UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

IDENTIFICATION OF SIMULTANEOUSLY CONGESTED TRANSMISSION LINES IN POWER MARKET

A THESIS

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE

By

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Norman, Oklahoma
2014

IDENTIFICATION OF SIMULTANEOUSLY CONGESTED TRANSMISSION LINES IN POWER MARKET

A THESIS APPROVED FOR THE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

BY

[redacted]

First, I would like to thank my family, Morteza, Maryam and Dorina that always support me during my entire life. Next, I need to thank my friends and colleagues in the Electric Energy Risk Research Laboratory, Dr. H. Hosseini, D. Rodriguez and Dr. D. Wu, who provided me with an honest friendship. Also, I want to thank Dr. Ch. Y. Tang and Dr. J. P. Havlicek that agree to serve on my thesis committee. Finally, I would like to thank my advisor, Dr. J. N. Jiang, for his knowledge and helpful guidance during my studies. To be a part of his group and doing research under his supervision was an invaluable experience.

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Abstract

The number and the locations of transmission lines that are likely to get simultaneously congested is a useful piece of information for power system operation, and activities in the power markets. This study presents, a novel algorithm for estimation of a smaller set of transmission lines that have high probability of being simultaneously congested. To this end, first, the maximum number of simultaneously congested lines is investigated. Then, the algorithm and the corresponded criteria are developed to identify such a smaller group of lines. The algorithm is developed based on two mathematical methods of operation research, which are modified with respect to the features of optimal power flow problem. Moreover, using the insights obtained from the study of the characteristics of frequently congested lines distribution, an index is developed to prioritize the lines with respect to their probability of being congested. The proposed algorithm is also illustrated by simulation of a power market based system.

Chapter 1: Introduction

1.1 Background

In countries with privatized electricity industries, the power systems are dominated by the deregulated electricity markets and their corresponded constraints. In these power markets, the market participants have to deal with the constraints of the optimal dispatch, which can affect their revenues significantly. The transmission lines' capacity constraints are one of the most important constraints that make market complicated. Not only the transmission lines' capacities, but also the transmission system's topology, play important roles in economic dispatch of the generation units and determination of the locational marginal prices. When single or multiple transmission lines approaching their maximum transfer capacity limits, they can be considered as the candidates to be the congested lines. Congestion is an important factor in power cost determination. Since, it sets restrictions on power transfers, specifically, the power that comes from the low cost generation zones to the load zones.

1.2 Study of Congestion Problem

Congestion takes place in the power transmission system, when there is insufficient transmission capacity in order to optimally dispatch power to supply demand. Now a days, by increasing the privatization in power industry all over the world, congestion prediction becomes more important for market participants. Since, it can result in noticeable impacts on system operation conditions and outcomes of the corresponding market, e.g., the final electricity cost. Figure 1 shows two simultaneously congested lines in a power system.

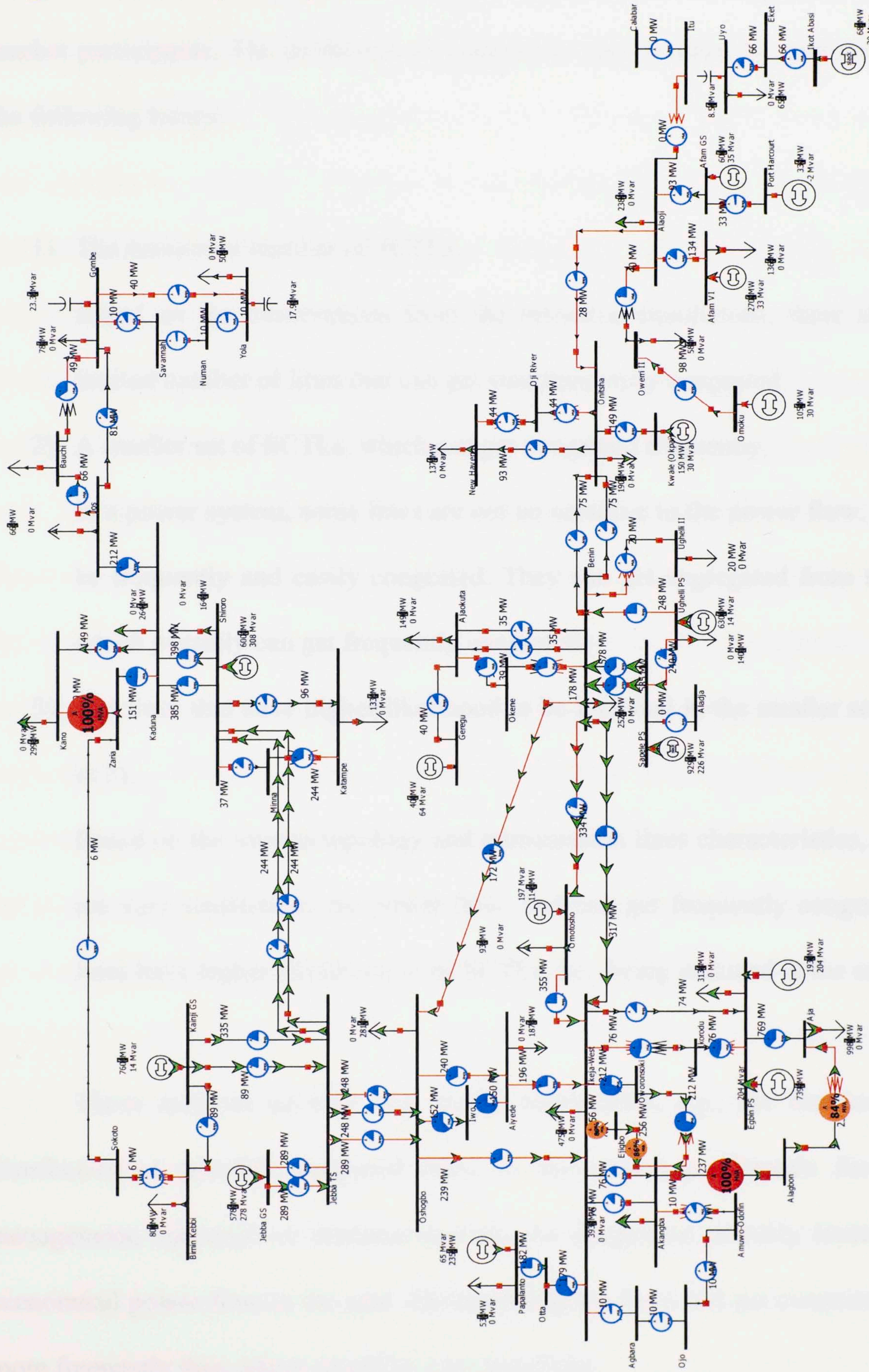

Figure 1. A Part of a Real Power System with Two Simultaneous Congested Lines

Predication of the possible number and the locations of *Simultaneously Congested Transmission Lines or Interfaces* (SCTLs) is a challenging problem to the market participants. The prediction of congestion may involve the analyses regarding the following issues:

1) The maximum number of SCTLs,

   Based on the observations from the intensive simulations, there should be a limited number of lines that can get simultaneously congested.

2) A smaller set of SCTLs, which can get congested frequently,

   In a power system, some lines are not so sensitive to the power flow, and cannot be frequently and easily congested. They can get segregated from the SCTLs, which possibly can get frequently congested.

3) The lines that have higher likelihood to be included in the smaller set identified in 2),

   Based on the system topology and transmission lines characteristics, some lines are very sensitive to the power flow, and can get frequently congested. These lines have higher likelihood to be SCTLs, i.e., being included in the smaller set.

These analyses provide very useful information, e.g., the locations and the distribution of possible congested lines, to the market participants for their risk management, operation or decision making. As congestion possibly limits the more economical power flow in the grid, distinguishing the lines that get congested easier or more frequently than others would be very beneficial.

1.3 Review of Previous Works

Over the last decade a large number of researches have been conducted regarding congestion in electricity market. With respect to approaches they followed, the studies can be generally categorized in two different clusters, power engineering, and optimization oriented prospects. In Current literature, reference [1] described an approach to prioritize lines with respect to their chance of getting congested from the engineering point of view. Based on line factor calculation, this method determines which lines get congested preceding others during single or multiple power-trading paths. In some other publications, boundary of feasible region in optimal dispatch has been considered. Reference [2] discussed computing the boundary of feasible region based on optimization techniques. Using MATLAB optimization toolbox, they studied the impact of various security constraints on boundary of feasible region to identify the ones that have important roles in boundary of the feasible region determination. The work done in [3] is developing a method to identify the boundaries of power flow feasible region that can be used for binding constraints identification. They consider an optimization problem, which its objective is to determine the minimum distance from an external point to the boundary of the feasible region. In [4] congestion management by optimal transmission switching is studied. In order to relieve the congestions, they employed transmission switching to change the network topology, which consequently yields lower prices and higher market efficiency. This paper demonstrates the important role of transmission system's topology in congestion occurrence, which we are going to focus on. In [5] they proposed a methodology to promote fair competition for short-term transmission planning by using assessment of transmission line congestion cost index.

Their method minimizes the distance between market equilibrium point and operation point by redispatching power based on historical bidding data. Also, they illustrate determination of optimal location and capacity of transmission line from the average total congestion cost index under transmission line constraints. This work also emphasizes on the importance of transmission system in congestion management problem. References [6] and [7] represent analyses in the PJM (part of the Easter Interconnection power grid, including Delaware, Illinois, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia, the Washington, D.C. [8]) and MISO (Midcontinent Independent System Operator, Inc.) markets about the transmission congestion. The articles employs the flow-gates model as a market based model for congestion management studies. In this thesis, it is tried to make an improvement to the previous research activities carried out about the congestion, by proposing an algorithm to identify the SCTLs in the transmission system.

1.4 Objective and Contribution

The objective of this study is to identify a smaller set of lines that can get congested frequently. To this end, the possible maximum number of simultaneously congested lines is also investigated.

In this study, we are particularly interested in two types of transmission lines:

1) the lines that come from the cheaper generation zones,

2) the lines from expensive generation zones,

For the first type of lines, they are supposed to transfer more power generated because of their low production costs, which may cause congestions on a set of transmission lines to the load zones. For the second type of lines, for the similar reason, they are less likely to get congested because of their high costs. However, sometimes it is hard to identify these lines quickly by the market participants due to the uncertainties in operating conditions and the market competitions.

In order to identify the smaller set of lines that can get congested frequently, a feasible region is considered, in such a way that keeping the operating point in that region guarantees system's secure operation under the steady-state condition. In this article, mentioning 'feasible region' refers to OPF security region; and its boundary is determined by several constraints there exist in optimal dispatch.

To determine and analyze this region, an optimization problem is considered to be solved. Power system operation or power market normally can be formulated as a linear optimization problem, i.e., OPF, subject to certain equality and inequality constraints. In OPF, there exist several constraints corresponded to system state and the physical equipment's operating limitations; such as lines thermal limit, generation capacity limit, bus voltage and transmission lines' capacity. Among these constraints, transmission lines and generation units' capacity constraints play more important roles in boundary of the feasible region determination. Hence, the OPF problem considered in this study would be subjected to these constraints.

