AN EMPIRICAL STUDY OF SHELLSORT

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PREFACE

This thesis empirically studies the average running time characteristics of the Shellsort algorithm, and then determines what the optimum initial increment should be for two of the more widely used sequences of increments. A Shellsort algorithm due to Pratt, with an average running time of O(N*lg(N)**2) is also investigated briefly. A proof of the "worst case" permutation using the original Shellsort algorithm is given. Programs were written in FORTRAN, compiled using FORTRAN H, and run on an IBM 370/168 and on an IBM 1130.

I would like to thank my thesis adviser, Dr. J. P. Chandler, and the other members of my committee, Dr. G. E. Hedrick and Dr. D. W. Grace, and all of the faculty for their help and encouragement throughout my graduate study. I would also like to express my appreciation for support and understanding of my parents, Rudolph and Grace Bauer, my friends, and Denise.

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LIST OF SYMBOLS

A	number of left-to-right minima encountered in the intermediate sorting operations
B	number of moves
C	numeric constant
FIT(J)	fitted value of Y(J)
H(J)	Jth increment size
lg	logarithm base 2
log	logarithm base 10
N	number of elements in a list
NDF	number of degrees of freedom
O( )	exact order of the behavior of a function
P	power parameter in a power function
R(J)	Jth element in a list
S	sum of the increments: H(1), H(2), ...
T	number of passes used in Shellsort
y(J)	ordinate of the Jth raw data point
Y(J)	lg(y(J))
YSIG(J)	expected error (sigma) of Y(J)

CHAPTER I

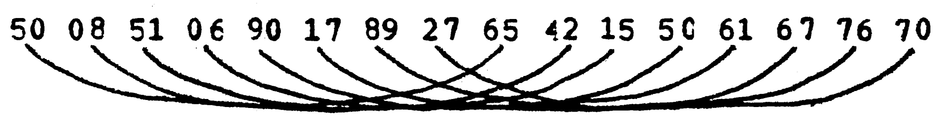
INTRODUCTION

Shellsort - The Algorithm

In the late 1950's, large high-speed random access memories were developed. Their size and speed demand an "in place" internal sorting technique that is both efficient and portable. The straight insertion sort is often used, but the algorithm has a running time of O(N**2), where N is the number of elements to be sorted. In the insertion sort, for each J between 2 and N, the Jth element, R(J), is inserted into the already sorted segment of the list, R(1), R(2), ..., R(J-1), by comparing R(J) with these elements (from right to left) and moving the larger elements to the left until the proper position for R(J) is found. The reason for the low speed of the insertion sort is that each comparison removes at most one inversion from the file. The two-way merge sort is faster, but it requires twice as much memory as an "in place" sort. In order to overcome these difficulties, Shell (10) proposed an algorithm in which the list is divided into groups. R(J) is the Jth element in the list, and H is known as the increment size. The first group is formed from the R(1), R(1+H), R(1+2H), ... elements in the list. The second group is formed frcm the R(2) ,R(2+H), R(2+2H), ..., elements in the list. This continues until every element is in some group. This group is then sorted using a straight insertion sort. A smaller value of H is chosen, and the process is repeated. At the end of the sort, H has a value of 1, which places all of the elements into one group. The last pass is therefore simply an insertion sort on the entire list, which ensures that the list is sorted.

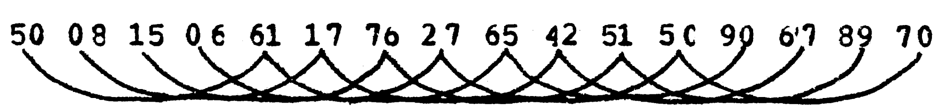
An example of Shellsort is given in Knuth (7). Suppose a list of sixteen elements is to be sorted. For the first pass, H will have a value of 8. Eight groups of two are formed, namely (R(1),R(9)), (R(2),R(10)), (R(3),R(12)), ..., (R(8),R(16)). Each member cf the group is separated by the value of H.

First pass H=8



An insertion sort is performed on these groups. Note that some of the elements may travel a great distance within the list, which is the reason for the efficiency of Shellsort. On the second pass, H will be given a value of 4. Four groups of four will be formed, namely (R(1),R(5),R(9),R(13)), ..., (R(4),R(8),R(12),R(16)). Again, each group is sorted using straight insertion.

Second pass H=4



On the third pass, H will have a value of 2.

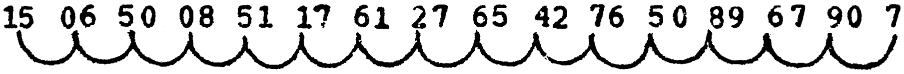
Third pass H=2

50 08 15 06 61 17 51 27 65 42 76 50 90 67 89 70

On the fourth pass, H will have a value of 1.

Fourth pass H=1

15 06 50 08 51 17 61 27 65 42 76 50 89 67 90 70



The result is a sorted list.

Final sorted list

06 08 15 17 27 42 50 50 51 61 65 67 70 76 89 90

On the final pass (H=1), a straight insertion sort is performed to ensure that the entire list is sorted. The sequence of H values 8, 4, 2, 1 is not mandatory, so long as the final H value is 1. Some sequences of H values are more efficient than others, and they will be discussed later.

The original Shellsort was formalized by Boothroyd (2) in Program A (Figure 1). Boothroyd calculates the inital increment by repeated integer multiplication as shown in the first executable statement. This avoids the roundoff that can occur if logarithms are used. Note that the sort within the groups is accomplished by interchanging adjacent pairs within each group as long as an element is out-of-order. This was done in order to simplify the algorithm, with the realization that it decreases the efficiency of the sort compared to an insertion method to be discussed below (personal communication from J. Boothroyd to J. P. Chandler).

```
procedure Shellsort (a,n); value n; real array a; integer n;
comment a[1] through a[n] of a[1:n] are rearranged in
  ascending order. The method is that of D. A. Shell,
  (A high-speed sorting procedure, Comm. ACM 2 (1959),
  30-32) with subsequences chosen as suggested by T. N.
  Hibbard (An empirical study minimal storage sorting,
  SDC report SP-982). Subsequences depend on m(1)
  the first operative value of m. Here m(1) = 2**k-1
  for 2**k <= n <= 2**(k+1). To implement Shell's original
  choice of m(1) = floor(n/2) change the first statement to
  m := n;
begin integer i,j,k,m; real w;
  for i := 1 step i until n do m := 2 x i - 1;
  for m := m / 2 while m ¬= 0 do
    begin k := n - m;
      for j := 1 step 1 until k do;
        begin for i := j step -m until 1 do
          begin if a[i+m] => a[i] then go to 1;
            w := a[i]; a[i] := a[i+m]; a[i+m] := w;
          end i;
      1:end j
  end m
end Shellsort
```

Figure 1. Program A, by J. Boothroyd

Hibbard (5) recoded the algorithm in order to improve the efficiency (see Program B, Figure 2). Instead of performing pairwise interchanges, Hibbard stores the out-of-order element in a temporary location, then compares the temporary value with the preceding elements within the group in order to find the correct position of the temporary value. The elements within the group are shifted by one position in order to make room for the eventual reinsertion of the temporary value into the group.

```
procedure C(x,n); array x(1:n);
comment Shell's method using Hibbard's increments
begin integer d,i,j;
  d := 2 ** entier log(2)n - 1
C1:  if d <= 0 the go to exit;  i :=1;
C2:  j := i;  y := x[i+d];
C3:  if y < x[j] then go to C4;
C5:  x[j+d] := y;  i := i + 1;
     if i + d <= n then go to C2;
     d := (d - 1) / 2;  go to C1;
C4:  x[j+d] := x[j];  j:= j - d;
     if j > 0 the go to C3;  go to C5;
exit: end
```

Figure 2. Program B, by T. N. Hibbard

The following definitions describe the basic operations in Hibbard's version of Shellsort:

Comparison - Any examination that is made between any element in the list and either another element in the list or a saved value is a "comparison".

Save - If a comparison is made between two elements in the list and they are determined to be out-of-order then the appropriate element is stored in a temporary location. The saved value is later reinserted when its proper position is found. The operation of storing an out-of-order element and subsequently reinserting into the list constitutes one "save" or "save/insertion".

Move - Once an out-of-order element has been found and its value has been stored in a temporary location, it is necessary to move elements within the list in order to provide a place to reinsert the saved element in its correct position. Each element that is shifted constitutes a "move".

Stepping through the following example will illustate these definitions:

Suppose that in the following group of elements, "5" is to be inserted into its proper position.

```
Comparisons=0    Saves=0    Moves=0

        3...2...6...8...9...5
```

A comparison is made between "9" and "5".

```
Comparisons=1    Saves=0    Moves=0

        3...2...6...8...9...5
                        |   |
                        | ? |
                        -----
```

The comparison determines that "9" and "5" are out-of-order, so a save is made. "5" is stored in a temporary location, TEMPVAL.

```
Comparisons=1    Saves=1    Moves=0

3...2...6...8...9...5      TEMPVAL=5
                    |              ^
                    |              |
                    ----------------
```

"9" is moved to make room for a possible reinsertion. "5" is destroyed, but it has been saved in TEMPVAL.

```
        Comparisons=1    Saves=1    Moves=1

       3...2...6...8...9...9      TEMPVAL=5
                       |   ^
                       |   |
                       L---J
```

A comparison is made between "8" and TEMPVAL.

```
        Comparisons=2    Saves=1    Moves=1

       3...2...6...8...9...9      TEMPVAL=5
                   |                      |
                   |           ?          |
                   L----------------------J
```

The comparison determines that "8" and TEMPVAL are out-of-order, so another move is made to make room for possible insertion.

```
        Comparisons=2    Saves=1    Moves=2

       3...2...6...8...8...9      TEMPVAL=5
                   |   ^
                   |   |
                   L---J
```

"6" and TEMPVAL are compared.

```
        Comparisons=3    Saves=1    Moves=2

       3...2...6...8...8...9      TEMPVAL=5
               |                          |
               |             ?            |
               L--------------------------J
```

"6" and TEMPVAL are out-of-order, so another move is made.

```
        Comparisons=3    Saves=1    Moves=3

       3...2...6...6...8...9      TEMPVAL=5
               |   ^
               |   |
               L---J
```

"2" and TEMPVAL are compared.

```
        Comparisons=4   Saves=1   Moves=3

       3...2...6...6...8...9      TEMPVAL=5
           |                          |
           |            ?             |
           L--------------------------J
```

"2" and TEMPVAL are in-order, so TEMPVAL is inserted into the group. Note that the number of saves does not change.

```
        Comparisons=5   Saves=1   Moves=4

       3...2...5...6...8...9      TEMPVAL=5
               ^                      |
               |                      |
               L----------------------J
```

If Boothroyd's algorithm is used, then M interchanges require M+1 comparisons and 3*M replacement operations. In order to accomplish the same results, true insertion requires one save (two replacement operations) and M moves (M+1 comparisons and M replacement operations), for a total of M+2 replacement operations and M+1 comparisons. This decreases the number of replacement operations by nearly two-thirds. Chandler and Harrison (3) indicated that this results in a 17% decrease in the average running time relative to Boothroyd's algorithm for a Shellsort written in Fortran on a CDC 6400 computer.

An important feature of Program B is its use of an "unconditional" save operation, in which Program B executes the save operation before an initial comparison is made. If the elements are determined to be in-order, then the saved element is reinserted into its original position. The save operation can be avoided when the elements are in-order if the comparison is made before the save, as is done in Program C (Figure 3).

Consider the types of operations required by Programs B and C. In Program C, an in-order comparison (a comparison in which the two elements are determined to be in the correct relative order) requires two subscript references and one comparison, whereas Program B requires two subscript references, one comparison, and two replacement operations. In Program C, an out-of-order comparison (a comparison in which the two elements are determined not to be in the correct relative order) requires three subscript references, one comparison, and one replacement operation. Program B uses uses two subscript references, one comparison, and one replacement operation. It is obvious that an in-order comparison is faster in Program C than in Program B, because the save operation is not made. On the other hand, Program B may be slightly faster than Program C for an out-of-order comparison, because the comparison in Program C requires both operands to be subscripted, whereas Program B has only one subscripted operand. In a straight insertion sort, it seldom happens that an item to be inserted is already in the proper position, so an "unconditional save" may well be faster than making a comparsion before the save is made. However in Shellsort, the groups in passes following the

```
      SUBROUTINE SHELL(JR,N)
      DIMENSION JR(100)
C
C                SHELLSORT
C
C                INPUT PARAMETERS
C                   JR    INTEGER ARRAY TO BE SORTED
C                   N     NUMBER OF ELEMENTS IN ARRAY JR
C
C                RETURNS ARRAY JR IN ASCENDING ORDER.
C
C                SUBROUTINES CALLED
C                   NEXTH RETURNS THE  NEXT INCREMENT.
C
C                JCOMP, JMOVE, AND JSAVE ARE THE COUNTERS FOR
C                COMPARISONS, MOVES, AND INSERTIONS/SAVES.
C                JCOMP=0
C                JMOVE=0
C                JSAVE=0
C
      IF(N.LT.2) RETURN
      JH=0
      CALL NEXTH(JH)
   10 NMJH=N-JH
      DO 40 K=1,NMJH
        JHPK=JH+K
C               JCOMP=JCOMP+1
        IF(JR(JHPK).GT.JR(K)) GO TO 40
        JTEMP=JR(JHPK)
C               JMOVE=JMOVE+1
        JR(JHPK)=JR(K)
        J=K-JH
        IF(J.LE.0) GO TO 30
C               JCOMP=JCOMP+1
   20   IF(JTEMP.GT.JR(J)) GO TO 30
C               JMOVE=JMOVE+1
        JHPJ=JH+J
        JR(JHPJ)=JR(J)
        J=J-JH
        IF(J.GT.0) GO TO 20
C               JSAVE=JSAVE+1
   30   JHPJ=JH+J
        JR(JHPJ)=JTEMP
   40 CONTINUE
      CALL NEXTH(JH)
      IF(JH.GE.1) GO TO 10
      RETURN
      END
```

Figure 3. Program C

first pass are partially ordered, so it is not clear that the unconditional save will increase the speed. For example, consider the final pass of Shellsort. Since the final pass is an insertion sort, one might expect the order to be O(N**2) for a random list. This cannot be the case since the average running time of Shellsort is much less than O(N**2), indicating that the list must have been partially ordered by the initial passes. Table I is an empirical comparison between Programs B and C for various number of passes. The table was generated using an IBM 1130, in order to avoid the timing problems caused by a multi-programmed system. Program B is slightly faster than Program C in an insertion sort (H=1) due to the many out-of-order comparisons that are made. Program C is faster than Program B in a multi-pass Shellsort, due to the ordering that is done in the initial passes. Table I indicates that Program C should be used on all multi-pass Shellsorts.

Shellsort - Increments and Running Times

The optimum set of increments (H(T), H(T-1), H(T-2), ..., H(1)) for a given value of N is not known, but many sequences have been suggested. Representative sequences are given in Table II. Shell (10) introduced the algorithm in 1959, but his version can result in a very poor worst case running time (4) of O(N**2). The worst case running time of Shell's version is investigated in Chapter IV. When the

TABLE I

AVERAGE RUNNING TIMES FOR PROGRAMS B AND C

Increments	No. of Passes	Ave. Running Time(Sec) Program B	Program C	Percent Gain Program C
(3**K-1)/2	1	234.20	234.51	-0.13
(3**K-1)/2	2	70.43	70.40	+0.04
(3**K-1)/2	3	27.80	27.60	+0.71
(3**K-1)/2	4	16.57	16.21	+2.17
(3**K-1)/2	5	13.90	13.37	+3.81
(3**K-1)/2	6	13.61	12.92	+5.07
2**K-1	1	234.19	234.51	-0.14
2**K-1	2	87.91	87.89	+0.02
2**K-1	3	41.83	41.60	+0.55
2**K-1	4	24.61	24.19	+1.71
2**K-1	5	17.66	17.07	+3.34
2**K-1	6	15.14	14.36	+5.15
2**K-1	7	14.25	13.26	+6.95
2**K-1	8	14.12	12.93	+8.43
2**K-1	9	14.43	13.12	+9.08

This table was generated on an IBM 1130. All counters were removed from the programs. 1000 random elements were sorted.

binary representation of N contains long strings of zeros, then there is little interaction between the groups, which results in many sorted but distinct groups. This forces the final pass to do much of the sorting, thus degrading the sort time. For example, if H equals 8, 4, 2, 1, each pass combines pairs of groups that have not interacted at all in previous passes. Frank and Lazarus (4) have shown that this can be eliminated if the increments are relatively prime. Papernov and Stasevich (8) proved that the worst case running time using Hibbard's increments is bounded by O(N**1.5). This was generalized by Pratt (9) to include most "fuzzy" geometric series obeying certain coprimeness properties. Yao (11) derived the asymptotic form of a three-pass Shellsort using fixed increments. Pratt (9) also suggested a Shellsort algorithm that has a worst case and average case running time of O(N*(lg(N)**2)) ("lg" symbolizes logarithm base 2), which will be discussed in Chapter III.

Knuth (7) estimated the total running time of Program B using a MIX program (MIX is an assembler language for a hypothetical machine). He shows that the execution time can be expressed in terms of five parameters, namely: the size of the file, N; the number of passes, T; the sum of the increments, S; the number of left-to-right minima encountered in the intermediate sorting operations, A; and the number of moves, B. Using this terminology, the number of comparisons is B + NT - S - A. The total running time for the MIX program is (9B + 10NT + 13T - 10S - 3A + 1) cycles.

TABLE II

AVERAGE NO. OF COMPARISONS FOR VARIOUS FAMILIES OF INCREMENTS

Author(s)	Stated Number of Comparisons	Increments
Shell (10)		floor(N/2**K))
Frank and Lazarus (4)		add 1 to Shell's even increments
Papernov and Stasevich (8)	1.09*N**1.27	2**K+1
Hibbard (5)	1.22*N**1.26	2**K-1
Knuth (7)	1.12*N**1.28	(2**K-(-1**K))/3
Knuth (7)	1.66*N**1.25	(3**K-1)/2

CHAPTER II

The Optimum Initial Increment

Previous Work

Once a sequence of increments has been chosen, how is the initial increment, H(T), chosen for a given value of N? Hibbard (5) suggested using the largest increment less than N. This seems rather high, due to the fact that an out-of-order element in the first pass moves almost N places, which is greater than the expected average total displacement of N/3 for a random input list. Shell (10) required the first increment to be floor(N/2), while Knuth (7) suggested that the initial increment be no greater than N/3.

Table III contains some typical total running times using the MIX assembler language as measured by Knuth (7). Several families of increments are shown, using various numbers of passes. Examination of Table III indicates that as the number of passes decreases for a given family of increments, the total average running time decreases, passes through a minimum, then increases. This minimum represents the number of passes, T, that gives the fastest sorting time for that particular family of increments. For example, Table III gives the MIX times when Hibbard's increments

(2**K-1) are used. The MIX times for nine, eight, and seven passes are 142238, 139435, and 140072, respectively. This indicates that if Hibbard's increments are used to sort 1000 random elements, eight passes should be used rather than the nine passes that Hibbard's algorithm actually uses. There is no intuitive way to know in advance the optimum number of passes to use, but this can be determined empirically. If Table III could be expanded to include all values of N, then it would easy to determine the optimum initial increment. It is impossible to include all values of N in Table III, but it is possible to estimate the value of N at which each pass becomes justified. These values will be called "crossover values". The first crossover value is the value of N at which a two-pass Shellsort is more efficient than a one-pass Shellsort. The second crossover value is the value of N at which a three-pass Shellsort is more efficient than a two-pass Shellsort, and so on.

Method of Investigation

In order to investigate Shellsort, one must establish a criterion to measure its efficiency. Any measurement using real or relative time on a given computer is not very useful in drawing general conclusions, due to the differences in instruction execution time between different machines. Fortunately, the number of comparison, move, and save operations is machine independent. Program C (Chapter I, Figure 3) indicates how these operations can be counted.

TABLE III

SHELLSORT MIX TIME FOR 1000 RANDOM ELEMENTS

Increments	A	B	T	S	MIX Time (Cycles)
1	6	249750	1	1	2257734
17 1	65	41667	2	18	394652
60 6 1	158	26361	3	67	266141
140 20 4 1	262	21913	4	165	234829
256 64 16 4 1	362	20459	5	341	229700
576 192 48 16 4 1	419	20088	6	837	231240
729 243 81 27 9 3 1	378	18533	7	1093	224826
512 246 128 64 32 16 8 4 2 1	493	16435	10	1023	236336
500 250 125 63 31 15 7 3 1	525	7600	9	994	147003
501 251 125 63 31 15 7 3 1	575	7200	9	997	143223
511 255 127 63 31 15 7 3 1	550	7100	9	1013	142238
255 127 63 31 15 7 3 1	450	7300	8	502	139435
127 63 31 15 7 3 1	300	8150	7	247	140072
63 31 15 7 3 1	200	9700	6	120	145579
31 15 7 3 1	125	13600	5	57	171521
513 257 129 65 33 17 9 5 3	575	6600	10	1032	147486
257 129 65 33 17 9 5 3	450	6800	9	519	144778

Table III (Continued)

129	65	33	17	9	5	300	7600	8	262	144985
3	1									
65	33	17	9	5	3	200	9400	7	133	152762
1										
33	17	9	5	3	1	125	13000	6	68	176024
683	341	171	85	43	21	500	7050	10	1364	148441
11	5	3	1							
341	171	85	43	21	11	500	7300	9	681	147508
5	3	1								
255	63	15	7	3	1	400	8600	6	344	132839
257	65	17	5	3	1	400	8700	6	348	133699
341	85	21	5	3	1	425	9300	6	456	137944
610	377	233	144	89	55	550	7400	14	1595	189183
34	21	13	8	5	3					
2	1									
377	144	55	21	8	3	475	8800	7	609	141777
1										
365	122	41	14	5	2	450	8300	7	550	137942
1										
364	121	40	13	4	1	450	9200	6	543	136099
	121	40	13	4	1	275	9900	5	179	136551

Comparisons predominate as N becomes large, because the number of comparisons consists of the number of in-order comparisons plus the number of moves (out-of-order comparisons). Thus, the number of moves can never be greater than the number of comparisons. Likewise, the comparisons predominate in an in-order list, because the number of comparisons is NT - S, while the number of moves and saves is zero.

The following outline can be used to investigate Shellsort:

1. Choose one family of increments to study, such as Hibbard's family, Knuth's family, etc.
2. Sort random lists with various values of N using Program C (Figure 3). Adjust subroutine NEXTH so that one pass is used. Repeat the sort with different random lists in order to obtain a good statistical average of the number of comparisons, moves, and saves required for a given value of N.
3. Repeat step 2 with NEXTH adjusted for two passes, three passes, etc.
4. Fit the number of comparisons to a curve as a function of N for each pass.
5. Determine the crossover points as the intersections of the fitted curves of adjacent numbers of passes.
6. Fit the crossover values found in step 5 to a curve as a function of N.

In order to generate the crossover values, several design parameters need to be considered:

1. Many families of increments have been suggested, and it would be impractical to study all of them. Hibbard's increments (2**K-1) and Knuth's increments ((3**K-1)/2) were studied because of their efficiency and popularity.
2. Only random lists containing up to 20000 elements were considered. Larger lists would require a large amount of computing time.
3. Fifty random lists were sorted for each value of N in order to determine the average number of comparisons, moves, and saves. This number of repetitions yielded an acceptable standard deviation.
4. The values of N were chosen to give a fairly even distribution of data points on a log scale. More closely spaced values of N were used in the vicinity of the crossover values.

Results of the sorting are found in Appendix A and Appendix B. Since the number of comparisons predominates, it can be used as one indication of the relative efficiency of various Shellsorts. Examining the appendices show that for a given value of N, the use of Knuth's increments results in approximately two to three percent fewer comparisons than the use of Hibbard's increments.

In order to determine how well the various fits agree with the data, two methods were used to measure the "goodness of fit": the value of chi-square and the runs test. Chi-square is the sum over all of the data points of

((Y(J)-FIT(J)/YSIG(J))**2

where Y(J) is the ordinate of the Jth data point, FIT(J) is the fitted value, and YSIG(J) is the expected standard error of Y(J). If the model FIT(J) is linear in all parameters, then the expected minimum value of chi-square is equal to the number of degrees of freedom, NDF, of the system. The number of degrees of freedom of a system is equal to the number of data points minus the number of parameters in the fit. The expected standard error of chi-square is the square root of 2*NDF. If the model is nonlinear these expected values are only approximate, but the test is still widely used for nonlinear models.

The runs test examines the number of runs of positive and negative values in the residuals (Y(J)-FIT(J)) of the fit. For example, if the signs of successive residuals are ++-+--+-, then there are six runs of successive residuals having the same sign. For a large number of data points, the expected number of runs for a good fit approaches half the number of data points. Bennett and Franklin (1) give details on the use of the runs test.

Fitting the Results of Each Pass

Initially, an attempt was made to fit the number of comparisons to a power function of N for each pass:

number of comparisons = C * (N ** P)

This system is linearized by using the log function:

$$lg(\text{number of comparisons}) = lg(C) + P * lg(N)$$

so that a straight line on a log-log plot corresponds to a power function in the raw data. Unless otherwise specified, the log-log system is used, and the term "raw data" refers to the original data.

The one-pass Shellsort is simply an insertion sort, so the data were compared to the asymptotic behavior of the insertion sort, 0.25*N**2. The one-pass Shellsort deviates sharply from 0.25*N**2 for small values of N, but it approaches 0.25*N**2 for large values of N. Figure 8 (Appendix D) shows the deviation of the one-pass Shellsort from its asymptotic behavior of 0.25*N**2. The first few raw data points were checked by sorting all of the permutations of a list for a given N, then averaging the operations, as shown in Table IV. Examination of the appendices shows that the data agree well with Table IV.

For each number of passes greater than one, a straight line was fitted to the log-log plot of lg(comparisons) verses lg(N). The results are shown in Table V. On small and intermediate numbers of passes, the fit is poor, as judged by the number of runs. The error is great at large values of N. On large number of passes, the fit is better, but this is due to the smaller range of N. The results in Table V are comparable to those found by Knuth (7) in Table II (Chapter I).

TABLE IV

AVERAGE BEHAVIOR OF INSERTION SORT FOR
ALL PERMUTATIONS OF N

N	Comparisons	Moves	Saves
2	2/2	1/2	1/2
3	16/6	9/6	7/6
4	118/24	72/24	46/24
5	926/120	600/120	326/120
6	7956/720	5400/720	2556/720
7	75132/5040	52920/5040	22212/5040
8	777456/40320	564480/40320	212976/40320
9	8771184/362880	6531840/362880	2239344/362880

After examining the operation of the Shellsort as N becomes large, the asymptotic behavior of the Shellsort with a fixed set of increments can be established.

Theorem: The average and worst case asymptotic behavior of Shellsort using a fixed set of increments is proportional to N**2.

Proof: Let the increment for the first pass be a fixed value, H(T). Let N be the number of elements in the list. On the first pass, the list is divided into H(T) groups, each group containing H(T)/N elements. An insertion sort is performed on each group, thus the bound for the average running time in sorting each group is proportional to (N/H(T))**2. Since there are H(T) groups, then the average running time for the first pass is proportional to H(T)*((N/H(T))**2). H(T) is a constant, so the bound for the average running time for the first pass is O(N**2), which becomes the bound for the entire Shellsort. This is also an upper bound because the maximum number of inversions in a list is (N*(N-1))/2, which is O(N**2), and it is not possible for Shellsort to increase the number of inversions.

TABLE V

LEAST SQUARES FIT OF POWER FUNCTION TO NUMBER OF COMPARISONS

Increments	Passes	Fit	Rurs	Runs(Expected)
2**K-1	1	0.25*N**2.00	2	18
2**K-1	2	0.41*N**1.77	3	18
2**K-1	3	0.50*N**1.65	3	20
2**K-1	4	0.52*N**1.59	3	45
2**K-1	5	0.67*N**1.51	3	25
2**K-1	6	1.03*N**1.41	3	24
2**K-1	7	1.49*N**1.34	3	22
2**K-1	8	2.04*N**1.27	3	20
2**K-1	9	2.62*N**1.25	3	18
2**K-1	10	3.06*N**1.22	6	14
2**K-1	11	3.43*N**1.21	9	10
2**K-1	12	3.43*N**1.21	7	6
2**K-1	13	3.60*N**1.21	3	2
(3**K-1)/2	1	0.25*N**2.00	2	21
(3**K-1)/2	2	0.39*N**1.75	3	21
(3**K-1)/2	3	0.49*N**1.63	3	23
(3**K-1)/2	4	0.67*N**1.49	3	23
(3**K-1)/2	5	1.17*N**1.37	3	23
(3**K-1)/2	6	1.93*N**1.29	3	18
(3**K-1)/2	7	2.62*N**1.24	7	12
(3**K-1)/2	8	3.14*N**1.22	7	7
(3**K-1)/2	9	3.06*N**1.22	4	2

The theorem makes it apparent that the fitted curve for each fixed number of passes contains an N**2 term, plus some other term that diminishes as N becomes large. The error in the fit of the one-pass Shellsort (see Figure VII, Appendix D) suggests that the other term is a function resembling a gamma density function, which are of the form:

F(X) = A*(T**B) * EXP-(C*T)

If the log of the data is considered instead of the raw data, then the log of the data must be multiplied by some appropriate weight in order to give the correct statistical distribution of the error in lg(Y). The calculation of the weights is done in the following manner:

Let:

LOG	-	natural logarithm function
y(J)	-	Jth raw data ordinate
Y(J)	-	lg(y(J))
SD(J)	-	standard deviation of the Jth raw data ordinate
REP	-	number of repetitions used to generate a raw data point
SIGMA	-	the expected standard error of a data ordinate
SQRT	-	square root function
W(J)	-	weight of the Jth data point

The following relations are true by definition:

SIGMA(y(J)) = SD(J) / SQRT(REP)
W(J) is proportional to 1/SIGMA(Y)**2

The following relation is approximately true from a linear expansion:

SIGMA(Y(J)) = SIGMA(y(J)) * dY/dy

Differentiating dY/dy and substituting yields:

SIGMA(Y(J)) = SIGMA(y(J)) / (y*log(2))

Substituting again yields:

W(J) = [y(J)*LOG(2)*SQRT(REP) / SD(J)] ** 2

Since all of the linear fits to the lg(comparisors) are unsatifactory, the data were weighted and then fitted to a gamma density function plus a line of slope 2.0. (A line of slope 2.0 in the linearized systems corresponds to the function N**2 in the raw data.) The origin of the gamma density function is held to zero, because the line fits the first point exactly. This yields an excellent fit; chi-square has a value of 30 (expected 32), and the number of runs is 19 (expected 18). Table VI shows the data and the fit.

