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THE ELECTRIC DIPOLE MOMENT OF THE TAU LEPTON AS A SIGNATURE OF CP VIOLATION

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THE ELECTRIC DIPOLE MOMENT OF THE TAU LEPTON
AS A SIGNATURE OF CP VIOLATION

A THESIS APPROVED FOR THE
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BY

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To
the memory of my grandmother

Frances Saulinskas
1917–2018

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Abstract

We calculate a one-loop level electric dipole moment (EDM) of the tau lepton that arises from scalar/pseudoscalar Higgs mixing in a type II two Higgs doublet model. Numerical results at $m_0 = 125$ GeV give an EDM of 3.66×10^{-24} e cm for $\tan\beta = 1$ and 2.33×10^{-21} e cm for $\tan\beta = 30$. The predicted EDM is still far below the current best experimental limit of $|d_\tau| < 3.9 \times 10^{-17}$ e cm; however, it can be much larger than the tau EDM of the Standard Model.

Chapter 1

Introduction

At present, one of the unanswered foundational questions in physics is the origin of the observed baryon asymmetry of the universe, i.e. why there is more matter than antimatter. The problem* traces its roots to 1928, when P. A. M. Dirac [2] derived the quantum mechanical equation for a relativistic electron—the Dirac equation. The Dirac equation predicted something like an electron but with a positive charge, which would turn out to be the positron. This idea of antimatter and the mathematical structures behind it began the modern importance of symmetries in theoretical physics. Yet, outside of the physics laboratory, antimatter is exceedingly rare. How does the mathematical symmetry become broken to give the observed matter-dominated universe?

In 1967, A. D. Sakharov [3] proposed three conditions that would explain the origin of baryon asymmetry as a result of the evolution of the universe. These conditions are violation of baryon number, violation of C and CP symmetries, and non-equilibrium thermal interactions of particles in the form of a first-order phase transition. Therefore, if we can find signatures of CP violation, we could possibly explain the baryon asymmetry.

One such candidate for signatures of CP violation is an elementary particle with an electric dipole moment (EDM). As we will show in Chapter 2, the Hamiltonian that describes the EDM of an elementary particle is proportional to the particle's spin and the electric field. Since spin and the electric field behave oppositely under both time-reversal T and spatial inversion P symmetries, an elementary particle with an EDM would violate these symmetries. Furthermore, the CPT theorem states that T violation is equivalent to CP violation, thus making EDM's a possible signature of

*For a more complete history of the baryon asymmetry problem (up to 2003), see the article by H. R. Quinn [1].

CP violation.

Although no particle EDM's have been observed to date [4], several experiments have established upper limits for various particles. The most stringent limit has been established for the electron. The second generation of the Advanced Cold Molecule Electron EDM (ACME II) experiment used electron spin precession in an electric field to establish an upper limit on the EDM of the electron of $|d_e| < 1.1 \times 10^{-29}$ e cm [5].

Violation of CP symmetry was discovered in K^0 decays in 1964 by J. H. Christenson et al. [6] and in B meson decays in 2001 by the BaBar [7] and Belle [8] collaborations. The LHCb collaboration announced discovery of CP violation in D^0 decays in March 2019 [9].

Although the Standard Model of particle physics produces CP violation via the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, it is too weak to explain the observed baryon asymmetry [10]. (The Standard Model currently allows for an electron EDM of order 10^{-41} e cm [11].) Therefore, any measured EDM significantly larger than this value would provide evidence for new physics beyond the Standard Model.

In this thesis, we calculate a one-loop level electric dipole moment of the tau lepton that arises from scalar/pseudoscalar Higgs mixing in a type II two Higgs doublet model. As the tau is the most massive of the charged leptons, the one-loop diagrams will have the dominant contribution to its EDM, whereas the electron and muon require two-loop level diagrams. Similar calculations have been carried out for the electron and muon by Barger, Das, and Kao [12].

Chapter 2

The Electric Dipole Moment of an Elementary Particle

The idea that an elementary particle could have an electric dipole moment (EDM) goes back to 1950, when E. M. Purcell and N. F. Ramsey [13] realized that violations of parity and time-reversal symmetries could give rise to an EDM. To understand why, let us look at EDM's from two perspectives, first from the general definition of an EDM, and then from the interaction Lagrangian that produces the EDM of an elementary particle. In both cases, we will see that an elementary particle EDM requires violation of time-reversal and parity symmetries.

At sufficiently large scales, any localized charge distribution will appear to be entirely concentrated at a single point. If the charge distribution contains more of one charge than the other, describing the distribution as a point charge will be a good first approximation. But what if there are equal numbers of positive and negative charges, canceling each other out so there is no overall charge? Or, perhaps a simple point charge description is not good enough because we are not sufficiently far away. How do we overcome these problems?

The next simplest approximation would be two equal and opposite charges separated by some distance: a dipole. If that still is not good enough, we can use four charges—two positive and two negative—to create a quadrupole. In fact, we can keep building up successively more complicated charge distributions to better approximate our actual charge distribution through a series of multipoles. How do we do this?

To demonstrate our multipole approximation, we follow the method given in Griffiths [14]. Let us start with the general form of the scalar potential of an arbitrary localized charge distribution:

$$V(\mathbf{x}) = \frac{1}{4\pi} \int \frac{1}{|\mathbf{x} - \mathbf{x}'|} \rho(\mathbf{x}') \, d^3x' \, . \tag{2.1}$$

Using the law of cosines, we may write

$$|\mathbf{x} - \mathbf{x}'| = x\sqrt{1 + \frac{x'}{x}\left(\frac{x'}{x} - 2\cos\theta\right)} \tag{2.2}$$

$$\equiv x\sqrt{1+\varepsilon} \, . \tag{2.3}$$

Now, if we are sufficiently far away from the charge distribution, then $\varepsilon \ll 1$. Thus, we can use the binomial approximation to write

$$\frac{1}{x\sqrt{1+\varepsilon}} = \frac{1}{x}\left(1 - \frac{1}{2}\varepsilon + \frac{3}{8}\varepsilon^2 - \frac{5}{16}\varepsilon^3 + \cdots\right) . \tag{2.4}$$

Substituting our definition of ε and grouping powers of (x'/x), we obtain

$$\frac{1}{x\sqrt{1+\varepsilon}} = \frac{1}{x}\left[1 + \frac{x'}{x}\cos\theta + \left(\frac{x'}{x}\right)^2 \cdot \frac{1}{2}(3\cos^2\theta - 1) + \left(\frac{x'}{x}\right)^3 \cdot \frac{1}{2}(5\cos^3\theta - 3\cos\theta) + \cdots\right] \tag{2.5}$$

$$= \frac{1}{x}\sum_{i=0}^{\infty}\left(\frac{x'}{x}\right)^i P_i(\cos\theta) \, , \tag{2.6}$$

where $P_i(z)$ is the i^{th} Legendre polynomial of z. Thus, the potential can be written as

$$V(\mathbf{x}) = \frac{1}{4\pi}\sum_{i=0}^{\infty}\frac{1}{x^{i+1}}\int (\mathbf{x}')^i P_i(\cos\theta)\rho(\mathbf{x}') \, d^3x' \, , \tag{2.7}$$

where

$$\int (\mathbf{x}')^i P_i(\cos\theta)\rho(\mathbf{x}') \, d^3x' \tag{2.8}$$

is the i^{th} multipole moment. The zeroth moment is simply the total charge. The first moment is the dipole moment $\mathbf{d}$:

$$\mathbf{d} = \int \mathbf{x}' \rho(\mathbf{x}')\, d^3x' \,. \tag{2.9}$$

Now that we have our electric dipole moment, let us examine its symmetry properties. If time is reversed, $\mathbf{d}$ remains unchanged; $\mathbf{d}$ is thus even under time-reversal T symmetry. On the other hand, $\mathbf{d}$ is odd under spatial inversion (parity P), as the locations of the positive and negative charges flip and thus cause $\mathbf{d}$ to change sign.

We can also deduce the symmetry properties of $\mathbf{d}$ by examining the symmetries of the individual terms on the right-hand side of Eq. (2.9). All three terms, i.e. $\mathbf{x}'$, $\rho(\mathbf{x}')$, and d^3x, are invariant under time-reversal, so $\mathbf{d}$ *must* be even under T. Likewise, $\mathbf{d}$ must be odd under P because only $\mathbf{x}'$ is odd under P; the others are even. (Charge density is even under parity even though the locations of the positive and negative charges flip. If this seems counter-intuitive, consider the analogy of mass density. If we spatially invert a teacup or a coffee mug by flipping it upside down, the density remains the same even though the location of the mass within the base of the cup has moved.)

However, an elementary particle also has spin. If an elementary particle were to have an EDM, $\mathbf{d}$ must lie along the particle's axis of spin [15, 16]; otherwise it would be averaged to zero by the act of spinning. Spin behaves the opposite way of $\mathbf{d}$: reversing time reverses the direction of spin, whereas spatial inversion does nothing. Spin is thus odd under T and even under P.