At the optimal solution, it is observed that there is only a limited number of simultaneously congested lines, while most constraints that are not binding can be seen as redundant ones. So, in order to simplify the boundary of the feasible region determination, the redundant constraints should be identified and eliminated. This way, the remaining constraints would be potential candidates to be the binding constraints, i.e., congested transmission lines. As the binding constraints in optimization are equivalent with the congested lines in power market, the constraints that are more likely to be binding are associated with the transmission lines are those that can get frequently congested.

In this study, we put forth an algorithm to identify the transmission lines that have high probabilities of getting congested simultaneously by using some advanced methods in identification the redundancies. In the past, some methods have been developed in operation research for redundant constraints identification, and determination of feasible region of the optimal solution, [9]- [13]. In addition, there are efforts reported to reduce the number of inequality constraints using constraint combination\alteration techniques. These methods apply algebraic procedures to reduce the number of constraints by generating an equivalent set of inequalities with less number of constraints, [14]. These methods might be used to find the redundant constraints but sometimes the results are not accurate, and they need to be modified to be applied to solve OPF problems.

In this study, two methods are proposed based on the features of OPF, and they are applied jointly to develop a novel algorithm in order to screen the lines that can get frequently congested. These two methods are termed the ***Maximization*** and the ***Comparative*** methods, which the main basis for the ***Intersection Algorithm***.

1.5 Overview

In this thesis, by solving the OPF problem, congestion is studied from a new prospect. In this regard, the possible maximum number of SCTLs is investigated. Then, based on the OPF features and the concept of congestion, an algorithm is proposed for identification of the SCTLs. In addition, some insights about the distribution and location of the SCTLs is presented.

The aforementioned studies are organized in the following order:

1) In Chapter 2, it is shown that, under normal operating conditions, only a limited number of specific lines can get congested simultaneously.
2) In Chapter 3, an analytical study is presented to show the maximum number of SCTLs.
3) In Chapter 4, the algorithm is presented to distinguish the lines with high probability of congestion.
4) In Chapter 5, we provide the insights obtained from the study of the characteristics of distribution of SCTLs.
5) In Chapter 6, the simulation results of a case study are included.
6) In Chapter 7, the conclusions of the study are presented.

Chapter 2: Congestion Problem Explanation

In this chapter, the focus would be on the illustration of congestion importance in the power market, the explanation of the problem, and investigation of the possible maximum number of SCTLs. The discussion is carried out by assessment of congestion importance in electricity pricing, introducing generation shift factor, intensive simulations to create as much as possible simultaneous congestions, and then verification of the observations by a real power system.

Section 2.1 An Example of Congestion Importance in Power Market

In this section, a reason that why congestion plays an important role in power market study is exemplified. One of the dominating factors in energy market operation, planning and pricing is determination of Locational Marginal Prices (LMP), which can be calculated based on OPF or DC-OPF power flow simulations. The transmission providers calculates LMPs based on the Marginal Energy Components (MEC), which are the marginal cost of energy, based on the offers and bids selected in the day-ahead market. This way, the LMP_i that is the LMP at bus i would be as follows,

$$LMP_i = MEC_s + MCC_i + MLC_i \qquad \textbf{(2-1)}$$

Where,

$MEC_s =$ the LMP component representing the marginal cost of energy at the slack bus.

MEC_i = the LMP component representing the marginal cost of congestion at bus i relative to the slack bus.

MLC_i = the LMP component representing the marginal cost of losses at bus i relative to the slack bus.

In (2-1), MCC_i of the LMP at bus i is calculating by the equation below,

$$MCC_i = -\left(\sum_{k=1}^{k} SF_{ik} \cdot \lambda_k\right) \quad \textbf{(2-2)}$$

Where,

k = the number transmission constraints.

SF_{ik} = the shift factor at bus i on the transmission constraint k.

λ_k = the shadow price of the constraint k.

Equation (2-1) shows that LMP calculation is dependent on the marginal cost of congestion (MCC_i), which requires identification of congestions in the system [15]. The LMP calculation, presented here, is just one of the several applications of congestion identification. Moreover, (2-2) shows that the marginal cost of congestion calculation is based on the shift factor, which is a structural-related factor. In next chapters, this concept will be used to develop the method for identification of the congested lines.

Section 2.2 Generation Shift Factor

Generation Shift Factor (GSF) is one of the power system sensitivity factors that represents the affection of injecting 1 *MW* power into one of the system's buses, on the transmission lines' flow change as the consequence. In Figure 2, if F_{12}, F_{13} and F_{23} be the initial lines flows, *GSF* is useful to find the new line flows, F'_{12}, F'_{13} and F'_{23}, by determination of line flow changes as the result of injection 1 *MW* to the bus 2.

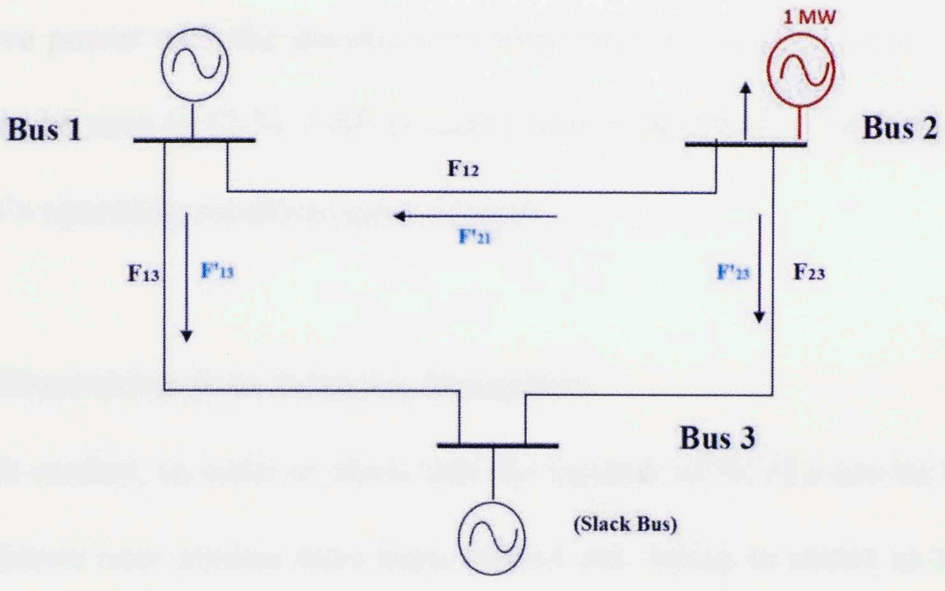

Figure 2. A Simple Power System

GSF can be calculated by the following equation, which only includes the structural- related factors,

$$GSF_{i,l} = \frac{\Delta f_l}{\Delta P_{G_i}} = \frac{X_{ji} - X_{ki}}{x_{jk}} \quad \textbf{(2-3)}$$

Where,

$$X_{ji} = \frac{\partial \delta_j}{\partial P_{Gi}} \quad , \quad X_{ki} = \frac{\partial \delta_k}{\partial P_{Gi}}$$

Δf_l = change in *MW* power flow on line l, when a change in generation takes place at bus i.

ΔP_{Gi} = amount of generation change in active power at bus i.

X_{ji} = the affection of injecting P_{Gi} MW to bus i, on the voltage angle at bus k. It is also called the electrical distance between the buses i and j.

GSF calculation is based on DC load flow method, which means, it can be used only for active power with the assumptions there exist for the DC Load flow analysis. Also, as it can be seen in (2-3), *GSF* is a structural-related factor and remains constant when system's operating condition gets changed.

Section 2.3 Observation from Intensive Simulation

In this section, in order to show that the number of SCTLs can be very limited, several simulation case studies have been carried out, trying to create as many SCTLs as possible. It is found that the first congested line is most likely the one that comes from a low cost generation zone. Then, by change of the load and offer prices of generation units, it is tried to create more SCTLs. This is done carefully without causing any diverges of power flow or other violations.

In the study, it is found that in the power system, there are some dependencies between transmission lines in terms of congestion. For example, there exist an operating point that there are no more SCTLs can be created without violating the OPF security constraints. Another example is, the transmission lines with Generation Shift Factors (GSF) equal to zero for one or multiple generation units ($GSF_{j,P_{Gi}} = 0$) only get congested by the generation at specific locations. That is, if some other lines get congested preceding them, they probably can never get congested. Since, the required generation increase, by the specific generation units that can make them congested, may result in violation of the security constraints of the congested lines.

As it will be further explained in detail, we can take the advantage of the dependency of congested lines in determination of the maximum number of SCTLs, or segregate the redundant lines that are less likely to be congested, so that a smaller set of lines with high likelihood of congestion can be identified.

To illustrate it, consider the Table 1, which includes the excerpt of the data for the IEEE 39-bus system. For the complete data see the Appendix A. There can be seen that there exist some lines such as *2-3* which can get congested easily in comparison with other lines. By a generation increase at bus 31 (G_{31}) equal to 250 MW, it reaches its transmission capacity limit preceding others; however, some lines such as *25-2* and *2-1* require much more *MW* increase in the aforementioned bus in order to get congested. So, these lines cannot get congested without violating at least line *2-3* transmission capacity constraint; since, the injection increase would increase line *2-3*'s line flow as well.

Furthermore, there are some other lines that can get congested only depending on generation change at a specific bus. For example, congestion at line 12-13 is only dependent on MW increase at bus 31, approximately equal to 1925 MW. It means, it can never get congested; because, by 250 MW increase in G_{31}, line 2-3 reaches to its limit and more generation increase would not possible, due to line 2-3's transmission capacity constraint.

Figure 3, shows IEEE 39-bus system with seven simultaneously congested lines. Under this operating condition, creation of more congestions may not be feasible without violating single or multiple transmission constraints. Identification of this kind of lines not only reduces the dimension of the OPF problems' constraints, but also helps to come up with a congestion pattern for extra monitoring on this group of lines.

