UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

BARYOGENESIS SCENARIOS FOR NATURAL SUPERSYMMETRIC PARTICLE PHYSICS MODELS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

DOCTOR OF PHILOSOPHY

By

Yifan Zhang
Norman, Oklahoma
2019

BARYOGENESIS SCENARIOS FOR NATURAL SUPERSYMMETRIC PARTICLE PHYSICS MODELS

A DISSERTATION APPROVED FOR THE
HOMER L. DODGE DEPARTMENT OF PHYSICS AND ASTRONOMY

BY THE COMMITTEE CONSISTING OF

Dr. Howard Baer, Chair

Dr. Shihshu Walter Wei

Dr. Xinyu Dai

Dr. Kieran Mullen

Dr. Kuver Sinha

Acknowledgements

I wish to express my gratitude to my advisor, Dr. Howard Baer, for being an excellent mentor, for encouraging to explore and work in physics, for his guidance, unconditional support and patience during all these years. His timely, scholarly advice has helped me to a very great extent to accomplish this work. I am grateful for having had the opportunity to study with him.

This dissertation would not have been possible without the support of many people. I gratefully thank my collaborators Kyu Jung Bae and Hasan Serce for their hard work. I would like to express my sincere appreciation to my committee members who patiently read this work. I deeply thank all my professors, especially Dr. Chung Kao and Dr. Shihshu Walter Wei, for teaching me the fundamentals of physics and math which served as a good basis for the present work. Special thanks to my family for their understanding and support throughout all these years. Finally, it is a pleasure to thank all my friends who supported me during all my studies.

Contents

Abstract

Supersymmetric models with radiatively-driven electroweak naturalness require a light higgsino of mass $\sim 100 - 300$ GeV. Naturalness in the QCD sector is invoked via the Peccei-Quinn (PQ) axion leading to mixed axion-higgsino dark matter. The SUSY DFSZ axion model provides a solution to the SUSY μ problem and the Little Hierarchy $\mu \ll m_{3/2}$ may emerge as a consequence of a mismatch between PQ and hidden sector mass scales. The traditional gravitino problem is now augmented by the axino and saxion problems, since these latter particles can also contribute to overproduction of WIMPs or dark radiation, or violation of BBN constraints. We compute regions of the T_R vs. $m_{3/2}$ plane allowed by BBN, dark matter and dark radiation constraints for various PQ scale choices f_a. These regions are compared to the values needed for thermal leptogenesis, non-thermal leptogenesis, oscillating sneutrino leptogenesis and Affleck-Dine leptogenesis. The latter three are allowed in wide regions of parameter space for PQ scale $f_a \sim 10^{10} - 10^{12}$ GeV which is also favored by naturalness: $f_a \sim \sqrt{\mu M_P/\lambda_\mu} \sim 10^{10} - 10^{12}$ GeV. These f_a values correspond to axion masses somewhat above the projected ADMX search regions. AD baryogenesis can generate appropriate baryon asymmetry and dark matter, while AD baryogenesis without R-parity in SUSY DFSZ model can also avoid the problem of overproduction of dark matter.

Chapter 1

Introduction

We live in a world that is made of matter, i.e., baryons and leptons. Astrophysical observations show us there are no large regions of anti-matter in our universe. Assuming there were initial net baryon and lepton number, inflation in the early universe will dilute away all existing baryon and lepton asymmetries. So the baryon and lepton asymmetries have to be generated after inflation. The Standard Model (SM) tells us that matter and anti-matter are produced in pairs, and no baryon asymmetry can be generated perturbatively within the SM. The electroweak baryogenesis (EWBG) within the SM required a Higgs mass $m_H < 50$ GeV, but now that $m_H \simeq 125$ GeV is discovered, SM EWBG is excluded by experimental data. Thus, the origin of baryon asymmetry is one of biggest mysteries in particle physics and cosmology, and seems to require physics beyond the SM.

There are more problems that the SM cannot solve. First of all, the Higgs mass is unstable under quantum corrections, so we would expect its mass to be far above 125 GeV. Also, observational data shows that 85% of the matter in our universe is dark matter which is massive and doesn't interact with the electromagnetic field, but the SM does not include any such candidate for dark matter. The best way to solve those problems is the supersymmetric extended SM (SUSY). SUSY implies that for every particle in the SM there is a corresponding superpartner of opposite (i.e., fermionic or bosonic) symmetry. The lightest SUSY particle (LSP) in the minimal SUSY (MSSM) is naturally a good dark matter candidate.

The strong CP problem is a problem that neither the SM nor the SUSY can solve. In strong interactions, Charge-Parity (CP) symmetry violation could occur, but it is never observed in experiment. This is known as the strong CP problem. Peccei-Quinn (PQ) theory is the most popular solution to it. PQ theory proposed a global $U(1)_{PQ}$ symmetry that is spontaneously broken, giving us a new particle: the axion. The axion is super weakly interacting with ordinary matter, and is also a good candidate for dark matter. The axion term in the Lagrangian can dynamically cancel the CP-violating term.

There is also a μ problem: the superpotential μ-parameter should be at the weak energy scale since it gives mass to Higgs, but it is also expected at the Planck scale since it is supersymmetric. PQ-charged Higgs fields can also solve the SUSY μ problem, so the higgsino could be the LSP which is also a good candidate for dark matter. Then we expect axion-higgsino mixed dark matter. We will use the SUSY version of the PQ axion models. The lack of evidence for SUSY at the Large Hadron Collider (LHC) has pushed the SUSY particle mass limits higher. But still the energy of the accelerator is not high enough to cover all possible SUSY energy scales. The big bang of the universe is naturally at much higher energy scales, so we can build cosmological models, compare the results with observational data, and then test our cosmological models as well as particle physics models.

We assume that in the early universe there was an inflationary stage, in which the universe expanded exponentially and became very cold. The inflaton is the field that causes the inflation. After the inflation ended, the universe expansion slowed down, and the universe was reheated by inflaton decay which reheated the universe to a high temperature. This reheat temperature, T_R, is an critical parameter for most baryogenesis models. The gravitino, which is the superpartner of graviton, can be thermally produced in the early universe at a rate proportional to T_R. If the reheat temperature is too high, then the gravitino problem occurs: too many gravitinos can lead to too much dark matter or violate Big Bang Nucleosynthesis (BBN) bounds via their late decays. In the case of natural SUSY (natural means all contribution to the weak scale are comparable to or less than the weak scale) with mixed axion-higgsino dark matter, then similar constraints arise from axino and saxion production: weakly interacting massive particles (WIMP) or axions can be overproduced, or light element abundances can be destroyed by late decaying axinos and saxions.

The sphaleron process is a mechanism that can convert lepton asymmetry to baryon asymmetry. We can consider baryogenesis via leptogenesis. First, the simplest leptogenesis involves thermally produced right-handed neutrinos followed by asymmetric neutrino decay to leptons versus anti-leptons. This is called thermal leptogenesis (THL). The production of the heavy right-handed neutrino requires a high reheating temperature, so the THL is constrained. Secondly, we can also have non-thermally produced right-handed neutrino through inflaton decay, called non-thermal leptogenesis (NTHL). The produced lepton asymmetry is inflation model dependent, i.e., it depends on the inflaton mass and reheat temperature. But not all the inflation models require a high reheat temperature, so the gravitino problem can be avoided in this case. Thirdly, the right-handed sneutrino, which is the scalar superpartner of right-handed neutrino, can be mainly produced via coherent oscillations (OSL). The sneutrino decay temperature can be considered as the reheat temperature. The fourth leptogenesis mechanism is via an Affleck-Dine condensate, called Affleck-Dine leptogenesis (ADL). The scalar AD field carries lepton number. It generates net lepton asymmetry through the coherent oscillation along a flat direction which violates lepton number. The lepton asymmetry is almost independent of the reheat temperature, but it is sensitive to the mass of the lightest neutrino. We consider Affleck-Dine baryogenesis with and without R-parity. The SUSY R-parity violation can explain the smallness of neutrino mass without introducing new particles, and can avoid the gravitino problem. The smallness of R-parity violation is explained by the couplings between the PQ fields and the baryon number violating terms, so that it is consistent with the unobservable proton decay.

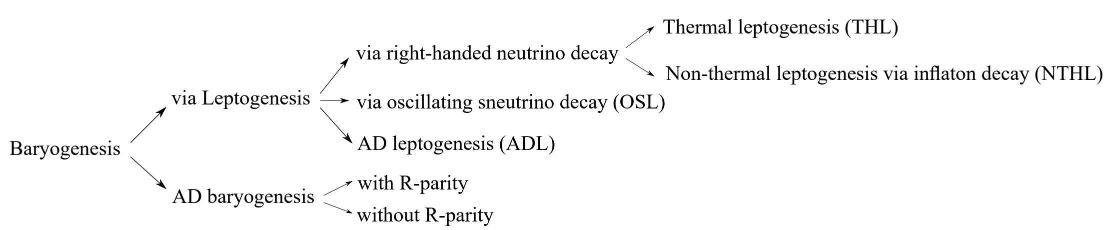

Figure 1.1: Logic structure of attempts to solve baryogenesis.

The aim of this thesis is to re-examine the origin of the baryon asymmetry in different baryogenesis scenarios, and assess their plausibility in the context of natural SUSY with mixed axion-higgsino dark matter. In this thesis, first, we review the SM, SUSY, axion models, SUSY naturalness and the standard model of cosmology. Then we investigate more details of the different baryogenesis scenarios. We calculate the mixed axion-WIMP dark matter abundance, and the constraints on leptogenesis in the T_R vs. $m_{3/2}$ planes assuming a natural SUSY spectrum, as well as corresponding results in the T_R vs. f_a planes. We vary the PQ scale f_a from values favored by naturalness ($f_a \sim \sqrt{\mu M_P}$ where naturally μ should be at weak energy scale) $f_a \sim 10^{10} - 10^{12}$ GeV to much higher values. While the thermal leptogenesis mechanism is quite constrained depending on $m_{3/2}$ and T_R, the latter three mechanisms appear plausible over a wide range of T_R, $m_{3/2}$ and f_a values which are consistent with naturalness. We also investigate the AD baryogenesis with and without R-parity, and calculate the baryon asymmetry without R-parity in SUSY DFSZ model, and show the baryon asymmetry in the $m_{3/2}$ vs. T_R plane.

Chapter 2

The Standard Model and the Supersymmetry

2.1 The Standard Model (SM)

The Standard Model has been a highly successful framework for describing particle physics. It is a non-Abelian gauge theory that describes strong, weak and electromagnetic interactions of elementary particles. In the SM, all the forces are mediated by exchange of the gauge fields of the corresponding local symmetry group. The symmetry group of the SM is

$$SU(3)_C \times SU(2)_L \times U(1)_Y \tag{2.1}$$

with the electroweak symmetry $SU(2)_L \times U(1)_Y$ spontaneously broken by the non-zero vacuum expectation value (VEV) of the complex scalar Higgs field $< H > \neq 0$ down to $U(1)_{EM}$. $SU(3)_C$ is unbroken.

The electroweak theory is based on the $SU(2) \times U(1)$ gauge symmetry. The

Lagrangian is

$$\mathcal{L}_{\text{EW}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \tag{2.2}$$

The gauge part is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^i_{\mu\nu} W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{2.3}$$

where W^i_μ, $i = 1, 2, 3$ is the $SU(2)$ gauge field, and B_μ is the $U(1)$ gauge field with field strength tensors

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2.4}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu. \tag{2.5}$$

B field is associated with the weak hypercharge $\frac{1}{2}Y = Q_{\text{em}} - T^3$, where Q_{em} is the electric charge operator and T^3 is the third component of weak isospin. The B and W_3 fields together mix to form the photon and Z boson.

The matter term is

$$\begin{aligned} \mathcal{L}_{\text{matter}} = \sum_{\text{generations}} & (\bar{Q} i \not{D} Q + \bar{L} i \not{D} L + \bar{u}_R i \not{D} u_R \\ & + \bar{d}_R i \not{D} d_R + \bar{e}_R i \not{D} e_R). \end{aligned} \tag{2.6}$$

At this stage, EW gauge bosons and fermions are massless.

The Higgs part is

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi) \tag{2.7}$$

where the Higgs scalar field $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is SU(2) complex doublet. The gauge covariant derivative is

$$D_\mu \phi = (\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + \frac{ig'}{2}B_\mu)\phi. \tag{2.8}$$

The Higgs potential $V(\phi)$ is

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2. \tag{2.9}$$

The mass parameter $-\mu^2 < 0$ leads to spontaneous electroweak symmetry breaking (EWSB). The quartic coupling λ is the Higgs scalar self-coupling parameter. $\lambda > 0$ so that there is a minimum in the potential.

The $\mathcal{L}_{\text{Yukawa}}$ term is

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{\text{generations}} [\lambda_u \epsilon^{ab} \bar{Q}_a \phi^\dagger_b u_R + \lambda_d \bar{Q} \phi d_R + \lambda_e \bar{L} \phi e_R] + h.c. \tag{2.10}$$

where the matrices λ contain the Yukawa couplings between the Higgs scalar ϕ and the fermions.

The strong interaction theory is called quantum chromodynamics (QCD). It is based on $SU(3)$. The Lagrangian is

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{i=\text{flavors}} \bar{q}_i (i\not{D} - m_i) q_i \tag{2.11}$$

where

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu \tag{2.12}$$

is the field strength tensor of the gluon fields G^a_μ. $f^{abc}(a, b, c = 1, \cdots, 8)$ are the structure constants, g_s is the QCD gauge coupling constant, and

$$D_\mu = \partial_\mu + i g_s \frac{\lambda^a}{2} G^a_\mu \tag{2.13}$$

and q_i contains a color triplet of quarks of flavor i.

The SM fields are:

1. Gauge bosons: The gauge bosons have *spin* $= 1$, so they are vector particles They have the representation of group $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\begin{array}{rllll} \text{gluons} & G_\mu: & (8,1,0) & SU_C(3) & g_s \text{ or } g_3 \\ \text{weak bosons} & W_\mu: & (1,3,0) & SU_L(2) & g \text{ or } g_2 \\ \text{abelian boson} & B_\mu: & (1,1,0) & U_Y(1) & g' \text{ or } g_1 = \sqrt{\frac{5}{3}} g' \end{array} \tag{2.14}$$

where g_3, g_2 and g_1 are the coupling constants.

2. Fermions: The fermions have *spin* $= 1/2$; they are matter fields. The fermions are left-right asymmetric: i.e., the SM is a chiral theory - it distinguishes handedness. The fermions come in three generations $i = 1, 2, 3$. For quarks

$$Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix}_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad (3, 2, 1/3) \tag{2.15}$$

and

$$\begin{aligned} U_{iR} &= u_R,\ c_R,\ t_R, \quad (3, 1, 4/3) \\ D_{iR} &= d_R,\ s_R,\ b_R, \quad (3, 1, -2/3). \end{aligned} \tag{2.16}$$

For leptons

$$L_i = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad (1, 2, -1) \tag{2.17}$$

and

$$E_{iR} = e_R, \ \mu_R, \ \tau_R, \quad (1, 1, -2) \tag{2.18}$$

where there are 3 colors for each quark.

3. Higgs Boson: The Higgs boson has $spin = 0$, so it is a scalar boson, with weak hypercharge $Y = 1$. The Higgs field is a 4-component complex doublet field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (1, 2, 1) \tag{2.19}$$

which is introduced to give masses to other particles via spontaneous EWSB.

A non-zero VEV of the Higgs field ϕ^0 will spontaneously break the symmetry of $SU_L(2)$ and $U(1)_Y$ groups, so that all the particles can obtain their masses. Let

$$<\phi>= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = |\mu|/\sqrt{\lambda}. \tag{2.20}$$

Then

$$v = 2m_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV} \tag{2.21}$$

where G_F is the Fermi coupling parameter determined by muon decay measurements. Three components of ϕ are eaten by $W^\pm$ and Z so these fields gain mass.

This leaves one neutral physical Higgs boson H. The physical Higgs mass is

$$m_H = \sqrt{2\mu^2}. \tag{2.22}$$

This Higgs boson was discovered by LHC in 2012. The measured value of Higgs mass is $m_H \simeq 125.09 \pm 0.21$ GeV, so $\lambda \simeq 0.13$ and $|\mu| \simeq 88.8$ GeV [1]. As a result,

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{EWSB} SU(3)_C \times U(1)_{EM}.$$

2.2 Motivation for the Supersymmetry (SUSY)

The Standard Model has been successful in describing the elementary particles and the fundamental interactions. It explains why baryon number and lepton number are conserved. But it has some problems. For example, the SM

- has the hierarchy problem;
- cannot explain the baryon asymmetry in the universe;
- cannot explain the accelerating expansion of the universe which apparently requires dark energy;
- cannot explain the existence of dark matter;
- does not include gravitation;
- does not explain CP conservation in the strong interactions.

SUSY has the potential to provide the explanations for some of the phenomenon which SM can not explain while improving other explanations. For example, the hierarchy problem, nonconvergent of the running coupling constants, and candidates of dark matter.

In the SM, the Higgs squared mass parameter receives large radiative corrections from particles, especially the top quark, that coupled to the Higgs field. Fig. 2.1 shows an example of one-loop corrections to m_H.

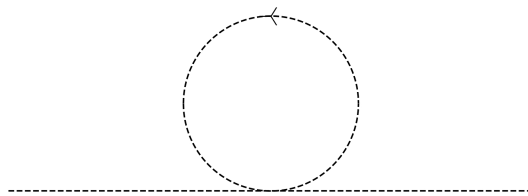

Figure 2.1: An example of quadratic divergent Feynman diagram of the corrections to the Higgs boson mass in the SM.

The one-loop radiative corrections to the Higgs boson mass in the SM can be expressed in terms of the Lorentz invariant cut-off Λ. For example, the correction of Higgs self-interaction

$$\delta m_H^2 =< H|\frac{g^2 m_H^2}{32 M_W^2} H^4 |H > \simeq \frac{c}{16\pi^2}\Lambda^2 \tag{2.23}$$

where M_W is the mass of gauge boson, c depends on coupling constants of the SM. The correction is quadratically divergent. If Λ is at 10^{16} GeV (GUT scale) or 10^{19} GeV (Planck scale), it is unnatural for m_H to be of order of m_{EW}. This is the Hierarchy Problem. The bare mass has to be fine-tuned to cancel the very large loop corrections. These quadratic divergences can be removed with SUSY, which introduces supersymmetric partners to the SM particles.

SUSY suggested that for every boson (fermion), there is a fermion (boson) supersymmetric partner for all SM particles. We add an “s” to superpartners of fermions (like slepton, squark) and an “ino” to superpartners of bosons (like wino, bino). These super particles have the same quantum numbers as their partners; however, their spins are differed by 1/2. They serve as the new perturbatively coupled degrees of freedom that act to cancel the quadratic divergences. [2] Then the divergences from the fermion loops will cancel the divergences from the boson loops. This cancellation will occur to all orders and for all values of particle masses.

Another main motivation of SUSY is the Grand Unification. The grand unification theory (GUT) suggests that at some high energy scale, all interactions unify to a single interaction, associated with a single gauge group G_{GUT}, and the three running couplings unify to a single running coupling constant. [3]

$$\begin{array}{cccccl} & & \text{Low Energy} & \rightarrow & \text{High Energy} \\ SU_C(3) & SU_L(2) & U_Y(1) & \rightarrow & G_{GUT} = SU(5) \text{ or } SO(10) \\ \text{gluons} & W,\ Z & \text{photon} & \rightarrow & \text{gauge bosons} \\ \text{quarks} & \text{leptons} & & \rightarrow & \text{fermions} \\ g_3 & g_2 & g_1 & \rightarrow & g_{GUT} \end{array}$$

The evolution of running coupling constants is a function of the renormalization energy scale Q defined by the Beta function: [4]

$$\beta_g = Q\frac{\partial g_i}{\partial Q} \quad \text{with } \alpha_i = \frac{g_i^2}{4\pi}. \tag{2.24}$$

Applying renormalization group equations (RGE) for the three couplings:

$$\beta_g \equiv \frac{dg_i}{dt} = \frac{1}{16\pi^2} b_i g_i^3, \quad t = \ln(\frac{Q}{Q_0}) \tag{2.25}$$

where Q_0 is some reference scale. The solution is

$$\frac{1}{g_i^2(Q)} = \frac{1}{g_i^2(Q_0)} - \frac{b_i}{8\pi^2}\ln(\frac{Q}{Q_0}). \tag{2.26}$$

For the SM, the coefficients b_i are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + n_g \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + n_H \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} \tag{2.27}$$

where n_g is the number of generations of matter multiplets and n_H is the number of Higgs doublets. For the SM, $n_g = 3$ and $n_H = 1$ give $b_i = (41/10, -19/6, -7)$. For the MSSM, the coefficients b_i are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + n_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + n_H \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix} \tag{2.28}$$

where $n_g = 3$ and $n_H = 2$ in the minimal SUSY standard model (MSSM) give $b_i = (33/5, 1, -3)$. The value of α_i at $Q = m_Z = 91.1876 \pm 0.0021$ GeV can be calculated from the experiment results [1], so it can be used as the reference scale Q_0.

Fig. 2.2 shows that in the SM, the three lines cannot unify at one point, and in the MSSM-2 Higgs Doublet model, the SUSY particles change the slopes of the coupling evolution curves above the SUSY scale $M_{\text{SUSY}} \simeq 1$ TeV, so it is possible for the couplings to be unified at about $M_{\text{GUT}} \simeq 10^{16}$ GeV. The value of α_3^{-1} is a little smaller than the unification point of α_1^{-1} and α_2^{-1}. This can be explained as there exists new particles around 10^{16} GeV. [5]

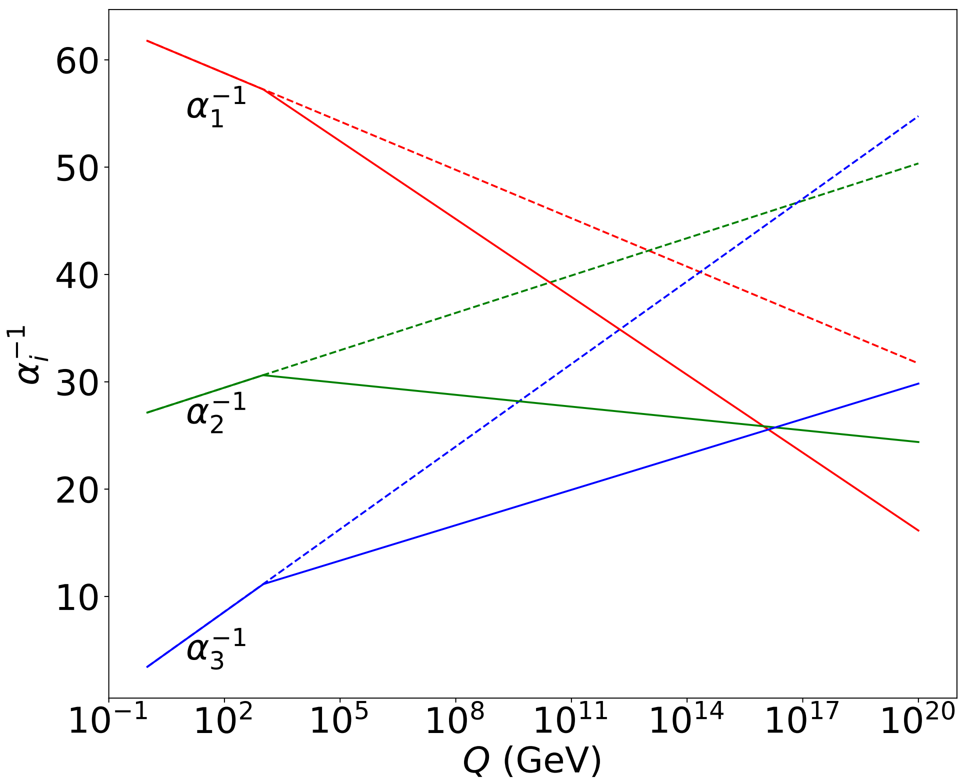

Figure 2.2: Evolution of the inverse of the three coupling constants as a function of the logarithm of energy in the SM (dot line) and in the MSSM-2 Higgs Doublet model (solid line).

SUSY also has the potential to include gravity. The graviton has *spin* 2, but other gauge bosons like photon, gluons, W and Z bosons have *spin* 1. *Spin* 2 and *spin* 1 gauge fields can be unified only within SUSY algebra. Q is a SUSY generator, then

$$Q|\text{boson} >= |\text{fermion} > \quad \text{and} \quad Q|\text{fermion} >= |\text{boson} > . \tag{2.29}$$

Applying the SUSY generator to graviton

$$spin\ 2 \to spin\ 3/2 \to spin\ 1 \to spin\ 1/2 \to spin\ 0.$$

So in SUSY, it is natural to unify all the matters and forces. [3]

2.3 SUSY algebra and Lagrangians

The SUSY algebra is a supersymmetric extension of the Poincare algebra. The commutation relations of the Poincare group are

$$[P_\mu, P_\nu] = 0, \tag{2.30a}$$

$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu), \tag{2.30b}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}). \tag{2.30c}$$

The Super-Poincare algebra contains additional SUSY spinorial generators Q and the conjugate $\bar{Q}$:

$$[P_\mu, Q_a] = [P_\mu, \bar{Q}_a] = 0, \tag{2.31a}$$

$$[M_{\mu\nu}, Q_a] = -(\frac{1}{2}\sigma_{\mu\nu})_{ab}Q_b, \tag{2.31b}$$

$$\{Q_a, \bar{Q}_b\} = 2(\gamma^\mu)_{ab}P_\mu, \tag{2.31c}$$

$$\{Q_a, Q_b\} = -2(\gamma^\mu C)_{ab}P_\mu, \tag{2.31d}$$

$$\{\bar{Q}_a, \bar{Q}_b\} = 2(C^{-1}\gamma^\mu)_{ab}P_\mu \tag{2.31e}$$

where $P_\mu = i\partial_\mu$ is the energy-momentum operator, and $M_{\mu\nu}$ is the second rank angular momentum generator with $M_{ij} = \epsilon_{ijk}J_k$ and $M_{0i} = -M_{i0} = -K_i$, J_i is the rotation generator, K_i is the boost generator. C is the charge conjugation matrix.

We now introduce new quantum spacetime coordinates θ_a with $a = 1, 2, 3, 4$, which are anticommuting Grassmann variables:

$$\{\theta_a, \theta_b\} = 0, \quad \{\bar{\theta}_a, \bar{\theta}_b\} = 0, \quad \theta_a^2 = 0, \quad \bar{\theta}_b^2 = 0 \tag{2.32}$$

where $\bar{\theta} = \theta^T C$. θ is also a four component Majorana spinor. The four commuting superspace Minkowski spacetime coordinates x^μ and four anticommuting coordinates θ_a form the Superspace. In some cases, it is convenient to consider two components of θ and $\bar{\theta}$ as independent variables. Now we move to two-component notation. The general superfield denoted by $\hat{\Phi}(x, \theta)$ is a function of x and θ.

The SUSY group element translation is similar to the ordinary translation

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q})}. \tag{2.33}$$

So the supertranslation in superspace is

$$\begin{aligned} x^\mu &\rightarrow x^\mu + i\theta\sigma_\mu\bar{\epsilon} - i\epsilon\sigma_\mu\bar{\theta}, \\ \theta &\rightarrow \theta + \epsilon, \\ \bar{\theta} &\rightarrow \bar{\theta} + \bar{\epsilon} \end{aligned} \tag{2.34}$$

where ϵ and $\bar{\epsilon}$ are Grassmannian transformation parameters. The supercharge generator in form of differential operator in the superspace is

$$Q_a = \frac{\partial}{\partial \theta_a} - i\sigma^{\mu}_{ab}\bar{\theta}^b \partial_\mu, \tag{2.35a}$$

$$\bar{Q}_b = -\frac{\partial}{\partial \bar{\theta}_b} + i\theta_a \sigma^{\mu}_{ab}\partial_\mu. \tag{2.35b}$$

The left chiral superfield satisfies the condition:

$$\bar{D}\Phi = 0 \tag{2.36}$$

where $\bar{D} = -\frac{\partial}{\partial \bar{\theta}} - i\theta\sigma^\mu\partial_\mu$ is a covariant derivative. The right antichiral superfield is $\Phi^\dagger$. Similarly, it satisfies

$$D\Phi^\dagger = 0 \tag{2.37}$$

where $D = -\frac{\partial}{\partial \theta} - i\sigma^\mu\bar{\theta}\partial_\mu$. The expansion of a chiral superfield is

$$\begin{aligned}\Phi(y,\theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ &= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x) \\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)\end{aligned} \tag{2.38}$$

where $y = x + i\theta\sigma\bar{\theta}$, and $\Box = \partial_\mu\partial^\mu = \partial^2/\partial t^2 - \partial^2/\partial x^2 - \partial^2/\partial y^2 - \partial^2/\partial z^2$. A is $spin = 0$ scalar field with mass dimension $[A] = 1$, ψ is two-component Weyl spinor field with mass dimension $[\psi] = 3/2$. F is scalar field with mass dimension $[F] = 2$. It is an auxiliary field and has no physical meaning. It can be eliminated by Euler-Lagrange equations, but it is useful to write supersymmetric variations as linear transformations on the fields. [2] The number of degrees of freedom of bosonic and fermionic fields are exactly same in the superfield.

Under SUSY transformation, the bosonic and fermionic fields convert into each other

$$\begin{aligned}
\delta_\varepsilon A &= \sqrt{2}\varepsilon\psi, \\
\delta_\varepsilon \psi &= i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F, \\
\delta_\varepsilon F &= i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi.
\end{aligned} \tag{2.39}$$

The variation of F is a total derivative, so it will disappear when integrated over superspace.

