VARIABLE-GAIN CONTROL OF LONGITUDINAL TENSION IN A WEB TRANSPORT SYSTEM

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ABSTRACT

Fixed-gain and variable-gain PID control of longitudinal tension in the winding section of a simple web transport system were evaluated. An open-loop mathematical model for the web transport system was derived and used for the design of the PID controllers. The winding roll radius is a time-varying parameter in the model.

The fixed-gain PID controller designed for a particular winding roll radius did not meet the desired specifications, whereas the variable-gain PID controller compensated for the time-varying parameter and produced accurate tension control. In comparison with other controllers, the variable-gain PID controller is easy to implement and shows promise for applications where the time-varying parameters are easily measured.

NOMENCLATURE

A, A_n	Cross-sectional area of web
a_n, b_n	Constants
B_{fn}	Rotary friction constant of bearing
dm	Mass of an infinitesimal web element
dx	Length of an infinitesimal web element
E	Modulus of elasticity
e	Error = $T_{2ref} - T_2$
C_2	Motor torque/speed constant
h	Thickness of web
J_n	Polar moment of inertia of roll or roller

J_{n0}	Polar moment of inertia of roll or roller when $R_b = 1$
K_p	Proportional gain in a PID controller
K_i	Integral gain in a PID controller
K_d	Derivative gain in a PID controller
K_2	Motor constant
k_r	Constant
L	Length of web span
R_n	Radius of roll or roller
R_{n0}	Initial radius of winding roll
R_b	Build-up ratio = R_n/R_{n0}
r	Constant
s	Laplace operator
t	Time
t_{n0}	Steady-state value of web tension
t_n	Web tension = $t_{n0} + T_n$
T_n	Change in web tension from a steady-state operating value
T_{2ref}	Reference tension
u_{n0}	Steady-state value of input to a drive motor
u_n	Input to motor = $u_{n0} + U_n$
U_n	Change in input to a drive motor from a steady-state operating value
v_{n0}	Steady-state operating value of web velocity
v_n	Web velocity = $v_{n0} + V_n$
V_n	Change in web velocity from a steady-state operating value
w	Width of web
x_n	Locations along web
α_n, β_n	Constants
ε_{n0}	Steady-state operating value of web strain
ε_n	Web strain $=\varepsilon_{n0} + \epsilon_n$
ϵ_n	Change in web strain from a steady-state operating value
ω_n	Natural frequency associated with desired system characteristic equation
ζ	Damping ratio associated with desired system characteristic equation
ρ	Density of web
τ	Motor torque
Ω	Motor angular velocity

Subscripts:

o	Steady-state operating condition
n	0,1,2,3, . . .
u	Condition in unstretched web
x,y,z	Cartesian coordinates

INTRODUCTION

The web material may have to pass through several consecutive processing sections in the manufacture of an intermediate or final product. Different web tension levels and accuracies may be required in the different processing sections. If severe tension variations occur, rupture of the material during processing or degradation of product quality may occur, resulting in significant economic loss. In order to minimize the potential for loss, it is important to monitor and control the tension[1] within the desired limit in a moving web.

There is an extensive literature concerning mathematical modeling of web transport systems and web tension control (1) - (12). Campbell (1), Brandenburg (2), King (3) conducted fundamental background studies of the longitudinal dynamics of a moving web. Working early in the field, Campbell did not consider the tension in the entering span when he developed a mathematical model for the longitudinal dynamics of a web span. His model does not predict "tension transfer". In contrast, King, Brandenburg, and Shelton (7) considered the tension in the entering span when they developed mathematical models for tension in a web span. Brandenburg and Shelton assumed that the strain in the web is very small, but King did not. Campbell, King, and Shelton did not take into account "non-ideal effects" (e.g., changes in cross-sectional area, temperature, and moisture in the web; viscoelastic characteristics of the web; slippage between the web and rollers, etc.) on the tension variation in a span. Brandenburg considered the effects of area change resulting from strain change, temperature change, and register error.