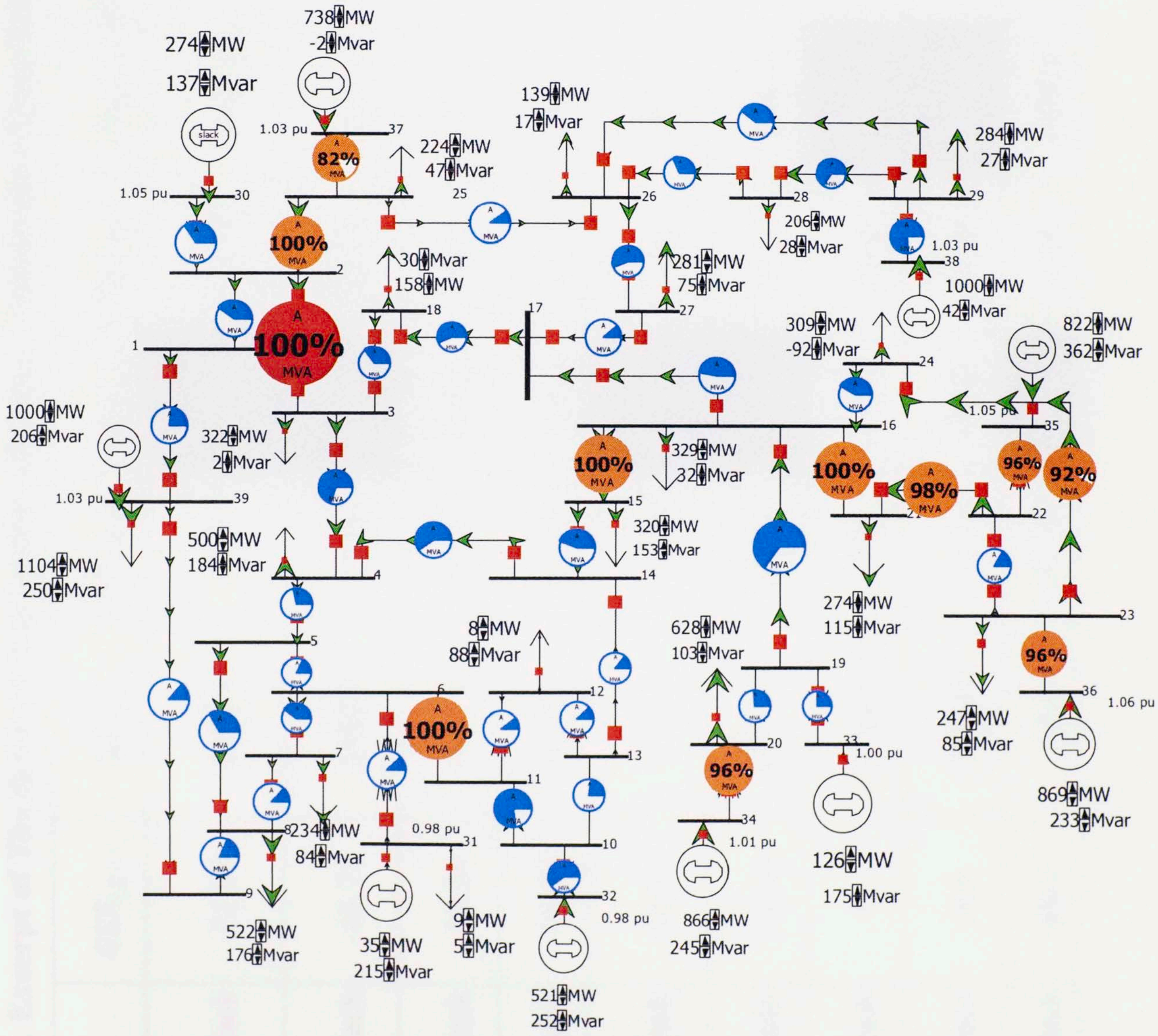

Figure 3. IEEE 39-Bus System with 7 Congested Lines

Table 1. Excerpt of The IEEE 39-Bus System GSF and Transmission Lines Data

	$GSF_{i,2-1}$	Δf_{2-1}	$GSF_{i,2-3}$	Δf_{2-3}	$GSF_{i,25-2}$	Δf_{25-2}	$GSF_{i,12-13}$	Δf_{12-13}	$GSF_{i,16-19}$	Δf_{16-19}
G_{31}= Slack	-21.07	2292.8	-63.2	**250**	15.72	1495.5	2.57	1925.6	0	No Impact
G_{32}= Slack	-18.71	2582.0	-64.46	**245.1**	16.38	1396.9	0	No Impact	0	No Impact
G_{33}= Slack	-10.24	4717.7	-63.63	248.8	26.13	899.7	0	No Impact	-100	**51.6**
G_{34}= Slack	-10.24	4717.7	-63.63	248.8	26.13	899.7	0	No Impact	-100	**51.6**
G_{35}= Slack	-10.24	4717.7	-63.63	**248.3**	26.13	899.7	0	No Impact	0	No Impact
G_{36}= Slack	-10.24	4717.7	-63.63	**248.3**	26.13	899.7	0	No Impact	0	No Impact
G_{37}= Slack	0	No Impact	-7.39	2138	91.6	**256.6**	0	No Impact	0	No Impact
G_{38}= Slack	-4.8	10064.5	-35.14	449.6	60.06	**391.4**	0	No Impact	0	No Impact
G_{39}= Slack	-60.9	-60.9	-31.34	**504.1**	7.76	3029.6	0	No Impact	0	No Impact

Section 2.4 Real Evidence of Limited Number of SCTLs

In the electricity markets, it can also be confirmed that the number of SCTLs is limited. By going through all the Security Constrained Economic Dispatch (SCED) shadow prices and binding transmission constraints reports published by Electric Reliability Council of Texas (ERCOT), the maximum number of constraints that were simultaneously active, i.e., being binding, during the entire year of 2013 was only 14, comparing to over 10,000 lines in ERCOT. Moreover, it only happened in two days of March 3^{rd} and June 4^{th}. In all other days, the maximum number of active constraints are less than 14. In fact, the second largest number of SCTLs in year 2013 was only 9, which occurred on the evening of January 29^{th} and in the middle of the day on September 27th, [16].

Chapter 3: Maximum Number of Simultaneous Congested Lines

In this chapter, the OPF problem formulation along with the concept of constraint redundancy in this problem are presented. Then, the possible maximum number of SCTLs is investigated. The investigation is carried out based on the properties of the linear systems and degeneracy in Linear Programing Problem (LPP).

Section 3.1 Optimal Dispatch Model Formulation

In an interconnected power system, the objective is to find the best generation dispatch pattern that minimizes the power cost. The desired dispatch pattern can be found through solving an optimization problem, called *Optimal Power Flow*, OPF. The objective function of OPF problem is summation of cost functions of all generation units, which is subjected to a set of specific equality and inequality constraints.

OPF is one of the fundamental methods for static power flow calculations, [17]. The objective of the OPF problem is to satisfy the constraints of power system, e.g., transmission lines capacity limits, and setting the decision variables to their optimum values in order to minimize the objective function. The objective function of the OPF problem is the summation of offer prices of the all generation units, which is subjected to a set of constraints.

Over the last decades, several efforts have been made to improve OPF calculations by making that more mathematical rather than being a power engineering analysis, [18][19]. With respect to past studies concerning the impact of OPF constraints on boundary of feasible region, generation capacity ($P_{Gi,min} \leq P_{Gi} \leq P_{Gi,max}$), and transmission lines capacity constraints play significant roles in boundary of feasible region determination. So, the general OPF can be formulated as a linear programing problem, described in equations (3-1)-(3-7) below,

$$min \sum_{i=1}^{k} a_i p_{Gi} \qquad \textbf{(3-1)}$$

$$\text{S.t. } \sum_{i=1}^{K} p_i - \sum_{i=1}^{N} D_i - loss = 0 \qquad \textbf{(3-2)}$$

$$p_i - P_{Gi,max} \leq 0 \quad \forall\, i \in \{1, \dots, K\} \qquad \textbf{(3-3)}$$

$$P_{Gi,min} - p_i \leq 0 \quad \forall\, i \in \{1, \dots . K\} \qquad \textbf{(3-4)}$$

$$f_j - f_{j,max} \leq 0 \quad \forall\, i \in \{1, \dots, L\} \qquad \textbf{(3-5)}$$

$$-f_{j,max} - f_j \leq 0 \;\; \forall\, i \in \{1, \dots, L\} \qquad \textbf{(3-6)}$$

$$f_j = \sum_{i=1}^{N} GSF_{j,i}\,(p_i - D_i) \qquad \textbf{(3-7)}$$

Where,

a_i = Cost function coefficient for generator i.

P_{Gi} = Output of generator number i.

f_j = Line flow for line number j.

L = Number of transmission lines.

k = Number of generation units.

D_i = Demand at bus number i.

$GSF_{j,i}$ = Generation shift factor for line j as the consequence of generation change at bus i, also known as *Power Transfer Distribution Factor* (PTDF).

$f_{j\,max}$ = Maximum transmission capacity of line j.

$P_{Gi,max}$ = The maximium MW output capacity of the generation unit i.

$P_{Gi,min}$ = The minimum MW output capacity of the generation unit i.

In this version of OPF formulation, (3-2), is the power balance equality constraint. Also, in (3-7) the *GSF*s for generation unit i are calculated by assuming that generation unit as buyer and other generation units as seller. As *GSF* is calculated by system structural- factors, it remains constant and independent from system operating condition.

In order to take advantage of LP properties, by using the calculated *GSF* matrix, the transmission and generation capacity constraints are rewritten in standard form of $A_{ij}x_j \leq b_i$.. With the knowledge of:

a) $A_{ij} = GSF_{j,i}$, which is GSF for line number j as the consequence of generation change at generation unit number i,
b) $x_j = \Delta P_{Gi}$, which is the change of generation at generation unit i,
c) $b_i = \Delta f_j$, which is equal to summation of all partial line flow changes happen at line j, as the result of simultaneous power injection changes at each of the generation units,

All transmission constraints can be rewritten in form of a system of linear inequalities as follows,

In addition, the generators' capacity constraints make the decision

$$GSF_{j,1}.\Delta P_{G1} + GSF_{j,2}.\Delta P_{G2} + \cdots + GSF_{j,i}.\Delta P_{Gi} \leq \Delta f_j$$

$$= \sum_{i=1}^{NG} GSF_{j,i}\,.\Delta P_{Gi} \leq \Delta f_j \tag{3-8}$$

variables (ΔP_{Gi}) bounded. So, by using (3-8), the constraints (3-3) to (3-6) can be rewritten in form of a system of linear inequalities, as follows:

$$[GSF]_{j\times i}\,.\,[\Delta P_{Gi}]_{i\times 1} \leq \left[\Delta f_j\right]_{j\times 1} \tag{3-9}$$

$$P_{Gi,min} \leq \Delta P_{Gi} \leq P_{Gi,max} \tag{3-10}$$

Having the OPF constraints in the above standard form, by segregation of the frequently binding constraint from those that does not impact the boundary of the feasible region, the maximum number of SCTLs can be obtained in the way presented below.

Section 3.2 Redundant and Binding Constraints in OPF

A constraint is redundant if it can be eliminated from the system of linear inequalities without affecting the feasible region. For example, let $GSF_m, i.\Delta P_{Gi} \leq \Delta f_{\mathrm{m}}$ be the m^{th} constraint and equation (3-11) represents the feasible region of the optimization problem with constraints described by (3-9) and (3-10) above. If equation

(3-12) below defines the feasible region associated with the same optimization problem with a smaller set of constraints. Then the m^{th} constraint, $GSF_{m,i}.\Delta P_{Gi} \leq \Delta f_j$ is a redundant constraint if and only if $S = S'$.

$$S = \{\Delta P_{Gi} \in \mathbb{R}^i | GSF_{j,i}\, \Delta P_{Gi} \leq \Delta f_j \,, \Delta P_{Gi} \geq 0\} \tag{3-11}$$

$$S' = \{\Delta P_{Gi} \in \mathbb{R}^i |\, GSF_{j,i}\, \Delta P_{Gi} \leq \Delta f_j, \Delta P_{Gi} \geq 0 \,, j \neq m\} \tag{3-12}$$

On the other hand, if a constraint passes through the optimum point, it is a binding constraint. In OPF, a line gets congested if the transmission constraint associated with that be a binding constraint, vice versa. As we are seeking the maximum number of simultaneous congested lines, the binding constraints should be segregated from the redundant constraints.