If T and P symmetries hold for an elementary particle, then there is a degeneracy [15, 17]: a particle with an EDM parallel to its spin is actually the same as a particle whose EDM is antiparallel—just rotate the particle's axis 180°, and it looks the same as the other particle. Therefore, the only way that a particle can have an

EDM is if T and P symmetries are violated.

We can also derive the T and P violation requirements from the interaction Lagrangian for a fermion [16]:

$$\mathscr{L}_I = -i\frac{d_f}{2}\bar{\psi}\gamma_5\sigma_{\mu\nu}\psi F^{\mu\nu}\,, \tag{2.10}$$

where

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]\,. \tag{2.11}$$

Substituting in,

$$\mathscr{L}_I = \frac{d_f}{4}\bar{\psi}\gamma_5(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\psi(\partial^\mu A^\nu - \partial^\nu A^\mu) \tag{2.12}$$

$$= \frac{d_f}{2}\bar{\psi}\gamma_5(g_{\mu\nu} + \gamma_\mu\gamma_\nu)\psi(\partial^\mu A^\nu - \partial^\nu A^\mu) \tag{2.13}$$

$$= \frac{d_f}{2}\bar{\psi}\gamma_5\gamma_\mu\gamma_\nu\psi F^{\mu\nu}\,. \tag{2.14}$$

Using the antisymmetric properties of $F^{\mu\nu}$ and the Clifford algebra $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu = 2g_{\mu\nu}$, we can reduce the number of terms to six:

$$\mathscr{L}_I = \frac{d_f}{2}\bar{\psi}\gamma_5\left[\gamma_0\gamma_1(F^{01} - F^{10}) + \gamma_0\gamma_2(F^{02} - F^{20}) + \gamma_0\gamma_3(F^{03} - F^{30})\right.$$
$$\left. +\gamma_1\gamma_2(F^{12} - F^{21}) + \gamma_1\gamma_3(F^{13} - F^{31}) + \gamma_2\gamma_3(F^{23} - F^{32})\right]\psi\,. \tag{2.15}$$

In the chiral basis [18], this becomes

$$\mathscr{L}_I = d_f\bar{\psi}\left[\begin{pmatrix}\sigma^1 & 0\\ 0 & \sigma^1\end{pmatrix}E_x + \begin{pmatrix}\sigma^2 & 0\\ 0 & \sigma^2\end{pmatrix}E_y + \begin{pmatrix}\sigma^3 & 0\\ 0 & \sigma^3\end{pmatrix}E_z\right.$$
$$\left. +i\begin{pmatrix}\sigma^1 & 0\\ 0 & -\sigma^1\end{pmatrix}B_x + i\begin{pmatrix}\sigma^2 & 0\\ 0 & -\sigma^2\end{pmatrix}B_y + i\begin{pmatrix}\sigma^3 & 0\\ 0 & -\sigma^3\end{pmatrix}B_z\right]\psi\,, \tag{2.16}$$

where σ^i are the Pauli spin matrices, or in vector notation,

$$\mathscr{L}_I = d_f \bar{\psi} \left[\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \cdot \mathbf{E} + i \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix} \cdot \mathbf{B} \right] \psi \, . \tag{2.17}$$

Switching to the Hamiltonian formulation ($\mathscr{H} = -\mathscr{L}$), Eq. (2.17) reduces to the non-relativistic single-particle Hamiltonian [19]

$$H_I = -d_f \boldsymbol{\sigma} \cdot \mathbf{E} \, . \tag{2.18}$$

As before, spin $\boldsymbol{\sigma}$ is odd under T and even under P. Meanwhile, $\mathbf{E}$ is even under T and odd under P. Thus, our Lagrangian is odd under both T and P. By comparison, the magnetic dipole moment Hamiltonian is

$$H_I = -\mu_f \boldsymbol{\sigma} \cdot \mathbf{B} , \tag{2.19}$$

which is even under T because both the spin and the magnetic field are T odd. Since magnetic dipole moments do not violate T symmetry, they have been experimentally observed for decades.

Parity violation was experimentally observed in 1956 by C. S. Wu et al. [20] in weak interactions.* It is T violations that are currently of interest [15].

*Grodzins [21] claims that an experiment conducted by Cox, McIlwraith, and Kurrelmeyer [22] in 1928 retrospectively showed evidence of parity violations in weak interactions. At that time, however, no one would have thought to look for violations of symmetries in nature.

Chapter 3

Violation of Time Reversal Symmetry and CPT Invariance

In the previous chapter, we explored how the EDM Hamiltonian violates P and T symmetries. Furthermore, we noted that it was T violations that are currently of experimental interest. At this point, two questions come to mind. First, since one cannot simply reverse the direction of time, how does one find T violations? Second, why are EDM's a possible candidate for signatures of CP violation if they are dependent on T symmetry violations?

The intuitive answer to the first question is to measure reactions that can be run in both directions. However, this is very difficult to do with weak interactions, where we expect T violations are most likely to appear [23].

The answer is the CPT theorem. According to the CPT theorem, all systems are invariant under the combined symmetry of parity P, time reversal T, and charge conjugation C. (A formal statement and the proof of the theorem may be found in Ref. [23].) Therefore, a violation of any one of these three symmetries is equivalent to violation of the product of the other two. Violation of T symmetry is thus equivalent to violation of CP symmetry. Hence, an elementary particle with an EDM would be evidence of CP violation, which also answers the second question. Conversely, any non-Standard-Model physics that involves a CP-violating interaction can predict an EDM that, at least in principle, can be measured or ruled out by experiment.

The CPT theorem is of fundamental importance in quantum field theory—in fact, it is an inherent property of all quantum field theories* [23]. Therefore, tests of

*R. Penrose [24] points out some subtleties that may question the validity of the CPT theorem when attempting to unify quantum field theory with general relativity. Should the theorem fail, all of quantum field theory would need to be rewritten [23].

its validity are also important. For example, CPT invariance requires that a particle and its antiparticle must have the same mass and lifetime; current experimental limits confirm this to about one part in 10^{18} [25].

Chapter 4

CP Violation in Two Higgs Doublet Models

In the Standard Model, CP violation arises solely from the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [26]. As we noted in Chapter 1, however, the Standard Model is unable to produce CP violations of sufficient magnitude to satisfy the Sakharov conditions for the baryon asymmetry of the universe. The addition of one (or more) Higgs doublets to the Standard Model could produce sufficiently large CP violations. A common extension of the Standard Model is the two Higgs doublet model (2HDM).

There are several types of 2HDM's, the most popular of which is type II, first proposed by J. F. Donoghue and L. F. Li [27]. In type II 2HDM's, one Higgs doublet (called ϕ_2 by convention) couples to up-type quarks (u, c, t) while the other doublet ϕ_1 couples to down-type quarks (d, s, b) and charged leptons (e, μ, τ).

Now, there are actually three basis sets used to write these doublets: the Higgs basis Φ_i, the Yukawa basis ϕ_i, and the Mass basis (H^0, h^0). They are written

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{H_1+v+iG^0}{\sqrt{2}} \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{H_2+iA^0}{\sqrt{2}} \end{pmatrix} \tag{4.1}$$

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\pi_1}{\sqrt{2}} \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\pi_2}{\sqrt{2}} \end{pmatrix} \tag{4.2}$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos(\beta-\alpha) & -\sin(\beta-\alpha) \\ \sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}. \tag{4.3}$$

Here, G^+ and G^0 are Goldstone bosons, H^+ is a charged scalar Higgs, H_1 and H_2 are neutral scalar Higgs, and A^0 is a pseudoscalar Higgs. The Standard Model

Higgs vacuum expectation value and 2HDM vacuum expectation vaules are given by $v = 246$ GeV and $v_1,\ v_2$, respectively.

We can translate from one basis to another through rotations in vector space, e.g. from the Yukawa basis to the Higgs basis:

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{4.4}$$

We can use this to derive relations for the expectation values:

$$\begin{pmatrix} v \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \tag{4.5}$$

which gives $v = \sqrt{v_1^2 + v_2^2}$ and $\tan\beta = |v_2/v_1|$.

