WEAK BOSON DECAYS INTO TWO QUARKS PLUS A PHOTON OR A GLUON

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PREFACE

This study is concerned with the decays of intermediate vector bosons into particular decay schemes. Much attention is given to the existence of zeros in certain amplitudes. Also, Dalitz plots are obtained for many three-body decays.

I wish to express my great appreciation to my thesis adviser, Dr. Karnig O. Mikaelian, first for his guidance and assistance throughout this study, and also for many enlightening discussions. Also, I would like to thank Dr. Mark Samuel and Katsunori Mita for their computer programming assistance, and to thank the committee members, Dr. Mark Samuel, Dr. N. V. V. J. Swamy, and Dr. Geoffrey P. Summers, for their assistance in the final manuscript preparation.

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CHAPTER I

INTRODUCTION

With the building of colliding beam machines which will be able to produce energetic W bosons, the possibility of recording decay products will facilitate the study of various decay schemes. The processes we have studied are three body decay distributions. Where bodies in the final state have color, like quarks and gluons, we have hadron jets.

If SU(2) x U(1) is the correct group unifying weak and electromagnetic interactions, we expect to find the W^- boson with the properties expected from the Weinberg - Salam model. A good test of the model is to produce the W^- bosons and study the decay modes. The prediction of a zero value in the differential cross-section of $W^- \to q\bar{q}\gamma$, for K = 1 (1), could lead to a good confirmation of this theory, and confirmation of other parameters such as K, quark charges, couplings, etc. If the anomylous magnetic moment of the W bosons, K, does not equal one, no zero could be expected in the angular distribution (2).

The first process, $W^- \to q\bar{q}\gamma$, like the crossed process, $q\bar{q} \to W^-\gamma$, has been found to contain a factorization in its amplitude (2). Briefly the factorization is produced as follows: three Feynman diagrams contribute to the process, $W^- \to q\bar{q}\gamma$, so we can write $M_{fi} = Q_i M_A + Q_j M_B + Q_w M_C$, where Q_i, Q_j, and Q_w are the electric charges of q, $\bar{q}$, and W. We find, after some algebra, $M_{fi} = F(Q_i, S, T, U)(M_A + M_B)$ after setting $Q_i = Q_w - Q_j$. For F = 0 we have a zero in our cross section. We have examined this particular zero as it occurs in Dalitz plots. We have found the analytic expression for this partial decay probability. There has been an attempt to explain the zero theoretically (3), but as yet we cannot present a physical reason for the existence of the zero.

Next, we have examined briefly the process $W^- \rightarrow q\bar{q}\gamma$ where the quarks possess spin zero; there are four Feynman diagrams here; thus we have determined that in this process we have a factorization also. Thereby, we have found a zero in the physical region of this process also. Notable is the "occurrence" that this factorization is identical to that one found previously for spin one-half quarks.

Then, since $W^- \rightarrow q\bar{q}g$ is similar to $W^- \rightarrow q\bar{q}\gamma$, we examined this three jet decay with gluon jet and produced Dalitz plots. We note that this process has color matrices (4,5), and had there been a factorization, it would have necessarily been a function of these matrices. We did not find a factorization though, since a factorization seems plausible in processes with three or more diagrams, and this process has only two diagrams. We found also the analytic expression for this partial decay probability.

These processes constitute groundwork for a more complete understanding of the nature of the W boson and its relationships with other particles.

CHAPTER II

W^- DECAY TO TWO QUARKS AND A PHOTON

Refering to Figure 1, and using the Feynman rules given in Appendix A; we find the matrix element for

$$W^-(p) \rightarrow q_i(q_1) + \bar{q}_j(q_2) + \gamma(k)$$

where

$$p = q_1 + q_2 + k.$$

$$M_{fi} = e\bar{u}(q_1)(-iQ_i\gamma_\mu)\left(\frac{i}{\ell_1}\right)\left(-\frac{ig}{2\sqrt{2}}\gamma_\nu(1-\gamma_5)\right)v(q_2)\varepsilon_k^\mu\,\varepsilon_p^\nu$$

$$+ e\bar{u}(q_1)\left(\frac{-ig}{2\sqrt{2}}\gamma_\nu(1-\gamma_5)\right)\left(\frac{i}{-\ell_2}\right)(-iQ_j\gamma_\mu)v(q_2)\varepsilon_k^\mu\,\varepsilon_p^\nu$$

$$+ e\bar{u}(q_1)\left(\frac{-ig}{2\sqrt{2}}\gamma_\alpha(1-\gamma_5)\right)v(q_2)\left(\frac{i(h^\alpha h^\beta - g^{\alpha\beta})}{h^2 - M_W^2}\right)(-i(Q_i - Q_j))$$

$$\times\left[g_{\nu\beta}(-p-h)_\mu + g_{\mu\nu}(2p-h)_\beta - g_{\mu\beta}(p-2h)_\nu\right]\varepsilon_k^\mu\,\varepsilon_p^\nu$$

where g is the weak coupling, e is the electromagnetic coupling, quark masses have been dropped, and where $\ell_1 = p-q_2 = q_1+k$, $\ell_2 = p-q_1 = q_2+k$, and $h = p-k = q_1 + q_2$. After considerable simplification using the gamma matrix and spinor relations, and the transversality condition for both of the bosons:

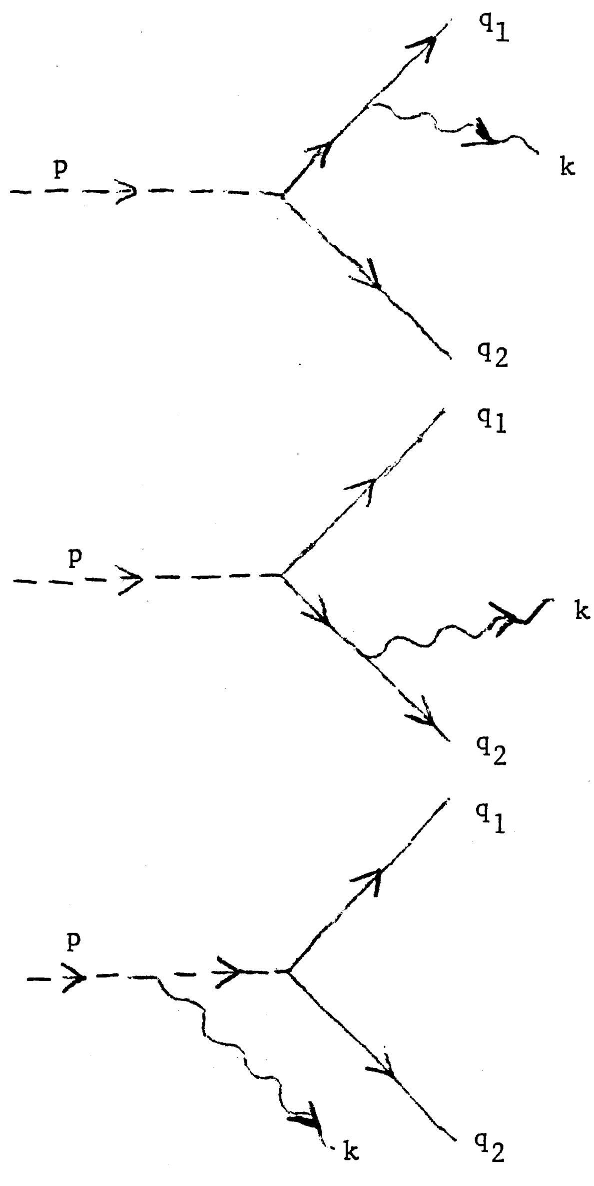


Figure 1. Diagrams for $W^- \to q\bar{q}\gamma$

$$M_{fi} = M_{\mu\nu}\,\varepsilon_k^{\mu}\varepsilon_p^{\nu} = \frac{-ieg}{2\sqrt{2}}\left(Q_i + \frac{k\cdot q_1}{k\cdot h}\right)$$

$$\times\ \bar{U}(q_1)\left\{\frac{1}{\ell_1^{\,2}}\gamma_\mu\,\not{\ell}_1\,\gamma_\nu - \frac{1}{\ell_2^{\,2}}\gamma_\nu\,\not{\ell}_2\,\gamma_\mu\right\}(1-\gamma_5)v(q_2)\,\varepsilon_k^{\mu}\,\varepsilon_p^{\nu}$$

and choosing:

