A MATHEMATICAL EXPRESSION OF STEM FORM AND

VOLUME FOR LOBLOLLY PINE IN

SOUTHERN BRAZIL

By

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A MATHEMATICAL EXPRESSION OF STEM FORM AND

VOLUME FOR LOBLOLLY PINE IN

SOUTHERN BRAZIL

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CHAPTER I

THE RESEARCH PROBLEM

Introduction

Following the world population growth as well as the changing liv-
ing standards of society, demand for natural resources is constantly
increasing. This naturally reflects on higher values for forest products
and consequently for wood. In addition, modern manufacturing methods
require more precise and detailed information on raw material supply.

In the context of multi-product intensive management of pine planta-
tions in southern Brazil, several questions have been asked regarding
the comparative efficiency of different stand density, thinning and prun-
ing methods and choice of rotation length. Currently however, additional
and important questions are being studied with respect to alternative
methodologies for the estimation of important parameters that will quan-
tify and describe the raw material being produced according to different
end-uses.

Dealing with even-aged industrial forests and because of their
production oriented management structures, forest inventory data of tim-
berland in southern Brazil must now be collected by more sophisticated
procedures. These are necessary to provide the forest manager with more
accurate estimates of the wood available by detailed size classes. These
measurements are also essential to the decision-maker with respect to
policy formation and planning of forest operations directly related to

the industry's needs. The information allows managers to make a more comprehensive analysis and consequently to suggest a more efficient use of wood.

Wood volume of a stand is the sum of the volume of each individual tree included in that stand. The volume of a tree has traditionally been defined by height, diameter at breast height and form, and can be estimated through several alternative ways, i.e. water displacement, graphical methods, the use of formulas such as Smalian's, Huber's and Newton's, or the integration of a taper model.

It is the objective of this paper to describe the development of a mathematical expression of tree form or taper, and the transformation of such a model into a tree volume function.

Data for this study were obtained in a loblolly pine plantation located in southern Brazil. This is an introduced conifer of increasing importance for the regional forestry activities of that country.[1]

## Justification

Some of the fundamental reasons for this research project are implied in the introduction and statement of the objective. This author believes however, that some other assumptions must be properly explained.

The shape and conceptual sectioning of the stem of a conifer is illustrated with Figure 1. The stems and stumps of trees closely resemble certain geometrical solids (Hush et al., 1972) and their cubic volume may be estimated by applying formulas either before or after they are

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[1]All species cited in this paper are listed in Appendix A with their common and scientific names.

Tip — Cone or paraboloid

Upper

logs — Frustums of paraboloid

Butt
log — Frustums of neiloid
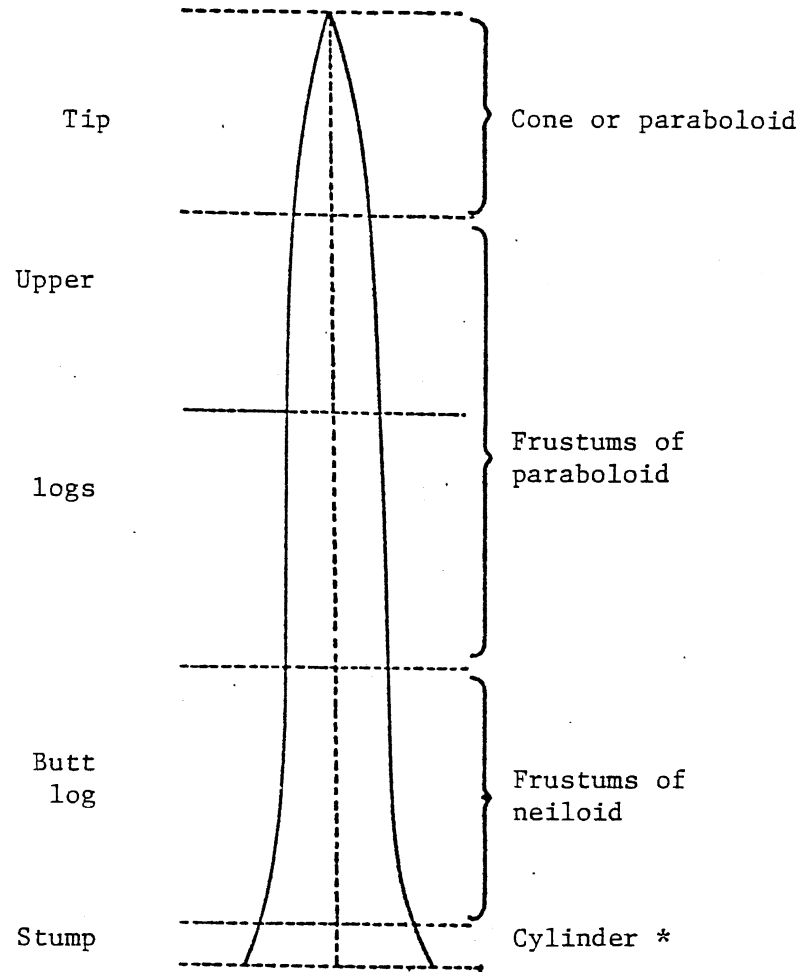
Stump — Cylinder *