When the number of comparisons for multiple passes is fitted to a gamma density function plus a line of slope 2.0, the origin of the gamma density function is allowed to move, since there is nothing to suggest where it should be positioned. This yields a good fit for a Shellsort using two to four passes, as shown in Table VI. In these passes, the origin of the gamma density function becomes increasingly more negative. In the five-pass Shellsort, the origin of the gamma density function moves to negative infinity. When this happens the gamma density function approaches a Gaussian function. A Gaussian function is a function of the form:

F(X) = A * EXP(-((T-B)**2) / (2*(C**2)))

In view of this, the data for five to thirteen passes were fitted using a Gaussian function plus a line of slope 2.0. Table VI shows that some of these fits are good while others are mediocre. None of the fits are really poor compared to the fits in Table V. The Shellsort with thirteen passes does not have enough data points in order to fit all of the parameters, so some of the parameters were estimated and held constant, as indicated by th values -14.5 and 7.0 in Subroutine HBCOM (Figure 4).

TABLE VI

GOODNESS OF FIT FOR SHELLSORT USING
HIBBARD'S INCREMENTS

Pass	Type of Fit	Chi Square	Chi Square (Expected)	Runs	Runs (Expected)
1	GAMMA+LINE	30.0	32 ± 8.0	19	18
2	GAMMA+LINE	32.1	31 ± 7.8	20	16
3	GAMMA+LINE	26.2	35 ± 8.4	20	20
4	GAMMA+LINE	56.3	40 ± 8.9	20	23
5	GAUSS+LINE	86.8	46 ± 9.6	17	25
6	GAUSS+LINE	151.0	43 ± 9.3	13	24
7	GAUSS+LINE	114.6	40 ± 8.9	23	22
8	GAUSS+LINE	50.0	36 ± 8.4	15	20
9	GAUSS+LINE	24.8	31 ± 7.9	18	18
10	GAUSS+LINE	23.6	24 ± 6.8	22	14
11	GAUSS+LINE	15.7	15 ± 5.6	12	10
12	GAUSS+LINE	13.8	8 ± 4.0	8	6
13	GAUSS+LINE	3.2	2 ± 2.0	3	2

Subroutine HBCOM, Figure 4, consolidates the fits into a subroutine that facilitates plotting the data. HBCOM contains all of the information found in these curve fits for a Shellsort using Hibbard's increments.

Figure 5 through Figure 17 (Appendix D) illustrate the importance of the gamma density function and the Gaussian function. In these figures, the linear part of the fitted curve (FLIN in HBCOM) is subtracted from the curve or data point, leaving the gamma density function or the Gaussian function. The vertical bars indicate one standard error in the data points. The curves are the values returned by

```
      SUBROUTINE HBCOM (NPASS,AL2N,AL2CM,FLIN)
C
C  COMPUTES FITTED VALUES FOR PLOTTING LG(COMPARISONS) VS.
C  LG(N) FOR SHELLSORT USING HIBBARD-S INCREMENTS.
C  NPASS IS THE NUMBER OF PASSES  AL2N IS LG(N).
C  AL2CM IS LG(COMPARISONS).  FLIN IS THE ASYMPTOTE.
C
      DIMENSION JTYPF(13),APAR(5,13)
      DATA JTYPF/4*0,9*1/, APAR/
     1      -2.     , .28704, 2.2983, 1.3135, 1.     ,
     2      -3.5610, 1.9265, 2.0476, 1.9798, -.27668,
     3      -4.7741, 3.0804, 2.3694, 2.6064, -3.6736,
     4      -5.8657, 3.9366, 2.7514, 3.3492, -17.226,
     5      -6.9026, 4.7756, 2.9164, 4.1637, 0.,
     6      -7.9730, 5.4376, 3.5495, 4.4890, 0.,
     7      -9.1144, 6.2637, 3.9031, 5.0108, 0.,
     8      -10.392, 7.3219, 4.0117, 5.7516, 0.,
     9      -11.800, 8.4507, 4.1543, 6.5578, 0.,
     X      -13.018, 9.3787, 4.3508, 7.2045, 0.,
     E      -13.399, 8.8791, 5.6840, 6.7422, 0.,
     T      -14.027, 8.8873, 6.4344, 6.7861, 0.,
     3      -14.5   , 9.1628, 6.6096, 7.0    , 0.
C
      QEXP(ARG)=EXP(ARG)
      QLOG(ARG)=ALOG(ARG)
      QMAX1(A,B)=AMAX1(A,B)
      QMIN1(A,B)=AMIN1(A,B)
C
      IF(NPASS.LT.1 .OR. NPASS.GT.13) STOP
      EXPLM=100.
      UNITR=1
      RTWO=2
C                    COMPUTE THE LINEAR PART OF THE FIT.
      FLIN=APAR(1,NPASS)+RTWO*AL2N
      IF(JTYPF(NPASS).GT.0) GO TO 30
C
C                    USE A LINE PLUS A GAMMA DENSITY.
C                    THE GAMMA DENSITY HAS BEEN
C                    REPARAMETERIZED IN ORDER TO
C                    DECREASE THE CORRELATION
C                    BETWEEN THE COEFFICIENTS.
   10 THAT=APAR(3,NPASS)-APAR(5,NPASS)
      CGAM=-THAT/APAR(4,NPASS)**2
      BGAM=-CGAM*THAT
      ALAGM=QLOG(APAR(2,NPASS))
      TT=AL2N-APAR(5,NPASS)
      FEXP=0.
      IF(TT.LE.0.) GO TO 20
      POWLN=ALAGM+BGAM*(UNITR+QLOG(TT/THAT))+CGAM*TT
      POWLN=QMAX1(-EXPLM,QMIN1(EXPLM,POWLN))
      FEXP=QEXP(POWLN)
   20 AL2CM=FLIN+FEXP
```

Figure 4. Subroutine HBCOM

```
      RETURN
C                   USE A LINE PLUS A GAUSSIAN.
   30 ARG=-(AL2N-APAR(3,NPASS))**2/(RTWO*APAR(4,NPASS)**2)
      ARG=QMAX1(-EXPLM,QMIN1(EXPLM,ARG))
      AL2CM=FLIN+APAR(2,NPASS)*QEXP(ARG)
      RETURN
      END
```

Figure 4. (Continued)

HBCOM, minus the linear terms (FLIN). The curves are asymptotic to zero, indicating that the raw data are asymptotically proportional to N**2.

Figure 18 through 20 (Appendix D) present another view of the data. Figure 18 is a plot of the output from HBCOM. The family of similiar curves represents a one-pass Shellsort, two-pass Shellsort, etc, from left to right.

Figure 19 (Appendix D) is simply Figure 18 with 1.25*(lg(N)-1.0) subtracted from it. This roughly corresponds to dividing the raw data by N**1.25. Note that there is a lower "envelope" which consists of segments of the passes that give the smallest number of comparisons. One would want to adjust the number of Hibbard's increments to follow the envelope in order to use the minimum number of comparisons. It is important to note that the envelope has a negative slope for large values of N, which indicates that the envelope does not increase as fast as N**1.25, as N goes to infinity. The lower curve is a plot of N*lg(N), which is the order of the average running time of quicksort and merge sort. The upper curve is the .315*N*(lg(N)**2) behavior of Shellsort using Pratt's increments, which are discussed in Chapter III. The envelope is similiar in shape to Pratt's curve, but there is no evidence that they are the same function. The only thing certain from the plots is that if the behavior of Shellsort is asymptotically a power function, then the power is less than 1.25.

Figure 20 (Appendix D) is an enlargement of Figure 19. It clearly shows some of the effects of the gamma density function or Gaussian function. As the number of passes becomes large, the left end of each curve should just touch the curve of the previous number of passes because the introduction of the next pass can contribute at most one more comparison, since the left end of each curve corresponds to the point at which H(T) is equal to N-1. The curves in Figure 20 do not exactly touch with the previous curves, and presumably this is due to statistical sampling error in the data or to the number of passes not yet being large enough.

Determination of Crossover Values

Examination of Table VI shows that some of the fits are not very good. A six-pass Shellsort has a chi-square of 151.0 (expected 43.0) with only 13 runs (expected 24). In order to improve the accuracy of the crossover values, a cubic polynomial was fitted to the log-log plot of the raw data only in the vicinity of the crossover points. The results are summarized in Table VII. All of these fits are very good; the number of runs is as large as expected.

The ratios in Table VII are the initial increment of the sequence, H(T), divided by the corresponding crossover value. This indicates at what fraction of N another pass is justified. The ratios using Knuth's increments are so rough that there is no really smooth pattern in the crossover

TABLE VII

CROSSOVER VALUES USING HIBBARD'S AND KNUTH'S INCREMENTS

Passes (A-B)	Comparisons Crossover Value	Comparisons Crossover Ratio	MIX Time Crossover Value	MIX Time Crossover Ratio	Range of N Pass A	Range of N Pass B
			Hibbard's Increments			
1-2	7	.46	11	.29	3-20	4-20
2-3	16	.45	23	.32	4-25	8-28
3-4	44	.34	63	.24	8-100	16-100
4-5	89	.35	123	.25	16-2000	33-2000
5-6	207	.31	271	.23	33-5000	70-5000
6-7	431	.29	566	.22	70-10000	130-10000
7-8	852	.30	1112	.23	130-20000	280-20000
8-9	1667	.31	2164	.24	280-20000	550-20000
9-10	3197	.32	4283	.24	550-20000	1100-20000
10-11	5663	.36	7653	.27	1100-20000	2200-20000
11-12	14713	.28	18471	.22	2200-20000	4192-20000
12-13	19868	.41	>20000	<.41	4192-20000	9000-20000
			Knuth's Increments			
1-2	6	.75	10	.40	3-20	5-20
2-3	14	1.0	16	.81	5-28	14-28
3-4	51	.79	77	.52	26-200	44-200
4-5	198	.61	264	.46	44-660	122-660
5-6	510	.71	762	.48	122-1200	365-1200
6-7	1342	.81	1782	.61	365-7000	1094-7000
7-8	3561	.92	3281	1.0	1093-20000	3281-20000
8-9	19992	.49	>20000	<0.5	3281-20000	9042-20000

values as a function of N. It seems reasonable to estimate the crossover values to be approximately 0.75*N. The ratios using Hibbard's increments are much smoother, and it is these points that will be fitted in the next section. One might estimate that these crossover values are approximately equal to 0.32*N, for N between 45 and 15000.

The log of the total running time as calculated using Knuth's MIX program (9B + 10NT - 13T - 10S - 3A + 1) was also fitted to a cubic polynomial. This is done in order to compare the use of comparisons verses total MIX execution time as a means of judging the efficiency of Shellsort. Again, data when Knuth's increments are used are somewhat ragged, but the crossover value can be estimated to be approximately 0.50*N, for N between 50 and 1500. Results from Hibbard's increments show the crossover value to be about 0.24*N.

Fitting the Crossover Values Verses N

The number of comparisons is not a smooth function N, since it is composed of segments of the curves for various numbers of passes (see Figure 20). The crossover values themselves, however, may conceivably lie on a smooth curve. Knowledge of this curve might facilitate calculation of the optimum initial increment in a production program. Since Knuth's crossover values are few and are rather ragged, Hibbard's crossover values are used, as shown in Table VIII.

TABLE VIII

CROSSOVER POINTS USING HIBBARD'S INCREMENTS

Passes	Lg(N)	Y or Lg(Comparisons)	YSIG
1-2	2.689	3.654	.040
2-3	3.963	5.781	.022
3-4	5.476	7.984	.010
4-5	6.477	9.358	.008
5-6	7.687	10.947	.006
6-7	8.755	12.312	.006
7-8	9.733	13.537	.006
8-9	10.703	14.738	.006
9-10	11.642	15.887	.006
10-11	12.467	16.888	.006
11-12	13.845	18.558	.006

The crossover point from 12 to 13 passes is not used since it is ill-determined due to the lack of data with N greater than 20000. The estimated standard error, YSIG, in the lg(comparisons) is taken to be the larger of the errors in the lg(comparisons) of the data for the two curves near the crossover value. This probably overestimates YSIG somewhat, as it ignores the smoothing effect of the cubic fits. The errors in successive lg(comparisons) values are probably correlated to some extent, because they have one cubic fit in common. The first few points have a larger value of YSIG than do the other points. The weights in the fits are YSIG**(-2), so the first points are weighted considerably less than the rest of the points. For example, the first point was weighted (0.006/0.040)**2 = 0.0225 times as much as the last point, etc.

The data in Table VIII were fitted to a straight line. This corresponds to a power function in N:

comparisons = C * (N ** P)

or

lg(comparisons) = lg(C) + P * lg(N)

The fit is unacceptable; chi-square is 1745 (expected 9). The fitted value of P is 1.26 plus or minus 0.01 (one standard error).

Peterson and Russell (7) chose to eliminate from consideration data points with N less than 100. However, Figure 22 (Appendix D) shows that the envelope is nonlinear throughout the entire range, so all of the data were fitted to the sum of a straight line plus an exponential in lg(N). This gives an excellent fit: chi-square is 1.4 for 7 degrees of freedom, and there are 7 runs through 11 data points. The slope (P) of the straight line is 1.196 plus or minus 0.005 (one standard error). This is the asymptotic value of the power (P) in the power function.

For comparison, another function was fitted that is asymptotically linear in lg(N), namely the sum of a straight line and a decreasing power function in lg(N). This gives a slightly better fit: a chi-square of 0.9 for 6 degrees of freedom, and 8 runs through 11 data points. The asymptotic power is 1.178 plus or minus 0.03 (one standard error). In view of these two fits, the value of P could be estimated to be 1.195 plus or minus 0.005 (one standard error). This is less than previous estimates by Shell (10), and by Peterson and Russell (7), but agrees with Figure 22 (Appendix D) which shows that P must be less than 1.25.

A complexity curve of the form:

lg(C * N**A * lg(N)**B)

was also fitted to the data in Table VIII. This gives only a mediocre fit: chi-square has a value of 14 for 8 degrees of freedom, and 4 runs among 11 data points. (Recall that the YSIG are somewhat overestimated, so the chi-square is underestimated.) A has a value of 1.062 plus or minus 0.006 (one standard error), while B has a value of 1.25 plus or minus 0.04 (one standard error).

An attempt was made to fit the data in Table VIII to the form:

lg(C * N**A * lg(N)**E + D * N**E),

but the fit is so ill-conditioned that all of the errors in the parameters exceed one hundred percent, so this fit was discarded.

It can be concluded that if the average case of Shellsort has a time complexity that is asymptotically a power of N, then the power must be less than previously estimated, probably not exceeding 1.20. A complexity of the form O(C * N**A * lg(N)**B) cannot be ruled out, but the fit is not very good.

One must be careful in implementing some type of function that calculates a bound on the initial increment.

If a complicated function is used, the time to calculate the bound may become comparable to the time needed to sort a short list. Since the running times for the Shellsort are not highly sensitive to the initial increment, only an approximate figure is needed. In view of this, a function of the form C * N is probably best used as a bound for the initial increment, as suggested by Knuth. It seems reasonable that the initial increment should not exceed 0.24*N using Hibbard's increments, nor should it exceed 0.50*N using Knuth's increments. Program D (Figure 5) takes advantage of the calculation of this approximately optimum initial increment. This subroutine avoids wasting time on the unconditional saves in Program B. It uses save operations instead of pairwise interchanges. Thus it is the most efficienct version of Shellsort of which we are presently aware, for reasonable values of N.

```
      SUBROUTINE SHELL(JR,N)
      DIMENSION JR(100)
C
C                SHELLSORT
C
C                INPUT PARAMETERS
C                   JR    INTEGER ARRAY TC BE SORTED
C                   N     NUMBER OF ELEMENTS IN ARRAY JR
C
C                RETURNS ARRAY JR IN ASCENDING ORDER.
C
C                HIBBARD-S INCREMENTS ARE IMPLEMENTED BUT MAY
C                MODIFIED TO USE KNUTH-S INCREMENTS
C
      IF(N.LT.2) RETURN
      JH=1
C                REPLACE .24 BY .50 IF KNUTH-S
C                INCREMENTS ARE USED
      JHMAX=.24*FLOAT(N)
   10 IF(JH.GE.JHMAX) GO TO 20
C
C                REPLACE 2*JH BY (3*JH)-1 IF KNUTH-S
C                INCREMENTS ARE USED
      JH=2*JH
      GO TO 10
C                REPLACE (JH-1)/2 BY (JH-1)/3 IF KNUTH-S
C                INCREMENTS ARE USED
   20 JH=(JH-1)/2
   30 NMJH=N-JH
      DO 60 K=1,NMJH
        JHPK=JH+K
        IF(JR(JHPK).GT.JR(K)) GO TO 60
        JTEMP=JR(JHPK)
        JR(JHPK)=JR(K)
        J=K-JH
        IF(J.LE.0) GO TO 50
   40   IF(JTEMP.GT.JR(J)) GO TO 50
        JHPJ=JH+J
        JR(JHPJ)=JR(J)
        J=J-JH
        IF(J.GT.0) GO TO 40
   50   JHPJ=JH+J
        JR(JHPJ)=JTEMP
   60 CONTINUE
C                REPLACE JH/2 BY JH/3 IF KNUTH-S
C                INCREMENTS ARE USED
      JH=JH/2
      IF(JH.GE.1) GO TO 30
      RETURN
      END
```

Figure 5. Program D

CHAPTER III

PRATT'S VERSION OF SHELLSORT

Pratt's Algorithm

Pratt (9) has improved the running time of the Shellsort to O(N*(lg(N)**2)) by requiring the increments to be all of the integers of the form (2**P) * (3**Q) that are less than N. Program E (Figure 6) illustrates this algorithm.

```
for i := 2 ** floor(lg(n-1)), i/2 while i => 1 do
for j := i, (3*j)/2 while j mod 3 = 0 and j < n do
for k := 1 step 1 until j do
for l := k step j until n-j do
    if A[l] > A[l+j] then begin swap (A[l],A[l+j]);
                                l := l+j
                          end
```

Figure 6. Program E

Program E was implemented in Fortran; results are given in Appendix C. The upper curve in Figure 19 (Appendix D) is for Pratt's algorithm.

The increments have been chosen by Pratt in such a way that an out-of-order comparison results in exactly one move, so the interchange method is efficient in moving an out-of-order element to its correct position.

Running Time of Pratt's Algorithm

Pratt reported that the best, worst, and average number of comparisons of Program E is 0.315*N*(lg(N)**2), which is asymptotically better than any power function of N. The major drawback in using Pratt's sequence is that N has to be extremely large before Pratt's Shellsort is more efficient than other sequences. If 1.0*N**1.2 is taken to be the average number of comparisons of Shellsort, then N has to be greater than about 10**18 in order for Pratt's sequence to be faster than using other sequences. Although Pratt's sequnce is interesting from a theoretical standpoint, it is not very useful in actual implementation. The algorithm simply executes too many comparisons (see Appendix C) in relation to other sequences to be of practical use. This was previously indicated by Pratt, but it is presented here to estimate the point at which Pratt's sequence would become more efficient than the conventional Shellsorts.

CHAPTER IV

WORST CASE PERMUTATION OF THE ORIGINAL SHELLSORT

Introduction

It is useful to know the worst case execution time of an algorithm. Papernov and Stasevich (8) have shown that the the worst execution time for Shellsort using Hibbard's increments is O(N**1.5), but the permutation of the list that produces this worst case execution time apparently is not known. Frank and Lazaraus (4) have indicated that the worst execution time for Shellsort using Shell's increments is O(N**2). This occurs when N is some power of two because in that case there is little interaction between groups in the intermediate passes.

Proof of Worst Case Permutation

In order to find the permutation of the list that gives the maximum number of comparisons, the problem is limited to using Shell's original increments, and N must be a power of two. Knuth (7) provided some background information that will be useful in this proof. (In particular, see pages 86-88 and the answer to Exercise 16, Section 5.2.1). He demonstrated an analogy between the "lattice diagram" in

Figure 7 and a two-ordered list. A two-ordered list is a list in which R(J) is not greater than R(J+2). A two-ordered list can be represented as a path from the upper left corner point (0,0) to the lower right corner point (ceil(N/2),floor(N/2)). Each Kth step goes downwards if K is in an odd position in the list and goes to the right if K is in an even position. For example, the list:

2 1 3 4 6 5 7 10 8 11 9 12 14 13 15

is represented by the line made of the symbols (-┐|└) in Figure 7. The line of circles (o) corresponds to a completely sorted list. Figure 7 shows a natural connection between paths and inversions, namely that the area between the dotted path and the given path is equal to the number of inversions in the given permutation. Thus the above list has six inversions.

In order to find the worst case permutation of a list, all of the permutations were sorted with N equal to 2, 4, and 8. The following lists were found to be the worst case permutations:

Sequence	Comparisons	Interchanges
2 1	1	1
4 2 3 1	7	5
8 4 6 2 7 3 5 1	30	20

Careful examination of these permutations reveals a pattern which can be proved by induction. The first permutation is obviously the worst case for N equal to 2, since the only other permutation is in-order. Now, let the elements of a worst case permutation of length N be R(1),

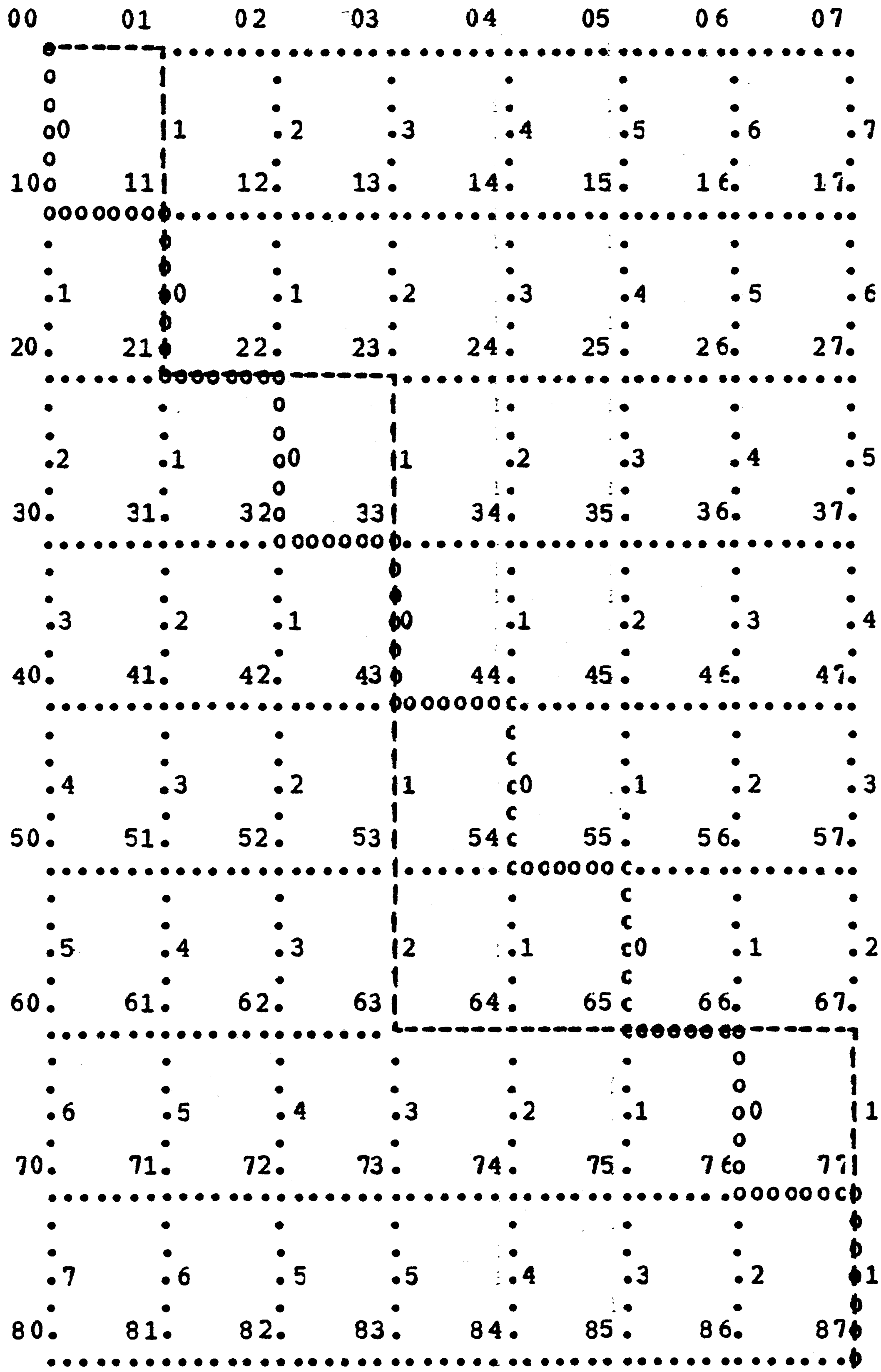


Figure 7. Correspondence Between Two-ordering and Paths in a Lattice

R(2), ..., R(N). The worst case permutation of length 2*N is found by intercalating each element, R(J), with R(J)+N on the left, resulting in:

R(1)+N, R(1), R(2)+N, R(2), R(3)+N, R(3), ..., R(N)+N, R(N)

As an example suppose the worst case for N equal to 4 is:

4 2 3 1

The worst case permutation for the next value of N has a length of 8 (2*N). This can be found by first adding N to each element:

4+4=8 2+4=6 3+4=7 1+4=5

Intercalating the above list yields:

8 4 6 2 7 3 5 1

The worst case permutation can also be generated by another method. If a list of length N is known to be the worst case permutation, then the worst case permutation of length 2*N can be generated by multiplying each element by 2, then concatenating this list on the right with itself minus 1. Thus the new list is

2*R || 2*R-1

where "||" symbolizes concatenation. This may be illustrated using the worst case permutation of length 4:

4 2 3 1

Multiply each element by 2:

8 4 6 2

Concatenate this list with itself minus 1:

8 4 6 2 7 3 5 1

This method lends itself to implementing a program that generates the worst case permutation, but it probably does not have any theoretical implications.

It must now be proved that this is the worst case for 2*N elements. When the list is sorted using Shell's increments, a two-ordered list results just before the last pass is begun. In other words, all of the odd-positioned elements have been sorted and all of the even-positioned elements have been sorted. It is known that this is the worst case except possibly on the last pass because all of the even-positioned elements were from the previous worst case permutation, and all of the odd-positioned elements have the same relative order as the even-positioned elements.

Note that on the last pass, the odd-positioned elements consist of the values 1 to N/2, while the even-positioned elements consist of the values (N/2)+1 to N (by construction). A path can now be traced in the lattice diagram (Figure 7), since this is a two-ordered list on the last pass. The first N/2 values are in odd positions, so N/2 steps are taken to the right (upper right corner). The last N/2 values are in even positions, so N/2 steps are taken down (lower right corner). Obviously this maximizes the distance between the steps of the permutation and the staircase line of circles (an in-order list), which indicates that the last pass has to sort the maximum number of inversions. Thus the entire permutation must have been the worst case for 2*N elements. This completes the proof of the worst case in the original Shellsort.

When Shell's increments are used on 2**K elements, the following numbers of operations occur in the worst case:

comparisons = (1/4)*N**2 + (5/4)*N*lg(N) - (9/4)*N + 2

moves or interchanges = (1/4)*N**2 + (1/4)*N*lg(N) - (1/4)*N

save/insertions = (1/2)*N*lg(N)

It is fortunate that the worst case of Shellsort is so complicated that it seldomly occurs. The worst case permutation for a "bubble" sort or an insertion sort is the reverse order list, which is probably more likely to happen than the worst case permutation for Shellsort. The worst case for the bubble sort or insertion sort is (1/2)*N**2 comparisons, while the leading term in the Shellsort is (1/4)*N**2 comparisons. Thus the original Shellsort is still better than the bubble sort or insertion sort in the worst case, and its worst case is much less likely to occur. And of course the average case for the original Shellsort is very much better than the average case for the bubble sort or insertion sort.

CHAPTER V

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

Summary and Conclusions

Program D (Figure 5) is one of the better methods of coding Shellsort. It roughly estimates the optimum number of increments for a given value of N for the families of increments 2**K-1 and (3**K-1)/2, and takes advantage of the natural partial order already in the list during the final passes by avoiding "unconditional save" operations. Empirically, it appears that the initial increment should not exceed about 0.24*N when Hibbard's increments are used and should not exceed about 0.50*N when Knuth's increments are used. The increase in speed by using the optimum initial increment is not very great, but the small improvement and its ease of calculation justify its use.

Though the exact form of the asymptotic average behavior of Shellsort is not known, evidence indicates that the asymptotic average behavior grows more slowly than N**1.2. In fact, the asymptotic average behavior may not be a power function at all, but this question cannot be resolved from our results. The results were obtained using sound numerical and statistical techniques, which lends credence to the new results, and may serve as a basis in future work.

Unconditional saves should be avoided when coding Shellsort. This can save as much as 9% in execution time.

The worst case list for the original Shellsort has been derived. It is quite complicated, so the chances of it occuring are quite small.

Suggestions for Future Work

There are many suggestions for future research on the Shellsort:

1. What is the order of the asymptoptic average behavior of the Shellsort?
2. What are the optimum increments for Shellsort? Perhaps the optimum increments could be found for some very small values of N, then be generalized for large N. Each value of N may have its own optimum set of increments, but some pattern might be discerned.
3. Can Pratt's increments be modified to make his version of the Shellsort useful?
4. What is the generalized worst case list for Shellsort, for other sets of increments and any value of N?

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APPENDIX A

EMPIRICAL RESULTS OF SHELLSORT USING HIBBARD'S INCREMENTS

Fifty repetitions were made using Program C with Hibbard's increments.