$$U = \ell_1^{\,2} = 2k\cdot q_1, \qquad t = \ell_2^{\,2} = 2k\cdot q_2$$

$$S = h^2 = 2q_1\cdot q_2, \qquad \text{with } S + t + U = M_W^2$$

$$M_{fi} = \frac{-ieg}{2\sqrt{2}}\left(Q_i + \frac{U}{t+u}\right)\bar{U}(q_1)\left\{\frac{1}{u}\gamma_\mu\not{\ell}_1\gamma_\nu - \frac{1}{t}\gamma_\nu\not{\ell}_2\gamma_\mu\right\}(1-\gamma_5)v(q_2)\,\varepsilon_k^{\mu}\,\varepsilon_p^{\nu}$$

and we note already the zero at $Q_i = \frac{-u}{t+u}$. Having checked gauge invariance by $M_{\mu\nu}k^{\mu} = 0$, it is worthwhile to check $M_{\mu\nu}P^{\nu}$ to look for simplification in the W^- polarization density; and we find $M_{\mu\nu}P^{\nu} = 0$. Now the tensor $\sum_{spin} \varepsilon_p^{\nu}\varepsilon_p^{\beta} = -g^{\nu\beta} + \frac{p^{\nu}p^{\beta}}{M_W^2}$ can be simplified into just $-g^{\nu\beta}$. We note in passing that if $M_{\mu\nu}P^{\nu} \neq 0$ we may construct what may be referred to as a polarization density projection operation (see Appendix B) which will change the original M_{fi} to a new M'_{fi} which will not change the physics and will obey $M'_{\mu\nu}P^{\nu} = 0$.

Now for the spin sum and average:

$$\frac{1}{3}\sum_{\text{all spins}} M_{\mu\nu}\,M^{*}_{\alpha\beta}\,\varepsilon_p^{\nu}\varepsilon_p^{\beta}\varepsilon_k^{\mu}\varepsilon_k^{\alpha} = \frac{1}{3}\sum_{\text{quarks spins}} M_{\mu\nu}M^{*}_{\alpha\beta}g^{\nu\beta}g^{\mu\alpha}$$

$$= \frac{1}{3}\sum_{\text{spins}} M_{\mu\nu}\,M^{*\mu\nu} \equiv \frac{1}{3}|\tilde{M}|^2$$

$$\text{and } |\overline{M_{fi}}|^2 = \frac{e^2g^2}{8}\left[\frac{1}{3}|\tilde{M}|^2(3)\right]$$

$$= \frac{e^2g^2}{8}\left(Q + \frac{u}{t+u}\right)^2 \sum_{\substack{\text{quark}\\ \text{spin}}} \left|\bar{u}\left(\frac{1}{U}\gamma_\mu \not{k}_1 \gamma_\nu - \frac{1}{t}\gamma_\nu \not{k}_2 \gamma_\mu\right)(1-\gamma_5)v\right|^2$$

The factor of three was inserted for the sum over three quark colors.

$$|\overline{M_{fi}}|^2 = \frac{e^2g^2}{8}\left(Q_i + \frac{u}{t+u}\right)^2 \left[\frac{2}{U^2}\,\mathrm{Tr}\,\not{q}_1\gamma_\mu\not{k}_1\gamma_\nu(1-\gamma_5)\not{q}_2\gamma^\nu\not{k}_1\gamma^\mu\right.$$

$$+ \frac{2}{t^2}\,\mathrm{Tr}\,\not{q}_1\gamma_\nu\not{k}_2\gamma_\mu(1-\gamma_5)\not{q}_2\gamma^\mu\not{k}_2\gamma^\nu$$

$$- \frac{2}{tu}\,\mathrm{Tr}\,\not{q}_1\gamma_\mu\not{k}_1\gamma_\nu(1-\gamma_5)\not{q}_2\gamma^\mu\not{k}_2\gamma^\nu$$

$$\left.- \frac{2}{tu}\,\mathrm{Tr}\,\not{q}_1\gamma_\nu\not{k}_2\gamma_\mu(1-\gamma_5)\not{q}_2\gamma^\nu\not{k}_1\gamma^\mu\right]$$

where Dirac spinors are normalized to 2m. Using the usual trace techniques;

$$|\overline{M_{fi}}|^2 = \frac{e^2g^2}{8}\left(Q + \frac{u}{t+u}\right)^2 \left[\frac{64}{u^2}\left(\frac{u}{2}\right)\left(\frac{t}{2}\right) + \frac{64}{t^2}\left(\frac{u}{2}\right)\left(\frac{t}{2}\right)\right.$$

$$\left.+ \frac{128}{tu}\left\{\frac{S}{2}\left(\frac{S}{2} + \frac{u}{2} + \frac{t}{2}\right)\right\}\right]$$

$$= 2e^2g^2\left(Q + \frac{u}{t+u}\right)^2 \left[\frac{t^2+u^2+2M_W^2S}{tu}\right]$$

where $Q \equiv Q_i$. The second factor, $t^2 + u^2 + 2M_W^2S$, is positive definite, but the first factor, $(Q + \frac{u}{t+u})^2$, can be zero (but of course never negative) when $Q = -\frac{u}{t+u}$. However, we defer the discussion of this point until after the phase space, since the zero is most significant in the context of the phase space.

The general differential decay rate is[6]:

$$d\Gamma = \frac{1}{2M_w} |\overline{M_{fi}}|^2 \frac{1}{2E_1} \frac{d^3q_1}{(2\pi)^3} \frac{1}{2E_2} \frac{d^3q_2}{(2\pi)^3} \frac{d^3k}{2E_k(2\pi)^3}$$

$$x \ (2\pi)^4 \delta^4 (P-q_1-q_2-k)$$

$$d\Gamma = \frac{1}{2M_w(2\pi)^5} |\overline{M_{fi}}|^2 \frac{d^3q_1}{2E_1} \frac{d^3q_2}{2E_k} \delta^3(\vec{q}_1 + \vec{q}_2 + k) \delta^6 (M-E_1-E_2-E_k)$$

where the rest frame of the decaying W has been chosen. Now

$$\int \frac{d^3P_n}{2E_n} = \int_{-\infty}^{\infty} d^4P_n \delta(P_n^2 - M_n^2)\theta(E_n)$$

$$d\Gamma = \frac{|\overline{M_{fi}}|^2}{2M_w(2\pi)^5} \int\int\int dq_1 dq_2 dk \ \delta(q_1^2-m_1^2)\delta(q_2^2-m_2^2)\delta(k^2)$$

$$x \ \ \theta(E_1)\theta(E_2)\theta(E_k)\delta^3(\vec{q}_1+\vec{q}_2+\vec{k})\delta^o(M-E_1-E_2-E_k)$$

where

$$\theta(x) \equiv \begin{cases} 1 \text{ for } x \geq o \\ 0 \text{ for } x < o \end{cases}$$

Now let

$$P.S. = \int\int\int dq_1 dq_2 dk \ \delta(q_1^2)\delta(q_2^2)\delta(k^2)\theta(E_1)\theta(E_2)\theta(E_k)$$

$$x \ \delta^3(\vec{q}_1+\vec{q}_2+\vec{k})\delta^o(M-E_1-E_2-E_k)$$

where the quark masses have again been dropped. We define the Jacobian needed to get the energy distribution of the two quarks:

$$J = \frac{\partial^2(P.S.)}{\partial E_1 \partial E_2} = \int\int d^3q_1 d^3q_2 \int dk \ \delta(q_1^2)\delta(q_2^2)\delta(k^2)$$

$$x \quad \theta(E_1)\theta(E_2)\theta(E_k) \quad \delta^3(\vec{q}_1+\vec{q}_2+\vec{k})\delta^o(M-E_1-E_2-E_k)$$

$$J = \iint d^3q_1 d^3q_2 \quad \delta(q_1^2)\delta(q_2^2)\theta(E_1)\theta(E_2)\theta(M-E_1-E_2)$$

$$x \quad \delta([M-E_1-E_2]^2 - [\vec{q}_1+\vec{q}_2]^2)$$

Substituting $d^3q_n = d\Omega|\vec{q}_n|^2 dq_n = d\Omega|\vec{q}_n|E_n dE_n$, and integrating the angular distribution of q_1 over all space and integrating $\cos\theta$ from 1 to -1 for angular distribution of q_2:

$$J = 8\pi^2 \iint |\vec{q}_1||\vec{q}_2| \frac{E_1}{2E_1} E_2 dE_2 \quad \theta(E_1)\theta(E_2)\theta(M-E_1-E_2)$$

$$x \quad d(\cos\theta)\delta(E_2^2-\vec{q}_2^{\,2})\delta(M_w^2+E_2^2+2E_2E_1-2M_w(E_1+E_2)-2|\vec{q}_1||\vec{q}_2| \ x \ \cos\theta)$$

$$= 2\pi^2 E_1 E_2 \int_{-1}^{1} d(\cos\theta)\delta(-[M-E_1-E_2]^2+E_1^2+E_2^2+2|\vec{q}_1||\vec{q}_2|\cos\theta)$$

$$x \quad \theta(E_1)\theta(E_2)\theta(M-E_1-E_2)$$

$$= \pi^2\theta(E_1)\theta(E_2)\theta(M-E_1-E_2)\theta(4|\vec{q}_1|^2|\vec{q}_2|^2-(-M^2+2M(E_1+E_2)-2E_1E_2)^2)$$

where the θ-functions give the physically allowed phase space. From the phase space we get;