To include CP violation in this 2HDM with a complex v_2, let us consider

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta e^{-i\theta} \\ -\sin\beta & \cos\beta e^{-i\theta} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{4.6}$$

The matrix is unitary; thus, $U^{-1} = U^{\dagger}$. That leads to

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta e^{i\theta} & \cos\beta e^{i\theta} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \tag{4.7}$$

or more explicitly,

$$\phi_1 = \cos\beta\Phi_1 - \sin\beta\Phi_2 \tag{4.8}$$

$$\phi_2 = e^{i\theta}(\sin\beta\Phi_1 + \cos\beta\Phi_2), \tag{4.9}$$

which leads to

$$\phi_1^0 = \cos\beta(H_1 + iG^0) - \sin\beta(H_2 + iA^0) \tag{4.10}$$

$$\phi_2^0 = e^{i\theta}[\sin\beta(H_1 + iG^0) + \cos\beta(H_2 + iA^0)]\,. \tag{4.11}$$

In our 2HDM, Higgs mixing will give the following terms [28]:

$$A_0(p^2) \equiv \frac{1}{v_1^* v_2}\langle \phi_1^{0\,*}\phi_2^0\rangle = \sum_n \frac{\sqrt{2}G_F Z_{0n}}{p^2 - m_n^2} \tag{4.12}$$

$$\tilde{A}_0(p^2) \equiv \frac{1}{v_1 v_2}\langle \phi_1^0\phi_2^0\rangle = \sum_n \frac{\sqrt{2}G_F \tilde{Z}_{0n}}{p^2 - m_n^2} \tag{4.13}$$

$$A_1(p^2) \equiv \frac{1}{v_1^2}\langle \phi_1^0\phi_1^0\rangle = \sum_n \frac{\sqrt{2}G_F Z_{1n}}{p^2 - m_n^2} \tag{4.14}$$

$$A_2(p^2) \equiv \frac{1}{v_2^2}\langle \phi_2^0\phi_2^0\rangle = \sum_n \frac{\sqrt{2}G_F Z_{2n}}{p^2 - m_n^2}\,, \tag{4.15}$$

where the sum is over the mass eigenstates of H_1, H_2, and A^0. Now, recall that the Feynman propagator for a spin zero field ϕ is [18]

$$D_F(x-y) = \langle 0|T\phi(x)\phi(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\Delta_F(p^2)e^{ip\cdot x}\,, \tag{4.16}$$

where

$$\Delta_F(p^2) = \frac{i}{p^2 - m^2 + i\varepsilon} \to A(p^2)\,. \tag{4.17}$$

From this, we can obtain the propagators [12]

$$\langle H_1 A\rangle = \frac{1}{2}\sum_n \frac{\sin(2\beta)\mathrm{Im}Z_{0n}}{p^2 - m_{\phi n}^2} \tag{4.18}$$

$$\langle H_2 A\rangle = \frac{1}{2}\sum_n \frac{\cos(2\beta)\mathrm{Im}Z_{0n} - \mathrm{Im}\tilde{Z}_{0n}}{p^2 - m_{\phi n}^2}\,. \tag{4.19}$$

Chapter 5

The Electric Dipole Moment of the Tau Lepton

We will now calculate the electric dipole moment of the tau lepton. We will follow the methods and notations of Ref. [18]. Our interaction Lagrangian density is

$$\mathscr{L}_I = -e\bar{\tau}\gamma^\mu\tau A_\mu - \frac{m_\tau}{v}\bar{\tau}\tau(H_1 - \tan\beta H_2) + i\frac{m_\tau}{v}\tan\beta\bar{\tau}\gamma_5\tau A^0\,, \tag{5.1}$$

where $\tan\beta = |v_2/v_1|$ and $v = \sqrt{v_1^2 + v_2^2}$ is the Standard Model Higgs field vacuum expectation value, which we derived in Chapter 4. Our two relevant Feynman diagrams are shown in Fig. 5.1.

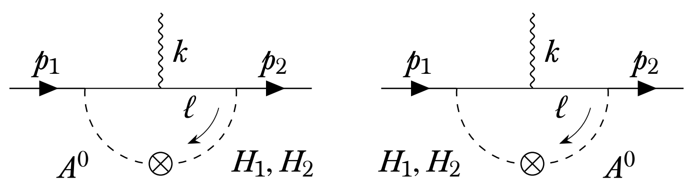

Fig. 5.1: The two one-loop Feynman diagrams that contribute to the electric dipole moment of the tau lepton. The crossed circle represents the conversion of the scalars H_1, H_2 into the pseudoscalar A^0 or vice versa.

Our transition matrix is

$$T^\mu = \bar{u}(p_2)\Gamma^\mu u(p_1)\,. \tag{5.2}$$

By the Feynman rules, the first diagram gives

$$\Gamma_1^\mu = \int \frac{d^4\ell}{(2\pi)^4}(-i)\left(\frac{m_\tau}{v}\right)\left(\frac{1}{2}\sum_n \frac{\sin(2\beta)\mathrm{Im}Z_{0n}}{\ell^2 - m_{\phi n}^2} - \tan\beta\frac{1}{2}\sum_n \frac{\cos(2\beta)\mathrm{Im}Z_{0n} - \mathrm{Im}\tilde{Z}_{0n}}{\ell^2 - m_{\phi n}^2}\right)$$
$$\times \frac{i[(\not\ell + \not p_2) + m_\tau]}{(\ell + p_2)^2 - m_\tau^2 + i\varepsilon}(-ie\gamma^\mu)\frac{i[(\not\ell + \not p_1) + m_\tau]}{(\ell + p_1)^2 - m_\tau^2 + i\varepsilon}(-1)\left(\frac{m_\tau}{v}\right)\tan\beta\gamma_5\,. \tag{5.3}$$

Likewise, the second diagram gives

$$\Gamma_2^\mu = \int \frac{d^4\ell}{(2\pi)^4}(-1)\left(\frac{m_\tau}{v}\right)\tan\beta\gamma_5\left(\frac{1}{2}\sum_n \frac{\sin(2\beta)\mathrm{Im}Z_{0n}}{\ell^2 - m_{\phi n}^2}\right.$$
$$\left. - \tan\beta\frac{1}{2}\sum_n \frac{\cos(2\beta)\mathrm{Im}Z_{0n} - \mathrm{Im}\tilde{Z}_{0n}}{\ell^2 - m_{\phi n}^2}\right)$$
$$\times \frac{i[(\not\ell + \not p_2) + m_\tau]}{(\ell + p_2)^2 - m_\tau^2 + i\varepsilon}(-ie\gamma^\mu)\frac{i[(\not\ell + \not p_1) + m_\tau]}{(\ell + p_1)^2 - m_\tau^2 + i\varepsilon}(-i)\left(\frac{m_\tau}{v}\right) . \tag{5.4}$$

For both diagrams, we can simplify the scalar/pseudoscalar mixing propagator and couplings

$$(-i)\left(\frac{m_\tau}{v}\right)^2\tan\beta\left(\frac{1}{2}\sum_n \frac{\sin(2\beta)\mathrm{Im}Z_{0n}}{\ell^2 - m_{\phi n}^2} - \tan\beta\frac{1}{2}\sum_n \frac{\cos(2\beta)\mathrm{Im}Z_{0n} - \mathrm{Im}\tilde{Z}_{0n}}{\ell^2 - m_{\phi n}^2}\right). \tag{5.5}$$

Using $\sin(2\beta) = 2\sin\beta\cos\beta$ and $\cos(2\beta) = \cos^2\beta - \sin^2\beta$,

$$= \frac{-i}{2}\left(\frac{m_\tau}{v}\right)^2\sum_n \frac{1}{\ell^2 - m_{\phi n}^2}\left(\sin(2\beta)\tan\beta\mathrm{Im}Z_{0n} - \cos(2\beta)\tan^2\beta\mathrm{Im}Z_{0n} + \tan^2\beta\mathrm{Im}\tilde{Z}_{0n}\right) \tag{5.6}$$

$$= \frac{-i}{2}\left(\frac{m_\tau}{v}\right)^2\sum_n \frac{1}{\ell^2 - m_{\phi n}^2}\left(\sin^2\beta(1 + \tan^2\beta)\mathrm{Im}Z_{0n} + \tan^2\beta\mathrm{Im}\tilde{Z}_{0n}\right) \tag{5.7}$$

$$= \frac{-i}{2}\left(\frac{m_\tau}{v}\right)^2\tan^2\beta\sum_n \frac{1}{\ell^2 - m_{\phi n}^2}\left(\mathrm{Im}Z_{0n} + \mathrm{Im}\tilde{Z}_{0n}\right) . \tag{5.8}$$

We assume the lightest neutral scalar Higgs dominates, so we drop the sum

$$\frac{-i}{2}\left(\frac{m_\tau}{v}\right)^2\tan^2\beta\frac{1}{\ell^2 - m_\phi^2}\left(\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0\right) . \tag{5.9}$$

We also note that $1/v^2 = \sqrt{2}G_F$, thus

$$\frac{-i}{2}m_\tau^2\sqrt{2}G_F\tan^2\beta\frac{1}{\ell^2 - m_\phi^2}\left(\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0\right) . \tag{5.10}$$

Substituting back in, our expressions for Γ^μ become

$$\Gamma^\mu_i = -\frac{1}{2} e m_\tau^2 \sqrt{2} G_F \tan^2 \beta I^\mu_i (\mathrm{Im} Z_0 + \mathrm{Im} \tilde{Z}_0) , \tag{5.11}$$

where

$$I^\mu_1 = \int \frac{d^4\ell}{(2\pi)^4} \frac{\left[\left(\not{\ell} + \not{p}_2\right) + m_\tau\right] \gamma^\mu \left[\left(\not{\ell} + \not{p}_1\right) + m_\tau\right] \gamma_5}{\left(\ell^2 - m_\phi^2\right)\left((\ell + p_1)^2 - m_\tau^2\right)\left((\ell + p_2)^2 - m_\tau^2\right)} \tag{5.12}$$

$$I^\mu_2 = \int \frac{d^4\ell}{(2\pi)^4} \frac{\gamma_5 \left[\left(\not{\ell} + \not{p}_2\right) + m_\tau\right] \gamma^\mu \left[\left(\not{\ell} + \not{p}_1\right) + m_\tau\right]}{\left(\ell^2 - m_\phi^2\right)\left((\ell + p_1)^2 - m_\tau^2\right)\left((\ell + p_2)^2 - m_\tau^2\right)} . \tag{5.13}$$

(The only difference between the two integrals is the location of the γ_5.)