N	Pass	Saves	Comparisons	Moves	Type
2	1	0.4199994E+00	0.1000000E+01	0.4199994E+00	AVE
2	1	0.4985690E+00	0.0000000E+00	0.4985690E+00	SD
3	1	0.1199990E+01	0.2639987E+01	0.1479988E+01	AVE
3	1	0.6700571E+00	0.4848724E+00	0.9089102E+00	SD
4	1	0.1759990E+01	0.4839988E+01	0.2759989E+01	AVE
4	1	0.7969308E+00	0.8417650E+00	0.1333401E+01	SD
5	1	0.2739988E+01	0.7519989E+01	0.4839988E+01	AVE
5	1	0.9216196E+00	0.1554976E+01	0.2279990E+01	SD
6	1	0.3479988E+01	0.1105999E+02	0.7399987E+01	AVE
6	1	0.9310931E+00	0.2170393E+01	0.2595122E+01	SD
7	1	0.4399987E+01	0.1505999E+02	0.1085999E+02	AVE
7	1	0.1142856E+01	0.2965163E+01	0.3625353E+01	SD
8	1	0.5359990E+01	0.1965982E+02	0.1443999E+02	AVE
8	1	0.1064450E+01	0.3611819E+01	0.4329102E+01	SD
9	1	0.6299990E+01	0.2351982E+02	0.1737985E+02	AVE
9	1	0.1073806E+01	0.4011436E+01	0.4421150E+01	SD
10	1	0.6759991E+01	0.2805981E+02	0.2085980E+02	AVE
10	1	0.1318004E+01	0.5467562E+01	0.6114579E+01	SD
12	1	0.9199990E+01	0.4345985E+02	0.3449982E+02	AVE
12	1	0.1178029E+01	0.8172123E+01	0.8795261E+01	SD
14	1	0.1065999E+02	0.5641982E+02	0.4521982E+02	AVE
14	1	0.1117757E+01	0.9333018E+01	0.9621984E+01	SD
16	1	0.1259999E+02	0.7251984E+02	0.5959982E+02	AVE
16	1	0.1160573E+01	0.9474418E+01	0.9893289E+01	SD
18	1	0.1429999E+02	0.8763985E+02	0.7287982E+02	AVE
18	1	0.1488047E+01	0.1063062E+02	0.1114400E+02	SD
20	1	0.1633986E+02	0.1108399E+03	0.9447980E+02	AVE
20	1	0.1437255E+01	0.1776300E+02	0.1817177E+02	SD
24	1	0.1989983E+02	0.1570198E+03	0.1370398E+03	AVE
24	1	0.1343920E+01	0.1807292E+02	0.1835359E+02	SD
28	1	0.2395981E+02	0.2169598E+03	0.1927198E+03	AVE
28	1	0.1470232E+01	0.1922469E+02	0.1964896E+02	SD
33	1	0.2879980E+02	0.2959568E+03	0.2670574E+03	AVE
33	1	0.1784298E+01	0.3240022E+02	0.3302872E+02	SD
38	1	0.3361983E+02	0.3874573E+03	0.3538374E+03	AVE
38	1	0.1839349E+01	0.3424690E+02	0.3467635E+02	SD
44	1	0.3951982E+02	0.5237371E+03	0.4840576E+03	AVE
44	1	0.1656636E+01	0.4507510E+02	0.4496933E+02	SD
50	1	0.4555984E+02	0.6589568E+03	0.6133374E+03	AVE
50	1	0.1692237E+01	0.5538014E+02	0.5564510E+02	SD
60	1	0.5535986E+02	0.9316567E+03	0.8762170E+03	AVE
60	1	0.1711392E+01	0.7681360E+02	0.7731062E+02	SD
70	1	0.6513983E+02	0.1278398E+04	0.1213198E+04	AVE
70	1	0.1628961E+01	0.9092099E+02	0.9147310E+02	SD
80	1	0.7539981E+02	0.1661177E+04	0.1586117E+04	AVE

80	1	0.1840561E+01	0.1228985E+03	0.1233560E+03	SD
90	1	0.8523982E+02	0.2119637E+04	0.2035357E+04	AVE
90	1	0.1954165E+01	0.1343805E+03	0.1348322E+03	SD
100	1	0.9447978E+02	0.2555277E+04	0.2460298E+04	AVE
100	1	0.2251874E+01	0.1751108E+03	0.1752604E+03	SD
120	1	0.1147598E+03	0.3719997E+04	0.3605537E+04	AVE
120	1	0.1732765E+01	0.2113506E+03	0.2116170E+03	SD
130	1	0.1243198E+03	0.4293379E+04	0.4168754E+04	AVE
130	1	0.2094087E+01	0.2836257E+03	0.2836260E+03	SD
140	1	0.1345798E+03	0.5012266E+04	0.4878234E+04	AVE
140	1	0.1917612E+01	0.2841489E+03	0.2840139E+03	SD
160	1	0.1544798E+03	0.6642398E+04	0.6488453E+04	AVE
160	1	0.2296773E+01	0.3636174E+03	0.3636604E+03	SD
180	1	0.1743398E+03	0.8179414E+04	0.8005141E+04	AVE
180	1	0.1813867E+01	0.4150212E+03	0.4148611E+03	SD
200	1	0.1945798E+03	0.1022669E+05	0.1003261E+05	AVE
200	1	0.2259582E+01	0.4721633E+03	0.4728093E+03	SD
300	1	0.2936172E+03	0.2276500E+05	0.2247103E+05	AVE
300	1	0.2293550E+01	0.6890349E+03	0.6893486E+03	SD
500	1	0.4930574E+03	0.6284736E+05	0.6235398E+05	AVE
500	1	0.2368553E+01	0.2124254E+04	0.2124976E+04	SD
1000	1	0.9923174E+03	0.2504056E+06	0.2494132E+06	AVE
1000	1	0.2795255E+01	0.5053590E+04	0.5054285E+04	SD
2000	1	0.1991817E+04	0.1001660E+07	0.9996677E+06	AVE
2000	1	0.2413516E+01	0.1582999E+05	0.1583023E+05	SD
4	2	0.1859989E+01	0.5199990E+01	0.2299989E+01	AVE
4	2	0.8083711E+00	0.8571416E+00	0.1073804E+01	SD
5	2	0.3039989E+01	0.7879991E+01	0.3719990E+01	AVE
5	2	0.1105826E+01	0.9178467E+00	0.1385637E+01	SD
6	2	0.4239989E+01	0.1133999E+02	0.5699988E+01	AVE
6	2	0.1238166E+01	0.1598594E+01	0.2296845E+01	SD
7	2	0.5139989E+01	0.1443999E+02	0.7079988E+01	AVE
7	2	0.1277909E+01	0.1579974E+01	0.1893487E+01	SD
8	2	0.6339987E+01	0.1839983E+02	0.9259991E+01	AVE
8	2	0.1437256E+01	0.2089802E+01	0.2663501E+01	SD
9	2	0.7799989E+01	0.2221983E+02	0.1153999E+02	AVE
9	2	0.1603566E+01	0.2971204E+01	0.3465326E+01	SD
10	2	0.9099988E+01	0.2663980E+02	0.1419999E+02	AVE
10	2	0.1693322E+01	0.2833892E+01	0.3428556E+01	SD
12	2	0.1167999E+02	0.3617982E+02	0.2031984E+02	AVE
12	2	0.1659109E+01	0.3690525E+01	0.4259142E+01	SD
14	2	0.1393999E+02	0.4567982E+02	0.2575980E+02	AVE
14	2	0.1983706E+01	0.5441477E+01	0.5694810E+01	SD
16	2	0.1711980E+02	0.5683983E+02	0.3365984E+02	AVE
16	2	0.1923407E+01	0.5665350E+01	0.6133226E+01	SD
18	2	0.1957980E+02	0.6907982E+02	0.4239980E+02	AVE
18	2	0.2268587E+01	0.8098464E+01	0.8043231E+01	SD
20	2	0.2281984E+02	0.8259984E+02	0.5197983E+02	AVE
20	2	0.1996820E+01	0.8312736E+01	0.9175114E+01	SD
24	2	0.2827982E+02	0.1118998E+03	0.7373984E+02	AVE
24	2	0.2303843E+01	0.1191504E+02	0.1256618E+02	SD
28	2	0.3465981E+02	0.1445999E+03	0.9859982E+02	AVE
28	2	0.2715115E+01	0.1543779E+02	0.1528368E+02	SD
33	2	0.4213983E+02	0.1904998E+03	0.1354598E+03	AVE

33	2	0.2814003E+01	0.2268343E+02	0.2328452E+02	SD
38	2	0.5011980E+02	0.2332198E+03	0.1685998E+03	AVE
38	2	0.2745261E+01	0.2564377E+02	0.2657141E+02	SD
44	2	0.5901981E+02	0.3046777E+03	0.2280398E+03	AVE
44	2	0.2691546E+01	0.2918434E+02	0.2964821E+02	SD
50	2	0.6815981E+02	0.3799773E+03	0.2924170E+03	AVE
50	2	0.2675370E+01	0.3158902E+02	0.3211038E+02	SD
60	2	0.8375983E+02	0.5202971E+03	0.4126367E+03	AVE
60	2	0.3684276E+01	0.4486119E+02	0.4553294E+02	SD
70	2	0.9943982E+02	0.6670371E+03	0.5399375E+03	AVE
70	2	0.3405645E+01	0.5787706E+02	0.5803494E+02	SD
80	2	0.1142998E+03	0.8382974E+03	0.6916174E+03	AVE
80	2	0.4828697E+01	0.6800133E+02	0.6849661E+02	SD
90	2	0.1322198E+03	0.1041358E+04	0.8744778E+03	AVE
90	2	0.4102206E+01	0.8476134E+02	0.8558119E+02	SD
100	2	0.1478998E+03	0.1278857E+04	0.1093037E+04	AVE
100	2	0.4141627E+01	0.8658658E+02	0.8744611E+02	SD
120	2	0.1793798E+03	0.1694877E+04	0.1469177E+04	AVE
120	2	0.4948258E+01	0.1137198E+03	0.1138994E+03	SD
130	2	0.1958798E+03	0.1978597E+04	0.1733178E+04	AVE
130	2	0.5445981E+01	0.1335385E+03	0.1336878E+03	SD
140	2	0.2101798E+03	0.2302377E+04	0.2037717E+04	AVE
140	2	0.5192955E+01	0.1465906E+03	0.1462437E+03	SD
160	2	0.2431798E+03	0.2905197E+04	0.2600556E+04	AVE
160	2	0.5181156E+01	0.1779044E+03	0.1775095E+03	SD
180	2	0.2768372E+03	0.3621737E+04	0.3276717E+04	AVE
180	2	0.5429821E+01	0.2536463E+03	0.2551374E+03	SD
200	2	0.3088572E+03	0.4424219E+04	0.4040617E+04	AVE
200	2	0.5287369E+01	0.2472826E+03	0.2473683E+03	SD
240	2	0.3740371E+03	0.6246215E+04	0.5783012E+04	AVE
240	2	0.5844847E+01	0.3801018E+03	0.3809622E+03	SD
280	2	0.4397981E+03	0.8177113E+04	0.7634180E+04	AVE
280	2	0.5979604E+01	0.4331826E+03	0.4328247E+03	SD
300	2	0.4711768E+03	0.9281461E+04	0.8698574E+04	AVE
300	2	0.6447875E+01	0.5000127E+03	0.5008972E+03	SD
500	2	0.8001973E+03	0.2457464E+05	0.2359399E+05	AVE
500	2	0.8668248E+01	0.1164571E+04	0.1164098E+04	SD
1000	2	0.1627077E+04	0.9390569E+05	0.9192650E+05	AVE
1000	2	0.1083401E+02	0.2851168E+04	0.2852372E+04	SD
2000	2	0.3279637E+04	0.3598443E+06	0.3558668E+06	AVE
2000	2	0.1300047E+02	0.8400449E+04	0.8400602E+04	SD
5000	2	0.8254055E+04	0.2183616E+07	0.2173644E+07	AVE
5000	2	0.2445987E+02	0.3068503E+05	0.3068600E+05	SD
8	3	0.6299988E+01	0.1855980E+02	0.8499988E+01	AVE
8	3	0.1555107E+01	0.1842358E+01	0.2349551E+01	SD
9	3	0.7359989E+01	0.2297984E+02	0.9979989E+01	AVE
9	3	0.1735110E+01	0.2015137E+01	0.2630195E+01	SD
10	3	0.8699986E+01	0.2729980E+02	0.1177999E+02	AVE
10	3	0.1775684E+01	0.2082476E+01	0.2443563E+01	SD
12	3	0.1247999E+02	0.3689984E+02	0.1715981E+02	AVE
12	3	0.1809720E+01	0.3271681E+01	0.3627301E+01	SD
14	3	0.1531999E+02	0.4693982E+02	0.2257983E+02	AVE
14	3	0.2590714E+01	0.3644468E+01	0.4338210E+01	SD
16	3	0.1823981E+02	0.5637981E+02	0.2635983E+02	AVE

16	3	0.2395218E+01	0.3816662E+01	0.4374133E+01	SD
18	3	0.2143982E+02	0.6739983E+02	0.3181984E+02	AVE
18	3	0.2900773E+01	0.4894796E+01	0.5366877E+01	SD
20	3	0.2561984E+02	0.7969984E+02	0.3917986E+02	AVE
20	3	0.2633271E+01	0.5059816E+01	0.5564768E+01	SD
24	3	0.3183984E+02	0.1036998E+03	0.5189980E+02	AVE
24	3	0.3781560E+01	0.6975882E+01	0.8051473E+01	SD
28	3	0.4011981E+02	0.1299598E+03	0.6729985E+02	AVE
28	3	0.3360244E+01	0.7811427E+01	0.8403703E+01	SD
33	3	0.4901982E+02	0.1652198E+03	0.8861981E+02	AVE
33	3	0.3656069E+01	0.8999513E+01	0.9400141E+01	SD
38	3	0.5803979E+02	0.1999398E+03	0.1091398E+03	AVE
38	3	0.4772173E+01	0.1269479E+02	0.1398536E+02	SD
44	3	0.7003979E+02	0.2461598E+03	0.1385998E+03	AVE
44	3	0.4948548E+01	0.1178780E+02	0.1254375E+02	SD
50	3	0.8279980E+02	0.3019773E+03	0.1776998E+03	AVE
50	3	0.5742766E+01	0.1449498E+02	0.1594280E+02	SD
60	3	0.1021799E+03	0.3948777E+03	0.2411398E+03	AVE
60	3	0.5822858E+01	0.1872986E+02	0.1933589E+02	SD
70	3	0.1241398E+03	0.4961975E+03	0.3145974E+03	AVE
70	3	0.7171404E+01	0.2833202E+02	0.2954089E+02	SD
80	3	0.1435398E+03	0.6039172E+03	0.3917371E+03	AVE
80	3	0.5682802E+01	0.2309767E+02	0.2486598E+02	SD
90	3	0.1646198E+03	0.7254968E+03	0.4839570E+03	AVE
90	3	0.7526491E+01	0.3496999E+02	0.3593231E+02	SD
100	3	0.1870998E+03	0.8570574E+03	0.5875371E+03	AVE
100	3	0.6188375E+01	0.3448038E+02	0.3600610E+02	SD
120	3	0.2299198E+03	0.1123217E+04	0.7948372E+03	AVE
120	3	0.8002132E+01	0.5368874E+02	0.5456841E+02	SD
130	3	0.2517798E+03	0.1284757E+04	0.9260774E+03	AVE
130	3	0.8671505E+01	0.5746851E+02	0.5850824E+02	SD
140	3	0.2735769E+03	0.1443577E+04	0.1056817E+04	AVE
140	3	0.8194287E+01	0.6482109E+02	0.6594434E+02	SD
160	3	0.3164968E+03	0.1795437E+04	0.1348057E+04	AVE
160	3	0.1057958E+02	0.7277132E+02	0.7374550E+02	SD
180	3	0.3609370E+03	0.2175958E+04	0.1669957E+04	AVE
180	3	0.1010248E+02	0.9446922E+02	0.9508049E+02	SD
200	3	0.4046973E+03	0.2566818E+04	0.2000577E+04	AVE
200	3	0.1084666E+02	0.1017076E+03	0.1027434E+03	SD
240	3	0.4933567E+03	0.3530957E+04	0.2847577E+04	AVE
240	3	0.1005425E+02	0.1261708E+03	0.1269994E+03	SD
280	3	0.5840972E+03	0.4522812E+04	0.3720817E+04	AVE
280	3	0.1156787E+02	0.1743988E+03	0.1752457E+03	SD
330	3	0.6927971E+03	0.6032273E+04	0.5080570E+04	AVE
330	3	0.1368260E+02	0.2348400E+03	0.2362674E+03	SD
380	3	0.8043169E+03	0.7754719E+04	0.6654055E+04	AVE
380	3	0.1783891E+02	0.2588733E+03	0.2584771E+03	SD
440	3	0.9425369E+03	0.1004120E+05	0.8761855E+04	AVE
440	3	0.1415240E+02	0.3155356E+03	0.3170049E+03	SD
500	3	0.1076257E+04	0.1253493E+05	0.1107608E+05	AVE
500	3	0.1837997E+02	0.4968315E+03	0.4972573E+03	SD
550	3	0.1195357E+04	0.1480695E+05	0.1319923E+05	AVE
550	3	0.1818217E+02	0.4371228E+03	0.4375781E+03	SD
600	3	0.1304617E+04	0.1737279E+05	0.1561593E+05	AVE

600	3	0.1737784E+02	0.6756018E+03	0.6770154E+03	SD
700	3	0.1535837E+04	0.2310455E+05	0.2104818E+05	AVE
700	3	0.2111794E+02	0.7107705E+03	0.7124150E+03	SD
800	3	0.1761317E+04	0.2958884E+05	0.2723432E+05	AVE
800	3	0.2419299E+02	0.7822507E+03	0.7816497E+03	SD
900	3	0.1993137E+04	0.3670433E+05	0.3405019E+05	AVE
900	3	0.2685744E+02	0.1210747E+04	0.1211072E+04	SD
1000	3	0.2218757E+04	0.4471726E+05	0.4176193E+05	AVE
1000	3	0.2527945E+02	0.1244315E+04	0.1244104E+04	SD
2000	3	0.4532828E+04	0.1645739E+06	0.1586244E+06	AVE
2000	3	0.3957210E+02	0.3916821E+04	0.3918119E+04	SD
5000	3	0.1147225E+05	0.9678763E+06	0.9529346E+06	AVE
5000	3	0.6066010E+02	0.1312766E+05	0.1312899E+05	SD
10000	3	0.2307388E+05	0.3778021E+07	0.3748082E+07	AVE
10000	3	0.1046441E+03	0.4311162E+05	0.4311302E+05	SD
16	4	0.1845985E+02	0.5669980E+02	0.2579982E+02	AVE
16	4	0.2082101E+01	0.3609807E+01	0.4000008E+01	SD
18	4	0.2113982E+02	0.6817979E+02	0.2997983E+02	AVE
18	4	0.2762803E+01	0.4202514E+01	0.4925955E+01	SD
20	4	0.2437984E+02	0.8029982E+02	0.3487981E+02	AVE
20	4	0.3076508E+01	0.5003037E+01	0.6049569E+01	SD
24	4	0.3241982E+02	0.1059798E+03	0.4691985E+02	AVE
24	4	0.4071248E+01	0.6001660E+01	0.7148114E+01	SD
28	4	0.4053983E+02	0.1309598E+03	0.5865984E+02	AVE
28	4	0.5027303E+01	0.8566570E+01	0.9832469E+01	SD
33	4	0.5083983E+02	0.1670598E+03	0.7627985E+02	AVE
33	4	0.4709352E+01	0.8262055E+01	0.9920141E+01	SD
38	4	0.6187985E+02	0.2038398E+03	0.9529977E+02	AVE
38	4	0.5169127E+01	0.1114899E+02	0.1192020E+02	SD
44	4	0.7517981E+02	0.2466198E+03	0.1166398E+03	AVE
44	4	0.4671602E+01	0.1032703E+02	0.1185753E+02	SD
50	4	0.8897983E+02	0.2957573E+03	0.1433398E+03	AVE
50	4	0.5596977E+01	0.1253009E+02	0.1354898E+02	SD
60	4	0.1143798E+03	0.3844771E+03	0.1951398E+03	AVE
60	4	0.7202283E+01	0.1523577E+02	0.1651463E+02	SD
70	4	0.1387998E+03	0.4737175E+03	0.2457598E+03	AVE
70	4	0.6452320E+01	0.1850964E+02	0.1885226E+02	SD
80	4	0.1610798E+03	0.5671970E+03	0.3010371E+03	AVE
80	4	0.6291670E+01	0.1873538E+02	0.1957909E+02	SD
90	4	0.1870998E+03	0.6641575E+03	0.3597771E+03	AVE
90	4	0.9187946E+01	0.2161844E+02	0.2353737E+02	SD
100	4	0.2128198E+03	0.7662571E+03	0.4237173E+03	AVE
100	4	0.8407221E+01	0.2728372E+02	0.2798474E+02	SD
120	4	0.2607771E+03	0.9851973E+03	0.5644373E+03	AVE
120	4	0.1053117E+02	0.3630037E+02	0.3802423E+02	SD
130	4	0.2907371E+03	0.1107797E+04	0.6494377E+03	AVE
130	4	0.9367195E+01	0.3496466E+02	0.3657089E+02	SD
140	4	0.3127173E+03	0.1219037E+04	0.7209973E+03	AVE
140	4	0.1019425E+02	0.4189432E+02	0.4280656E+02	SD
160	4	0.3658367E+03	0.1484197E+04	0.9085374E+03	AVE
160	4	0.1173060E+02	0.5693832E+02	0.5828725E+02	SD
180	4	0.4194170E+03	0.1739237E+04	0.1084657E+04	AVE
180	4	0.1265036E+02	0.5737761E+02	0.5940266E+02	SD
200	4	0.4753167E+03	0.2039157E+04	0.1305617E+04	AVE

200	4	0.1485342E+02	0.6616791E+02	0.6873444E+02	SD
240	4	0.5816377E+03	0.2651417E+04	0.1761717E+04	AVE
240	4	0.1769194E+02	0.8672659E+02	0.8846573E+02	SD
280	4	0.6952373E+03	0.3359757E+04	0.2312297E+04	AVE
280	4	0.1758659E+02	0.1064323E+03	0.1074334E+03	SD
330	4	0.8321370E+03	0.4277309E+04	0.3031437E+04	AVE
330	4	0.1826096E+02	0.1366921E+03	0.1373940E+03	SD
380	4	0.9774773E+03	0.5357926E+04	0.3915516E+04	AVE
380	4	0.2601933E+02	0.1730053E+03	0.1731740E+03	SD
440	4	0.1136297E+04	0.6720719E+04	0.5040844E+04	AVE
440	4	0.2024957E+02	0.1622206E+03	0.1623431E+03	SD
500	4	0.1306397E+04	0.8201254E+04	0.6281840E+04	AVE
500	4	0.2624770E+02	0.2098557E+03	0.2096895E+03	SD
550	4	0.1448277E+04	0.9553113E+04	0.7434262E+04	AVE
550	4	0.2744995E+02	0.2219333E+03	0.2234719E+03	SD
600	4	0.1587177E+04	0.1101815E+05	0.8700930E+04	AVE
600	4	0.3028607E+02	0.2642563E+03	0.2651514E+03	SD
700	4	0.1875497E+04	0.1409558E+05	0.1138197E+05	AVE
700	4	0.2842804E+02	0.2761272E+03	0.2765645E+03	SD
800	4	0.2167717E+04	0.1776534E+05	0.1465370E+05	AVE
800	4	0.2999835E+02	0.3709353E+03	0.3727610E+03	SD
900	4	0.2446837E+04	0.2175065E+05	0.1824140E+05	AVE
900	4	0.3219133E+02	0.5117336E+03	0.5141309E+03	SD
1000	4	0.2736817E+04	0.2589663E+05	0.2198830E+05	AVE
1000	4	0.4203735E+02	0.5773452E+03	0.5773367E+03	SD
1100	4	0.3025237E+04	0.3052465E+05	0.2621737E+05	AVE
1100	4	0.4527785E+02	0.5941479E+03	0.5939714E+03	SD
1200	4	0.3311877E+04	0.3551464E+05	0.3080793E+05	AVE
1200	4	0.4366058E+02	0.6704097E+03	0.6716433E+03	SD
1400	4	0.3884277E+04	0.4663643E+05	0.4113176E+05	AVE
1400	4	0.5164406E+02	0.9557141E+03	0.9569314E+03	SD
1600	4	0.4462484E+04	0.5895939E+05	0.5265556E+05	AVE
1600	4	0.5117778E+02	0.1230588E+04	0.1231055E+04	SD
1800	4	0.5053820E+04	0.7296706E+05	0.6586594E+05	AVE
1800	4	0.4993384E+02	0.1158404E+04	0.1160771E+04	SD
2000	4	0.5628328E+04	0.8833669E+05	0.8043925E+05	AVE
2000	4	0.6470706E+02	0.1620853E+04	0.1619651E+04	SD
2200	4	0.6239176E+04	0.1056772E+06	0.9697969E+05	AVE
2200	4	0.5633151E+02	0.1737216E+04	0.1738687E+04	SD
2400	4	0.6797172E+04	0.1232057E+06	0.1137081E+06	AVE
2400	4	0.6451218E+02	0.1770075E+04	0.1771794E+04	SD
2800	4	0.7962578E+04	0.1630208E+06	0.1519262E+06	AVE
2800	4	0.6767947E+02	0.2323474E+04	0.2324328E+04	SD
5000	4	0.1444505E+05	0.4863076E+06	0.4664227E+06	AVE
5000	4	0.1154720E+03	0.6532793E+04	0.6530848E+04	SD
10000	4	0.2915362E+05	0.1838976E+07	0.1799103E+07	AVE
10000	4	0.1412474E+03	0.1951638E+05	0.1951657E+05	SD
15000	4	0.4402224E+05	0.4043029E+07	0.3983161E+07	AVE
15000	4	0.1820346E+03	0.2868085E+05	0.2868042E+05	SD
20000	4	0.5883139E+05	0.7098109E+07	0.7018243E+07	AVE
20000	4	0.2454973E+03	0.4742243E+05	0.4742128E+05	SD
33	5	0.5065981E+02	0.1667798E+03	0.7461983E+02	AVE
33	5	0.4405987E+01	0.8352661E+01	0.9102348E+01	SD
38	5	0.6099985E+02	0.2055599E+03	0.9015984E+02	AVE

38	5	0.4290482E+01	0.9169943E+01	0.1008806E+02	SD
44	5	0.7519984E+02	0.2540996E+03	0.1117798E+03	AVE
44	5	0.4598107E+01	0.8741677E+01	0.9405334E+01	SD
50	5	0.8953981E+02	0.3040774E+03	0.1353398E+03	AVE
50	5	0.4477628E+01	0.1105620E+02	0.1213797E+02	SD
60	5	0.1178798E+03	0.3896572E+03	0.1781198E+03	AVE
60	5	0.7302894E+01	0.1456135E+02	0.1580961E+02	SD
64	5	0.1255398E+03	0.4212771E+03	0.1911198E+03	AVE
64	5	0.6791953E+01	0.1648703E+02	0.1683363E+02	SD
67	5	0.1343798E+03	0.4500176E+03	0.2048198E+03	AVE
67	5	0.7639561E+01	0.1643355E+02	0.1709715E+02	SD
70	5	0.1418398E+03	0.4765376E+03	0.2189198E+03	AVE
70	5	0.9015524E+01	0.2136183E+02	0.2283194E+02	SD
80	5	0.1674998E+03	0.5702971E+03	0.2657771E+03	AVE
80	5	0.8406136E+01	0.2045534E+02	0.2092880E+02	SD
90	5	0.1976198E+03	0.6678176E+03	0.3172969E+03	AVE
90	5	0.1029736E+02	0.2606866E+02	0.2762883E+02	SD
100	5	0.2251398E+03	0.7638174E+03	0.3667375E+03	AVE
100	5	0.9793788E+01	0.2570746E+02	0.2565669E+02	SD
120	5	0.2780176E+03	0.9585171E+03	0.4658569E+03	AVE
120	5	0.9838191E+01	0.2866982E+02	0.2900929E+02	SD
130	5	0.3107976E+03	0.1072637E+04	0.5318767E+03	AVE
130	5	0.1059689E+02	0.3574144E+02	0.3698778E+02	SD
140	5	0.3391570E+03	0.1189437E+04	0.6027170E+03	AVE
140	5	0.1178597E+02	0.4270164E+02	0.4369510E+02	SD
160	5	0.4026565E+03	0.1422997E+04	0.7404373E+03	AVE
160	5	0.1757503E+02	0.4885812E+02	0.5061447E+02	SD
180	5	0.4558770E+03	0.1635917E+04	0.8535977E+03	AVE
180	5	0.1433394E+02	0.4691219E+02	0.4745764E+02	SD
200	5	0.5238767E+03	0.1911957E+04	0.1033678E+04	AVE
200	5	0.1686525E+02	0.6264336E+02	0.6304330E+02	SD
240	5	0.6428577E+03	0.2418277E+04	0.1345077E+04	AVE
240	5	0.1700995E+02	0.6918126E+02	0.7033434E+02	SD
280	5	0.7773176E+03	0.2981877E+04	0.1714857E+04	AVE
280	5	0.2361926E+02	0.9866519E+02	0.1006803E+03	SD
330	5	0.9422371E+03	0.3757717E+04	0.2246457E+04	AVE
330	5	0.2774683E+02	0.1163656E+03	0.1170025E+03	SD
380	5	0.1092957E+04	0.4499043E+04	0.2740577E+04	AVE
380	5	0.2537929E+02	0.1211793E+03	0.1226490E+03	SD
440	5	0.1294378E+04	0.5555805E+04	0.3503577E+04	AVE
440	5	0.2820245E+02	0.1587727E+03	0.1590413E+03	SD
500	5	0.1491957E+04	0.6638656E+04	0.4287121E+04	AVE
500	5	0.3081816E+02	0.1707204E+03	0.1696116E+03	SD
550	5	0.1656197E+04	0.7600254E+04	0.5002500E+04	AVE
550	5	0.3260106E+02	0.2005804E+03	0.2018772E+03	SD
600	5	0.1833457E+04	0.8672418E+04	0.5827586E+04	AVE
600	5	0.3669504E+02	0.2057439E+03	0.2047258E+03	SD
700	5	0.2159177E+04	0.1081255E+05	0.7471516E+04	AVE
700	5	0.4146927E+02	0.2827893E+03	0.2837998E+03	SD
800	5	0.2505498E+04	0.1319557E+05	0.9359070E+04	AVE
800	5	0.3882397E+02	0.3050217E+03	0.3064473E+03	SD
900	5	0.2842037E+04	0.1569142E+05	0.1135936E+05	AVE
900	5	0.4394774E+02	0.3417725E+03	0.3422622E+03	SD
1000	5	0.3186017E+04	0.1847063E+05	0.1364156E+05	AVE