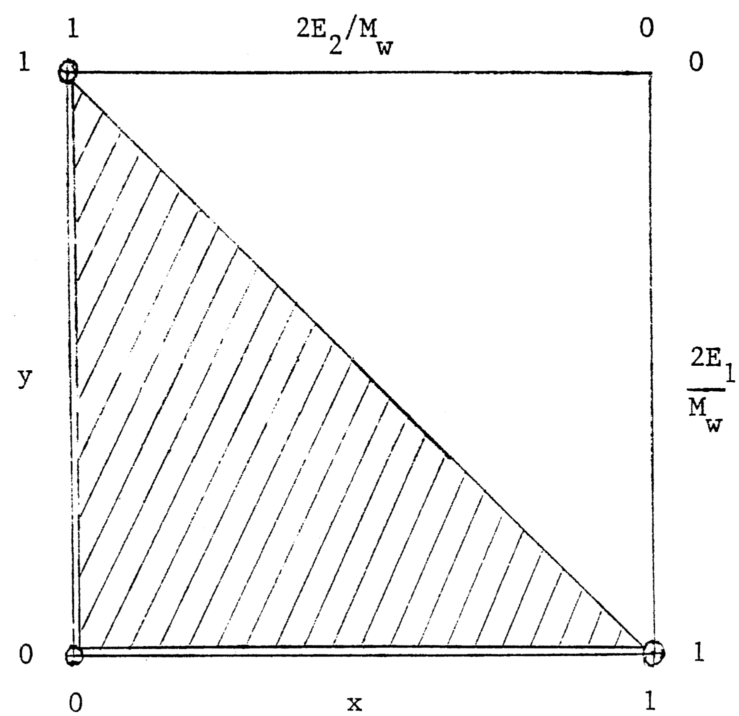
$$\frac{\partial^2\Gamma(W^-\to q\bar{q}\gamma)}{\partial E_1 \ \partial E_2} = \frac{\pi^2}{2M_w(2\pi)^5} |\overline{M_{fi}}|^2$$

where the range of E_1 and E_2, i.e. phase space, is given by the above θ-functions. Defining $x = 1 - \frac{2E_2}{M_w}$ and $y = 1 - \frac{2E_1}{M_w}$, we get for our factorization in terms of x and y; $(Q + \frac{x}{x+y})$. And we get for our differential decay width; $\frac{d^2\Gamma}{\partial x\ \partial y} = \frac{e^2 g^2 M_W}{16(2\pi)^3} (Q + \frac{x}{x+y})^2 [\frac{(x-1)^2+(y-1)^2}{xy}]$.

We get $\frac{\partial^2\Gamma}{\partial x\ \partial y} = 0$ when $Q = \frac{-x}{x+y}$ or $x = -(\frac{Q}{Q+1})y$ (a straight line in Dalitz plot phase space).

Now in order to integrate our differential decay width, we note that where we dropped the quark masses it is now necessary to impose a lower bound upon the quark energies. In addition, there is an infrared divergence for the photon energy, so it is convenient to impose the same lower bound on all three decay energies. This is equivalent to saying not only do we have an infrared divergence for the photon, but some energy must reside in the quark masses besides.

Graphing the allowed phase space:



and using the limits on x and y:

$$\Gamma = \frac{e^2 g^2 M_W}{16(2\pi)^3} \int_\varepsilon^{1-2\varepsilon} dy \int_\varepsilon^{1-y-\varepsilon} dx \ (Q + \frac{x}{x+y})^2 \ [\frac{(x-1)^2+(y-1)^2}{xy}]$$

Next, the Dalitz plot is obtained by computer methods (an analytic expression for the total decay rate will be given later); the program divided the phase space into small parts and provided a histogram of decay probability (see Appendix C). In our histogram, where the relation $Q = \frac{-x}{x+y}$ is satisfied, that is, when the energies of the quark and antiquark are such that

$$E_q = (\frac{2Q+1}{2Q}) M_W - (\frac{Q+1}{Q}) E_{\bar{q}}$$

then our probability of finding such an event is nil. For clarity, all probabilities in the Dalitz plots have been given as the percentage of the total probability for the respective process.

Now returning to integrate $d\Gamma$:

$$\Gamma(W \to q\bar{q}\gamma) = \frac{e^2 g^2 M_W}{16(2\pi)^3} \int_\varepsilon^{1-2\varepsilon} dx \int_\varepsilon^{1-x-\varepsilon} dy \ (Q^2 + \frac{2Qx}{x+y} + \frac{x^2}{(x+y)^2})$$
$$\times \ [\frac{(x-1)^2+(y-1)^2}{xy}]$$

To evaluate these integrals we can use the symmetry between x and y, dx and dy, to make the integrals much simpler. This is only valid for the identical cutoff ε we have imposed for both dx and dy.

For the first integral, we define X by

$$\int dx \int dy \ [\frac{(x-1)^2+(y-1)^2}{xy}] = 2 \int dx \int dy \ \frac{(x-1)^2}{xy} \equiv 2X$$

For the second integral:

$$2Q\int dx\int dy\ \frac{x}{(x+y)}\left[\frac{(x-1)^2+(y-1)^2}{xy}\right] = Q\int dx\int dy\ \frac{(x+y)}{(x+y)}\left[\frac{(x-1)^2+(y-1)^2}{xy}\right]$$

$$= 2Q\ \int dx\int dy\ \frac{(x-1)^2}{xy} = 2QX$$

For the third integral:

$$\int dx\int dy\left(\frac{x^2}{(x+y)^2}\right)\left(\frac{(x-1)^2+(y-1)^2}{xy}\right) = \int dx\int dy\ \frac{\frac{1}{2}x^2+\frac{1}{2}y^2}{(x+y)^2}\left(\frac{(x-1)^2+(y-1)^2}{xy}\right)$$

$$= \int dx\int dy\ \frac{\left[(x^2+2xy+y^2)-2xy\right]}{(x+y)^2}\left(\frac{(x-1)^2}{xy}\right)$$

$$= X - 2\int dx\int dy\ \frac{(x-1)^2}{(x+y)^2}$$

For the total integration

$$\Gamma(W^-\to q\bar{q}\gamma) = \frac{e^2g^2M_W}{16(2\pi)^3}\left[(2Q^2+2Q+1)X-2\int dx\int dy\ \frac{(x-1)^2}{(x+y)^2}\right]$$

Performing the integrals (see Appendix D)

$$\Gamma(W^-\to q\bar{q}\gamma) = \frac{e^2g^2M_W}{16(2\pi)^3}\{(Q^2+Q+\tfrac{1}{2})\ \left[2\ell n^2\varepsilon - 3|\ell n\varepsilon| + \frac{5}{2} - \frac{\pi^2}{3}\right.$$

$$- 6\varepsilon - \frac{9}{2}\varepsilon^2 + 6\varepsilon|\ell n\varepsilon| + 4\ell n(1-\varepsilon)|\ell n\varepsilon| - 3\ell n(1-2\varepsilon) + 6\varepsilon\ell n(1-2\varepsilon)$$

$$\left. + 4Li_2\left(\frac{\varepsilon}{1-\varepsilon}\right) + 2\ell n^2(1-\varepsilon)\right]$$

$$+ \frac{1}{(1-\varepsilon)}\left[\frac{11}{3} - 11\varepsilon - \varepsilon^2 + 3\varepsilon^3\right] + 2(\varepsilon^2+2\varepsilon+1)\ell n\left(\frac{2\varepsilon}{1-\varepsilon}\right)\}$$

where

$$Li_2(x) = -\int_0^x \frac{\ln(1-z)}{z}\,dz = \frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \cdots \text{ for } |x| \leq 1$$

If we keep only terms of order ε:

$$\Gamma(W^- \to q\bar{q}\gamma) \cong \frac{e^2 g^2 M_W}{16(2\pi)^3} \{(Q^2+Q+\tfrac{1}{2})[2\ell n^2\varepsilon - 3|\ell n\varepsilon| + \frac{5}{2} - \frac{\pi^2}{3}$$

$$+ 2\varepsilon|\ell n\varepsilon| + 4\varepsilon + O(\varepsilon^2)]$$

$$+ \frac{11}{3} - \frac{22}{3}\varepsilon + 2\ell n(2) + 4\varepsilon\ell n(2) + O(\varepsilon^2)\}$$

Using $\Gamma_o \equiv \Gamma(W^- \to q\bar{q}) = \frac{M_W g^2}{16\pi}$ we have $\frac{\Gamma(W^- \to q\bar{q}\gamma)}{\Gamma_o}$ as a function of ε (see Table I) where we have chosen $Q = -1/3$, corresponding to the charge of the d-quark ($W^- \to d\,\bar{u}\,\gamma$).