*the stump is usually treated as a cylinder,
but has the geometric form of a frustum of
neiloid.
Source:  Husch et al., 1972, p. 121.

Figure 1.  Geometric Forms Frequently Observed
in Portions of Tree Stems

felled. However, accurate volume estimation of standing trees has traditionally been demonstrated to be one of the most difficult tasks in forest mensuration. Usually what has been suggested is to measure several diameters along the stem of felled trees and compute an estimate of the volume through the use of any one of the accepted formulas, such as those of Smalian, Huber or Newton. Later, a regression analysis is run for volume prediction purposes having diameter at breast height ($d_{1.30}$), and height (h), as the independent variables.[2]

For some industrial activities the information on the volume of bulk wood is sufficient. In the production of quality timber however, volume must be given according to some merchantability limits, such as logs with some given diameters or lengths. This type of data can certainly be produced with the traditional methods. However the prediction ability is always conditioned by the way the field data was collected for the analysis. Investigators and industry recognize the need for a more flexible system.

With the availability of high-speed data processing equipment, the study of another approach to the problem is possible. That is the direct investigation of the form of tree stems. A taper model is a mathematical description of the longitudinal stem profile and, under the assumption that the cross-section is circular at all points along the stem, its volume can be determined by integration. Once the taper model is defined, volume can be obtained for any section of the stem.

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[2]In this paper special emphasis was made on the use of symbols recommended by the International Union of Forestry Research Organizations--IUFRO. All the measurements and computations were done with the metric system.

CHAPTER II

LITERATURE REVIEW

Considerable work has been done on the factors involved in volume
estimation because of their importance and complexity. Several investi-
gators have been studying the relationship between stem form and volume
by means of searching for an adequate taper function. The objective of
any taper study is to quantify the relationship between stem form and
some easy-to-measure tree characteristics such as diameter at breast
height and tree total height. That relationship must be expressed with
as much simplicity as possible.

The historical development of the most significant contributions to
this field can be organized into three different major groups of tree
taper studies: the early stage of oversimplified models, the use of
polynomials and the recently developed use of principal component
analysis, PCA. An appreciation of taper and volume systems is also
included in this review.

## The Early Oversimplified Models

In studying tree stem form, the concept of its expression with a
mathematical function was first generated in Sweeden by Höjer (1903)
(cited from Behre, 1923). Working with Norway spruce, he was the first
investigator to suggest a model for the determination of the diameter of

a tree at any point along its stem. The equation he suggested has the form:

$$\frac{d}{d_{4.5}} = b_1 \log \frac{b_2 + \ell/h}{b_2} \qquad (2.1)$$

where d is the estimated diameter at distance $\ell$ from the tip of a stem with total height h, $d_{4.5}$ is the diameter at the breast height (dbh, measured at 4.5 feet from the ground), and $b_1$ and $b_2$ are coefficients to be determined.