Section 3.3 Discussions and Explanation about Maximum Number of SCTLs

In this section, the investigation of the maximum number of SCTLs is presented. This study is carried out from two perspectives, linear systems properties and degeneracy in OPF problem.

Section 3.3.1 Linear System Perspective

The impact of independency of constraints on the possible maximum number of SCTLs is investigated by performing algebraic manipulations, so that redundant constraints in the set of constraints describe in Equation (3-9) can be identified and eliminated. In order to be able to use linear systems' properties, the system of inequalities in (3-9) is transformed to a system of inequalities,

$$[\widehat{GSF}].[\widehat{\Delta P_{Gi}}] - [y_j] = [\widehat{\Delta f_j}] \quad \textbf{(3-13)}$$

Where,

$[\widehat{GSF}] = [GSF, -I] \in \mathbb{R}^{j\times(i+q)}$, (q= I matrix dimension)

$[\widehat{\Delta f_j}] \quad \in \mathbb{R}^{j\times 1}$

$[\widehat{\Delta P_{Gi}}] = [\Delta P_{G1}, \Delta P_{G2}, \dots, \Delta P_{Gi}, y_1, y_2, \dots, y_i]^T \in \mathbb{R}^{(i+q)\times 1}$

$[y_j] \quad \in \mathbb{R}^{j\times 1}$, which are slack variables

Having the transformed linear equalities, redundancies can be found by using linear algebraic concepts. It is known that if condition of $rank([\ \widehat{GSF}\ \widehat{\Delta f_j}]) = rank(\ \widehat{GSF})$ holds with $rank(\ \widehat{GSF}) = j' < j$, there exist at least one redundant constraint, which can be eliminated from system (3-13), [14].

Knowing that some of the constraints are redundant, we can apply some algebraic techniques, e.g., *Singular Value Decomposition* (SVD) method, to $[\widehat{GSF}]$ in order to eliminate the redundancies by constructing a new set of constraints, [14]. The new set of constraints is made of a smaller number of linearly independent equalities, which the number of them is equal to the rank of $[\widehat{GSF}]$. It includes only the constraints that determine the boundary of the feasible region.

Moreover, using linear independency and rank of a matrix concepts, the maximum number of the linearly independent equations in (3-13), which are associated with the frequently congested lines (binding constraints), can be determined. In the power system, the number of transmission lines are always much bigger than the number of generation units ($j \gg i$). So, the rank of $[\widehat{GSF}]$ would be equal to its column rank, which is the number of generation units excluding the slack bus. It means that the maximum number of SCTLs defined in (3-13) is equal to the rank of $[\widehat{GSF}]$.

Section 3.3.2 Degeneracy in Linear Programming

Using the concept of degeneracy in LP, the reason that why there can be only a few SCTLs in the solution to an OPF problem is explained.

Definition of non-degeneracy:

A solution of a linear programming problem with i decision variables is degenerate if more than i inequality constraints are binding at the optimum point. Thus, degeneracy denotes that there is at least one redundant constraint in the OPF LPP. So non-degeneracy is referred to the situation that the maximum number of binding inequality constraints is, which is equal to the number of decision variables.

Proposition:

For a linear OPF problem as described in Equations (3-1)-(3-7), if it has a unique optimal solution, the problem is non-degenerate, i.e., the theoretical number of binding inequality constraints would be less than or equal to the number of decision variables.

Proof:

In matrix $[GSF]_{j\times i}$ the row independence is possible only when $j \leq i$. Then, if the number of constraints exceed the number of decision variables, the surplus number of constraints are dependent constraints. Also, the Lagrange Multipliers (λ_j) associated with them will be zero. Thus, as a constraint is either non-binding or its associated λ_j is positive, the surplus constraints, which the number of them is equal to $j - i$, would not be binding at least in a case that the problem is non-degenerate.

This proposition is useful; since, according to the non-degeneracy concept, we know that an OPF problem should be a non-degenerate LPP. In case of degeneracy in the OPF problem, the number of the binding constraints is larger than the number of decision variables. Under this condition, the optimal value of the objective function can be achieved by an infinite number of solutions. Such solutions result in several problems; such as, cycling or divergence in OPF iterations, or arbitrary OPF solutions, which are undesirable for a market oriented dispatch. For example, in a set of transmission lines in series, with the same line parameters and transferring the same flow, each line can be a candidate to be binding. The corresponding constraints of these lines are redundant; since, they are significantly identical.

Based on the proposition and the discussions in Section 3.3, the possible maximum number of SCTLs would be equal to the number of decision variables (ΔP_{Gi}), which is the same as the number of generation units. As the OPF has both transmission and generation constraints, this number includes the number of binding generation constraints plus the number of binding transmission constraints. (This conclusion is derived with respect to two assumptions: non-degeneracy and $[GSF]$ matrix row independency.)

That is, the possible maximum number of binding inequalities

= number of binding generators + number of binding lines

= number of decision variables (ΔP_{Gi})

However, in the power market, the number of binding lines are much less than the total number of generation units. In a real power market, some generation units are scheduled at their maximums; so, their outputs cannot be decision variables. Thus, they should not be considered in determination of the theoretical maximum number of SCTLs. Another reason for having less number of SCTLs in practice is that there are many transmission constraints that are off the boundary of the feasible region. So, they cannot be decision variables neither.

In the next section, an algorithm to identify and eliminate the redundant constraints in the OPF problem is proposed.

Chapter 4: Identification of Possible SCTLs

In this chapter, an optimization based algorithm is proposed to predict the possible SCTLs by identifying the redundancies in the OPF problem. The algorithm, called the *Intersection Algorithm,* is developed based on two methods, which are the *Maximization* and the *Comparative* elimination methods. It considers the intersection of the non-redundant constraints determined by the two methods, as the possible SCTLs, which are likely to get congested frequently.

Section 4.1 Maximization Method

In this method, the redundant constraints in (3-3)-(3-6) can be identified by comparing the possible maximum line flow increase of each line with the required line flow increase for congestion. The superiorities of this method to other similar LP techniques can be summarized as follows:

a) It is an objective function independent technique, which uses the minimum and maximum capacity limits of generation units to calculate the Maximization criterion (M_j) for each constraint.

b) It can be used to identify the redundancies in equality constraints in addition to the inequalities; however, most of the LP methods are only for inequalities.

c) It take into account the minimum and the maximum allowed values of the decision variables.

For the j^{th} line, the Maximization criterion, which is the maximum line flow change at that line, can be represented as,

$$M_j = \sum_{i \in P_j} GSF_{ji}\ \Delta P_{Gi,max} + \sum_{i \in N_j} GSF_{ji}\ \Delta P_{Gi,min} \tag{4-1}$$

Where,

M_j = The criterion of the Maximization method

$\Delta P_{Gi,max}$ = Remaining generation capacity.

$\Delta P_{Gi,min}$ = Generation decrease to the minimum limit.

$P_j = \{i; GSF_{ji} > 0\}$

$N_j = \{i; GSF_{ji} < 0\}$

By calculating (4-1) for all transmission lines, the redundancies can be identified with respect to the following criterion:

If maximum line flow change of a transmission line is not larger than its specified limit, i.e., $M_j \leq \Delta f_j$, then, the line's constraint would be redundant. This method is shown by path "a" on the flowchart, in Figure 4.

Section 4.1.1 Derivation of Maximization Method

In order to evaluate if a line has the capability of getting congested, the Maximization method assesses the line's corresponded constraint for likelihood of being a binding constraint. This assessment is based on the structure and the operational condition of the system.

Let M_j be an arbitrary transmission constraint for line j,

$$M_j = GSF_{j,1}.\Delta P_{G1} + GSF_{j,2}.\Delta P_{G2} + \cdots + GSF_{j,i}.\Delta P_{Gi} \leq \Delta f_j \quad \textbf{(4-2)}$$

This constraint indicates that the line flow change should be within the specified limit, when the outputs of the generation units change. So, it is redundant if the maximum line flow increase cannot meet the specified limit. According to (4-2), the maximum possible line flow increase can be represented with the following optimization problem,

$$Max\ M_j = GSF_{j,1}.\Delta P_{G1} + GSF_{j,2}.\Delta P_{G2} + \cdots + GSF_{j,i}.\Delta P_{Gi} \quad \textbf{(4-3)}$$

$$s.t. \quad P_{Gi,min} \leq \Delta P_{Gi} \leq P_{Gi,max} \quad \textbf{(4-4)}$$

In the maximization problem above, the objective function consists of two terms. Since, the constant coefficient (*GSF*) is positive in the first and negative in the second term, ΔP_{Gi} in the first term should be set to its maximum value, and ΔP_{Gi} in the second term should be set to its minimum value. This way, at the optimal solution, the first and the second terms reach their maximum and minimum, respectively. So, the optimal solution can be represented as,

$$Max\ M_j = \sum_{i \in P_j} GSF_{j,i}\,.\Delta P_{Gi} + \sum_{i \in N_j} GSF_{j,i}\ .\Delta P_{Gi} \quad \textbf{(4-5)}$$

Where, $P_j = \{i; GSF_{ji} > 0\}$ and $N_j = \{i; GSF_{ji} < 0\}$

Where,

$M_j =$ the maximum line flow increase for the j^{th} line,

$\Delta P_{Gi,max} = P_{Gi,max} - P_{current}$,

$\Delta P_{Gi,min} = P_{current} - P_{Gi,min}$.

Thus, assuming that M_j is the maximum possible line flow increase at line j, if $M_j < \Delta f_j$ holds, the j^{th} constraint is redundant.

Proposition:

If the change of generation pattern is a reliable control variable to maximize the change of power flow at line j, then, the j^{th} constraint that satisfies the Maximization method criterion, is redundant.

Proof:

If M_j is the maximum possible line flow increase at line j under the studied operating point, the line constraints with $M_j < \Delta f_j$ are redundant. In a power system, based on the OPF and operating condition, it is reasonable to assume that line flows are dependent variables of the generation pattern. So, maximizing the generation for positive GSFs and minimizing the generation for negative GSFs of a line yields the maximum flow at the line of interest. Thus, plugging in $\Delta P_{Gi,max}$ and $\Delta P_{Gi,min}$ in the first and the second terms of (4-5) guarantees M_j to be the maximum flow increase at the studied operating point.

Section 4.2 Comparative Method

This method is also used to identify the redundant line constraints in (3-3)-(3-6). The superiority of this method to other methods is the minimized computational burden; since, c_{ji} is the only parameter that should be computed and investigated. It eliminates the redundancies by comparing the rows of the $[C]_{j\times i}$ matrix, which can be constructed by the following factor for each transmission line,

$$c_{ji} = \frac{GSF_{j,i}}{\Delta f_j} \quad \begin{cases} j = (1,2, \dots, NL) \\ i = (1,2, \dots, NG) \end{cases} \qquad \textbf{(4-6)}$$

Where,

c_{ji} = the comparison factor of the Comparative method

NL = the number of transmission lines

NG = the number of generation units

With (4-6), the redundant line constraints can be identified as follows:

If there are lines k and l such that $c_{ki} \le c_{li}$ $(i = 1, \dots, NG)$ holds, then the constraint for the k^{th} line would be redundant. This method is shown by path "b" on the flowchart, in Figure 4).