1000	5	0.4264937E+02	0.4063281E+03	0.4078599E+03	SD
1100	5	0.3513777E+04	0.2125947E+05	0.1593298E+05	AVE
1100	5	0.4497264E+02	0.4643125E+03	0.4664854E+03	SD
1200	5	0.3867537E+04	0.2444129E+05	0.1861729E+05	AVE
1200	5	0.5789186E+02	0.4909011E+03	0.4944834E+03	SD
1400	5	0.4556543E+04	0.3110345E+05	0.2428406E+05	AVE
1400	5	0.5959999E+02	0.6143347E+03	0.6171069E+03	SD
1600	5	0.5241406E+04	0.3848954E+05	0.3067559E+05	AVE
1600	5	0.8075883E+02	0.7315132E+03	0.7332502E+03	SD
1800	5	0.5964555E+04	0.4697826E+05	0.3816645E+05	AVE
1800	5	0.7583994E+02	0.1000602E+04	0.1000625E+04	SD
2000	5	0.6633816E+04	0.5558873E+05	0.4578271E+05	AVE
2000	5	0.7786572E+02	0.8062063E+03	0.8053342E+03	SD
2200	5	0.7346121E+04	0.6526721E+05	0.5446487E+05	AVE
2200	5	0.9185666E+02	0.1080050E+04	0.1080910E+04	SD
2400	5	0.8054555E+04	0.7544131E+05	0.6364085E+05	AVE
2400	5	0.7487819E+02	0.9742488E+03	0.9745596E+03	SD
2800	5	0.9460652E+04	0.9822150E+05	0.8442475E+05	AVE
2800	5	0.9231157E+02	0.1424021E+04	0.1424583E+04	SD
3300	5	0.1122259E+05	0.1299847E+06	0.1136939E+06	AVE
3300	5	0.1248008E+03	0.2019524E+04	0.2016332E+04	SD
3800	5	0.1297261E+05	0.1666401E+06	0.1478540E+06	AVE
3800	5	0.1164103E+03	0.1935138E+04	0.1937643E+04	SD
4200	5	0.1441711E+05	0.1983734E+06	0.1775906E+06	AVE
4200	5	0.1284402E+03	0.2316952E+04	0.2316545E+04	SD
4400	5	0.1513301E+05	0.2169462E+06	0.1951641E+06	AVE
4400	5	0.1078663E+03	0.2637463E+04	0.2640229E+04	SD
5000	5	0.1726830E+05	0.2721385E+06	0.2473611E+06	AVE
5000	5	0.1474384E+03	0.2978099E+04	0.2980559E+04	SD
6000	5	0.2078317E+05	0.3774309E+06	0.3476564E+06	AVE
6000	5	0.1863696E+03	0.4563590E+04	0.4565031E+04	SD
7000	5	0.2436235E+05	0.5007091E+06	0.4659414E+06	AVE
7000	5	0.1703903E+03	0.5022207E+04	0.5022203E+04	SD
8000	5	0.2796676E+05	0.6404506E+06	0.6006872E+06	AVE
8000	5	0.2015422E+03	0.5997187E+04	0.5998941E+04	SD
9000	5	0.3153010E+05	0.7975935E+06	0.7528327E+06	AVE
9000	5	0.1765126E+03	0.6543398E+04	0.6544730E+04	SD
10000	5	0.3512131E+05	0.9688683E+06	0.9191082E+06	AVE
10000	5	0.2179236E+03	0.8770934E+04	0.8773289E+04	SD
15000	5	0.5310049E+05	0.2087752E+07	0.2013011E+07	AVE
15000	5	0.2743655E+03	0.1708037E+05	0.1708215E+05	SD
20000	5	0.7108631E+05	0.3621554E+07	0.3521816E+07	AVE
20000	5	0.3034658E+03	0.2188389E+05	0.2188392E+05	SD
64	6	0.1263799E+03	0.4243774E+03	0.1937198E+03	AVE
64	6	0.7785076E+01	0.1834209E+02	0.1915883E+02	SD
67	6	0.1334998E+03	0.4516570E+03	0.2044398E+03	AVE
67	6	0.8612416E+01	0.1879854E+02	0.1969267E+02	SD
70	6	0.1415998E+03	0.4812371E+03	0.2168398E+03	AVE
70	6	0.7639845E+01	0.1625009E+02	0.1801283E+02	SD
80	6	0.1669598E+03	0.5770969E+03	0.2557993E+03	AVE
80	6	0.9724714E+01	0.1982927E+02	0.2100240E+02	SD
90	6	0.1956598E+03	0.6747771E+03	0.2985967E+03	AVE
90	6	0.8984350E+01	0.2255278E+02	0.2332121E+02	SD
100	6	0.2242398E+03	0.7769368E+03	0.3464172E+03	AVE

100	6	0.1023509E+02	0.2695061E+02	0.2762354E+02	SD
120	6	0.2865571E+03	0.9875371E+03	0.4504370E+03	AVE
120	6	0.1264147E+02	0.3448538E+02	0.3558229E+02	SD
128	6	0.3131572E+03	0.1077337E+04	0.4985366E+03	AVE
128	6	0.1327743E+02	0.4148801E+02	0.4249284E+02	SD
130	6	0.3221167E+03	0.1106417E+04	0.5152571E+03	AVE
130	6	0.1269698E+02	0.3823972E+02	0.3884377E+02	SD
135	6	0.3289570E+03	0.1139217E+04	0.5190369E+03	AVE
135	6	0.1183895E+02	0.2776280E+02	0.2974503E+02	SD
140	6	0.3497576E+03	0.1210857E+04	0.5615972E+03	AVE
140	6	0.1325954E+02	0.3731348E+02	0.3874565E+02	SD
160	6	0.4086372E+03	0.1420017E+04	0.6573171E+03	AVE
160	6	0.1355549E+02	0.4054773E+02	0.4170581E+02	SD
180	6	0.4812173E+03	0.1664937E+04	0.7927373E+03	AVE
180	6	0.1598530E+02	0.4950247E+02	0.5056445E+02	SD
200	6	0.5446169E+03	0.1904277E+04	0.9152568E+03	AVE
200	6	0.1435974E+02	0.5703531E+02	0.5747375E+02	SD
240	6	0.6829175E+03	0.2402277E+04	0.1186077E+04	AVE
240	6	0.1695604E+02	0.6367596E+02	0.6638850E+02	SD
280	6	0.8173770E+03	0.2910337E+04	0.1461157E+04	AVE
280	6	0.2466764E+02	0.9099091E+02	0.9429210E+02	SD
330	6	0.9958369E+03	0.3584597E+04	0.1846257E+04	AVE
330	6	0.2972116E+02	0.1090068E+03	0.1094717E+03	SD
380	6	0.1182817E+04	0.4320855E+04	0.2291896E+04	AVE
380	6	0.3077872E+02	0.1288208E+03	0.1309332E+03	SD
440	6	0.1402838E+04	0.5210977E+04	0.2832077E+04	AVE
440	6	0.3349353E+02	0.1343714E+03	0.1331168E+03	SD
500	6	0.1626397E+04	0.6159156E+04	0.3430677E+04	AVE
500	6	0.3588121E+02	0.1722410E+03	0.1755078E+03	SD
550	6	0.1815017E+04	0.6992879E+04	0.3963596E+04	AVE
550	6	0.3636649E+02	0.2196003E+03	0.2200451E+03	SD
600	6	0.1997997E+04	0.7778152E+04	0.4457855E+04	AVE
600	6	0.4215813E+02	0.1904683E+03	0.1919323E+03	SD
700	6	0.2393457E+04	0.9588238E+04	0.5680352E+04	AVE
700	6	0.4800960E+02	0.2390263E+03	0.2400492E+03	SD
800	6	0.2765717E+04	0.1144623E+05	0.6942340E+04	AVE
800	6	0.4727170E+02	0.3007048E+03	0.3016326E+03	SD
900	6	0.3162198E+04	0.1341921E+05	0.8321191E+04	AVE
900	6	0.3990817E+02	0.2677898E+03	0.2686663E+03	SD
1000	6	0.3580117E+04	0.1569616E+05	0.1000680E+05	AVE
1000	6	0.5726262E+02	0.4253718E+03	0.4264514E+03	SD
1100	6	0.3953757E+04	0.1779839E+05	0.1151340E+05	AVE
1100	6	0.7278246E+02	0.4911421E+03	0.4883701E+03	SD
1200	6	0.4361754E+04	0.2010162E+05	0.1322331E+05	AVE
1200	6	0.7455917E+02	0.4820706E+03	0.4829280E+03	SD
1400	6	0.5133418E+04	0.2480984E+05	0.1674205E+05	AVE
1400	6	0.7675781E+02	0.6308665E+03	0.6324299E+03	SD
1600	6	0.5975422E+04	0.3028792E+05	0.2102937E+05	AVE
1600	6	0.8456303E+02	0.6455298E+03	0.6478337E+03	SD
1800	6	0.6753238E+04	0.3576702E+05	0.2531496E+05	AVE
1800	6	0.9557635E+02	0.7499133E+03	0.7492817E+03	SD
2000	6	0.7594473E+04	0.4210159E+05	0.3045741E+05	AVE
2000	6	0.1378826E+03	0.1124729E+04	0.1128194E+04	SD
2200	6	0.8375500E+04	0.4803761E+05	0.3519927E+05	AVE

2200	6	0.1066453E+03	0.1016327E+04	0.1017476E+04	SD
2400	6	0.9221031E+04	0.5489929E+05	0.4086404E+05	AVE
2400	6	0.1360809E+03	0.1267929E+04	0.1271321E+04	SD
2800	6	0.1084091E+05	0.6931175E+05	0.5288531E+05	AVE
2800	6	0.1409554E+03	0.1381470E+04	0.1379502E+04	SD
3300	6	0.1293017E+05	0.8973125E+05	0.7031512E+05	AVE
3300	6	0.1513827E+03	0.1738329E+04	0.1735736E+04	SD
3800	6	0.1498250E+05	0.1119385E+06	0.8953000E+05	AVE
3800	6	0.1727571E+03	0.1897723E+04	0.1902281E+04	SD
4200	6	0.1668793E+05	0.1318958E+06	0.1070937E+06	AVE
4200	6	0.1901349E+03	0.2266815E+04	0.2269207E+04	SD
4400	6	0.1753847E+05	0.1426349E+06	0.1166381E+06	AVE
4400	6	0.1985559E+03	0.2565813E+04	0.2562422E+04	SD
5000	6	0.2001429E+05	0.1748124E+06	0.1452206E+06	AVE
5000	6	0.2217535E+03	0.2488098E+04	0.2492157E+04	SD
6000	6	0.2416029E+05	0.2358452E+06	0.2002651E+06	AVE
6000	6	0.2568130E+03	0.3220372E+04	0.3220875E+04	SD
7000	6	0.2835461E+05	0.3069774E+06	0.2654121E+06	AVE
7000	6	0.2666855E+03	0.3528403E+04	0.3529844E+04	SD
8000	6	0.3266275E+05	0.3848599E+06	0.3373012E+06	AVE
8000	6	0.2808003E+03	0.4040209E+04	0.4041102E+04	SD
9000	6	0.3679391E+05	0.4721312E+06	0.4185786E+06	AVE
9000	6	0.2646592E+03	0.4757598E+04	0.4756684E+04	SD
10000	6	0.4124659E+05	0.5703427E+06	0.5107966E+06	AVE
10000	6	0.4214348E+03	0.6897004E+04	0.6900324E+04	SD
15000	6	0.6241071E+05	0.1172313E+07	0.1082791E+07	AVE
15000	6	0.4573933E+03	0.9694375E+04	0.9693844E+04	SD
20000	6	0.8368687E+05	0.1981616E+07	0.1862118E+07	AVE
20000	6	0.4860620E+03	0.1406457E+05	0.1405990E+05	SD
128	7	0.3124170E+03	0.1072717E+04	0.4922173E+03	AVE
128	7	0.1343889E+02	0.3754378E+02	0.3797513E+02	SD
130	7	0.3160972E+03	0.1085817E+04	0.4922974E+03	AVE
130	7	0.1403636E+02	0.3510570E+02	0.3662636E+02	SD
135	7	0.3320776E+03	0.1149557E+04	0.5207571E+03	AVE
135	7	0.1261957E+02	0.3343547E+02	0.3517346E+02	SD
140	7	0.3482175E+03	0.1216657E+04	0.5552969E+03	AVE
140	7	0.1282273E+02	0.3457820E+02	0.3420175E+02	SD
160	7	0.4096570E+03	0.1445397E+04	0.6514570E+03	AVE
160	7	0.1526999E+02	0.4680444E+02	0.4826685E+02	SD
180	7	0.4719172E+03	0.1673897E+04	0.7518171E+03	AVE
180	7	0.1542841E+02	0.4308745E+02	0.4506616E+02	SD
200	7	0.5536370E+03	0.1947076E+04	0.8973174E+03	AVE
200	7	0.1953723E+02	0.5680923E+02	0.5741670E+02	SD
240	7	0.6926572E+03	0.2436017E+04	0.1133377E+04	AVE
240	7	0.1965234E+02	0.6674675E+02	0.6858452E+02	SD
256	7	0.7566768E+03	0.2645017E+04	0.1241977E+04	AVE
256	7	0.2033641E+02	0.7060268E+02	0.7016017E+02	SD
260	7	0.7670371E+03	0.2696936E+04	0.1263377E+04	AVE
260	7	0.1822849E+02	0.6503935E+02	0.6521008E+02	SD
270	7	0.8054775E+03	0.2828777E+04	0.1330197E+04	AVE
270	7	0.1896828E+02	0.7397545E+02	0.7354234E+02	SD
280	7	0.8320571E+03	0.2927857E+04	0.1360197E+04	AVE
280	7	0.2393803E+02	0.7048431E+02	0.7385919E+02	SD
330	7	0.1028897E+04	0.3668377E+04	0.1767378E+04	AVE

330	7	0.2764413E+02	0.1093315E+03	0.1104480E+03	SD
355	7	0.1114958E+04	0.3956037E+04	0.1889577E+04	AVE
355	7	0.2242444E+02	0.7959824E+02	0.8352910E+02	SD
380	7	0.1225897E+04	0.4341695E+04	0.2116557E+04	AVE
380	7	0.2628896E+02	0.9856763E+02	0.9857649E+02	SD
440	7	0.1458077E+04	0.5213980E+04	0.2581837E+04	AVE
440	7	0.3743126E+02	0.1397584E+03	0.1443957E+03	SD
500	7	0.1701358E+04	0.6078391E+04	0.3042297E+04	AVE
500	7	0.3405122E+02	0.1241158E+03	0.1284453E+03	SD
550	7	0.1908437E+04	0.6920637E+04	0.3543078E+04	AVE
550	7	0.4667024E+02	0.2170017E+03	0.2187340E+03	SD
600	7	0.2100057E+04	0.7643672E+04	0.3926557E+04	AVE
600	7	0.4019287E+02	0.1809657E+03	0.1826710E+03	SD
700	7	0.2525937E+04	0.9325316E+04	0.4926527E+04	AVE
700	7	0.4955791E+02	0.2701245E+03	0.2703430E+03	SD
800	7	0.2949177E+04	0.1095754E+05	0.5879094E+04	AVE
800	7	0.4991054E+02	0.2319919E+03	0.2329169E+03	SD
900	7	0.3388977E+04	0.1279590E+05	0.7029973E+04	AVE
900	7	0.5897031E+02	0.3570127E+03	0.3584031E+03	SD
1000	7	0.3810837E+04	0.1457367E+05	0.8117156E+04	AVE
1000	7	0.6860721E+02	0.4046140E+03	0.4070847E+03	SD
1100	7	0.4258719E+04	0.1655804E+05	0.9411191E+04	AVE
1100	7	0.8286168E+02	0.4371484E+03	0.4341653E+03	SD
1200	7	0.4691273E+04	0.1845847E+05	0.1062923E+05	AVE
1200	7	0.7654926E+02	0.5296848E+03	0.5302905E+03	SD
1400	7	0.5585574E+04	0.2244621E+05	0.1323452E+05	AVE
1400	7	0.8001160E+02	0.5475957E+03	0.5481787E+03	SD
1600	7	0.6489469E+04	0.2689645E+05	0.1630222E+05	AVE
1600	7	0.8816029E+02	0.6512334E+03	0.6539587E+03	SD
1800	7	0.7412570E+04	0.3157816E+05	0.1960181E+05	AVE
1800	7	0.1234083E+03	0.9239980E+03	0.9272429E+03	SD
2000	7	0.8328117E+04	0.3632198E+05	0.2295508E+05	AVE
2000	7	0.1233522E+03	0.9875613E+03	0.9882271E+03	SD
2200	7	0.9262277E+04	0.4132057E+05	0.2656462E+05	AVE
2200	7	0.1499173E+03	0.1228753E+04	0.1228284E+04	SD
2400	7	0.1014227E+05	0.4621726E+05	0.3006612E+05	AVE
2400	7	0.1433111E+03	0.1177169E+04	0.1172401E+04	SD
2800	7	0.1198608E+05	0.5706216E+05	0.3813492E+05	AVE
2800	7	0.1807305E+03	0.1560809E+04	0.1559738E+04	SD
3300	7	0.1440007E+05	0.7233712E+05	0.4993487E+05	AVE
3300	7	0.1483635E+03	0.1519457E+04	0.1522111E+04	SD
3800	7	0.1675718E+05	0.8860587E+05	0.6272155E+05	AVE
3800	7	0.1908210E+03	0.2111268E+04	0.2111979E+04	SD
4200	7	0.1858660E+05	0.1014597E+06	0.7278944E+05	AVE
4200	7	0.1961116E+03	0.1917918E+04	0.1922781E+04	SD
4400	7	0.1954486E+05	0.1086486E+06	0.7858256E+05	AVE
4400	7	0.2401112E+03	0.2632194E+04	0.2631902E+04	SD
5000	7	0.2241280E+05	0.1310164E+06	0.9676006E+05	AVE
5000	7	0.2261209E+03	0.2518804E+04	0.2518713E+04	SD
6000	7	0.2727171E+05	0.1730182E+06	0.1317852E+06	AVE
6000	7	0.2996919E+03	0.3665096E+04	0.3666299E+04	SD
7000	7	0.3204128E+05	0.2176589E+06	0.1694519E+06	AVE
7000	7	0.3163108E+03	0.4116840E+04	0.4116059E+04	SD
8000	7	0.3695729E+05	0.2685372E+06	0.2133488E+06	AVE

8000	7	0.3653066E+03	0.4704809E+04	0.4707004E+04	SD
9000	7	0.4169714E+05	0.3216965E+06	0.2595121E+06	AVE
9000	7	0.3882769E+03	0.5363238E+04	0.5363180E+04	SD
10000	7	0.4657883E+05	0.3805640E+06	0.3113984E+06	AVE
10000	7	0.4384050E+03	0.5836996E+04	0.5836141E+04	SD
15000	7	0.7117356E+05	0.7386310E+06	0.6345217E+06	AVE
15000	7	0.6537312E+03	0.1068071E+05	0.1068155E+05	SD
20000	7	0.9601150E+05	0.1208141E+07	0.1069061E+07	AVE
20000	7	0.7777371E+03	0.1656268E+05	0.1656355E+05	SD
256	8	0.7564773E+03	0.2645777E+04	0.1241717E+04	AVE
256	8	0.2144141E+02	0.6467500E+02	0.6671011E+02	SD
260	8	0.7662373E+03	0.2685097E+04	0.1251337E+04	AVE
260	8	0.2530801E+02	0.8196266E+02	0.8390591E+02	SD
270	8	0.8000574E+03	0.2824278E+04	0.1310197E+04	AVE
270	8	0.2144611E+02	0.7681779E+02	0.7796143E+02	SD
280	8	0.8386772E+03	0.2968097E+04	0.1378117E+04	AVE
280	8	0.2282436E+02	0.7009254E+02	0.7377426E+02	SD
330	8	0.1020077E+04	0.3659297E+04	0.1688337E+04	AVE
330	8	0.2359171E+02	0.7907846E+02	0.7987943E+02	SD
355	8	0.1118817E+04	0.4021397E+04	0.1862857E+04	AVE
355	8	0.3143384E+02	0.1090787E+03	0.1119503E+03	SD
380	8	0.1225377E+04	0.4401223E+04	0.2058258E+04	AVE
380	8	0.2775568E+02	0.1124050E+03	0.1125026E+03	SD
440	8	0.1468757E+04	0.5283156E+04	0.2497737E+04	AVE
440	8	0.3278291E+02	0.1202898E+03	0.1213675E+03	SD
500	8	0.1723977E+04	0.6125192E+04	0.2906057E+04	AVE
500	8	0.4013397E+02	0.1345735E+03	0.1366273E+03	SD
512	8	0.1793577E+04	0.6401555E+04	0.3094337E+04	AVE
512	8	0.4404660E+02	0.1767188E+03	0.1784123E+03	SD
520	8	0.1809197E+04	0.6464859E+04	0.3095597E+04	AVE
520	8	0.3509666E+02	0.1608393E+03	0.1606891E+03	SD
535	8	0.1869797E+04	0.6715875E+04	0.3225937E+04	AVE
535	8	0.4290704E+02	0.1910042E+03	0.1918035E+03	SD
550	8	0.1930137E+04	0.6929574E+04	0.3324477E+04	AVE
550	8	0.4160204E+02	0.1751524E+03	0.1732876E+03	SD
575	8	0.2038837E+04	0.7322590E+04	0.3523657E+04	AVE
575	8	0.4270972E+02	0.1868514E+03	0.1911901E+03	SD
600	8	0.2140558E+04	0.7725121E+04	0.3731558E+04	AVE
600	8	0.4328607E+02	0.1734923E+03	0.1757670E+03	SD
700	8	0.2598898E+04	0.9386039E+04	0.4632980E+04	AVE
700	8	0.4538965E+02	0.2482646E+03	0.2520320E+03	SD
800	8	0.3044557E+04	0.1103846E+05	0.5521980E+04	AVE
800	8	0.5917740E+02	0.3213589E+03	0.3224172E+03	SD
900	8	0.3496858E+04	0.1277071E+05	0.6475090E+04	AVE
900	8	0.7412129E+02	0.3614189E+03	0.3636484E+03	SD
1000	8	0.3955097E+04	0.1451629E+05	0.7452641E+04	AVE
1000	8	0.6706699E+02	0.3569268E+03	0.3593171E+03	SD
1100	8	0.4417516E+04	0.1618580E+05	0.8339113E+04	AVE
1100	8	0.7388042E+02	0.3466628E+03	0.3456738E+03	SD
1200	8	0.4895277E+04	0.1811579E+05	0.9488102E+04	AVE
1200	8	0.7730502E+02	0.4088508E+03	0.4091572E+03	SD
1400	8	0.5861582E+04	0.2197680E+05	0.1179439E+05	AVE
1400	8	0.9662894E+02	0.6879031E+03	0.6831274E+03	SD
1600	8	0.6854992E+04	0.2604602E+05	0.1429479E+05	AVE

1600	8	0.1162398E+03	0.7501277E+03	0.7532371E+03	SD
1800	8	0.7875332E+04	0.3025857E+05	0.1693737E+05	AVE
1800	8	0.1005374E+03	0.7088286E+03	0.7111760E+03	SD
2000	8	0.8820312E+04	0.3430419E+05	0.1940691E+05	AVE
2000	8	0.1044445E+03	0.8318345E+03	0.8325728E+03	SD
2200	8	0.9839445E+04	0.3882001E+05	0.2234196E+05	AVE
2200	8	0.1568894E+03	0.1222854E+04	0.1228862E+04	SD
2400	8	0.1084545E+05	0.4318709E+05	0.2513493E+05	AVE
2400	8	0.1681967E+03	0.1399385E+04	0.1397930E+04	SD
2800	8	0.1291257E+05	0.5269187E+05	0.3148211E+05	AVE
2800	8	0.1638502E+03	0.1304781E+04	0.1305351E+04	SD
3300	8	0.1553213E+05	0.6548103E+05	0.4030682E+05	AVE
3300	8	0.2054673E+03	0.1749342E+04	0.1746746E+04	SD
3800	8	0.1806684E+05	0.7819062E+05	0.4905436E+05	AVE
3800	8	0.2345417E+03	0.2151356E+04	0.2154573E+04	SD
4200	8	0.2020176E+05	0.8956762E+05	0.5725951E+05	AVE
4200	8	0.2348166E+03	0.2136459E+04	0.2138818E+04	SD
4400	8	0.2129886E+05	0.9527656E+05	0.6138510E+05	AVE
4400	8	0.2706863E+03	0.2701495E+04	0.2702204E+04	SD
5000	8	0.2443957E+05	0.1125774E+06	0.7391162E+05	AVE
5000	8	0.3409224E+03	0.3267866E+04	0.3267777E+04	SD
6000	8	0.2973286E+05	0.1437924E+06	0.9717000E+05	AVE
6000	8	0.3386902E+03	0.3351874E+04	0.3355607E+04	SD
7000	8	0.3522143E+05	0.1788026E+06	0.1242219E+06	AVE
7000	8	0.2877756E+03	0.3575770E+04	0.3574706E+04	SD
8000	8	0.4058950E+05	0.2149459E+06	0.1523996E+06	AVE
8000	8	0.4453208E+03	0.5516832E+04	0.5520211E+04	SD
9000	8	0.4601022E+05	0.2542460E+06	0.1837242E+06	AVE
9000	8	0.4358198E+03	0.6535684E+04	0.6535746E+04	SD
10000	8	0.5154347E+05	0.2960251E+06	0.2175385E+06	AVE
10000	8	0.4752434E+03	0.6966184E+04	0.6971145E+04	SD
15000	8	0.7897031E+05	0.5350828E+06	0.4166943E+06	AVE
15000	8	0.7681238E+03	0.1215604E+05	0.1215461E+05	SD
20000	8	0.1071659E+06	0.8382614E+06	0.6799362E+06	AVE
20000	8	0.9008418E+03	0.1634617E+05	0.1635203E+05	SD
512	9	0.1788197E+04	0.6360594E+04	0.3053477E+04	AVE
512	9	0.3625253E+02	0.1354387E+03	0.1388559E+03	SD
520	9	0.1805057E+04	0.6467535E+04	0.3086017E+04	AVE
520	9	0.3916217E+02	0.1738363E+03	0.1772671E+03	SD
535	9	0.1865438E+04	0.6692145E+04	0.3181857E+04	AVE
535	9	0.3662196E+02	0.1480245E+03	0.1495787E+03	SD
550	9	0.1923577E+04	0.6911898E+04	0.3270357E+04	AVE
550	9	0.3914474E+02	0.1630171E+03	0.1658220E+03	SD
575	9	0.2044757E+04	0.7406012E+04	0.3548657E+04	AVE
575	9	0.3958876E+02	0.1961573E+03	0.1964701E+03	SD
600	9	0.2134597E+04	0.7750363E+04	0.3673238E+04	AVE
600	9	0.5106926E+02	0.2008773E+03	0.2010245E+03	SD
700	9	0.2583957E+04	0.9450250E+04	0.4519773E+04	AVE
700	9	0.4517326E+02	0.2372334E+03	0.2354471E+03	SD
800	9	0.3048198E+04	0.1117407E+05	0.5403855E+04	AVE
800	9	0.6271953E+02	0.3084446E+03	0.3108442E+03	SD
900	9	0.3544477E+04	0.1292731E+05	0.6323277E+04	AVE
900	9	0.6276364E+02	0.3078635E+03	0.3107324E+03	SD
1000	9	0.4033977E+04	0.1468679E+05	0.7263680E+04	AVE

1000	9	0.6955074E+02	0.3824319E+03	0.3797771E+03	SD
1024	9	0.4142102E+04	0.1502859E+05	0.7401660E+04	AVE
1024	9	0.7950705E+02	0.3635623E+03	0.3708562E+03	SD
1040	9	0.4212074E+04	0.1531525E+05	0.7549793E+04	AVE
1040	9	0.7609087E+02	0.3192627E+03	0.3264485E+03	SD
1070	9	0.4354117E+04	0.1585543E+05	0.7822742E+04	AVE
1070	9	0.8541011E+02	0.4521606E+03	0.4556675E+03	SD
1100	9	0.4496277E+04	0.1648330E+05	0.8188109E+04	AVE
1100	9	0.6573074E+02	0.4036875E+03	0.4062393E+03	SD
1150	9	0.4743898E+04	0.1739804E+05	0.8663156E+04	AVE
1150	9	0.8906552E+02	0.4727476E+03	0.4750891E+03	SD
1200	9	0.4974176E+04	0.1827633E+05	0.9104914E+04	AVE
1200	9	0.8577553E+02	0.4907397E+03	0.4939685E+03	SD
1400	9	0.5985965E+04	0.2212827E+05	0.1123185E+05	AVE
1400	9	0.9860612E+02	0.6579868E+03	0.6600466E+03	SD
1600	9	0.7029789E+04	0.2604909E+05	0.1342871E+05	AVE
1600	9	0.9275093E+02	0.6088362E+03	0.6064915E+03	SD
1800	9	0.8066770E+04	0.3003577E+05	0.1565691E+05	AVE
1800	9	0.1187144E+03	0.7469185E+03	0.7462935E+03	SD
2000	9	0.9127414E+04	0.3409271E+05	0.1798448E+05	AVE
2000	9	0.1007097E+03	0.8032908E+03	0.8048218E+03	SD
2200	9	0.1015741E+05	0.3816173E+05	0.2027844E+05	AVE
2200	9	0.1362587E+03	0.9951621E+03	0.9883018E+03	SD
2400	9	0.1128091E+05	0.4264752E+05	0.2301803E+05	AVE
2400	9	0.1306396E+03	0.1063128E+04	0.1063391E+04	SD
2800	9	0.1350481E+05	0.5178267E+05	0.2862342E+05	AVE
2800	9	0.1781823E+03	0.1493605E+04	0.1493466E+04	SD
3300	9	0.1622225E+05	0.6309275E+05	0.3550907E+05	AVE
3300	9	0.2086917E+03	0.1738383E+04	0.1740946E+04	SD
3800	9	0.1904090E+05	0.7516431E+05	0.4316071E+05	AVE
3800	9	0.2368702E+03	0.2037215E+04	0.2027040E+04	SD
4200	9	0.2127615E+05	0.8483494E+05	0.4928143E+05	AVE
4200	9	0.2482860E+03	0.2296059E+04	0.2300000E+04	SD
4400	9	0.2248854E+05	0.9059681E+05	0.5327494E+05	AVE
4400	9	0.2815996E+03	0.2737860E+04	0.2740363E+04	SD
5000	9	0.2589941E+05	0.1056807E+06	0.6301873E+05	AVE
5000	9	0.3138445E+03	0.2781122E+04	0.2787220E+04	SD
6000	9	0.3178931E+05	0.1335906E+06	0.8201706E+05	AVE
6000	9	0.3367361E+03	0.3112174E+04	0.3107905E+04	SD
7000	9	0.3758962E+05	0.1621559E+06	0.1016531E+06	AVE
7000	9	0.4617187E+03	0.4331172E+04	0.4327195E+04	SD
8000	9	0.4352191E+05	0.1928473E+06	0.1234286E+06	AVE
8000	9	0.4454209E+03	0.5621547E+04	0.5629117E+04	SD
9000	9	0.4945517E+05	0.2247736E+06	0.1464006E+06	AVE
9000	9	0.4553245E+03	0.5654738E+04	0.5653426E+04	SD
10000	9	0.5551898E+05	0.2587897E+06	0.1714649E+06	AVE
10000	9	0.5430916E+03	0.5920262E+04	0.5917656E+04	SD
15000	9	0.8589662E+05	0.4466856E+06	0.3145570E+06	AVE
15000	9	0.7675505E+03	0.1147066E+05	0.1147257E+05	SD
20000	9	0.1165434E+06	0.6606922E+06	0.4837119E+06	AVE
20000	9	0.1022370E+04	0.1529253E+05	0.1528220E+05	SD
1024	10	0.4155137E+04	0.1507038E+05	0.7446328E+04	AVE
1024	10	0.7325050E+02	0.3690190E+03	0.3724436E+03	SD
1040	10	0.4206516E+04	0.1532444E+05	0.7542164E+04	AVE