The special case of $W \to e\,\bar{\nu}\,\gamma$, where now we have a color singlet with $Q = -1$, will give us

$$\frac{\Gamma(W^- \to e\,\bar{\nu}\,\gamma)}{\Gamma_o(W^- \to e\bar{\nu})}$$

as a function of ε (see Table II).

Now, since the factorization we have is dependent upon charge, it is natural to conclude that the factorization in this case and in any other case will depend upon one set of group theoretical factors, in the present case, charge factors. To see how independent the factorization is of non-group factors, we can ask how other factors might change the existing factorization. For instance, we already know that if $K \neq 1$, no zero should be found (2). We could replace the quarks by particles

TABLE I

THE DECAY WIDTH RATIO, $\frac{\Gamma(W^- \to q\bar{q}\gamma)}{\Gamma_o}$

ε	.09	.12	.15	.18	.21
$\frac{\Gamma(W^- \to q\bar{q}\gamma)}{\Gamma_o}$	3.3×10^{-4}	1.8×10^{-4}	9.8×10^{-5}	5.3×10^{-5}	2.8×10^{-5}

TABLE II

THE DECAY WIDTH RATIO, $\frac{\Gamma(W^- \to e\bar{\nu}\gamma)}{\Gamma_o}$

ε	.09	.12	.15	.18	.21
$\frac{\Gamma(W^- \to e\bar{\nu}\gamma)}{\Gamma_o}$	1.4×10^{-3}	8.8×10^{-4}	5.4×10^{-4}	3.3×10^{-4}	1.9×10^{-4}

retaining their charged nature; also, we might replace the W by another spin one or spin zero charged particle. Any of these changes may or may not have a factorization in their amplitude. One change we have tried is to use spin zero quarks.

CHAPTER III

W DECAY TO SPIN ZERO QUARKS AND PHOTON

To examine the case of spin zero quarks, we refer to the diagrams in Figure 2 and the Feynman rules in Appendix A and get:

$$M_{fi} = -ieg[\frac{Q_i}{\ell_1^2}\{(2q_1+k)_\mu (P-2q_2)_\nu - \frac{1+Q_i}{\ell_2^2}\{(2q_2+k)_\mu (2q_1-P)_\nu\}$$

$$- (2Q_i+1) g_{\mu\nu} + \frac{1}{h^2-M_w^2}\{(q_1-q_2)_\nu (-2P)_\mu + (P+k)\cdot(q_1-q_2) g_{\mu\nu}$$

$$+ 2(q_1-q_2)_\mu (q_1+q_2)_\nu\}]\varepsilon_k^\mu \varepsilon_P^\nu$$

We first check the gauge invariance: $M_{\mu\nu}k^\mu=0$. Then, using the transversality condition, and dropping the quark masses, we find the factorization:

$$M_{fi} = 4ieg(Q_i + \frac{u}{t+u})\{\frac{1}{u} q_{2\nu}\, q_{1\mu} + \frac{1}{t} q_{1\nu}\, q_{2\mu} + \frac{g_{\mu\nu}}{2}\}\varepsilon_k^\mu \varepsilon_P^\nu$$

(spin zero)

cf. $$M_{fi} = \frac{-ieg}{2\sqrt{2}} (Q_i + \frac{u}{t+u})\bar{u}(\gamma_\mu \frac{\not{\ell}_1}{u} \gamma_\nu - \gamma_\nu \frac{\not{\ell}_2}{t} \gamma_\mu) v\, \varepsilon_k^\mu \varepsilon_P^\nu$$

(spin one-half)

where s,t, and u are same in each. While the second factor of each amplitude is different, the first factor of each is the same, $Q+ \frac{u}{t+u}$.

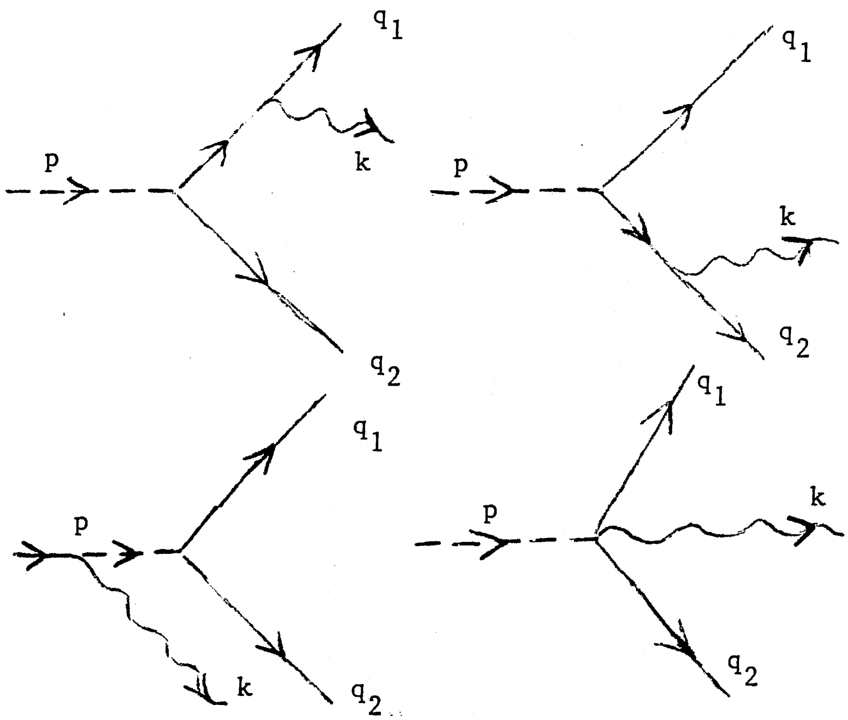


Figure 2. Diagrams for $W^- \to \phi_i \phi_j^+ \gamma$

We do not continue this problem since we were interested only in seeing if the factorization property depended on the spin of the quarks--we do not know the factorization to be completely independent of the quarks spin, but in this case the factorization is the same as for the spin one-half case. We expect that the Dalitz plot with spin-0 quarks will be different from the spin-½ case considered in the previous section, yet we also see that the decay probability will vanish along the line

$$E_q = \left(\frac{2Q+1}{2Q}\right) M_w - \left(\frac{Q+1}{Q}\right) E_{\bar{q}}$$

for both.

To see what the group factors will give in the way of a factorization for gluon vertices, we now study the process $W^- \rightarrow q\bar{q}g$.

CHAPTER IV

DECAY TO TWO QUARKS AND A GLUON

For the process: $W^-(p) \rightarrow q_i(q_1) + q_j(q_2) + g(k)$

where

$$P = q_1 + q_2 + k$$

Referring to the diagrams (Figure 3), the matrix elements are obtained; (see Appendix A for Feynman rules)

$$M_{fi} = \bar{U}(q_1)(-ig_s\gamma_\mu T^a_{ji})(\frac{i}{\not{\ell}_1})(\frac{-ig}{2\sqrt{2}}\gamma_\nu(1-\gamma_5))v(q_2)\epsilon^\mu_k\epsilon^\nu_P$$

$$+ \bar{U}(q_1)(\frac{-ig}{2\sqrt{2}}\gamma_\nu(1-\gamma_5))(\frac{i}{-\not{\ell}_2})(-ig_s\gamma_\mu T_{ji}{}^a)v(q_2)\epsilon^\mu_k\epsilon^\nu_p$$

$$= \frac{-gg_s T_{ji}{}^a}{2\sqrt{2}}\bar{U}(q_1)[\gamma_\mu \frac{1}{\not{\ell}_1}\gamma_\nu - \gamma_\nu \frac{1}{\not{\ell}_2}\gamma_\mu](1-\gamma_5)v(q_2)\epsilon^\mu_k\epsilon^\nu_p$$

Here g_s stands for the strong coupling, and we note that the gluon couples only to the colored quarks and not to the W boson.