Section 4.2.1 Derivation of Comparative Method

In development of this method, it is assumed that small *GSF*s have insignificant impacts on the line flows; so, they can be ignored in compare with the large *GSF*s. Under this condition, as we are interested to increase the line flow, positive large *GSF*s would be associated with positive ΔP_{Gi}s, and negative large *GSF*s would be associated with negative ΔP_{Gi}s. Thus, the product of each *GSF* with its corresponded ΔP_{Gi} would be positive. Considering this assumption the method is developed as follows:

By dividing the both sides of (3-8) by Δf_j, the line constraints can be rewritten as the inequality below,

$$\sum_{i=1}^{NG} c_{ji} \,.\, \Delta P_{Gi} \leq 1 \tag{4-7}$$

Where, $i = 1, \ldots, NG \quad , \quad j = 1, \ldots, NL \quad , \quad \Delta P_{Gi} > 0$

With respect to (4-7), if there exist lines k and l, such that $c_{ki} \leq c_{li}$ holds, for an arbitrary value of ΔP_{Gi}, the k^{th} constraint would be redundant. Note that the right hand side of the inequality (4-7) is equal to one. So, when this inequality is satisfied for c_{li}, which is bigger than c_{ki}, it will be definitely satisfied for c_{ki}, for the same value of ΔP_{Gi}.

$$\sum_{i=1}^{NG} c_{li} \,.\, \Delta P_{Gi} \leq 1 \Longrightarrow \sum_{i=1}^{NG} c_{ki} \,.\, \Delta P_{Gi} \leq 1$$

This means the l^{th}constraints does not let the k^{th} constraint to impact the boundary of the feasible region, and the k^{th} constraint can be eliminated.

Section 4.3 Intersection Algorithm

Each of these methods identify some constraints as non-redundant; however, their results are not always the same. For example, the Maximization method might over-estimate the non-redundant constraints; or the Comparative might under-estimate the non-redundant constraints. In order to improve the precision of redundancy elimination process, the results from these methods are used for the Intersection algorithm development.

Through simulation observations, it is found that it is more efficient to search for frequently congested lines based on the intersection of the results from the Maximization and the Comparative elimination methods. Either methods can identify some redundancies per se. Thus, it is useful to search for the frequently congested lines in the non-redundant line constraints, which are identified based on one of the two methods. However, the number of non-redundant constraints by both methods is still relatively large. It is observed that most of the binding lines' constraints are in the intersection of the remaining non-redundant line constraints identified with the two methods. Thus, it would be more efficient to search for the frequently congested lines from the intersection.

The intersection algorithm is proposed based on the two methods to improve the efficiency of searching for the binding line constraints. Figure 4 presents the flowchart of the proposed algorithm, which is shown by path "c" on the flowchart. The proposed algorithm can identify most of the line that are likely to the SCTLs. The number of the identified lines is usually smaller than the maximum number of SCTLs, which is described in the previous chapter.

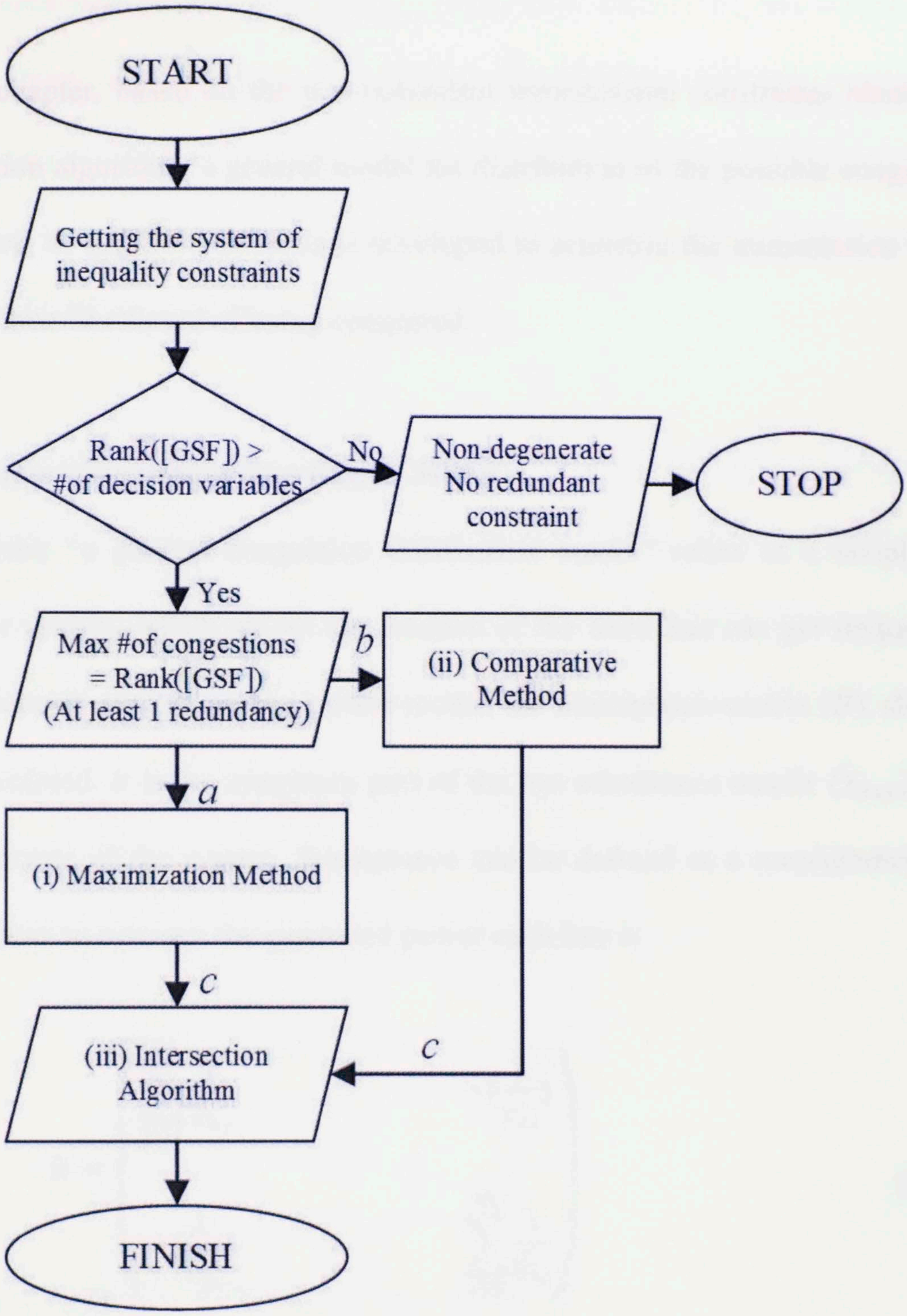

Figure 4. Determination of maximum number of binding constraints algorithm

Chapter 5: Distribution and Locations of SCTLs

In this chapter, based on the non-redundant transmission constraints identified by the Intersection algorithm, a general model for distribution of the possible congested lines is presented. In addition, an index is developed to prioritize the transmission lines with respect to their likelihood of being congested.

Section 5.1 General Congestion Distribution Model

The phrase "a general congestion distribution model" refers to a simplified model of power system, which shows the location of the lines that can get frequently congested. In order to explain the congestion model, the Susceptance matrix (B), shown in (5-1), is considered. It is the imaginary part of the bus admittance matrix (Y_{bus}) and express the structure of the system. Susceptance can be defined as a measurement to show how sensitive to transmit the generated power each line is.

$$B = \begin{pmatrix} \sum_{j=1}^{NL} \frac{1}{x_{1j}} & \cdots & -(\frac{1}{x_{1i}}) \\ \vdots & \ddots & \vdots \\ -(\frac{1}{x_{i1}}) & \cdots & \sum_{j=1}^{NL} \frac{1}{x_{ij}} \end{pmatrix} \quad \textbf{(5-1)}$$

Where,

x_{ij} = the reactance of line ij.

Not surprisingly, the observations show that the b_{kk} values associated with the transfer buses are commonly smaller than the b_{kk} values for the generation and load buses. With respect to Susceptance definition, it shows that the transfer buses are less sensitive to power flow rather than generator and load buses. For this reason, the generator and load buses should have more deterministic roles in congestion occurrence. In other words, the lines that connect generation buses to the load buses are expected to be the lines that may form the congestion model. They probably can get congested preceding the other lines.

In order to investigate if the aforementioned lines can create a congestion distribution model for the SCTLs, the simplified power system model in Figure 5 is considered. It shows the aggregation of the adjacent generation units and the aggregation of the adjacent loads, which construct a generation zone (source) and a load

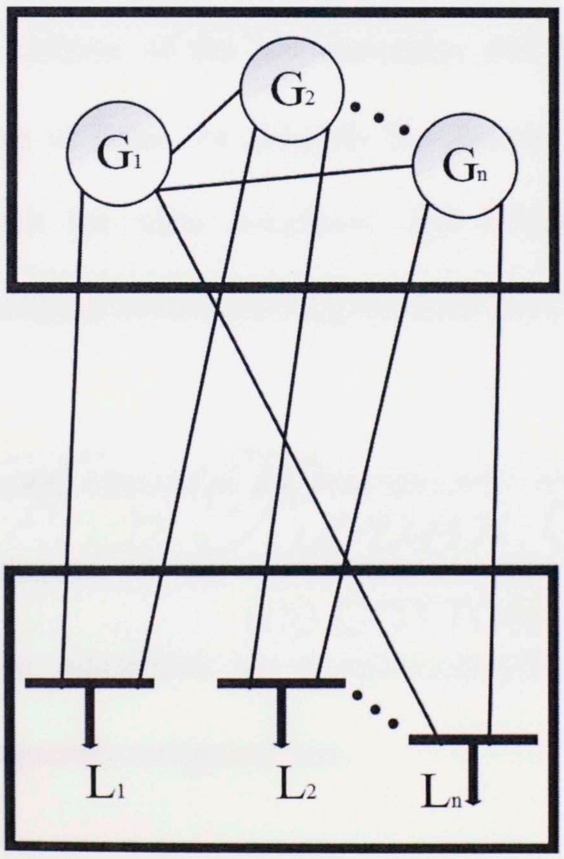

Figure 5. Source and sink model

zone (sink), respectively. The generation and the load zones are connected by the transmission lines of interest.

With respect to Equation (2-3), one of the most important characteristics of these lines is the positive GSFs for all generation units that exist in the generation zone. Since, with all the same line parameters, any MW injection increase in the generation zone results in increase of their power flows. The flows of this group of lines increase more in comparison with other lines in the system as the consequence of generation/load level increase. So, due to this high sensitivity, they can be the main candidates that can get congested frequently.

In order to verify if this model can demonstrate the distribution of the frequently congested lines, the criteria of the Maximization and the Comparative methods are studied for this group of lines. To simplify the discussion, the lines in Figure 5 are assumed lossless with the same reactance, and MVA capacities. However, these assumptions will be relaxed before drawing the final conclusion.

Section 5.1.1 Discussion 1 based on the Maximization Method Criterion

Based on criterion of the Maximization approach, if $M_j \geq \Delta f_j$ holds, the j^{th}constraint will be identified as a non-redundant constraint, which can be corresponded to a frequently congested line.