A study related to tension control in a multi-span web transport system was reported by W. Wolfermann and D. Schroder (8) in 1987. In their technique, optimal output feedback was applied to control the speed of the driven rollers. A decentral observer was designed which is able to decouple the drives from the web tension acting on the driven rollers. The observer reconstructs the web tensions acting on the driven rollers and uses this information to improve the speed control of the driven rollers. This method leads to considerable improvement in the speed responses of the driven rollers. However, the reference inputs used in the control system are the desired "speeds of the driven rollers" rather than the desired "tensions" in the web spans. That is, the web tensions are still controlled in open loop by the relation of the speeds of the driven rollers (draw control). This control method cannot reject disturbances due to "tension transfer" from adjacent web spans and interaction between adjacent web spans through an intermediate driven roller.

In the "draw control" scheme, tension in a web span is controlled in an open-loop fashion by controlling the velocities of rollers at either end of the web span. Tension is very sensitive to the velocity difference between the ends of the web span. For example, a change in the velocity difference of 0.1% of the operating velocity results in a tension variation of 42 lbs (187 N) in a Polypropylene web (E=350,000 lbs/in^2 (2.4*10^9 N/m^2), h=0.001 in. (2.5*10^{-5}m), w=120 in. (3.0 m)). This means that for a nominal operating web tension of 0.5 pli (88.0 N/m), the tension variation is 70% of the operating web tension . If an encoder with 0.1% accuracy (typical) is used to sense the velocity of the roller at one end of the span,

[1] Unless stated otherwise throughout this paper, the term "tension" refers to the longitudinal tension in a web span.

accurate control of web tension cannot be achieved. On the other hand, if a load cell with a 10% accuracy or better is available to measure tension, feedback control of web tension can result in greatly improved accuracy.

In this paper, an idealized mathematical model for a single-span web transport system is derived. This open-loop dynamic model is used as the basis for the design of closed-loop systems for the control of longitudinal tension in the web. Two specific examples are presented to illustrate the use of closed-loop control in a web transport system with a time-varying parameter.

A fixed-gain and a variable-gain PID controller were designed to control tension in a winding section that has a winding roll with a time-varying radius. The dynamic performance of the system with a fixed-gain PID controller and with a variable-gain PID controller was determined through computer simulation using the idealized dynamic model. Only the variable-gain PID controller produced accurate tension control for all values of the winding roll radius.

DERIVATION OF THE MATHEMATICAL MODEL

To facilitate the modeling and analysis of web transport systems, the concept of a "primitive element" was established (13). Examples of primitive elements are a free web span, a roller, a roll, a web interacting with a free roller, a web interacting with a fixed roller, etc.. Dynamic models may be derived for these primitive elements using a first principles approach involving the law of conservation of mass, Hooke's law, and Newton's law of motion. A mathematical model for a single-span system can be obtained using these models.

Tension- Web Velocity Relationship

Consider the single-span system shown in Figure 1. The assumptions listed below facilitate the derivation of an "idealized" mathematical model for the single-span system.

1. The length of contact region between the web material and the rollers is negligible compared to the length of free web span between the rollers (i.e., the tension variations in the contact region are negligible).
2. The thickness of the web is very small compared to the radius of the rollers.
3. There is no slippage between the web material and the rollers.
4. There is no change in the temperature or humidity within the web span.
5. There is no change in the density and modulus of elasticity within the web span.
6. There is no change in the cross-sectional area of the web span.
7. The strain in the web span is very small (strain $<<$ 1).
8. The strain is uniform within the web span.
9. The web is perfectly elastic.
10. The machine direction stress prevails.
11. There is no change in the moments of inertia of the rollers.
12. The dynamics of the tension sensors are considered to be negligible compared to the dynamics of the system.