Jonson (1910, 1911) reported that Höjer's model was adequate for spruce stands growing from local seed sources. However, diameters were overestimated in the upper sections of stems obtained from compartments established with imported seeds. Jonson added a "biological constant" to Höjer's model and produced a taper function and cubic volume tables for Scots pine.

$$\frac{d}{d_{4.5}} = b_1 \log \frac{b_2 + \ell/(h - b_3)}{b_2} \qquad (2.2)$$

This transformed model however, will not produce any diameter along a portion of the tip of the stem equivalent to $b_3$. Hence an inconsistency was in fact introduced (cited from Demaerschalk, 1971a).

Following the lead of the European investigators and working with ponderosa pine, Behre (1923, 1927, 1935) suggested a new model that has the form of the ordinary hyperbola:

$$\frac{d}{d_{4.5}} = \frac{\ell/h}{b_1 + b_2(\ell/h)} \qquad (2.3)$$

In this function, d, $d_{4.5}$, $\ell$ and h have the same meaning as in Höjer's model and, $b_1$ and $b_2$ are coefficients obtained by regression analysis. Under the assumption that the form of the stem of conifers can be expressed with a single equation applicable to all species, diameters

and heights, Behre's model became the basis for the board-foot volume tables prepared by Mesavage and Girard (1946) and Girard and Bruce (1947, 1949).

According to Bruce and Schumacher (1950), Behre's equation gives a satisfactory result for several species but it fails to fit that portion of the tree stem where the butt swell begins. Correction factors were suggested by the authors of those earlier tables and they were used as a standard reference by many forest operators. Residuals of predicted volumes were considered not significant in practice when the tables were used with native species of western United States.

Atterbury (1973) reported the work done by Wicoff in 1966 to estimate cubic volume of wood on Crown Zellerbach timberland in Northwestern United States. In those studies Wicoff claimed that if the convex hyperboloid described with Behre's equation fits the stem above the form point (i.e. the inflection point as defined by that author), then a concave hyperboloid should be adequate for the portion of the stem below that level. The results obtained with this approach were considered adequate for the objectives of the company at that time. Recently (Liu, 1973), the same company provided loblolly pine data for a principal component analysis.[1] This is an indication that currently a more flexible and reliable mathematical expression of taper is needed.

The main difficulties of the practical use of the equations suggested by Höjer, Jonson and Behre were summarized by Bruce, Curtis and Vancoevering (1968). The computational work involved was excessive and

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[1]Since that was also the technique used in this study, PCA is more completely described in Chapter III, Materials and Methods.

the resulting oversimplified equations did not satisfactorily describe the butt swell and tip.

Among the objectives of those early studies to describe stem form, was also the attempted development of a universal taper function. This author believes it will be hard to accomplish such an objective with a simple model because of the several sources of variability that may influence tree stem form: different species, past management practices, age and growth rates among others. In regard to the observed adjustment of those functions, it must be said that only the inclusion of more coefficients to a basic model, following an empirical method or iterative process, will not necessarily improve its diameter prediction ability. It may even introduce a more serious bias. At that stage the natural extension of taper model building was obviously towards the addition of more independent variables by using transformations of diameter at breast height and tree total height. This necessarily includes the use of polynomial functions to investigate stem form.

## The Use of Polynomials

In reviewing the previous work done on stem form, this author observed that the literature contains an extremely significant amount of taper studies using polynomial regression analysis. This reflects what was considered by several investigators a reasonable approach to the problem. A review follows:

To describe stem form above breast height of loblolly pine, Matte (1949) suggested the model

$$\frac{d^2}{d^2_{4.5}} = b_o \frac{h_d^2}{h^2} + b_1 \frac{h_d^3}{h^3} + b_2 \frac{h_d^4}{h^4} \qquad (2.4)$$

where d is the diameter in inches at height $h_d$ from the ground in feet. The dbh outside bark ($d_{4.5}$) is measured in inches at 4.5 feet from the ground. Tree total height (h) is measured in feet.