The Grassmannian expansion of a real vector superfield V is:

$$\begin{aligned}
V(x,\theta,\bar{\theta}) = {} & C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) \\
& + \frac{i}{2}\theta\theta[M(x) + iN(x)] - \frac{i}{2}\bar{\theta}\bar{\theta}[M(x) - iN(x)] \\
& - \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}[\lambda(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)] \\
& - i\bar{\theta}\bar{\theta}\theta[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[G(x) + \frac{1}{2}\Box C(x)]
\end{aligned} \tag{2.40}$$

with $V = V^\dagger$ and $\Box = \partial_\mu\partial^\mu$. v_μ is the vector gauge field and λ is the Majorana spinor field. G is a auxiliary field. C, χ, M, N have no physical meaning and can be removed. The supergauge transformation of V is

$$V \to V + \Phi + \Phi^\dagger. \tag{2.41}$$

The transformation of the components are

$$\begin{aligned}
C &\rightarrow C + A + A^*, \\
\chi &\rightarrow \chi - i\sqrt{2}\psi, \\
M + iN &\rightarrow M + iN - 2iF, \\
v_\mu &\rightarrow v_\mu - i\partial_\mu(A - A^*), \\
\lambda &\rightarrow \lambda, \\
G &\rightarrow G.
\end{aligned} \tag{2.42}$$

The Wess-Zumino gauge is $C = \chi = M = N = 0$, so V becomes

$$\begin{aligned}
V &= -\theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}G(x), \\
V^2 &= -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}v_\mu(x)v^\mu(x), \\
V^3 &= 0, \quad etc.
\end{aligned} \tag{2.43}$$

The field strength tensors in the Wess-Zumino gauge are

$$\begin{aligned}
W_a &= -\frac{1}{4}\bar{D}^2 e^V D_a e^{-V}, \\
\bar{W}_b &= -\frac{1}{4}D^2 e^V \bar{D}_a e^{-V}.
\end{aligned} \tag{2.44}$$

The strength tensor satisfies the equation

$$\bar{D}_b W_a = 0, \quad D_a \bar{W}_b = 0, \tag{2.45}$$

so it is a chiral superfield. The component expression is:

$$W_a = T^i\left(-i\lambda_a^i + \theta_a D^i - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_a F^i_{\mu\nu} + \theta^2(\sigma^\mu D_\mu\bar{\lambda}^i)_a\right) \tag{2.46}$$

where

$$
\begin{aligned}
F^i_{\mu\nu} &= \partial_\mu v^i_\nu - \partial_\nu v^i_\mu + f^{ijk} v^j_\mu v^k_\nu \\
D_\mu \bar{\lambda}^i &= \partial \bar{\lambda}^i + f^{ijk} v^j_\mu \bar{\lambda}^k
\end{aligned}
\tag{2.47}
$$

In Abelian gauge, the field strength tensor can be simplified as

$$
\begin{aligned}
W_a &= -\frac{1}{4}\bar{D}^2 D_a V \\
\bar{W}_b &= -\frac{1}{4}D^2 \bar{D}_a V
\end{aligned}
\tag{2.48}
$$

The general SUSY invariant Lagrangian is

$$
\mathcal{L} = \Phi_i^\dagger \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} + [(\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k)|_{\theta\theta} + h.c.]. \tag{2.49}
$$

The first term is a kinetic term when $\Phi_i^\dagger \Phi_i$ is the Kahler potential. It can be expanded about $\theta\theta\bar{\theta}\bar{\theta}$ to the ordinary kinetic form. The second set of terms is the superpotential W so it has to be a chiral field. It can be expanded about $\theta\theta$ to get an ordinary potential.

The Lagrangian density can be rewritten as an integral over superspace:

$$
\mathcal{L} = \int d^2\theta d^2\bar{\theta} \Phi_i^\dagger \Phi_i + \int d^2\theta [\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k] + h.c. \tag{2.50}
$$

with Grassmannian integration

$$
\int d\theta_a = 0, \quad \int \theta_a d\theta_b = \delta_{ab}. \tag{2.51}
$$

This leads to

$$\mathcal{L} = i\partial_\mu\bar{\psi}_i\bar{\sigma}^\mu\psi_i + A_i^*\Box A_i + F_i^*F_i + [\lambda_i F_i + m_{ij}(A_iF_j - \frac{1}{2}\psi_i\psi_j) + y_{ijk}(A_iA_jF_k - \psi_i\psi_jA_k) + h.c.]. \tag{2.52}$$

The constraints are

$$\frac{\partial\mathcal{L}}{\partial F_k^*} = F_k + \lambda_k^* + m_{ik}^*A_i^* + y_{ijk}^*A_i^*A_j^* = 0 \tag{2.53a}$$

$$\frac{\partial\mathcal{L}}{\partial F_k} = F_k^* + \lambda_k + m_{ik}A_i + y_{ijk}A_iA_j = 0 \tag{2.53b}$$

The auxiliary fields F and F^* can be eliminated now, so

$$\mathcal{L} = i\partial_\mu\bar{\psi}_i\bar{\sigma}^\mu\psi_i + A_i^*\Box A_i - \frac{1}{2}m_{ij}\psi_i\psi_j - \frac{1}{2}m_{ij}^*\bar{\psi}_i\bar{\psi}_j - y_{ijk}\psi_i\psi_jA_k - y_{ijk}^*\bar{\psi}_i\bar{\psi}_jA_k^* - V(A_i, A_j) \tag{2.54}$$

where $V = F_k^*F_k$ is the scalar potential. The general form of the scalar potential V is defined as

$$V = V_D + V_F = \frac{1}{2}D^aD^a + F_i^*F_i \tag{2.55}$$

where

$$D^a = -gT_{ij}^aA_i^*A_j, \tag{2.56a}$$

$$F_i = -\frac{\partial W}{\partial A_i}. \tag{2.56b}$$

In the Wess-Zumino gauge

$$W^aW_a|_{\theta\theta} = -2i\lambda\sigma^\mu D_\mu\bar{\lambda} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D^2 + i\frac{1}{4}F^{\mu\nu}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma} \tag{2.57}$$

where $D_\mu = \partial_\mu + igv_\mu$. The gauge invariant Lagrangian is

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}(\int d^2\theta W^a W_a + \int d^2\bar{\theta}\bar{W}^b\bar{W}_b) \\ &= \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\bar{\lambda}.\end{aligned} \tag{2.58}$$

The gauge transformation of matter chiral superfields is

$$\Phi \ \rightarrow \ e^{-ig\Lambda}\Phi, \quad \Phi^\dagger \ \rightarrow \ \Phi^\dagger e^{ig\Lambda^\dagger}, \quad V \ \rightarrow \ V + i(\Lambda - \Lambda^\dagger) \tag{2.59}$$

where Λ is a gauge parameter. Then the gauge invariant kinetic term is

$$\Phi_i^\dagger \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \ \rightarrow \ \Phi_i^\dagger e^{gV}\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}}. \tag{2.60}$$

So the gauge invariant Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{inv}} = \frac{1}{4}\int d^2\theta W^a W_a + \frac{1}{4}\int d^2\bar{\theta}\bar{W}^b\bar{W}_b + \int d^2\theta d^2\bar{\theta}\Phi_i^\dagger e^{gV}\Phi_i \\ + \int d^2\theta(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k) + h.c..\end{aligned} \tag{2.61}$$

The Lagrangian of SUSY QED is

$$\begin{aligned}\mathcal{L}_{\text{SQED}} = \frac{1}{4}\int d^2\theta W^a W_a + \frac{1}{4}\int d^2\bar{\theta}\bar{W}^b\bar{W}_b \\ + \int d^4\theta(\Phi_+^\dagger e^{gV}\Phi_+ + \Phi_-^\dagger e^{-gV}\Phi_-) \\ + \int d^2\theta m\Phi_+\Phi_- + \int d^2\bar{\theta} m\Phi_+^\dagger\Phi_-^\dagger\end{aligned} \tag{2.62}$$

where the superfield Φ_+ is the left-handed fermion and Φ_- is the right-handed fermion.

The non-Abelian SUSY Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{SUSY YM}} = &\frac{1}{4}\int d^2\theta \ \mathrm{Tr}(W^a W_a) + \frac{1}{4}\int d^2\bar{\theta} \ \mathrm{Tr}(\bar{W}^a \bar{W}_a) \\ &+ \int d^2\theta d^2\bar{\theta}\bar{\Phi}_{ia}(e^{gV})^a_b \Phi^b_i + \int d^2\theta \mathcal{W}(\Phi_i) + \int d^2\bar{\theta}\bar{\mathcal{W}}(\bar{\Phi}_i)\end{aligned} \tag{2.63}$$

where $\mathcal{W}$ is a superpotential.

2.4 The minimal supersymmetric standard model (MSSM)

The MSSM is the simplest supersymmetric version of the SM. It contains the smallest number of added particles and parameters, but includes soft SUSY breaking terms. The MSSM is also characterized by the $SU(3)_C \times SU(2)_L \times U(1)_Y$ group. The MSSM is R-parity conserving by definition, so the lightest supersymmetric particle (LSP) is stable. We define the new quantum number $R = (-1)^{3B+L+2S}$, where B is the baryon number, L is the lepton number, and S is the spin. Then particles have $R = 1$, and sparticles have $R = -1$. An essential feature of MSSM is the presence of lots of scalar fields, some contain baryon number and some lepton number. In SUSY, the number of fermionic and bosonic degrees of freedom are equal, and in the SM they are not.

2.4.1 The superfields and Lagrangian of the MSSM

The MSSM predicts a host of new particles, like squarks, sleptons, charginos and neutralinos, that ought to exist around the TeV scale. In addition to the superpartners for each SM particle, another left-chiral scalar Higgs doublet superfield and its superpartner were introduced in the MSSM. The VEV of scalar

Bosons	Fermions	$SU_C(3)$	$SU_L(2)$	$U_Y(1)$
g	$\tilde{g}$	8	0	0
$W^{\pm}, Z$	$\tilde{W}^{\pm}, \tilde{Z}$	1	3	0
B	$\tilde{B}$	1	1	0
$\tilde{L} = \begin{pmatrix} \tilde{\nu}_e \\ \tilde{e} \end{pmatrix}_L$	$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	1	2	-1
$\tilde{E} = \tilde{e}_R$	$E = e_R$	1	1	2
$\tilde{Q} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L$	$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	1/3
$\tilde{U} = \tilde{u}_R$	$U = u_R^c$	$\bar{3}$	1	-4/3
$\tilde{D} = \tilde{d}_R$	$D = d_R^c$	$\bar{3}$	1	2/3
$H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	$\tilde{H}_u = \begin{pmatrix} \tilde{h}_u^+ \\ \tilde{h}_u^0 \end{pmatrix}$	1	2	1
$H_d = \begin{pmatrix} h_d^- \\ h_d^0 \end{pmatrix}$	$\tilde{H}_d = \begin{pmatrix} \tilde{h}_d^- \\ \tilde{h}_d^0 \end{pmatrix}$	1	2	-1

Table 2.1: The superfields and group representations of the MSSM in four-component notation

components h_u^0 and h_d^0 of the two Higgs doublet give masses to up-type quarks and down-type quarks, respectively. The superfields of the MSSM are shown in Table 2.1.

The Higgs sector of the MSSM has eight degrees of freedom. Three of them are the Goldstone bosons which are absorbed by $W^{\pm}$, Z, so there are five fields left. Therefore, in the MSSM, there are five physical Higgs bosons: two charged, three neutral.

The Lagrangian of the MSSM can be written in form of

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{Breaking}} \tag{2.64}$$

where

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Yukawa}} \tag{2.65}$$

where

$$\begin{aligned}\mathcal{L}_{\text{Gauge}} = &\sum_{SU(3),SU(2),U(1)} \frac{1}{4}\left(\int d^2\theta \ \text{Tr}W^a W_a + \int d^2\bar{\theta} \ \text{Tr}\bar{W}^b \bar{W}_b\right) \\ &+ \sum_{\text{Matter}} \int d^2\theta d^2\bar{\theta} \Phi_i^\dagger e^{g_3\hat{V}_3 + g_2\hat{V}_2 + g_1\hat{V}_1} \Phi_i\end{aligned} \tag{2.66}$$

and

$$\mathcal{L}_{\text{Yukawa}} = \int d^2\theta (\mathcal{W}_{\text{A}} + \mathcal{W}_{\text{F}}) + h.c.. \tag{2.67}$$

where

$$\mathcal{W}_{\text{A}} = y_u Q H_u U^c + y_d Q H_d D^c + y_e L H_d E^c + \mu H_u H_d \tag{2.68}$$

and

$$\mathcal{W}_{\text{F}} = (\lambda^L L L E^c + \lambda^{L\prime} L Q D^c + \mu' L H_u) + \lambda^B U^c D^c D^c. \tag{2.69}$$

The first three terms of $\mathcal{W}_{\text{F}}$ violate lepton number L, and the last term violates baryon number B. Since the B or L violation is not observed in experiments, the $\mathcal{W}_{\text{F}}$ term needs to be set to zero. It can be avoided by introducing R-parity

$$R = (-1)^{3(B-L)+2S} \tag{2.70}$$

where S is the spin of the field. Then the ordinary particles have $R = 1$, and the superpartners have $R = -1$. The superpotential $\mathcal{W}$ is required to be invariant under R-parity. So the $\mathcal{W}_{\text{A}}$ term is allowed, and the $\mathcal{W}_{\text{F}}$ term is forbidden.

2.4.2 Breaking of SUSY in the MSSM

From observation, the experiments didn't find any superparticle with exactly the same mass as its superpartner, so the SUSY must be a broken symmetry.

The SUSY is expected to be spontaneously broken. Spontaneous SUSY breaking develops a non-zero VEV $< 0|F|0 >$ or $< 0|D|0 > \neq 0$ to break SUSY, but does not cause reappearance of quadratic divergences. So spontaneous SUSY breaking is a type of soft SUSY breaking. There is no compelling theory of SUSY breaking yet, so it is more practical to write the expected result of SUSY breaking as soft SUSY breaking terms explicitly in the Lagrangian.

There is no good field that can break the SUSY within the MSSM, so the MSSM must be extended to include new fields. The most common choices are *hidden sector + messengers*. The MSSM matters are the *visible sector*. In the hidden sector, SUSY is spontaneously broken, and the hidden sector does not interact with MSSM fields directly, so could contain dark matter candidates. The hidden sector communicates with the visible sector through the exchange of messengers which mediates the soft SUSY breaking. There are several popular SUSY breaking mechanism:

1. Gravity mediated SUSY breaking:

In this mechanism, the SUSY breaking effects are mediated from the hidden sector to the visible sector via the supergravity (SUGRA) interactions. Supergravity is a generic consequence of local supersymmetry. To reduce the number of free parameters, we assume that at some high energy scale, all the spin 0 particle masses equal to m_0, all the spin 1/2 particle or gaugino masses equal to $m_{1/2}$, and all the cubic and quadratic terms proportional to trilinear coupling A and bilinear coupling B corresponding to the Yukawa superpotential.

For an F type SUSY breaking in the hidden sector, there are some scalar fields that can develop nonzero VEVs for their F component, which leads to spontaneous SUSY breaking at scale $\sqrt{F}$. Then the spin 3/2 gravitino becomes massive through the super-Higgs effect, and mediates the SUSY breaking to the visible sector. The SUSY breaking scale in the visible sector is [7]:

$$M_{\mathrm{SUSY}} \sim \frac{< F >}{M_{Planck}} \sim m_{3/2} \tag{2.71}$$

where $m_{3/2}$ is the gravitino mass. We want $M_{\mathrm{SUSY}} \sim 1$ TeV, so $\sqrt{< F >} \sim 10^{11}$ GeV. The lightest SUSY particle (LSP) could be the lightest neutralino.

2. Gauge mediated SUSY breaking (GMSB):

In this mechanism, gauge interactions mediate the SUSY breaking effects in the hidden sector to the visible sector. The messengers are new chiral supermultiplets which are charged under the SM gauge group. The messengers couple directly to the hidden sector so that they get mass at tree level, and also couple indirectly to the visible sector through the SM gauge interaction.

For the simplest model, the messenger fields are a set of left-handed chiral supermultiplets ψ transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ but not part of the MSSM. The messenger couple to the hidden sector through the superpotential:

$$\mathcal{W}_{messenger} = \phi\psi\bar{\psi} \tag{2.72}$$

where ϕ is a gauge-singlet chiral supermultiplet. The scalar component and auxiliary F component of ϕ acquire non-zero VEVs:

$$< \phi >= m_{\phi}, \quad < F_{\phi} >\neq 0 \tag{2.73}$$

Gauge loops transmit SUSY breaking to the MSSM fields. The gauginos receive masses at one-loop order of the messenger fields, and the MSSM scalars receive masses at two-loop order, so their masses are comparable. The masses of gauginos and scalars only depend on the gauge couplings, so there are no flavor violating problems. For any model of GMSB, the LSP is the gravitino, whose mass should be around SUSY breaking scale.

3. Anomaly mediated SUSY breaking (AMSB)

In this mechanism, the SUSY breaking is mediated through the conformal anomaly. In the hidden sector, the SUSY is spontaneously breaking by $< F > \neq 0$. The scalar auxiliary component of the supergravity multiplet obtains a non-zero VEV

$$< F_\phi > \sim \frac{< F >}{M_{Planck}} \sim m_{3/2} \tag{2.74}$$

The original MSSM Lagrangian is scale invariant, so the F_ϕ term has no effect. When we apply quantum effects, the couplings become scale dependent. The conformal anomaly breaks the scale invariance, so the SUSY breaking effect appears in the visible sector. Anomaly mediation is flavor blind in general [8]. It generates sparticle soft masses proportional to the gravitino mass, and they only depend on the overall scale F_ϕ.

All these mechanisms have similar results, but they have their own advantages and disadvantages for solving problems in different conditions.

2.4.3 The Higgs potential

The MSSM Higgs potential is defined by superpotential and the SUSY breaking terms. The Higgs potential is

$$V_{\text{Higgs}} = V_{\text{tree}} + \Delta V \tag{2.75}$$

where the tree level potential is

$$\begin{aligned}V_{\text{tree}}(H_u, H_d) =& m_1^2|H_u|^2 + m_2^2|H_d|^2 - m_3^2(H_u H_d + h.c.) \\ &+ \frac{g^2+g'^2}{8}(|H_u|^2 - |H_d|^2)^2 + \frac{g^2}{2}|H_u^+ H_d|^2\end{aligned} \tag{2.76}$$

where $m_1^2 = m_{H_u}^2 + \mu^2$ and $m_2^2 = m_{H_d}^2 + \mu^2$. The simplest model assumes scalar mass universality so that at the GUT scale $m_1^2 = m_2^2 = m_0^2 + \mu_0^2,\ m_3^2 = -B\mu_0$. The radiative correction of Higgs potential is

$$\Delta V = \sum_i \frac{(-1)^{2s_i}}{64\pi^2}(2s_i+1)c_i m_i^4 \left[\log\left(\frac{m_i^2}{Q^2}\right) - \frac{3}{2}\right] \tag{2.77}$$

where m_i are the mass of all the fields that coupled to the Higgs field, and $c_i = c_{col}c_{cha}$, with $c_{col} = 3,\ 1$ for colored and uncolored particles, and $c_{cha} = 2,\ 1$ for charged and neutral particles, and s_i is the spin quantum number.

The Higgs self-interaction coupling is defined by the gauge interactions. The potential satisfies

$$\begin{aligned}\frac{1}{2}\frac{\delta V}{\delta H_u} &= m_1^2 v_1 - m_3^2 v_2 + \frac{g^2+g'^2}{4}(v_1^2 - v_2^2)v_1 = 0 \\ \frac{1}{2}\frac{\delta V}{\delta H_d} &= m_2^2 v_2 - m_3^2 v_1 + \frac{g^2+g'^2}{4}(v_1^2 - v_2^2)v_2 = 0\end{aligned} \tag{2.78}$$

where

$$\begin{aligned}&< H_u > \equiv v_1 = v\cos\beta, \quad &< H_d > \equiv v_2 = v\sin\beta, \\ &v^2 = v_1^2 + v_2^2, \quad &\tan\beta \equiv \frac{v_1}{v_2}.\end{aligned} \tag{2.79}$$

Write in terms of v^2 and $\sin 2\beta$

$$v^2 = \frac{4(m_1^2 - m_2^2\tan^2\beta)}{(g^2+g'^2)(\tan^2\beta - 1)}, \quad \sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2} \tag{2.80}$$

We can see that if $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, v^2 are negative, then the minimum does not exist. Real positive solutions exist only if

$$m_1^2 + m_2^2 > 2m_3^2, \quad m_1^2 m_2^2 < m_3^4 \tag{2.81}$$

which is not valid at GUT scale. So the spontaneous breaking of the $SU(2)$ gauge invariance in the SM does not happen in the MSSM. The the tree level potential parameters become running parameters given by the RG equations which are the function of the energy scale. The running of the parameters leads to "radiative spontaneous symmetry breaking". This only occurs for top quark masses $m_t \sim 100-200$ GeV. Since the measured top quark mass $m_t = 173.1 \pm 0.6$ GeV [1], then radiative EW symmetry breaking does occur.

2.5 Strong CP problem, Peccei-Quinn (PQ) symmetry and axion

The QCD Lagrangian for N flavors when the quark masses $m_f \to 0$ has a global symmetry $U(N)_{Vector} \times U(N)_{Axial}$. We take the case $N = 2$ for two light quarks u and d. From experiments, we know $U(2)_V = SU(2)_{Isospin} \times U(1)_{Baryon}$. The axial symmetry $SU(2)_A$ and $U(1)_A$ are broken down spontaneously by quark condensate, and should give us four Nambu-Goldstone bosons. So in addition to the three pions, we should have another boson. But the mass of pion $m_\pi \simeq 0$, and the mass of meson eta $m_\eta \gg m_\pi$, so we cannot say the fourth boson is η since the fourth boson should have similar mass as pions. The missing fourth boson is the $U(1)_A$ problem. [9] This problem can be resolved if we approximate the QCD path integral in the classical limit $\hbar \to 0$, then we can solve the semi-classical field equations by summing over a kind of configuration called instanton. [10] This introduces a new QCD vacuum phase parameters θ and makes the $U(1)_A$ not a real symmetry of QCD. The more physical parameter is $\bar{\theta} \equiv \theta + \arg \det M$ where M is the quark mass matrix, because the computations are done in mass basis. The $U(1)_A$ is explicitly broken by an effective breaking term

$$\mathcal{L} \sim \bar{\theta} \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a} \tag{2.82}$$

which is caused by instanton, even though the $U(1)_A$ is assumed to be broken spontaneously. For a non-zero $\bar{\theta}$, CP symmetry will be violated, and the neutron electric dipole moment (EDM) will be non-zero. The current EDM experiment shows $\bar{\theta} \ll 10^{-10}$ rad. [11] So the solution of $U(1)_A$ problem bring us a new problem: Why $\bar{\theta}$ is so small? This is the strong CP problem.

The most popular solution of the strong CP problem is the Peccei-Quinn theory. The theory introduces a new global chiral $U(1)_{PQ}$ symmetry. This symmetry is spontaneously broken, and the Goldstone boson of this broken symmetry is axion a. Under $U(1)_{PQ}$ symmetry, the axion translates [12]

$$a \rightarrow a + \alpha f_a \tag{2.83}$$

where f_a is the axion decay constant. The effective axion-gluon-gluon interaction term is

$$\mathcal{L}_{\text{eff}} = \bar{\theta} \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a} + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}. \tag{2.84}$$

The effective potential has a minimum at $< a >= -f_a \bar{\theta}$. The CP-violating $\bar{\theta}$ term can be canceled by the dynamical axion term. The axion mass can be estimated by [13]

$$m_a^2 \simeq \frac{m_\pi^2 f_\pi^2}{f_a^2}. \tag{2.85}$$

In the original PQ model $f_a \sim v \simeq 240$ GeV was ruled out by experiments, but $f_a \gg v$ is still viable. The axion mass and interactions are suppressed by f_a, so in *invisible axion models*, large f_a makes the axion mass too small to be easily measured in experiments. f_a is constrained in the *axion window* 10^9 GeV $\leq f_a \leq 10^{12}$ GeV by cosmology data. [14] The lower limit comes from stellar cooling bounds while upper limit avoids overproduction of axion dark matter.

2.6 KSVZ and DFSZ models

Two invisible axion models are KSVZ (Kim, Shifman, Vainshtein and Zakharov) [15,16] and DFSZ (Dine, Fischler, Srednicki and Zhitnisky) [17,18]. These models introduce new fields to the SM. Both of them include a new SM singlet scalar field S carrying global PQ charge. The S field develop a large VEV with $f_a =< S >$ that is much larger than the electroweak scale. The coupling of the S to Higgs field should not give large mass correction to the Higgs boson while solving the strong CP problem. KSVZ model introduces a scalar field S, and a superheavy vector-like PQ charge carrying quark Q with $M_Q \sim f_a$ and couples to S. The Higgs fields do not carry PQ charge. The axion does not interact with leptons, but only interacts with light quarks through the strong and EM anomaly terms. [15, 16] DFSZ model adds a PQ charge carrying singlet field S to the two Higgs doublet model (2HDM) which has two scalar doublets, ϕ_u and ϕ_d with hypercharge -1 and $+1$, respectively. The Higgs doublets are charged under the PQ symmetry.

ϕ_u couples only to right-handed charge $2/3$ quarks, ϕ_d only to right-handed charge $-1/3$ quarks and to right-handed charged leptons. [17,18] The axion field should have a mass $m_a \sim 620\ \mu\text{eV}\left(\frac{10^{10}\ \text{GeV}}{f_a/N}\right)$ where N is the color anomaly of the PQ symmetry. $N = 1$ for KSVZ and $N = 6$ for DFSZ. The axion can be produced via axion field coherent oscillations in the early universe and serves as a candidate for cold dark matter. [19–24]

The large f_a could give large correction to the Higgs mass, bringing in the hierarchy problem. So we consider both SUSY and PQ symmetry. In the PQ augmented Minimal Supersymmetric Standard Model (PQMSSM), the axion superfield is

$$A = \frac{s + ia}{\sqrt{2}} + \sqrt{2}\theta\tilde{a} + \theta^2 F_a \tag{2.86}$$

where s is the *saxion*, the scalar superpartner of axion, with R-parity even and *spin*-0. a is the *pseudoscalar axion* field. θ is the spinorial Grassmann coordinates. $\tilde{a}$ is the *axino* field, the fermionic partner of axion, with R-parity odd and *spin*-1/2. F_a is the axion auxiliary field. Under a $U(1)_{PQ}$ transformation, the PQ scalar field Lagrangian is invariant. The axion superfield transforms as

$$A \rightarrow A + i\alpha v_{PQ} \tag{2.87}$$

below the PQ breaking scale $v_{PQ} = f_a/\sqrt{2}$, where α is a real parameter. The kinetic and self-couplings terms are

$$\mathcal{L}_{as\tilde{a}} = \left(1 + \frac{\sqrt{2}\xi}{v_{PQ}} s\right)\left(\frac{1}{2}\partial^\mu a \partial_\mu a + \frac{1}{2}\partial^\mu s \partial_\mu s + \frac{i}{2}\bar{\tilde{a}}\partial\!\!\!/\tilde{a}\right) \tag{2.88}$$

where $\xi = \Sigma_i q_i^3 v_i^2 / v_{PQ}^2$, and q_i are the PQ charges, v_i are the VEVs of PQ fields S_i, and $v_{PQ} = \sqrt{\Sigma_i q_i^2 v_i^2}$. ξ is 0 or 1 depending on if saxion decays into axino and axion. [25]

2.6.1 SUSY KSVZ model

The heavy quark superfields Q and Q^c with PQ charges are introduced in the SUSY KSVZ model. The superpotential of KSVZ model is

$$W_{\mathrm{KSVZ}} = \lambda S Q Q^c. \tag{2.89}$$

The heavy quark acquires mass of $m_Q \simeq \lambda v_{PQ}/\sqrt{2}$ after PQ symmetry breaking. The axion superfield couples to QCD gauge fields leading to thermal production rates for axinos and saxions proportional to the reheat temperature T_R after inflation. The thermal production rates for axino, saxion and axion are calculated to be [26]

$$\frac{\rho_{\tilde{a}}^{TP}}{s} \simeq 0.9\times 10^{-5} g_s^6 \,\ln\left(\frac{3}{g_s}\right)\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)^2\left(\frac{T_R}{10^8\mathrm{GeV}}\right) m_{\tilde{a}} \tag{2.90}$$

$$\frac{\rho_{s}^{TP}}{s} \simeq 1.3\times 10^{-5} g_s^6 \,\ln\left(\frac{1.01}{g_s}\right)\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)^2\left(\frac{T_R}{10^8\mathrm{GeV}}\right) m_{s} \tag{2.91}$$

$$\frac{\rho_{a}^{TP}}{s} \simeq 18.6\times g_s^6 \,\ln\left(\frac{1.501}{g_s}\right)\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)^2\left(\frac{T_R}{10^{14}\mathrm{GeV}}\right) m_{a}. \tag{2.92}$$

In the KSVZ model, axino and saxion decay primarily to gauge bosons and gauginos. [27] For heavy axino scenario $m_{\tilde{a}} \sim 100$ TeV, the KSVZ heavy PQ charged matter fields could be the gauge mediation messengers. [28]

2.6.2 SUSY DFSZ model

The SUSY DFSZ model introduces a PQ singlet superfield S carrying PQ charges -1, and the Higgs doublet superfields H_u and H_d carries PQ charges $+1$, so the PQ superfield can directly couple to the Higgs superfields. The superpotential is

$$W_{\mathrm{DFSZ}} = \lambda \frac{S^2}{M_P} H_u H_d. \tag{2.93}$$

An advantage of this model is that it provides a simple solution of the SUSY μ Problem: μ is supersymmetric, so it is expected to be at the order of Planck scale, but it gives mass to Higgs, so it should be around weak scale. The general way to solve the problem is to forbid the μ term, and then regenerate it at soft SUSY breaking scale. There are some popular solutions to the μ problem: i.e., Next-to-Minimal Supersymmetric Standard Model (NMSSM) [29–31], Giudice-Masiero (GM) [32], and Kim-Nilles (KN) [33] solutions. In the KN solution, PQ charge assignments to the Higgs fields forbid the usual μ term at tree level. After the PQ symmetry breaking by a VEV of the scalar component of S, where $< S > \sim f_a$, an effective μ term

$$\mu \sim \lambda f_a^2 / M_P \sim \lambda m_{3/2} \tag{2.94}$$

is generated. For $\lambda \sim 1$ and $f_a \sim 10^{10} - 10^{11}$ GeV, then μ is around weak scale. For small λ, μ can be around weak scale while $m_{\tilde{q}} \sim m_{3/2} \sim 10$ TeV. The μ term arises from PQ symmetry breaking, and $m_{3/2}$ might arise from hidden sector SUSY breaking. So the PQ breaking scale is much smaller than the hidden sector mass scale $f_a \ll m_{\mathrm{hidden}}$ leads to a Little Hierarchy Problem (LHP): μ is much smaller than the SUSY particle masses $\mu \ll m_{3/2}$.