The law of conservation of mass for the control volume (see Figure 1) can be written as:

$$\frac{d}{dt}\left[\int_{x_1}^{x_2} \rho_2(x,t)A_2(x,t)dx\right] = \rho_1(t)A_1(t)v_1(t) - \rho_2(t)A_2(t)v_2(t) \tag{1}$$

Consider the infinitesimal web element shown in Figure 2. The stretched and unstretched states of the web are related by the following equations:

$$dx = (1 + \varepsilon_x)\, dx_u \tag{2}$$
$$w = (1 + \varepsilon_y)\, w_u \tag{3}$$
$$h = (1 + \varepsilon_z)\, h_u \tag{4}$$
$$dm = \rho\ w\ h\ dx = \rho_u\ w_u\ h_u\ dx_u \tag{5}$$

The subscript u indicates the unstretched state of the web.

Combining equations (1) through (5) gives the following nonlinear strain-web velocity relationship:

$$L\frac{d}{dt}[\varepsilon_2(t)] = - v_2(t)\varepsilon_2(t) + v_1(t)\varepsilon_1(t) + v_2(t) - v_1(t) \tag{6}$$

Equation (6) can be linearized for the case when all variables undergo small perturbations from an initial steady-state value. Let $\varepsilon = \varepsilon_0 + \epsilon$ and $v = v_0 + V$ in equation (6), where ϵ and V are small perturbations from initial steady-state operating values; the subscript 0 denotes an initial steady-state operating value. With these definitions, the linearized strain-velocity relation is:

$$\frac{d}{dt}[\epsilon_2(t)] = - \frac{v_{20}}{L}\epsilon_2(t) + \frac{v_{10}}{L}\epsilon_1(t) + \frac{V_2(t)}{L} - \frac{V_1(t)}{L} \tag{7}$$

Under assumptions (4) through (10), Hooke's law can be written as:

$$T_1 = AE\epsilon_1,\ T_2 = AE\epsilon_2 \tag{8}$$

Combining equations (7) and (8) gives the tension-velocity relationship:

$$\frac{d}{dt}[T_2(t)] = - \frac{v_{20}}{L}T_2(t) + \frac{v_{10}}{L}T_1(t) + \frac{AE}{L}(V_2(t) - V_1(t)) \tag{9}$$

Tension-Roller Velocity Relationship

It is assumed that the roller at each end of a free span is driven by a motor. A motor characteristic linearized around an operating point is shown in Figure 3. Under assumptions (3) and (11) (i.e., the roller tangential velocity equals to the web velocity at contact), the following linearized equation can be derived from the torque balance on the roller at position 2 in Figure 1:

$$\frac{dV_2}{dt} = -\frac{B_{f2}+C_2}{J_2}V_2 + \frac{R_2^2}{J_2}(T_3 - T_2) + \frac{R_2}{J_2}K_2U_2 \tag{10}$$

CLOSED-LOOP CONTROL OF LONGITUDINAL TENSION

Consider the single-span system with feedback control of tension as shown in Figure 4. The PID controller, commonly used in industry, will be used here.

PID Controller

The PID controller is described by the following equation:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \tag{11}$$

The linearized mathematical model given by equations (9) and (10) can be used for the single-span system. Figure 5 shows a block diagram of the closed-loop system. The closed-loop transfer function can be written as follows:

$$T_2 = \frac{G_cG_p}{1+G_cG_p}T_{2ref} + \frac{G_1}{1+G_cG_p}T_3 + \frac{G_2}{1+G_cG_p}T_1 - \frac{G_3}{1+G_cG_p}V_1 \tag{12}$$

For the special case when $T_1 = V_1 = T_3 = 0$, the closed-loop transfer function reduces to:

$$\frac{T_2}{T_{2ref}} = \frac{G_cG_p}{1+G_cG_p} \tag{13}$$

or

$$\frac{T_2}{T_{2ref}} = \frac{b_2s^2 + b_1s + b_0}{s^3 + a_2s^2 + a_1s + a_0} = \frac{b_2s^2 + b_1s + b_0}{F(s)} \tag{14}$$