Osumi (1959), cited by Demaerschalk (1971a), reported a similar model to the one suggested by Matte. Using the same symbols, the form of his suggested equation is:

$$\frac{d}{d_{4.5}} = b_o \frac{h_d}{h} + b_1 \frac{h_d^2}{h^2} + b_2 \frac{h_d^3}{h^3} \tag{2.5}$$

According to these authors, both models were considered adequate for the data independently collected. It is worthwhile to say at this point that to some extent, the evaluation of a taper model involves a subjective judgement of the prediction ability of the function. What is considered a reasonable bias or acceptable residual by one investigator will not be necessarily true for another researcher.

In studying generalized taper functions, Newnham (1958) believed that a quadratic parabola was adequate for the description of the bole shape.

Prodan (1965) found that a satisfactory taper function should be:

$$\frac{d}{d_{1.30}} = \frac{(h_d/h)^2}{b_o + b_1 \frac{h_d}{h} + b_2 \frac{h_d^2}{h^2}} \tag{2.6}$$

In this equation, d is the predicted diameter at height $h_d$ of a tree with total height h. The diameter at breast height ($d_{1.30}$) is measured in centimeters at 1.30 m from the ground level.

Munro (1966) developed a linear transformation of the quadratic paraboloid for the prediction of diameter inside bark ($d_{ib}$) at any given height ($h_d$) in feet from the ground.

$$d_{ib} = d_{4.5} \sqrt{b_1 + b_2 \frac{h_d}{h-4.5}} \qquad (2.7)$$

As in the previous models $d_{4.5}$ is the dbh at 4.5 feet from the ground and h is total tree height in feet. Kozak and Smith (1966) reported that Munro's model failed to describe the stem at the butt and tip but, after integration, satisfied many of the needs with respect to merchantable volume.

An improved model was suggested by Munro in 1968:

$$\frac{d^2}{d_{4.5}^2} = b_1 + b_2 \frac{h_d}{h} + b_3 \frac{h_d^2}{h^2} \qquad (2.8)$$

Here the symbols have the same meaning as in Munro's previous model. Diameters were overestimated in the upper third of tree height and Kozak et al. (1959) suggested an improvement for Munro's last model by imposing the condition that diameter be zero at the tip. The new model simplified to:

$$\frac{d^2}{d_{4.5}^2} = b_1 (\frac{h_d}{h} - 1) + b_2 (\frac{h_d^2}{h^2} - 1) \qquad (2.9)$$

In a study of taper and volume for red alder, Bruce et al. (1968) derived prediction equations expressing ratio of squared upper stem inside bark diameters to squared $d_{4.5}$ outside bark as a function of $d_{4.5}$, h, and the 3/2, 3rd, 32nd and 40th powers of relative length. This was necessary to obtain a model that could tentatively fit the several inflection points along the stem.

Demaerschalk (1971a) reported a logarithmic model:

$$\log d = b_o + b_1 \log d_{4.5} + b_2 \log h_d + b_3 \log h \qquad (2.10)$$

He suggested that by using this function, no conditioning was necessary to insure that the estimated diameter at the top be zero and that no negative diameter estimates occurred.

Demaerschalk (1971b) developed a compatible system based on the combined variable volume equation of Spurr (1952) and existing volume tables for British Columbia species. In conflict with those results, Evert (1971) found seriously biased estimates of true volume when both logarithmic and the combined variable approach were used.