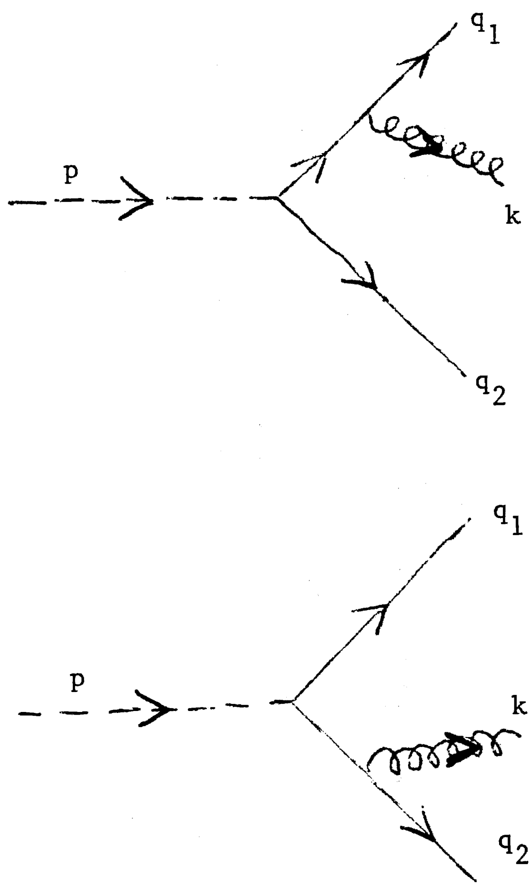


Figure 3. Diagrams for $W^- \rightarrow q\bar{q}g$

We have the same factor $\gamma_\mu \frac{1}{\not{l}_1} \gamma_\nu - \gamma_\nu \frac{1}{\not{l}_2} \gamma_\mu$ as we did in the photon case because this process is similar to diagrams (A) and (B) in Figure 1. Noting that a diagram like (C) in Figure 1 does not exist for this process, we can now understand why there is no outside factorization dependent upon gauge factors. It seems if the diagram (C) did exist, for instance, by replacing the W^- by a gluon, then we might get a factorization.

Carrying out the spin averaging and summing:

$$|\tilde{M}_{fi}|^2 = \frac{1}{3} \sum_{spins} |M_{fi}|^2 = \frac{2}{3} T^a_{ji} T^{a*}_{ji} g^2 g_s^2 \left(\frac{t^2+u^2+2M_W^2 s}{tu}\right)$$

where the last step has been accomplished from the great similarity of this process to $W^- \to q\bar{q}\gamma$.

Performing the color sum:

$$\sum_{ij} T_a^{ij} T_a^{ji} = \sum_{i,j} \frac{1}{2}(\delta_{ii}\delta_{jj} - \frac{1}{N}\delta_{ij}\delta_{ji}) = 4, \text{ for } N = 3$$

$$|\overline{M_{fi}}|^2 = \frac{8}{3} g^2 g_s^2 \left(\frac{t^2+u^2+2M_W^2 s}{tu}\right)$$

$$\Gamma(W^- \to q\bar{q}g) = \frac{g^2 g_s^2 M_W}{12(2\pi)^3} \int_\varepsilon^{1-2\varepsilon} dy \int_\varepsilon^{1-y-\varepsilon} \frac{(x-1)^2 + (y-1)^2}{xy}$$

where the phase space integral is identical to X encountered in $W^- \to q\bar{q}\gamma$. Now we integrate on computer to get the dalitz diagram (see Appendix C). And the total decay width is

$$\Gamma(W^- \to q\bar{q}g) = \frac{g^2 g_s^2 M_W}{12(2\pi)^3} [2|\ln\varepsilon|^2 - 3|\ln\varepsilon| + \frac{5}{2} - \frac{\pi^2}{3} - 6\varepsilon$$

$$- 9/2\ \varepsilon^2 + 6\varepsilon|\ell n\varepsilon| + 4\ell n(1-\varepsilon)\ell n\varepsilon| - 3\ell n(1-2\varepsilon)$$

$$+ 6\varepsilon\ell n(1-2\varepsilon) + 4Li_2(\frac{\varepsilon}{1-\varepsilon}) + 2\ell n^2(1-\varepsilon)]$$

Using $\Gamma_o = \Gamma(W^- \rightarrow q\bar{q}) = \frac{M_W g^2}{16\pi}$, we have $\frac{\Gamma(W^- \rightarrow q\bar{q}g)}{\Gamma_o}$ as a function of ε (see Table III).

Noting the $W^- f_i f_j$ vertex equals $\frac{-ig}{2\sqrt{2}} \gamma_\mu (1-\gamma_5)$, and the $Z^o f_i f_i$ vertex equals $\frac{-ig}{2} \frac{M_z}{M_w} \gamma_\mu (a_i - b_i\gamma_5)$

$a_e = -\frac{1}{2} + 2x$, $a_u = a_c = \frac{1}{2} - 4/3x$, $a_d = a_s = -\frac{1}{2} + 2/3x$

$b_e = b_d = b_s = -\frac{1}{2}$, $b_a = b_c = \frac{1}{2}$,

"where high energy neutrino-physics data point to a weinberg angle corresponding to $x \equiv \sin^2\theta_w \simeq 0.3$ or $\theta_w \simeq 33^o$."[7]

Substitution with the Z^o vertex and simple algebra give:

$$\Gamma(Z^o \rightarrow q_i\bar{q}_i g) = \frac{g^2 g_s^2 (a_i^2+b_i^2) M_z^3}{12(2\pi)^3 M_w^2} [2|\ell n\varepsilon|^2 - 3|\ell n\varepsilon| + 5/2 - \frac{\pi^2}{3}$$

$$- 6\varepsilon - \frac{9}{2}\varepsilon^2 + 6\varepsilon|\ell n\varepsilon| + 4\ell n(1-\varepsilon)\ell n\varepsilon| - 3\ell n(1-2\varepsilon)$$

$$+ 6\varepsilon\ell n(1-2\varepsilon) + 4Li_2(\frac{\varepsilon}{1-\varepsilon}) + 2\ell n^2(1-\varepsilon)]$$

$$= (a_i^2+b_i^2) \frac{M_z^3}{M_w^3} \Gamma(W \rightarrow q\bar{q}g)$$

Also taking into account the relative couplings we can find ($\alpha_s = g_s^2/4\pi$) $\frac{\alpha_s}{\alpha} \frac{\Gamma(W^- \rightarrow q\bar{q}\gamma)}{\Gamma(W^- \rightarrow q\bar{q}g)}$ as a function of ε. This is given in Table IV.

TABLE III

THE DECAY WITH RATIO $\frac{\alpha\Gamma(W^- \to q\bar{q}g)}{\alpha_s \quad \Gamma_0}$

ε	.09	.12	.15	.18	.21
$\frac{\alpha \quad \Gamma(W^- \to q\bar{q}g)}{\alpha_s \quad \Gamma_0}$	6.7×10^{-3}	4.2×10^{-3}	2.7×10^{-3}	1.7×10^{-3}	9.9×10^{-4}

TABLE IV

THE DECAY WIDTH RATIO, $\frac{\alpha_s\Gamma(W^- \to q\bar{q}\gamma)}{\alpha \; \Gamma(W^- \to q\bar{q}g)}$

ε	.0	.13	.16	.19	.22
$\frac{\alpha_s\Gamma(W^- \to q\bar{q}\gamma)}{\alpha \; \Gamma(W^- \to q\bar{q}g)}$	4.6×10^{-2}	4.0×10^{-2}	3.4×10^{-2}	3.0×10^{-2}	2.8×10^{-2}

Finally, the ratio of Z^o decay into a gluon versus a photon (plus $q + \bar{q}$) is

$$\frac{d\Gamma(Z^o \to q\bar{q}g)}{d\Gamma(Z^o \to q\bar{q}\gamma)} = \frac{\Gamma(Z^o \to q\bar{q}g)}{\Gamma(Z^o \to q\bar{q}\gamma)} = \frac{4}{3}\frac{\alpha_s}{\alpha Q_i^2} .$$

CHAPTER V

SUMMARY AND CONCLUSIONS

With the expected branching ratios, $\frac{\Gamma(W\to q\bar{q}\gamma)}{\Gamma_o}$, $\frac{\Gamma(W\to q\bar{q}g)}{\Gamma_o}$, and $\frac{\Gamma_w(\to q\bar{q}\gamma)}{\Gamma_w(\to \bar{q}qg)}$, it will be feasible to look at the smallest decay, $W^- \to q\bar{q}\gamma$, and see the expected zero somewhere in the amplitude's angular distribution, depending upon the charges of the decay products, the quarks. If the zero is found, it will confirm many expectations about the theory SU(2)xU(1) and other parameters associated with the theory; but now we can only wait in anticipation of the experiments. Further investigation might be made to find why only specific parameter values seem to have a factorization, and why the quarks of spin-o gave the same factorization as spin-1/2 quarks. Investigation of other particle processes may provide still other observable factorizations. We note that the factorization we found was independent of the spin in the sense that the factorization was found before the spin sum-average was completed. Thus if only particular spins are chosen for the initial and final states, we would still have identically the same zero. This is very unlike processes where a zero appears that is very spin dependent, such as those processes involving leptons and corresponding anti-neutrinos. It would be interesting to know exactly why some processes which have charge factors do contain zeros and why similar processes with different charge factors do not contain zeros; only further exploration might tell us the exact nature of these factorizations.