A model proposed by Murayama, Suzuki and Yanagida (MSY) [34] showed us the LHP is not a real problem, but just a reflection of the mis-match between PQ breaking scale and hidden sector mass scales. In the MSY model, PQ symmetry is broken by a PQ scalar X radiatively driven to negative mass-squared value. It is very similar to electroweak symmetry being broken by Higgs mass-squared $m^2_{H_u}$ radiative corrections driven to negative. The radiatively-broken PQ symmetry generates a 100 GeV scale μ and an intermediate mass for right-hand Majorana neutrino $m_N \sim 10^{11}$ GeV. Although we get different PQ scales by setting $m_{3/2}$ to different masses at Planck scale, the PQ scalar X is driven to negative mass-squared values independent of $m_{3/2}$ mass. [35]

In the SUSY DFSZ model, the axion supermultiplet couples directly to the Higgs fields below the PQ symmetry breaking scale. The superpotential is non-linearly realized

$$W = \mu e^{c_H A/v_{PQ}} H_u H_d \tag{2.95}$$

where c_H is the PQ charge of the Higgs bilinear operator $H_u H_d$, and and $c_H = 2$ since both H_u and H_d carry PQ charge $+1$. Higgs fields transform as

$$H_u H_d \to e^{-i c_H \alpha} \tag{2.96}$$

where α is an arbitrary real number. The interaction term of axion supermultiplet and Higgs fields is

$$\mathcal{L}_{\mathrm{DFSZ}} = \int d^2\theta (1 + B\theta^2) \mu e^{c_H A/v_{PQ}} H_u H_d \tag{2.97}$$

where θ is the Grassmann coordinate. B is the soft SUSY breaking term in the Higgs sector. $1 + B\theta^2$ is a SUSY breaking spurion field; it parameterizes the

SUSY breaking and determines the operators invariant under the symmetry. Due to the direct coupling of axion to Higgs superfields, the thermal production rates of axions, saxions and axinos are independent of T_R in the SUSY DFSZ model. The saxion and axino thermal yields are [26]

$$Y_s^{TP} \simeq 10^{-7}\zeta_s \left(\frac{\mu}{\mathrm{TeV}}\right)^2 \left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)^2 \tag{2.98}$$

$$Y_{\tilde{a}}^{TP} \simeq 10^{-7}\zeta_{\tilde{a}} \left(\frac{\mu}{\mathrm{TeV}}\right)^2 \left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)^2 \tag{2.99}$$

where the ζ_i are model-dependent constants of order unity determined by the mass spectrum. The axions and saxions can be produced via coherent oscillations. The saxions decay mainly to Higgsino pairs or axion pairs when $\xi \sim 1$. The axinos decay mainly into Higgsino and Higgs, or Higgsino and gauge bosons. In the SUSY DFSZ model, for a given v_{PQ}, saxion and axino have larger decay rates, and many more decay final states than in the SUSY KSVZ model. [36]

2.7 SUSY Naturalness

The discovery at LHC of the Higgs boson [37] with mass $m_H = 125.09 \pm 0.21$ GeV is within the range that MSSM predicted $m_H \simeq 115 \sim 135$ GeV [38]. This adds credence to SUSY. But at the same time, there is no SUSY particle signal at LHC yet, pushing the mass limits to gluino mass $m_{\tilde{g}} \gtrsim 2$ TeV and top squark mass $m_{\tilde{t}_1} \gtrsim 1$ TeV within context of simplified decay models. [39] The Higgs, squark and gluino mass limits have raised concern for the naturalness of the SUSY model. Weak scale SUSY can solve the gauge hierarchy problem via the cancellation of quadratic divergences by introducing new superparticles, but these new particles might not reduce loop corrections to Higgs mass enough to make the renormalized Higgs mass completely natural.

2.7.1 Naturalness measurements

To see if a model is natural or fine-tuned, we have several ways to quantify naturalness: 1. the electroweak measure Δ_{EW}, 2. the Higgs mass fine-tuning measure Δ_{HS}, 3. the traditional EENZ/BG measure Δ_{BG}. [40] These three measures should agree with each other without overestimate the fine-tunings if applied properly.

1. The electroweak measure Δ_{EW} [41] requires that there are no unnatural cancellations in deriving the value of m_Z from the weak scale scalar potential

$$\begin{aligned} \frac{m_Z^2}{2} &= \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \\ &\simeq -(m_{H_u}^2 + \Sigma_u^u) - \mu^2. \end{aligned} \tag{2.100}$$

   where Σ_u^u and Σ_d^d are the one-loop corrections of particles that couple to the Higgs doublets, such as $\tilde{t}_{1,2},\ \tilde{b}_{1,2},\ \tilde{\tau}_{1,2},\ \tilde{W}_{1,2},\ \tilde{Z}_{1-4},\ h,\ H,\ H^{\pm},\ W^{\pm},\ Z$, and t. Here we assume $\tan\beta$ is large, so the $m_{H_d}^2$ and Σ_d^d terms are suppressed by $\tan^2\beta - 1$. The largest contribution of Σ_u^u comes from top squarks $\tilde{t}_{1,2}$. So we can get the reduced expression of electroweak fine-tuning parameter Δ_{EW}:

$$\Delta_{EW} = max\left(\left|\frac{-m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1}\right|, \left|\frac{-\Sigma_u^u(\tilde{t}_{1,2})\tan^2\beta}{\tan^2\beta - 1}\right|, |-\mu^2|\right) / \left(\frac{m_Z^2}{2}\right) \tag{2.101}$$

   where $m_{H_u}^2$ is a small negative value. It means the largest corrections should be comparable to $m_Z^2/2$, that is $|m_{H_u}^2|, \mu^2 \sim m_Z^2/2$. The biggest advantage of Δ_{EW} is model-independent. [42]

2. The Higgs mass fine-tuning measure Δ_{HS} [43] compares the radiative correction $\delta m^2_{H_u}$ of the $m^2_{H_u}$ soft term to the physical Higgs mass m^2_h

$$\Delta_{HS} = \frac{\delta m^2_{H_u}}{m^2_h/2} \tag{2.102}$$

where

$$m^2_h \sim \mu^2 + m^2_{H_u}(\Lambda) + \delta m^2_{H_u}. \tag{2.103}$$

Λ is some high energy cutoff. $\delta m^2_{H_u}$ can be determined from the RGE:

$$\frac{dm^2_{H_u}}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right) \tag{2.104}$$

where $t = \ln(Q^2/Q_0^2)$, $S = m^2_{H_u} - m^2_{H_d} + \mathrm{Tr}(m^2_Q - m^2_L - 2m^2_U + m^2_D + m^2_E)$ and $X_t = m^2_{Q_3} + m^2_{U_3} + m^2_{H_u} + A_t^2$. M_1 is the bino mass parameter, M_2 is the wino mass parameter, A_t is the top quark trilinear coupling. Neglecting gauge terms, $m^2_{H_u}$ contribution to X_t, and S, and integrating from m_{SUSY} to the cutoff Λ, we can get: [35]

$$\delta m^2_{H_u}|_{rad} \sim -\frac{3f_t^2}{8\pi^2}(m^2_{Q_3} + m^2_{U_3} + A_t^2)\ln(\Lambda^2/m^2_{\mathrm{SUSY}}). \tag{2.105}$$

3. The EENZ/BG measure Δ_{BG} was proposed by Ellis, Enquist, Nanopoulos, and Zwirner, [44] then later studied by Barbieri and Giudice [45]. The Δ_{BG} measures the variation in m^2_Z due to the variation of high scale parameter p_i:

$$\Delta_{BG} \equiv \max[c_i] \text{ where } c_i = \left|\frac{\partial \ln m^2_Z}{\partial \ln p_i}\right| = \left|\frac{p_i}{m^2_Z}\frac{\partial m^2_Z}{\partial p_i}\right| \tag{2.106}$$

where c_i are the sensitivity coefficients. We write m_Z^2 in weak scale, and then in high scale

$$\begin{aligned} m_Z^2 &\sim -2\mu^2(\text{weak}) - 2m_{H_u}^2(\text{weak}) \\ &\sim -2\mu^2(\Lambda) + a \cdot m_{3/2}^2 \end{aligned} \tag{2.107}$$

where a is some real number. [46] Since the high scale SUSY parameters are *not independent*, we can combine the *dependent* soft term contributions of m_Z^2 in terms of $m_{3/2}^2$. The naturalness requires no large cancellations in m_Z^2, that is $\mu^2 \sim a \cdot m_{3/2}^2 \sim m_Z^2$. Since μ do not evolve much from Λ to weak scale $\mu(\Lambda) \sim \mu(\text{weak})$, so $-2m_{H_u}^2(\text{weak}) \simeq a \cdot m_{3/2}^2$. The low value of Δ_{BG} requires a low value of $m_{H_u}^2$, leads to the same requirements as a low value of Δ_{EW}.

2.7.2 Radiatively-driven natural supersymmetry (RNS)

Since the top quark is so massive, it must have a large Yukawa coupling y_{u3}. Then large y_{u3} can drive the soft SUSY breaking Higgs mass $m_{H_u}^2$ to negative value at weak scale $\sim -m_Z^2 \sim -(100^2 \sim 300^2)\ (\text{GeV})^2$ so EW symmetry breaks properly, and keep the higgsino mass low with $|\mu| \simeq 100 \sim 300$ GeV, so Δ_{BG} is low. This is Radiatively-driven natural supersymmetry (RNS). [42] RNS does not require large cancellations at the electroweak scale when constructing $m_Z = 91.2$ GeV while keeping the light Higgs mass at $m_H = 125$ GeV. The measured Higgs mass requires highly mixed TeV-scale top squarks $\tilde{t}$ while LHC requires gluinos $\tilde{g}$ at multi-TeV scale. Electroweak gauginos are between $300 \sim 1200$ GeV, gluinos $\tilde{g}$ between $2 \sim 6$ TeV, top squarks $\tilde{t}_1$ around 1-3 TeV. The first/second generation matter scalars may exist between $10 \sim 30$ TeV and can potentially solve the

SUSY flavor, CP and gravitino $\tilde{G}$ problems by decoupling. [47]

The RNS is based on MSSM without adding extra matter and keep features such as gauge coupling unification and radiative electroweak symmetry breaking due to a large top quark mass. The model can be realized in the SUSY GUT type models with non-universal Higgs masses (NUHM). But it cannot work in the mSUGRA/CMSSM models where scalar mass universality is assumed since μ is not a free parameter in these models. For example, mSUGRA/CMSSM have the following parameters:

$$m_0, m_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu) \tag{2.108}$$

but the two-extra-parameter non-universal Higgs model (NUHM2) has the parameters:

$$m_0, m_{1/2}, A_0, \tan\beta, \mu, m_A \tag{2.109}$$

where m_0 is the soft SUSY-breaking scalar mass, $m_{1/2}$ is the soft SUSY-breaking gaugino mass, A_0 is the trilinear supersymmetry-breaking parameter, m_A is the pseudoscalar Higgs boson mass. Low electroweak fine-tuning is obtained due to large cancellations between $m^2_{H_u}(\Lambda = M_{\mathrm{GUT}})$ and $\delta m^2_{H_u}$,

$$m^2_{H_u}(M_{\mathrm{SUSY}}) = m^2_{H_u}(\Lambda) + \delta m^2_{H_u}. \tag{2.110}$$

The radiatively-induced low fine-tuning at the electroweak scale can be at the $\Delta^{-1}_{EW} \sim 3 - 10\%$ level. [47]

2.7.3 Dark matter with SUSY naturalness

In a highly natural model where the electroweak sector is stabilized by SUSY, the QCD sector is stabilized by the axion, the μ problem is resolved by PQ-charged Higgs fields and the Little Hierarchy $\mu \ll m_{3/2}$ emerges from radiative PQ breaking, then the dark matter is expected to be composed of two dark matter particles: an axion-higgsino admixture. The axinos can be produced thermally. The saxions can be produced thermally and via coherent oscillations. The axions can be produced thermally, via saxion decays, and via coherent oscillation. The higgsino-like weakly interacting massive particles (WIMP) are produced thermally, and also can be produced via axino and saxion production/decay in the early universe. The saxion decays into axions increases the effective number of additional neutrinos ΔN_{eff}, so the energy density which is parameterized by ΔN_{eff} increases. We can get an estimate of the mixed axion-higgsino dark matter via simultaneously solving eight coupled Boltzmann equations which track the abundance of radiation, WIMPs, thermal- and oscillation-produced axions, thermal- and oscillation-produced saxions, axinos and gravitinos. [48, 49] See Appendix B for coupled Boltzmann equations. The results are model dependent. In the SUSY KSVZ model, thermal production of axinos and saxions is proportional to the reheat temperature T_R. [50–52] The decay modes arise from heavy quark induced loop diagrams due to the superpotential term

$$W_{\text{KSVZ}} = m_Q e^{A/f_a} Q Q^c \tag{2.111}$$

where Q is the intermediate scale heavy quark superfield with $m_Q \sim f_a$. In the SUSY DFSZ model, the axion superfield has tree level couplings which are proportional to the SUSY μ parameter [53–55]

$$W_{\mathrm{DFSZ}} = \mu e^{-2A/f_a} H_u H_d. \tag{2.112}$$

Due to this interaction, thermal production of axions, axinos and saxions is independent of T_R unless $T_R \lesssim \mu$. [56] Decays also dominantly proceed through this tree level coupling so the axino and saxion tend to be shorter lived than in the KSVZ case.

Chapter 3

The Standard Model of Cosmology

From observations, we know the universe is expanding, the distances between galaxies increase. The distance between two objects is proportional to $R(t)$, the "scale factor" or "radius" of the universe. We use the Hubble parameter to describe the cosmological expansion rate:

$$H(t) = \frac{\dot{R}(t)}{R(t)}. \tag{3.1}$$

The present value today is [57]

$$H_0 = 67.80 \pm 0.77 \ \frac{km}{s \cdot Mpc} \sim 10^{-42} \text{ GeV}. \tag{3.2}$$

This leads to the big bang theory, the most popular cosmological model for the development of the early universe. The simplest model suggests that all spaces was contained in a single point. After the big bang, the universe went through an accelerating phase called inflation. To generate the appropriate amount of inflation, the *new inflation* (*slow-roll inflation*) suggests that the inflaton potential has to be nearly flat, so the inflaton cannot be too heavy. The inflation occurs when the scalar field rolled much slower than the expansion of the universe. During the inflation phase, the universe became flat, homogeneous and very cold. To exit inflation, the inflaton reached a steeper position of the potential and headed toward the minimum of the potential. The inflaton began to oscillate about the minimum and decay, and inflation ended. The inflaton must transfer its energy to a radiation dominated plasma at a temperature sufficient to allow standard nucleosynthesis. The decay of the inflaton produced a lot of relativistic particles, which made the universe radiation-dominated (RD), and reheated the universe to temperature T_R. The universe had to reheat to at least 4 MeV to produce enough light nuclei in the standard Big Bang Nucleosynthesis (BBN). [26] This is the starting point of thermal cosmology.

3.1 The big bang theory

The big bang theory assumes the universe is homogeneous and isotropic. It leads to the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{3.3}$$

where the coordinates (t, r, θ, ϕ) are the comoving polar coordinates which follow the cosmological expansion. k is the spatial curvature constant. $k = -1, 0, 1$ corresponding to a universe which is open, flat and closed respectively. So the physical distance $d_H(t)$ can be calculated through comoving distance

$$d_H(t) = R(t) \int_0^r \frac{dr}{\sqrt{1-kr^2}} = R(t) \int_0^t \frac{dt'}{R(t')}. \tag{3.4}$$

The energy momentum tensor T_μ^ν with energy density ρ and pressure p is

$$T_\mu^\nu = diag(-\rho, p, p, p). \tag{3.5}$$

Then we can get the continuity equation

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0. \tag{3.6}$$

The Einstein equations for the FRW universe is

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = 8\pi G T_\mu^\nu \tag{3.7}$$

where R_μ^ν is the Ricci tensor and R is the scalar curvature. $G \equiv M_P^{-2}$ is the Newton's constant. T_μ^ν is the energy momentum tensor. The 00-component give us the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2}. \tag{3.8}$$

If the universe is flat, $k = 0$, the corresponding energy density is critical energy density

$$\rho_c = \frac{3H^2}{8\pi G} = 3M_P^2 H^2 \tag{3.9}$$

where M_P is the reduced Planck mass given by

$$M_P^2 = \frac{1}{8\pi G} = \frac{m_P^2}{8\pi} \tag{3.10}$$

where m_P is the Planck mass, and

$$M_P \simeq 2.44 \times 10^{18} \text{ GeV}, \qquad m_P \simeq 1.22 \times 10^{19} \text{ GeV}. \tag{3.11}$$

Then we can get the Friedmann acceleration equation [156]

$$H^2 + \dot{H} = \frac{\ddot{R}(t)}{R(t)} = -\frac{1}{6M_P^2}(\rho + 3p). \tag{3.12}$$

The Hubble parameter H and ρ_c are functions of time. The density parameter Ω is

$$\Omega \equiv \frac{\rho}{\rho_c}. \tag{3.13}$$

The current value of Ω is around $\Omega_0 = 1$.

If the equation of state is of the form $p = w\rho$, Eq.(3.6) give us

$$\rho \propto R^{-3(1+w)}. \tag{3.14}$$

Plug Eq.(3.14) into Eq.(3.8), we get

$$R(t) \propto t^{2/3(1+w)}. \tag{3.15}$$

So, for matter dominated (MD) universe,

$$w = 0, \quad p = 0, \quad \rho \propto R^{-3}, \quad R \propto t^{2/3}, \quad H = \frac{2}{3}t^{-1}, \quad \rho \propto t^{-2}, \quad d_H(t) = 3t \tag{3.16}$$

where $\rho \propto R^{-3}$ is due to Hubble expansion in 3D. For radiation dominated (RD) universe,

$$w = 1/3, \quad p = \frac{1}{3}\rho, \quad \rho \propto R^{-4}, \quad R \propto t^{1/2}, \quad H = \frac{1}{2}t^{-1}, \quad \rho \propto t^{-2}, \quad d_H(t) = 2t \tag{3.17}$$

where $\rho \propto R^{-4}$ is due to expansion in 3D and redshift by the expansion.

For an inflationary universe

$$p = -\rho, \qquad R(t) \propto e^{Ht}, \qquad d_H(t) = \frac{1}{H}(e^{Ht} - 1) \tag{3.18}$$

with $\rho = constant$, and H is approximately constant.

For RD, the energy density is

$$\rho = \frac{\pi^2}{30}\left(N_b + \frac{7}{8}N_f\right)T^4 \equiv cT^4 \tag{3.19}$$

where T is the cosmic temperature. N_f and N_b are the number of degrees of freedom of fermion and boson, respectively. The entropy density s is

$$s = \frac{2\pi^2}{45}g_* T^3. \tag{3.20}$$

where $g_* = N_b + \frac{7}{8}N_f$ is effective number of massless degrees of freedom.

If the universe evolves adiabatically or in thermal equilibrium, then $sR^3 =$

constant, so

$$R(t)T = constant. \tag{3.21}$$

From Friedmann Eq.(3.8) with $k = 0$, Eq.(3.19) and Eq.(3.21), we got [155]

$$T = \left(\frac{M_P}{\sqrt{c/3}\; t} \right)^{1/2} \propto t^{-1/2} \quad \text{for radiation domination} \tag{3.22}$$

where c is some constant. For MD, Eq.(3.21) still holds, so

$$T \propto t^{-2/3} \quad \text{for matter domination.} \tag{3.23}$$

3.2 Inflation

The big bang theory has been a successful theory, but it is limited to the time when the universe is cool enough that the low energy scale physics is well understood. We believe that the universe went through a period of "inflation". During the inflation, the scale of the universe expanded from a small Hubble volume by a factor of order at least 10^{26}. It solves the two major problems in the big bang theory: The horizon and the flatness problems. And also there are some observational data to support the inflation theory. Any asymmetry generated before inflation would be diluted away by the inflation, so the baryon asymmetry we observe today has to be generated after the inflation.

There are different ways to define inflation

$$\ddot{R} > 0 \qquad \Leftrightarrow \qquad \frac{d(H^{-1}/R)}{dt} < 0 \qquad \Leftrightarrow \qquad p < -\frac{\rho}{3} \qquad \Leftrightarrow \qquad \dot{\phi}^2 < V(\phi). \tag{3.24}$$

However, there is no known field, neither ordinary matter $p = 0$ nor radiation

$p = \rho/3$, satisfy this condition. We need to find another kind of field as inflaton.

Let's consider a scalar field ϕ with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{3.25}$$

where $V(\phi)$ is the effective potential. And the energy-momentum tensor for the scalar field is

$$T^\mu_\nu = \partial^\mu\phi\partial_\nu\phi - \delta^\mu_\nu\left[\frac{1}{2}(\partial_\lambda\phi\partial^\lambda\phi) - V(\phi)\right]. \tag{3.26}$$

Since we assume the scalar field is homogeneous and isotropic, ϕ only depends on time. So the energy density and the pressure are

$$T^0_0 = \rho \qquad = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3.27a}$$

$$T^i_j = -p\delta^i_j = -\left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right]\delta^i_j. \tag{3.27b}$$

Apply Euler-Lagrange equation

$$D_\mu(\partial^\mu\phi) = -V'(\phi) \tag{3.28}$$

where D_μ is the covariant derivative. Let's assume ϕ is homogeneous, so $\nabla\phi = 0$. Now we have the equation of motion (EOM)

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{3.29}$$

When the ϕ is in a slow roll approximation, the frictional term $3H\dot{\phi}$ is dominant.

$$\left|\frac{\ddot{\phi}}{3H\dot{\phi}}\right| \ll 1. \tag{3.30}$$

The $\ddot{\phi}$ can be neglected, so the slow roll EOM become

$$\dot{\phi} = -\frac{V'(\phi)}{3H}. \tag{3.31}$$

So we can get

$$\ddot{\phi} = -\frac{1}{3H(\phi)}V''(\phi)\dot{\phi} + \frac{H'(\phi)}{3H^2(\phi)}V'(\phi)\dot{\phi}. \tag{3.32}$$

From Eq.(3.9), we have

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi}{3m_P^2}\rho = \frac{1}{3M_P^2}\rho. \tag{3.33}$$

If the potential energy $V(\phi)$ dominates the energy density ρ, we have

$$H^2 = \frac{1}{3M_P^2}V(\phi). \tag{3.34}$$

So

$$H'(\phi) = \frac{1}{2}\frac{V'(\phi)}{3M_P^2} = \frac{V'(\phi)}{2V(\phi)}H. \tag{3.35}$$

Plug Eq.(3.35) and Eq.(3.34) into Eq.(3.32), then Eq.(3.30) become

$$\left|\frac{\ddot{\phi}}{3H\dot{\phi}}\right| = \left|-\frac{1}{3}M_P^2\frac{V''(\phi)}{V(\phi)} + \frac{1}{3}\frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2\right| \ll 1. \tag{3.36}$$

We define the Potential Slow Roll Parameters ϵ_V and η_V, and the slow roll conditions are

$$\epsilon_V \equiv \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1, \tag{3.37}$$

$$\eta_V \equiv M_P^2\frac{V''(\phi)}{V(\phi)} \qquad \ll 1. \tag{3.38}$$

If $V(\phi)$ is flat enough, the slow roll conditions can be satisfied. We can plug different types of $V(\phi)$ into the slow roll EOM to get different types of inflation.

3.3 Reheat

During the inflation, the matter and the existing radiation were diluted to a very low density by the inflation, so the temperature was very low. At the end of inflation, $\ddot{\phi}$ became dominant in Eq.(3.29). The slow-roll condition was not satisfied anymore, and Eq.(3.29) switched from overdamped to underdamped. ϕ started the coherent oscillation about the true minimum of the potential. The inflaton decayed into conventional matter and radiation, reheating the universe. Introducing the decay rate Γ_ϕ of the inflaton, then the Eq.(3.29) becomes

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi\dot{\phi} + V'(\phi) = 0. \tag{3.39}$$

The rate of the energy transferred from the inflaton is given by Eq.(3.6) [155]

$$\dot{\rho_\phi} = \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = -(3H + \Gamma_\phi)\dot{\phi}^2. \tag{3.40}$$

For simple harmonic oscillations, we have [6]

$$\frac{1}{2} < \dot{\phi}^2 > = < V(\phi) > = \frac{1}{2} < \rho > . \tag{3.41}$$

So we have

$$\dot{\rho_\phi} = -(3H + \Gamma_\phi)\rho_\phi. \tag{3.42}$$

The H term is the radiation diluted by inflation, and the Γ_ϕ term is the energy of inflaton decayed into radiation. The evolution of the energy density of radiation is described by

$$\dot{\rho}_r = -4H\rho_r + \Gamma_\phi \rho_\phi + \Gamma_{th}(\rho_r - \rho_r^{eq}) \tag{3.43}$$

where ρ_r is the energy density of radiation, ρ_r^{eq} is the equilibrium energy density, and Γ_{th} is the reaction rate for thermalization of the radiation.

When $t \sim \Gamma_\phi^{-1}$, the inflaton decayed and reheated the universe to the reheat temperature T_R, although no supercooling and reheating actually took place. The T_R is given by Eq.(3.22)

$$T_R = \left(\frac{45}{4\pi^3 \left(N_b + \frac{7}{8} N_f \right)} \right)^{1/4} (\Gamma_\phi M_P)^{1/2} \tag{3.44}$$

where $N_b + \frac{7}{8} N_f = \frac{427}{4}$ for SM, and $\frac{915}{4}$ for SUSY. After that, the universe entered the RD era.

Chapter 4

Baryogenesis via Leptogenesis

From the observation of present universe, we know that there are much more matter than antimatter. Planck Collaboration gave the ratio of the components of the universe's total energy density [57]. $\Omega_{dark\,matter} = 0.268$ is the ratio of dark matter to the critical energy density. $\Omega_{dark\,energy} = 0.683$ is the ratio of dark energy, a kind of negative pressure, which causes the acceleration of the expansion of present universe. $\Omega_B = 0.049$ is the ratio of the ordinary baryonic matter. So we can see about 85% of matter is dark matter, which is non-baryonic non-luminous. The contribution of neutrinos and photons is small and negligible. The inflation diluted both n_B and the photon number density n_γ as $R(t)^{-3}$. So we use their ratio

$$\eta \equiv \frac{n_B}{n_\gamma} \tag{4.1}$$

to measure the asymmetry. n_γ can be calculated from the Boltzmann distribution $n_\gamma = 2\frac{\xi(3)}{\pi^2}T^3$, where T is the temperature. From the present values of microwave background [57], $T = T_0 = 2.725 \pm 0.002$ K, we get $n_\gamma \approx 411$ cm^{-3}. The current critical density is

$$\rho_c = \frac{3}{8\pi^2} m_P^2 H_0^2 = 1.88 h^2 \times 10^{-29} \text{ g} \cdot \text{cm}^{-3} \tag{4.2}$$

where $h \equiv H_0/100 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, and $m_P \equiv G_N^{-1/2} = 1.22 \times 10^{19}$ GeV is the Planck mass. The current net baryon number density is

$$n_B = \frac{\Omega_B}{m_B} \rho_c = 1.1 \times 10^{-5} \Omega_B h^2 \text{ cm}^{-3} \tag{4.3}$$

and $\eta = 2.737 \times 10^{-8} \Omega_B h^2$ where $\Omega_B \equiv \rho_B/\rho_c$ is the ratio of baryon density and the critical density. The measured primordial deuterium and hydrogen abundances require $\Omega_B h^2 = 0.024 \pm 0.001$. [58] The ratio of baryons to photons is: [57]

$$\eta = \frac{n_B}{n_\gamma} = (6.103 \pm 0.38) \times 10^{-10}. \tag{4.4}$$

η is determined both from light element production in BBN and also from CMB measurements. This tells us that there is no large regions of antimatter in the universe, because if the universe was matter-antimatter symmetric, the baryon to photon ratio at 1 GeV should be: [59]

$$\frac{n_b}{n_\gamma} = \frac{n_{\bar{b}}}{n_\gamma} \simeq 10^{-18}. \tag{4.5}$$

The entropy density s scales as $R(t)^{-3}$. So we can also use the ratio of the baryon number density and entropy density

$$\eta_B \equiv \frac{n_B}{s} \tag{4.6}$$

to measure the baryon asymmetry, where

$$s = \frac{2\pi^2}{45} g_{*S,T} T^3 = 7.04 \left(\frac{g_{*S,T}}{3.91} \right) n_\gamma, \tag{4.7}$$

and

$$g_{*S,T} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T} \right)^3 \tag{4.8}$$

is the total effective number of massless degrees of freedom at the temperature T. So $\eta = 1.8 g_{*S,T} \frac{n_B}{s}$.

4.1 Conditions for baryogenesis

In order to produce the baryon asymmetry of the Universe (BAU), Andrei Sakharov suggested that three conditions must be satisfied:

1. Baryon number (B) violation.
2. C and CP (charge conjugation and parity) violation.
3. Thermal inequilibrium.

The big bang theory said the universe starting with all quantum number zero, and now the baryon number is not zero. Obviously, the baryon number must be violated. There must be some baryon number non-conserving interactions. But until now, there is no experiment shows that such interactions exist. For any theory that contains the baryon number violation must be constrained by the current lower bound of the proton lifetime (τ_P) [60]

$$\tau_P \gtrsim 10^{31} \sim 10^{33} \text{ years} \tag{4.9}$$

C and CP symmetry must be violated so that the matter and antimatter can generate at different rates, and net baryon number can be generated. We know that C symmetry is violated by weak interactions, and CP symmetry can be violated in kaon decays and the strange B^0 meson decays [61].

When the universe approaches thermal equilibrium, the baryon number density tends to be zero, the baryon asymmetry could be washed out. The baryon number non-conserving interactions must be able to occur in thermal inequilibrium. The universe was in thermal equilibrium at the beginning of the big bang, and remain in thermal equilibrium as long as the particles interaction rate is higher than the expansion rate of the universe. When the universe cooled down, and photons were free as the electrons were bounded to atoms, then the universe was out of thermal equilibrium.