1040	10	0.7413823E+02	0.3903198E+03	0.3882097E+03	SD
1070	10	0.4362191E+04	0.1597160E+05	0.7893641E+04	AVE
1070	10	0.8045404E+02	0.4219524E+03	0.4293782E+03	SD
1100	10	0.4482734E+04	0.1639557E+05	0.8026773E+04	AVE
1100	10	0.8179588E+02	0.4200876E+03	0.4210969E+03	SD
1150	10	0.4732816E+04	0.1743352E+05	0.8577500E+04	AVE
1150	10	0.6781062E+02	0.3878123E+03	0.3834111E+03	SD
1200	10	0.4953578E+04	0.1833839E+05	0.8996113E+04	AVE
1200	10	0.7283908E+02	0.4326316E+03	0.4333101E+03	SD
1400	10	0.5953277E+04	0.2222973E+05	0.1098150E+05	AVE
1400	10	0.9584406E+02	0.5971807E+03	0.6033127E+03	SD
1600	10	0.7010090E+04	0.2606460E+05	0.1293642E+05	AVE
1600	10	0.9501302E+02	0.5319978E+03	0.5365613E+03	SD
1800	10	0.8110734E+04	0.3025272E+05	0.1526220E+05	AVE
1800	10	0.1119856E+03	0.7885696E+03	0.7858931E+03	SD
2000	10	0.9231160E+04	0.3423907E+05	0.1739254E+05	AVE
2000	10	0.1293459E+03	0.8395503E+03	0.8405781E+03	SD
2048	10	0.9514598E+04	0.3534509E+05	0.1806301E+05	AVE
2048	10	0.1368958E+03	0.9314692E+03	0.9310708E+03	SD
2050	10	0.9511234E+04	0.3528448E+05	0.1798106E+05	AVE
2050	10	0.1251856E+03	0.8541729E+03	0.8569761E+03	SD
2200	10	0.1029948E+05	0.3853134E+05	0.1975823E+05	AVE
2200	10	0.1376722E+03	0.1141163E+04	0.1134636E+04	SD
2400	10	0.1141227E+05	0.4307160E+05	0.2234098E+05	AVE
2400	10	0.1630604E+03	0.1205567E+04	0.1204385E+04	SD
2600	10	0.1251436E+05	0.4703150E+05	0.2436934E+05	AVE
2600	10	0.1806104E+03	0.1276276E+04	0.1282166E+04	SD
2800	10	0.1368164E+05	0.5159202E+05	0.2702442E+05	AVE
2800	10	0.1609832E+03	0.1159163E+04	0.1167750E+04	SD
3300	10	0.1659682E+05	0.6297105E+05	0.3355924E+05	AVE
3300	10	0.2242840E+03	0.1671886E+04	0.1678302E+04	SD
3800	10	0.1951518E+05	0.7447350E+05	0.4019359E+05	AVE
3800	10	0.2762717E+03	0.2129678E+04	0.2131941E+04	SD
4200	10	0.2199730E+05	0.8464975E+05	0.4649229E+05	AVE
4200	10	0.2116218E+03	0.1818953E+04	0.1815935E+04	SD
4400	10	0.2310304E+05	0.8884669E+05	0.4871504E+05	AVE
4400	10	0.2179624E+03	0.2012691E+04	0.2004401E+04	SD
5000	10	0.2687327E+05	0.1051574E+06	0.5916586E+05	AVE
5000	10	0.2638738E+03	0.2668323E+04	0.2667664E+04	SD
6000	10	0.3292360E+05	0.1297730E+06	0.7395719E+05	AVE
6000	10	0.3499575E+03	0.3144090E+04	0.3146086E+04	SD
7000	10	0.3921587E+05	0.1567361E+06	0.9106587E+05	AVE
7000	10	0.4006790E+03	0.3948690E+04	0.3952000E+04	SD
8000	10	0.4556871E+05	0.1847261E+06	0.1091866E+06	AVE
8000	10	0.4395242E+03	0.4862832E+04	0.4875215E+04	SD
9000	10	0.5203866E+05	0.2150375E+06	0.1296143E+06	AVE
9000	10	0.4881292E+03	0.5450895E+04	0.5448723E+04	SD
10000	10	0.5851821E+05	0.2457899E+06	0.1504660E+06	AVE
10000	10	0.5166021E+03	0.5545468E+04	0.5540711E+04	SD
15000	10	0.9139669E+05	0.4100302E+06	0.2651287E+06	AVE
15000	10	0.7908884E+03	0.1193018E+05	0.1192957E+05	SD
20000	10	0.1248684E+06	0.5931597E+06	0.3985282E+06	AVE
20000	10	0.1040558E+04	0.1845821E+05	0.1845821E+05	SD
2048	11	0.9518125E+04	0.3548137E+05	0.1819405E+05	AVE

2048	11	0.1315915E+03	0.8694941E+03	0.8702029E+03	SD
2050	11	0.9521285E+04	0.3529123E+05	0.1798859E+05	AVE
2050	11	0.1354409E+03	0.9067622E+03	0.9041621E+03	SD
2200	11	0.1027528E+05	0.3841159E+05	0.1948311E+05	AVE
2200	11	0.1286447E+03	0.8360764E+03	0.8356624E+03	SD
2400	11	0.1139713E+05	0.4293384E+05	0.2187909E+05	AVE
2400	11	0.1730333E+03	0.1305818E+04	0.1301110E+04	SD
2600	11	0.1250362E+05	0.4743314E+05	0.2425773E+05	AVE
2600	11	0.1494489E+03	0.1024481E+04	0.1027936E+04	SD
2800	11	0.1367148E+05	0.5195619E+05	0.2668413E+05	AVE
2800	11	0.2138323E+03	0.1667253E+04	0.1675812E+04	SD
3300	11	0.1665760E+05	0.6358863E+05	0.3311722E+05	AVE
3300	11	0.2386032E+03	0.1998159E+04	0.2002611E+04	SD
3800	11	0.1963753E+05	0.7458587E+05	0.3895778E+05	AVE
3800	11	0.2058395E+03	0.1787444E+04	0.1790009E+04	SD
4192	11	0.2219093E+05	0.8490131E+05	0.4521191E+05	AVE
4192	11	0.2783110E+03	0.2429723E+04	0.2436284E+04	SD
4200	11	0.2223166E+05	0.8516762E+05	0.4538871E+05	AVE
4200	11	0.2891040E+03	0.2453561E+04	0.2457626E+04	SD
4300	11	0.2274086E+05	0.8685681E+05	0.4599648E+05	AVE
4300	11	0.2452686E+03	0.1902370E+04	0.1905032E+04	SD
4400	11	0.2340343E+05	0.8973362E+05	0.4779003E+05	AVE
4400	11	0.2602573E+03	0.2112939E+04	0.2114941E+04	SD
5000	11	0.2706421E+05	0.1044963E+06	0.5612484E+05	AVE
5000	11	0.3279648E+03	0.2844574E+04	0.2851091E+04	SD
6000	11	0.3350289E+05	0.1294944E+06	0.7055350E+05	AVE
6000	11	0.3744641E+03	0.3862441E+04	0.3861770E+04	SD
7000	11	0.4000121E+05	0.1561491E+06	0.8644331E+05	AVE
7000	11	0.4292034E+03	0.4553031E+04	0.4545348E+04	SD
8000	11	0.4673905E+05	0.1843232E+06	0.1039227E+06	AVE
8000	11	0.5015825E+03	0.5047859E+04	0.5053766E+04	SD
9000	11	0.5340091E+05	0.2120469E+06	0.1208244E+06	AVE
9000	11	0.4838665E+03	0.4844273E+04	0.4846457E+04	SD
10000	11	0.6014460E+05	0.2406969E+06	0.1387286E+06	AVE
10000	11	0.5040457E+03	0.5984082E+04	0.5974023E+04	SD
15000	11	0.9501644E+05	0.3956642E+06	0.2394642E+06	AVE
15000	11	0.6648535E+03	0.9518043E+04	0.9529000E+04	SD
20000	11	0.1307641E+06	0.5655217E+06	0.3548971E+06	AVE
20000	11	0.1005222E+04	0.1383978E+05	0.1383297E+05	SD
4192	12	0.2203057E+05	0.8383994E+05	0.4405907E+05	AVE
4192	12	0.2476042E+03	0.2031479E+04	0.2040848E+04	SD
4200	12	0.2213371E+05	0.8444350E+05	0.4456410E+05	AVE
4200	12	0.2911455E+03	0.2522703E+04	0.2527667E+04	SD
4300	12	0.2272493E+05	0.8698800E+05	0.4593076E+05	AVE
4300	12	0.2387243E+03	0.2133710E+04	0.2140394E+04	SD
4400	12	0.2337278E+05	0.8977319E+05	0.4755479E+05	AVE
4400	12	0.2623669E+03	0.2081474E+04	0.2088377E+04	SD
5000	12	0.2705879E+05	0.1052349E+06	0.5600765E+05	AVE
5000	12	0.3271060E+03	0.3025807E+04	0.3030109E+04	SD
6000	12	0.3346104E+05	0.1304366E+06	0.6974119E+05	AVE
6000	12	0.3382776E+03	0.3105361E+04	0.3102426E+04	SD
7000	12	0.4006215E+05	0.1563272E+06	0.8423925E+05	AVE
7000	12	0.3878164E+03	0.3537077E+04	0.3538577E+04	SD
8000	12	0.4725418E+05	0.1862502E+06	0.1029245E+06	AVE

8000	12	0.5423787E+03	0.5265504E+04	0.5275328E+04	SD
9000	12	0.5391121E+05	0.2130852E+06	0.1180767E+06	AVE
9000	12	0.4274912E+03	0.5122664E+04	0.5119781E+04	SD
10000	12	0.6088866E+05	0.2425149E+06	0.1358084E+06	AVE
10000	12	0.5325059E+03	0.6777320E+04	0.6764770E+04	SD
15000	12	0.9687375E+05	0.3947451E+06	0.2296061E+06	AVE
15000	12	0.7454990E+03	0.9977652E+04	0.1000270E+05	SD
20000	12	0.1343551E+06	0.5609917E+06	0.3370528E+06	AVE
20000	12	0.1048819E+04	0.1615142E+05	0.1615290E+05	SD

APPENDIX B

EMPIRICAL RESULTS OF SHELLSORT USING KNUTH'S INCREMENTS

Fifty repetitions were made using Program C with Knuth's increments.

N	Pass	Saves	Comparisons	Moves	Type
2	1	0.4199994E+00	0.1000000E+01	0.4199994E+00	AVE
2	1	0.4985690E+00	0.0000000E+00	0.4985690E+00	SD
3	1	0.1199990E+01	0.2639987E+01	0.1479988E+01	AVE
3	1	0.6700571E+00	0.4848724E+00	0.9089102E+00	SD
4	1	0.1759990E+01	0.4839988E+01	0.2759989E+01	AVE
4	1	0.7969308E+00	0.8417650E+00	0.1333401E+01	SD
5	1	0.2739988E+01	0.7519989E+01	0.4839988E+01	AVE
5	1	0.9216196E+00	0.1554976E+01	0.2279990E+01	SD
6	1	0.3479988E+01	0.1105999E+02	0.7399987E+01	AVE
6	1	0.9310931E+00	0.2170393E+01	0.2595122E+01	SD
7	1	0.4399987E+01	0.1505999E+02	0.1085999E+02	AVE
7	1	0.1142856E+01	0.2965163E+01	0.3625353E+01	SD
8	1	0.5359990E+01	0.1965982E+02	0.1443999E+02	AVE
8	1	0.1064450E+01	0.3611819E+01	0.4329102E+01	SD
9	1	0.6299990E+01	0.2351982E+02	0.1737985E+02	AVE
9	1	0.1073806E+01	0.4011436E+01	0.4421150E+01	SD
10	1	0.6759991E+01	0.2805981E+02	0.2085980E+02	AVE
10	1	0.1318004E+01	0.5467562E+01	0.6114579E+01	SD
12	1	0.9199990E+01	0.4345985E+02	0.3449982E+02	AVE
12	1	0.1178029E+01	0.8172123E+01	0.8795261E+01	SD
14	1	0.1065999E+02	0.5641982E+02	0.4521982E+02	AVE
14	1	0.1117757E+01	0.9333018E+01	0.9621984E+01	SD
16	1	0.1259999E+02	0.7251984E+02	0.5959982E+02	AVE
16	1	0.1160573E+01	0.9474418E+01	0.9893289E+01	SD
18	1	0.1429999E+02	0.8763985E+02	0.7287982E+02	AVE
18	1	0.1488047E+01	0.1063062E+02	0.1114400E+02	SD
20	1	0.1633986E+02	0.1108399E+03	0.9447980E+02	AVE
20	1	0.1437255E+01	0.1776300E+02	0.1817177E+02	SD
24	1	0.1989983E+02	0.1570198E+03	0.1370398E+03	AVE
24	1	0.1343920E+01	0.1807292E+02	0.1835359E+02	SD
28	1	0.2395981E+02	0.2169598E+03	0.1927198E+03	AVE
28	1	0.1470232E+01	0.1922469E+02	0.1964896E+02	SD
33	1	0.2879980E+02	0.2959568E+03	0.2670574E+03	AVE
33	1	0.1784298E+01	0.3240022E+02	0.3302872E+02	SD
38	1	0.3361983E+02	0.3874573E+03	0.3538374E+03	AVE
38	1	0.1839349E+01	0.3424690E+02	0.3467635E+02	SD
40	1	0.3549985E+02	0.4317971E+03	0.3960769E+03	AVE
40	1	0.1619366E+01	0.4868425E+02	0.4906624E+02	SD
44	1	0.3951982E+02	0.5237371E+03	0.4840576E+03	AVE
44	1	0.1656636E+01	0.4507510E+02	0.4496933E+02	SD
50	1	0.4555984E+02	0.6589568E+03	0.6133374E+03	AVE
50	1	0.1692237E+01	0.5538014E+02	0.5564510E+02	SD
60	1	0.5535986E+02	0.9316567E+03	0.8762170E+03	AVE
60	1	0.1711392E+01	0.7681360E+02	0.7731062E+02	SD
70	1	0.6513983E+02	0.1278398E+04	0.1213198E+04	AVE

70	1	0.1628961E+01	0.9092059E+02	0.9147310E+02	SD
80	1	0.7539981E+02	0.1661177E+04	0.1586117E+04	AVE
80	1	0.1840561E+01	0.1228985E+03	0.1233560E+03	SD
90	1	0.8523982E+02	0.2119637E+04	0.2035357E+04	AVE
90	1	0.1954165E+01	0.1343805E+03	0.1348322E+03	SD
100	1	0.9447978E+02	0.2555277E+04	0.2460298E+04	AVE
100	1	0.2251874E+01	0.1751108E+03	0.1752604E+03	SD
120	1	0.1147598E+03	0.3719997E+04	0.3605537E+04	AVE
120	1	0.1732765E+01	0.2113506E+03	0.2116170E+03	SD
122	1	0.1173598E+03	0.3813057E+04	0.3696257E+04	AVE
122	1	0.1723282E+01	0.2230545E+03	0.2229359E+03	SD
130	1	0.1243198E+03	0.4293379E+04	0.4168754E+04	AVE
130	1	0.2094087E+01	0.2836257E+03	0.2836260E+03	SD
140	1	0.1345798E+03	0.5012266E+04	0.4878234E+04	AVE
140	1	0.1917612E+01	0.2841489E+03	0.2840139E+03	SD
160	1	0.1544798E+03	0.6642398E+04	0.6488453E+04	AVE
160	1	0.2296773E+01	0.3636174E+03	0.3636604E+03	SD
180	1	0.1743398E+03	0.8179414E+04	0.8005141E+04	AVE
180	1	0.1813867E+01	0.4150212E+03	0.4148611E+03	SD
200	1	0.1945798E+03	0.1022669E+05	0.1003261E+05	AVE
200	1	0.2259582E+01	0.4721633E+03	0.4728093E+03	SD
300	1	0.2936172E+03	0.2276500E+05	0.2247103E+05	AVE
300	1	0.2293550E+01	0.6890349E+03	0.6893486E+03	SD
365	1	0.3583772E+03	0.3360196E+05	0.3324376E+05	AVE
365	1	0.1689413E+01	0.1053991E+04	0.1054202E+04	SD
400	1	0.3932778E+03	0.4038664E+05	0.3999318E+05	AVE
400	1	0.2339029E+01	0.1431796E+04	0.1432243E+04	SD
450	1	0.4438569E+03	0.5136229E+05	0.5091946E+05	AVE
450	1	0.2148210E+01	0.1532523E+04	0.1533091E+04	SD
1094	1	0.1085997E+04	0.3019499E+06	0.3008589E+06	AVE
1094	1	0.2240455E+01	0.5532348E+04	0.5531840E+04	SD
500	1	0.4930574E+03	0.6284736E+05	0.6235398E+05	AVE
500	1	0.2368553E+01	0.2124294E+04	0.2124976E+04	SD
1000	1	0.9923174E+03	0.2504056E+06	0.2494132E+06	AVE
1000	1	0.2795255E+01	0.5053590E+04	0.5054285E+04	SD
2000	1	0.1991817E+04	0.1001660E+07	0.9996677E+06	AVE
2000	1	0.2413516E+01	0.1582999E+05	0.1583023E+05	SD
3281	1	0.3271397E+04	0.2691398E+07	0.2688124E+07	AVE
3281	1	0.2799219E+01	0.3389569E+05	0.3389539E+05	SD
5	2	0.2559987E+01	0.7719989E+01	0.3979990E+01	AVE
5	2	0.8843279E+00	0.1278391E+01	0.1392251E+01	SD
6	2	0.3579988E+01	0.1071999E+02	0.5579989E+01	AVE
6	2	0.1011968E+01	0.1629320E+01	0.1738986E+01	SD
7	2	0.4519989E+01	0.1361999E+02	0.6899988E+01	AVE
7	2	0.1147134E+01	0.1883222E+01	0.1982061E+01	SD
8	2	0.6179989E+01	0.1729982E+02	0.9359990E+01	AVE
8	2	0.1438393E+01	0.2340856E+01	0.2981293E+01	SD
9	2	0.7019989E+01	0.2113983E+02	0.1135999E+02	AVE
9	2	0.1584232E+01	0.2312428E+01	0.2797662E+01	SD
10	2	0.8519990E+01	0.2559984E+02	0.1423999E+02	AVE
10	2	0.1619268E+01	0.2920693E+01	0.3408022E+01	SD
12	2	0.1133999E+02	0.3527983E+02	0.2053981E+02	AVE
12	2	0.1636444E+01	0.4333241E+01	0.4845383E+01	SD
14	2	0.1427999E+02	0.4453981E+02	0.2671980E+02	AVE

14	2	0.1852080E+01	0.5470535E+01	0.5989909E+01	SD
16	2	0.1705980E+02	0.5609981E+02	0.3477983E+02	AVE
16	2	0.2179796E+01	0.7324148E+01	0.7423890E+01	SD
18	2	0.1959979E+02	0.6549982E+02	0.4017978E+02	AVE
18	2	0.2213133E+01	0.7882439E+01	0.8568322E+01	SD
20	2	0.2239983E+02	0.7847981E+02	0.4971982E+02	AVE
20	2	0.2329934E+01	0.8878423E+01	0.9426512E+01	SD
24	2	0.2825984E+02	0.1044998E+03	0.6837981E+02	AVE
24	2	0.3141602E+01	0.1078022E+02	0.1159320E+02	SD
28	2	0.3485983E+02	0.1339998E+03	0.9031979E+02	AVE
28	2	0.2934727E+01	0.1394447E+02	0.1462792E+02	SD
33	2	0.4243985E+02	0.1732198E+03	0.1195198E+03	AVE
33	2	0.2483898E+01	0.1545536E+02	0.1641579E+02	SD
38	2	0.5035979E+02	0.2176398E+03	0.1558798E+03	AVE
38	2	0.2746111E+01	0.2000278E+02	0.2051837E+02	SD
40	2	0.5371982E+02	0.2428598E+03	0.1768398E+03	AVE
40	2	0.3356631E+01	0.2361682E+02	0.2450848E+02	SD
44	2	0.5959979E+02	0.2808171E+03	0.2067798E+03	AVE
44	2	0.3522504E+01	0.2879932E+02	0.2963948E+02	SD
50	2	0.6991982E+02	0.3511770E+03	0.2660974E+03	AVE
50	2	0.3463186E+01	0.3592842E+02	0.3617609E+02	SD
60	2	0.8657985E+02	0.4739570E+03	0.3689573E+03	AVE
60	2	0.3753016E+01	0.4896837E+02	0.4933247E+02	SD
70	2	0.1023198E+03	0.6059971E+03	0.4821372E+03	AVE
70	2	0.3961232E+01	0.5706126E+02	0.5785437E+02	SD
80	2	0.1199998E+03	0.7770376E+03	0.6340972E+03	AVE
80	2	0.4342464E+01	0.8517896E+02	0.8485663E+02	SD
90	2	0.1352598E+03	0.9199980E+03	0.7560376E+03	AVE
90	2	0.3379489E+01	0.8285333E+02	0.8287778E+02	SD
100	2	0.1525198E+03	0.1107297E+04	0.9252974E+03	AVE
100	2	0.4713149E+01	0.8455984E+02	0.8586136E+02	SD
120	2	0.1850598E+03	0.1534437E+04	0.1313077E+04	AVE
120	2	0.4913153E+01	0.1141766E+03	0.1145924E+03	SD
122	2	0.1883998E+03	0.1563857E+04	0.1338217E+04	AVE
122	2	0.5379499E+01	0.1159173E+03	0.1171480E+03	SD
130	2	0.2019398E+03	0.1746117E+04	0.1504517E+04	AVE
130	2	0.3824778E+01	0.1375736E+03	0.1378023E+03	SD
140	2	0.2192198E+03	0.2000497E+04	0.1739497E+04	AVE
140	2	0.4348523E+01	0.1652251E+03	0.1655927E+03	SD
160	2	0.2536198E+03	0.2570218E+04	0.2269857E+04	AVE
160	2	0.4910989E+01	0.1746526E+03	0.1756514E+03	SD
180	2	0.2870972E+03	0.3145117E+04	0.2805177E+04	AVE
180	2	0.5932405E+01	0.2510092E+03	0.2511245E+03	SD
200	2	0.3203970E+03	0.3797177E+04	0.3416938E+04	AVE
200	2	0.5106671E+01	0.2866504E+03	0.2873696E+03	SD
240	2	0.3889963E+03	0.5239098E+04	0.4780168E+04	AVE
240	2	0.6037111E+01	0.3624751E+03	0.3628477E+03	SD
280	2	0.4552773E+03	0.6953055E+04	0.6414555E+04	AVE
280	2	0.6987483E+01	0.4344285E+03	0.4345769E+03	SD
300	2	0.4913970E+03	0.7902828E+04	0.7325012E+04	AVE
300	2	0.5142990E+01	0.5226350E+03	0.5223384E+03	SD
365	2	0.6033572E+03	0.1122140E+05	0.1051396E+05	AVE
365	2	0.6438973E+01	0.5371631E+03	0.5382671E+03	SD
400	2	0.6628765E+03	0.1322282E+05	0.1244504E+05	AVE

400	2	0.6106894E+01	0.6081013E+03	C.6085264E+03	SD
450	2	0.7497573E+03	0.1658344E+05	0.1570679E+05	AVE
450	2	0.7598495E+01	0.9685557E+03	C.9684653E+03	SD
500	2	0.8354172E+03	0.2018520E+05	C.1920865E+05	AVE
500	2	0.8424823E+01	0.1232774E+04	0.1233239E+04	SD
1000	2	0.1702317E+04	0.7478119E+05	C.7280794E+05	AVE
1000	2	0.1005819E+02	0.2749723E+04	C.2749971E+04	SD
1094	2	0.1867417E+04	0.8936706E+05	0.8720644E+05	AVE
1094	2	0.9955269E+01	0.3129830E+04	0.3130547E+04	SD
2000	2	0.3437198E+04	0.2831827E+06	C.2792120E+06	AVE
2000	2	0.1376194E+02	0.7742957E+04	0.7743285E+04	SD
3281	2	0.5666312E+04	0.7404134E+06	C.7338824E+06	AVE
3281	2	0.1866125E+02	0.1793469E+05	0.1793503E+05	SD
9842	2	0.1710092E+05	0.6377386E+07	0.6357739E+07	AVE
9842	2	0.2561116E+02	0.7958775E+05	C.7958831E+05	SD
14	3	0.1349999E+02	0.4397984E+02	C.2459982E+02	AVE
14	3	0.1876275E+01	0.5172488E+01	0.5202040E+01	SD
16	3	0.1637982E+02	0.5337984E+02	C.2861981E+02	AVE
16	3	0.2019197E+01	0.5710312E+01	C.6110837E+01	SD
18	3	0.1935982E+02	0.6481982E+02	0.3483981E+02	AVE
18	3	0.2505191E+01	0.6980137E+01	C.7051897E+01	SD
20	3	0.2299983E+02	0.7553981E+02	C.4135982E+02	AVE
20	3	0.2784805E+01	0.6812977E+01	0.7585592E+01	SD
24	3	0.3091980E+02	0.9931981E+02	0.5603983E+02	AVE
24	3	0.3029354E+01	0.8057038E+01	C.8866258E+01	SD
28	3	0.3755983E+02	0.1254398E+03	0.7135982E+02	AVE
28	3	0.3693108E+01	0.9870897E+01	0.1043882E+02	SD
33	3	0.4659981E+02	0.1598798E+03	C.9267981E+02	AVE
33	3	0.4090800E+01	0.1412424E+02	0.1502162E+02	SD
38	3	0.5651981E+02	0.1975598E+03	0.1172598E+03	AVE
38	3	0.5087962E+01	0.1407432E+02	C.1571400E+02	SD
40	3	0.6039983E+02	0.2065398E+03	0.1209598E+03	AVE
40	3	0.4304710E+01	0.1542749E+02	C.1649234E+02	SD
44	3	0.6769980E+02	0.2406198E+03	C.1436798E+03	AVE
44	3	0.4811769E+01	0.1630557E+02	0.1655222E+02	SD
50	3	0.8015984E+02	0.2860769E+03	0.1718198E+03	AVE
50	3	0.5708399E+01	0.2039987E+02	0.2113672E+02	SD
60	3	0.1012798E+03	0.3714971E+03	0.2298998E+03	AVE
60	3	0.4932333E+01	0.2434848E+02	C.2498340E+02	SD
70	3	0.1228198E+03	0.4609575E+03	0.2906375E+03	AVE
70	3	0.5920142E+01	0.2496156E+02	0.2564517E+02	SD
80	3	0.1449998E+03	0.5576570E+03	0.3602573E+03	AVE
80	3	0.7428556E+01	0.3496603E+02	C.3586456E+02	SD
90	3	0.1671598E+03	0.6651372E+03	0.4390369E+03	AVE
90	3	0.6290088E+01	0.3381430E+02	C.3472952E+02	SD
100	3	0.1895598E+03	0.7713777E+03	C.5164771E+03	AVE
100	3	0.6411603E+01	0.3814423E+02	0.3878328E+02	SD
120	3	0.2329198E+03	0.9949373E+03	C.6820972E+03	AVE
120	3	0.8412415E+01	0.4713258E+02	C.4831657E+02	SD
122	3	0.2406598E+03	0.1030017E+04	0.7115967E+03	AVE
122	3	0.6974031E+01	0.4877705E+02	0.4823030E+02	SD
130	3	0.2572380E+03	0.1117777E+04	C.7765776E+03	AVE
130	3	0.8620205E+01	0.5303439E+02	0.5436365E+02	SD
140	3	0.2794570E+03	0.1261397E+04	C.8899773E+03	AVE

140	3	0.8844044E+01	0.6362793E+02	0.6473965E+02	SD
160	3	0.3258967E+03	0.1517217E+04	0.1088657E+04	AVE
160	3	0.8919198E+01	0.5227582E+02	0.5237073E+02	SD
180	3	0.3750369E+03	0.1816717E+04	0.1329097E+04	AVE
180	3	0.1017366E+02	0.7320700E+02	0.7477777E+02	SD
200	3	0.4188169E+03	0.2135277E+04	0.1589017E+04	AVE
200	3	0.1167356E+02	0.9706001E+02	0.9759460E+02	SD
240	3	0.5117773E+03	0.2787637E+04	0.2122057E+04	AVE
240	3	0.1194073E+02	0.1025883E+03	0.1038560E+03	SD
280	3	0.6123970E+03	0.3586317E+04	0.2803297E+04	AVE
280	3	0.1227087E+02	0.1283593E+03	0.1288638E+03	SD
330	3	0.7286975E+03	0.4616852E+04	0.3686118E+04	AVE
330	3	0.1584245E+02	0.1503846E+03	0.1518065E+03	SD
365	3	0.8102371E+03	0.5391590E+04	0.4357695E+04	AVE
365	3	0.1561957E+02	0.2020376E+03	0.2032500E+03	SD
380	3	0.8480977E+03	0.5785617E+04	0.4708973E+04	AVE
380	3	0.1617798E+02	0.1871720E+03	0.1878738E+03	SD
400	3	0.8988569E+03	0.6283512E+04	0.5145426E+04	AVE
400	3	0.1596567E+02	0.2452881E+03	0.2465441E+03	SD
440	3	0.9941765E+03	0.7312613E+04	0.6056008E+04	AVE
440	3	0.1517598E+02	0.2559543E+03	0.2572812E+03	SD
450	3	0.1017977E+04	0.7617531E+04	0.6331555E+04	AVE
450	3	0.1456261E+02	0.2370937E+03	0.2375052E+03	SD
500	3	0.1139237E+04	0.9018258E+04	0.7584457E+04	AVE
500	3	0.1655109E+02	0.2577615E+03	0.2590615E+03	SD
550	3	0.1266437E+04	0.1058503E+05	0.9001660E+04	AVE
550	3	0.1685332E+02	0.2999868E+03	0.2995151E+03	SD
600	3	0.1389257E+04	0.1233512E+05	0.1060346E+05	AVE
600	3	0.1848082E+02	0.4233784E+03	0.4249531E+03	SD
700	3	0.1637117E+04	0.1603149E+05	0.1400135E+05	AVE
700	3	0.1991231E+02	0.4580037E+03	0.4578433E+03	SD
800	3	0.1876657E+04	0.2001568E+05	0.1768820E+05	AVE
800	3	0.2434000E+02	0.6864639E+03	0.6879185E+03	SD
900	3	0.2125437E+04	0.2447129E+05	0.2184456E+05	AVE
900	3	0.2008762E+02	0.6041487E+03	0.6058455E+03	SD
1000	3	0.2371637E+04	0.2953513E+05	0.2661061E+05	AVE
1000	3	0.2722406E+02	0.8336687E+03	0.8339001E+03	SD
1094	3	0.2604657E+04	0.3463379E+05	0.3142674E+05	AVE
1094	3	0.2173822E+02	0.9621753E+03	0.9642478E+03	SD
2000	3	0.4861391E+04	0.1021732E+06	0.9625762E+05	AVE
2000	3	0.4027464E+02	0.2249113E+04	0.2248865E+04	SD
3281	3	0.8056910E+04	0.2562143E+06	0.2464605E+06	AVE
3281	3	0.5845679E+02	0.4914883E+04	0.4914441E+04	SD
5000	3	0.1239689E+05	0.5680166E+06	0.5531112E+06	AVE
5000	3	0.6757372E+02	0.8175238E+04	0.8174398E+04	SD
9842	3	0.2460615E+05	0.2091391E+07	0.2061970E+07	AVE
9842	3	0.8019727E+02	0.2766982E+05	0.2767023E+05	SD
15000	3	0.3764270E+05	0.4732328E+07	0.4687431E+07	AVE
15000	3	0.9699803E+02	0.4737289E+05	0.4737400E+05	SD
20000	3	0.5026467E+05	0.8309740E+07	0.8249855E+07	AVE
20000	3	0.1099666E+03	0.7174744E+05	0.7174694E+05	SD
44	4	0.6969983E+02	0.2407598E+03	0.1403798E+03	AVE
44	4	0.4348333E+01	0.1593421E+02	0.1584215E+02	SD
50	4	0.8091980E+02	0.2907573E+03	0.1676199E+03	AVE