Now, we must integrate Eqs. (5.12) and (5.13). We will make use of the Feynman parameterization

$$\frac{1}{d_1 d_2 d_3} = \Gamma(3) \int_0^1 dx \int_0^x dy \frac{1}{\left[x\left(d_2 - d_1\right) + y\left(d_3 - d_2\right) + d_1\right]^3} , \tag{5.14}$$

with

$$d_1 = \ell^2 - m_\phi^2, \quad d_2 = (\ell + p_1)^2 - m_\tau^2, \quad d_3 = (\ell + p_2)^2 - m_\tau^2. \tag{5.15}$$

After some algebra, we can write the denominator of Eq. (5.14) as

$$\ell^2 + 2\ell(p_1 x + p_2 y - p_1 y) + m_\phi x^2 - m_\phi^2 . \tag{5.16}$$

Next, we complete the square using

$$q = \ell + \left[p_1(x - y) + p_2 y\right] . \tag{5.17}$$

Thus, our denominator in Eq. (5.12) can be written as

$$\frac{1}{d_1 d_2 d_3} = \Gamma(3) \int_0^1 dx \int_0^x dy \frac{1}{\left[q^2 - [p_1(x-y) + p_2 y]^2 + (x-1)m_\phi^2 + i\varepsilon\right]^3} \tag{5.18}$$

$$\equiv 2 \int_0^1 dx \int_0^x dy \frac{1}{\left(q^2 - p^2 + M^2\right)^3} \,. \tag{5.19}$$

Next, we must rewrite the numerator in terms of our new variables p and q. Let us start with the numerator of Eq. (5.12).

$$N_1^\mu = \left[\left(\not{\ell} + \not{p}_2\right) + m_\tau\right] \gamma^\mu \left[\left(\not{\ell} + \not{p}_1\right) + m_\tau\right] \gamma_5 \tag{5.20}$$

$$= \Big(\not{\ell}\gamma^\mu\not{\ell} + \not{\ell}\gamma^\mu\not{p}_1 + \not{\ell}\gamma^\mu m_\tau + \not{p}_2\gamma^\mu\not{\ell} + \not{p}_2\gamma^\mu\not{p}_1$$

$$+ \not{p}_2\gamma^\mu m_\tau + m_\tau\gamma^\mu\not{\ell} + m_\tau\gamma^\mu\not{p}_1 + m_\tau^2\gamma^\mu\Big) \gamma_5 \,. \tag{5.21}$$

The algebra will be the least tedious if we apply the Dirac equation to our numerator as soon as possible. From our Feynman diagrams, we have

$$(\not{p}_1 - m_\tau)u(p_1) = 0 \Rightarrow \not{p}_1 u(p_1) = m_\tau u(p_1) \tag{5.22}$$

$$\bar{u}(p_2)(\not{p}_2 - m_\tau) = 0 \Rightarrow \bar{u}(p_2)\not{p}_2 = m_\tau \bar{u}(p_2) \,. \tag{5.23}$$

We also need to make note of the algebra of γ_5, which leads to

$$\not{p}_1\gamma_5 u(p_1) = \gamma^\mu p_{1\mu}\gamma_5 u(p_1) = \gamma^\mu\gamma_5 p_{1\mu} u(p_1)$$

$$= -\gamma_5\gamma^\mu p_{1\mu} u(p_1) = -\gamma_5\not{p}_1 u(p_1)$$

$$= -m_\tau\gamma_5 u(p_1) \,. \tag{5.24}$$

Thus, applying the Dirac equation to Eq. (5.21),

$$
\begin{aligned}
\bar{u}(p_2)N_1^\mu u(p_1) &= \bar{u}(p_2)\left(\not{\ell}\gamma^\mu\not{\ell} + \not{\ell}\gamma^\mu\not{p}_1 + \not{\ell}\gamma^\mu m_\tau + \not{p}_2\gamma^\mu\not{\ell} + \not{p}_2\gamma^\mu\not{p}_1\right. \\
&\qquad \left. + \not{p}_2\gamma^\mu m_\tau + m_\tau\gamma^\mu\not{\ell} + m_\tau\gamma^\mu\not{p}_1 + m_\tau^2\gamma^\mu\right)\gamma_5 u(p_1) \\
&= \bar{u}(p_2)\left(\not{\ell}\gamma^\mu\not{\ell} - \not{\ell}\gamma^\mu m_\tau + \not{\ell}\gamma^\mu m_\tau + m_\tau\gamma^\mu\not{\ell} - m_\tau^2\gamma^\mu\right. \\
&\qquad \left. + m_\tau^2\gamma^\mu + m_\tau\gamma^\mu\not{\ell} - m_\tau^2\gamma^\mu \ + m_\tau^2\gamma^\mu\right)\gamma_5 u(p_1) \\
&= \bar{u}(p_2)\left(\not{\ell}\gamma^\mu\not{\ell} + 2m_\tau\gamma^\mu\not{\ell}\right)\gamma_5 u(p_1)\,. \qquad (5.25)
\end{aligned}
$$

Now, we substitute $\ell = q - p$ back in:

$$
\begin{aligned}
N_1^\mu &= \left[(\not{q} - \not{p})\gamma^\mu(\not{q} - \not{p}) + 2m_\tau\gamma^\mu(\not{q} - \not{p})\right]\gamma_5 \qquad (5.26)\\
&= \left[\not{q}\gamma^\mu\not{q} - \not{q}\gamma^\mu\not{p} - \not{p}\gamma^\mu\not{q} + \not{p}\gamma^\mu\not{p} + 2m_\tau\gamma^\mu\not{q} - 2m_\tau\gamma^\mu\not{p}\right]\gamma_5\,. \qquad (5.27)
\end{aligned}
$$

All terms with only one power of q are odd functions and thus will integrate to zero; we are left with

$$
N_1^\mu = \left[\not{q}\gamma^\mu\not{q} + \not{p}\gamma^\mu\not{p} - 2m_\tau\gamma^\mu\not{p}\right]\gamma_5\,. \qquad (5.28)
$$

Next, we use the identity

$$
\not{p}\gamma^\mu + \gamma^\mu\not{p} = 2p^\mu \qquad (5.29)
$$

to obtain

$$
N_1^\mu = \left[\not{q}\gamma^\mu\not{q} + 2p^\mu\not{p} - \gamma^\mu\not{p}\not{p} - 2m_\tau\gamma^\mu\not{p}\right]\gamma_5\,. \qquad (5.30)
$$

Furthermore,

$$\begin{aligned}
\not{p}\not{p} &= \gamma^\mu p_\mu \gamma^\nu p_\nu = \gamma^\mu \gamma^\nu p_\mu p_\nu \\
&= (2g^{\mu\nu} - \gamma^\nu \gamma^\mu) p_\mu p_\nu \\
&= 2p_\mu p^\mu - \gamma^\nu \gamma^\mu p_\mu p_\nu = 2p^2 - \not{p}\not{p} \\
&= p^2 = m_\tau^2\,.
\end{aligned} \tag{5.31}$$

Recalling our definition of p,

$$\begin{aligned}
N_1^\mu = \Big\{ &\not{q}\gamma^\mu \not{q} + 2[(x-y)p_1^\mu + y p_2^\mu][(x-y)\not{p}_1 - y\not{p}_2] \\
&- 2m_\tau \gamma^\mu [(x-y)\not{p}_1 - y\not{p}_2] - m_\tau^2 \gamma^\mu \Big\} \gamma_5\,.
\end{aligned} \tag{5.32}$$

Again using Eq. (5.29),

$$\begin{aligned}
N_1^\mu = \Big\{ &\not{q}\gamma^\mu \not{q} + 2[(x-y)p_1^\mu + y p_2^\mu][(x-y)\not{p}_1 - y\not{p}_2] \\
&- 2m_\tau \gamma^\mu (x-y)\not{p}_1 - 2m_\tau y(2p_2^\mu - \not{p}_2\gamma^\mu) - m_\tau^2 \gamma^\mu \Big\} \gamma_5\,.
\end{aligned} \tag{5.33}$$