As the GSFs of the lines of interest in Figure 5 are assumed to be all positive, the second term of M_j can be ignored; and M_j would be as follows,

$$M_j = \sum_{i \in P_j} GSF_{ji} \ \Delta P_{Gi,max} \tag{5-2}$$

$$\text{Where, } P_j = \{i; GSF_{ji} > 0\}$$

With respect to (5-2), by considering the summation of all positive coefficients of each constraint (*GSF*s) multiplied by the residual capacity of the generation units, this redundancy elimination method identifies the constraints with all positive (GSF) coefficients as non-redundant. In other words, it means by output increase of any generation unit ($\Delta P_{Gi,max}$), M_j would be increased; however, generation increase of generation units with negative *GSF*s reduce the flow of line j.

The intuition behind this discussion is that when a line's GSFs are positive and closer to one rather than other lines, that line transfers more flow in comparison with the other lines. Therefore, it will have higher probability of getting congested, where all the lines have same transmission capacity limits. In a large power system with hundreds or thousands of lines with negative or positive *GSF*s, this condition is satisfied for the group of lines that connect the generation zones to the load zones in Figure 5.

Section 5.1.2 Discussion 2 based on the Comparative Method Criterion

The Comparative method, which is based on the C matrix, identifies those constraints as non-redundant that their associated rows in the C matrix have all entries bigger than or equal to their corresponding entries in other rows.

As the absolute values of GSF and Δf_j are usually different for each constraint, the discussion can be focused on the GSFs signs, when Δf_j is positive. Based on this approach if a line has a negative GSF even for one generation unit, that line will be identified as redundant in comparison with the group of lines in Figure 5, with all positive GSFs. Similar to the discussion of the Maximization method, based on the Comparison method criterion, when a line has more positive and bigger GSFs, it would have higher probability of getting frequently congested. This condition is also satisfied for the lines of interest in the Figure 5.

Based on discussions (1) and (2), the lines that connect a generation zone to a load zone, satisfy the criteria of the Maximization and the Comparative methods to be the frequently congested lines. This result remains consistent by having different line parameters and transfer capacities. Thus, the validity of explained model even is verified by the two proposed methods, and this group of lines would be probably some of those that can get frequently congested.

Section 5.2 Congestion Prioritization Index

In this section, we relax the assumptions about the equal line parameters and transmission capacities to develop a prioritization index to prioritize the lines with respect to their likelihood of getting congested. Under this condition, the flow of each line would be dependent on its transmission capacity and its GSFs. With respect to (2-3), the total line flow change at the k^{th} line (Δf_k), as a result of generation change at generation zone i can be calculated as $GSF_{k,i}.\Delta P_{Gi}$. However, this is only for aggregated model in which there is only one generation zone, i.

In order to consider generation change of all generation units at different locations, the IEEE 14-bus system is considered, Figure 6. Let line 6-13 be an arbitrary line chosen to develop the prioritization index. Any generation change at each generator bus will have an impact on the flow of this line, either in the agreeing or in the opposite

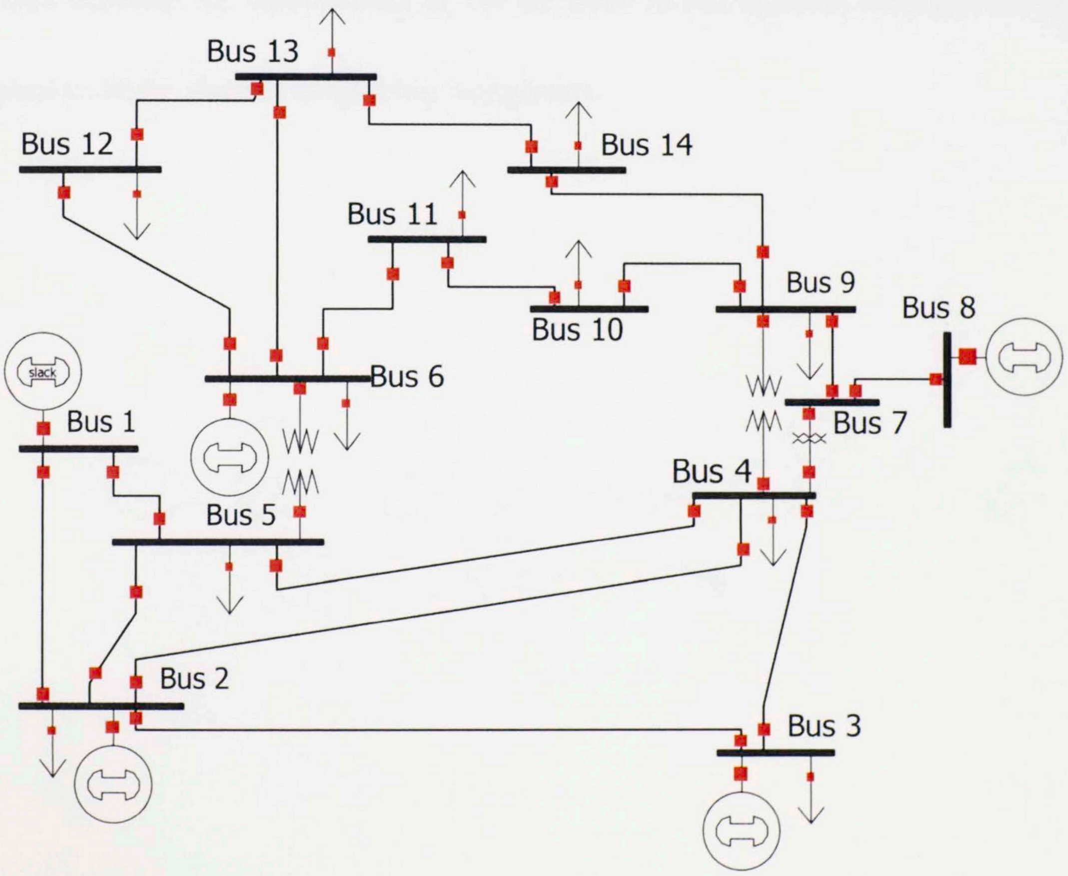

Figure 6. IEEE 14-bus system

direction. So, the summation of all GSFs of this line associated with all generation units would be the GSF of the all generation zones for this line. Moreover, to take the current line flow and its capacity limit into account, the line flow change (Δf_k) is summed with f_j and their summation is divided by the maximum transmission capacity of the line, respectively. The developed congestion prioritization index (α_l), which is the absolute value of the explained measurement, is as following,

$$\alpha_l = \left| \frac{f_l + \sum_{i=1}^{NG} GSF_{l,i} . \Delta P_{Gi}}{f_{max,l}} \right| \quad \textbf{(5-3)}$$

The line with the largest α_l value is expected to get congested preceding other lines. In this manner, by calculating α_j for all lines in the system, they can be prioritized with respect to their chance of getting congested.

Chapter 6: Case Study

In this chapter, the IEEE-39 bus system is simulated to implement the proposed algorithm Figure 7. It is a standard system for testing new methods; since, it represents a reduced model of the power system in New England. This system has 10 generators and 46 lines, and operates under OPF condition with all generators available for

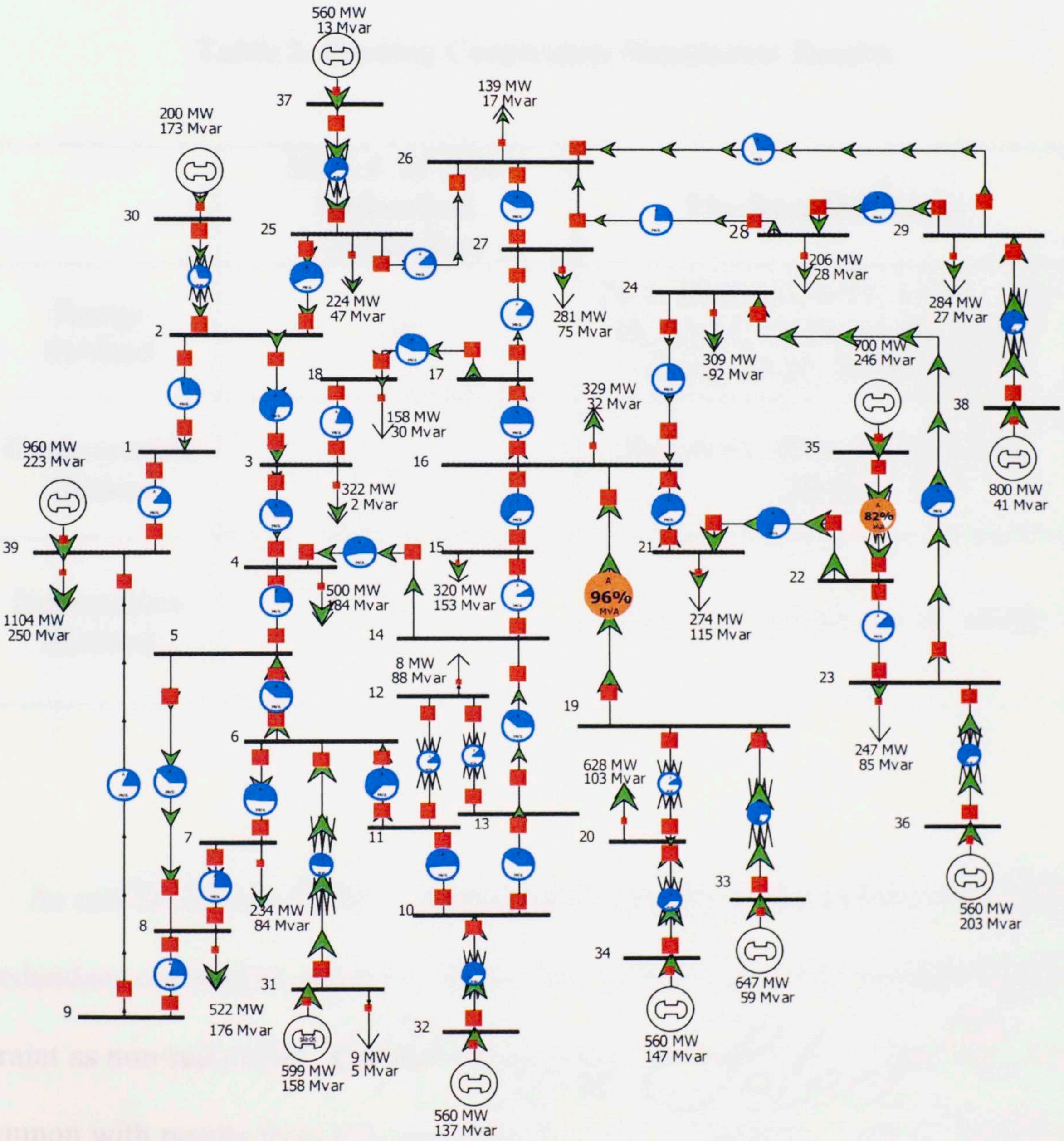

Figure 7. IEEE 39-bus system

Automatic Generation Control. The data of the system is available in appendix B.

The maximum number of simultaneous binding constraints is expected to be equal or less than $rank([GSF]) = 9$. In order to investigate it, the Maximization and the Comparative methods and the Intersection algorithm are simulated; and the results are presented in Table 2.