50	4	0.4685333E+01	0.2266292E+02	0.2286415E+02	SD
60	4	0.1028798E+03	0.3673770E+03	0.2100399E+03	AVE
60	4	0.5393231E+01	0.1930713E+02	0.2079237E+02	SD
70	4	0.1263798E+03	0.4553169E+03	0.2638977E+03	AVE
70	4	0.5122565E+01	0.2253789E+02	0.2310442E+02	SD
80	4	0.1519798E+03	0.5476768E+03	0.3222976E+03	AVE
80	4	0.6072628E+01	0.2915302E+02	0.2839844E+02	SD
90	4	0.1744798E+03	0.6354370E+03	0.3718774E+03	AVE
90	4	0.8899080E+01	0.3158990E+02	0.3170699E+02	SD
100	4	0.2006398E+03	0.7339165E+03	0.4331167E+03	AVE
100	4	0.7665872E+01	0.2881403E+02	0.2993929E+02	SD
120	4	0.2547198E+03	0.9339172E+03	0.5639172E+03	AVE
120	4	0.8923816E+01	0.4064372E+02	0.4078159E+02	SD
122	4	0.2579775E+03	0.9533572E+03	0.5750574E+03	AVE
122	4	0.9809258E+01	0.4311842E+02	0.4359738E+02	SD
130	4	0.2795574E+03	0.1034577E+04	0.6243965E+03	AVE
130	4	0.9502198E+01	0.3735625E+02	0.3799760E+02	SD
140	4	0.3055371E+03	0.1141717E+04	0.6953770E+03	AVE
140	4	0.7877174E+01	0.4299692E+02	0.4450493E+02	SD
160	4	0.3622773E+03	0.1369277E+04	0.8477371E+03	AVE
160	4	0.1197474E+02	0.5844534E+02	0.5993683E+02	SD
180	4	0.4178767E+03	0.1592817E+04	0.9946975E+03	AVE
180	4	0.1391576E+02	0.6357535E+02	0.6531621E+02	SD
200	4	0.4739768E+03	0.1835937E+04	0.1163317E+04	AVE
200	4	0.1281479E+02	0.6960129E+02	0.7156738E+02	SD
240	4	0.5861572E+03	0.2322557E+04	0.1497777E+04	AVE
240	4	0.1288809E+02	0.7715247E+02	0.7996213E+02	SD
280	4	0.7047974E+03	0.2882037E+04	0.1900897E+04	AVE
280	4	0.1560226E+02	0.1090926E+03	0.1090495E+03	SD
330	4	0.8482168E+03	0.3606417E+04	0.2433037E+04	AVE
330	4	0.1621570E+02	0.1215384E+03	0.1207644E+03	SD
365	4	0.9545972E+03	0.4113531E+04	0.2804397E+04	AVE
365	4	0.1758128E+02	0.1338038E+03	0.1328825E+03	SD
380	4	0.9944373E+03	0.4341758E+04	0.2972097E+04	AVE
380	4	0.1687578E+02	0.1419415E+03	0.1418830E+03	SD
400	4	0.1060857E+04	0.4657656E+04	0.3210197E+04	AVE
400	4	0.1693083E+02	0.1523583E+03	0.1534295E+03	SD
440	4	0.1181897E+04	0.5347187E+04	0.3743918E+04	AVE
440	4	0.2287955E+02	0.2187581E+03	0.2195726E+03	SD
450	4	0.1205757E+04	0.5473516E+04	0.3829677E+04	AVE
450	4	0.2386749E+02	0.1729995E+03	0.1747522E+03	SD
500	4	0.1359857E+04	0.6324797E+04	0.4487270E+04	AVE
500	4	0.2056334E+02	0.1900065E+03	0.1892964E+03	SD
550	4	0.1513897E+04	0.7247598E+04	0.5212852E+04	AVE
550	4	0.2122865E+02	0.2035942E+03	0.2044082E+03	SD
600	4	0.1666937E+04	0.8235555E+04	0.6004320E+04	AVE
600	4	0.2618025E+02	0.2621074E+03	0.2631279E+03	SD
700	4	0.1977458E+04	0.1028774E+05	0.7660930E+04	AVE
700	4	0.3122816E+02	0.3288984E+03	0.3300227E+03	SD
800	4	0.2292237E+04	0.1251727E+05	0.9498734E+04	AVE
800	4	0.2709320E+02	0.3459626E+03	0.3490261E+03	SD
900	4	0.2611117E+04	0.1492359E+05	0.1150859E+05	AVE
900	4	0.2801926E+02	0.4081992E+03	0.4080674E+03	SD
1000	4	0.2913857E+04	0.1734020E+05	0.1353097E+05	AVE

1000	4	0.3206332E+02	0.4628936E+03	0.4650388E+03	SD
1094	4	0.3195597E+04	0.1972871E+05	0.1554368E+05	AVE
1094	4	0.3072852E+02	0.4928447E+03	0.4928862E+03	SD
1100	4	0.3225937E+04	0.1992939E+05	0.1572186E+05	AVE
1100	4	0.4149724E+02	0.4907666E+03	0.4915332E+03	SD
1200	4	0.3538197E+04	0.2271150E+05	0.1810709E+05	AVE
1200	4	0.3110806E+02	0.3950303E+03	0.3988328E+03	SD
1400	4	0.4177883E+04	0.2901836E+05	0.2362096E+05	AVE
1400	4	0.3873912E+02	0.6505000E+03	0.6520562E+03	SD
1600	4	0.4800312E+04	0.3542731E+05	0.2923639E+05	AVE
1600	4	0.4921208E+02	0.9040767E+03	0.9063386E+03	SD
1800	4	0.5441969E+04	0.4268006E+05	0.3569328E+05	AVE
1800	4	0.5620677E+02	0.1024061E+04	0.1022911E+04	SD
2000	4	0.6087145E+04	0.5057271E+05	0.4278937E+05	AVE
2000	4	0.5886142E+02	0.1207632E+04	0.1208385E+04	SD
2200	4	0.6731035E+04	0.5907604E+05	0.5049866E+05	AVE
2200	4	0.5188538E+02	0.1084447E+04	0.1084798E+04	SD
2400	4	0.7368754E+04	0.6813594E+05	0.5876173E+05	AVE
2400	4	0.5621761E+02	0.1321393E+04	0.1318948E+04	SD
2800	4	0.8641680E+04	0.8730756E+05	0.7633669E+05	AVE
2800	4	0.6678008E+02	0.1455824E+04	0.1457081E+04	SD
3281	4	0.1020831E+05	0.1141339E+06	0.1012424E+06	AVE
3281	4	0.7003049E+02	0.1864726E+04	0.1865204E+04	SD
5000	4	0.1577121E+05	0.2354166E+06	0.2156678E+06	AVE
5000	4	0.1035059E+03	0.2949839E+04	0.2944580E+04	SD
9842	4	0.3156137E+05	0.7907220E+06	0.7516373E+06	AVE
9842	4	0.1278343E+03	0.8347711E+04	0.8349242E+04	SD
10000	4	0.3207125E+05	0.8151585E+06	0.7754392E+06	AVE
10000	4	0.1549083E+03	0.7839828E+04	0.7839887E+04	SD
15000	4	0.4848881E+05	0.1724905E+07	0.1665204E+07	AVE
15000	4	0.1877542E+03	0.2089825E+05	0.2089825E+05	SD
20000	4	0.6497440E+05	0.2968712E+07	0.2889027E+07	AVE
20000	4	0.2557112E+03	0.1978149E+05	0.1978216E+05	SD
122	5	0.2586575E+03	0.9544570E+03	0.5751167E+03	AVE
122	5	0.1070085E+02	0.4509894E+02	0.4784717E+02	SD
130	5	0.2794575E+03	0.1031917E+04	0.6131367E+03	AVE
130	5	0.9965610E+01	0.4375316E+02	0.4533075E+02	SD
140	5	0.3049175E+03	0.1148677E+04	0.6817573E+03	AVE
140	5	0.9866168E+01	0.4593427E+02	0.4572157E+02	SD
160	5	0.3626172E+03	0.1376617E+04	0.8196770E+03	AVE
160	5	0.1149763E+02	0.5257544E+02	0.5408250E+02	SD
180	5	0.4225974E+03	0.1593737E+04	0.9485972E+03	AVE
180	5	0.1075168E+02	0.5117587E+02	0.5277711E+02	SD
200	5	0.4826372E+03	0.1830577E+04	0.1096537E+04	AVE
200	5	0.1033634E+02	0.5811519E+02	0.5911098E+02	SD
240	5	0.6133369E+03	0.2309757E+04	0.1404897E+04	AVE
240	5	0.1449638E+02	0.8824390E+02	0.8749942E+02	SD
280	5	0.7335566E+03	0.2824857E+04	0.1726297E+04	AVE
280	5	0.1904530E+02	0.1109610E+03	0.1087328E+03	SD
330	5	0.9001770E+03	0.3493057E+04	0.2165177E+04	AVE
330	5	0.1669960E+02	0.1143542E+03	0.1150875E+03	SD
365	5	0.1015817E+04	0.3907617E+04	0.2419257E+04	AVE
365	5	0.2258649E+02	0.1317802E+03	0.1341193E+03	SD
380	5	0.1063477E+04	0.4132797E+04	0.2571157E+04	AVE

380	5	0.2035818E+02	0.1514462E+03	0.1522015E+03	SD
400	5	0.1131497E+04	0.4426898E+04	0.2769537E+04	AVE
400	5	0.1916132E+02	0.1370739E+03	0.1368423E+03	SD
440	5	0.1265417E+04	0.4946449E+04	0.3101518E+04	AVE
440	5	0.2383412E+02	0.1448683E+03	0.1458784E+03	SD
450	5	0.1305197E+04	0.5168656E+04	0.3275617E+04	AVE
450	5	0.2396956E+02	0.1438755E+03	0.1442858E+03	SD
500	5	0.1476257E+04	0.5880773E+04	0.3748737E+04	AVE
500	5	0.2314189E+02	0.1803971E+03	0.1801782E+03	SD
550	5	0.1647677E+04	0.6608070E+04	0.4235855E+04	AVE
550	5	0.2279768E+02	0.1804334E+03	0.1845168E+03	SD
600	5	0.1832878E+04	0.7446895E+04	0.4837738E+04	AVE
600	5	0.2976276E+02	0.2124216E+03	0.2155157E+03	SD
700	5	0.2176977E+04	0.8936832E+04	0.5843465E+04	AVE
700	5	0.2765648E+02	0.2398249E+03	0.2425399E+03	SD
800	5	0.2534657E+04	0.1070742E+05	0.7128199E+04	AVE
800	5	0.3354686E+02	0.3629734E+03	0.3636348E+03	SD
900	5	0.2901856E+04	0.1244505E+05	0.8380477E+04	AVE
900	5	0.3364366E+02	0.3294866E+03	0.3327224E+03	SD
1000	5	0.3268277E+04	0.1432083E+05	0.9766711E+04	AVE
1000	5	0.4266055E+02	0.4924692E+03	0.4946309E+03	SD
1094	5	0.3626357E+04	0.1616025E+05	0.1115027E+05	AVE
1094	5	0.3716840E+02	0.3969709E+03	0.3983635E+03	SD
1100	5	0.3632997E+04	0.1619054E+05	0.1114823E+05	AVE
1100	5	0.4614503E+02	0.4869158E+03	0.4856831E+03	SD
1200	5	0.4021977E+04	0.1828320E+05	0.1275345E+05	AVE
1200	5	0.4542876E+02	0.5172617E+03	0.5171633E+03	SD
1400	5	0.4770152E+04	0.2245643E+05	0.1594578E+05	AVE
1400	5	0.5622211E+02	0.8068491E+03	0.8108728E+03	SD
1600	5	0.5508187E+04	0.2663635E+05	0.1914168E+05	AVE
1600	5	0.5040793E+02	0.7937659E+03	0.7936758E+03	SD
1800	5	0.6278117E+04	0.3154877E+05	0.2306393E+05	AVE
1800	5	0.5426955E+02	0.8393594E+03	0.8385557E+03	SD
2000	5	0.7072570E+04	0.3654576E+05	0.2707857E+05	AVE
2000	5	0.6139233E+02	0.1230870E+04	0.1233546E+04	SD
2200	5	0.7818199E+04	0.4137027E+05	0.3091055E+05	AVE
2200	5	0.6088637E+02	0.1005191E+04	0.1007933E+04	SD
2400	5	0.8588969E+04	0.4673312E+05	0.3528421E+05	AVE
2400	5	0.6984094E+02	0.1194204E+04	0.1196505E+04	SD
2800	5	0.1012995E+05	0.5813385E+05	0.4469916E+05	AVE
2800	5	0.7966698E+02	0.1574420E+04	0.1577286E+04	SD
3281	5	0.1201201E+05	0.7292544E+05	0.5711257E+05	AVE
3281	5	0.8929726E+02	0.1894600E+04	0.1898760E+04	SD
3300	5	0.1208140E+05	0.7333969E+05	0.5742712E+05	AVE
3300	5	0.8026883E+02	0.1667193E+04	0.1668284E+04	SD
3800	5	0.1404156E+05	0.9033544E+05	0.7194131E+05	AVE
3800	5	0.1104490E+03	0.2617965E+04	0.2616465E+04	SD
4200	5	0.1562704E+05	0.1047992E+06	0.8441775E+05	AVE
4200	5	0.1122338E+03	0.2291861E+04	0.2291496E+04	SD
4400	5	0.1638198E+05	0.1112556E+06	0.8987587E+05	AVE
4400	5	0.1194240E+03	0.2197591E+04	0.2198997E+04	SD
5000	5	0.1876797E+05	0.1354059E+06	0.1110426E+06	AVE
5000	5	0.1230440E+03	0.2839064E+04	0.2838512E+04	SD
6000	5	0.2276008E+05	0.1796429E+06	0.1503002E+06	AVE

6000	5	0.1541787E+03	0.4499309E+04	0.4505191E+04	SD
7000	5	0.2673580E+05	0.2263116E+06	0.1919919E+06	AVE
7000	5	0.1421859E+03	0.3428411E+04	0.3425821E+04	SD
8000	5	0.3074507E+05	0.2798032E+06	0.2405025E+06	AVE
8000	5	0.1741856E+03	0.5368918E+04	0.5370418E+04	SD
9000	5	0.3475038E+05	0.3375946E+06	0.2933056E+06	AVE
9000	5	0.2014506E+03	0.6232871E+04	0.6235203E+04	SD
9842	5	0.3811424E+05	0.3900106E+06	0.3415269E+06	AVE
9842	5	0.2049997E+03	0.7864273E+04	0.7868234E+04	SD
10000	5	0.3875820E+05	0.4016624E+06	0.3523819E+06	AVE
10000	5	0.1956306E+03	0.6876922E+04	0.6881707E+04	SD
15000	5	0.5896742E+05	0.7826536E+06	0.7084251E+06	AVE
15000	5	0.3017244E+03	0.1097835E+05	0.1097968E+05	SD
20000	5	0.7914700E+05	0.1274395E+07	0.1175195E+07	AVE
20000	5	0.3110186E+03	0.1544575E+05	0.1544705E+05	SD
365	6	0.1022717E+04	0.3953097E+04	0.2466257E+04	AVE
365	6	0.2096370E+02	0.1116682E+03	0.1129673E+03	SD
380	6	0.1065796E+04	0.4142844E+04	0.2566617E+04	AVE
380	6	0.1988434E+02	0.1328699E+03	0.1323102E+03	SD
400	6	0.1127097E+04	0.4438234E+04	0.2742977E+04	AVE
400	6	0.2099869E+02	0.1491180E+03	0.1474372E+03	SD
440	6	0.1261617E+04	0.4985191E+04	0.3067678E+04	AVE
440	6	0.2436182E+02	0.1505069E+03	0.1512926E+03	SD
450	6	0.1291897E+04	0.5111238E+04	0.3137017E+04	AVE
450	6	0.2552571E+02	0.1964161E+03	0.1978977E+03	SD
500	6	0.1469517E+04	0.5858891E+04	0.3604657E+04	AVE
500	6	0.2186337E+02	0.1556376E+03	0.1558368E+03	SD
550	6	0.1660138E+04	0.6591477E+04	0.4071272E+04	AVE
550	6	0.2613370E+02	0.1800909E+03	0.1802050E+03	SD
600	6	0.1854417E+04	0.7382875E+04	0.4595570E+04	AVE
600	6	0.2694788E+02	0.2276424E+03	0.2297088E+03	SD
700	6	0.2249017E+04	0.8975039E+04	0.5651770E+04	AVE
700	6	0.3089510E+02	0.2668638E+03	0.2684148E+03	SD
800	6	0.2616137E+04	0.1058891E+05	0.6696594E+04	AVE
800	6	0.3535503E+02	0.4013103E+03	0.4025166E+03	SD
900	6	0.2996437E+04	0.1209599E+05	0.7630590E+04	AVE
900	6	0.3863567E+02	0.2784109E+03	0.2791392E+03	SD
1000	6	0.3412997E+04	0.1387580E+05	0.8856211E+04	AVE
1000	6	0.3841827E+02	0.4017161E+03	0.4026687E+03	SD
1100	6	0.3829577E+04	0.1564365E+05	0.1006893E+05	AVE
1100	6	0.4512943E+02	0.4669849E+03	0.4658701E+03	SD
1094	6	0.3809697E+04	0.1560016E+05	0.1005594E+05	AVE
1094	6	0.4594702E+02	0.4062261E+03	0.4072517E+03	SD
1200	6	0.4221414E+04	0.1750721E+05	0.1134387E+05	AVE
1200	6	0.4568895E+02	0.5409407E+03	0.5411611E+03	SD
1400	6	0.5062879E+04	0.2120633E+05	0.1390216E+05	AVE
1400	6	0.5241064E+02	0.6381741E+03	0.6370071E+03	SD
1600	6	0.5906035E+04	0.2505846E+05	0.1659152E+05	AVE
1600	6	0.6309601E+02	0.7814927E+03	0.7841277E+03	SD
1800	6	0.6753055E+04	0.2897273E+05	0.1935095E+05	AVE
1800	6	0.6450252E+02	0.1108586E+04	0.1109321E+04	SD
2000	6	0.7603953E+04	0.3291202E+05	0.2212336E+05	AVE
2000	6	0.6109334E+02	0.9478594E+03	0.9476130E+03	SD
2200	6	0.8493156E+04	0.3735800E+05	0.2540463E+05	AVE

2200	6	0.6302803E+02	0.1116961E+04	0.1118277E+04	SD
2400	6	0.9353020E+04	0.4161411E+05	0.2848598E+05	AVE
2400	6	0.8394855E+02	0.1308964E+04	0.1312376E+04	SD
2800	6	0.1113125E+05	0.5116046E+05	0.3568832E+05	AVE
2800	6	0.9865845E+02	0.1676846E+04	0.1677918E+04	SD
3281	6	0.1329552E+05	0.6247272E+05	0.4417713E+05	AVE
3281	6	0.1046527E+03	0.2001375E+04	0.2001660E+04	SD
3300	6	0.1335823E+05	0.6313103E+05	0.4472573E+05	AVE
3300	6	0.9264554E+02	0.1992819E+04	0.1991669E+04	SD
3800	6	0.1563214E+05	0.7574081E+05	0.5438000E+05	AVE
3800	6	0.1057298E+03	0.2060303E+04	0.2063328E+04	SD
4200	6	0.1738705E+05	0.8487181E+05	0.6114045E+05	AVE
4200	6	0.1288602E+03	0.2587342E+04	0.2586990E+04	SD
4400	6	0.1835143E+05	0.9156875E+05	0.6665775E+05	AVE
4400	6	0.1463075E+03	0.2854002E+04	0.2853839E+04	SD
5000	6	0.2104540E+05	0.1077766E+06	0.7931906E+05	AVE
5000	6	0.1426401E+03	0.3491243E+04	0.3497786E+04	SD
6000	6	0.2566528E+05	0.1372259E+06	0.1028236E+06	AVE
6000	6	0.2203952E+03	0.4634426E+04	0.4631953E+04	SD
7000	6	0.3030059E+05	0.1692671E+06	0.1289179E+06	AVE
7000	6	0.1869851E+03	0.5069086E+04	0.5067820E+04	SD
8000	6	0.3492575E+05	0.2013321E+06	0.1550325E+06	AVE
8000	6	0.1947186E+03	0.5247855E+04	0.5245094E+04	SD
9000	6	0.3963395E+05	0.2376502E+06	0.1853977E+06	AVE
9000	6	0.2359204E+03	0.6040504E+04	0.6043820E+04	SD
9842	6	0.4360831E+05	0.2693768E+06	0.2121053E+06	AVE
9842	6	0.2352712E+03	0.6362121E+04	0.6370285E+04	SD
10000	6	0.4433516E+05	0.2737302E+06	0.2155132E+06	AVE
10000	6	0.2332060E+03	0.7115082E+04	0.7120031E+04	SD
15000	6	0.6799262E+05	0.4894576E+06	0.4013814E+06	AVE
15000	6	0.3915713E+03	0.1180857E+05	0.1181089E+05	SD
20000	6	0.9185962E+05	0.7466722E+06	0.6287050E+06	AVE
20000	6	0.4886624E+03	0.1660094E+05	0.1660118E+05	SD
1094	7	0.3815317E+04	0.1560944E+05	0.1006465E+05	AVE
1094	7	0.3190485E+02	0.4652031E+03	0.4648591E+03	SD
1100	7	0.3833937E+04	0.1567978E+05	0.1009436E+05	AVE
1100	7	0.4837837E+02	0.5236553E+03	0.5245439E+03	SD
1200	7	0.4229129E+04	0.1754500E+05	0.1128069E+05	AVE
1200	7	0.4744907E+02	0.5782063E+03	0.5739521E+03	SD
1400	7	0.5051375E+04	0.2124573E+05	0.1364991E+05	AVE
1400	7	0.5693967E+02	0.7531060E+03	0.7542354E+03	SD
1600	7	0.5925891E+04	0.2498173E+05	0.1609065E+05	AVE
1600	7	0.5676851E+02	0.7770215E+03	0.7812251E+03	SD
1800	7	0.6841137E+04	0.2872777E+05	0.1857847E+05	AVE
1800	7	0.6666553E+02	0.8843977E+03	0.8799678E+03	SD
2000	7	0.7741270E+04	0.3272401E+05	0.2129637E+05	AVE
2000	7	0.6921378E+02	0.9249880E+03	0.9248130E+03	SD
2200	7	0.8714016E+04	0.3713534E+05	0.2443416E+05	AVE
2200	7	0.7481187E+02	0.1390681E+04	0.1389357E+04	SD
2400	7	0.9581680E+04	0.4108813E+05	0.2702606E+05	AVE
2400	7	0.8727168E+02	0.1263262E+04	0.1260999E+04	SD
2800	7	0.1145130E+05	0.4960887E+05	0.3287909E+05	AVE
2800	7	0.8512555E+02	0.1254043E+04	0.1260254E+04	SD
3281	7	0.1384362E+05	0.6126569E+05	0.4137377E+05	AVE

3281	7	0.1118447E+03	0.2446336E+04	0.2456044E+04	SD
3300	7	0.1391202E+05	0.6087187E+05	0.4084963E+05	AVE
3300	7	0.1055180E+03	0.1781170E+04	0.1781242E+04	SD
3800	7	0.1629171E+05	0.7250569E+05	0.4908263E+05	AVE
3800	7	0.1317288E+03	0.2566359E+04	0.2574999E+04	SD
4200	7	0.1832152E+05	0.8215162E+05	0.5605188E+05	AVE
4200	7	0.1357051E+03	0.2877495E+04	0.2880894E+04	SD
4400	7	0.1933907E+05	0.8699494E+05	0.5955304E+05	AVE
4400	7	0.1287285E+03	0.2334583E+04	0.2341406E+04	SD
5000	7	0.2228988E+05	0.1022382E+06	0.7069881E+05	AVE
5000	7	0.1677881E+03	0.3407462E+04	0.3405752E+04	SD
6000	7	0.2739315E+05	0.1282110E+06	0.8985000E+05	AVE
6000	7	0.1787944E+03	0.4842461E+04	0.4845812E+04	SD
7000	7	0.3244789E+05	0.1540091E+06	0.1088194E+06	AVE
7000	7	0.2200474E+03	0.4724715E+04	0.4721055E+04	SD
8000	7	0.3767515E+05	0.1820414E+06	0.1299957E+06	AVE
8000	7	0.1964545E+03	0.5273820E+04	0.5275699E+04	SD
9000	7	0.4291805E+05	0.2112309E+06	0.1523026E+06	AVE
9000	7	0.2765305E+03	0.6460930E+04	0.6455293E+04	SD
9842	7	0.4735229E+05	0.2346514E+06	0.1699363E+06	AVE
9842	7	0.3081121E+03	0.6948180E+04	0.6946551E+04	SD
10000	7	0.4816686E+05	0.2418491E+06	0.1760386E+06	AVE
10000	7	0.2773083E+03	0.8183723E+04	0.8187020E+04	SD
15000	7	0.7484131E+05	0.4033775E+06	0.3029927E+06	AVE
15000	7	0.3726001E+03	0.1094607E+05	0.1095196E+05	SD
20000	7	0.1018001E+06	0.5889891E+06	0.4538956E+06	AVE
20000	7	0.4413435E+03	0.1506447E+05	0.1506842E+05	SD
3281	8	0.1382439E+05	0.6054703E+05	0.4065783E+05	AVE
3281	8	0.1230009E+03	0.2321734E+04	0.2322756E+04	SD
3300	8	0.1390003E+05	0.6050401E+05	0.4046497E+05	AVE
3300	8	0.1138522E+03	0.2092306E+04	0.2095713E+04	SD
3800	8	0.1628318E+05	0.7272987E+05	0.4880572E+05	AVE
3800	8	0.1266753E+03	0.2307703E+04	0.2309148E+04	SD
4200	8	0.1827766E+05	0.8207931E+05	0.5510602E+05	AVE
4200	8	0.1296523E+03	0.2857020E+04	0.2852109E+04	SD
4400	8	0.1932354E+05	0.8763450E+05	0.5915117E+05	AVE
4400	8	0.1567108E+03	0.3054644E+04	0.3059657E+04	SD
5000	8	0.2235847E+05	0.1006696E+06	0.6775406E+05	AVE
5000	8	0.1359551E+03	0.2692270E+04	0.2693256E+04	SD
6000	8	0.2769828E+05	0.1250172E+06	0.8474531E+05	AVE
6000	8	0.1882116E+03	0.3844639E+04	0.3856279E+04	SD
7000	8	0.3307033E+05	0.1516075E+06	0.1037871E+06	AVE
7000	8	0.1785337E+03	0.4289977E+04	0.4286957E+04	SD
8000	8	0.3853776E+05	0.1787326E+06	0.1231902E+06	AVE
8000	8	0.2128242E+03	0.4810648E+04	0.4817465E+04	SD
9000	8	0.4409089E+05	0.2058942E+06	0.1427591E+06	AVE
9000	8	0.2293943E+03	0.6110359E+04	0.6113273E+04	SD
9842	8	0.4900986E+05	0.2317714E+06	0.1622716E+06	AVE
9842	8	0.2999448E+03	0.7185500E+04	0.7190180E+04	SD
10000	8	0.4989638E+05	0.2363331E+06	0.1655850E+06	AVE
10000	8	0.2782231E+03	0.6583812E+04	0.6585734E+04	SD
15000	8	0.7838294E+05	0.3835084E+06	0.2738927E+06	AVE
15000	8	0.4011931E+03	0.1210407E+05	0.1211167E+05	SD
20000	8	0.1079509E+06	0.5502411E+06	0.4015575E+06	AVE

20000	8	0.4610645E+03	0.1632942E+05	0.1632580E+05	SD
9842	9	0.4904980E+05	0.2322473E+06	0.1627559E+06	AVE
9842	9	0.2633020E+03	0.7181270E+04	0.7201215E+04	SD
10000	9	0.4981017E+05	0.2347467E+06	0.1638485E+06	AVE
10000	9	0.2556932E+03	0.6676367E+04	0.6684703E+04	SD
15000	9	0.7890769E+05	0.3879627E+06	0.2742056E+06	AVE
15000	9	0.3834666E+03	0.1138517E+05	0.1137442E+05	SD
20000	9	0.1098226E+06	0.5495724E+06	0.3939294E+06	AVE
20000	9	0.4919666E+03	0.1713050E+05	0.1713328E+05	SD

APPENDIX C

EMPIRICAL RESULTS OF SHELLSORT USING

PRATT'S INCREMENTS

Fifty repetitions were made using Program E with Pratt's increments.