We again apply the Dirac equation, which gives

$$\begin{aligned}
N_1^\mu = \Big\{ &\not{q}\gamma^\mu \not{q} + 2[(x-y)p_1^\mu + y p_2^\mu][(y-x)m_\tau - y m_\tau] \\
&- 2m_\tau \gamma^\mu (y-x) m_\tau - 2m_\tau y(2p_2^\mu - \not{p}_2\gamma^\mu) - m_\tau^2 \gamma^\mu \Big\} \gamma_5
\end{aligned} \tag{5.34}$$

$$= \Big\{ \not{q}\gamma^\mu \not{q} + 2m_\tau (x-y)(2y-x)p_1^\mu + 2m_\tau y(2y-x-2)p_2^\mu + m_\tau^2(2x-1)\gamma^\mu \Big\} \gamma_5\,. \tag{5.35}$$

Finally, we address the $\not{q}\gamma^\mu\not{q}$ term:

$$\begin{aligned}\not{q}\gamma^\mu\not{q} &= \not{q}\gamma^\mu\gamma^\nu q_\nu \\ &= \not{q}(2g^{\mu\nu} - \gamma^\nu\gamma^\mu)q_\nu \\ &= 2\not{q}q^\mu - q^2\gamma^\mu\,. \end{aligned} \tag{5.36}$$

As before, the term with one $\not{q}$ will integrate to zero. Thus the numerator of Eq. (5.12) is

$$N_1^\mu = \left\{-q^2\gamma^\mu + 2m_\tau(x-y)(2y-x)p_1^\mu + 2m_\tau y(2y-x-2)p_2^\mu + m_\tau^2(2x-1)\gamma^\mu\right\}\gamma_5\,. \tag{5.37}$$

For the second numerator, we can move the γ_5 term to the right:

$$N_2^\mu = \gamma_5\left[\left(\not{\ell}+\not{p}_2\right) + m_\tau\right]\gamma^\mu\left[\left(\not{\ell}+\not{p}_1\right) + m_\tau\right] \tag{5.38}$$

$$= \left[-\left(\not{\ell}+\not{p}_2\right) + m_\tau\right](-\gamma^\mu)\left[-\left(\not{\ell}+\not{p}_1\right) + m_\tau\right]\gamma_5 \tag{5.39}$$

$$= -\left\{\left[\left(\not{\ell}+\not{p}_2\right) - m_\tau\right]\gamma^\mu\left[\left(\not{\ell}+\not{p}_1\right) - m_\tau\right]\right\}\gamma_5\,. \tag{5.40}$$

Applying the same process that we used for the first numerator, we obtain

$$N_2^\mu = \left\{q^2\gamma^\mu + 2m_\tau(x-y)(x-2y-2)p_1^\mu + 2m_\tau y(x-2y)p_2^\mu + m_\tau^2(1-2x)\gamma^\mu\right\}\gamma_5\,. \tag{5.41}$$

Since we are only interested in the overall interaction and not the two diagrams individually, let us add the numerators to simplify our calculations:

$$I_1^\mu + I_2^\mu = 2\int_0^1 dx \int_0^x dy \int \frac{d^4q}{(2\pi)^4} \frac{N_1^\mu + N_2^\mu}{(q^2 - p^2 + M^2)^3} \tag{5.42}$$

$$= 2\int_0^1 dx \int_0^x dy \int \frac{d^4q}{(2\pi)^4} \frac{4m_\tau \left[(y-x)p_1^\mu - yp_2^\mu\right]\gamma_5}{(q^2 - p^2 + M^2)^3}\,. \tag{5.43}$$

Next, we write the numerator in symmetric and anti-symmetric parts:

$$I^\mu = 2\int_0^1 dx \int_0^x dy \int \frac{d^4q}{(2\pi)^4} \frac{4m_\tau \left[\frac{1}{2}(y-x-y)(p_1^\mu + p_2^\mu) + \frac{1}{2}(y-x+y)(p_1^\mu - p_2^\mu)\right]\gamma_5}{(q^2 - p^2 + M^2)^3} \tag{5.44}$$

$$= 4m_\tau \int_0^1 dx \int_0^x dy \int \frac{d^4q}{(2\pi)^4} \frac{\left[(-x)(p_1^\mu + p_2^\mu) + (2y-x)(p_1^\mu - p_2^\mu)\right]\gamma_5}{(q^2 - p^2 + M^2)^3}\,. \tag{5.45}$$

The symmetric part corresponds to the electric dipole form factor F_D from our tensor structure. The antisymmetric part becomes part of the anapole moment. (See Appendix B.)

Using the method outlined in Appendix A, the integral over q becomes

$$I^\mu = 4m_\tau \int_0^1 dx \int_0^x dy \frac{1}{(2\pi)^4}\pi^2 \frac{\Gamma(1)}{\Gamma(3)} \frac{\left[(-x)(p_1^\mu + p_2^\mu) + (2y-x)(p_1^\mu - p_2^\mu)\right]\gamma_5}{(-p^2 + M^2)} \tag{5.46}$$

$$= \frac{m_\tau}{8\pi^2} \int_0^1 dx \int_0^x dy \frac{\left[(-x)(p_1^\mu + p_2^\mu) + (2y-x)(p_1^\mu - p_2^\mu)\right]\gamma_5}{m_\phi^2 x - m_\phi^2 - m_\tau^2 x^2}\,. \tag{5.47}$$

At this point, we will separate out the antisymmetric part into a $G_K(p_1^\mu - p_2^\mu)\gamma_5$ term. Placing the minus sign in the denominator and letting $\rho = m_\tau^2/m_\phi^2$,

$$I^\mu = \frac{m_\tau}{8\pi^2} \int_0^1 dx \int_0^x dy \frac{x}{m_\phi^2(\rho x^2 - x + 1)}(p_1^\mu + p_2^\mu)\gamma_5 + G_K(p_1^\mu - p_2^\mu)\gamma_5 \tag{5.48}$$

$$= \frac{m_\tau}{8\pi^2 m_\phi^2} \int_0^1 dx \frac{x^2}{\rho x^2 - x + 1}(p_1^\mu + p_2^\mu)\gamma_5 + G_K(p_1^\mu - p_2^\mu)\gamma_5\,. \tag{5.49}$$

The last integral may be looked up in a table, e.g. Ref. [29], or computed via a computer algebra system. Dropping the antisymmetric term, our result is

$$I^\mu = \frac{m_\tau}{8\pi^2 m_\phi^2}\frac{1}{\rho}\left\{1 + \frac{1}{\rho(z_1 - z_2)}\right.$$
$$\left.\times\left[(z_1 - 1)\ln\left(\frac{z_1 - 1}{z_1}\right) - (z_2 - 1)\ln\left(\frac{z_2 - 1}{z_2}\right)\right]\right\}(p_1^\mu + p_2^\mu)\gamma_5\,, \tag{5.50}$$

where z_1 and z_2 are the positive and negative roots of $\rho x^2 - x + 1 = 0$, respectively. Applying the Gordon identity

$$\bar{u}(p_2)(p_1^\mu + p_2^\mu)\gamma_5 u(p_1) = \bar{u}(p_2) i\sigma_{\mu\nu}k^\nu\gamma_5 u(p_1)\,, \tag{5.51}$$

where $k^\nu = p_1^\nu - p_2^\nu$, we can write

$$\Gamma^\mu = -\frac{1}{2}em_\tau^2\sqrt{2}G_F\tan^2\beta I^\mu(\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0) \tag{5.52}$$
$$= -\frac{em_\tau\sqrt{2}G_F\tan^2\beta}{16\pi^2}\left\{1 + \frac{1}{\rho(z_1 - z_2)}\right.$$
$$\left.\times\left[(z_1 - 1)\ln\left(\frac{z_1 - 1}{z_1}\right) - (z_2 - 1)\ln\left(\frac{z_2 - 1}{z_2}\right)\right]\right\}(\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0)i\sigma_{\mu\nu}k^\nu\gamma_5 \tag{5.53}$$
$$\equiv F_D(k^2)i\sigma_{\mu\nu}k^\nu\gamma_5\,, \tag{5.54}$$

where in the last line we have defined the electric dipole form factor* $F_D(k^2)$. The electric dipole moment d_τ is given by $-F_D(0)$. Our final result is

$$d_\tau = \frac{em_\tau\sqrt{2}G_F\tan^2\beta}{16\pi^2}\left\{1 + \frac{1}{\rho(z_1 - z_2)}\right.$$
$$\left.\times\left[(z_1 - 1)\ln\left(\frac{z_1 - 1}{z_1}\right) - (z_2 - 1)\ln\left(\frac{z_2 - 1}{z_2}\right)\right]\right\}(\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0)\,, \tag{5.55}$$

*For a discussion of form factors, see Appendix B.

where

$$\mathrm{Im}Z_0 = \frac{1}{2}\sqrt{1+\left|\frac{v_1}{v_2}\right|^2}u_1u_3 + \frac{1}{2}\sqrt{1+\left|\frac{v_2}{v_1}\right|^2}u_2u_3 \tag{5.56}$$

$$\mathrm{Im}\tilde{Z}_0 = \frac{1}{2}\sqrt{1+\left|\frac{v_1}{v_2}\right|^2}u_1u_3 - \frac{1}{2}\sqrt{1+\left|\frac{v_2}{v_1}\right|^2}u_2u_3\,, \tag{5.57}$$

where the u_i are Lagrange multipliers subject to a unitarity constraint as given in Weinberg [28].