Early baryogenesis proposals like GUT scale baryogenesis which takes place before inflation is not favored since the inflation will dilute away the baryon asymmetry. Modern proposals for developing the BAU take place after the end of the inflation, at or after the reheating. The SM electroweak baryogenesis and first order phase transition requires a low Higgs mass $m_H \leq 50$ GeV, which is excluded by experimental data. However, the SUSY electroweak baryogenesis can relax the limits of Higgs, sparticles mass to higher value, and the naturalness limits the value $m_A \lesssim 4 - 8$ TeV. We are going to discuss several baryogenesis mechanisms: thermal leptogenesis [62–67], non-thermal leptogenesis [68–73], leptogenesis from oscillating sneutrino decay [74, 75], leptogenesis via AD condensate, and AD baryogenesis. [74, 76–78] If in a model where gravitinos can be thermally produced in the early universe at a rate proportional to the reheat temperature T_R, then it may have the cosmological gravitino problem. [79, 80] If T_R is too high, then too much gravitino could be generated from the thermal production. If gravitino decays to or is the LSP, then there will be too much dark matter. If the gravitino is too long-lived and decays after BBN, it could decay to photons, leptons or mesons, and destroy the successful BBN predictions of the light nucleus abundances. [81–83] In the case of natural SUSY with mixed axion-higgsino dark matter, then similar constraints arise from axino and saxion production: WIMPs or axions can be overproduced, or light element abundances can be destroyed by late decaying axinos and saxions. [84]

4.2 Leptogenesis

Sphaleron process is very important for baryogenesis via leptogenesis. If net lepton number is generated, then baryon asymmetry can be generated due to the sphaleron process. Sphaleron process can convert baryons and anti-leptons (or anti-baryons and leptons) into each other. The baryon and lepton violation rate equations are [85]

$$\dot{B}(t) = -\gamma(t)\left[B(t) + \eta(t)\sum_{i=1}^{n_G} L_i(t)\right] \tag{4.10a}$$

$$\dot{L}_i(t) = -\frac{\gamma(t)}{n_G}\left[B(t) + \eta(t)\sum_{i=1}^{n_G} L_i(t)\right] + f_i(t) \tag{4.10b}$$

where n_G is the number of generations, $f_i(t)$ is the sources of the lepton numbers, and $\eta(t) \simeq 0.52 \pm 0.03$. So the rate is proportional to the baryon number (lepton number) at time t. Sphaleron conserves the $B - L$, and violates the $B + L$, so we need $B - L$ violation process to get baryon number asymmetry. So if we have an initial $B-L$ value, then after the sphaleron, we have a final B

$$B_f = \frac{8N_f + 4N_\varphi}{22N_f + 13N_\varphi}(B-L)_i = \frac{28}{79}(B-L)_i \tag{4.11}$$

where N_f is generations of fermions, and N_φ is number of Higgs doublets. In the SM, $N_f = 3$, and $N_\varphi = 1$. Leptogenesis provides a way to generate lepton asymmetry that can be realized in seesaw mechanism for neutrino mass generation. Baryon or lepton asymmetry that created before electroweak phase transition could be washed out by sphaleron process. Sphaleron process is efficient at temperature

$$10^2\text{GeV} < T < 10^{12}\text{GeV}.$$

The wash out rate $k(T)$ obeys the equation

$$\frac{\partial n_B}{\partial t} = -k(T)n_B. \tag{4.12}$$

The wash out rate is high at high temperature, the rate is [86, 87]

$$k(T) = -\frac{13n_f}{2}\frac{\Gamma_{\text{sph}}(T)}{VT^3} \sim A(T)e^{-E_{\text{sph}}/T} \tag{4.13}$$

where n_f is the number of fermion generations, the prefactor $A(T)$ doesn't contain exponential dependence on T, the sphaleron energy E_{sph} is of order

$$E_{\text{sph}} = \frac{c}{g^2}M_W. \tag{4.14}$$

and the sphaleron rate per unit volume $\Gamma_{\text{sph}}(T)/V$ is

$$\Gamma_{\text{sph}}(T)/V = T^4 e^{-E_{\text{sph}}/T}/V. \tag{4.15}$$

We can solve the Boltzmann equation to determine the right-handed neutrinos decay and inverse decay. Sphaleron process can convert the remaining net lepton number to net baryon number.

4.2.1 Leptogenesis via right-handed neutrino decay

4.2.1.1 Thermal leptogenesis (THL)

Thermal leptogenesis introduces three intermediate mass scale right-handed singlet neutrinos N_i $(i = 1, 2, 3)$ so that the type I see-saw mechanism generates a very light SM neutrino mass. The superpotential is

$$W = \frac{1}{2}M_i N_i N_i + h_{i\alpha} N_i L_\alpha H_u \tag{4.16}$$

where we assume a basis for the N_i masses which is diagonal and real. α is the lepton doublet generation index and $h_{i\alpha}$ are the neutrino Yukawa couplings. The see-saw mechanism generates a spectrum of three sub-eV mass neutrinos m_1, m_2 and m_3 and three heavy neutrinos $M_1 < M_2 < M_3$. In GUT-type theories, the typically mass of third generation heavy neutrino is $M_3 \sim 10^{15}$ GeV. If the three generations heavy neutrino masses are hierarchical like the quark masses, then we have $M_1/M_3 \sim m_u/m_t \sim 10^{-5}$, and so $M_1 \sim 10^{10}$ GeV. [88]

After inflation, the universe reheats to a temperature $T_R \gtrsim M_1$ thus thermally produces heavy neutrinos N_1. The N_1 decay into LH_u and $\bar{L}\bar{H}_u$ asymmetrically due to interference between tree and loop level decay diagrams which include CP violating interactions. The CP asymmetry factor ϵ_1 is [89–91]

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to LH_u) - \Gamma(N_1 \to \bar{L}\bar{H}_u)}{\Gamma_{N_1}} \tag{4.17a}$$

$$\simeq \frac{3}{8\pi}\frac{M_1}{\langle H_u\rangle^2} m_{\nu_3}\delta_{\text{eff}} \tag{4.17b}$$

where $\langle H_u\rangle \simeq 174$ GeV $\sin\beta$. δ_{eff} is an effective CP-violating phase which depends on the MNS matrix elements and is expected to be $\delta_{\text{eff}} \sim 1$. For hierarchical heavy neutrinos

$$\epsilon_1 \sim 2 \times 10^{-10} \left(\frac{M_1}{10^6 \text{ GeV}}\right)\left(\frac{m_{\nu_3}}{0.05 \text{ eV}}\right)\delta_{\text{eff}}. \tag{4.18}$$

The ultimate lepton asymmetry can be calculated from a coupled Boltzmann equation. [92] When N_1 is in thermal equilibrium, its ratio of number density n_{N_1} to entropy density s is proportional to $1/g_*$ where the effective degrees of freedom $g_* = 232.5$ for the MSSM. The lepton number density n_L to entropy density s ratio is then

$$\frac{n_L}{s} = \kappa\epsilon_1\frac{n_{N_1}}{s} \simeq \kappa\frac{\epsilon_1}{240} \tag{4.19}$$

where κ is the coefficient for washout effects and the efficiency of N_1 thermal production. Numerical evaluations of κ imply $\kappa \simeq 0.05 - 0.3$.

The induced lepton asymmetry converts to baryon asymmetry via $B + L$ violating but $B - L$ conserving sphaleron interactions. The ultimate baryon asymmetry is then [93]

$$\frac{n_B}{s} \simeq 0.35\frac{n_L}{s} \simeq 0.3 \times 10^{-10}\left(\frac{\kappa}{0.1}\right)\left(\frac{M_1}{10^9 \text{ GeV}}\right)\left(\frac{m_{\nu_3}}{0.05 \text{ eV}}\right)\delta_{\text{eff}} \tag{4.20}$$

provided that T_R is large enough that the N_1 are efficiently produced by thermal interactions: $T_R \gtrsim M_1$. Naively, this requires $T_R \gtrsim 10^{10}$ GeV although detailed calculations allow for $T_R \gtrsim 1.5 \times 10^9$ GeV. [92] This rather large lower bound on T_R potentially leads to conflict with the gravitino problem and violation of BBN bounds or overproduction of dark matter. In the event that late decaying relics inject entropy after N_1 decay is complete, then n_L/s is modified by an entropy dilution factor r: $n_L/s \to n_L/(r \times s)$.

In some variant thermal leptogenesis scenarios, the lower bound of T_R can be relaxed to a lower value. In the simple scenario of thermal leptogenesis, the flavor dependence is normally neglected by assuming the alignment of final state leptons and anti-leptons, i.e., $CP(L) = \bar{L}$. In general, however, one can consider the case in which the final state leptons and anti-leptons are not aligned and thus the flavor effect must be taken into account. Depending on the temperature at which dominant lepton asymmetry is generated, flavor effect can enhance the final asymmetry by up to an order of magnitude. [94, 95] On the other hand, one can also consider the case of nearly degenerate right handed neutrinos rather than a hierarchical spectrum. If the mass difference is as small as its decay width, i.e., $(M_1 - M_2) \sim \Gamma_{N_1}$, the CP asymmetry factor is resonantly enhanced so that a successful leptogenesis scenario is possible with $O(\text{TeV})$ right handed neutrino mass. [96–98]

We will examine the viability of various leptogenesis scenarios for natural SUSY with mixed axion-higgsino dark matter. We do not specify the structure of the neutrino sector, and only consider the simplest scenarios for the thermal leptogenesis. If one considers a specific neutrino sector in which flavor and/or resonant effects are important, then bounds from thermal leptogenesis may be modified.

4.2.1.2 Non-thermal leptogenesis via inflaton decay (NTHL)

As an alternative to thermal leptogenesis, non-thermal leptogenesis posits a large branching fraction of the inflaton field χ into $N_1N_1 : \chi \to N_1N_1$ which is followed by asymmetric N_1 decay to (anti-)leptons as before. In this case, the N_1 number density to entropy density ratio is given by [68–73]

$$\frac{n_{N_1}}{s} \simeq \frac{\rho_{\rm rad}}{s}\frac{n_\chi}{\rho_\chi}\frac{n_{N_1}}{n_\chi} \tag{4.21a}$$

$$= \frac{3}{4}T_R \times \frac{1}{m_\chi} \times 2B_r = \frac{3}{2}B_r\frac{T_R}{m_\chi} \tag{4.21b}$$

where $\rho_{\rm rad}$ is the radiation density once reheating has completed. ρ_χ is the energy density stored in the inflaton field just before inflaton decay. Thus, $\rho_{\rm rad} \simeq \rho_\chi$ and $\rho_\chi \simeq m_\chi n_\chi$. Here also B_r is the inflaton branching fraction into N_1N_1. So the generated lepton asymmetry is inflation model dependent. The lepton number to entropy ratio is then given by $n_L/s \simeq \epsilon_1 n_{N_1}/2$ where ϵ_1 is CP asymmetry factor. The lepton number asymmetry is converted to a baryon asymmetry via sphaleron interactions as before

$$\frac{n_B}{s} \simeq 0.35\frac{n_L}{s} \simeq 0.5\times 10^{-10} B_r\left(\frac{T_R}{10^6\ \text{GeV}}\right)\left(\frac{2M_1}{m_\chi}\right)\left(\frac{m_{\nu_3}}{0.05\ \text{eV}}\right)\delta_{\rm eff} \tag{4.22}$$

The resultant baryon asymmetry can match data provided $m_\chi > 2M_1$ and that the branching fraction is nearly maximal. The key difference from THL is that NTHL only requires a relatively low reheat temperature $T_R \gtrsim 10^6$ GeV, so there is no gravitino problem for a wide range of gravitino mass. For $T_R \lesssim 10^6$ GeV, then $\rho_{\rm rad}$ and consequently ρ_χ are reduced so that there is insufficient energy stored in the inflaton field to generate the required n_{N_1} number density.

4.2.2 Leptogenesis from coherent oscillating right-handed sneutrino decay (OSL)

In the previous two mechanisms, right-handed neutrinos and sneutrinos are produced by thermal scattering or inflaton decay. On the other hand, for sneutrinos, coherent oscillation can be a dominant production process. The decay of oscillating right-handed sneutrino produces lepton asymmetry which is given by [75]

$$n_L = \epsilon_1 M_1 \left|\tilde{N}_{1d}\right|^2 \tag{4.23}$$

where $\tilde{N}_{1d}$ is the sneutrino amplitude when it decays.

Once the universe is dominated by sneutrino oscillation, pre-existing relics are mostly diluted away and the universe is reheated again by sneutrino decay at $H = \Gamma_{N_1}$, where Γ_{N_1} is the sneutrino decay rate. The decay temperature T_{N_1} is determined by

$$T_{N_1} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{M_P \Gamma_{N_1}} \tag{4.24}$$

where g_* is the number of degree of freedom at $T = T_{N_1}$. And the entropy density is

$$s = \frac{2\pi^2}{45} g_* T_{N_1}^3. \tag{4.25}$$

At the time of sneutrino decay, the energy density of sneutrino oscillation is dominantly transferred to radiation energy density, so one can find an additional relation as

$$\rho_{N_1} = M_1^2 \left|\tilde{N}_{1d}\right|^2 = \frac{\pi^2}{30} g_* T_{N_1}^4. \tag{4.26}$$

From these relations, one finds the lepton number to entropy ratio

$$\frac{n_L}{s} = \epsilon_1 \frac{\rho_{N_1}}{M_1}\frac{1}{s} = \frac{3}{4}\epsilon_1 \frac{T_{N_1}}{M_1} \tag{4.27a}$$

$$\simeq 1.5 \times 10^{-10} \left(\frac{T_{N_1}}{10^6 \text{ GeV}}\right)\left(\frac{m_{\nu_3}}{0.05 \text{ eV}}\right)\delta_{\text{eff}}. \tag{4.27b}$$

The baryon asymmetry is obtained via sphaleron process, and thus baryon number is given by $n_B/s \simeq 0.35 n_L/s$. Thus, enough baryon number can be generated for $T_{N_1} \gtrsim 10^6$ GeV.

In this scenario, it is interesting that the effective reheat temperature is $O(T_{N_1})$ for thermal relic particles, since sneutrino domination dilutes pre-existing particles when it decays. [75] It is assumed that inflaton decay after sneutrino oscillation starts. If sneutrino oscillation starts after inflaton decay, effective reheat temperature is given by $2T_{N_1}(T_R/T_{R_C})$ where T_{R_C} is the temperature at which sneutrino oscillation starts. Therefore, we will consider T_{N_1} a reheat temperature for production of gravitinos, axinos and saxions in the case of leptogenesis from oscillating sneutrino decay.

4.2.3 Affleck-Dine leptogenesis (ADL)

Affleck-Dine (AD) [76–78] leptogenesis makes use of the LH_u flat direction in the scalar potential. [74,99] This direction is lucrative in that it is not plagued by Q-balls which are problematic for flat directions carrying baryon number [100] and also because the rate for baryogenesis can be linked to the mass of the lightest neutrino, leading to a possible consistency check via observations of neutrinoless double beta decay ($0\nu\beta\beta$). [101]

In the case of the LH_u flat direction, F-flatness is only broken by higher dimensional operators which also give rise to neutrino mass via the see-saw mechanism:

$$W = \frac{1}{2M_i}(L_i H_u)(L_i H_u) \tag{4.28}$$

where M_i is the heavy neutrino mass scale. Here M_i contains neutrino Yukawa coupling, i.e., $1/M_i = y_{\nu i}^2/M_{N_i}$, so it can be larger than M_P for small $y_{\nu i}$. The most efficient direction is that for which $i = 1$ corresponding to the lightest neutrino mass: $m_{\nu_1} \sim \langle H_u \rangle^2 / M_1$ in a basis where the neutrino mass matrix is diagonal. The Affleck-Dine field ϕ then occurs as

$$\tilde{L}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi \\ 0 \end{pmatrix} \quad H_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi \end{pmatrix}. \tag{4.29}$$

The scalar potential is

$$V = V_{\text{SB}} + V_H + V_{\text{TH}} + V_F \tag{4.30}$$

where

$$V_{\text{SB}} = m_\phi^2 |\phi|^2 + \frac{m_{\text{SUSY}}}{8M}(a_m \phi^4 + h.c.) \tag{4.31a}$$

$$V_H = -c_H H^2 |\phi|^2 + \frac{H}{8M}(a_H \phi^4 + h.c.) \tag{4.31b}$$

$$V_{\text{TH}} = \sum_{f_k|\phi|<T} c_k f_k^2 T^2 |\phi|^2 + \frac{9\alpha_s^2(T)}{8} T^4 \ln\left(\frac{|\phi|^2}{T^2}\right) \tag{4.31c}$$

$$V_F = \frac{1}{4M^2}|\phi|^6 \tag{4.31d}$$

where $c_H \simeq |a_H| \simeq 1$, $|a_m| \sim 1$, $M \equiv \langle H_u \rangle^2 / m_{\nu_1}$, and $\langle H_u \rangle = 174$ GeV $\times \sin\beta$. We take $\sin\beta \approx 1$ for simplicity. The first contribution V_{SB} is the SUSY breaking contribution where $m_\phi^2 = (\mu^2 + m_{H_u}^2 + m_L^2)/2$. [102] The second contribution arises from SUSY breaking during inflation [77,78] where $3H_\mathrm{I}^2 m_\mathrm{GUT}^2 \simeq |F_\chi|^2$ with H_I being the Hubble constant during inflation and where F_χ is the inflaton F-term which fuels inflation, and χ is the inflaton field which dominated the energy density during inflation. In the expression V_H, the coefficient c_H is generally to be expected > 0. When $c_H H^2 > m_\phi^2$, this negative $-c_H H^2|\phi|^2$ term provides an instability of the potential at $|\phi| = 0$. Then a large VEV of ϕ can form at one of the four minimum points of the potential V

$$\langle\phi\rangle \simeq \sqrt{MH_\mathrm{I}} \tag{4.32}$$

$$\arg(\phi) = [(-\arg(a_H) + (2n+1)\pi]/4 \tag{4.33}$$

where $H_\mathrm{I} \sim 10^{13}$ GeV $\gg m_\phi$ and $m_{3/2}\,|a_m|$, and $n = 0 \sim 3$. See Fig. 4.1. The second term in V_H is the Hubble induced trilinear SUSY breaking term. The term V_F is the up-lifting F-term contribution arising from the higher dimensional operator W. Lastly, the term V_TH arises from thermal effects after the inflation ended. [103,104] The first term is generated when the light particle species which couple to the AD field are produced in the thermal plasma, while the plasma are produced by the oscillation and decay of the inflaton χ with temperature $T \simeq (T_R^2 M_G H)^{1/4}$. The second term is generated by effective gauge coupling running from heavy effective mass of particles which couple to the AD field. Here, f_k represents the Yukawa/gauge couplings of ϕ and c_k is expected ~ 1.

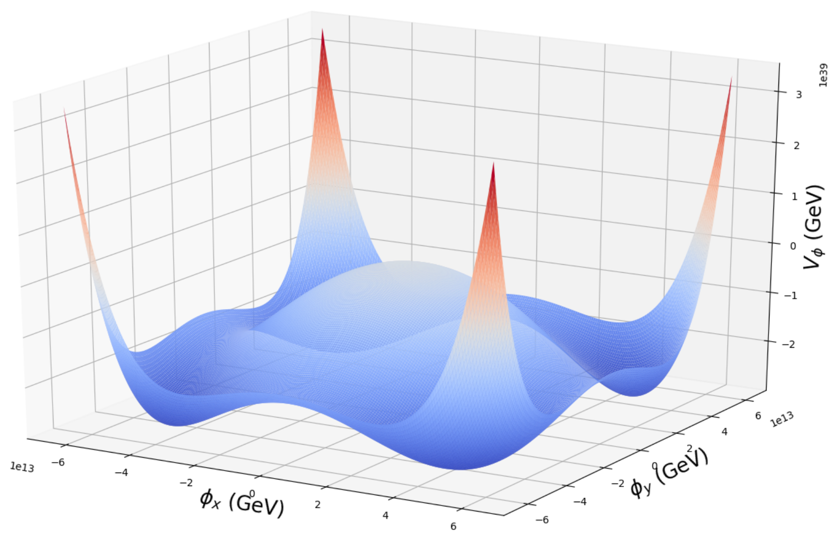

Figure 4.1: The four minimum points of the potential V are at $\langle\phi\rangle \simeq \sqrt{MH_I}$, $\arg(\phi) = [(-\arg(a_H) + (2n+1)\pi]/4$. We take $\arg(a_H) = 0$ in the figure.

After the end of the inflation, H started to decrease from H_I to a much smaller value, so $\langle\phi\rangle$ was getting smaller but still tracking the minimum of the potential at $\langle\phi\rangle \simeq \sqrt{MH}$. The absolute value of the negative H term would exceed by one of these three terms in the potential V:

$$m_\phi^2|\phi|^2, \quad \sum_{f_k|\phi|<T} c_k f_k^2 T^2 |\phi|^2, \quad a_g \alpha_S^2(T) T^4. \tag{4.34}$$

The potential V became dominated by the largest of above three terms. ϕ started to oscillate. The net lepton number is fixed when ϕ started to oscillate, after the end of inflation and before the end of reheat. The evolution of ϕ was determined by which term dominated the potential in the equation of motion.

The equation of motion for the AD field is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0 \tag{4.35}$$

which is the usual equation for a damped harmonic oscillator. Once the AD condensate forms, then the universe continues expansion and the Hubble-induced terms decrease. The minimum of the potential decreases as does the value of the condensate. When H decreases to a value [105]

$$H_{\rm osc} = \max\left[m_\phi, H_i, \alpha_s T_R \left(\frac{9M_P}{8M}\right)^{1/2}\right] \tag{4.36}$$

where

$$H_i = \min\left[\frac{1}{f_i^4}\frac{M_P T_R^2}{M^2}, (c_i^2 f_i^4 M_P T_R^2)^{1/3}\right]. \tag{4.37}$$

where the value of c_i and f_i are in the Table 4.1. Then the AD field begins to

c_i	$1/4$	$1/2$	$3/4$	$1/4$
f_i	$\sqrt{\frac{g_1^2+g_2^2}{2}}$	$\frac{g_2}{\sqrt{2}}$	$\frac{y_u}{\sqrt{2}}$	$\frac{y_{L1}}{\sqrt{2}}$

Table 4.1: The value of c_i and f_i where $y_u \simeq m_{\rm up}/\langle H_u\rangle$ and $y_{L1} \simeq 0.4 \times 0.061$.

oscillate, and a non-zero lepton number arises

$$n_L = \frac{i}{2}(\dot{\phi}^*\phi - \phi^*\dot{\phi}). \tag{4.38}$$

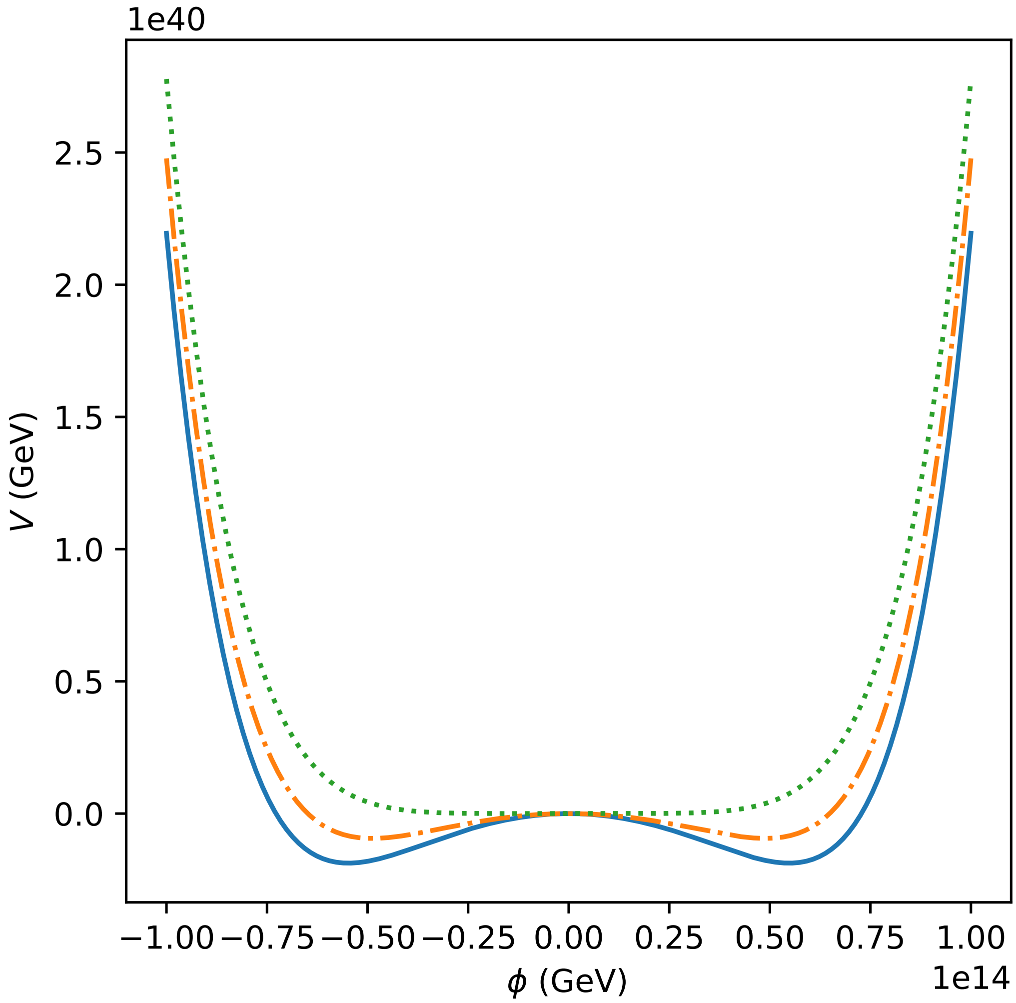

Figure 4.2: Shape of the potential of ϕ for different H before and after the inflation in ϕ vs. V_ϕ plane. The solid line is for $H = H_I$ during and at the end of the inflation, and the ϕ field stay at the minimum. The dashdotted line is for $H < H_I$ after the inflation, and the ϕ field keep tracking the minimum. The dotted line is for small H that the negative $H|\phi|^2$ term was exceeded by other positive $|\phi|^2$ term, and the ϕ field oscillates around the true minimum.

So we can get

$$\dot{n}_L = \frac{i}{2}(\ddot{\phi}^* \phi - \phi^* \ddot{\phi}), \tag{4.39}$$

and since $|\phi|^2 = \phi\phi^*$ and $|\phi|^6 = \phi^3\phi^{*3}$, we can obtain expressions for $\partial V/\partial\phi^*$ and $\partial V^*/\partial\phi$, then plug into Eq. (4.35), and use relation $\phi^4 - \phi^{*4} = 2i\mathrm{Im}(\phi^4)$ to get

$$\dot{n}_L + 3Hn_L = \frac{m_{\mathrm{SUSY}}}{2M}\mathrm{Im}(a_m\phi^4) + \frac{H}{2M}\mathrm{Im}(a_H\phi^4). \tag{4.40}$$

The first term on the RHS is dominant, and using

$$\frac{d}{dt}R^3 n_L = R^3\dot{n}_L + 3R^3 H n_L \tag{4.41}$$

where $R^3 n_L \propto t$ is the total lepton number. We can integrate from early times up to $t = 1/H_{\mathrm{osc}}$ to find

$$n_L = \frac{m_{\mathrm{SUSY}}}{2M}\mathrm{Im}(a_m\phi^4)t_{\mathrm{osc}} \tag{4.42a}$$

$$\simeq \frac{1}{3}(m_{\mathrm{SUSY}}|a_m|)MH_{\mathrm{osc}}\delta_{\mathrm{ph}} \tag{4.42b}$$

where $\delta_{\mathrm{ph}} = \sin(4\arg\phi + \arg a_m)$. We have used the relations, $\phi(t_{\mathrm{osc}}) \sim \sqrt{MH_{\mathrm{osc}}}$ and $t_{\mathrm{osc}} = 2/(3H_{\mathrm{osc}})$ for an oscillating field/matter-dominated universe. After ϕ started to oscillate, the net lepton number is fixed. During the inflaton oscillation dominated era, the produced lepton number density is diluted by an $(H/H_{\mathrm{osc}})^2$ factor. The entropy density at T_R is determined by the relation $3M_P^2 H_R^2 = \rho_{\mathrm{rad}} = s \times 3T_R/4$ where H_R is the Hubble parameter at T_R. The lepton number to entropy ratio is conserved once the era of reheat is completed

$$\frac{n_L}{s} = \frac{MT_R}{12M_P^2}\left(\frac{m_{\mathrm{SUSY}}|a_m|}{H_{\mathrm{osc}}}\right)\delta_{\mathrm{ph}}. \tag{4.43}$$

This quantity has the virtue of being T_R independent if H_{osc} is determined by the third (thermal) contribution in Eq.(4.36). The lepton asymmetry is then converted to a baryon asymmetry via sphaleron interactions $n_B/s \simeq 0.35(n_L/s)$ and replacing M by $\langle H_u \rangle^2 / m_{\nu 1}$, the baryon to entropy ratio is

$$\frac{n_B}{s} = 0.029 \times \frac{\langle H_u \rangle^2 T_R}{m_{\nu 1} M_P^2} \left(\frac{m_{\mathrm{SUSY}} |a_m|}{H_{\mathrm{osc}}} \right) \delta_{\mathrm{ph}}. \tag{4.44}$$

It is found [105] that $n_B/s \sim 10^{-10}$ can be developed roughly independent of T_R for $T_R \gtrsim 10^5$ GeV for $m_{\nu 1} \sim 10^{-9}$ eV and for $m_{\mathrm{SUSY}} |a_m| \sim 1$ TeV.

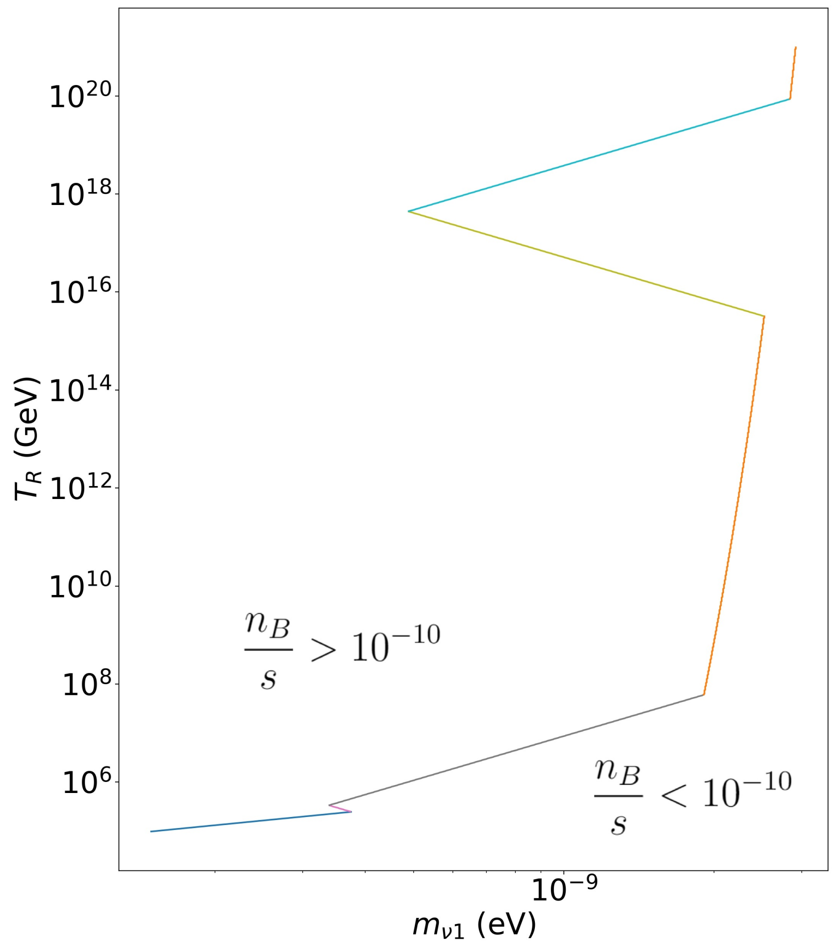

Figure 4.3: Plot of $n_B/s = 10^{-10}$ in $m_{\nu 1}$ vs. T_R plane.