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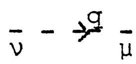
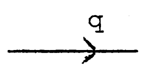
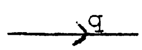
APPENDIX A

FEYNMAN RULES

<u>Propagators</u>

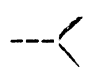
W =  = $-i(g_{\mu\nu}-q_\mu q_\nu/M^2)/(q^2-M^2)$

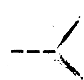
q_i =  = $i/\not{q}$

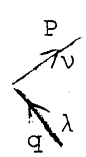
ϕ_i =  = i/q^2

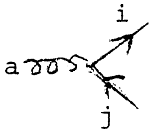
<u>Vertices</u>

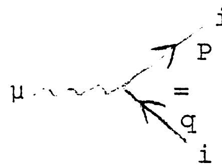
$\gamma q_i q_i$ = μ  = $-ie\,Q_i\gamma_\mu$

$W^{\pm}q_i q_j$ = μ  = $\frac{-ig}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)$

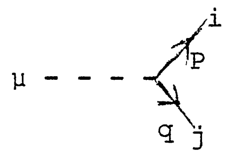
$Z^{o}q_i q_i$ = μ  = $\frac{-igM_z}{2\,M_w}\gamma_\mu(a_i-b_i\gamma_5)$

γWW = μ  = $-ie[-g_{\nu\lambda}(+q+P)_\mu + g_{\mu\nu}(2P-q)_\lambda + g_{\lambda\mu}(2q-P)_\nu]$

$g_a q_i q_j$ = μ, a  = $-ig_s\gamma_\mu T^a_{ji}$

$\gamma\phi_i\phi_i$ = 

$= -ieQ_i(P+q)_\mu$

$W\,\phi_i\phi_j^+$ = 

$= -ig(P-q)_\mu$

$W\,\gamma\,\phi_i\phi_j^+$ = 

$= +\,ieg(Q_i+Q_j)g_{\mu\nu}$

<u>Wave Functions</u>

$g,\gamma \quad = \quad \varepsilon_\mu,\ \varepsilon_\mu^+ \text{ where } \Sigma\varepsilon_\mu(q)\varepsilon_\nu^+(q) \quad = \quad -g_{\mu\nu}$

$W,Z \quad = \quad \varepsilon_\mu,\varepsilon_\mu^+ \text{ where } \Sigma\varepsilon_\mu(q)\varepsilon_\nu^+(q) \quad = \quad -g_{\mu\nu} + q_\mu q_\nu/M^2$

$q_i \quad = \quad u,\bar{u} \text{ where } \sum_s U_\alpha^{(s)}(q)\bar{u}_\beta^{(s)}(q) \ = \ (\not{q})_{\alpha\beta}$

$\phi_i \quad = \quad 1$

APPENDIX B

POLARIZATION DENSITY PROJECTION OPERATION

For $M_{\mu\nu} = M'_{\mu\nu} + \Omega_\mu P_\nu$ where $M'_{\mu\nu}$ is chosen such that $M'_{\mu\nu} P^\nu = 0$, we get from our desired requirement:

$$M_{\mu\nu} P^\nu = \Omega_\mu P_\nu P^\nu = \Omega_\mu M^2$$

$$M'_{\mu\nu} = M_{\mu\nu} - \frac{M_{\mu\alpha} P^\alpha}{M^2} P_\nu = M_{\mu\alpha}\left[g^\alpha_\nu - \frac{P^\alpha P_\nu}{M^2}\right]$$

where now $M'_{\mu\nu}$ fulfills our requirement $M'_{\mu\nu} P^\nu = 0$ and in all ways gives the same physics.