Chapter 6

Numerical Results and Experimental Limits

Our result for the tau EDM given in Eq. (5.55) has two free parameters, $\tan\beta$ and the CP-violating parameter $\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0$. One can, however, place a unitarity constraint on the latter parameter [28], which can be written in terms of the former as [12]

$$\left|\mathrm{Im}Z_0 + \mathrm{Im}\tilde{Z}_0\right| \leq (1/2)\cot\beta\sqrt{1+\tan^2\beta}\,, \tag{6.1}$$

which is shown as a function of $\tan\beta$ in Fig. 6.1.

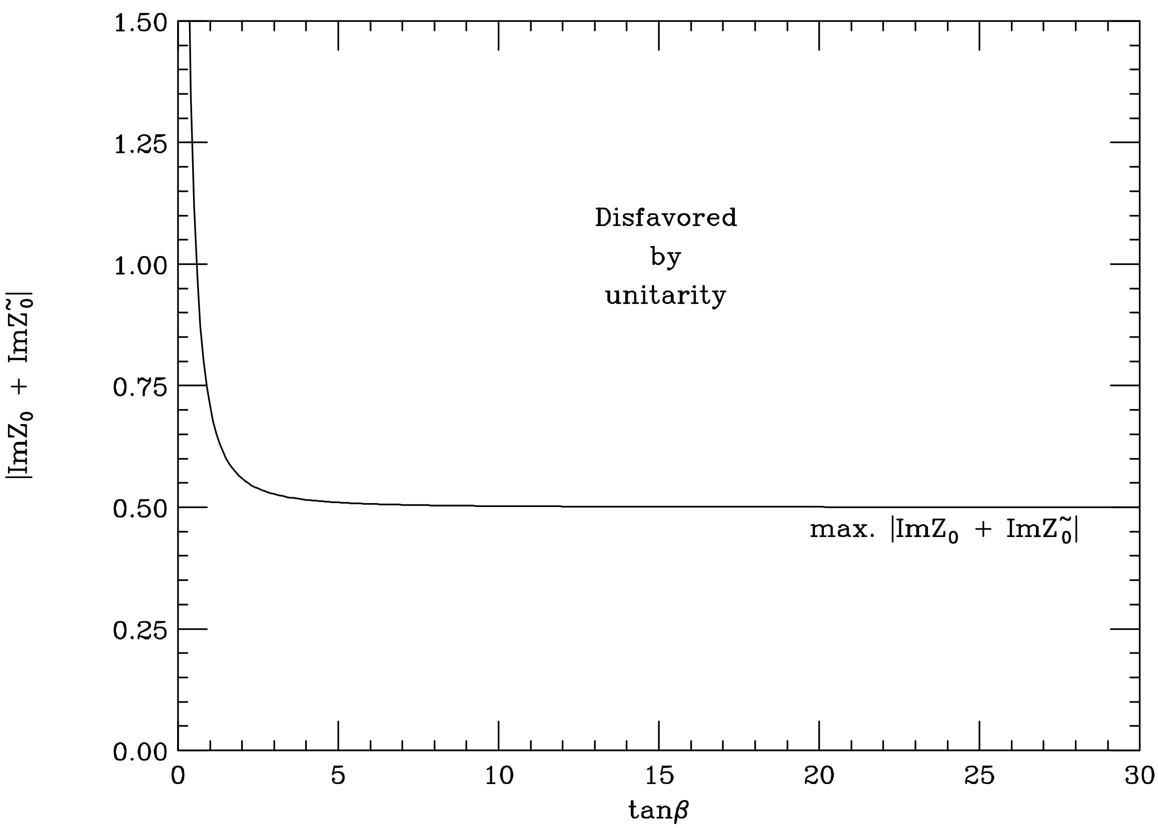

FIG. 6.1: Maximum CP-violating parameter allowed by unitarity constraint as a function of $\tan\beta$.

Figure 6.2 gives the numerical results for the tau EDM as a function of the domi-

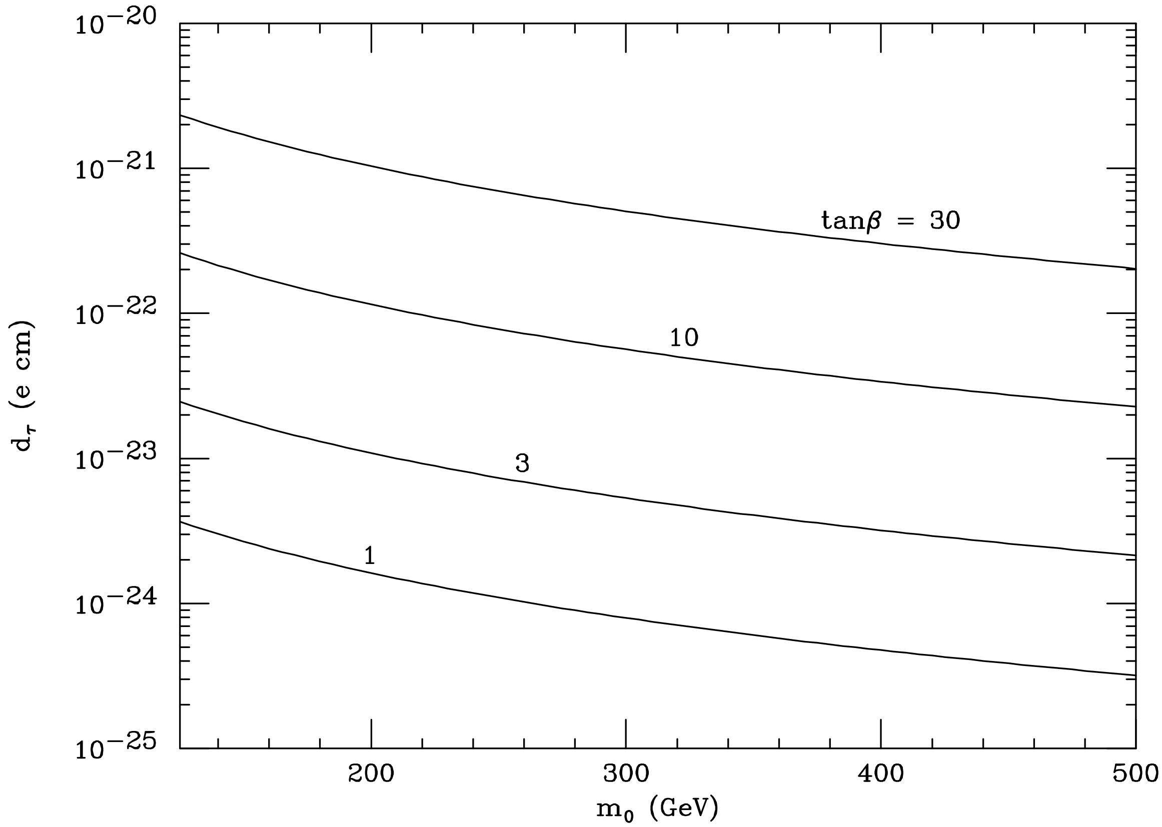

FIG. 6.2: Predicted tau EDM from Eq. (5.55) as a function of dominant scalar Higgs mass m_0 for various values of $\tan\beta$. For each value of $\tan\beta$, the corresponding maximum value of the CP-violating phase via Eq. (6.1) is assumed.

nant scalar Higgs mass m_0. Table 6.1 gives the EDM values at $m_0 = 125$ GeV, ranging from 3.66×10^{-24} e cm for $\tan\beta = 1$ to 2.33×10^{-21} e cm for $\tan\beta = 30$.

The current limit [4] for the EDM of the tau, $|d_\tau| < 3.9{\times}10^{-17}$ e cm, was established by the Belle Collaboration [30] in 2003. From Fig. 6.2, it is apparent that the predicted EDM is still far below the current best experimental limit.

TABLE 6.1: Tau EDM at $m_0 = 125$ GeV

$\tan\beta$	EDM d_τ (e cm)
1	3.66×10^{-24}
3	2.46×10^{-23}
10	2.60×10^{-22}
30	2.33×10^{-21}

Chapter 7

Conclusions

We have found that the one-loop EDM of the tau lepton can be much larger than predicted by the Standard Model but is still far below the current experimental upper limits.

The most immediate extension of the current work would be to calculate the tau EDM at the two-loop level. One such possibility would be the tau lepton analog of the two-loop diagram given in Ref. [31], which is shown in Fig. 7.1. At the two-loop level, there is only one scalar/pseudoscalar coupling with the tau, giving only one factor of m_τ/v compared to two factors at the one-loop level. Thus, due to the larger mass of the tau, these diagrams are not as important for the tau as they are for the electron and muon, where these diagrams dominate at low $\tan\beta$ [12].