A third-degree polynomial was suggested by Bennett and Swindel (1972) after an investigation of taper of planted slash pine:

$$d_h = b_1 \frac{d_{4.5} (h-h_d)}{h - 4.5} + b_2 \cdot (h - h_d) \cdot (h_d - 4.5) +$$

$$b_3 \ h(h - h_d)(h_d - 4.5) +$$

$$b_4 \ (h - h_d)(h_d - 4.5)(h + h_d + 4.5) \tag{2.11}$$

That model is essentially a third-degree polynomial in respect to $h_d$ and the results were considered adequate for diameter prediction with the data used. Characteristics of volume estimation with that model were not reported.[2]

Max and Burkhart (1976) applied the techniques of segmented polynomial regression to describe the stem of loblolly pine with a three models jointed function. The resulting model was used with an independent data set and when compared to a single quadratic taper function, the segmented polynomial was found superior.

A computerized system proposed by Demaerschalk and Kozak (1977) involved the use of two mathematical functions, one describing the upper

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[2]In this research project a third-degree polynomial was also used in a trial to express stem form. The independent variables of the model however, were defined differently. After unsatisfactory results were obtained, this author used principal component analysis.

bole and jointed at the inflection point with a second function for the lower portion. Biased diameter estimates at the butt level were detected by the authors, but the prediction of diameter inside bark was very accurate over most of the stem. That segmented model was also used by Waite (1977) with data obtained in northern Brazil with Gmelina plantations. Although good results were achieved for the estimation of diameter and volumes, that author concluded that the system of segmented equations originally proposed was not adequate for that species.

Cao (1978) reported a comprehensive review of taper functions where the efficiency of several previous and new models was compared. With respect to Demaerschalk and Kozak's study, Cao pointed out that the complexity of the system is the main drawback that makes it impractical.

To summarize, all investigators tried to describe tree taper with different models of varying degree of complexity. It is clear that each and every model produced biased or undesirable results when used out of the range of data upon which its development was based. The research studies are conflicting in their conclusions and there is yet no agreement on what should be the model for a reasonable universal taper function, if any.

## The Use of Principal Component Analysis

Polynomial regression analysis is just a special case of multiple regression analysis. Since the idea of multidimensional sources of variation has been present in the previous attempts to express stem form of trees, the investigation of the problem with a more flexible multivariate technique is reasonable.

Multivariate statistical analysis is concerned with data collected on several dimensions of the same individual, i.e. several variates (Morrison, 1976).

Principal component analysis (PCA) is one of the techniques under the general heading of multivariate methods. PCA is extremely useful for reduction of data dimensionality, analysis of the structure of multivariate systems and for descriptive purposes. It was first developed with the variance maximizing studies of Hotteling (cited from Morrison, 1976).

Fries (1965) and Fries and Matern (1966) introduced the use of principal component analysis in the investigation and development of tree taper curves. After an analysis of those studies, Kozak and Smith (1966) claimed that the use of simple functions, sorting and graphical methods, is adequate for many uses in forest operations and research. Of course the definition of what is an adequate prediction characteristic of a taper model must be considered in the decision process of what technique shall be used.

Evert (1971) mentioned some of the limitations involved in the attempts to increase the precision of volume estimate of standing trees. With regard to principal component analysis he had the following comment:

> A lot of progress has been made, using multivariate functions in terms of dbh and height only, or using measurements of upper-stem diameters as well . . . Unfortunately, obtaining solutions to multivariate functions by regression analysis is not without pitfalls. One of these pitfalls is possible bias resulting from the preparation of tree-volume equations from data already expressed as equations or as freehand curves (p. 353).

The data already expressed as equations that Evert refers to, are the eigenvector elements associated with the eigenvalues. However, it must be observed that the eigenvalues contain all the variability of the

original data matrix, and can not be a source of bias.[3]  In fact bias
will be introduced if an inadequate model is used to relate the eigen-
vector elements to the relative position of the original diameter
measurements along the stem of the sample trees.  That is, biases are
due to the model chosen, not to the technique or, in this case, to the
data.