where

$a_0 = \beta_0 K_i$, $\quad b_0 = \beta_0 K_i$,

$a_1 = \alpha_0 + \beta_0 K_p$, $\quad b_1 = \beta_0 K_p$,

$a_2 = \alpha_1 + \beta_0 K_d$, $\quad b_2 = \beta_0 K_d$,

and

$$\alpha_0 = \frac{v_{20}(B_{f2}+C_2)}{LJ_2} + \frac{AER_2^2}{LJ_2}, \quad \alpha_1 = \frac{v_{20}}{L} + \frac{B_{f2}+C_2}{J_2}$$

$$\beta_0 = K_2\frac{AER_2}{LJ_2}$$

The characteristic equation of the closed-loop tension control system is third order (see equation (14)). The third-order characteristic equation of the desired closed-loop tension control system can be rewritten as follows (14):

$$F(s) = (s + r)(s^2 + 2\zeta\omega_n + \omega_n^2) = 0 \tag{15}$$

where

$$r = k_r |\zeta\omega_n|$$

If $k_r > 10$, the third order system behavior is similar to that of a second order system with natural frequency ω_n and damping ratio ζ. The gains of the PID controller can be obtained as follows by comparing terms in equations (14) and (15). That is:

$$K_i = \frac{1}{\beta_0} k_r \zeta \omega_n^3 \tag{16}$$

$$K_p = \frac{1}{\beta_0}\left(\frac{2\zeta\omega_n\beta_0 K_i}{\omega_n^2} + \omega_n^2 - \alpha_0\right) \tag{17}$$

$$K_d = \frac{1}{\beta_0}\left(\frac{\beta_0 K_i}{\omega_n^2} + 2\zeta\omega_n - \alpha_1\right) \tag{18}$$

Using equations (16) through (18), a PID controller can be designed for a given set of open-loop system parameter values and to meet the desired specifications of the closed-loop tension control system.

DESIGN OF A CLOSED-LOOP CONTROL SYSTEM FOR A WINDING SECTION

Consider the winding section with feedback control of tension as shown in Figure 6. Two controller types will be evaluated: (1) a fixed-gain PID controller, and (2) a variable-gain PID controller.

The radius of the winding roll, R_2, and the inertia of winding roll, J_2, in equation (14) are time-varying. The moment of inertia of the winding roll can be expressed as a function of the radius of the winding roll if the density of the roll is known. A "build-up ratio", R_b, can be defined as:

$$R_b = \frac{R_2}{R_{20}} \tag{19}$$

The system parameters α_0, α_1, and β_0 in equations (16) through (18) can be rewritten as functions of the build-up ratio, R_b, as follows:

$$\alpha_0 = \frac{v_{20}(B_{f2}+C_2)}{LJ_{20}} \frac{1}{R_b^4} + \frac{AER_{20}^2}{LJ_{20}} \frac{1}{R_b^2} \tag{20}$$

$$\alpha_1 = \frac{v_{20}}{L} + \frac{B_{f2}+C_2}{J_{20}} \frac{1}{R_b^4} \tag{21}$$

$$\beta_0 = K_2 \frac{AER_{20}}{LJ_{20}} \frac{1}{R_b^3} \tag{22}$$

It is assumed that the winding roll has a uniform density.

Fixed-Gain PID Control

A set of fixed-gains for the PID controller can be obtained at a particular instant of time or for a particular build-up ratio by using equations (16) through (22). Since the build-up ratio changes as the parameters (radius and inertia of the roll) in the winding section change, a fixed-gain PID controller designed for a particular build-up ratio cannot meet the desired specifications (see example later).

Variable-Gain PID Control

A simple variable-gain control technique is proposed to overcome the deficiency of the fixed-gain PID controller. The concept of variable-gain control is to continuously update the gains of the PID controller as the parameters change. A schematic diagram of a variable-gain tension control system is shown in Figure 7. The radius R_2 (and build-up ratio, R_b) is time-varying. The gains in the controller are functions of the build-up ratio, R_b. The resulting controller gains can be determined using the algorithm contained in equations (16) through (22), where α_0, α_1, and β_0 are functions of the build-up ratio and therefore functions of time.