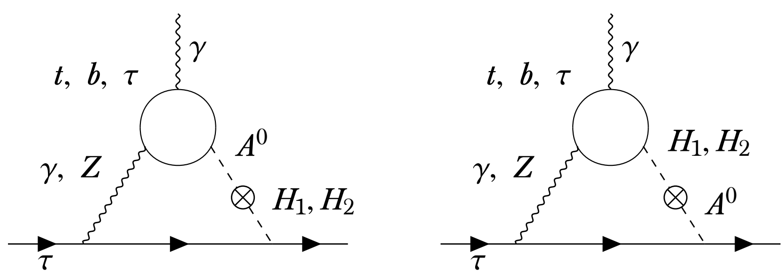

Fig. 7.1: Two-loop Feynman diagrams for the EDM of the tau lepton. Note that at the two-loop level, there is only one direct scalar/pseudoscalar interaction versus the two interactions at the one-loop level.

At this point, it is still unclear whether any particular EDM—if observed—will be of sufficiently large magnitude to explain baryon asymmetry. Nonetheless, the search for EDM's continues to offer the hope that one day we will have finally answered one of the foundational questions about our universe. Likewise, it has been said that electric dipole moment experiments have taught us more about the fundamental forces of nature than any other type of experiment [17]. Whether true or not, it is clear that the limits imposed by EDM experiments—or any EDM's observed in the future—will continue to push the limits of our understanding of the nature of physical reality.

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Appendix A

Loop Integrals in N Dimensions

This appendix is based heavily on that of Raymond [32].

Suppose we have an N-dimensional integral

$$I = \int d^N \ell \, F(\ell^2) \,, \tag{A.1}$$

where $F(\ell^2)$ is an arbitrary integrand that depends only on the lengths of the individual ℓ_μ, with $\mu = 1$ to N. Of course, if the coordinates are rectangular, we could just compute N successive integrations like normal. However, this clearly will not work for other coordinate systems. Furthermore, if N is large, then this process would be very tedious even in rectangular coordinates. Therefore, we wish to find a simple and more general method to perform the integrations.

Let us try moving to N-dimensional spherical coordinates, i.e.

$$(\ell_1, \ell_2, \ldots, \ell_N) \to (L, \phi, \theta_1, \theta_2, \ldots, \theta_{N-2}) \,, \tag{A.2}$$

where $L = |\ell| = \sqrt{\ell_\mu \ell^\mu}$. Then in N dimensions, our Jacobian becomes

$$d^N \ell = (L^{N-1} \, dL)(d\phi)(\sin\theta_1 \, d\theta_1 \; \sin^2\theta_2 \, d\theta_2 \cdots \sin^{N-2}\theta_{N-2} \, d\theta_{N-2}) \tag{A.3}$$

$$= (L^{N-1} \, dL)(d\phi) \prod_{i=1}^{N-2} \sin^i \theta_i \, d\theta_i \tag{A.4}$$

with the usual limits

$$0 \le L \le +\infty \,, \quad 0 \le \phi \le 2\pi \,, \quad 0 \le \theta_i \le \pi \,. \tag{A.5}$$

Thus, our integral becomes

$$I = 2\pi \prod_{i=1}^{N-2} \int_0^{\pi} \sin^i \theta_i \, d\theta_i \int_0^{\infty} L^{N-1} F(L^2) \, dL \, . \tag{A.6}$$

Next, we note integral 858.46 from Dwight [29]:

$$\int_0^{\pi} \sin^i \theta_i \, d\theta_i = \sqrt{\pi} \frac{\Gamma\left(\frac{i+1}{2}\right)}{\Gamma\left(\frac{i}{2}+1\right)} \, . \tag{A.7}$$

Recall that we have $N-2$ of these integrals, so when multiplied together, we obtain

$$\begin{aligned} I &= 2\pi (\pi^{1/2})^{N-2} \frac{\Gamma(1)}{\Gamma\left(\frac{3}{2}\right)} \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma(2)} \cdots \frac{\Gamma\left(\frac{N-2+1}{2}\right)}{\Gamma\left(\frac{N-2}{2}+1\right)} \int_0^{\infty} L^{N-1} F(L^2) \, dL \\ &= \frac{2\pi^{N/2}}{\Gamma(N/2)} \int_0^{\infty} L^{N-1} F(L^2) \, dL \, . \end{aligned} \tag{A.8}$$

Now, since L is just a magnitude, we can let $x = L^2$ and write

$$\begin{aligned} \int_0^{\infty} L^{N-1} F(L^2) \, dL &= \int_0^{\infty} (x^{1/2})^N x^{-1/2} F(x) \frac{1}{2} x^{-1/2} \, dx \\ &= \frac{1}{2} \int_0^{\infty} x^{(N-2)/2} F(x) \, dx \, . \end{aligned} \tag{A.9}$$

Combining Eq. (A.9) with Eq. (A.8), we arrive at

$$I = \frac{\pi^{N/2}}{\Gamma(N/2)} \int_0^{\infty} x^{(N-2)/2} F(x) \, dx \, . \tag{A.10}$$

Thus, we have reduced an N-dimensional integral into a one-dimensional integral times a factor involving pi and a gamma function. However, we can generalize things a bit further. A more general form of $F(x)$ is

$$F(x) = (x + a^2)^{-m} \, , \tag{A.11}$$

where m is an integer greater than or equal to 2. Then we have

$$\int d^N x\,(x+a^2)^{-m} = \frac{\pi^{N/2}}{\Gamma(N/2)}\int_0^\infty x^{(N-2)/2}(x+a^2)^{-m}\,dx\,. \tag{A.12}$$

Letting $x = a^2 y \Rightarrow dx = a^2\,dy$,

$$\int d^N x\,(x+a^2)^{-m} = (a^2)^{-m+N/2}\frac{\pi^{N/2}}{\Gamma(N/2)}\int_0^\infty y^{N/2-1}(1+y)^{-m}\,dy\,. \tag{A.13}$$

Now we can play a trick with the exponents. If we let $p = m - N/2$, we have one of the definitions of the beta function [33]:

$$\int_0^\infty y^{N/2-1}(1+y)^{-N/2+p}\,dy = \mathrm{B}(N/2, p) = \frac{\Gamma(N/2)\Gamma(p)}{\Gamma(N/2+p)}\,. \tag{A.14}$$

Putting m back in, we have

$$\int d^N x\,(x+a^2)^{-m} = \frac{\pi^{N/2}}{\Gamma(N/2)}\frac{\Gamma(N/2)\Gamma(p)}{\Gamma(N/2+p)}(a^2)^{-m+N/2} \tag{A.15}$$

$$= \pi^{N/2}\frac{\Gamma(m-N/2)}{\Gamma(m)}(a^2)^{-m+N/2}\,. \tag{A.16}$$

If we go back to our original variable, we have an equation that facilitates computing loop integrals:

$$\int d^N \ell \frac{1}{(\ell^2+a^2)^m} = \pi^{N/2}\frac{\Gamma(m-N/2)}{\Gamma(m)}\frac{1}{(a^2)^{m-N/2}}\,. \tag{A.17}$$

Appendix B

Form Factors and Tensor Structure of Elementary Fermions

In Chapter 5, we briefly mentioned form factors but did not discuss them. Let us look at the general tensor structure of Γ^μ for our EDM calculation. Our transition matrix is given by

$$T^\mu = \bar{u}(p_2)\Gamma^\mu u(p_1)\,. \tag{B.1}$$

At tree level in QED (Fig. B.1), we simply have

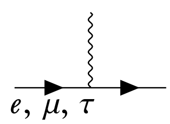

Fig. B.1: Tree-level QED interaction

$$\Gamma^\mu = -ie\gamma^\mu\,. \tag{B.2}$$

At higher orders with electroweak corrections, our interaction is no longer a simple vertex interaction but a series of more complicated interactions as represented by the

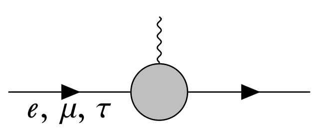

Fig. B.2: Higher-order interactions of fermions gray circle in Fig. B.2. Thus, we can have six possible types of terms, giving

$$\Gamma^\mu = \gamma^\mu(A + B\gamma_5) + (p_2^\mu - p_1^\mu)(C + D\gamma_5) + (p_1^\mu + p_2^\mu)(E + F\gamma_5)\,, \tag{B.3}$$

where the coefficients A–F are functions $f(p^2)$. (Recall that at the end of the calculation these are just numbers, so γ^μ commutes with them.) We can use the Gordon identities

$$\bar{u}(p_2)\gamma^\mu u(p_1) = \bar{u}(p_2)\left(\frac{p_1^\mu + p_2^\mu}{2m} + \frac{i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})}{2m}\right)u(p_1) \tag{B.4}$$

$$\bar{u}(p_2)(p_1^\mu + p_2^\mu)\gamma_5 u(p_1) = -\bar{u}(p_2)i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})\gamma_5 u(p_1) \tag{B.5}$$

to write

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu(A + B\gamma_5) + (p_2^\mu - p_1^\mu)(C + D\gamma_5)\\ &\quad + E[2m\gamma^\mu - i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})] - Fi\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})\gamma_5\,.\end{aligned} \tag{B.6}$$