Principal component analysis was used by Liu (1973) in the develop-
ment of a taper model for loblolly pine data obtained from Crown
Zellerbach Company.  The developed function was reported to be an effec-
tive diameter predictor for any height along the stem regardless of tree
size.

Liu and Keister (1978) described the procedure by which PCA was
applied to define stem taper of loblolly and slash pine.  A single
eigenvalue absorbed 99 percent of the total variance of the data (dia-
meter measurements collected at the same proportional heights for all
trees).  To obtain a taper model, regression analysis was used, the
first eigenvector being the dependent variable, and the corresponding
proportional heights and their powers the independent variables.  Total
and merchantable volume functions were derived by integration of the
taper model.  No significant difference was observed when actual volume
of sample trees was estimated by using both the volume function obtained
and Smalian's procedure.

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[3]The terminology is more completely explained in the description of
the statistical method, Chapter III, p.

Taper and Volume Systems

A comparative analysis of different compatible systems of taper and volume was provided by the study of Munro and Demaerschalk (1974). Taper based and volume-based systems were defined, and from that study, the following statement is pertinent: "A good tree taper and volume system should be unbiased for all tree sizes in the estimation of diameter at any height, height for any diameter, volume of any section and total tree volume" (p. 198).

A taper and volume system is called taper-based when volume is obtained after an adequate taper function is defined. On the other hand, the system is called volume-based if a taper model is derived from an existing and accepted volume function. To be considered a compatible system, volume obtained after the integration of a taper function must be equal to the actual volume also estimated, but by using a traditional procedure. Currently, it is a broadly accepted philosophy that the development of a compatible taper-based system of taper and volume must achieve the desirable characteristics synthesized with Munro and Demaerschalk's statement.

However, the investigation of tree stem form is complex, and Demaerschalk (1973b) has shown that an equation considered best for taper is not necessarily best for volume. This statement needs an appropriate interpretation. In agreement with Demaerschalk, this author adds: a bias considered not significant for diameter prediction may have different effects when volume is computed by integration of a given taper model. That is, volume is affected differently if that bias occurs in the upper sections of the stem or close to the ground. However, unbiased volume

estimates for any section of the stem will only be obtained if an accurate taper model is defined. Hence, the degree of accuracy of the taper model must be properly identified previous to the volume computation.

After an elaborate review of stem form, Grosenbaugh (1966, p. 456) recognized the complexity of the subject and concluded: "Quantification of tree form requires measurement of tree heights and diameters along as much of the stem as possible. No single pair of coordinates is of much value in assessment of tree form."

In a stem form investigation, however, several diameters are measured along the stem of sample trees. In the model building process, those measurements are used in a way such that the taper function is developed upon those diameters and heights but, for practical uses, the expression is a function of dbh and tree total height.

CHAPTER III

MATERIAL AND METHODS

Study Area and Data

The tree measurement information for this study consists basically
of 188 stems obtained from four compartments planted with loblolly pine
at Irati National Forest in southern Brazil (Figure 2). The description
of this site is summarized as follows (Golfari, 1978):

Irati National Forest, Irati County, State of Parana, Brazil

Latitude: 25°27'S

Longitude: 50°35'W

Altitude: 850 m (2,790 feet)

Climate: annual precipitation . . . . . . 1442 mm (58 inches)

mean annual temperature . . . . 17.2°C (63°F)

Soil: dark reddish latosol

Table I is the joint frequency distribution of the sample trees
with respect to dbh and height. Volume of trees has a nearly linear
association with height but is greatly related to the square of diameter
at breast height. Thus sample trees were randomly collected within each
diameter class rather than randomly distributed over the entire target
population. Age range of the data was from 5 to 19 years.