APPENDIX C

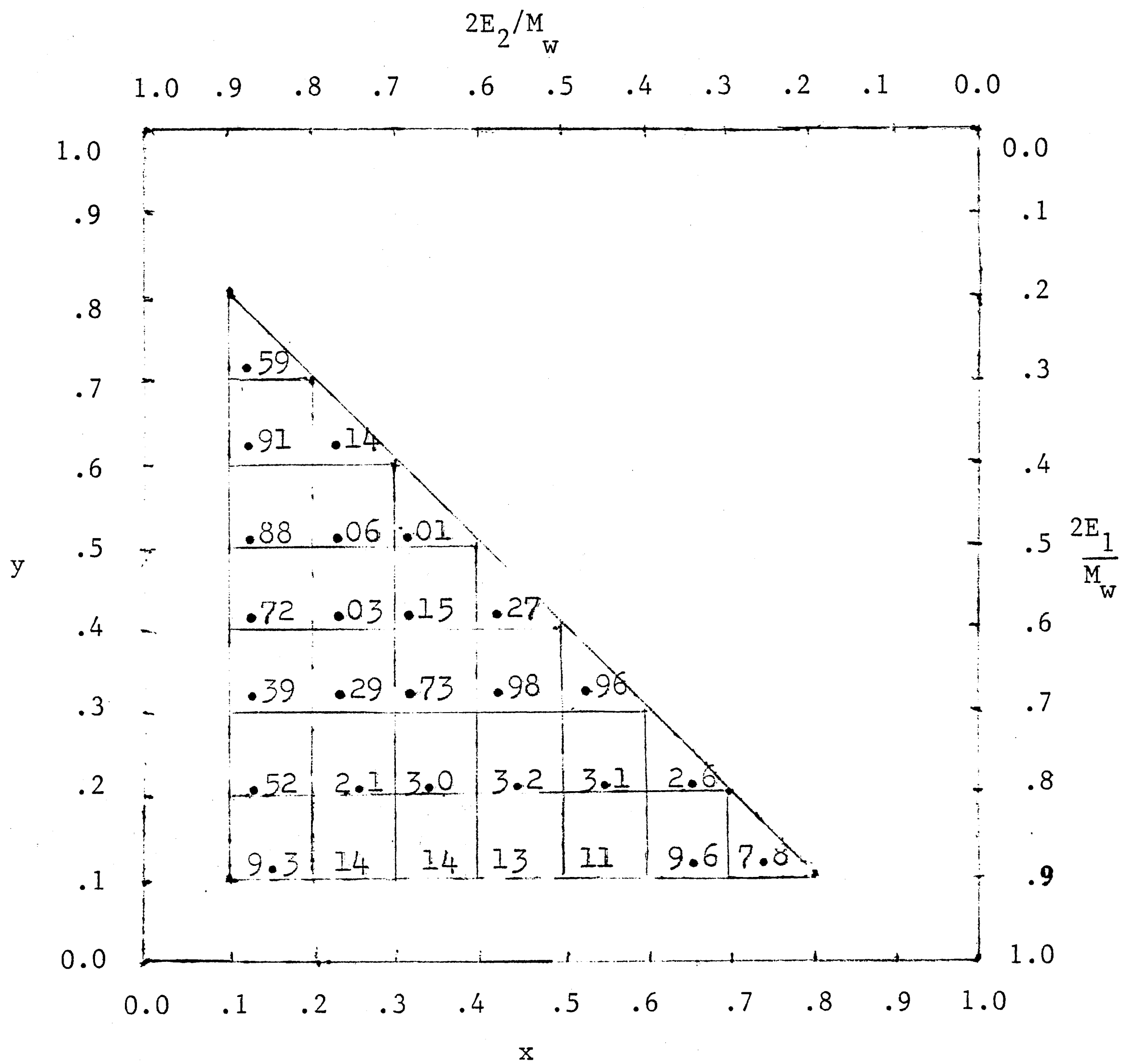
COMPUTER PROGRAM AND GENERATED

DALITZ PLOTS

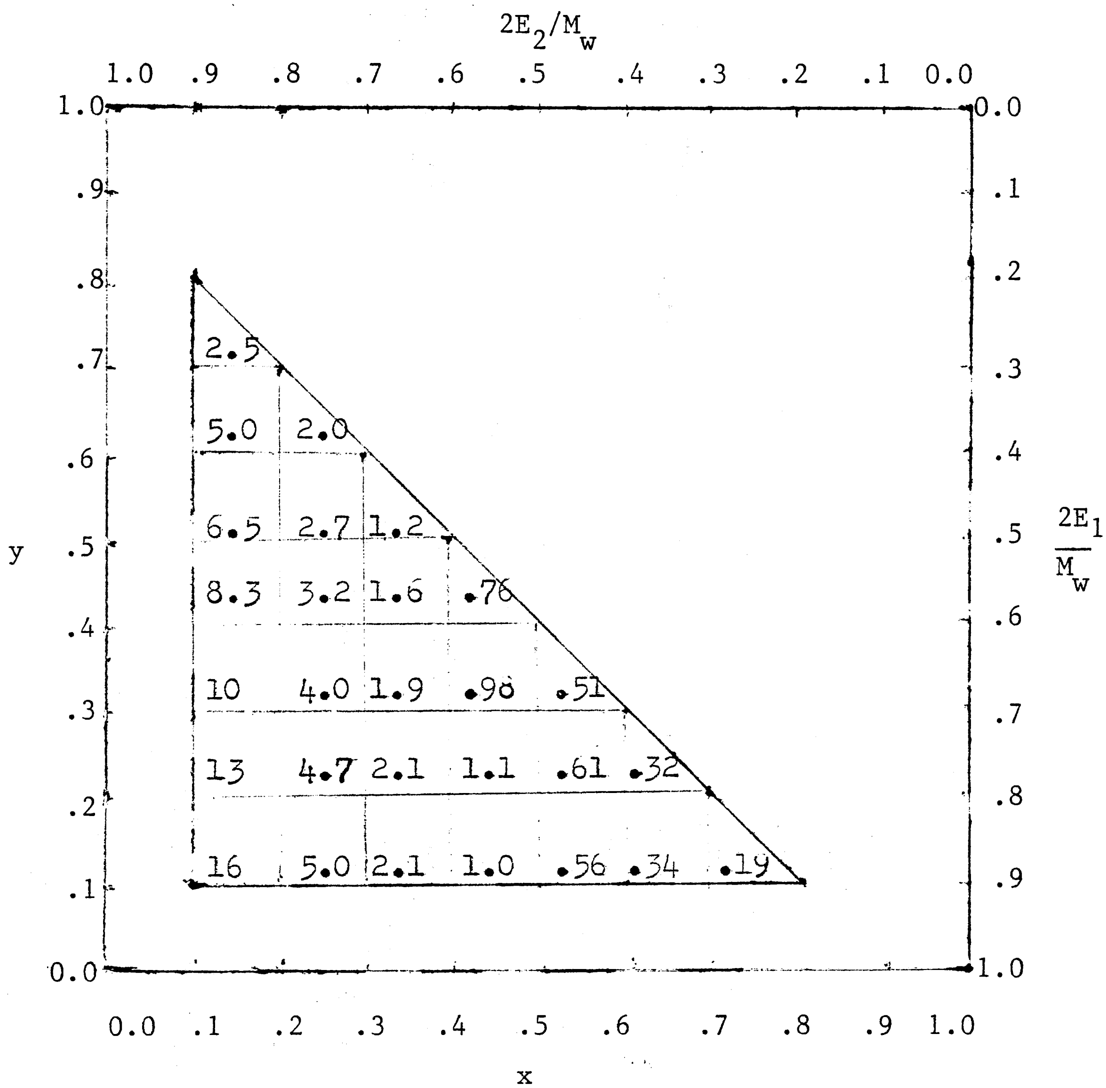
Computer Program

```
      DIMENSION W(50),Z(50)
      DIMENSION ARAY(30,30)
      DIVY=14.
      PI=3.14159
      M=7
      W(1)=0.4179591837
      W(2)=0.3813300505
      W(3)=W(2)
      W(4)=0.2797053915
      W(5)=W(4)
      W(6)=0.1294849662
      W(7)=W(6)
      Z(1)=0.0
      Z(2)=0.4058451514
      Z(3)=-Z(2)
      Z(4)=0.7415311856
      Z(5)=-Z(4)
      Z(6)=0.9491079123
      Z(7)=-Z(6)
      DIMENSION QP(50),EP(50)
      WRITE(6,80)
   80 FORMAT(1H0,3HEPS,10X,1HQ,10X,5HTOTAL)
      QP(1)=-1.0
      QP(2)=-1.0/3.0
      QP(3)=-1.0/2.0
      QP(4)=-2.0/3.0
      QP(5)=-3./4.
      QP(6)=0.0
      QP(7)=1.0
      EP(1)=.0001
      EP(2)=0.001
      EP(3)=0.01
      EP(4)=0.05
      EP(5)=0.1
      EP(6)=0.15
      EP(7)=0.2
      EP(8)=1.0/3.0
      DO 60 J=4,7
      EPS=EP(J)
      DO 70 K=1,2
      Q=QP(K)
      DIVX=14.
      DDIVX=DIVX
      TOTAL=0.0
      YMAX=1.0-2.0*EPS
      YMIN=EPS
      DELY=YMAX-YMIN
      YL=YMIN
      DJ=0.0
      JJ=DJ
      SUM=0.0
   10 YU=YMIN+DELY*(DJ+1.0)/DIVY
      C=(YU-YL)/2.0
      D=(YU+YL)/2.0
      I=1
      Y=C*Z(I)+D
      XMAX=1.0-EPS-Y
      XMIN=EPS
```