Table 2. Binding Constraints Simulation Results

	Max. # of Non-Redundant Constraints	**The Specified Lines**
Range Method	14	25-2, 18-3, 6-7, 6-11, 11-10, 13-10, 13-14, 15-14, 16-15, 24-16, 17-18, 17-27, 37-25, 26-27
Comparative Method	6	25-2, 6-11, 39-9, 11-10, 13-14, 17-18
Intersection Method	5	25-2, 6-11, 11-10, 13-14, 17-18

As can be seen in Table 2, Maximization method overestimates the number of non-redundant constraints equal to 14; however, the Comparative method identifies 6 constraint as non-redundant. The Intersection algorithm, identifies 5 constraints that are in common with results from the two methods as non-redundant, Figure 8 and Figure 9. In Figure 8, the red, blue and purple lines are associated with non-redundant constraints, identified by the Maximization and Comparative methods and Intersection algorithm, respectively. These lines can be considered as those which have high probability of being congested simultaneously (SCTLs).

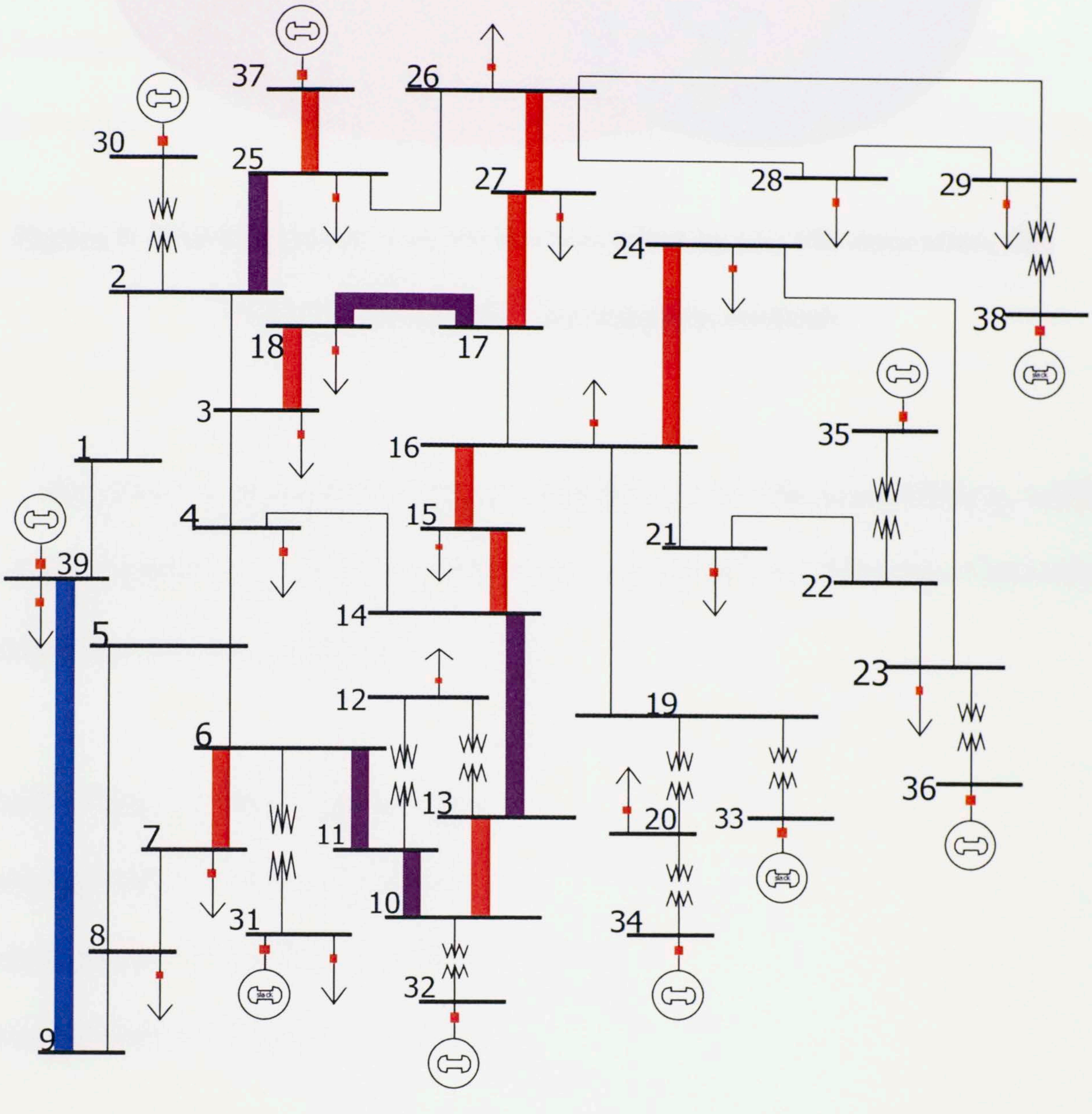

Figure 8. Representation of lines with high chance of getting congested in IEEE 39-Bus System

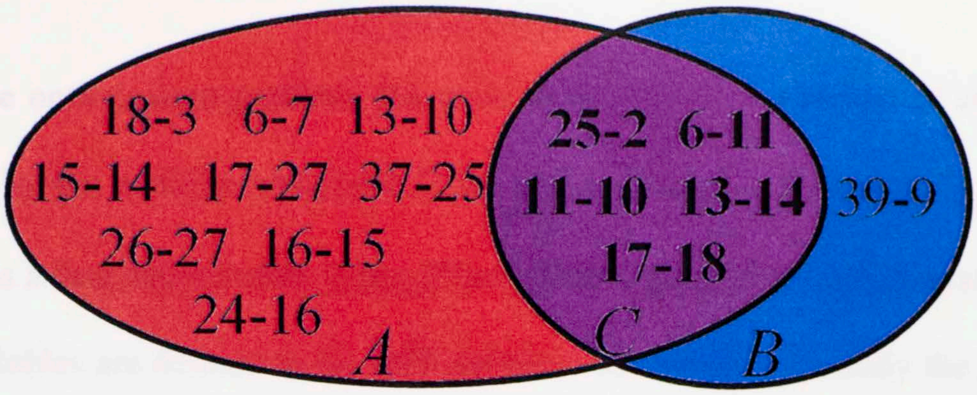

Figure 9. Non-redundant constraints identified by (A) Maximization, (B) Comparative and (C) Intersection methods

Moreover, in discussion around degeneracy in LP, the proposition is verified with various linear cost functions and generator limits and the following observations are obtained at every simulated case:

7 binding lines + 2 binding generators

= 4 binding lines + 5 binding generators

= 3 binding lines + 6 binding generators

= 2 binding lines + 7 binding generators

= 9

Based on the simulation results, it can be concluded that in case of a non-degenerate solution, the number of simultaneously congested lines (binding constraints) would be always less than or equal to the number of decision variables, which the generation units in the OPF problem. This conclusion is agree with the result obtained from intersection algorithm as well.

In the optimization problem discussed in this study, the impact of slack bus is represented by the equality constraint (3-2), and its output variations $\Delta P_{G_{Slack}}$ is not considered as a decision variable. Thus, if there exist i generation units in system, $i-1$ decision variables are defined in the OPF problem. Moreover, as usually the number of lines (j) is significantly more than number of generation units (i), $max\ rank([GSF]) = i-1$ holds and this represents that the max number of constraints that can be simultaneously binding -SCTLs- is equal or smaller than number of generation units in the system minus one. This theory is implemented successfully by the IEEE-39 bus system. Since, the Intersection algorithm identifies 5 lines as the candidates to be the SCTLs.

Conclusions

In this study, a new algorithm for identification of the simultaneously congested lines in power market is proposed. To this end, the theoretical maximum number of possible simultaneously congested lines is studied. The study implies that if there exists one unique optimal solution, the theoretical limit is the number of the dispatch decisions of all generation units. Then, the algorithm is developed to identify a smaller set of lines based on the two methods, Maximization and Comparative, which one concerns the absolute impact of generation dispatch decisions, and one concerns the relative sensitivities of power flow between different transmission lines. Each of these methods are sufficient conditions for identification of redundant line constraints. In Intersection algorithm, first some redundant constraints are eliminated based on the two methods, then, a smaller set of lines are searched in the intersection of the two sets of remaining constraints after the methods are applied. In addition, using the algorithm and based on the intensive simulation results a general congestion distribution model is explained, which expresses the lines connecting the generation and load zones are those which can get frequently congested. The proposed algorithm is illustrated in a power market using the IEEE-39 bus system.

References

[1] A. Sekar, "Identify Overloaded Transmission Lines In Ttc And Atc Determinations," *Ieee Pes Power Syst. Conf. Expo. 2004.*, Pp. 1333–1337.

[2] L. Hu, J. Zhao, J. H. Member, X. Guo, And Q. Zeng, "Computing The Boundary Of Simple Power System Feasible Region Based On The Optimization Method," Pp. 1–6, 2012.

[3] Y. Yu, S. Member, P. Li, H. Jia, S. T. Lee, And P. Zhang, "Computation Of Boundary Of Power Flow Feasible Region With Hybrid Method," Pp. 1–7.

[4] M. Khanabadi, H. Ghasemi, And I. Member, "Transmission Congestion Management Through Optimal Transmission Switching," Pp. 1–5, 2011.

[5] K. Y. Lee And S. Manuspiya, "Network Congestion Assessment For Short-Term Transmission Planning Under Deregulated Environment," *2001 Ieee Power Eng. Soc. Winter Meet. Conf. Proc. (Cat. No.01ch37194)*, Vol. 3, No. C, Pp. 1266–1271, 2001.

[6] A. L. Ott, "Can Flowgates Really Work?," Pjm, 2000.

[7] "Joint Miso-Pjm Planning Study," Miso- Pjm Ipsac 3rd Meeting, December 14, 2012, 2012.

[8] Wikipedia, "Pjm Interconnection." [Online]. Available: Http://En.Wikipedia.Org/Wiki/Pjm_Interconnection.

[9] H. Greenberg, "An Algorithm For Determining Redundant Inequalities And All Solutions To Convex Polyhedra," *Numer. Math.*, Vol. 24, No. 1, Pp. 19–26, Feb. 1975.

[10] B. Stott And O. Alsaç, "Optimal Power Flow — Basic Requirements For Real-Life Problems And Their Solutions," Pp. 1866–1876, 2012.

[11] S. Z. G. L. Thompson, F. M. Tonge, "Techniques For Removing Nonbinding Constraints And Extraneous Variables From Linear Programming Problems," *Manage. Sci.*, Vol. 26, Pp. 588–608, 1966.

[12] J. Telgen, "Identifying Redundant Constraints And Implicit Equalities In Systems Of Linear Constraints *," Vol. 29, No. 10, Pp. 1209–1222, 2013.

[13] G. V Cox, "A Redundant Linear Constraint Elimination Technique By," No. September, 2006.

[14] W.-S. L. Andreas Antoniou, *Practical Optimization- Algorithms And Engineering Applications*. University Of Victoria, British Columbia, Canada: Springer Us, 2007, Pp. 265–319.

[15] F. E. Tariff, "Day-Ahead Locational Marginal Prices," *Miso*, 2014. [Online]. Available: Https://Www.Misoenergy.Org/_Layouts/Miso/Ecm/Download.Aspx?Id=19269.