The so-called Hohenadl's method of tree measurement and volume
estimation (Hohenadl, 1936) was used in providing the data needed for
this study. Appendix B contains a review on the method as well as a

MATO GROSSO

SÃO PAULO

MINAS GERAIS

ESPIRITO SANTO

RIO DE JANEIRO

PARANÁ

LOCATION OF IRATI NATIONAL FOREST

ADEQUATE REGION FOR INTRODUCTION OF
LOBLOLLY AND SLASH PINE (ADEQUATE
THERMAL AND HYDRIC CONDITIONS)

SANTA CATARINA

RIO GRANDE
DO SUL
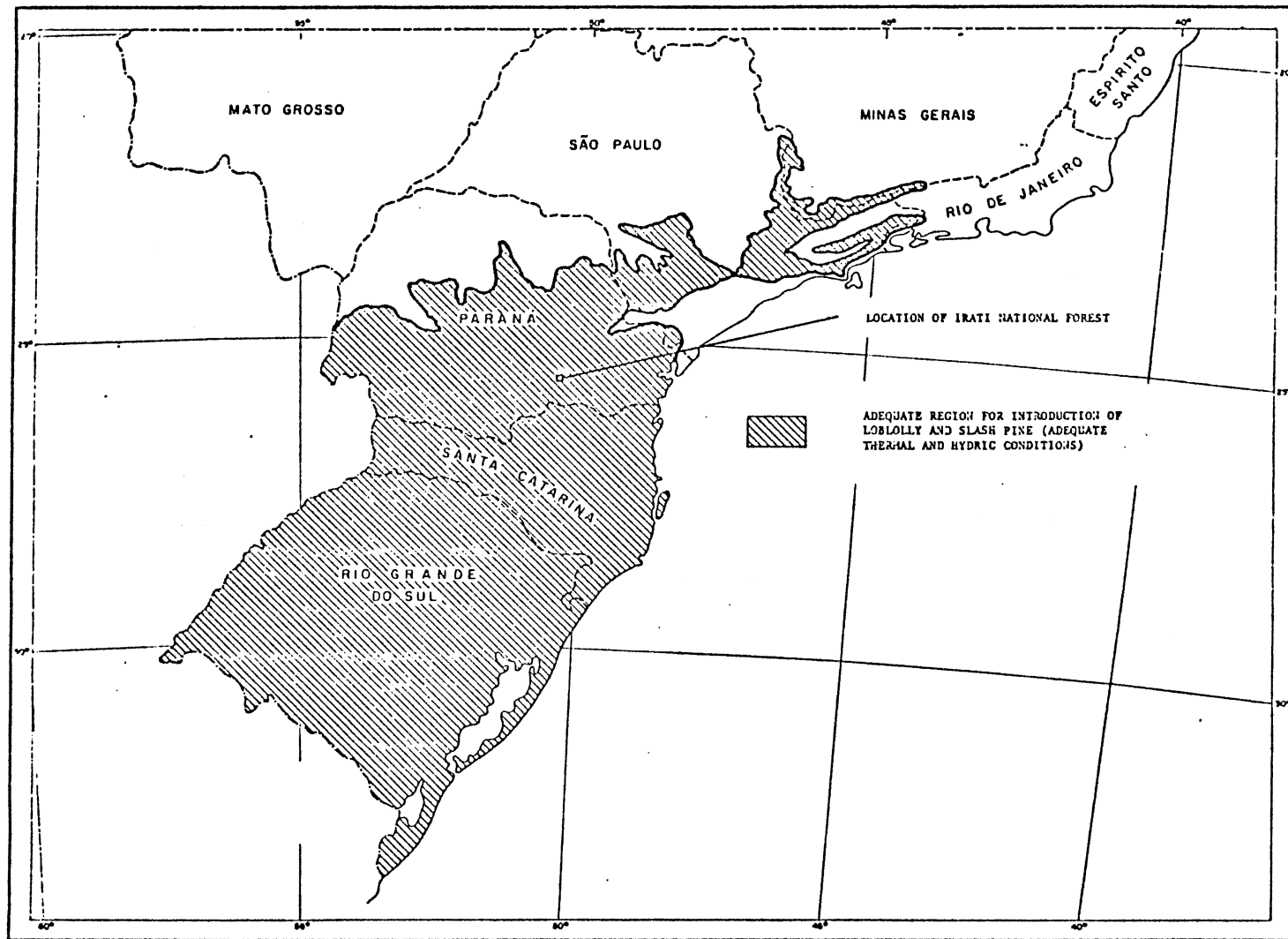
Source:   Golfari et al., (1978, p. 45)