In Fig. 4.3, we chose $|a_m| = 1$, $\delta_{\rm ph} = 1$, $m_{\rm up} = 3$ MeV, $\sin\beta = 0.995$, $m_\phi = m_{3/2} = 1$ TeV and $n_B/s = 10^{-10}$. The blue segment of the line is when $H_{\rm osc} = m_\phi$. The pink segment is when $H_{\rm osc} = 4M_P T_R^2/(M^2 y_u^4)$. The black segment is when $H_{\rm osc} = (9y_u^4 M_P T_R^2/64)^{1/3}$. The orange segment is when $H_{\rm osc} = \alpha_s T_R\sqrt{9M_P/(8M)}$. The yellow segment is when $H_{\rm osc} = 4M_P T_R^2/(M^2 y_{L1}^4)$. The cyan segment is when $H_{\rm osc} = (y_{L1}^4 M_P T_R^2/64)^{1/3}$. However, $T_R > 10^{12}$ GeV is not favored, we will focus on $T_R < 10^{12}$ GeV. We can see that $f_i = y_u/\sqrt{2}$ played an important role in this situation. The thermal effect terms greatly reduced the baryon asymmetry dependence of T_R.

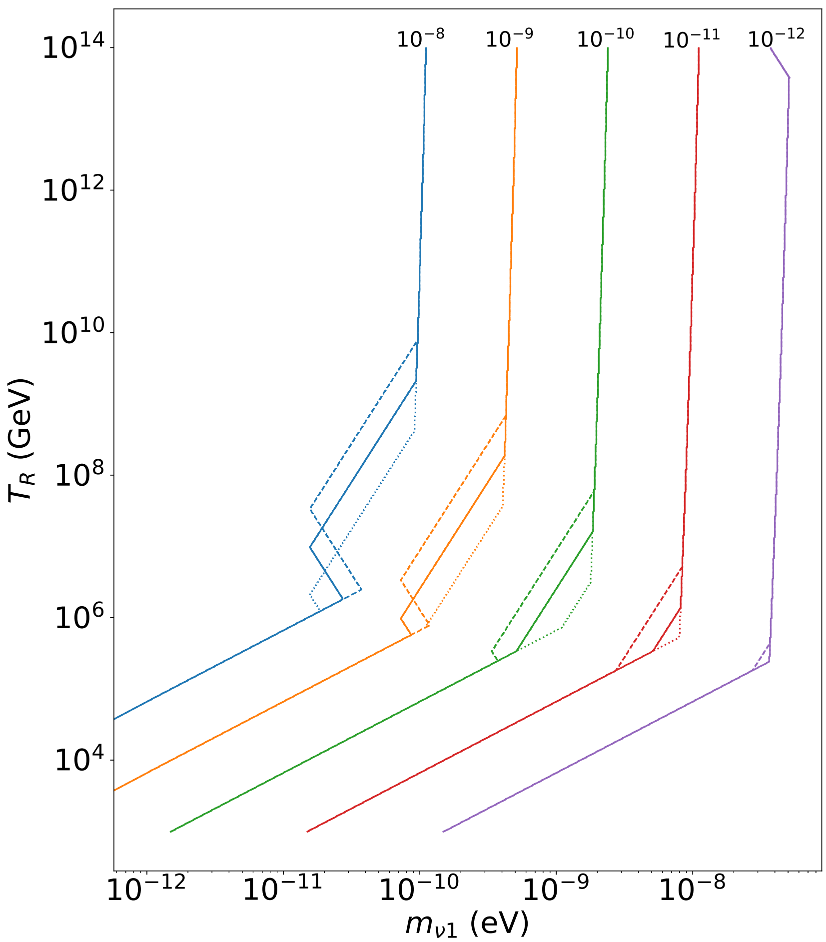

Figure 4.4: Plot of $n_B/s = 10^{-8}$, 10^{-9}, 10^{-10}, 10^{-11}, 10^{-12} and $m_{\rm up} = 1.5$ (dot line), 2.2 (solid), 3 (dash) MeV in $m_{\nu 1}$ vs. T_R plane. Here, we assume $m_\phi = m_{3/2} = 1$ TeV.

In Fig. 4.4, we showed plots of $n_B/s = 10^{-8},\ 10^{-9},\ 10^{-10},\ 10^{-11},\ 10^{-12}$ and $m_{\rm up} = 1.5,\ 2.2,\ 3$ MeV. We can see that lighter $m_{\nu 1}$ produce more baryon asymmetry. Heavier $m_{\rm up}$ gives more impact on the thermal effect. For small baryon asymmetry, thermal term $T^2|\phi|^2$ took place early, but less impact on the baryon asymmetry. For $n_B/s = 10^{-12}$, the $T^2|\phi|^2$ term even has no contribution for $m_{\rm up} \lesssim 2.2$ MeV.

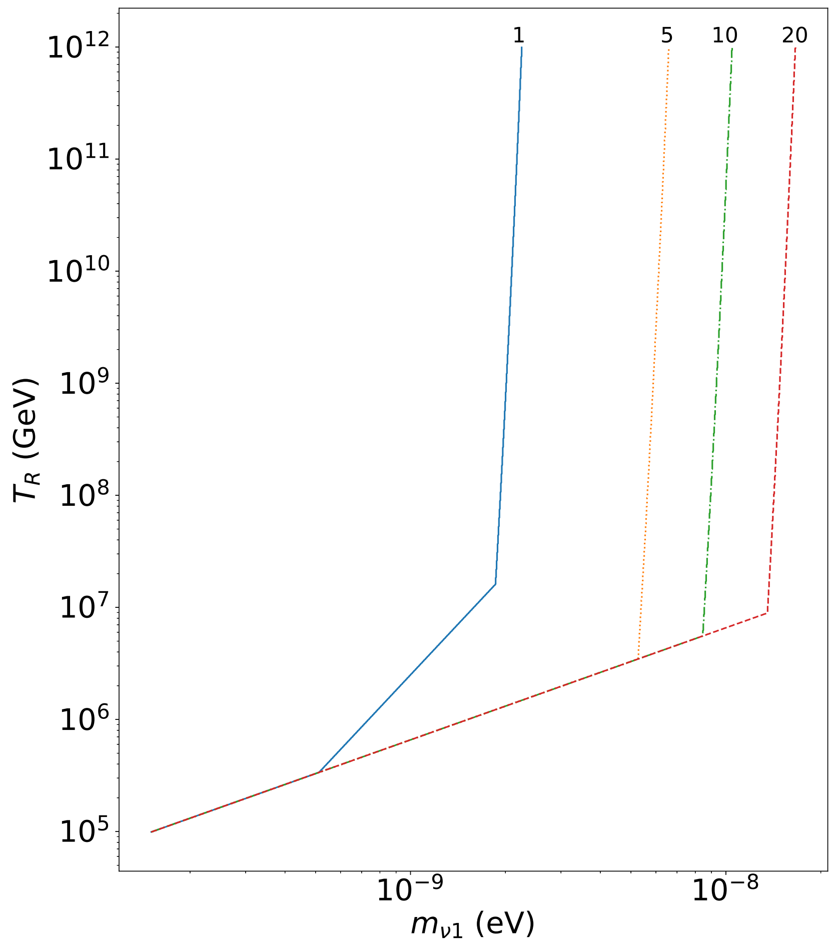

Figure 4.5: Plot of $m_\phi = m_{3/2} = 1,\ 5,\ 10,\ 20$ TeV where $n_B/s = 10^{-10}$, $m_{\rm up} = 2.2$ MeV in $m_{\nu 1}$ vs. T_R plane.

In Fig. 4.5, we compared $m_{3/2} = m_\phi = 1,\ 5,\ 10$ and 20 TeV in the $m_{\nu 1}$ vs. T_R plane where we took $n_B/s = 10^{-10}$ and $m_{\rm up} = 2.2$ MeV. The solid/dotted/dashdotted/dash lines are for $m_{3/2} = 1,\ 5,\ 10,\ 20$ TeV, respectively. Heavier $m_{3/2}$ and m_ϕ produce lower baryon asymmetry. This is because the larger value of m_ϕ, the later thermal term exceeds the m_ϕ value and dominates $H_{\rm osc}$, so later oscillation of ϕ field and higher T_R are needed to start oscillation.

4.3 Constraints in the T_R vs. $m_{3/2}$ plane for various f_a

To compute the mixed axion WIMP dark matter abundance in SUSY axion models, we adopt the eight coupled Boltzmann equation computation. [48, 49, 106] See Appendix B. The Boltzmann equations track the energy densities of the various constituents of the early universe while accounting for: 1. Hubble expansion and dilution, 2. particle production and annihilation from scattering reactions, 3. particle production from decay and inverse decay processes, 4. particle disappearance due to decays and 5. particle production via bosonic coherent motion (BCM). The Boltzmann equations allow for species which may be in or out of thermal equilibrium. The number densities n_i and energy densities ρ_i for thermal species $i = a, s, \tilde{a}$ obey the following equations

$$\begin{aligned}\frac{dn_i}{dt} + 3Hn_i = &\sum_{\rm j\in MSSM} \left(\bar{n}_i\bar{n}_j - n_i n_j\right) \langle\sigma v\rangle_{ij} - \Gamma_i m_i \frac{n_i}{\rho_i}\left(n_i - \bar{n}_i \sum_{i\to a+b} \mathcal{B}_{ab}\frac{n_a n_b}{\bar{n}_a\bar{n}_b}\right)\\ &+ \sum_a \Gamma_a \mathcal{B}_i m_a \frac{n_a}{\rho_a}\left(n_a - \bar{n}_a \sum_{a\to i+b} \frac{\mathcal{B}_{ib}}{\mathcal{B}_i}\frac{n_i n_b}{\bar{n}_i\bar{n}_b}\right),\end{aligned} \tag{4.45}$$

$$\begin{aligned}\frac{d\rho_i}{dt} + 3H(\rho_i + P_i) = &\sum_{\rm j\in MSSM} \left(\bar{n}_i\bar{n}_j - n_i n_j\right) \langle\sigma v\rangle_{ij} \frac{\rho_i}{n_i} - \Gamma_i m_i \left(n_i - \bar{n}_i \sum_{i\to a+b} \mathcal{B}_{ab}\frac{n_a n_b}{\bar{n}_a\bar{n}_b}\right)\\ &+ \sum_a \Gamma_a \mathcal{B}_i \frac{m_a}{2}\left(n_a - \bar{n}_a \sum_{a\to i+b} \frac{\mathcal{B}_{ib}}{\mathcal{B}_i}\frac{n_i n_b}{\bar{n}_i\bar{n}_b}\right)\end{aligned} \tag{4.46}$$

where $\mathcal{B}_{ab} \equiv BR(i \to a+b)$, $\mathcal{B}_{ib} \equiv BR(a \to i+b)$, $\mathcal{B}_i \equiv \sum_b \mathcal{B}_{ib}$. Here number densities in thermal equilibrium are denoted by $\bar{n}_i$. The zero temperature decay widths are denoted by Γ_i. Among terms on the RHS of Eq. (4.45), the first term describes the scattering processes of the species of concern with ordinary MSSM particles. On the other hand, the second term shows particle disappearance (production) via decay (inverse decay) processes while the third term represents particle production (disappearance) via decay (inverse decay) of heavier particles. The same explanation also holds for the ρ_i equation in Eq. (4.46). The BCM components of the axion and saxion are simply determined by their initial energy density and decay widths as follows

$$\frac{dn_i^{\rm BCM}}{dt} + 3Hn_i^{\rm BCM} = -\Gamma_i m_i \frac{n_i^{\rm BCM}}{\rho_i^{\rm BCM}} n_i^{\rm BCM}, \tag{4.47}$$

$$\frac{d(\rho_i^{\rm BCM}/n_i^{\rm BCM})}{dt} = 0. \tag{4.48}$$

The initial amplitudes are parametrized as $\theta_i = a_0/f_a$ and $\theta_s = s_0/f_a$. In the following analyses, we consider $\theta_s = 1$ as its natural initial condition while the axion amplitude is adjustable to complete the dark matter density if the Higgsino-like neutralino is underabundant.

In this treatment, one begins at temperature $T = T_R$ and tracks the energy densities of radiation, WIMPs, gravitinos, axinos, saxions (BCM- and thermally-produced) and axions (BCM-, thermally- and saxion decay-produced). Whereas WIMPs quickly reach thermal equilibrium at $T = T_R$, the axinos, saxions, axions and gravitinos do not, even though they are still produced thermally. In SUSY KSVZ, the axino, axion and saxion thermal production rates are all proportional to T_R while in SUSY DFSZ model they are largely independent of T_R. The calculation depends sensitively on the sparticle mass spectrum, on the reheat temperature T_R, on the gravitino mass $m_{3/2}$ and on the PQ model (KSVZ or DFSZ), the PQ parameters f_a, the axion mis-alignment angle θ_i, the saxion angle θ_s and on a parameter ξ_s which accounts for the model dependent saxion to axion coupling. [25] Here, we adopt the choices $\xi_s = 0$ ($s \to aa,\ \tilde{a}\tilde{a}$ decays turned off) or $\xi_s = 1$ ($s \to aa,\ \tilde{a}\tilde{a}$ decays turned on).

In order to solve the coupled Boltzmann equations, it is important to know the axino, saxion and gravitino decay rates. The gravitino decay rates are adopted from Ref. [107] while the axino and saxion decay rates are given in Ref. [108,109] for SUSY KSVZ and in Ref. [36] for SUSY DFSZ. The axino decays via loops involving the heavy quark Q field such that $\tilde{a} \to g\tilde{g}$, $\tilde{Z}_i\gamma$ and $\tilde{Z}_iZ$ in SUSY KSVZ. In SUSY DFSZ, the axino couples directly to Higgs superfields yielding faster decay rates into gauge/Higgs boson plus gaugino/higgsino states. In SUSY KSVZ, the saxion decays via $s \to gg,\ \tilde{g}\tilde{g}$ and, when $\xi_s = 1$, also to aa and $\tilde{a}\tilde{a}$ (if kinematically allowed). The decay $s \to aa$ leads to production of dark radiation as parametrized by $\Delta N_{\rm eff}$ which is the effective number of neutrinos. In SUSY DFSZ, the saxion decays directly to gauge- or Higgs-boson pairs or to gaugino/higgsino pairs. [36] If $\xi_s = 1$, then also $s \to aa$ or $\tilde{a}\tilde{a}$. In the case where axinos or saxions decay to SUSY particles (leading to WIMPs), then WIMPs may re-annihilate.

For the SUSY mass spectrum, we generate a natural SUSY model within the context of the 2-extra parameter non-universal Higgs (NUHM2) model with $m_0 = 5$ TeV, $m_{1/2} = 0.7$ TeV, $A_0 = -8.4$ TeV and $\tan\beta = 10$. We take $\mu = 125$ GeV and $m_A = 1$ TeV. The spectrum is generated using IsaSUGRA 7.84. [110] The value of $m_{\tilde{g}} = 1.8$ TeV, so now the model is slightly below LHC13 constraints. The value of $m_h = 125$ GeV and $\Delta_{EW} = 20$ so the model is highly natural.

Higgsino-like WIMPs with mass $m_{\tilde{Z}_1} = 115.5$ GeV are thermally underproduced so that $\Omega^{\rm TP}_{\tilde{Z}_1} h^2 = 0.007$ using IsaReD. [111] In all frames, we take $m_{\tilde{a}} = m_s = m_{3/2}$ as is roughly expected in gravity-mediated SUSY breaking models. [25,112] Since we take $m_{\tilde{a}} = m_s$, then $s \to \tilde{a}\tilde{a}$ decays are never a factor in our results.

The results of the dark matter abundance calculation for natural SUSY may be found in Ref. [48, 49, 106] where the relic abundance of WIMPs and axions are plotted typically versus the PQ scale f_a. At low $f_a \sim 10^9 - 10^{11}$ GeV, then the axion supermultiplet couplings are sufficiently large that axinos and saxions decay well before neutralino freeze-out so that the thermally-produced neutralino abundance is valid while axions make up the remainder of dark matter via axionic BCM. As f_a increases, then axinos and saxions decay more slowly. If they decay after neutralino freeze-out, then they may add a non-thermal component to the neutralino relic abundance. If a sufficient amount of neutralinos are produced at the axino/saxion decay temperature, then they may re-annihilate yielding again an enhanced abundance. At very large $f_a \sim 10^{13} - 10^{15}$ GeV, then saxion production via BCM can be huge. Saxion decays to SUSY particles may bolster the neutralino abundance to values far beyond the measured DM abundance in which case the parameter choices are excluded. However, if saxions dominantly decay to SM particles then entropy dilution occurs which can reduce the abundance of any relics present during decay. In either case, saxion decays after the onset of BBN can lead to disolution of light elements and such cases would be ruled out by BBN limits on late decaying neutral relics. [113] If ξ_s is large, then $s \to aa$ decay can produce large amounts of dark radiation, frequently violating observational limits on $\Delta N_{\rm eff}$.

It is reasonable to ask: is it sufficient to present results based on a single SUSY benchmark point? In our case, it is for the following reasons. We restrict our analysis to natural SUSY where $\mu \sim 100-300$ GeV but where the remaining sparticles lie in the multi-TeV regime as required by recent LHC search limits and by the measured value of m_h. Now the sparticle mass spectrum enters the dark matter abundance calculation mainly through the decay widths (lifetimes) of the axinos and saxions. For natural SUSY, in the KSVZ case the axino decays dominantly to $\tilde{g}g$ when this mode is open and where $m_{\tilde{g}} \lesssim 2-4$ TeV is bounded from above by naturalness. [42, 114, 115] Since this decay mode is almost always open, the axino decay width mainly depends on $m_{\tilde{a}}(\equiv m_{3/2})$ and f_a and not on the SUSY spectrum. In SUSY KSVZ, the saxion mainly decays as $s \to gg$ (and $s \to aa$ in the $\xi_s = 1$ case). Thus, the saxion decays are rather independent of natural SUSY spectrum variations. In the DFSZ case, the axino decays to higgsino+Higgs or higgsino+vector boson and since μ is required small, these decay modes always dominate and again the axino decay pattern depends mainly on μ, $m_{\tilde{a}}$ and f_a and not upon variations in the natural SUSY spectra. In SUSY DFSZ, the saxion decays dominantly into higgsino pairs (or into aa for $\xi_s = 1$) and as these modes are always open, is again quite independent of the natural SUSY spectrum.

In all the ensuing plots, the light-blue region corresponds to the parameter space where all BBN, DM and dark radiation constraints are satisfied. The red region corresponds to BBN excluded region, gray to overproduction of dark matter and brown to $\Delta N_{\rm eff} > 1$. Red and brown solid lines show the boundaries of excluded regions due to BBN and dark radiation respectively.

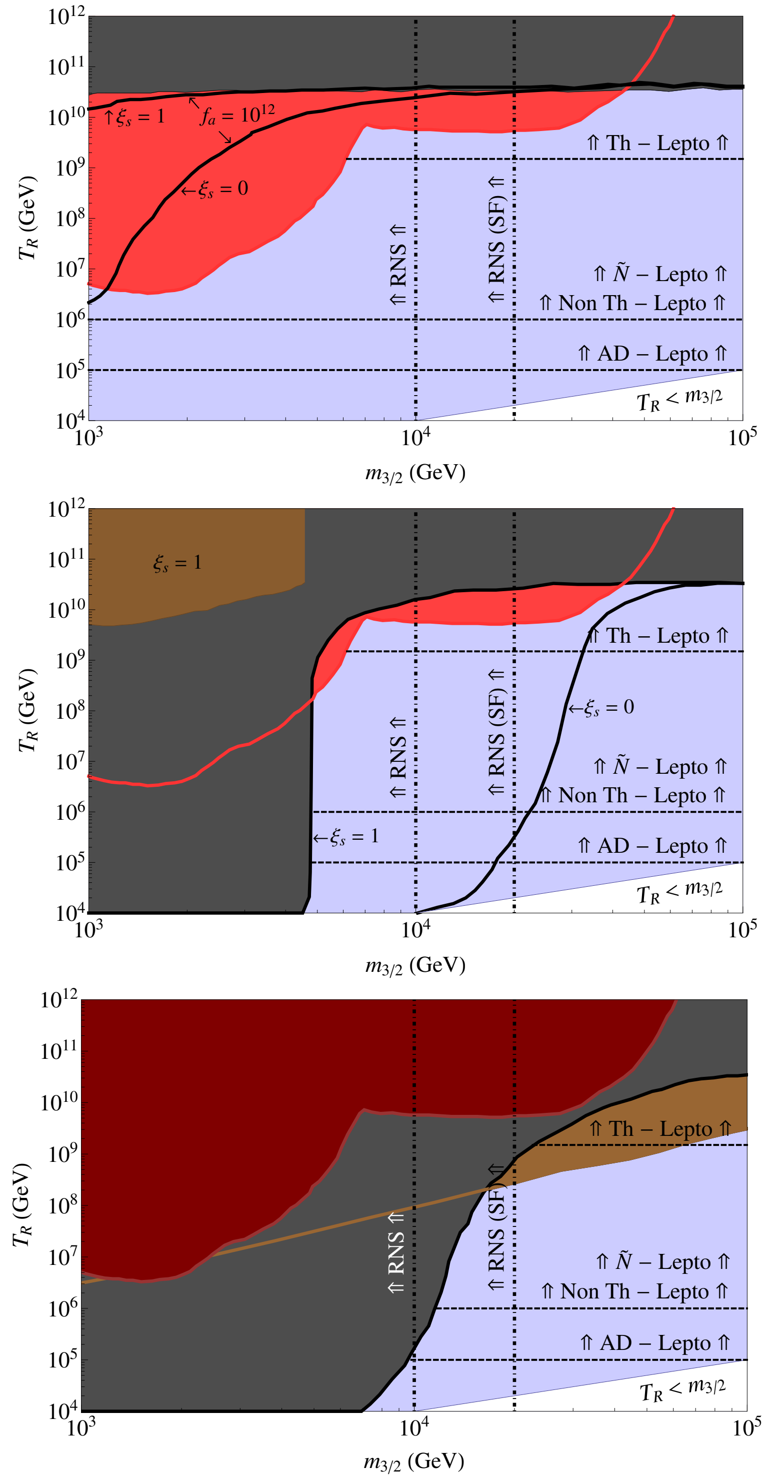

Figure 4.6: Plot of allowed regions in T_R vs. $m_{3/2}$ plane in the SUSY DFSZ axion model for *a*) $f_a = 10^{11}$ and 10^{12} GeV, *b*) $f_a = 10^{13}$ GeV, for $\xi_s = 0$ and 1 and *c*) $f_a = 10^{14}$ GeV for $\xi_s = 1$. For $f_a = 10^{11}$ GeV, $T_R > 10^{11}$ GeV is forbidden to avoid PQ symmetry restoration. We take $m_s = m_{\tilde{a}} \equiv m_{3/2}$ in all plots.

4.3.1 SUSY DFSZ model

Our first results of allowed regions in the T_R vs $m_{3/2}$ plane are shown in Fig. 4.6. In frame a), we first take $f_a = 10^{11}$ GeV and 10^{12} GeV and show allowed and excluded regions. For lower values of f_a, DM density is enhanced by gravitino decay only and BBN constraints are violated by late-decaying gravitinos since axinos and saxions are short-lived. For $f_a < 10^{11}$ GeV, BBN bounds and DM exclusion contours can be read from Fig. 4.6 once the region $T_R > f_a$ is omitted. As we increase f_a to 10^{11} GeV, then the axino and saxion decay rates are suppressed and they decay later. However, they still typically decay before neutralino freeze-out and thus do not change the picture.

The gray band at the top of frame a) is forbidden due to overproduction of WIMP dark matter due to thermal gravitino production and decay well after WIMP freeze-out. This occurs for $T_R \gtrsim 3 \times 10^{10}$ GeV when $f_a = 10^{11}$. The red-shaded region occurs due to violation of BBN constraints on late-decaying neutral relics. In the case of frame a), this comes again from gravitino production along with decay after the onset of BBN. Here, we use a digitized version of BBN constraints from Jedamzik [113] which appear in the $\Omega_X h^2$ vs. τ_X plane where X stands for the quasi-stable neutral particle, $\Omega_X h^2$ is its would-be relic abundance had it not decayed and τ_X is its lifetime. The curves also depend on the X-particle hadronic branching fraction B_h and on the mass m_X. Ref. [113] presents results for $m_X = 0.1$ and 1 TeV and we extrapolate between and beyond these values for alternative mass cases. Together, the red- and gray-shaded regions constitute the well-known gravitino problem: thermal gravitino production, which is proportional to T_R [116], can lead to overproduction of decay-produced WIMPs or violations of BBN constraints.

In comparison, we also show several lines. The black vertical lines show the upper bound on gravitino mass from naturalness ($\Delta_{\rm EW} < 30$) that arises on m_0 in the NUHM2 model (labelled "RNS" at $m_{3/2} = 10$ TeV for universal generations and "RNS SF" at $m_{3/2} = 20$ TeV for split families [42, 114]) if m_0 were directly determined by $m_{3/2}$, i.e., $m_0 = m_{3/2}$. In our numerical study, however, the SUSY spectrum is fixed in order to examine various leptogenesis scenarios in the context of a natural SUSY model. We scan $m_{3/2}$ values independently of the SUSY spectrum in order to separate out constraints from the gravitino problem (augmented by the axino and saxion problems). Thus larger $m_{3/2}$ may be allowed if one can find a UV model to realize the natural SUSY spectrum that we show here. Although these bounds are not directly applicable to our numerical results, we regard the parameter space with $m < 10$ TeV (or 20 TeV) as the natural gravitino mass region. In addition, we show the regions where various leptogenesis mechanisms can account for the BAU. The region above $T_R = 1.5 \times 10^9$ GeV is where thermal leptogenesis (THL) can occur. From the plot, we see the viable region, colored as light-blue, is bounded by $m_{3/2} \gtrsim 5$ TeV by BBN, by $m_{3/2} \lesssim 10$ TeV by naturalness and by 1.5×10^9 GeV$< T_R < 5 \times 10^9$ GeV by BBN and by successful baryogenesis. Thus, THL is viable only in a highly restricted region of parameter space. In contrast, non-thermal leptogenesis (NTHL) and sneutrino leptogenesis (OSL) are viable in a much larger region bounded from below by $T_R \gtrsim 10^6$ GeV while Affleck-Dine LH_u flat-direction leptogenesis (ADL) is viable in an even larger region for $T_R \gtrsim 10^5$ GeV. These latter three leptogenesis regions are fully viable for $m_{3/2} > 1$ TeV.

As f_a is increased to 10^{12} GeV, then decays of axino and saxion are suppressed even further. In this case, the DM-excluded region expands to the black contours labelled by $f_a = 10^{12}$ GeV and $\xi_s = 0$ or 1. The $\xi_s = 1$ region is smaller than the

$\xi_s = 0$ region because for $\xi_s = 1$ the saxion decay width increases due to $s \to aa$ and the saxion lifetime is quicker. The important point is that SUSY electroweak naturalness expects $f_a \sim \sqrt{\mu M_P/\lambda_\mu} \sim 10^{10} - 10^{12}$ GeV and for these values then there are wide swaths of parameter space which support NTHL, OSL and ADL, and even THL is viable in some small region.

Instead, if we increase f_a to $\sim 10^{13}$ GeV as in frame b), then we are somewhat beyond the natural value of f_a, but also now the DM-forbidden region has increased greatly so that only values of $m_{3/2} \gtrsim 5$ TeV are allowed for $\xi_s = 1$, while for $\xi_s = 0$ then all of natural gravitino mass region is forbidden. For low values of $m_{3/2}(= m_s \Rightarrow$ long-lived saxions) and at high T_R, the decay $s \to aa$ produces too much dark radiation for $\xi_s = 1$ case only. This region is colored brown and triply excluded by DM, BBN and dark radiation constraints. In frame c), with $f_a = 10^{14}$ GeV, then natural gravitino mass region is mostly forbidden by overproduction of WIMPs for $\xi_s = 1$ and totally forbidden for $\xi_s = 0$ (not shown in the Fig. 4.6c). In addition, the brown-shaded region ($\Delta N_{\rm eff} > 1$) has extended and imposes an additional excluded region for $m_{3/2} \gtrsim 15$ TeV and $T_R \gtrsim 10^8$ GeV.

These results have important implications for axion detection. Currently, the ADMX experiment is exploring regions of $f_a/N \gtrsim 10^{12}$ GeV. Future plans include an exploration of regions down to $f_a/N \gtrsim 10^{11}$ GeV. To make a complete exploration of the expected locus of the axion in natural SUSY, then such experiments should also aim for exploration down to $f_a/N \sim 10^{10}$ GeV. For even smaller $f_a/N < 10^{10}$ GeV values, then axion BCM-production requires θ_i values very close to π and the axion production rates would be considered as fine-tuned. [35]

4.3.2 SUSY KSVZ model

In this subsection, we show baryogenesis-allowed regions in the T_R vs. $m_{3/2}$ plane for the SUSY KSVZ model. We regard the SUSY KSVZ model as less lucrative in that one loses the DFSZ solution to the SUSY μ problem and the connection with electroweak naturalness. In addition, if the exotic heavy quark field Q is not an element of a complete GUT multiplet, then one loses gauge coupling unification. Nonetheless, it is instructive to view these results for comparison to the SUSY DFSZ case.