The locations of the poles of the closed-loop transfer function change with time. The variable-gain control algorithm continuously updates and places these closed-loop poles at locations in the left half s-plane such that the design specifications are met for all values of R_b.

EXAMPLES OF FIXED-GAIN AND VARIABLE-GAIN PID CONTROL

For illustrative purposes, it is desired to design a closed-loop control system which satisfies the following specifications:

Steady-State Error	$e_{ss} = 0$ lbf
Damping Ratio of Closed-Loop System (equation (15))	$\zeta = 0.7$
Natural Frequency of Closed-Loop System (equation (15))	$\omega_n = 10$ rad/sec
Real root (equation (15))	$r = 13\lvert\zeta\omega_n\rvert$

Fixed-Gain PID Control

An example is solved to illustrate the dynamic performance of a winding section with fixed-gain PID control of longitudinal tension (see Figure 6). It is desired to obtain the response of the tension, T_2, to a step change in the reference tension, T_{2ref}. Parameter values and conditions of the system for simulation are shown in the Appendix. The controller was designed for $R_b = 1.0$. Simulation results are shown in Figure 8 for different values of the build-up ratio. Equation (15) was used to obtain the "desired response". The desired response and the response for $R_b = 1.0$ are identical. Since the build-up ratio changes as the parameters (radius and inertia of the roll) in the winding section change, the fixed-gain PID controller designed for $R_b = 1.0$ cannot meet the desired specifications for tension control in the winding section for all values of R_b.

Variable-Gain PID Control

An example is solved to illustrate the dynamic performance of a winding section with variable-gain PID control of longitudinal tension (see Figure 7). Parameter values and conditions of the system for simulation are shown in the Appendix. The step responses of the tension control system with a variable-gain PID controller are compared with the step responses for the tension control system with a fixed-gain PID controller in Figure 9. The system with a variable-gain controller satisfies the desired specifications for all values of R_b; the system with a fixed-gain controller does not satisfy the desired specifications except for $R_b = 1.0$.

CONCLUSIONS

The advantage of a variable-gain PID controller compared to a fixed-gain PID controller is that it compensates for time-varying parameters. The advantage of variable-gain PID control compared to other "adaptive control" techniques is its simplicity. The variable-gain PID control is easy to implement and shows promise for applications where the time-varying parameters are easily measured. A weakness of this approach is that it may not be robust against measurement noise. An error in measurement directly affects the gains of the controller.

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APPENDIX

System Conditions and Parameter Values for Simulation

System Conditions:

$T_n(0)$	= 0.0 (lbf), n = 1, 2
$V_n(0)$	= 0.0 (ft/sec), n = 1, 2

Parameter Values:

A	= 0.12 (in^2)
E	= 350,000 (lbf/in^2)
J_{20}	= 94.0 ($lbf\text{-}in\text{-}sec^2$)
J_2	= 94.0 R_b^4 ($lbf\text{-}in\text{-}sec^2$)
L	= 120 (in)
R_b^4	= 1 to 1.75
v_{n0}	= 1,000 (ft/min), n = 1, 2

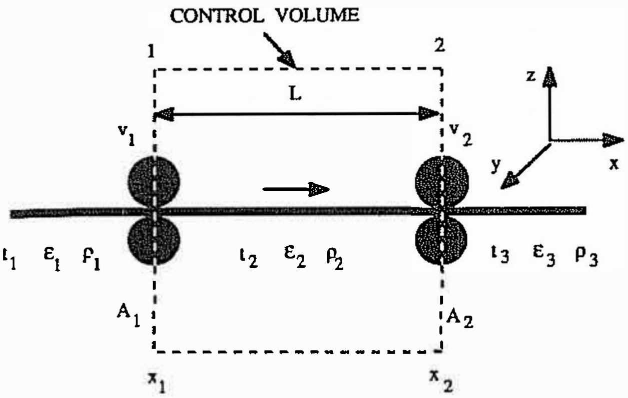


Figure 1. Single-Span System.