[16] Ercot, "Congestion Report," 2013. [Online]. Available: Http://Www.Ercot.Com/Mktinfo/Rtm/Index.

[17] W. F. T. H. W. Dormmel, "Optimal Power Flow Soultions," *Power Appar. Syst. Ieee Trans. Power Syst.*, Vol. Pas-87, Pp. 1866–9510, 2007.

[18] F. D. G. M. Huneault, "A Survey Of The Optimal Power Flow Literature," *Ieee Trans. Power Syst.*, Vol. 6, No. 2, Pp. 726–770, 1991.

[19] H. Unive-Sity, "Ieee Transactions On Power Systems, Vol. 14, No. 1, February 1999 105," Vol. 14, No. 1, Pp. 105–111, 1999.

[20] A. B. Ihrig, "Congestion In The Iso-Ne Electricity Markets," 2002.

Appendix A: Generation Shift Factor Data

Table 3. GSFs for All Trading Paths of IEEE 39-Bus System

#	Line	G_{31}	G_{32}	G_{33}	G_{34}	G_{35}	G_{36}	G_{37}	G_{38}	G_{39}
1	2-1	-21.07	-18.71	-10.24	-10.24	-10.24	-10.24	0	-4.8	-60.9
2	1-39	-21.07	-18.71	-10.24	-10.24	-10.24	-10.24	0	-4.8	-60.9
3	2-3	-63.2	-64.46	-63.63	-63.63	-63.63	-63.63	-7.39	-35.14	-31.34
4	25-2	15.72	16.83	26.13	26.13	26.13	26.13	91.6	60.06	7.76
5	30-2	-100	-100	-100	-100	-100	-100	-100	-100	-100
6	3-4	-53.98	-52.35	-19.23	-19.23	-19.23	-19.23	0	-5.75	-26.92
7	18-3	9.22	12.11	44.4	44.4	44.4	44.4	6.18	29.39	4.43
8	4-5	-50.41	-28.86	0	0	0	0	0	0	-26.2
9	4-14	-3.58	-23.49	-19.84	-19.84	-19.84	-19.84	0	-6.81	0
10	5-6	-56.19	-35.49	-4.57	-4.57	-4.57	-4.57	0	0	-4.18
11	5-8	5.79	6.64	5.18	5.18	5.18	5.18	0	2.5	-22.02
12	6-7	15.28	12.07	5.06	5.06	5.06	5.06	0	2.3	-17.08
13	6-11	28.52	-47.57	-9.64	-9.64	-9.64	-9.64	0	-3.74	12.9
14	6-31	-100	0	0	0	0	0	0	0	0
15	7-8	15.28	12.07	5.06	5.06	5.06	5.06	0	2.3	-17.08
16	8-9	21.07	18.71	10.24	10.24	10.24	10.24	0	4.8	-39.1
17	39-9	-21.07	-18.71	-10.24	-10.24	-10.24	-10.24	0	-4.8	39.1
18	11-10	25.96	-47.78	-8.77	-8.77	-8.77	-8.77	0	-3.4	11.74
19	13-10	-25.96	-52.22	8.77	8.77	8.77	8.77	0	3.4	-11.74

#	Line	G_{31}	G_{32}	G_{33}	G_{34}	G_{35}	G_{36}	G_{37}	G_{38}	G_{39}
20	10-32	0	-100	0	0	0	0	0	0	0
21	12-11	-2.57	0	0	0	0	0	0	0	0
22	12-13	2.57	0	0	0	0	0	0	0	0
23	13-14	28.52	52.43	-9.64	-9.64	-9.64	-9.64	0	-3.74	12.9
24	15-14	-24.94	-28.94	29.47	29.47	29.47	29.47	2.22	10.54	-12.18
25	16-15	-24.94	-28.94	29.47	29.47	29.47	29.47	2.22	10.54	-12.18
26	17-16	-24.94	-28.94	-70.53	-70.53	-70.53	-70.53	2.22	10.54	-12.18
27	16-19	0	0	-100	-100	0	0	0	0	0
28	16-21	0	0	0	0	-64.74	-52.44	0	0	0
29	24-16	0	0	0	0	35.26	47.56	0	0	0
30	17-18	9.22	12.11	44.4	44.4	44.4	44.4	6.18	29.39	4.43
31	17-27	15.72	16.83	26.13	26.13	26.13	26.13	-8.4	-39.94	7.76
32	19-20	0	0	0	-100	0	0	0	0	0
33	19-33	0	0	-100	0	0	0	0	0	0
34	20-34	0	0	0	-100	0	0	0	0	0
35	21-22	0	0	0	0	-64.74	-52.44	0	0	0
36	22-23	0	0	0	0	35.26	-52.44	0	0	0
37	22-35	0	0	0	0	-100	0	0	0	0
38	24-23	0	0	0	0	-35.26	-47.56	0	0	0
39	23-36	0	0	0	0	0	-100	0	0	0
40	25-26	-15.72	-16.83	-26.13	-26.13	-26.13	-26.13	8.4	-60.06	-7.76
41	37-25	0	0	0	0	0	0	100	0	0

#	Line	G_{31}	G_{32}	G_{33}	G_{34}	G_{35}	G_{36}	G_{37}	G_{38}	G_{39}
42	26-27	-15.72	-16.83	-26.13	-26.13	-26.13	-26.13	8.4	39.94	-7.76
43	26-28	0	0	0	0	0	0	0	-50	0
44	26-29	0	0	0	0	0	0	0	-50	0
45	28-29	0	0	0	0	0	0	0	-50	0
46	29-38	0	0	0	0	0	0	0	-100	0

Appendix B: IEEE 39-Bus System Data

Table 4. IEEE 39-Bus System (Line Parameters)

From	To	R	X	B
1	2	0.0035	0.0411	0.6987
1	39	0.001	0.025	0.75
2	3	0.0013	0.0151	0.2572
2	25	0.007	0.0086	0.146
3	4	0.0013	0.0213	0.2214
3	18	0.0011	0.0133	0.2138
4	5	0.0008	0.0128	0.1342
4	14	0.0008	0.0129	0.1382
5	6	0.0002	0.0026	0.0434
5	8	0.0008	0.0112	0.1476
6	7	0.0006	0.0092	0.113
6	11	0.0007	0.0082	0.1389
7	8	0.0004	0.0046	0.078
8	9	0.0023	0.0363	0.3804
9	39	0.001	0.025	1.2
10	11	0.0004	0.0043	0.0729
10	13	0.0004	0.0043	0.0729
13	14	0.0009	0.0101	0.1723
14	15	0.0018	0.0217	0.366
15	16	0.0009	0.0094	0.171
16	17	0.0007	0.0089	0.1342
16	19	0.0016	0.0195	0.304
16	21	0.0008	0.0135	0.2548
16	24	0.0003	0.0059	0.068
17	18	0.0007	0.0082	0.1319
17	27	0.0013	0.0173	0.3216
21	22	0.0008	0.014	0.2565
22	23	0.0006	0.0096	0.1846
23	24	0.0022	0.035	0.361
25	26	0.0032	0.0323	0.513
26	27	0.0014	0.0147	0.2396
26	28	0.0043	0.0474	0.7802
26	29	0.0057	0.0625	1.029
28	29	0.0014	0.0151	0.249

Table 5. IEEE 39-Bus System (Transformers Data)

From	To	R	X	MVA
2	30	0	0.0181	100
31	6	0	0.025	100
10	32	0	0.02	100
12	11	0.0016	0.0435	100
12	13	0.0016	0.0435	100
19	20	0.0007	0.0138	100
19	33	0.0007	0.0142	100
20	34	0.0009	0.018	100
22	35	0	0.0143	100
23	36	0.0005	0.0272	100
25	37	0.0006	0.0232	100

Table 6. IEEE 39-Bus System (Generation Units Data)

Bus	MW	MVAR	Max MVAR	Min MVAR	V	MVA	X	P_{max}	P_{min}
30	200	173.14	250	-50	1.047	100	0.031	500	0
31	600	157.76	999	-100	0.98	150	0.0697	999	0
32	560	136.81	240	-70	0.983	100	0.0531	700	0
33	647	58.63	130	-70	0.997	200	0.0436	700	0
34	560	147.48	220	-70	1.012	300	0.132	700	0
35	700	246.46	350	-70	1.049	400	0.05	700	0
36	560	203.15	430	-70	1.063	100	0.049	700	0
37	560	13.06	110	-70	1.027	300	0.057	700	0
38	800	40.94	110	-70	1.026	100	0.057	1000	0
39	960	223.21	310	-120	1.03	100	0.06	1200	0

Table 7. IEEE 39-Bus System (Bus Data)

Bus Number	Voltage	Angle	Load		Shunt
			MW	MVAR	MVAR
1	1.05345	-10.3857			
2	1.06163	-9.4819	8.5	88	
3	1.05234	-9.7466	32	2.4	
4	1.01442	-9.682	50	14	-100
5	0.98993	-9.6114			-200
6	0.98947	-9.536			
7	0.99023	-9.8678	33.8	14	
8	0.99091	-9.9445	52	16	
9	0.99609	-10.494			-200
10	1.01898	-9.244			
11	1.00951	-9.3408			
12	1.02169	-9.3163			
13	1.02168	-9.292			
14	1.02818	-9.3945			
15	1.05801	-9.1307	32	13	
16	1.06974	-8.8696	32.4	2.3	
17	1.07032	-9.6048			
18	1.06368	-9.8088	58	10	
19	1.07172	-7.2934			
20	1.01416	-7.0557	68	13	
21	1.07448	-8.3239	74	15	
22	1.07563	-7.7489			
23	1.07378	-7.8235	27.5	4.6	
24	1.07144	-8.8189	38.6	-2.2	
25	1.06657	-9.4992	24	7.2	
26	1.08605	-10.7823	39	7	
27	1.08034	-10.5667	81	5.5	
28	1.08197	-11.5348	26	7.6	
29	1.07367	-11.5471	83.5	6.9	
30	1.0475	-8.8605			
31	0.982	-8.7931	9.2	4.6	
32	0.9831	-8.326			
33	0.9972	-6.2199			
34	1.0123	-6.0605			
35	1.0493	-7.0047			
36	1.0635	-6.8852			
37	1.0278	-7.9267			
38	1.0265	-11.0716			
39	1.03	-10.92	114	25	

Table 8. IEEE 39-Bus System (Optimal Schedule for the Base Case)

Generator Number	Cost Function ($/h)	Available for AGC	Available for AVR	Participation Factor
30	$0.0193P^2 + 6.9P$	Yes	Yes	10.0
31	$0.0111P^2 + 3.7P$	Yes	Yes	10.0
32	$0.0104P^2 + 2.8P$	Yes	Yes	10.0
33	$0.0088P^2 + 4.7P$	Yes	Yes	10.0
34	$0.0128P^2 + 2.8P$	Yes	Yes	10.0
35	$0.0094P^2 + 3.7P$	Yes	Yes	10.0
36	$0.0099P^2 + 4.8P$	Yes	Yes	10.0
37	$0.0113P^2 + 3.6P$	Yes	Yes	10.0
38	$0.0071P^2 + 3.7P$	Yes	Yes	10.0
39	$0.0064P^2 + 3.9P$	Yes	Yes	10.0