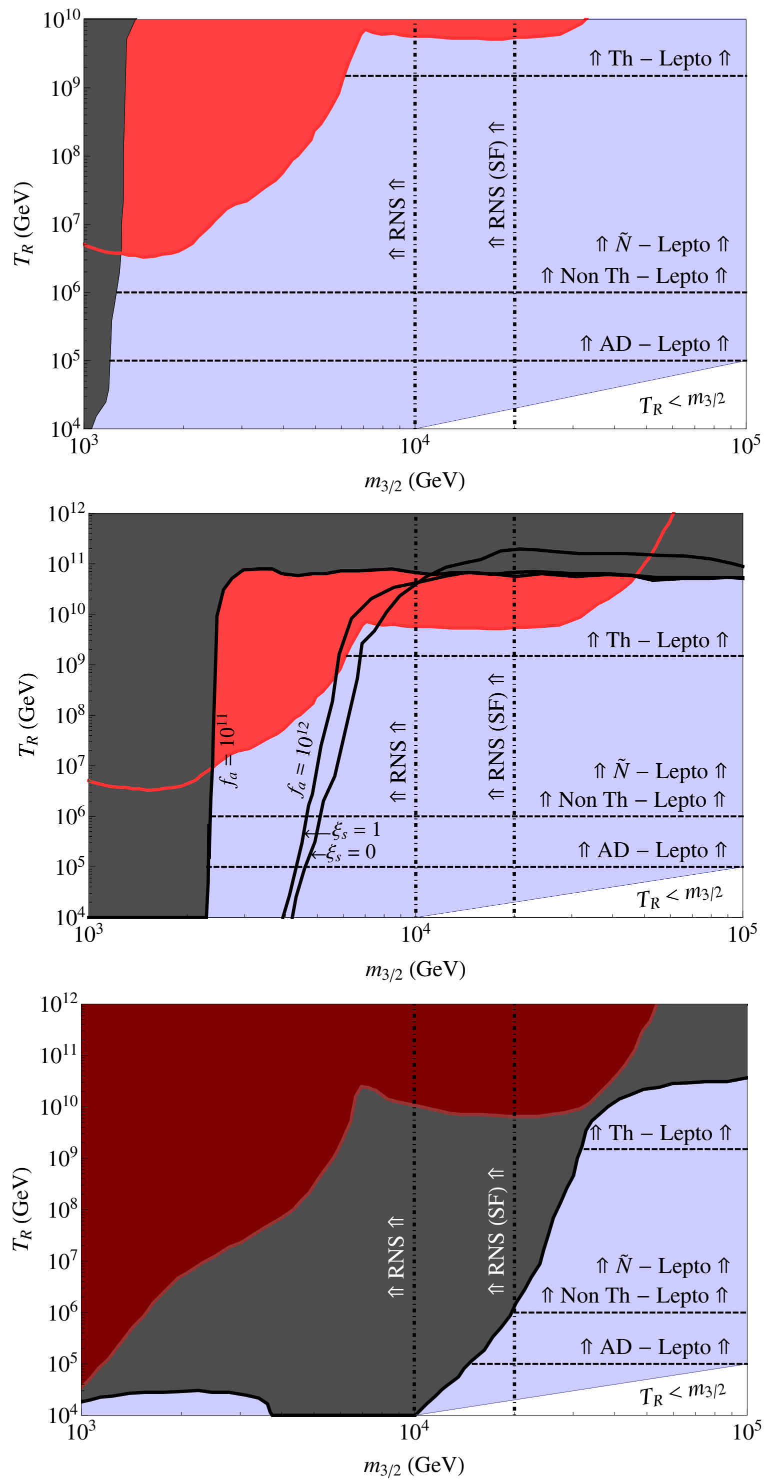

Figure 4.7: Plot of allowed regions in T_R vs. $m_{3/2}$ plane in the SUSY KSVZ axion model for *a*) $f_a = 10^{10}$ GeV, *b*) 10^{11} and 10^{12} GeV for ξ_s =0 and 1 and *c*) $f_a = 10^{13}$ GeV for $\xi_s = 0$. For $f_a = 10^{11}$ GeV, $T_R > 10^{11}$ GeV is forbidden to avoid PQ symmetry restoration. We take $m_s = m_{\tilde{a}} = m_{3/2}$ in all plots.

In Fig. 4.7a), we show results for $f_a = 10^{10}$ GeV. Even for f_a as low as 10^{10} GeV, the gray-shaded WIMP-overproduction region occupies the region with $m_{3/2} \lesssim 1.3$ TeV. In this region, since $m_{\tilde{a}} = m_{3/2}$, then thermal axino production followed by decay after neutralino freeze-out leads to WIMP over production across a wide range of T_R values. This is because the axino decay is suppressed by Q-mediated loops as compared to SUSY DFSZ. As f_a is increased to 10^{11} GeV (Fig. 4.7b)), then the DM-forbidden region expands out to $m_{3/2} \sim 2$ TeV region. For $f_a = 10^{12}$ GeV (Fig. 4.7b)), then the DM-forbidden region expands out to $m_{3/2} \sim 4$ TeV. Even for this high value of f_a, there is still room for leptogenesis in natural SUSY models for each of the cases of THL, NTHL, OSL and ADL. For this case only, we have found that there exists some mild entropy dilution r of n_L due to thermal axino production for $T_R \sim 10^{10} - 10^{11}$ GeV by up to a factor of 2. Since these T_R values are beyond the lower limit, our plots hardly change. Alternatively, the THL lower bound on T_R may be interpretted as a lower bound on T_R/r.

For the SUSY KSVZ model with $f_a = 10^{13}$ GeV as shown in Fig. 4.7c), then the DM forbidden region has expanded to exclude all viable natural SUSY parameter space except for a tiny slice with $m_{3/2} \sim 15 - 20$ TeV and $T_R < 10^6$ GeV where ADL might still function.

4.4 Constraints in the T_R vs. f_a plane for fixed $m_{3/2}$

In this section, we examine the DM constraints on baryogenesis in the T_R vs. f_a plane for fixed natural $m_{3/2}$ values to gain further insights on axion decay constant dependence of the constraints for T_R between $10^4 - 10^{12}$ GeV. On these planes, in the yellow region labelled $T_R > f_a$ we expect PQ symmetry to be restored during reheating which leads to generation of separate domains with different θ values and the appearance of domain walls and associated problems. In this case, axion oscillations including the anharmonic effect must be averaged over separate domains. [117, 118] As before, we do not consider this region.

4.4.1 SUSY DFSZ model

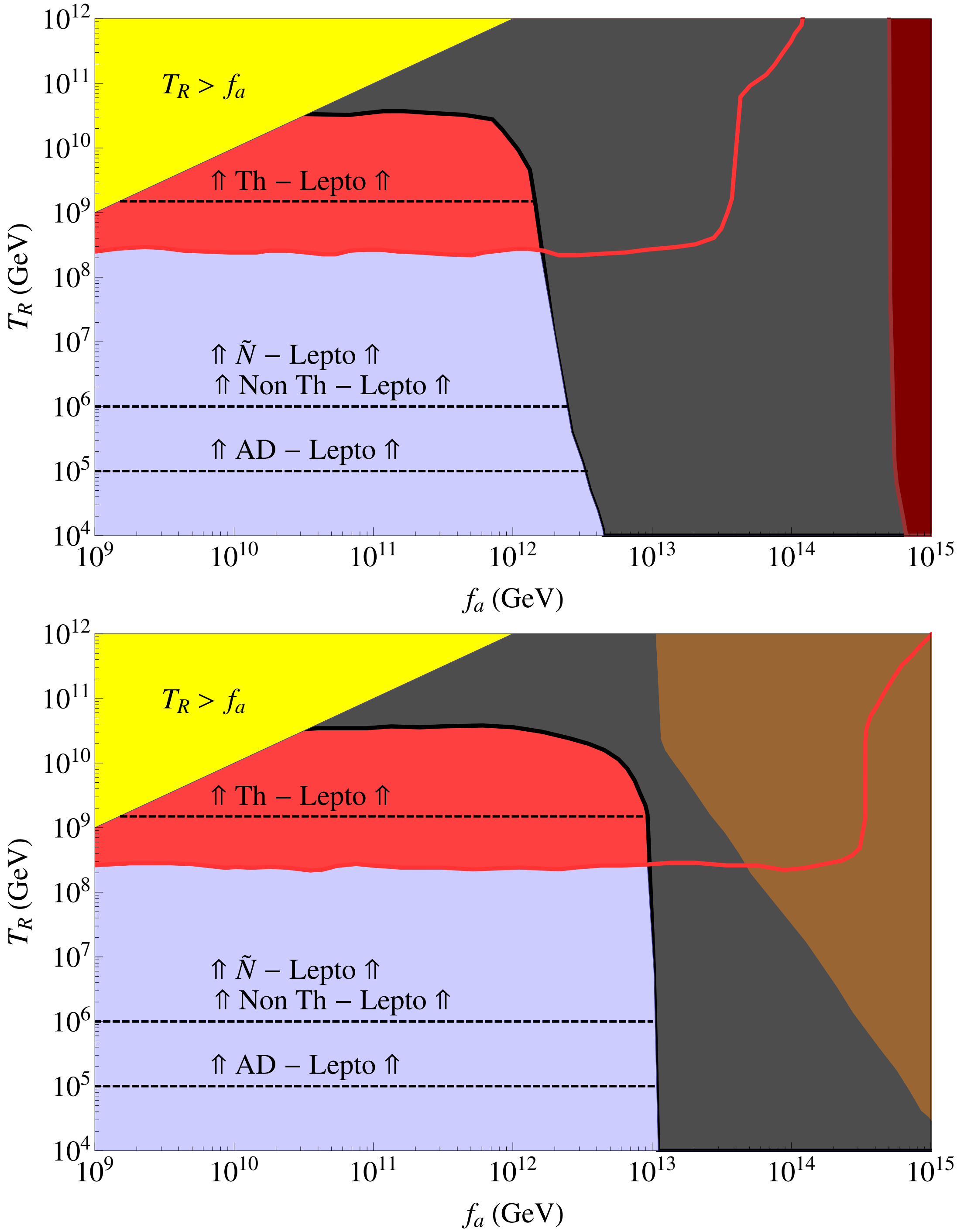

Figure 4.8: Plot of allowed regions in T_R vs. f_a plane in the SUSY DFSZ axion model for $m_{3/2} = 5$ TeV and with *a*) $\xi_s = 0$ and *b*) $\xi_s = 1$. We take $m_s = m_{\tilde{a}} = m_{3/2}$ in all plots.

In Fig. 4.8, we plot allowed and forbidden regions for baryogenesis in SUSY DFSZ model in the T_R vs. f_a plane for $m_{3/2} = 5$ TeV. In frame a), with $\xi_s = 0$, the gray-shaded region still corresponds to WIMP overproduction and sets an upper limit of $f_a \lesssim 10^{12}$ GeV. The red-shaded region corresponds to violation of BBN constraints from late decaying gravitinos and bounds T_R from above: $T_R \lesssim 2 \times 10^8$ GeV which excludes the possibility of THL. Still, large regions of natural SUSY parameter space are consistent with NTHL, OSL and with ADL. The BBN bound kicks in again at $f_a \sim 6 \times 10^{14}$ due to long-lived saxions. For the case of $\xi_s = 1$ shown in Fig. 4.8b), then $s \to aa$ is turned on. This leads to the brown dark radiation excluded region at very large f_a values and large T_R. In addition, we note for this case that the red-shaded BBN forbidden region has actually expanded compared to frame a). This is because for $\xi_s = 0$, the BCM-produced saxions inject considerable entropy into the cosmic soup at large f_a thus diluting the gravitino abundance. For $\xi_s = 1$, then the saxion decays more quickly leading to less entropy dilution of gravitinos and thus more restrictive BBN bounds. Thus, the BBN constraints are actually more severe for $\xi_s = 1$. In addition, for frame b), we see WIMP overproduction bounds are less severe with $f_a \lesssim 10^{13}$ GeV being required for the allowed regions. These are due to a reduced $s \to$ SUSY branching fractions for the $\xi_s = 1$ case.

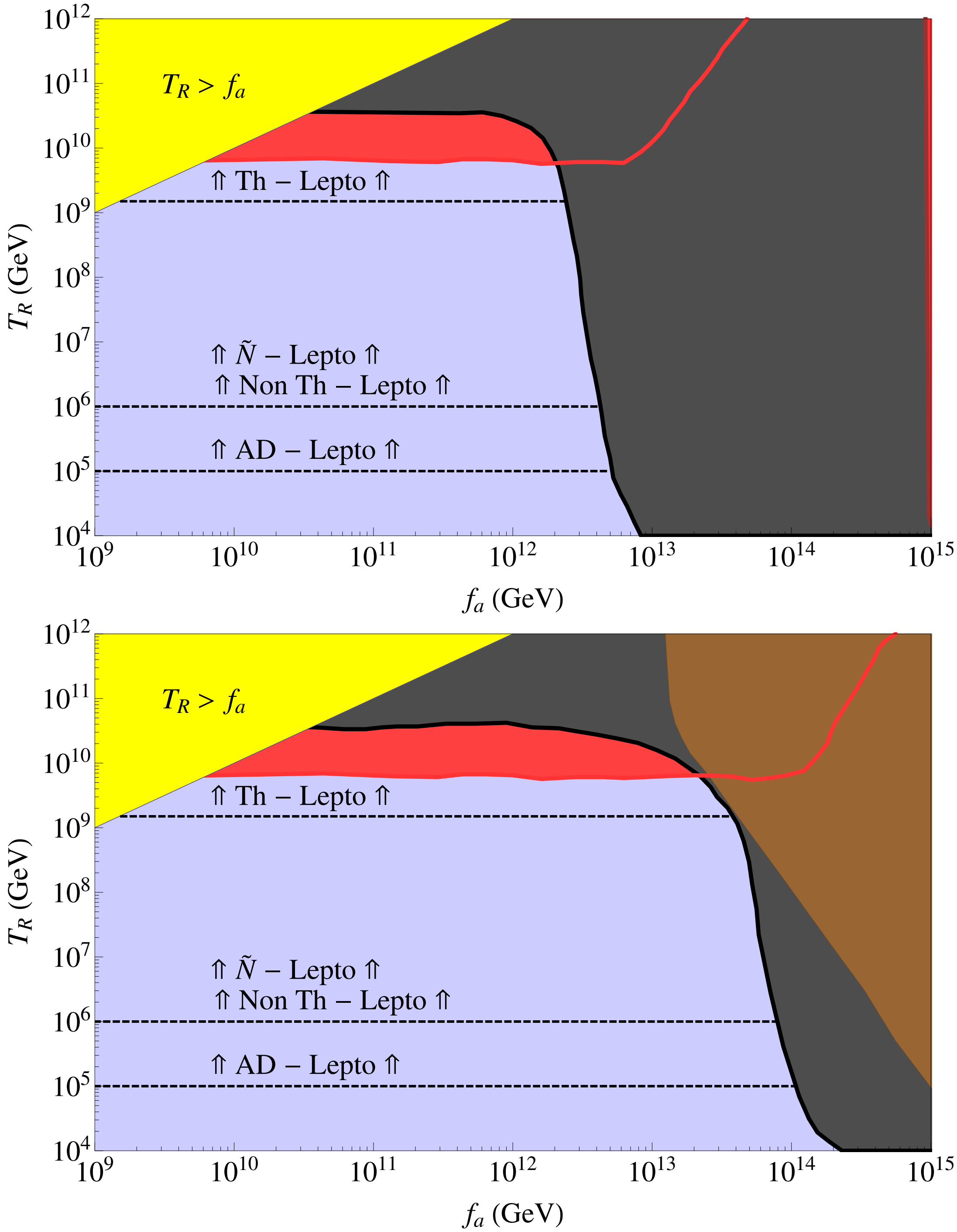

Figure 4.9: Plot of allowed regions in T_R vs. f_a plane in the SUSY DFSZ axion model for $m_{3/2} = 10$ TeV and with $a)$ $\xi_s = 0$ and $b)$ $\xi_s = 1$.

In Fig. 4.9, we show allowed and excluded regions in the T_R vs. f_a plane for $m_{3/2} = 10$ TeV. In the case of $\xi_s = 0$ shown in frame a), the larger gravitino mass causes the gravitinos to decay more quickly so that BBN constraints are diminished: in this case, the THL scenario with $T_R > 1.5 \times 10^9$ GeV is allowed in contrast to the previous case with $m_{3/2} = 5$ TeV. In addition, broad swaths of parameter space are allowed for the NTHL, OSL and ADL scenarios with $f_a \lesssim 5 \times 10^{12}$ GeV. For larger f_a values, then axino and saxion production followed by late decays leads to too much WIMP dark matter. For the case with $\xi_s = 1$ shown in frame b), we see again the BBN constraints are somewhat enhanced due to diminished entropy dilution of gravitinos at large f_a. In addition, a dark radiation forbidden region has appeared. Most importantly, the DM-allowed region occurs for $f_a \lesssim 10^{14}$ GeV so that large swaths of parameter space are open for baryogenesis. This is because, since we take $m_{\tilde{a}} = m_s = m_{3/2}$, then the axinos and saxions are also shorter-lived and tend to decay earlier - frequently before WIMP freeze-out - so DM overproduction is more easily avoided.

For even larger values of $m_{3/2}$ up to $m_{3/2} \sim 25$ TeV, we would expect to see a very similar BBN constraint since BBN bounds are not sensitive to any changes in $m_{3/2}$ for 7 TeV $\lesssim m_{3/2} \lesssim 25$ TeV (see Fig. 4.6). As $m_{3/2}$ increases and reaches beyond $m_{3/2} \sim 65$ TeV, then gravitino decays much sooner and does not violate BBN constraints at all. However DM production highly depends on f_a and the DM exclusion picture would look different up to a maximum f_a after which the whole parameter space is excluded by too much DM.

4.4.2 SUSY KSVZ model

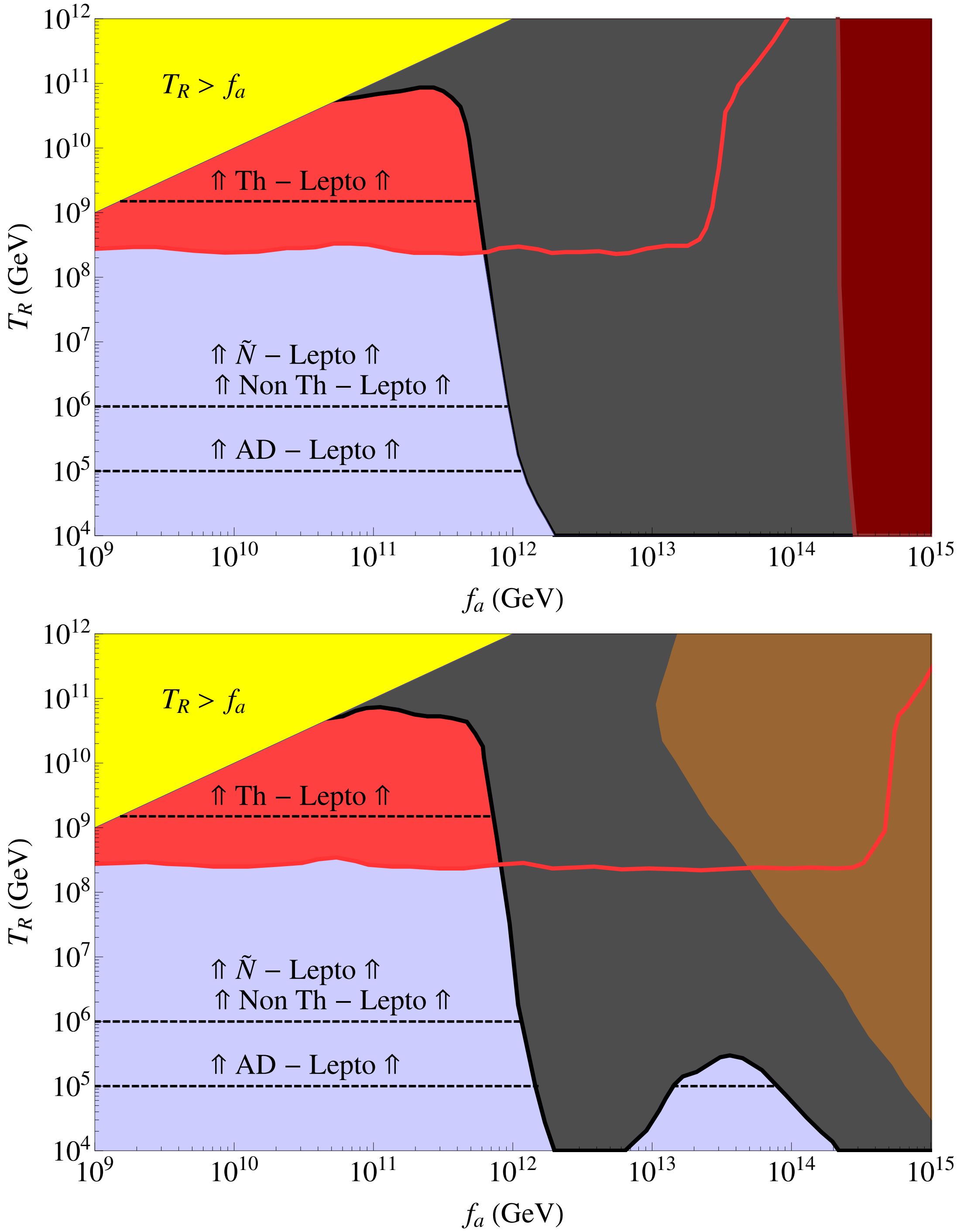

Figure 4.10: Plot of allowed regions in T_R vs. f_a plane in the SUSY KSVZ axion model for $m_{3/2} = 5$ TeV and with $a)$ $\xi_s = 0$ and $b)$ $\xi_s = 1$.

In this subsection, we show corresponding results in the T_R vs. f_a plane for SUSY KSVZ. In Fig. 4.10, we show the plane for $m_{3/2} = 5$ TeV and a) $\xi_s = 0$. Here, we see that THL is ruled out due to the severe BBN bounds arising from gravitino production and decay which restrict $T_R \lesssim 2 \times 10^8$ GeV while the DM restriction rules out $f_a \gtrsim 10^{12}$ GeV. The NTHL, OSL and ADL are still viable baryogenesis mechanisms over a wide range of T_R and f_a values. In frame 4.10b) for $\xi_s = 1$, the DM forbidden region is similar with a $f_a < 10^{12}$ GeV restriction. However, the BBN restricted region has increased because there is less entropy dilution from saxion decay of the gravitinos abundance. The expanded BBN region lies in the already DM and dark radiation excluded region so provides no additional constraint. Since saxions decay earlier for $\xi_s = 1$ compared to $\xi_s = 0$, then they inject neutralinos at a higher decay temperature T_s^D; as a consequence, a small DM-allowed region appears at high $f_a \sim 10^{13} - 10^{14}$ GeV and $T_R \sim 10^5$ GeV which is barely consistent with ADL.

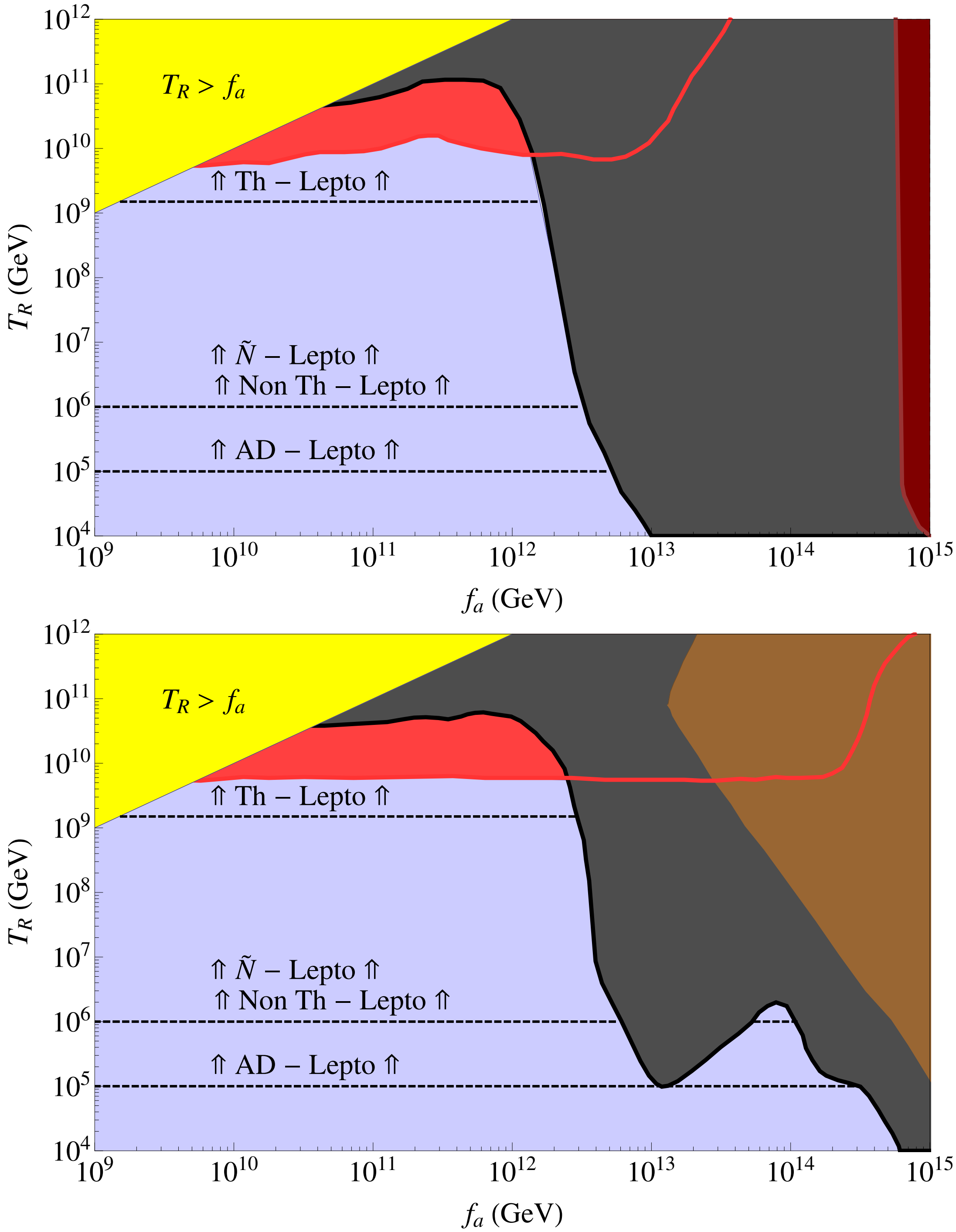

Figure 4.11: Plot of allowed regions in T_R vs. f_a plane in the SUSY KSVZ axion model for $m_{3/2} = 10$ TeV and with a) $\xi_s = 0$ and b) $\xi_s = 1$.

In Fig. 4.11a), we show the same T_R vs. f_a plane with $\xi_s = 0$, but this time for a higher value of $m_{3/2} = m_s = m_{\tilde{a}} = 10$ TeV. The higher value of $m_{3/2}$ means gravitinos decay more quickly and at higher temperature so that the BBN bound on T_R is given by $T_R \gtrsim 4 \times 10^9$ so that THL is again viable. Also, the DM-allowed region has moved to a higher f_a bound of $f_a \lesssim 2 \times 10^{12}$ GeV. In this frame, all four baryogenesis mechanisms are possible. In Fig. 4.11b), we show the same plane for $\xi_s = 1$. Here a prominent dark radiation excluded region appears at large $f_a \gtrsim 10^{13} - 10^{14}$ GeV, although this region is already excluded by WIMP overproduction and by BBN. The larger saxion width arising from the additional $s \to aa$ decay mode means the saxion decay at higher temperatures leading to some possible allowed regions appearing at $f_a \sim 10^{14}$ GeV and $T_R \sim 10^5$ GeV which admits ADL. Otherwise, large regions of viable parameter space exists for $f_a \lesssim 2 \times 10^{12}$ GeV and for $T_R \lesssim 4 \times 10^9$ GeV where all four leptogenesis mechanisms are possible.

Chapter 5

Affleck-Dine (AD) Baryogenesis

In the Affleck-Dine (AD) model, some scalar fields do not enter the superpotential, but lift flat direction of the potential. They receive soft masses in the SUSY breaking vacuum. As long as $H \gg m$, the contribution to the soft mass squared of the flat direction is negative, the flat direction ϕ is excited. After the inflation, when $H \ll m$, ϕ begins to oscillate about the minimum of the potential [119]:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{5.1}$$

If the scalar fields carries baryon and lepton number, then it can produce baryon asymmetry at low energy and temperature level which is required by inflation, and could produce matter and dark matter simultaneously.

5.1 AD baryogenesis with R-parity

In SUSY, squarks and sleptons can carry baryon and lepton number. Lets start with a Lagrangian of a single complex scalar field which carries a $U(1)$ charge:

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2, \tag{5.2}$$

and the current density

$$j_B^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*). \tag{5.3}$$

Now add some quartic coupling as interaction terms to the Lagrangian, which breaks the $U(1)$ symmetry at higher order:

$$\mathcal{L}_I = \lambda |\phi|^4 + \epsilon \phi^3 \phi^* + \delta \phi^4 + c.c.. \tag{5.4}$$

This violate CP and baryon number. Suppose at the very early time of the universe, $H \gg m$ and the field vacuum expectation value (VEV) $\langle \phi_0 \rangle \gg 0$, these terms are irrelevant.

We want to know the dynamics of the field. The full Lagrangian is

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 - \lambda |\phi|^4 - \epsilon \phi^3 \phi^* - \delta \phi^4 - c.c.. \tag{5.5}$$

The equation of motion of the field is

$$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} + m^2 \phi + \lambda \phi^2 \phi^* + \epsilon \phi^3 + 3\epsilon^* \phi \phi^{*2} + 4\delta^* \phi^{*3} = 0. \tag{5.6}$$

At the very early time, $H \gg m$ and $B = 0$. Eq. (5.6) is an equation of motion of an overdamped harmonic oscillator. H is the damping term. ϕ cannot oscillate due to large H. After the inflation, H is getting smaller. When $H \leq m$, ϕ became an underdamped oscillator, $\langle \phi \rangle$ continues to track the instantaneous minimum of the scalar potential, begins to oscillate.

The scalar potential has flat directions with vanishing quartic terms, at the renormalizable level. In cosmology, it requires the scalar fields in the flat directions have large VEV after inflation. Baryon number violating quartic terms are non-renormalizable interactions along flat directions. The effect of quartic terms are amplified by large field VEV. Then the scalars decay into ordinary particles with baryon number conserved.

Begin from a $U(1)$ group and two fields with opposite charge ϕ^+ and ϕ^-, and take the superpotential W to vanish. Then the scalar potential is D-term dominated: [120]

$$V = \frac{1}{2} D^2, \text{ where } D = g(\phi^{+*}\phi^+ - \phi^{-*}\phi^-). \tag{5.7}$$

The D field and the potential V will vanish if $\phi^+ = \phi^- = v$. The expectation value of the Higgs field and the slepton field are

$$H_u = \begin{pmatrix} 0 \\ v \end{pmatrix}, \; L_1 = \begin{pmatrix} v \\ 0 \end{pmatrix}. \tag{5.8}$$

The F term vanishes in this direction. The D term is

$$D_Y = g^{'2}(|H_u|^2 - |L_1|^2) = 0, \tag{5.9}$$

where Y is hypercharge.

Before we calculate the D term for $SU(2)$ to see if it vanishes, we define a matrix gauge field which could work for any $SU(N)$.

$$D^i{}_j = D^a(t^a)^i{}_j, \tag{5.10}$$

so in this example,

$$\begin{aligned} D^i{}_j &= \phi^{*i}\phi_j - \frac{1}{N}|\phi|^2\delta^i_j \\ &= \begin{pmatrix} |v|^2 & 0 \\ 0 & |v|^2 \end{pmatrix} - \frac{1}{2}|v|^2 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0. \end{aligned} \tag{5.11}$$

This field carries a lepton number, which can be converted to baryon number through the sphaleron processes. [121]

Higher order terms can lift the flat potential. For example,

$$W = \frac{1}{M}(H_u L_1)^2 \tag{5.12}$$

leads to

$$V_{lift} = \frac{\hat{\Phi}^6}{M^2}, \tag{5.13}$$

where the superfield $\hat{\Phi}$ has a flat direction that is parameterized by the VEV.