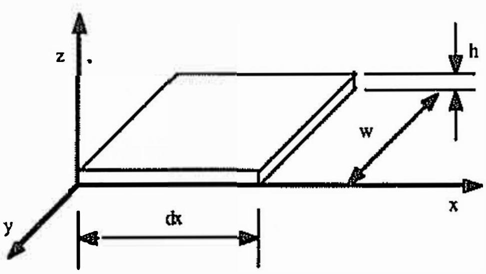


Figure 2. Infinitesimal Mass Element from a Web Span.

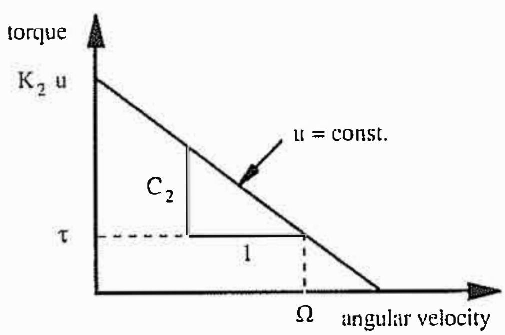


Figure 3. Motor Characteristic Linearized around an Operating Point.

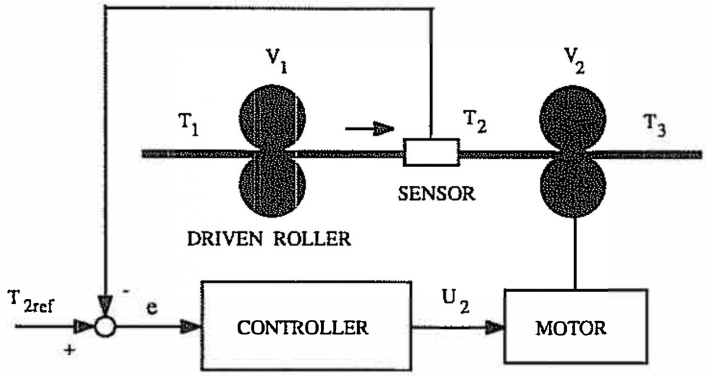
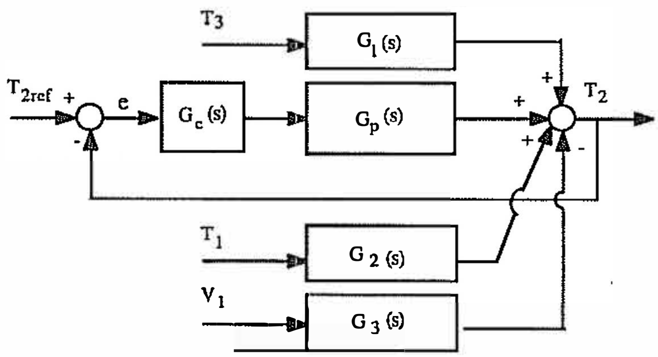


Figure 4. Single-Span System with Closed-Loop Tension Control.



$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \text{ (PID controller)}, \quad G_P(s) = \frac{1}{\Delta} K_2 \frac{AER_2}{LJ_2},$$

$$G_1(s) = \frac{1}{\Delta}\frac{AER_2^2}{LJ_2}, \quad G_2(s) = \frac{1}{\Delta}\frac{v_{20}}{L}\left(s + \frac{B_{f2}+C_2}{J_2}\right), \quad G_3(s) = \frac{1}{\Delta}\frac{AE}{L}\left(s + \frac{B_{f2}+C_2}{J_2}\right)),$$

$$\Delta = s^2 + \left(\frac{v_{20}}{L} + \frac{B_{f2}+C_2}{J_2}\right) s + \frac{v_{20}(B_{f2}+C_2) + AER_2^2}{LJ_2}$$

Figure 5. Block Diagram for a Single-Span System with Closed-Loop Tension Control.