N	Interchanges	Comparisons	Type
2	0.4199994E+00	0.1000000E+01	AVE
2	0.4985690E+00	0.0000000E+00	SD
5	0.3479989E+01	0.8479990E+01	AVE
5	0.1281577E+01	0.8388485E+00	SD
10	0.1079999E+02	0.3089984E+02	AVE
10	0.2424363E+01	0.1656759E+01	SD
20	0.3391982E+02	0.9917979E+02	AVE
20	0.4256251E+01	0.3739965E+01	SD
50	0.1238398E+03	0.4115574E+03	AVE
50	0.9896108E+01	0.8487765E+01	SD
100	0.3238572E+03	0.1128077E+04	AVE
100	0.1897282E+02	0.1800291E+02	SD
200	0.8088569E+03	0.2981177E+04	AVE
200	0.3790422E+02	0.3219229E+02	SD
500	0.2637097E+04	0.1025093E+05	AVE
500	0.9164096E+02	0.8959196E+02	SD
1000	0.6305035E+04	0.2537381E+05	AVE
1000	0.1800338E+03	0.1796776E+03	SD
2000	0.1482911E+05	0.6145547E+05	AVE
2000	0.2928613E+03	0.2807837E+03	SD
5000	0.4469401E+05	0.1938936E+06	AVE
5000	0.8613052E+03	0.8552341E+03	SD
10000	0.1025526E+06	0.4548204E+06	AVE
10000	0.1706500E+04	0.1700505E+04	SD
15000	0.1662295E+06	0.7437364E+06	AVE
15000	0.2494716E+04	0.2500478E+04	SD
20000	0.2323074E+06	0.1052895E+07	AVE
20000	0.2558284E+04	0.2562661E+04	SD