Now, letting $k = p_2 - p_1$ and rearranging terms,

$$\Gamma^\mu = (A + 2mE)\gamma^\mu - B\gamma^\mu\gamma_5 + Ck^\mu + Dk^\mu\gamma_5 - Ei\sigma^{\mu\nu}k_\nu - Fi\sigma^{\mu\nu}k_\nu\gamma_5\,. \tag{B.7}$$

Applying the Ward identity $k_\mu\Gamma^\mu = 0$, we have

$$\begin{aligned}0 = k_\mu\Gamma^\mu &= (A + 2mE)k_\mu\gamma^\mu + Bk_\mu\gamma^\mu\gamma_5 + Ck_\mu k^\mu\\ &\quad + Dk_\mu k^\mu\gamma_5 - Ei\sigma^{\mu\nu}k_\mu k_\nu - Fi\sigma^{\mu\nu}k_\mu k_\nu\gamma_5\,.\end{aligned} \tag{B.8}$$

The momentum $k_\mu k^\mu$ will not vanish by itself, so C must be zero. This gives

$$0 = (A+2mE)k_\mu\gamma^\mu + Bk_\mu\gamma^\mu\gamma_5 + Dk_\mu k^\mu\gamma_5 - Ei\sigma^{\mu\nu}k_\mu k_\nu - Fi\sigma^{\mu\nu}k_\mu k_\nu\gamma_5 \tag{B.9}$$

$$= (A+2mE)\not{k} + B\not{k}\gamma_5 + Dk^2\gamma_5 + E\frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)k_\mu k_\nu + F\frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)k_\mu k_\nu\gamma_5 \tag{B.10}$$

$$= (A+2mE)\not{k} - B\not{k}\gamma_5 - Dk^2\gamma_5 + E\frac{1}{2}(\not{k}\not{k} - \not{k}\not{k}) + F\frac{1}{2}(\not{k}\not{k} - \not{k}\not{k})\gamma_5 \tag{B.11}$$

$$= (A+2mE)(\not{p}_2 - \not{p}_1) + B(\not{p}_2 - \not{p}_1)\gamma_5 + Dk^2\gamma_5 \tag{B.12}$$

$$= B(2m)\gamma_5 + Dk^2\gamma_5\,. \tag{B.13}$$

The last line implies $B = -Dk^2/2m$. Plugging this back into Eq. (B.7), we obtain

$$\Gamma^\mu = (A+2mE)\gamma^\mu - \frac{D}{2m}k^2\gamma^\mu\gamma_5 + Dk^\mu\gamma_5 - Ei\sigma^{\mu\nu}k_\nu - Fi\sigma^{\mu\nu}k_\nu\gamma_5\,. \tag{B.14}$$

We just need to take care of the D terms. First, we write them in terms of a common coefficient:

$$-\frac{D}{2m}k^2\gamma^\mu\gamma_5 + Dk^\mu\gamma_5 = -\frac{D}{2m}(k^2\gamma^\mu - 2mk^\mu)\gamma_5\,. \tag{B.15}$$

Next, instead of using the Dirac equation to eliminate factors of $\not{p}_1$ and $\not{p}_2$ as we have before, we will use it to *add* a factor of $\not{p}_1$ and $\not{p}_2$:

$$\bar{u}(p_2)2mk^\mu\gamma_5 u(p_1) = \bar{u}(p_2)(\not{p}_2 - \not{p}_1)k^\mu\gamma_5 u(p_1)\,. \tag{B.16}$$

Thus,

$$
\begin{aligned}
(k^2\gamma^\mu - 2mk^\mu)\gamma_5 &= [k^2\gamma^\mu - (\not{p}_2 - \not{p}_1)k^\mu]\gamma_5 && \text{(B.17)} \\
&= (k^2\gamma^\mu - \not{k}k^\mu)\gamma_5 && \text{(B.18)} \\
&= (k^2\gamma^\mu - \gamma_\nu k^\mu k^\nu)\gamma_5 && \text{(B.19)} \\
&= (k^2 g^{\mu\nu}\gamma_\nu - \gamma_\nu k^\mu k^\nu)\gamma_5 && \text{(B.20)} \\
&= \gamma_\nu\gamma_5(k^2 g^{\mu\nu} - k^\mu k^\nu)\,, && \text{(B.21)}
\end{aligned}
$$

which gives us the overall tensor structure

$$\Gamma^\mu = (A + 2mE)\gamma^\mu - \frac{D}{2m}\gamma_\nu\gamma_5(k^2 g^{\mu\nu} - k^\mu k^\nu) - Ei\sigma^{\mu\nu}k_\nu - Fi\sigma^{\mu\nu}k_\nu\gamma_5\,. \tag{B.22}$$

Finally, we can rename our coefficients according to standard designations, giving

$$\Gamma^\mu = F_V\gamma^\mu + F_A\gamma_\nu\gamma_5(k^2 g^{\mu\nu} - k^\mu k^\nu) + F_M i\sigma^{\mu\nu}k_\nu + F_D i\sigma^{\mu\nu}k_\nu\gamma_5\,, \tag{B.23}$$

where k_ν is the photon momentum; F_V is the vector coupling (form factor), which behaves as the effective charge and becomes the standard charge when reduced back to tree-level; F_A is the axial vector coupling; F_M is the magnetic dipole form factor; and F_D is the electric dipole form factor, for which we derived a specific form in Chapter 5 according to our 2HDM model.

Appendix C

Derivation of the Gordon Identities

In this Appendix, we derive the Gordon identities

$$\bar{u}(p_2)\gamma^\mu u(p_1) = \bar{u}(p_2)\left(\frac{p_1^\mu + p_2^\mu}{2m} + \frac{i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})}{2m}\right)u(p_1)\,, \tag{C.1}$$

$$\bar{u}(p_2)[p_1^\mu + p_2^\mu + i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})]\gamma_5 u(p_1) = 0\,. \tag{C.2}$$

To do so, we need the Dirac equation, the relation $\not{p}\gamma^\mu + \gamma^\mu\not{p} = 2p^\mu$ and

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,. \tag{C.3}$$

For the second identity, we will also need $\not{p}\gamma_5 u(p) = -m\gamma_5 u(p)$. We will start with the first identity:

$$\text{R.H.S.} = \bar{u}(p_2)\left[\frac{p_2^\mu + p_1^\mu}{2m} + \frac{i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})}{2m}\right]u(p_1) \tag{C.4}$$

$$= \frac{1}{2m}\bar{u}(p_2)[p_2^\mu + p_1^\mu - \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(p_{2\nu} - p_{1\nu})]u(p_1) \tag{C.5}$$

$$= \frac{1}{2m}\bar{u}(p_2)[p_2^\mu + p_1^\mu - \frac{1}{2}(\gamma^\mu\not{p}_2 - \gamma^\mu\not{p}_1 - \not{p}_2\gamma^\mu + \not{p}_1\gamma^\mu)]u(p_1) \tag{C.6}$$

$$= \frac{1}{2m}\bar{u}(p_2)[p_2^\mu + p_1^\mu - \frac{1}{2}(\gamma^\mu\not{p}_2 - \gamma^\mu\not{p}_1 - 2p_2^\mu + \gamma^\mu\not{p}_2 + 2p_1^\mu - \gamma^\mu\not{p}_1)]u(p_1) \tag{C.7}$$

$$= \frac{1}{2m}\bar{u}(p_2)[2p_2^\mu - \gamma^\mu\not{p}_2 + \gamma^\mu\not{p}_1]u(p_1) \tag{C.8}$$

$$= \frac{1}{2m}\bar{u}(p_2)[\not{p}_2\gamma^\mu + \gamma^\mu\not{p}_1]u(p_1) \tag{C.9}$$

$$= \frac{1}{2m}\bar{u}(p_2)[2m\gamma^\mu]u(p_1) \tag{C.10}$$

$$= \bar{u}(p_2)\gamma^\mu u(p_1) \tag{C.11}$$

$$= \text{L.H.S.} \tag{C.12}$$

For the second identity, the steps of the derivation are the same up to Eq. (C.9). Thus, we have

$$\begin{aligned}
\text{L.H.S.} &= \bar{u}(p_2)\left[p_2^\mu + p_1^\mu + i\sigma^{\mu\nu}(p_{2\nu} - p_{1\nu})\right]u(p_1) && \text{(C.13)}\\
&\vdots \\
&= \bar{u}(p_2)[\not{p}_2\gamma^\mu + \gamma^\mu\not{p}_1]\gamma_5 u(p_1) && \text{(C.14)}\\
&= \bar{u}(p_2)[m\gamma^\mu - m\gamma^\mu]\gamma_5 u(p_1) && \text{(C.15)}\\
&= 0 && \text{(C.16)}\\
&= \text{R.H.S.} && \text{(C.17)}
\end{aligned}$$