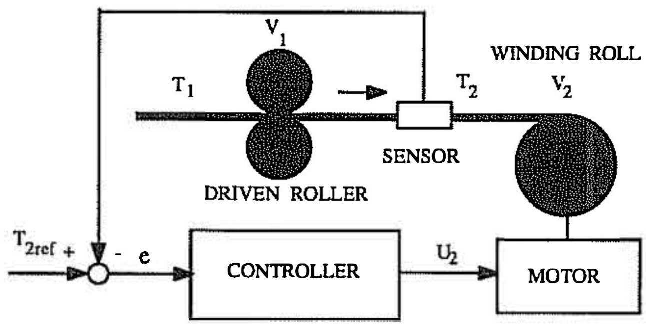


Figure 6. Winding Section with Closed-Loop Tension Control.

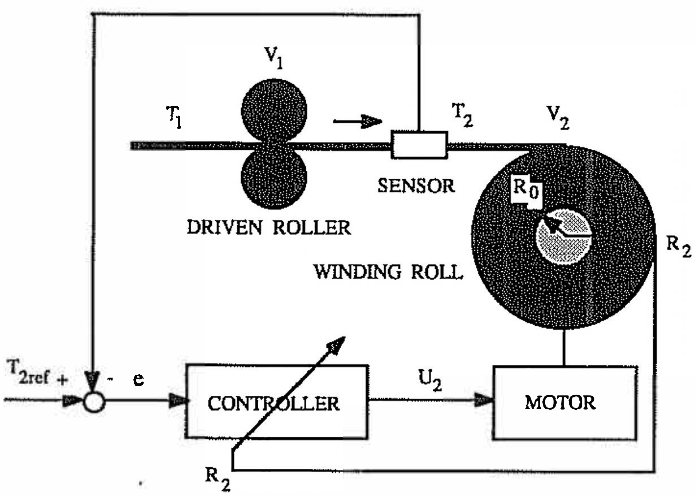


Figure 7. Winding Section with Closed-Loop Tension Control Using a Variable-Gain Controller.

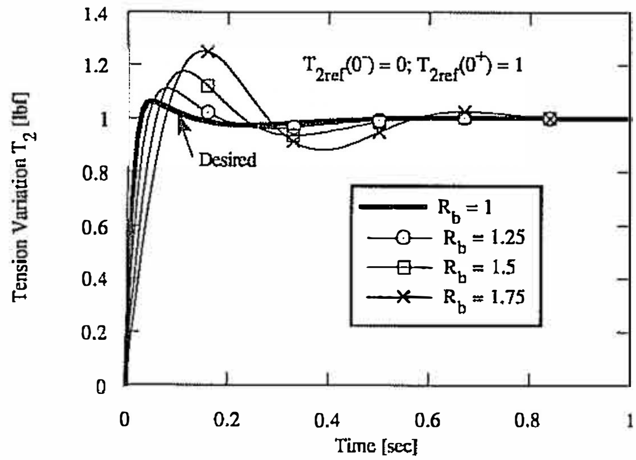


Figure 8. Step Responses for System with Fixed-Gain PID Controller.

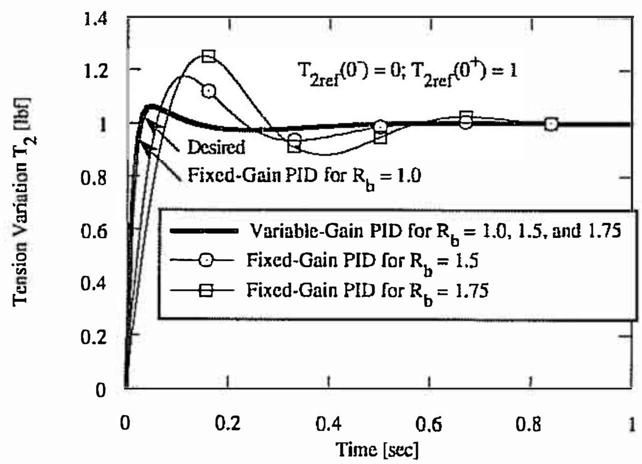


Figure 9. Step Responses for System with Fixed-Gain PID and Variable-Gain PID Controllers.