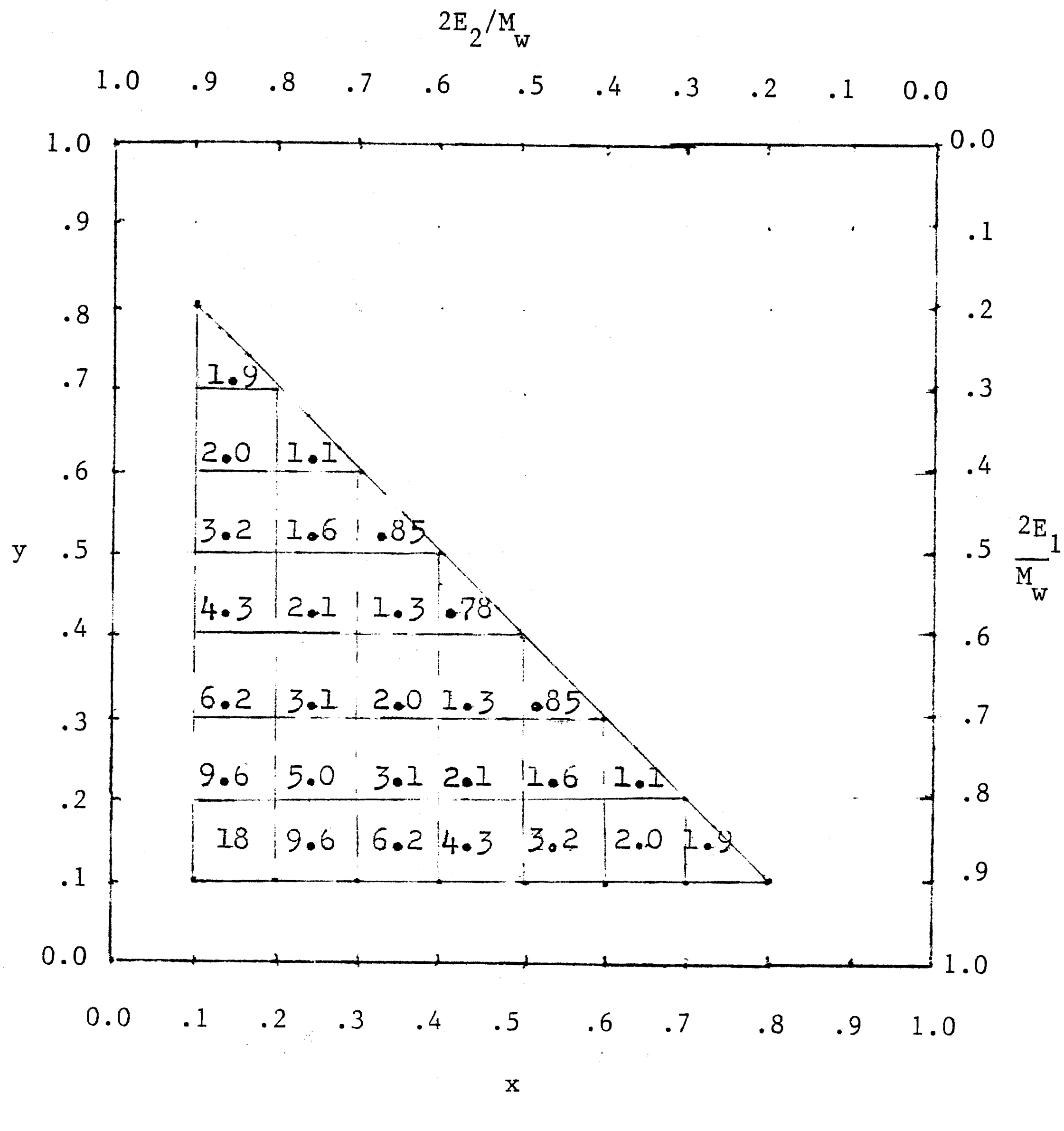
```
      DELX=XMAX-XMIN
   20 CONTINUE
      XL=XMIN
      DI=0.0
      II=DI
      Y=C*Z(I)+D
      XMAX=1.0-EPS-Y
      SUMIN=0.0
  100 XU=XMIN+DELX*(DI+1.0)/DIVX
      IF(XMAX .LE. XU) XU=XMAX
      A=(XU-XL)/2.0
      B=(XU+XL)/2.0
      N=1
  200 CONTINUE
      X=A*Z(N)+B
      SUMIN=SUMIN+W(N)*XYF(X,Y,Q)*A*W(I)*C
      N=N+1
      IF(N .LE. M) GO TO 200
      XL=XU
      DI=DI+1.0
      II=DI
      ARAY(JJ+1,II)=ARAY(JJ+1,II)+SUMIN
      SUM=SUM+SUMIN
      SUMIN=0.0
      IF(DI .LT. (DIVX-0.5)) GO TO 100
      I=I+1
      IF(I .LE. M) GO TO 20
      DIVX=DIVX-1.0
      YL=YU
      DJ=DJ+1.0
      JJ=DJ
      IF(DJ .LT. (DIVY-0.5)) GO TO 10
      TOTAL=SUM
      WRITE(6,50)EPS,Q,TOTAL
   50 FORMAT(1H0,E8.2,3X,E10.3,3X,E14.7)
      JJ=0
    9 JJ=JJ+1
      IF(JJ .GT. DIVY) GO TO 70
      II=0
   90 II=II+1
      IF(ARAY(JJ,II) .EQ. 0.0) GO TO 9
      SMIJ=ARAY(JJ,II)/TOTAL
      WRITE(6,900)JJ,II,SMIJ
  900 FORMAT(1H0,3HJJ=,I2,3X,3HII=,I2,3X,E14.7)
      ARAY(JJ,II)=0.0
      IF(II .LT. (DDIVX-.5)) GO TO 90
      IF(JJ .LT. (DIVY-.5)) GO TO 9
   70 CONTINUE
   60 CONTINUE
      STOP
      END


      FUNCTION XYF(X,Y,Q)
      XYF=(Q*Q+2.0*Q*X/(X+Y)+X*X/((X+Y)*(X+Y)))*((X-1.0)*(X-1.0)+(
     &Y-1.0)*(Y-1.0))/(Y*X)
      XYF=((X-1.)*(X-1.)+(Y-1.)*(Y-1.))/(Y*X)
      RETURN
      END
```



Dalitz plot for $W^- \rightarrow d\bar{u}\gamma$



Dalitz plot for $W^- \rightarrow e\bar{\nu}_e\gamma$



Dalitz plot for $W^- \rightarrow d\bar{u}g$

APPENDIX D

$$X = \int_{\varepsilon}^{1-2\varepsilon} dy \int_{\varepsilon}^{1-y-\varepsilon} \frac{(x-1)^2}{xy} = \int_{\varepsilon}^{1-2\varepsilon} \frac{dy}{y} \int_{\varepsilon}^{1-y-\varepsilon} dx(x-2 + \frac{1}{x})$$

$$X = \int_{\varepsilon}^{1-2\varepsilon} \frac{dy}{y} [\frac{x^2}{2} - 2x + \ell n\ x]\Big|_{\varepsilon}^{1-y-\varepsilon}$$

$$= \int_{\varepsilon}^{1-2\varepsilon} dy[y\{\frac{1}{4} - \frac{1}{2}\ \ell n\varepsilon\} + \{\frac{1}{2} + \frac{\varepsilon}{2} + \ell n\varepsilon\}$$

$$+ \frac{1}{y}\{- \frac{3}{4} + \frac{3\varepsilon}{2} - \ell n\varepsilon\} + \frac{y}{2}\ \ell n(1-y-\varepsilon)$$

$$- \ell n(1-y-\varepsilon) + \frac{1}{y}\ \ell n(1-y-\varepsilon)]$$

$$x \int_{\varepsilon}^{1-2\varepsilon} dy\ \ell n(1-y-\varepsilon) = [\frac{1-\varepsilon-y}{-1}\ \ell n(1-\varepsilon-y)-y]\Big|_{\varepsilon}^{1-2\varepsilon}$$

$$= -\varepsilon\ell n\varepsilon + (1-2\varepsilon)\ell n(1-2\varepsilon) - 1+3\varepsilon$$

$$x \int_{\varepsilon}^{1-2\varepsilon} dy\ y\ \ell n(1-y-\varepsilon) = (\ell n\varepsilon)[-\varepsilon + \frac{3}{2}\ \varepsilon^2] + \ell n(1-2\varepsilon)[\frac{1}{2} - \varepsilon]$$

$$- \frac{3}{4}\ (1-4\varepsilon + 3\varepsilon^2)$$

$$\int_{\varepsilon}^{1-2\varepsilon} dy\ \frac{1}{y}\ \ell n(1-y-\varepsilon) = \int_{\frac{\varepsilon}{1-\varepsilon}}^{\frac{1-2\varepsilon}{1-\varepsilon}} dy\ \frac{\ell n(1-\varepsilon-(1-\varepsilon)y)}{y}$$

$$= \int_{\frac{\varepsilon}{1-\varepsilon}}^{\frac{1-2\varepsilon}{1-\varepsilon}} dy \, \frac{[\ell n(1-\varepsilon) + \ell n(1-y)]}{y}$$

$$= \ell n(1-\varepsilon)\ell n\left[\frac{(1-2\varepsilon)/(1-\varepsilon)}{\varepsilon/(1-\varepsilon)}\right] - Li_2\left(\frac{1-2\varepsilon}{1-\varepsilon}\right) + Li_2\left(\frac{\varepsilon}{1-\varepsilon}\right)$$

$$= \ell n(1-\varepsilon)\ell n(1-2\varepsilon) - \ell n(1-\varepsilon)\ell n\varepsilon - Li_2\left(\frac{1-2\varepsilon}{1-\varepsilon}\right) + Li_2\left(\frac{\varepsilon}{1-\varepsilon}\right)$$

$$= -\ell n(1-2\varepsilon)|\ell n\varepsilon| - \frac{\pi^2}{6} + \ell n^2(1-\varepsilon) + 2Li_2\left(\frac{\varepsilon}{1-\varepsilon}\right)$$

Substituting everything:

$$X = |\ell n\varepsilon|^2 - \frac{3}{2}|\ell n\varepsilon| + \frac{5}{4} - \frac{\pi^2}{6} - 3\varepsilon - \frac{9}{4}\varepsilon^2 + 3\varepsilon|\ell n\varepsilon|$$

$$+ 2\ell n(1-\varepsilon)|\ell n\varepsilon| - \frac{3}{2}\ell n(1-2\varepsilon) + 3\varepsilon\ell n(1-2\varepsilon) + 2Li_2\left(\frac{\varepsilon}{1-\varepsilon}\right)$$

$$+ \ell n^2(1-\varepsilon)$$

$$\int dx \int dy \frac{(x-1)^2}{(x+y)^2} = \int_{\varepsilon}^{1-2\varepsilon} dx(x-1)^2 \int_{\varepsilon}^{1-x-\varepsilon} \frac{dy}{(y+x)^2}$$

$$= \int_{\varepsilon}^{1-2\varepsilon} dx(x-1)^2 \left[\frac{-1}{1-\varepsilon} + \frac{1}{x+\varepsilon}\right]$$

$$= -\frac{1}{2(1-\varepsilon)}\left[\frac{11}{3} - 11\varepsilon - \varepsilon^2 + 3\varepsilon^3\right] - (\varepsilon^2+2\varepsilon+1)\ell n\left(\frac{2\varepsilon}{1-\varepsilon}\right)$$

VITA

Theodore R. Grose

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Master of Science

Thesis: WEAK BOSON DECAYS INTO TWO QUARKS PLUS A PHOTON OR A GLUON

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