Another example is $e_1 L_2 L_3$ direction:

$$e_1 = \frac{1}{\sqrt{3}}\hat{\Phi},\ L_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} \hat{\Phi} \\ 0 \end{pmatrix},\ L_3 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 \\ \hat{\Phi} \end{pmatrix}. \tag{5.14}$$

Other scalars which are coupled to the AD field become massive if $\hat{\Phi}$ acquires a large VEV. Flat directions $\hat{\Phi}$ are lifted by SUSY breaking and non-renormalizable

(NR) operators:

$$W_{NR} = \frac{\hat{\Phi}^n}{M^{n-3}}, \quad n = 4, \ldots, 9. \tag{5.15}$$

$$V(\hat{\Phi}) = (m_{\hat{\Phi}}^2 - cH^2)|\hat{\Phi}|^2 + \frac{|\hat{\Phi}|^{2n-2}}{M^{2n-6}} + \left(A\frac{m_{3/2}\hat{\Phi}^n}{nM^{n-3}} + h.c. \right), \tag{5.16}$$

where $(m_{\hat{\Phi}}^2 - cH^2)|\hat{\Phi}|^2 + \frac{|\hat{\Phi}|^{2n-2}}{M^{2n-6}}$ are $U(1)$ preserving terms, in which, $m_{\hat{\Phi}}^2$ is soft mass or SUSY breaking mass, cH^2 is due to inflation, $\frac{|\hat{\Phi}|^{2n-2}}{M^{2n-6}}$ is non-renormalizable (NR) operator, and $A\frac{m_{3/2}\hat{\Phi}^n}{nM^{n-3}} + h.c.$ are $U(1)$ breaking terms, in which, $A\frac{m_{3/2}\hat{\Phi}^n}{nM^{n-3}}$ is NR operator, and A is of order of the gravitino mass $m_{3/2}$.

After inflation, the flat direction VEV slides down to

$$< \hat{\Phi}(t) > \sim (H(t)M^{n-3})^{1/(n-2)}. \tag{5.17}$$

When $H \simeq m_{\hat{\Phi}}$, the flat direction starts to oscillate with the initial amplitude:

$$\hat{\Phi}_0 \sim (m_{\hat{\Phi}} M^{n-3})^{1/(n-2)}. \tag{5.18}$$

The AD field has a baryon number q:

$$n_B = q n_{\hat{\Phi}} = -iq(\dot{\hat{\Phi}}^* \hat{\Phi} - \hat{\Phi}^* \dot{\hat{\Phi}}). \tag{5.19}$$

Using Eq. (5.1) equation of motion of $\hat{\Phi}$, the evolution of the baryon number is given by

$$\dot{n}_B + 3Hn_B = -2q \, \mathrm{Im} \left[\frac{\partial V}{\partial \hat{\Phi}} \hat{\Phi} \right]. \tag{5.20}$$

Only A-term contributes to the RHS.

$$\dot{n}_B(t) \sim \frac{2q}{m_{\hat{\Phi}}} \frac{|\hat{\Phi}|^n}{M^{n-3}} A m_{3/2} \delta_m, \tag{5.21}$$

where $\delta_m \equiv \sin[\arg(a_m) + n\theta]$. So the baryon asymmetry is

$$\frac{n_B}{s} \sim \frac{m_{3/2} T_R}{m_{\hat{\Phi}} M_P^2} (m_{\hat{\Phi}} M^{n-3})^{\frac{2}{n-2}\delta_m}. \tag{5.22}$$

The reheating temperature can be relatively low. For example,

$$n = 4: \quad \frac{n_B}{s} \sim 10^{-10} \left(\frac{m_{3/2}}{m_{\hat{\Phi}}}\right) \left(\frac{M}{M_P}\right) \left(\frac{T_R}{10^8 \mathrm{GeV}}\right), \tag{5.23}$$

$$n = 6: \quad \frac{n_B}{s} \sim 10^{-10} \left(\frac{m_{3/2}}{m_{\hat{\Phi}}}\right) \left(\frac{m_{\hat{\Phi}}}{\mathrm{TeV}}\right)^{-1/2} \left(\frac{M}{M_P}\right)^{3/2} \left(\frac{T_R}{10 \mathrm{GeV}}\right), \tag{5.24}$$

There are many other flat directions. An example that have both baryon and lepton number excited is [122]

$$\text{First generation}: \quad Q_1^1 = b, \ \bar{u}_2 = a, \ L_2 = b \tag{5.25a}$$

$$\text{Second generation}: \quad \bar{d}_1 = \sqrt{|b|^2 + |a|^2} \tag{5.25b}$$

$$\text{Third generation}: \quad \bar{d}_3 = a \tag{5.25c}$$

where a, b are the expectation values. We can check that D terms vanishes under $SU(3)$, $SU(2)$ and $U(1)$, and F terms also vanishes by simple algebra. So this is a flat direction. Higher order operators lift the flat direction. The leading term here is

$$W = \frac{1}{M^3} [Q^1 \bar{d}^2 L^1][\bar{u}^1 \bar{d}^2 \bar{d}^3], \tag{5.26}$$

where the upper index is flavor. The potential is lifted by

$$V_{lift} = \frac{\hat{\Phi}^{10}}{M^6}. \tag{5.27}$$

The scalar field condensate forms along the flat direction of the scalar potential. The baryon asymmetry can arise via NR soft SUSY breaking terms. The condensate can be homogeneous or inhomogeneous of charge Q. The homogeneous AD condensate is unstable to the spatial perturbation, because the flat direction potential often comes from a ϕ^2 potential. [123] When the potential is flatter than ϕ^2, there will be a negative pressure. There are more effects, which depend on some parameters, can make the condensate disappear more rapidly. Although we are more interested in AD baryogenesis, these effects can occur for any coherent oscillating scalar field along the flat direction with gauge interactions. This baryogenesis happen long after inflation, so the reheating temperature is not an important constraint.

5.1.1 Q-Ball

Since we assumed that the homogeneous AD condensate oscillates for a long time until it decays, the stability of oscillations under small perturbations, which are caused by the quantum fluctuations, becomes important. Under some circumstances, the AD condensate will be unstable and non-linear, forming condensate fragmentations.

If the potential of the AD field ϕ is flatter than the quadratic term, the AD condensate fragmentations could reach the lowest energy state and turn into non-topological solitons with a fixed charge Q, which may be stable or decay into fermions, called the Q-ball. Q-ball can form when a complex scalar field carries conserved charges, i.e., lepton or baryon numbers, with global $U(1)$ symmetry. In the MSSM, the squarks and sleptons which carry conserved baryon and lepton numbers can form Q-ball. [120] The stability of Q-balls depends on their energy to charge ratio. The Q-ball with the lowest energy to charge ratio is most stable. [124]

Let's consider a scalar potential $V(\phi)$, which has a global minimum $V(0) = 0$, where it has an unbroken global $U(1)$ symmetry at the minimum: $\phi \to e^{i\theta}\phi$. So

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - V(\phi). \tag{5.28}$$

The charge Q is a constant

$$\begin{aligned} Q &= \int \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}(i\phi) + \frac{\partial \mathcal{L}}{\partial \dot{\phi}^*}(-i\phi^*) \right) \mathrm{d}^3 x && \text{(5.29a)} \\ &= \frac{1}{2i} \int (\phi^* \dot{\phi} - \dot{\phi}^* \phi) \mathrm{d}^3 x. && \text{(5.29b)} \end{aligned}$$

The solution which minimizes the energy with $Q =$ constant gives

$$E = \int \left(\frac{1}{2}|\dot{\phi}|^2 + \frac{1}{2}|\nabla\phi|^2 + V(\phi) \right) \mathrm{d}^3 x. \tag{5.30}$$

We got

$$\begin{aligned} E_\omega &= E + \omega \left[Q - \frac{1}{2i} \int \left(\phi^* \dot{\phi} - \dot{\phi}^* \phi \right) \mathrm{d}^3 x \right] && \text{(5.31a)} \\ &= \int \left[\frac{1}{2}|\dot{\phi}|^2 + i\frac{\omega}{2}(\phi^* \dot{\phi} - \dot{\phi}^* \phi) + \frac{1}{2}|\nabla\phi|^2 + V(\phi) \right] \mathrm{d}^3 x + \omega Q && \text{(5.31b)} \\ &= \int \left[\frac{1}{2}|\phi -\dot{}\, i\omega\phi|^2 + \frac{1}{2}|\nabla\phi|^2 + V(\phi) - \frac{1}{2}\omega^2|\phi|^2 \right] \mathrm{d}^3 x + \omega Q, && \text{(5.31c)} \end{aligned}$$

where ω is a Lagrange multiplier. Use

$$\phi(x,t) = e^{i\omega t}\phi(x). \tag{5.32}$$

We minimize E_ω

$$E_\omega = \int \left[\frac{1}{2}|\nabla\phi|^2 + V_\omega(\phi)\right] \mathrm{d}^3x + \omega Q, \tag{5.33}$$

$$\Delta\phi(x) - \frac{\partial}{\partial\phi}V_\omega(\phi(x)) = 0, \tag{5.34}$$

$$\omega_0^2 = \min\left(\frac{2V(\phi_0)}{\phi^2}\right). \tag{5.35}$$

The minimum is

$$\frac{V(\phi)}{\phi^2} = \text{min.}, \text{ for } \phi = \phi_0 > 0. \tag{5.36}$$

If a minimum exists, then a Q-ball solution exists.

For a finite ϕ_0, if Q is large, then we can use a thin wall approximation for the Q-ball: [125]

$$\phi(x,t) = e^{i\omega t}\bar{\phi}(x) \tag{5.37}$$

where

$$\bar{\phi}(x) = \theta(R-x)\phi_0 \tag{5.38}$$

where $\theta(R-x)$ is step function. The mass of Q-ball is

$$M(Q) = \omega_0 Q. \tag{5.39}$$

If the potential $V(\phi) = \mu^4$ for large ϕ, then the minimum will be at infinity, then

$$M(Q) = \mu Q^{3/4}. \tag{5.40}$$

If $V(\phi)$ increase slower than ϕ^2, then the minimum can not be achieved at any finite ϕ_0,

$$M(Q) \sim \mu Q^{(3-p/2)/(4-p)}. \tag{5.41}$$

So the energy per unit charge decrease as the charge and VEV increase.

The Q-ball can not decay into scalars, but can decay into fermions. If the Q-ball doesn't have baryon number, then it is unstable and can decay into light neutrinos. [126] For a Q-ball that carries baryon number, it can be unstable or stable. If the baryon number is not very large, then the Q-ball can be unstable, and emit baryons and also neutralinos when out of equilibrium. [127–130] For unstable Q-ball, the decay rate is suppressed by the surface to volume ratio since the fermions fill up the Q-ball very fast. [126] If the baryon number is large enough, and the mass is relatively small, then the Q-ball doesn't have enough energy to decay into fermions. The mass per unit baryon number is

$$\frac{M(Q_B)}{Q_B} \sim M_S Q^{-1/4}, \tag{5.42}$$

where Q_B is baryon number, $M(Q_B)$ is the mass of the Q-ball, M_S is SUSY breaking scale. If $M(Q_B)/Q_B$ is less than proton mass, then the Q-ball is totally stable, and could survive until present day. [131]

The coherent scalar fields oscillation produce baryon asymmetry. Stable Q-balls can be a candidate of dark matter, and unstable Q-balls can decay and emit neutralinos when out of equilibrium, and thus contribute to the WIMP dark matter relic density. Q-balls can produce both ordinary and dark matter, so this could be the reason that their amounts are fairly close. [129, 130, 132, 133]

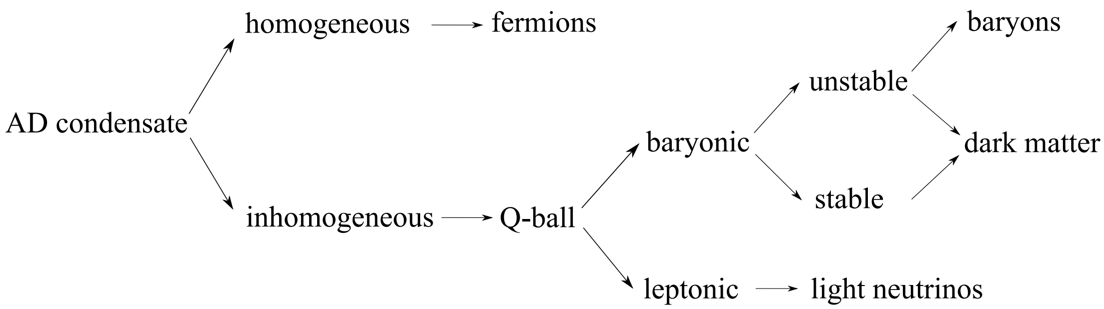

Figure 5.1: AD condensate and Q-ball decay

5.2 AD baryogenesis without R-parity in SUSY DFSZ

Let us introduce PQ symmetry $U(1)_{PQ}$ and PQ fields $S_0,\ S_1,\ S_2$ with PQ charge $0,\ 1,\ -1$, respectively, but do not carry baryon number. $H_u, H_d, \bar{u}_i, \bar{d}_i$ have PQ charge -1, Q_i has 2, $\bar{e}_i$ has 3, and L_i has -2. [134] In SUSY DFSZ, the PQ symmetry can replace the role of R-parity. Here, we assume the R-parity is not conserved. However, the R-parity violating interaction is small for phenomenological reasons. The size of baryon asymmetry and R-parity violation is controlled by the dynamics of PQ fields and the couplings between the PQ fields and the baryon/lepton number violating terms. [135, 136] The superpotential is [134]

$$W = W_{\text{MSSM}} + W_{\not{R}_p} + W_{\text{PQ}} \tag{5.43}$$

where

$$W_{\text{MSSM}} = y^u_{ij}\bar{u}_i Q_j H_u - y^d_{ij}\bar{d}_i Q_j H_d - y^e_{ij}\bar{e}_i L_j H_d + \frac{y_0 S_1^2}{M_P} H_u H_d, \tag{5.44}$$

$$W_{\not{R}_p} = \frac{y_i S_1^3}{M_P^2} L_i H_u + \frac{\gamma_{ijk} S_1}{M_P} L_i L_j \bar{e}_k + \frac{\gamma'_{ijk} S_1}{M_P} L_i Q_j \bar{d}_k + \frac{\gamma''_{ijk} S_1^3}{M_P^3} \bar{u}_i \bar{d}_j \bar{d}_k, \tag{5.45}$$

$$W_{\rm PQ} = \kappa S_0 \left(S_1 S_2 - f^2 \right). \tag{5.46}$$

where $i, j, k = 1, 2, 3$. $W_{\not{R}_p}$ is the R-violating term. $S_1 = S e^{A/f}, S_2 = S e^{-A/f}$ where A is the axion superfield. $W_{\rm PQ}$ is $U(1)_{\rm PQ}$ symmetry breaking term. The symmetry is spontaneously breaken at minimum $\langle S \rangle \simeq f$. After PQ breaking, the effective superpotential is

$$\begin{aligned} W_{\rm eff} =& W_{\rm Yukawa} + \mu_0 e^{2A/f} H_u H_d + \mu_i e^{3A/f} L_i H_u \\ &+ \lambda_{ijk} e^{A/f} L_i L_j \bar{e}_k + \lambda'_{ijk} e^{A/f} L_i Q_j \bar{d}_k + \lambda''_{ijk} e^{3A/f} \bar{u}_i \bar{d}_j \bar{d}_k \end{aligned} \tag{5.47}$$

where

$$\begin{aligned} &\mu_0 = \frac{y_0 f^2}{M_P}, \quad \mu_i = \frac{y_i f^3}{M_P^2} \\ &\lambda_{ijk} = \gamma_{ijk} \left(\frac{f}{M_P} \right), \quad \lambda'_{ijk} = \gamma'_{ijk} \left(\frac{f}{M_P} \right), \quad \lambda''_{ijk} = \gamma''_{ijk} \left(\frac{f}{M_P} \right)^3 \end{aligned} \tag{5.48}$$

where γ is of order unity. So the baryon number violating coupling constants λ''_{ijk} are greatly suppressed, while lepton number is not. The R-parity violating couplings are suppressed by the PQ charge assignment in the Froggatt-Nielsen mechanism. [137]

We consider the D-type AD field $\bar{u}\bar{d}\bar{d}$ direction

$$\bar{u} = \frac{1}{\sqrt{3}} \phi, \quad \bar{d} = \frac{1}{\sqrt{3}} \phi \tag{5.49}$$

since μ term give large contribution to the scalar potential of flat direction LH_u while $\bar{u}\bar{d}\bar{d}$ direction is not affected. The potential of the AD field and PQ fields are SUSY breaking model dependent. This model is realized in the SUSY DFSZ framework. The scalar potential for supergravity is [138]

$$V = V_{\mathrm{Hubble}} + V_{\mathrm{soft}} + V_{\mathrm{F}} + V_{\mathrm{A}} \tag{5.50}$$

where

$$V_{\mathrm{Hubble}} \simeq c_0 H^2 \left|S_0\right|^2 - c_1 H^2 \left|S_1\right|^2 + c_2 H^2 \left|S_2\right|^2 - c_3 H^2 |\phi|^2, \tag{5.51}$$

$$V_{\mathrm{soft}} = m_0^2 \left|S_0\right|^2 + m_1^2 \left|S_1\right|^2 + m_2^2 \left|S_2\right|^2 + m_\phi^2 |\phi|^2, \tag{5.52}$$

$$V_{\mathrm{F}} = |\kappa|^2 \left|S_1 S_2 - f^2\right|^2 + \left|\kappa S_0 S_2 - \frac{\gamma S_1^2 \phi^3}{M_P^3}\right|^2 + |\kappa|^2 \left|S_0 S_1\right|^2 + \frac{|\gamma|^2 \left|S_1\right|^6 |\phi|^4}{M_P^6}, \tag{5.53}$$

$$V_A = -\left(a_H H + a_m m_{3/2}\right) \frac{\gamma S_1^3 \phi^3}{3 M_P^3} + \left(b_H H + b_m m_{3/2}\right) \kappa S_0 \left(S_1 S_2 - f^2\right) + \text{h.c..} \tag{5.54}$$

During the inflation, the AD and PQ fields $\phi_i = S_0, S_1, S_2, \phi$ are at the minimum points with

$$m_{S_0} \simeq m_{S_2} \simeq \left\langle \hat{S}_1 \right\rangle \gg \left|m_{S_1}\right| \text{ and } \left|m_\phi\right| \simeq H \tag{5.55}$$

where $\hat{S}$ means the amplitude of S. At the time after the inflation and $H > m_\phi$, the inflaton oscillates and decays. S_0 and S_2 are heavier than S_1 and ϕ as long as $\left\langle \hat{S}_1 \right\rangle < f$, so S_0 and S_2 stay at the minimum. The EOMs of S_1 and ϕ are

$$\frac{d^2 S_1}{dt^2} + \frac{2}{t}\frac{dS_1}{dt} + \frac{\partial V}{\partial S_1^\dagger} = 0, \tag{5.56}$$

$$\frac{d^2 \phi}{dt^2} + \frac{2}{t}\frac{d\phi}{dt} + \frac{\partial V}{\partial \phi^\dagger} = 0. \tag{5.57}$$

When $H \simeq m_{3/2}$, ϕ_i fields start to oscillate around the minimum. V_ϕ does not depend on PQ fields. Baryon asymmetry is conserved after $S_1 \simeq f$. The baryon number density is

$$n_B = \frac{i}{3}\left(\frac{d\phi^*}{dt}\phi - \phi^*\frac{d\phi}{dt}\right) = \frac{2}{3}|\phi|^2\frac{d\theta_\phi}{dt}, \tag{5.58}$$

and the EOM is

$$\frac{dn_B}{dt} + 3Hn_B = \mathrm{Im}\left(\frac{\partial V}{\partial \phi}\phi\right). \tag{5.59}$$

The baryon number will be fixed at the beginning of the ϕ oscillation. So integrate from the end of inflation t_I to the beginning of the oscillation $t_{\rm osc}$

$$R(t)^3 n_B(t) = \int_{t_{\rm inf}}^{t_{\rm osc}} dt' R\left(t'\right)^3 \mathrm{Im}\left(\frac{\gamma a_m m_{3/2} S_1^3 \phi^3}{3M_P^3}\right), \tag{5.60}$$

we get

$$n_B\left(t_{\rm osc}\right) \simeq \frac{1}{3}\epsilon \hat{a}_m m_{3/2}\delta_{\rm eff}\left(\frac{m_{3/2}M_P^3}{\hat{\gamma}}\right)^{\frac{1}{2}} \tag{5.61}$$

where

$$\epsilon = \hat{S}_1\left(t_{\rm osc}\right)^3 \hat{\phi}\left(t_{\rm osc}\right)^3\left(\frac{m_{3/2}M_P^3}{\hat{\gamma}}\right)^{-\frac{3}{2}}. \tag{5.62}$$

The EOM of S_1 and ϕ are now

$$\begin{aligned}\frac{d^2S_1}{dt^2} + \frac{2}{t}\frac{dS_1}{dt} + 2m_{3/2}^2\hat{S}_1^2\hat{\phi}^6 S_1 + 3m_{3/2}^2\hat{S}_1^4\hat{\phi}^4 S_1 - a_m m_{3/2}^2 S_1^{\dagger 2}\phi^{\dagger 3}\\ + \left(m_1^2 - \frac{4c_1}{9t^2}\right)S_1 - \left(m_2^2 + \frac{4c_2}{m_{3/2}}\right)\frac{f^4}{m_{3/2}M_P^3\hat{S}_1^4}S_1 = 0\end{aligned} \tag{5.63}$$

$$\frac{d^2\phi}{dt^2} + \frac{2}{t}\frac{d\phi}{dt} + 3m_{3/2}^2\hat{S}_1^4\hat{\phi}^4\phi + 2m_{3/2}^2\hat{S}_1^6\hat{\phi}^2\phi - a_m m_{3/2}^2 S_1^{\dagger 3}\phi^{\dagger 2} + \left(m_\phi^2 - \frac{4c_3}{9t^2}\right)\phi = 0 \tag{5.64}$$

where $m_1 = m_2 = m_{3/2},\ \kappa = \gamma = 1$. t starts from $t_{\rm osc} = 2c_3^{1/2}/\left(3m_\phi\right)$.

The ratio of the baryon number to entropy density n_B/s after the reheating

$(t = t_{\mathrm{R}})$ is

$$\begin{aligned} \frac{n_B}{s} &= \frac{1}{s\left(t_{\mathrm{R}}\right)} \left(\frac{R\left(t_{\mathrm{osc}}\right)}{R\left(t_{\mathrm{R}}\right)} \right)^3 n_B\left(t_{\mathrm{osc}}\right) \\ &= \frac{\epsilon \hat{a}_m m_{3/2} \delta_{\mathrm{eff}} T_{\mathrm{R}}}{12 M_P^2 m_\phi^2} \left(\frac{m_\phi M_P^3}{\hat{\gamma}} \right)^{\frac{1}{2}} . \end{aligned} \tag{5.65}$$

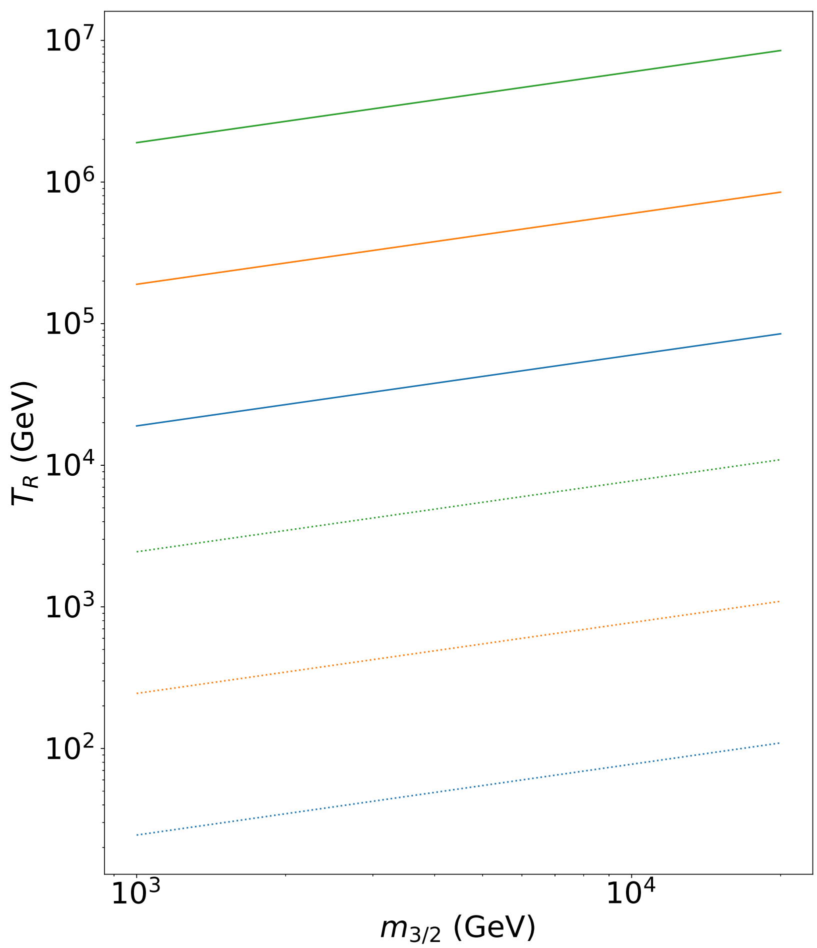

Figure 5.2: Log plot of $n_B/s = 10^{-11}$ (blue), 10^{-10} (orange), 10^{-9} (green) with $\epsilon = 3.1 \times 10^{-4}$ (solid), 2.4×10^{-1} (dotted).

In Fig. 5.2, we choose $\hat{a}_m = \delta_{\text{eff}} = \hat{\gamma} = 1$ and $m_\phi = m_{3/2}$. The blue lines are $n_B/s = 10^{-11}$. The orange lines are $n_B/s = 10^{-10}$. The blue lines are $n_B/s = 10^{-9}$. The solid lines are $\epsilon = 3.1 \times 10^{-4}$. The dotted lines are $\epsilon = 2.4 \times 10^{-1}$. We can see the preferred $n_B/s = 10^{-10}$ is at around $T_R = 10^5 \sim 10^6$ GeV for $m_{3/2} = 1 \sim 20$ TeV. This reheating temperature is relative low, which is good to avoid the gravitino problem.

As we talked in the AD model with R-parity, the AD field can form Q-ball. In the gravity mediated SUSY breaking, the late decay of unstable Q-balls could overproduce LSP if R-parity is conserved. Without R-parity, the LSP decay solves the problem.

AD baryogenesis without R-parity in SUSY DFSZ can generate right amount of baryon asymmetry, while avoids gravitino problem, explains the smallness of neutrino mass, keep the reheat temperature low for a wide range of gravitino mass. We could also extend this method to other flat direction.

Chapter 6

Conclusion

In this paper, we calculated the different baryogenesis scenarios within the SUSY axion KSVZ and DFSZ models, and natural SUSY models and investigated constraints on four compelling baryogenesis via leptogenesis scenarios within the framework of supersymmetric models with radiatively-driven naturalness (RNS). These models are especially attractive since they contain solutions to the gauge hierarchy problem (via SUSY), the strong CP problem (via the axion), the SUSY μ problem (for the case of the SUSY DFSZ axion) and the Little Hierarchy problem (where $\mu \sim 100-200$ GeV is generated from multi-TeV values of $m_{3/2}$). The characteristic, unambiguous signature of such models is the presence of light higgsinos $\widetilde{Z}_{1,2}$ and $\widetilde{W}_1^{\pm}$ with mass $\sim \mu$. In these models, the LSP is a higgsino-like WIMP which is thermally underproduced. The remainder of the dark matter abundance is filled by the axion. Indeed, over most of parameter space the axion forms the bulk of dark matter. [139]

The RNS spectra can be tested at the LHC via gluino pair production followed by cascade decays for $m_{\tilde{g}} \lesssim 2.5$ TeV (for 300-1000 fb^{-1} of integrated luminosity). [140] The cascade decay signatures will include an opposite-sign/same-flavor

(OSSF) dilepton mass edge bounded by $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 5 - 30$ GeV. [140, 141] In addition, a unique same-sign diboson signature arising from wino pair production emerges. [140, 142] For naturalness measure $\Delta_{\rm EW} < 30$, then $m_{\tilde{g}}$ may range up to 3-6 TeV so the gluino possibly could be beyond LHC reach. Monojet and monojet-plus-OSSF dilepton signatures are also possible. [143, 144] The crucial test of naturalness is light higgsino pair production at a linear e^+e^- collider such as ILC operating with $\sqrt{s} > 2m$(higgsino). [145] Regarding dark matter detection, while higgsino-like WIMPs make up likely only a fraction of dark matter, naturalness implies a large coupling of WIMPs to the Higgs boson so WIMP direct detection seems guaranteed at ton-scale noble liquid detectors. [35, 146, 147] The ADMX axion detection experiment may not be sensitive to axion mass and coupling considered in this thesis due to suppression of $a\gamma\gamma$ coupling due to higgsino contributions. [148]

In supersymmetric dark matter models, baryogenesis mechanisms are confronted by the gravitino problem: gravitinos which are thermally produced in the early universe can lead to overproduction of WIMPs or to violations of BBN constraints. In SUSY axion models, there are analogous problems arising from thermal axino production and decay and from thermal and oscillation-produced saxions. We calculated regions of the T_R vs. $m_{3/2}$ plane in the compelling RNS SUSY model with DFSZ axions and $\xi_s = 0$ and 1. Our main result is that the region of parameter space preferred by naturalness with $f_a \sim \sqrt{\mu M_P/\lambda_\mu} \sim 10^{10}-10^{12}$ GeV supports all four leptogenesis mechanisms. The thermal leptogenesis is perhaps less plausible since its allowed region is nestled typically between the constricted region of 7 TeV $< m_{3/2} <$10 TeV (or 20 TeV if one considers the naturalness in terms of the gravitino mass) and 1.5×10^9 GeV $< T_R < 4 \times 10^9$ GeV. The other NTHL, OSL and ADL mechanisms can freely operate over a broad region of parameter space for $f_a \lesssim 10^{12}$ GeV and $T_R \gtrsim 10^5$ GeV. We also evaluated all constraints in the T_R vs. f_a plane for fixed $m_{3/2} = 5$ and 10 TeV.

