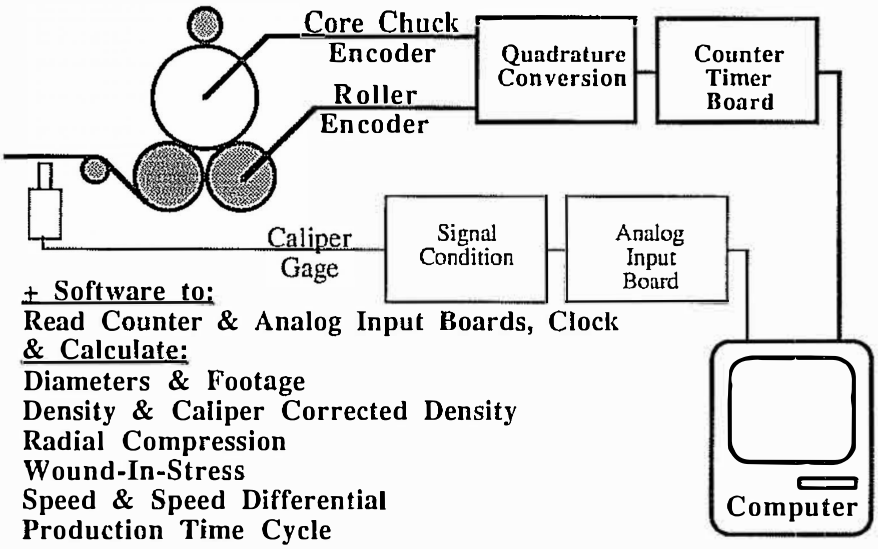


Figure 4. - DENSITY ANALYZER AND CALIPER ADDITION

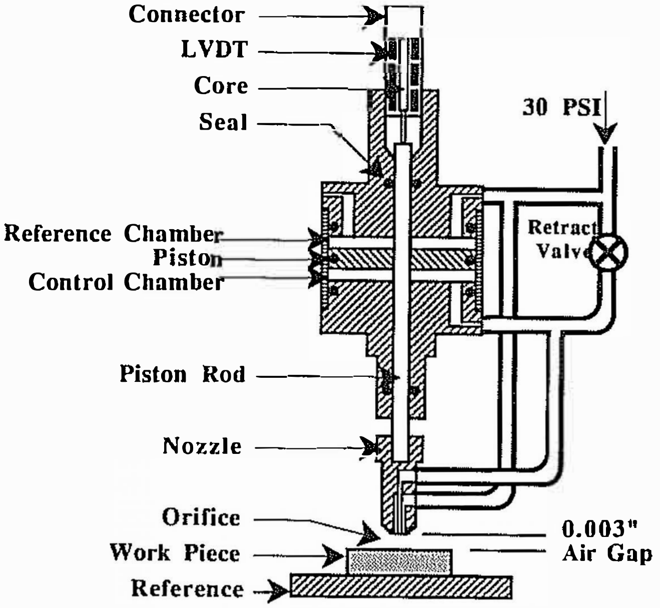


Figure 5. - NONCONTACTING LVDT CALIPER GAGE

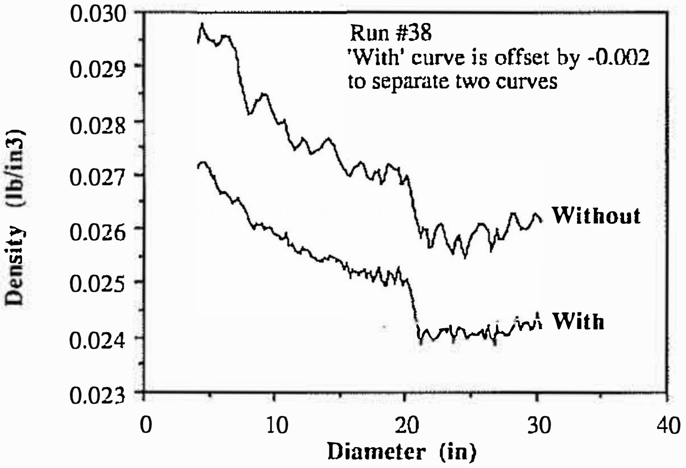


Figure 6 . - EFFECT OF CALIPER COMPENSATION ON DENSITY

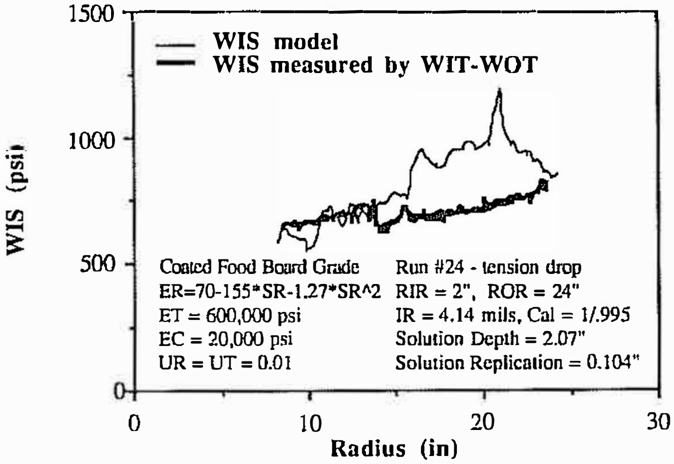


Figure 7 . - WIS FOR NIP DROP - CALCULATED VS MEASURED

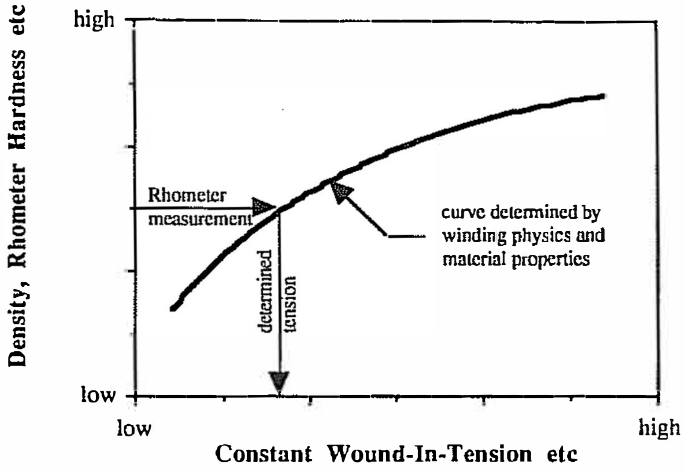


Figure 8. - WINDING DEPENDENCY

WOUND ROLL STRESSES FROM DISPLACEMENT MEASUREMENTS
D. Roisum

If the wound material has a high degree of surface roughness, is it possible to relate radial deformation of the roll to stresses and strains in the wound material?
Terry Gerhardt, Sonoco

Surface roughness poses additional difficulties in measuring and defining thickness. However, the principles remain the same.

What were roll and winding conditions for Beloit vs. New comparison? Speed? of on-line Measurement. Paper Weight/Caliper? Width?
Tim Walker, 3M

As I recall we did this on three grades, a bond paper very much similar to copy paper, newsprint and the other was lightweight coated magazine grades. In every case, we achieved essentially the same answers. In other words some measurement devices consistently ranked better than others. I should mention that there was a paper written on the resolutions of winding measurements a few years ago. I should also call your attention to the paper on caliper corrected density, which is a part of the outcome of this thesis, that will appear in the TAPPI Journal in the next few months, and I'll be speaking on that in the finishing conference.

Does this modeling technique yield information that is qualitatively different from a standard roll density monitor?
Alvin Penner, Quebec & Ontario Paper Company, Ltd.

Yes. One of the problems we have had with the density analyzer is that either changes in caliper or changes on the winder can make the same effects in uncorrected density. Once you get this graph of a density profile, you can't tell what caused it. It could be the web or it could be the winder. The only thing you know is something is different. This new measurement technique separates the effects of web and winder.