APPENDIX D

COMPUTER DATA PLOTS

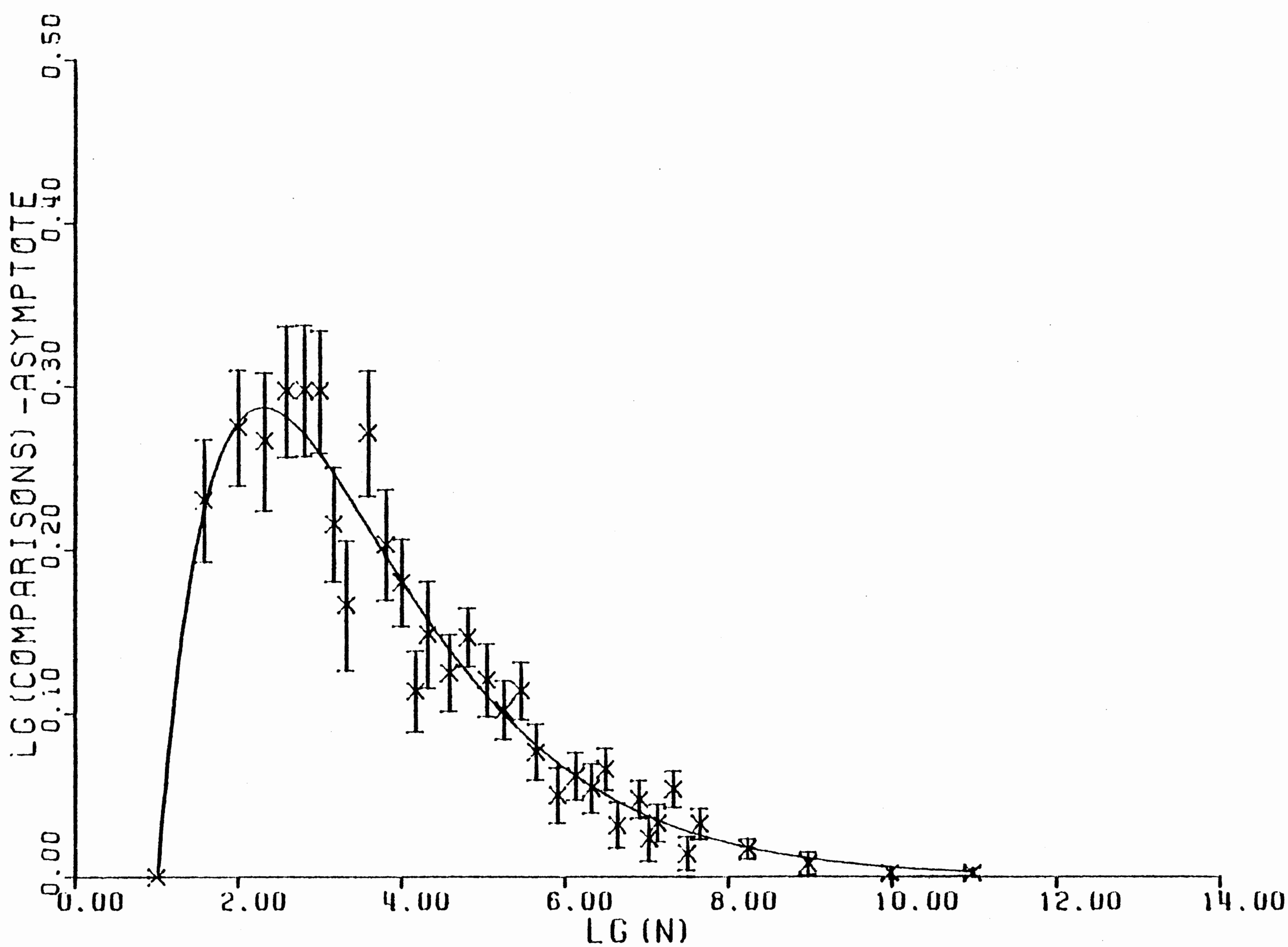

Figure 8. One-pass Hibbard's Shellsort with Known Asymptote Subtracted

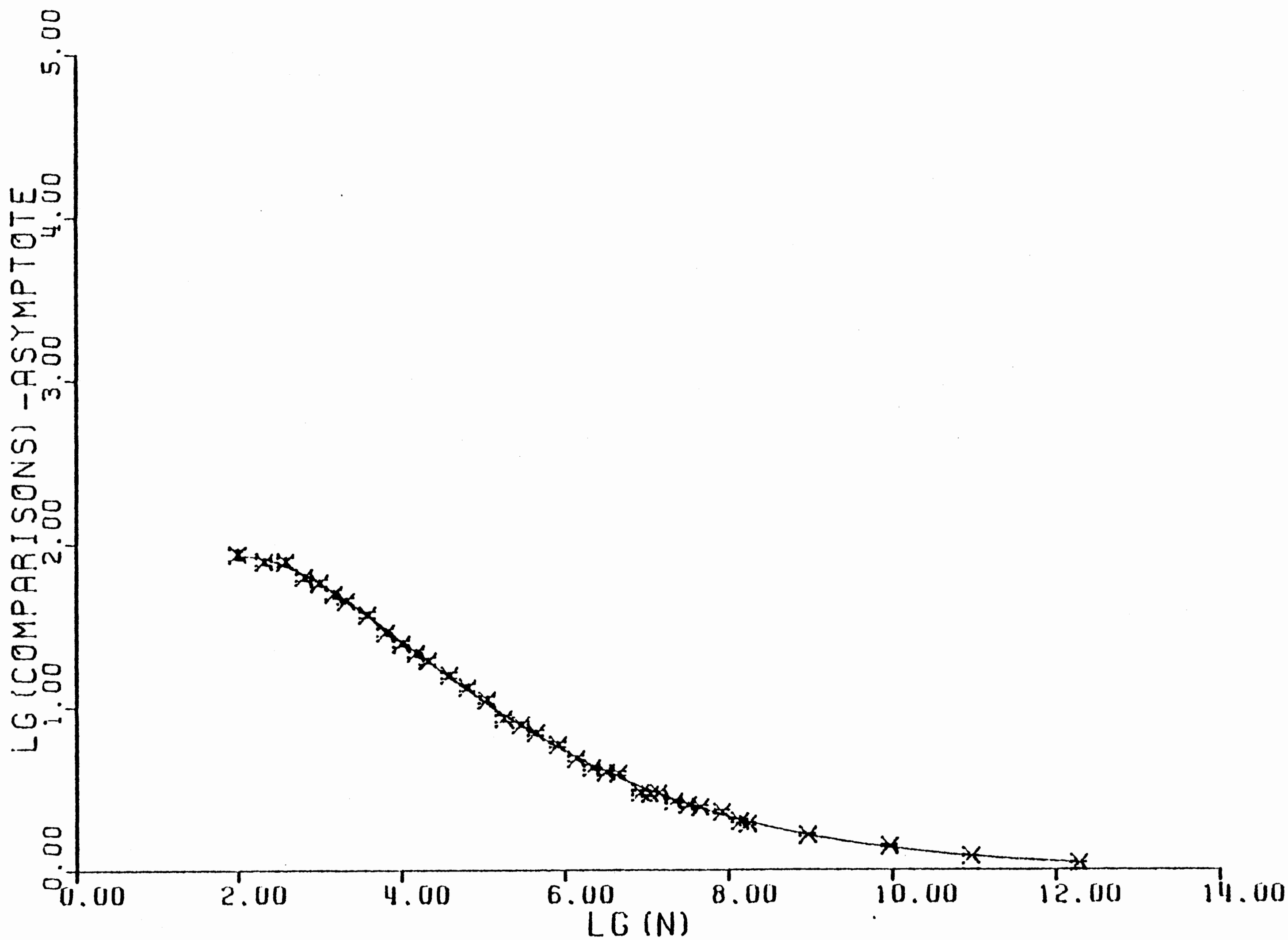

Figure 9. Two-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

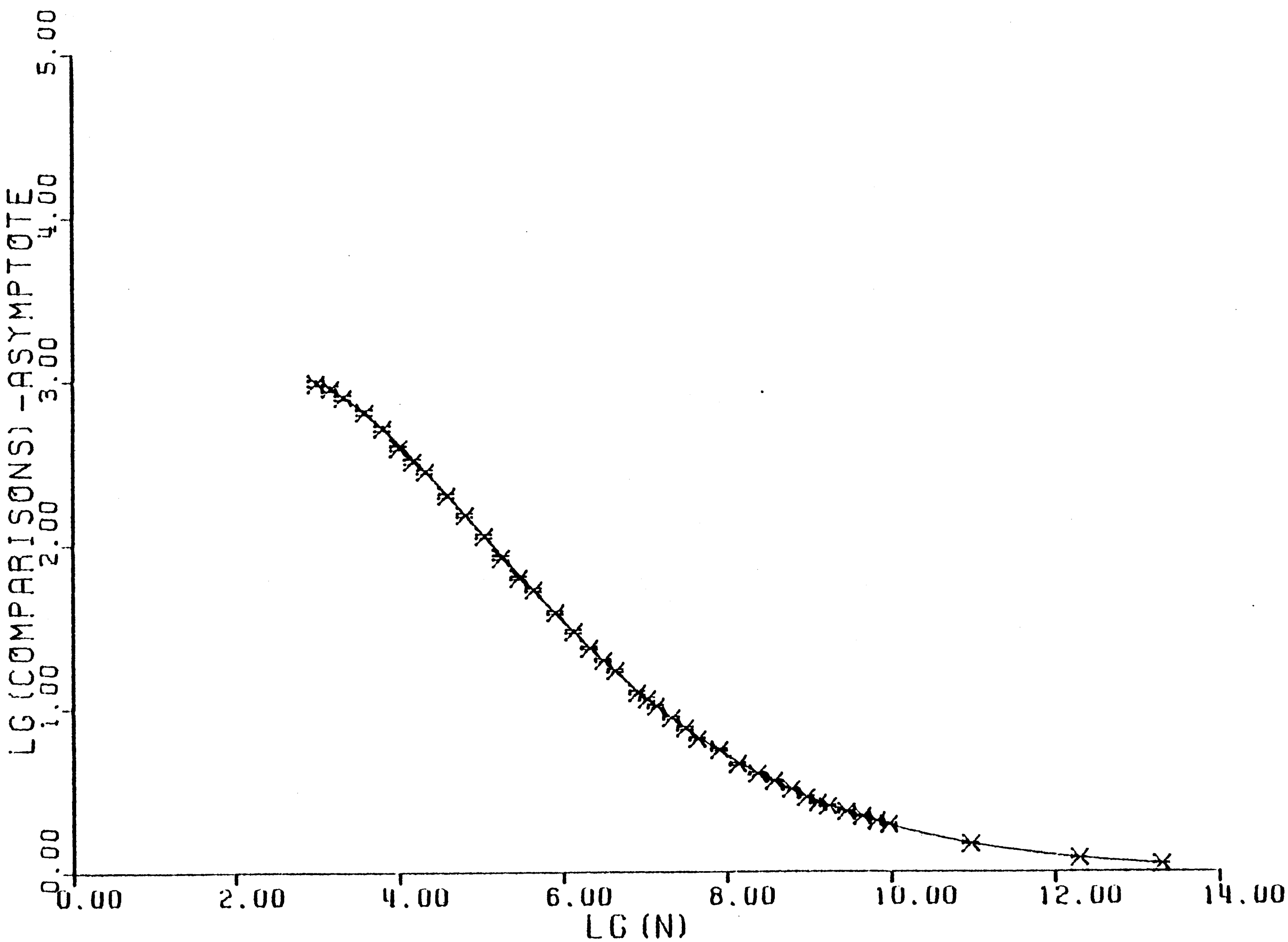

Figure 10. Three-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

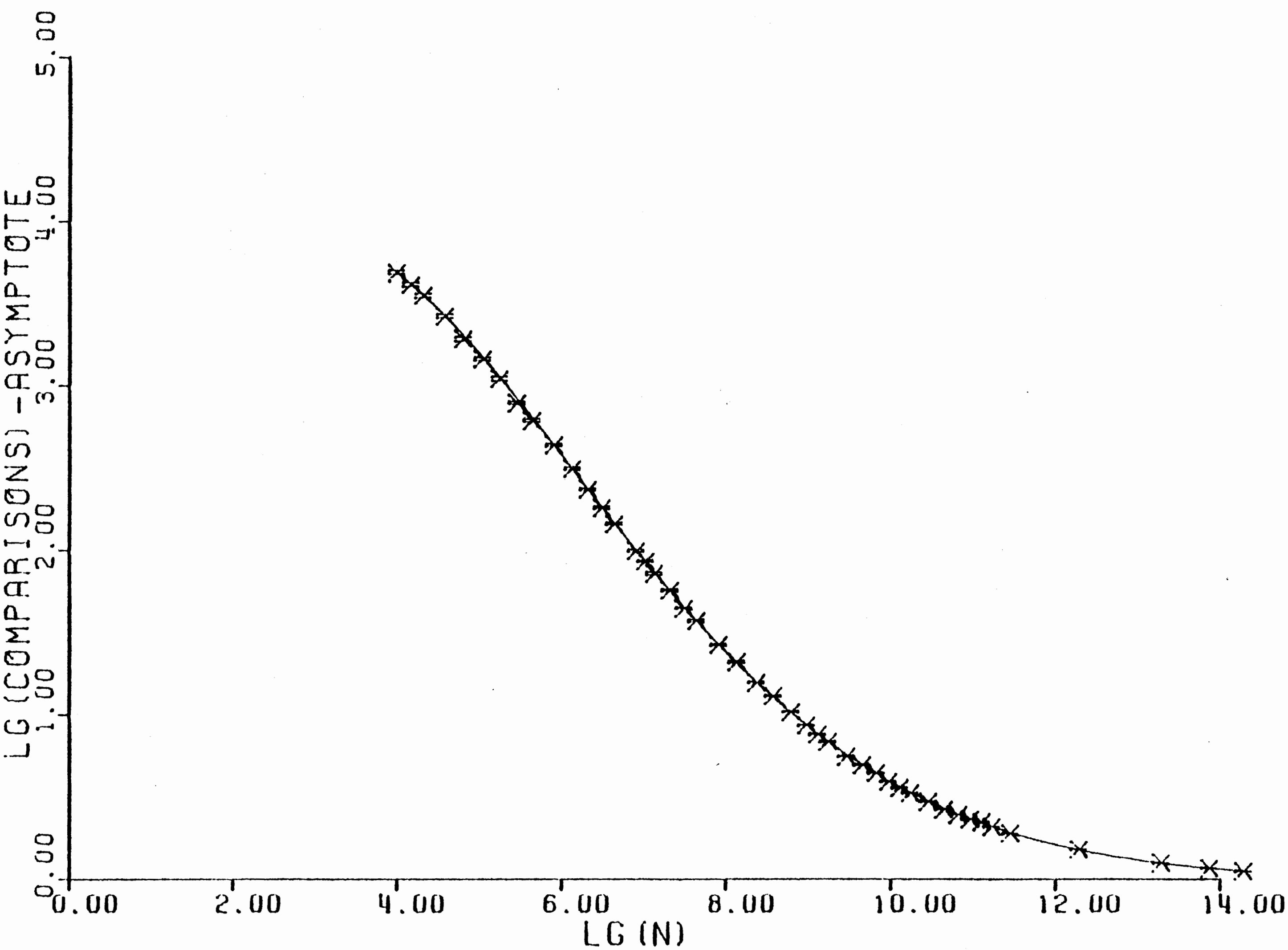

Figure 11. Four-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

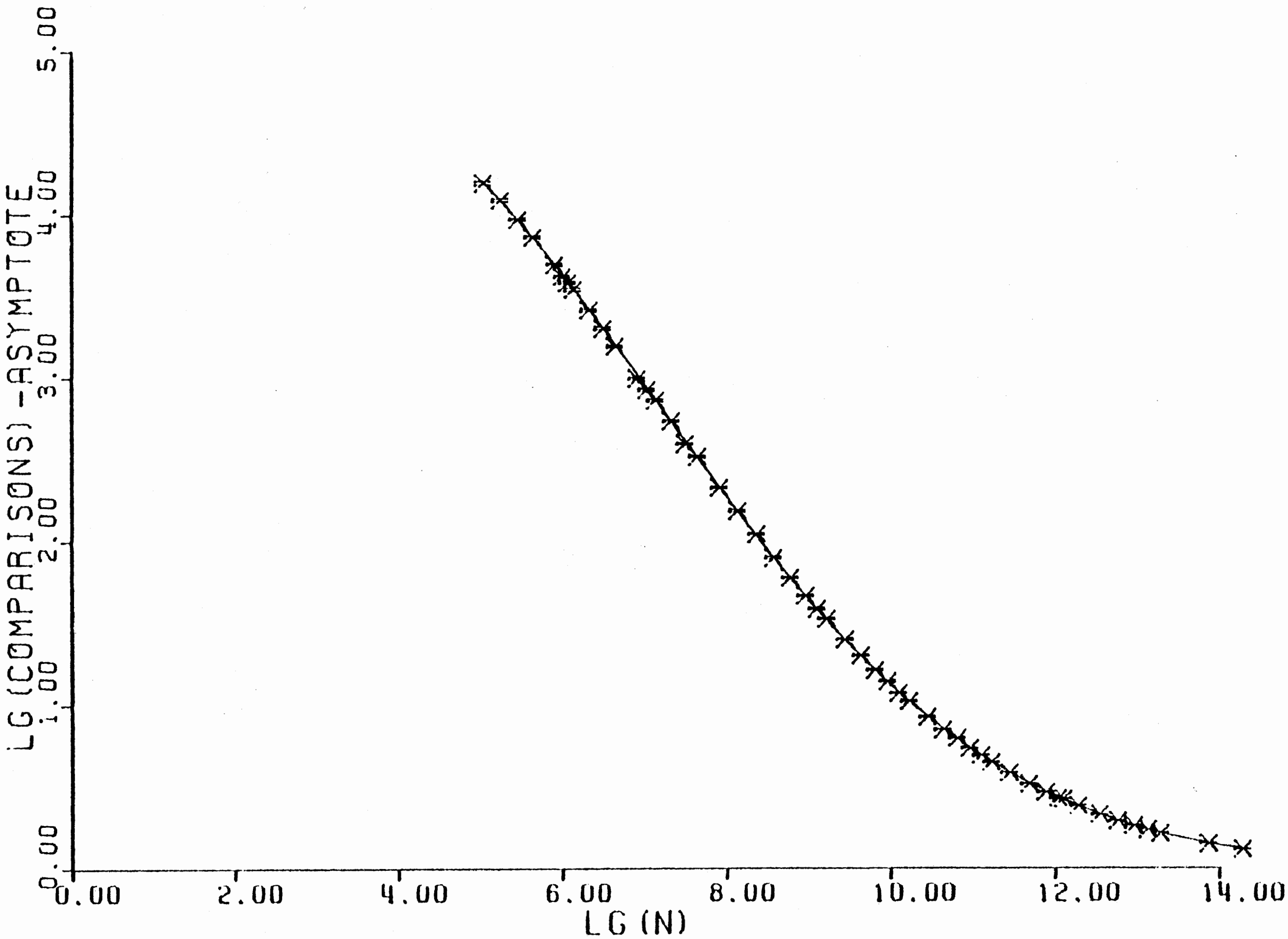

Figure 12. Five-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

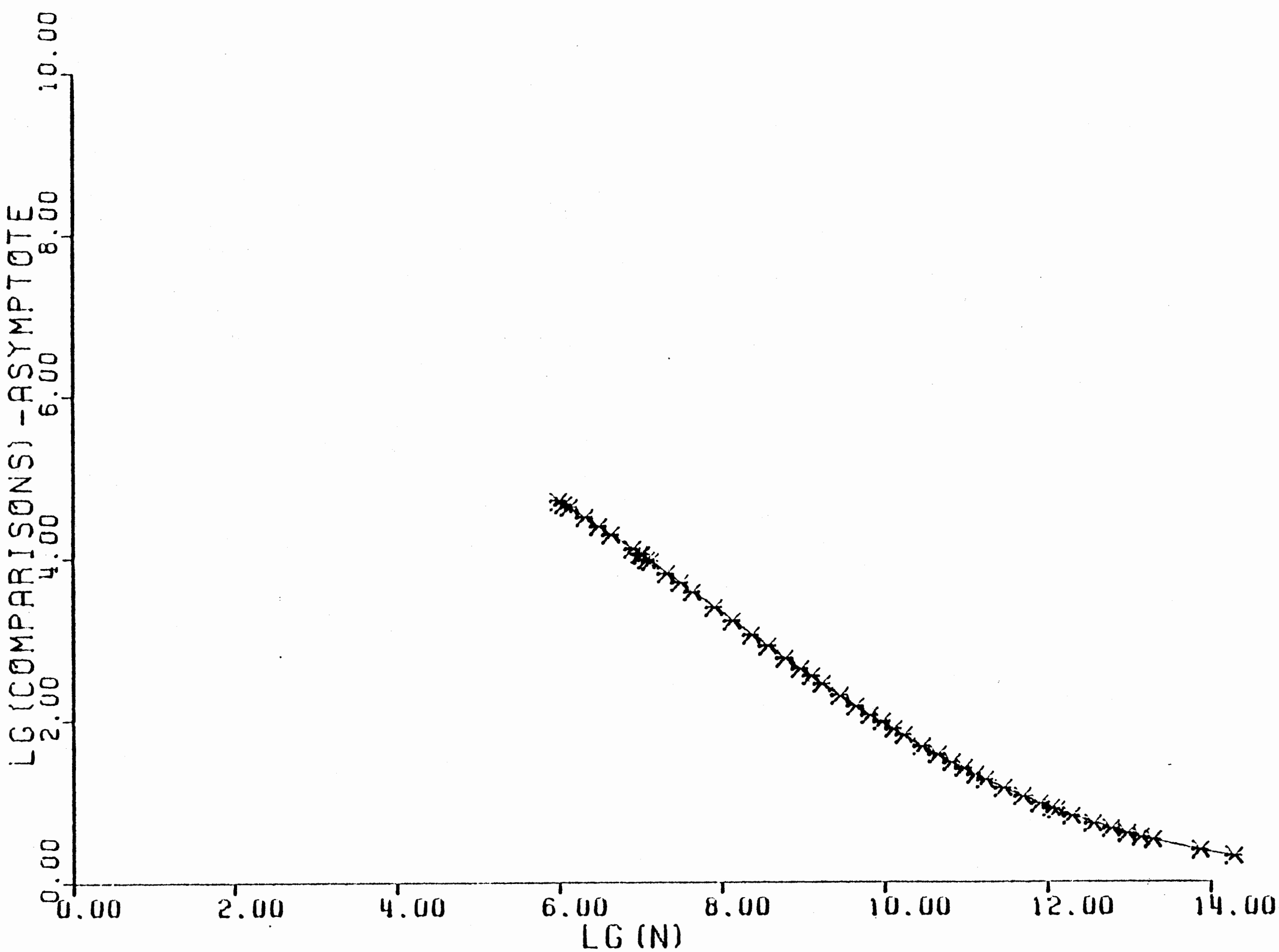

Figure 13. Six-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

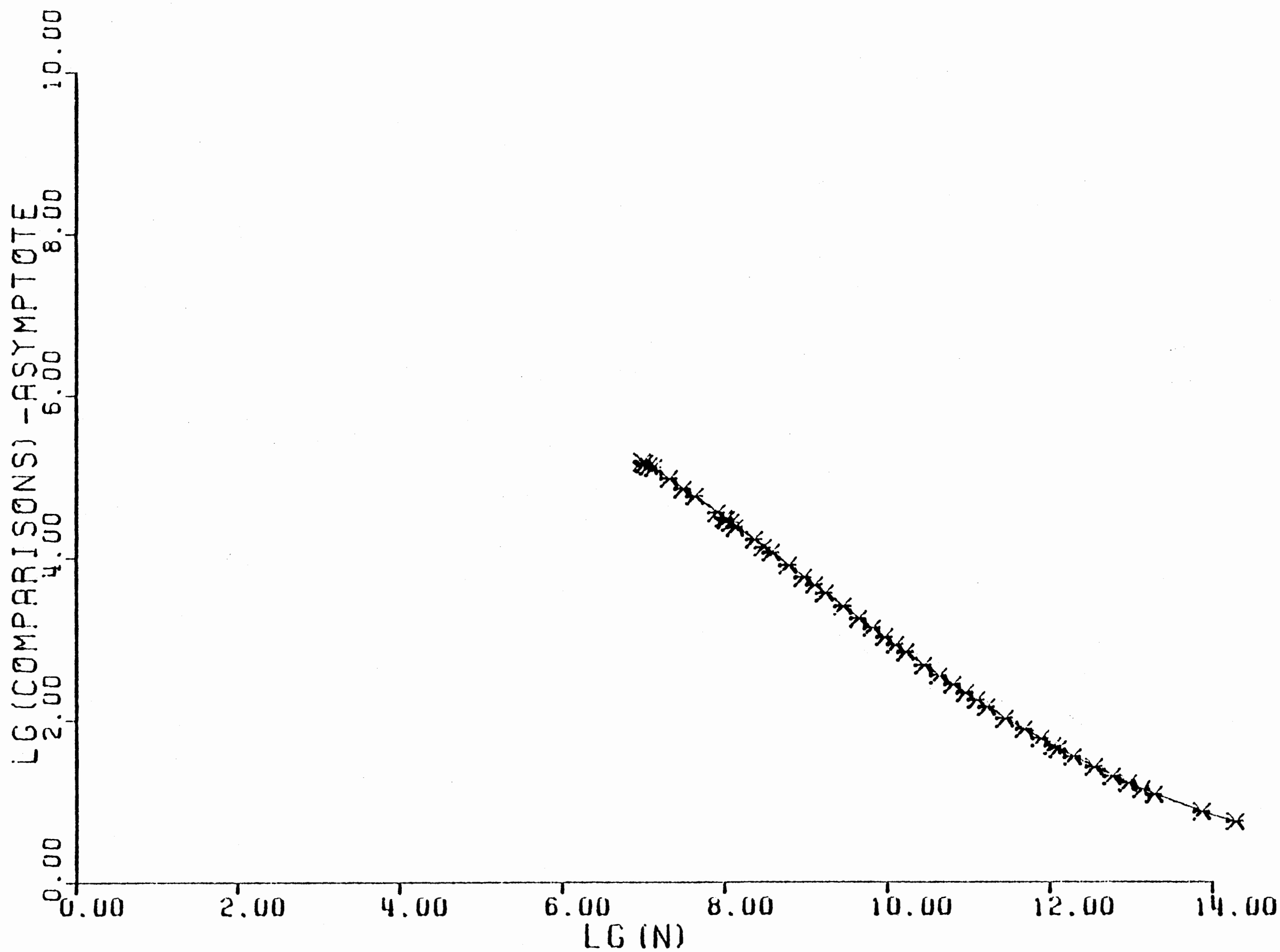

Figure 14. Seven-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

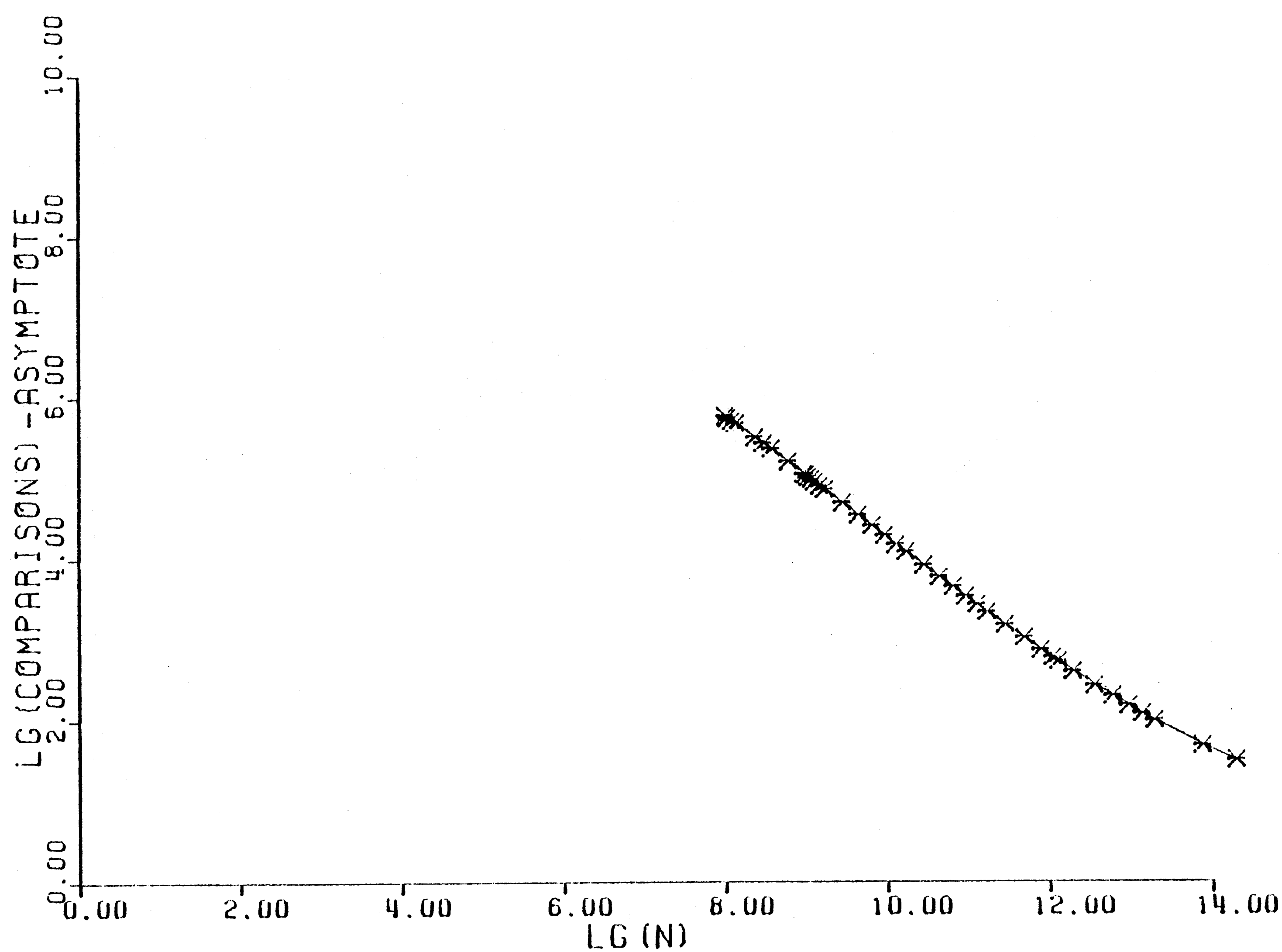

Figure 15. Eight-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

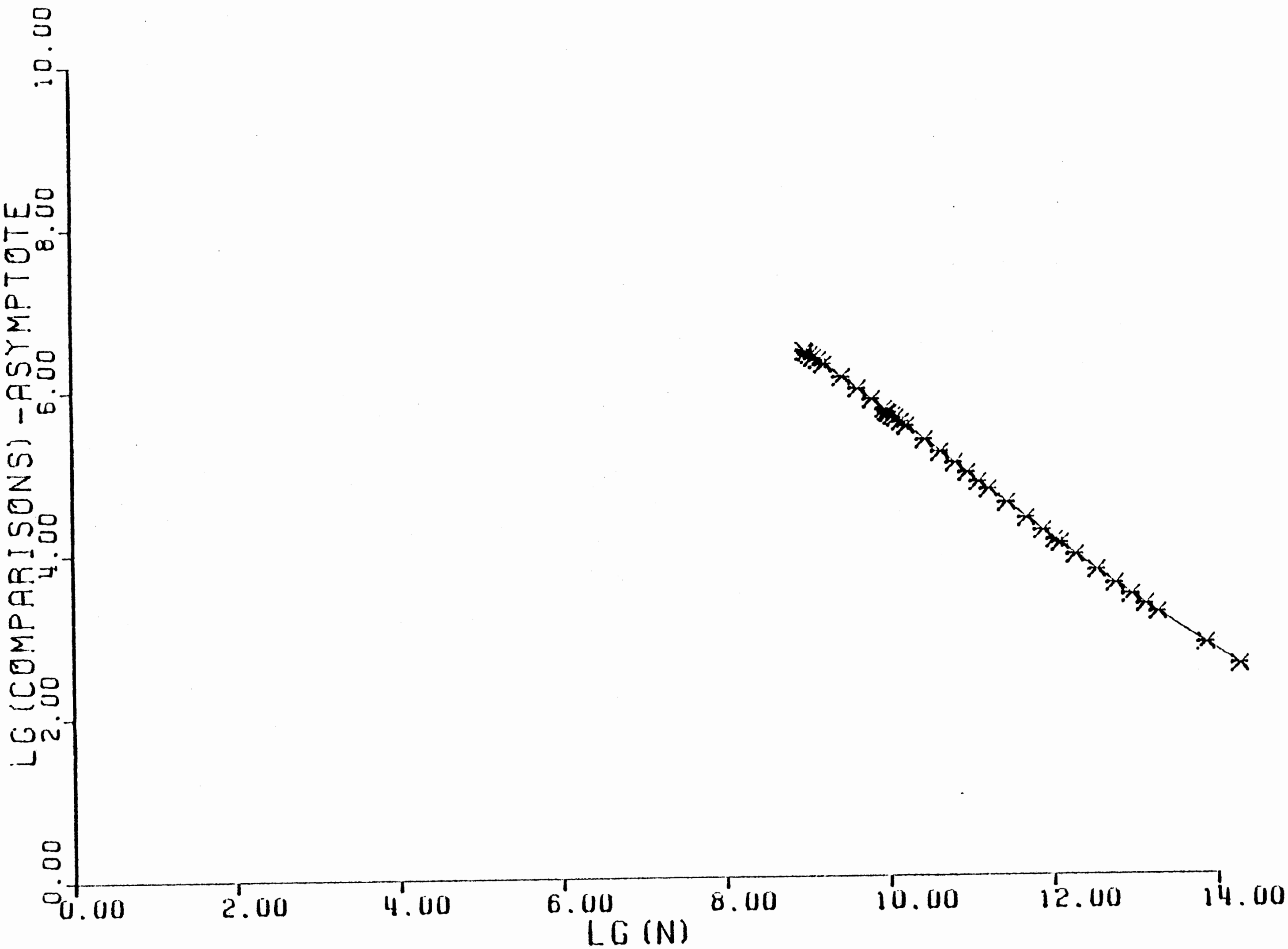

Figure 16. Nine-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

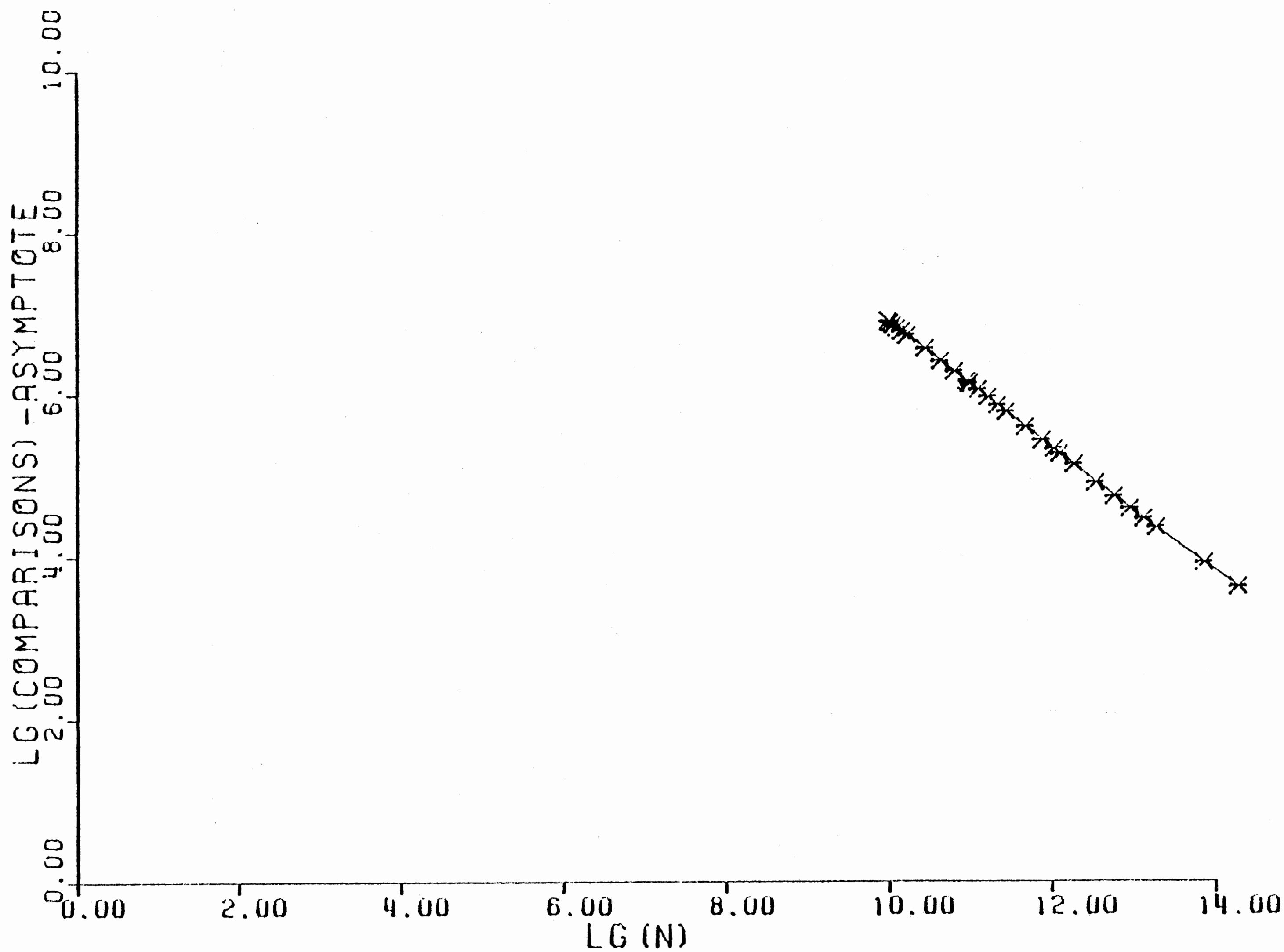

Figure 17. Ten-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

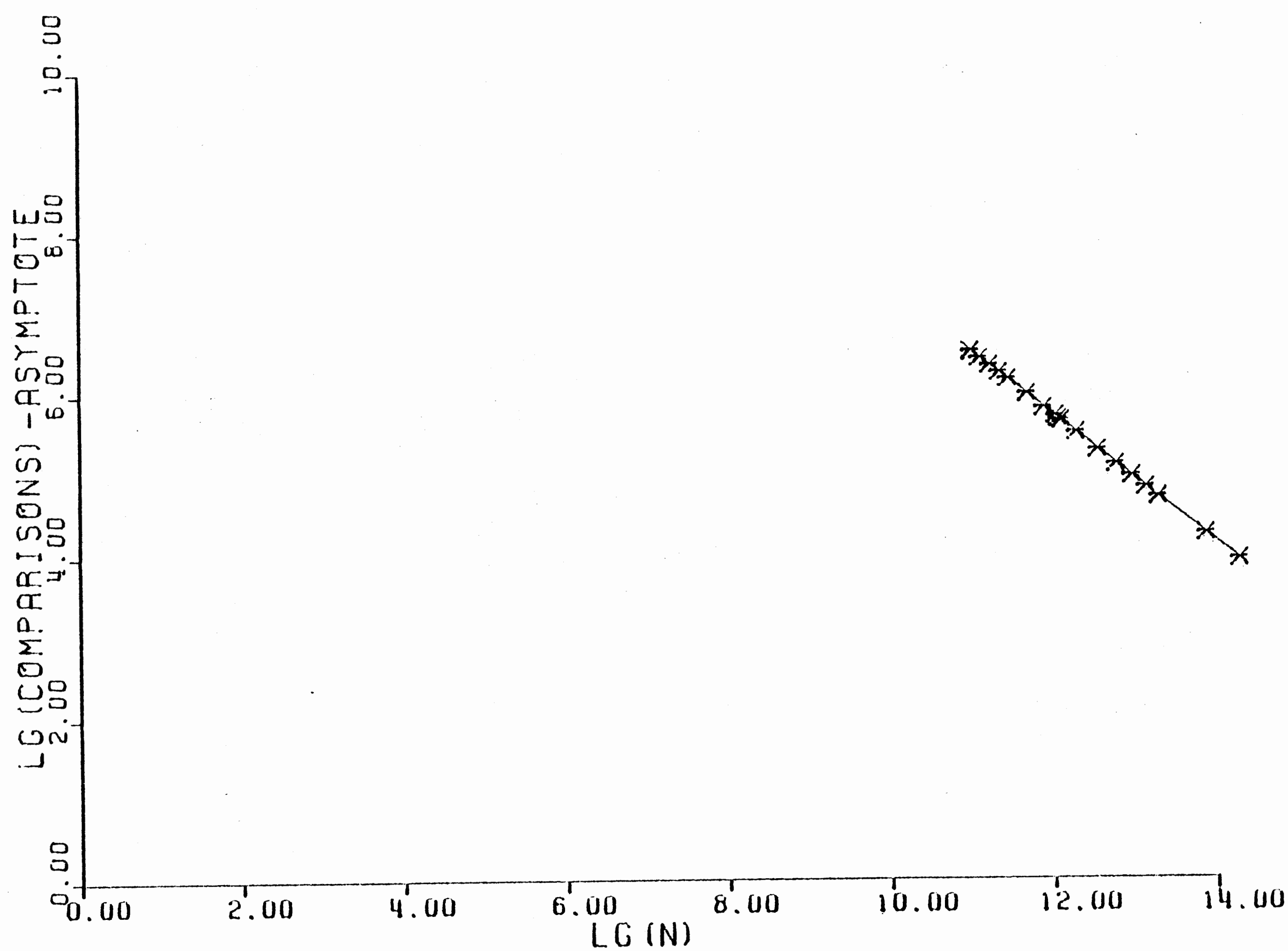

Figure 18. Eleven-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

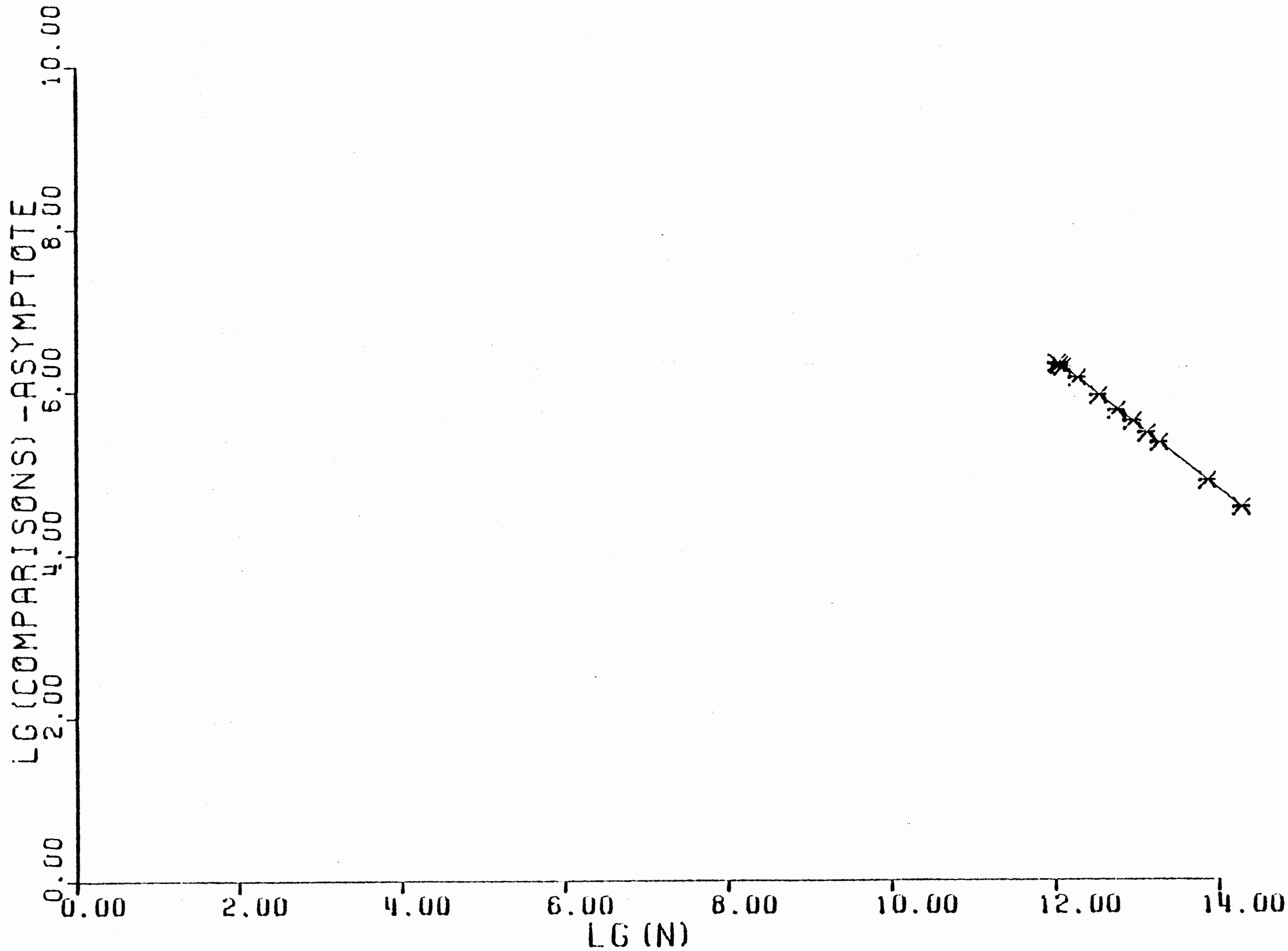

Figure 19. Twelve-pass Hibbard's Shellsort with Fitted Asymptote Subtracted

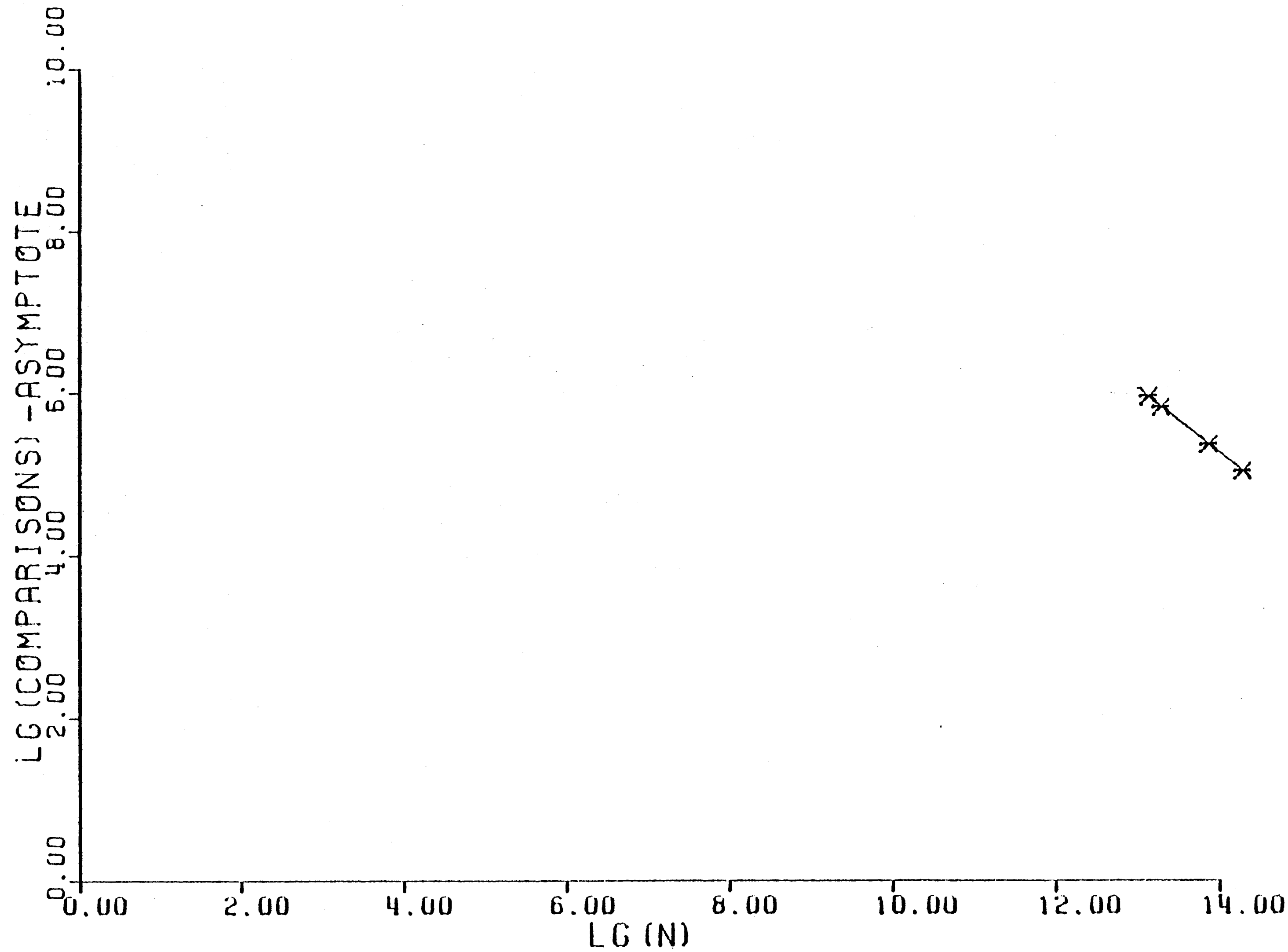

Figure 20. Thirteen-pass Hibbard's Shellsort with Estimated Asymptote Subtracted

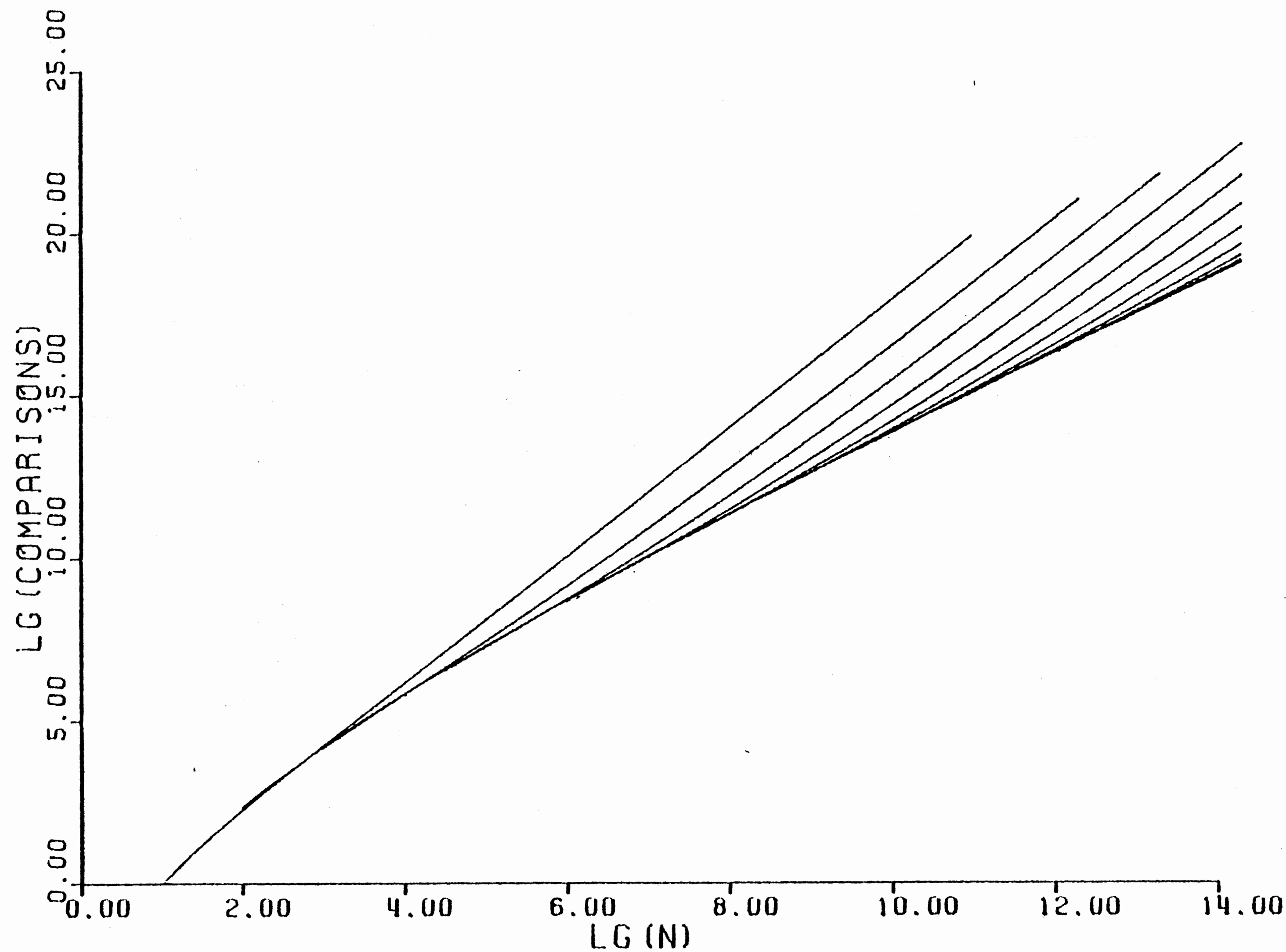

Figure 21. Data Derived from Subroutine HBCOM

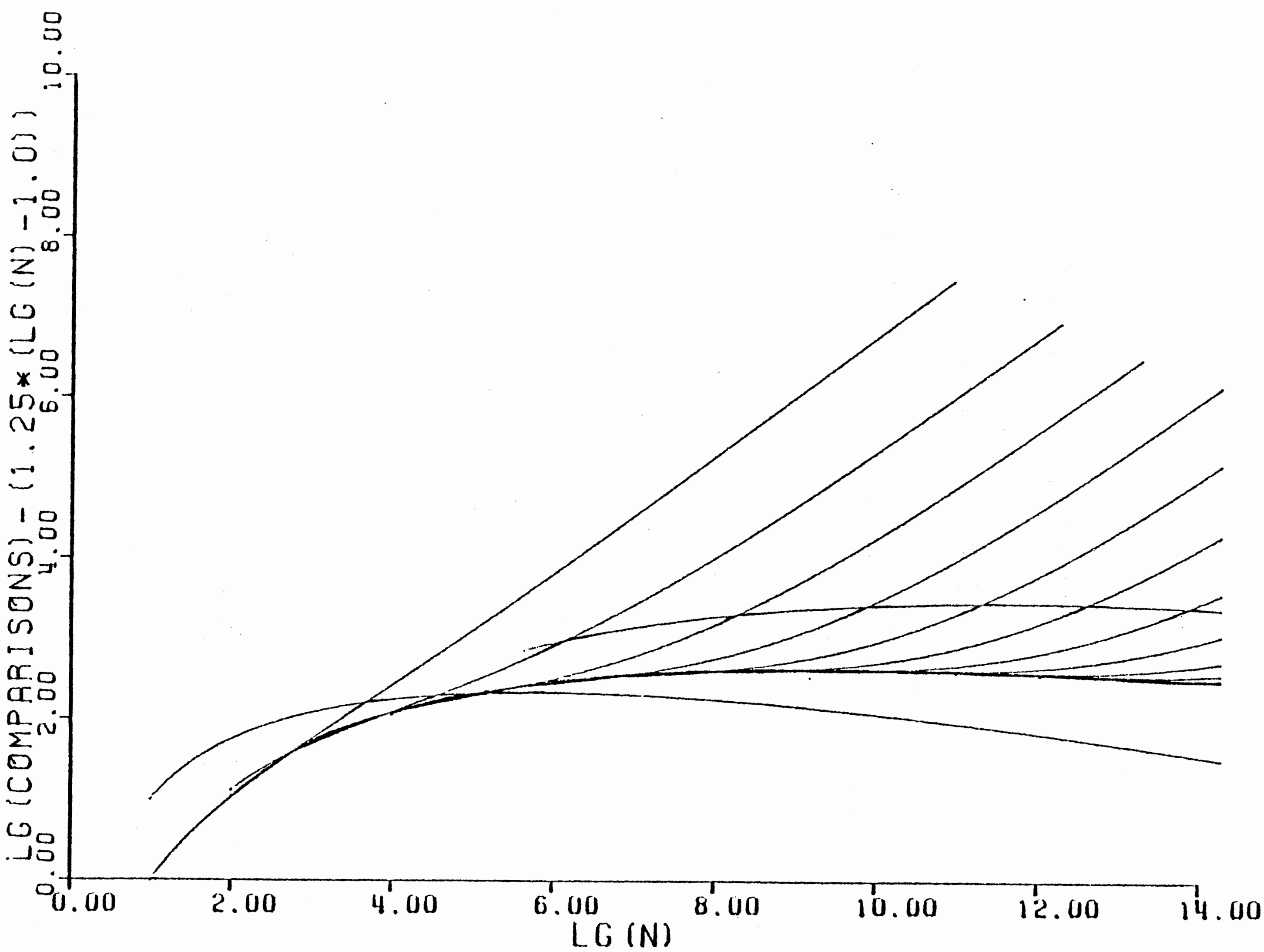

Figure 22. Data Derived from Subroutine HBCOM with 1.25*(lg(N)-1.0) Subtracted

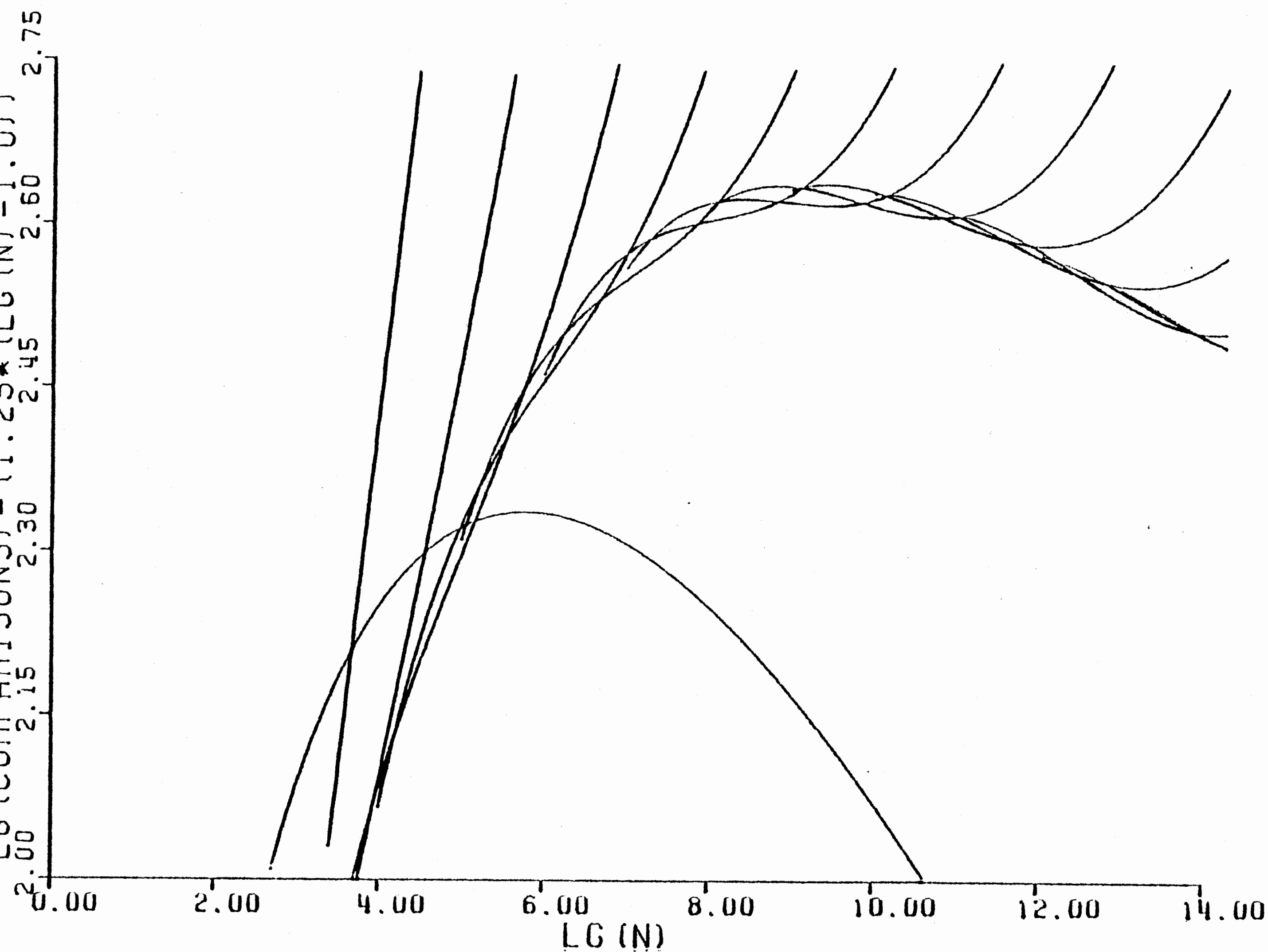

Figure 23. Magnification of Part of Figure 22

VITA

Bruce Lee Bauer

Candidate for the Degree of

Master of Science

Thesis: AN EMPIRICAL STUDY OF SHELLSORT

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