The broad allowed regions of parameter space basically favor the following:

1. Multi-TeV values of $m_{3/2}$ to avoid BBN constraints and to hasten saxion and axino decays. Since $m_{3/2}$ sets the scale for superpartner masses at LHC, these multi-TeV values of $m_{3/2}$ are also supported by LHC sparticle search constraints and the large value of $m_h \sim 125$ GeV at little cost to naturalness.

2. A value of $f_a \sim 10^{10} - 10^{12}$ GeV which suppresses WIMP over production from axino/saxion production. Such values of f_a lead to axion masses somewhat above the standard search region of ADMX and should motivate future axion search experiments to increase their search region to heavier axion masses.

3. Values of $T_R \sim 10^5 - 10^9$ GeV.

For completeness, we have also evaluated the leptogenesis allowed regions in the SUSY KSVZ model for which an alternative solution to the μ problem is needed. The loop-suppressed axino and saxion decay rates typically lead to more stringent constraints in this case although regions of parameter space can still be found where the various leptogenesis mechanisms are still possible.

Also, we investigated the baryon asymmetry in the AD model with R-parity conservation and AD model with R-parity violation in the SUSY DFSZ framework. The AD models can generate baryon asymmetry almost independent of reheat temperature, and also give candidates of dark matter. For AD model without R-parity, the AD field couples to the PQ fields. The R-parity violation term could naturally produce the appropriate amount of baryon asymmetry, and keep the reheat temperature T_R low at a wide range of $m_{3/2}$ to avoid the gravitino problem. And also the R-parity violation can avoid the problem of overproduction of LSPs from Q-ball decay.

The origin of matter anti-matter asymmetry is important for both particle physics and cosmology. We expect these models to be tested in the high luminosity LHC and cosmological observations. Especially, RNS models are expected to be confirmed in the next 5-10 years via direct higgsino pair production at LHC.

Appendices

Appendix A

The Einstein equations for the FRW universe

The Einstein equations for the Friedmann-Robertson-Walker(FRW) universe are

$$R_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{A.1}$$

where G_N is the Newtonian gravitational constant, $T_{\mu\nu}$ is the energy-momentum tensor and we are including a cosmological constant Λ. $R_{\mu\nu}$ is the Ricci tensor, and it has non-zero components

$$R_{00} = -3\frac{\ddot{R}}{R}, \quad R_{ij} = -\left[\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + \frac{2k}{R^2}\right] g_{ij} \tag{A.2}$$

and $\mathcal{R}$ is the corresponding curvature scalar

$$\mathcal{R} \equiv g^{\mu\nu}R_{\mu\nu} = -6\left[\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right] \tag{A.3}$$

For a perfect fluid with energy density ρ and pressure p, the non-vanishing components are

$$T_{00} = \rho \quad \text{and} \quad T_{ij} = -p\delta_{ij} \tag{A.4}$$

The corresponding Einstein equations are, from the 00-component

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G_N}{3}\rho + \frac{\Lambda}{3} \tag{A.5}$$

usually referred to as the "Friedmann" equation, and, from the ij-components

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = -8\pi G_N p + \Lambda \tag{A.6}$$

so we have

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{A.7}$$

In the case $\Lambda = 0$, this equation implies that $\ddot{R} < 0$ for all times. Then, the present positive $\dot{R}$ implies that $\dot{R}$ was always positive and, therefore, that R was always increasing. Consequently, ignoring the effects of quantum gravity, there was a past time when $R = 0$ - the moment of the "big bang".

Appendix B

The coupled Boltzmann equations

This part is mostly from Ref's [49]. The coupled Boltzmann equations track the number and energy densities of neutralinos $\tilde{Z}_1$, gravitinos $\tilde{G}$, saxions s, axinos $\tilde{a}$, axions a and radiation as a function of time starting at the reheat temperature

$T = T_R$ at the end of inflation until today. Coherent oscillating (CO) components are included separately for axions and saxions. In KSVZ, the thermal production of a, $\tilde{a}$, s and decay processes of $\tilde{a}$, s can be safely treated as taking place at distinct time scales. The inverse decay contributions are suppressed. In the DFSZ model, the decay widths of saxions, axions and axinos are larger. We can estimate the scattering cross section by

$$\sigma_{(I+J\to\tilde{a}+\cdots)}(s) \sim \frac{1}{16\pi s}|\mathcal{M}|^2 \sim \frac{g^2 c_H^2 \left|T_{ij}(\Phi)^a\right|^2}{2\pi s}\frac{M_\Phi^2}{v_{PQ}^2} \tag{B.1}$$

where Φ is a PQ- and gauge-charged matter supermultiplet, g the corresponding gauge coupling constant, $T_{ij}(\Phi)^a$ is the gauge-charge matrix of Φ and M_Φ its mass. For the DFSZ SUSY axion model, the heaviest PQ charged superfields are the Higgs doublets, so g is the $SU(2)$ gauge coupling, $M_\Phi = \mu$, and $\left|T_{ij}(\Phi)^a\right|^2 = \left(N^2-1\right)/2 = 3/2$. We can obtain the rate for the scattering contribution of axino (or saxion) production from the integration formula [149]

$$\left\langle \sigma_{(I+J\to\tilde{a}(s)+\cdots)} v\right\rangle n_I n_J \simeq \frac{T^6}{16\pi^4}\int_{M/T}^{\infty} dx K_1(x) x^4 \sigma\left(x^2T^2\right) \tag{B.2}$$

where the K_1 is the modified Bessel function, M is the threshold energy for the process (either the higgsino or saxion/axino mass) and we have assumed $T \gtrsim M$. Integrating over the Bessel function, we find that the axino (or saxion) production rate is proportional to [54]

$$\langle \sigma(I+J\to\tilde{a}(s)+\cdots)v\rangle \propto \left(\frac{\mu}{f_a}\right)^2 \frac{M^2}{T^4}K_2(M/T) \tag{B.3}$$

where we used $n_I n_J \propto T^6$ from the above expression (unlike the KSVZ case), production is maximal at $T \simeq M/3 \ll T_R$. Hence most of the thermal production of axinos and saxions takes place at $T \sim M$, resulting in thermal yields which are independent of T_R.

We can not neglect the inverse decay processes in DFSZ. So the Boltzmann equations for the number (n_i) and energy (ρ_i) densities of a thermal species $i(= a, s \text{ or } \tilde{a})$ reads:

$$\frac{dn_i}{dt} + 3Hn_i = \sum_{j \in \text{MSSM}} \left(\bar{n}_i \bar{n}_j - n_i n_j\right) \langle \sigma v \rangle_{ij} - \Gamma_i m_i \frac{n_i}{\rho_i} \left(n_i - \bar{n}_i \sum_{i \to a+b} \mathcal{B}_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b}\right) + \sum_a \Gamma_a \mathcal{B}_i m_a \frac{n_a}{\rho_a} \left(n_a - \bar{n}_a \sum_{a \to i+b} \frac{\mathcal{B}_{ib}}{\mathcal{B}_i} \frac{n_i n_b}{\bar{n}_i \bar{n}_b}\right) \tag{B.4}$$

$$\frac{d\rho_i}{dt} + 3H\left(\rho_i + P_i\right) = \sum_{j \in \text{MSSM}} \left(\bar{n}_i \bar{n}_j - n_i n_j\right) \langle \sigma v \rangle_{ij} \frac{\rho_i}{n_i} - \Gamma_i m_i \left(n_i - \bar{n}_i \sum_{i \to a+b} \mathcal{B}_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b}\right) + \sum_a \Gamma_a \mathcal{B}_i \frac{m_a}{2} \left(n_a - \bar{n}_a \sum_{a \to i+b} \frac{\mathcal{B}_{ib}}{\mathcal{B}_i} \frac{n_i n_b}{\bar{n}_i \bar{n}_b}\right) \tag{B.5}$$

where $\mathcal{B}_{ab} \equiv BR(i \to a + b)$, $\mathcal{B}_{ib} \equiv BR(a \to i + b)$, $\mathcal{B}_i \equiv \sum_b \mathcal{B}_{ib}$, $\bar{n}_i$ is the equilibrium density of particle species i and the Γ_i are the zero temperature decay widths. The MSSM particles that interact with axion, saxion and axino are denoted by subscript j. It is also convenient to use the above results to obtain a simpler equation for ρ_i/n_i:

$$\frac{d\left(\rho_i/n_i\right)}{dt} = -3H\frac{P_i}{n_i} + \sum_a \mathcal{B}_i \frac{\Gamma_a m_a}{n_i} \left(\frac{1}{2} - \frac{n_a}{\rho_a}\frac{\rho_i}{n_i}\right) \left(n_a - \bar{n}_a \sum_{a \to i+b} \frac{\mathcal{B}_{ib}}{\mathcal{B}_i} \frac{\mathcal{B}_i n_b}{\bar{n}_i \bar{n}_b}\right) \tag{B.6}$$

where P_i is the pressure density ($P_i \simeq 0$ (or $\rho_i/3$) for non-relativistic (or relativistic) particles). As discussed in Ref's [48], we track separately the CO-produced components of the axion and saxion fields since we assume the CO components do not have scattering contributions. Under this approximation, the equations for the CO-produced fields (axions and saxions) read:

$$\frac{dn_i^{\rm CO}}{dt} + 3Hn_i^{\rm CO} = -\Gamma_i m_i \frac{n_i^{\rm CO}}{\rho_i^{\rm CO}} n_i^{\rm CO} \quad \text{and} \quad \frac{d\left(\rho_i^{\rm CO}/n_i^{\rm CO}\right)}{dt} = 0. \tag{B.7}$$

The amplitude of the coherent oscillations is defined by the initial field values, which for the case of PQ breaking before the end of inflation is a free parameter for both the axion and saxion fields. We parametrize the initial field values by $\theta_i = a_0/f_a$ and $\theta_s = s_0/f_a$.

Finally, we must supplement the above set of simplified Boltzmann equations with an equation for the entropy of the thermal bath:

$$\frac{dS}{dt} = \frac{R^3}{T} \sum_i BR(i,X)\Gamma_i m_i \left(n_i - \bar{n}_i \sum_{i\to a+b} \mathcal{B}_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b} \right) \tag{B.8}$$

where R is the scale factor and $BR(i,X)$ is the fraction of energy injected in the thermal bath from i decays.

In order to solve the above equations, it is necessary to compute the values of the decay widths and annihilation cross sections. The MSSM particles are in thermal equilibrium in most cases, so we make a further approximation as $n_j \simeq n_j$. The value of $\langle \sigma v \rangle$ for thermal axino production is given in Ref's [55,56], while $\langle \sigma v \rangle$ for neutralino annihilation is extracted from IsaReD [111]. For thermal saxion and axion production, it is reasonable to expect annihilation/production rates similar to axino's, since supersymmetry assures the same dimensionless couplings. Hence we apply the result for axino thermal production from Ref's [55, 56] to saxions and axions. For the gravitino thermal production we use the result in Ref's [150]. The necessary saxion and axino partial widths and branching fractions can be found in Ref's [36], while the gravitino widths are computed in Ref's [107].

References

[1] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys.* **C40** 100001 (2016) and 2017 update.

[2] H. Baer and X. Tata, *Weak scale supersymmetry: from superfields to scattering events* (Cambridge University Press, Cambridge, 2006).

[3] D. Kazakov, *Particle physics and cosmology: the interface* (Springer, Dordrecht, 2005).

[4] Stephen P. Martin and Michael T. Vaughn, *Phys. Rev. D* **50** 2282 (1994).

[5] Stephen P. Martin, DOI:10.1142/9789812839657_0001, arXiv:hep-ph/9709356 (2016).

[6] D. Bailin and A. Love, *Cosmology in gauge field theory and string theory* (Institute of physics publishing, Bristol, 2004).

[7] Christopher Kolda, *Nuclear Physics* **B62A-C** 266-275 (1998).

[8] Yael Shadmi, arXiv:hep-th/0601076 (2006)

[9] S. Weinberg, *Phys. Rev. D* **11** 3583 (1975).

[10] G. t Hooft, *Phys. Rev. D* **14** 3432 (1976)

[11] J.M. Pendlebury et al., *Phys. Rev. D* **92** 092003 (2015).

[12] R. D. Peccei, Lect.Notes Phys.741:3-17 (2008), arXiv:hep-ph/0607268.

[13] Michael Dine, arXiv:hep-ph/0011376.

[14] H. T. Janka, W. Keil, G. Raffelt and D. Seckel, *Phys. Rev. Lett.* **76** 2621 (1996).

[15] J.E. Kim, *Phys. Rev. Lett.* **43** 103 (1979).

[16] M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, *Nucl. Phys.* **B166** 3 493-506 (1980).

[17] Michael Dine, Willy Fischler, Mark Srednicki, *Phys. Lett. B* **104** 3 199-202 (1981).

[18] A.P. Zhitnitskii, *Sov. J. Nucl. Phys.* **31** 260 (1980).

[19] L.F. Abbott, P. Sikivie, *Phys. Lett. B* **120** 133 (1983).

[20] J. Preskill, M.B. Wise, F. Wilczek, *Phys. Lett. B* **120** 127 (1983).

[21] M. Dine, W. Fischler, *Phys. Lett. B* **120** 137 (1983).

[22] M.S. Turner, *Phys. Rev. D* **33** 889 (1986).

[23] K.J. Bae, J.-H. Huh, J.E. Kim, *JCAP* **09** 005 (2008).

[24] L. Visinelli, P. Gondolo, *Phys. Rev. D* **80** 035024 (2009).

[25] E. J. Chun, A. Lukas *Phys. Lett. B* **357** 43 (1995).

[26] Howard Baer, Ki-Young Choi, Jihn E. Kim, Leszek Roszkowski, *Phys. Rep.* **555** 1 (2015).

[27] Kyu J. Bae, Howard A. Baer, Andre Lessa, Hasan Serce, *Front. Phys.* **3** 49 (2015).

[28] Kyu Jung Bae, Howard Baer, Eung Jin Chun, Chang Sub Shin, *Phys. Rev. D* **91** 075011 (2015).

[29] H. P. Nilles, M. Srednicki, D. Wyler, *Phys. Lett. B* **120** 346 (1983).

[30] J. M. Frere, D. R. T. Jones, S. Raby, *Nucl. Phys.* **B222** 11 (1983).

[31] J. P. Derendinger, C. A. Savoy, *Nucl. Phys.* **B237** 307 (1984).

[32] G. F. Giudice, A. Masiero, *Phys. Lett. B* **206** 480 (1988).

[33] J. E. Kim, H. P. Nilles, *Phys. Lett. B* **138** 150 (1984).

[34] H. Murayama, H. Suzuki, T. Yanagida, *Phys. Lett. B* **291** 418 (1992).

[35] Kyu Jung Bae, Howard Baer, Vernon Barger, Michael R. Savoy, Hasan Serce, *Symmetry* **7** 2 788-814 (2015).

[36] Kyu Jung Bae, Howard Baer, Eung Jin Chun, *JCAP* **12** 028 (2013).

[37] The ATLAS Collaboration, *Phys. Lett. B* **716** 1 30-61 (2012).

[38] M. S. Carena, H. E. Haber, *Prog. Part. Nucl. Phys.* **50** 63-152 (2003).

[39] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018).

[40] Kyu Jung Bae, Howard Baer, Vernon Barger, Dan Mickelson, Michael Savoy, *Phys. Rev. D* **90** 075010 (2014).

[41] H. Baer, V. Barger, P. Huang, A. Mustafayev, X. Tata, *Phys. Rev. Lett.* **109** 161802 (2012).

[42] Howard Baer, Vernon Barger, Peisi Huang, Dan Mickelson, Azar Mustafayev, Xerxes Tata, *Phys. Rev. D* **87** 115028 (2013).

[43] R. Harnik, G. D. Kribs, D. T. Larson, H. Murayama, *Phys. Rev. D* **70** 015002 (2004).

[44] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, F. Zwirner, *Mod. Phys. Lett. A* **1** 57 (1986).

[45] R. Barbieri, G. Giudice, *Nucl. Phys. B* **306** 63 (1988).

[46] Howard Baer, Vernon Barger, Dan Mickelson, Maren Padeffke-Kirkland, *Phys. Rev. D* **89** 115019 (2014).

[47] H. Baer, V. Barger, P. Huang, *et al. J. High Energ. Phys.* **12** 13 (2013).

[48] Howard Baer, Andre Lessa, Warintorn Sreethawong, *JCAP* **01** 036 (2012).

[49] Kyu Jung Bae, Howard Baer, Andre Lessa, Hasan Serce, *JCAP* **10** 082 (2014).

[50] Laura Covi, Hang Bae Kim, Jihn E. Kim, Leszek Roszkowski, *JHEP* **0105** 033 (2001).

[51] Arnd Brandenburg, Frank Daniel Steffen, *JCAP* **0408** 008 (2004).

[52] Alessandro Strumia, *JHEP* **1006** 036 (2010).

[53] Eung Jin Chun, *Phys. Rev. D* **84** 043509 (2011).

[54] Kyu Jung Bae, Kiwoon Choi, Sang Hui Im, *JHEP* **1108** 065 (2011).

[55] Kyu Jung Bae, Eung Jin Chun, Sang Hui Im, *JCAP* **1203** 013 (2012).

[56] Kyu Jung Bae, Kiwoon Choi, Sang Hui Im, *JHEP* **1108** 065 (2011).

[57] P. A. R. Ade *et. al.*, [Planck Collaboration 2015], *A&A* **594** A13 (2016), arXiv:1502.01589 [astro-ph.CO].

[58] Spergel D. N. *et. al., Astrophys. J. Suppl.* **148** 175 (2003), arXiv:astro-ph/0302209.

[59] A. G. Riess, *et al., Astrophys. J.* 607, 665 (2004).

[60] C. Amsler, *et al.* (Particle Data Group) *Physics Letters B* **667** 1-6 (2008).

[61] J. P. Lees, *et al.* (The BABAR Collaboration), *Phys. Rev. Lett.* **109** 211801 (2012).

[62] M. Fukugita, T. Yanagida, *Phys. Lett. B* **174** 45-47 (1986).

[63] Markus A. Luty, *Phys. Rev. D* **45** 455 (1992).

[64] B.A. Campbell, S. Davidson, K.A. Olive, *Phys. Lett. B* **303** 63 (1993).

[65] W. Buchmuller, P. Di Bari, M. Plumacher, *Nucl. Phys.* **B643** 367-390 (2002) [*Erratum-ibid* **B793** 362 (2008)].

[66] W. Buchmuller, P. Di Bari, M. Plumacher, *Phys. Lett. B* **547** 128-132 (2002).

[67] P. Di Bari, *AIP Conf.Proc.* **655** 208-219 (2003).

[68] K. Kumekawa, T. Moroi, T. Yanagida, *Prog. Theor. Phys.* **92** 437-448 (1994).

[69] G. Lazarides, *SpringerTractsMod. Phys.* **163** 227-245 (2000).

[70] G.F. Giudice, M. Peloso, A. Riotto, I. Tkachev, *JHEP* **9908** 014 (1999).

[71] T. Asaka, K. Hamaguchi, M. Kawasaki, T. Yanagida, *Phys. Lett. B* **464** 12-18 (1999).

[72] T. Asaka, K. Hamaguchi, M. Kawasaki, T. Yanagida, *Phys. Rev. D* **61** 083512 (2000).

[73] M. Kawasaki, Masahide Yamaguchi, T. Yanagida, *Phys. Rev. D* **63** 103514 (2001).

[74] Hitoshi Murayama, T. Yanagida, *Phys. Lett. B* **322** 349-354 (1994).

[75] K. Hamaguchi, Hitoshi Murayama, T. Yanagida, *Phys. Rev. D* **65** 043512 (2002).

[76] Ian Affleck, Michael Dine, *Nuclear Physics* **B249** 2 361-380 (1985).

[77] Michael Dine, Lisa Randall, Scott Thomas, *Phys. Rev. Lett.* **75** 398-401 (1995).

[78] Michael Dine, Lisa Randall, Scott Thomas, *Nucl. Phys.* **B458** 291-326 (1996).

[79] M. Yu. Khlopov, A. D. Linde, *Physics Letters B* **138** 4 265-268 (1984).

[80] J.R. Ellis, D.V. Nanopoulos, S. Sarkar, *Nucl. Phys.* **B259** 175 (1985).

[81] T. Moroi, H. Murayama, Masahiro Yamaguchi, *Phys. Lett. B* **303** 289-294 (1993).

[82] Richard H. Cyburt, John Ellis, Brian D. Fields, Keith A. Olive, *Phys. Rev. D* **67** 103521 (2003).

[83] Richard H. Cyburt, Brian D. Fields, Keith A. Olive, Tsung-Han Yeh, *Rev. Mod. Phys.* **88** 015004 (2016).

[84] Kyu Jung Bae, Howard Baer, Hasan Serce, Yifan Zhang, *JCAP* **01** 012 (2016).

[85] Y. Burnier, M. Laine, M. Shaposhnikov, *JCAP* **0602** 007 (2006).

[86] H. A. Kramers, *Physica* **7** 284 (1940).

[87] Hiren H. Patel, Michael J. Ramsey-Musolf, *J. High Energ. Phys.* **1107** 029 (2011), arXiv:1101.4665 [hep-ph].

[88] Rabindra N. Mohapatra, Goran Senjanovic, *Phys. Rev. Lett.* **44** 912 (1980).

[89] Marion Flanz, Emmanuel A. Paschos, Utpal Sarkar, *Phys. Lett. B* **345** 248-252 (1995) [*ERRATUM-ibid.* **B382** 447 (1996)].

[90] L. Covi, E. Roulet, F. Vissani, *Phys. Lett. B* **384** 169-174 (1996).

[91] W. Buchmuller, M. Plumacher, *Phys. Lett. B* **431** 354-362 (1998).

[92] M. Plumacher, *Nucl. Phys.* **B530** 207-246 (1998).

[93] W. Buchmuller, P. Di Bari, M. Plumacher, *Annals Phys.* **315** 305-351 (2005).

[94] A. Abada, S. Davidson, F-X.J. Michaux, M. Losada, A. Riotto, *JCAP* **0604** 004 (2006).

[95] Enrico Nardi, Yosef Nir, Esteban Roulet, Juan Racker, *JHEP* **0601** 164 (2006).

[96] Apostolos Pilaftsis, *Phys. Rev. D* **56** 5431-5451 (1997).

[97] Apostolos Pilaftsis, *Int. J. Mod. Phys.* **A14** 1811-1858 (1999).

[98] Apostolos Pilaftsis, Thomas E. J. Underwood, *Nucl. Phys.* **B692** 303-345 (2004).

[99] Tony Gherghetta, Chris Kolda, Stephen P. Martin, *Nucl. Phys.* **B468** 37-58 (1996).

[100] Kari Enqvist, John McDonald, *Phys. Lett. B* **425** 309-321 (1998).

[101] Masaaki Fujii, K. Hamaguchi, T. Yanagida, *Phys. Lett. B* **538** 107-114 (2002).

[102] Rouzbeh Allahverdi, Manuel Drees, Anupam Mazumdar, *Phys. Rev. D* **65** 065010 (2002).

[103] Rouzbeh Allahverdi, Bruce. A. Campbell, John Ellis, *Nucl. Phys.* **B579** 355-375 (2000).

[104] Alexey Anisimov, Michael Dine, *Nucl. Phys.* **B619** 729-740 (2001).

[105] Masaaki Fujii, K. Hamaguchi, T. Yanagida, *Phys. Rev. D* **63** 123513 (2001).

[106] Kyu Jung Bae, Howard Baer, Andre Lessa, *JCAP* **04** 041 (2013).

[107] Kazunori Kohri, Takeo Moroi, Akira Yotsuyanagi, *Phys. Rev. D* **73** 123511 (2006).

[108] H. Baer, A. Lessa, S. Rajagopalan and W. Sreethawong, *JCAP* **06** 031 (2011).

[109] H. Baer, S. Kraml, A. Lessa and S. Sekmen, *JCAP* **04** 039 (2011).

[110] Howard Baer, Frank E. Paige, Serban D. Protopescu, Xerxes Tata, arXiv:hep-ph/0312045.

[111] Howard Baer, Csaba Balazs, Alexander Belyaev, *JHEP* **0203** 042 (2002).

[112] Jihn E. Kim, Min-Seok Seo, *Nucl. Phys.* **B864** 2 296-316 (2012).

[113] Karsten Jedamzik, *Phys. Rev. D* **74** 103509 (2006).

[114] Howard Baer, Vernon Barger, Peisi Huang, Azar Mustafayev, Xerxes Tata, *Phys. Rev. Lett.* **109** 161802 (2012).

[115] Howard Baer, Vernon Barger, Michael Savoy, *Phys. Rev. D* **93** 035016 (2016).

[116] Josef Pradler, Frank Daniel Steffen, *Phys. Lett. B* **648** 224-235 (2007).

[117] Michael S. Turner, *Phys. Rev. D* **33** 889-896 (1986).

[118] Luca Visinelli, Paolo Gondolo, *Phys. Rev. D* **80** 035024 (2009).

[119] W. Buchmuller, R. D. Peccei, and T. Yanagida, *Ann. Rev. Nucl. Part. Sci.* **55** 311-355 (2005).

[120] Michael Dine, Alexander Kusenko, *Rev. Mod. Phys.* **76** 1 (2004), arXiv:hep-ph/0303065.

[121] V. A. Kuzmin, V. A. Rubakov and M. A. Shaposhnikov, *Phys. Lett. B* **155** 36 (1985).

[122] Tony Gherghetta, Chris Kolda, Stephen P. Martin, *Nucl. Phys. B* **468** 37-58 (1996).

[123] K. Enqvist, J. McDonald, *Nucl. Phys.* **B570** 407-422 (2000), arXiv:hep-ph/9908316.

[124] Alexander Kusenko, Mikhail Shaposhnikov, *Phys. Lett. B* **418** 46-54 (1998), arXiv:hep-ph/9709492.

[125] S. Coleman, **Nucl. Phys. B 262** 263 (1985).

[126] A. G. Cohen, S. R. Coleman, H. Georgi, A. Manohar, *Nucl. Phys. B* **272** 301 (1986).

[127] K. Enqvist, J. McDonald, *Phys. Lett. B* **425** 309 (1998).

[128] K. Enqvist, J. McDonald, *Phys. Lett. B* **440** 59 (1998).

[129] K. Enqvist, J. McDonald, *Nucl. Phys. B* **538** 321 (1999).

[130] M. Fujii, K. Hamaguchi, *Phys. Lett. B* **525** 143 (2002).

[131] J. Arafune, T. Yoshida, S. Nakamura, K. Ogure, *Phys. Rev. D* **62** 105013 (2000).

[132] M. Laine, M. E. Shaposhnikov, *Nucl. Phys. B* **532** 376 (1998).

[133] R. Banerjee, K. Jedamzik, *Phys. Lett. B* **484** 278 (2000).

[134] Kensuke Akita, Hajime Otsuka, *Phys. Rev. D* **99** 055035 (2019).

[135] K. J. Bae, H. Baer, K. Hamaguchi, K. Nakayama, *JHEP* **1702** 017 (2017).

[136] K. Akita, T. Kobayashi, H. Otsuka, *JCAP* **04** 042 (2017).

[137] C. D. Froggatt, H. B. Nielsen, *Nucl. Phys. B* **147** 277 (1979).

[138] H. P. Nilles, *Phys. Rept.* **110** 1 (1984).

[139] K.J. Bae, H. Baer, E.J. Chun, *Phys. Rev. D* **89** 031701 (2014).

[140] Howard Baer, Vernon Barger, Peisi Huang, Dan Mickelson, Azar Mustafayev, Warintorn Sreethawong, Xerxes Tata, *JHEP* **12** 013 (2013).

[141] B. Altunkaynak, H. Baer, V. Barger and P. Huang, *Phys. Rev. D* **92** 035015 (2015).

[142] Howard Baer, Vernon Barger, Peisi Huang, Dan Mickelson, Azar Mustafayev, Warintorn Sreethawong, Xerxes Tata, *Phys. Rev. Lett.* **110** 151801 (2013).

[143] H. Baer, A. Mustafayev, X. Tata, *Phys. Rev. D* **89** 055007 (2014).

[144] H. Baer, A. Mustafayev and X. Tata, *Phys. Rev. D* **90** 115007 (2014).

[145] H. Baer, V. Barger, D. Mickelson, A. Mustafayev, X. Tata, *JHEP* **06** 172 (2014).

[146] H. Baer, V. Barger, D. Mickelson, *Phys. Lett. B* **726** 330 (2013).

[147] H. Baer, V. Barger, D. Mickelson, X. Tata, arXiv:1306.4183.

[148] Kyu Jung Bae, Howard Baer, Hasan Serce, *JCAP* **06** 024 (2017).

[149] Kiwoon Choi, Kyuwan Hwang, Hang Bae Kim, Taekoon Lee, *Phys. Lett. B* **467** 211-217 (1999).

[150] Josef Pradler, Frank Daniel Steffen, *Phys. Lett. B* **648** 103 (1992).

[151] M. Fukugital and T. Yanagida, *Phys. Lett. B* **174** 1 45-47 (1986).

[152] B. C. Allanach, arXiv:1609.02015 [hep-ph] (2016).

[153] Kyu Jung Bae, Howard Baer, Koichi Hamaguchi, Kazunori Nakayama, *J. High Energ. Phys.* **2017:17** (2017).

[154] Jonathan Bagger, Takeo Moroi, Erich Poppitz, *J. High Energ. Phys.* **0004** 009 (2000).

[155] George Lazarides, arXiv:hep-ph/0204294.

[156] Andrew R Liddle, arXiv:astro-ph/9901124.

[157] Kyle Allison, Christopher T. Hill, Graham G. Ross, *Nuclear Physics* **B891** 613-626 (2015).

[158] Tetsutaro Higaki,Ryuichiro Kitano, *Phys. Rev. D* **86** 075027 (2012).

[159] Kyu Jung Bae, Howard Baer, Eung Jin Chun, Chang Sub Shin, *Phys. Rev. D* **91** 075011 (2015).

[160] Howard Baer, *AIP Conference Proceedings* **1604** 289 (2014).

[161] Margarita Garcia Perez, Pierre van Baal, *Nucl. Phys. Proc. Suppl.* **42** 575-577 (1995).

[162] Peter Minkowski, *Phys. Lett. B* **67** 4 421-428 (1977).

[163] Tsutomu Yanagida, *Progress of Theoretical Physics* **64** 3 1103-1105 (1980).

DEDICATION

to

My respectful parents

Rongfen Zhang and Chengying Tian

My lovely wife

Sherry Li

My adorable baby girl

Alison Zhang

For

Supporting me all the way.

Special thanks to

Dr. Howard Baer,

who always lifts me up, directs my way, and is patient enough to me.