Figure 2.   Location of Irati National Forest, State of Parana, Brazil

TABLE I

TREE DATA FREQUENCY DISTRIBUTION

| Diameter Class (cm) | Height Class (m) | | | | | | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | |
| 6 | 7 | 1 | | | | | | | | | | | 8 |
| 8 | 5 | 4 | | 2 | | | | | | | | | 11 |
| 10 | | 9 | 1 | | | | | | | | | | 10 |
| 12 | | 2 | 3 | 5 | 1 | | | | | | | | 11 |
| 14 | | | 4 | 4 | 2 | 1 | | | | | | | 11 |
| 16 | | | | | 4 | 1 | 3 | 2 | | | | | 10 |
| 18 | | | | | 2 | | 4 | 5 | | | | | 11 |
| 20 | | | | | 2 | 1 | 1 | 5 | 1 | | | | 10 |
| 22 | | | | | 1 | | 2 | 6 | 1 | | | | 10 |
| 24 | | | | | | | 1 | 5 | 3 | 1 | | | 10 |
| 26 | | | | | | | | 2 | 7 | 1 | | | 10 |
| 28 | | | | | | | | 1 | 9 | | | | 10 |
| 30 | | | | | | | | 3 | 5 | 2 | | 1 | 11 |
| 32 | | | | | | | | | 3 | | 8 | | 11 |
| 34 | | | | | | | | | 1 | 4 | 6 | | 11 |
| 36 | | | | | | | | | | 2 | 7 | 2 | 11 |
| 38 | | | | | | | | | | 2 | 5 | 4 | 11 |
| 40 | | | | | | | | | | | 7 | 4 | 11 |
| TOTAL | 12 | 16 | 8 | 11 | 12 | 3 | 11 | 29 | 30 | 12 | 33 | 11 | 188 |

Diameter class 6 cm  =  5.0 -- 6.9 cm
Height class  4  m  =  3.0 -- 4.9 m

detailed description of the field procedure used. In essence, the data

obtained from each sample tree were: $d_{1.30}$ (diameter at breast height,

outside bark), total height, and beginning from the tip, diameter inside

bark at 1/10, 3/10, 5/10, 7/10, 8.333/10, 9/10, and 9.667/10 along the

stem above a 15.0 cm standard stump height. Hence, all taper variability

was condensed in a 7 x 188 matrix: seven positional diameters and 188

trees. The field measurements originally recorded on forms were punched

on cards and SAS-79 edition was used all the way through the data

processing and statistical analysis.[1]

<p style="text-align:center">An Attempt With Polynomial Analysis</p>

It was believed that a third degree polynomial of the form $Y = b_0 +$

$b_1 x + b_2 x^2 + b_3 x^3$ could fit the data because the plot of some cubic

polynomials appears to follow a taper much like a tree stem.

It must be recalled that all polynomials used in previous investiga-

tions of stem form were developed in attempts to predict diameter at any

point along the stem as a function of dbh, tree total height and the

absolute position of the diameter being predicted. In this study how-

ever, the third degree polynomial was suggested to predict radii (diam-

eter) as a function of its relative position along a stem of unit length.

This approach has produced biased estimates for both diameter and volume.

The basic idea was to reduce geometrically the dimensions of all

trees in order to relate all radii (diameters) and their absolute posi-

tion to the smallest tree (Figure 3). That is to say, all measurements

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[1]Statistical Analysis System, SAS-79 edition is a collection of sub-
routines for data analysis prepared by SAS Institute Inc., Raleigh, NC.

The smallest tree
(total length = 4.2m)
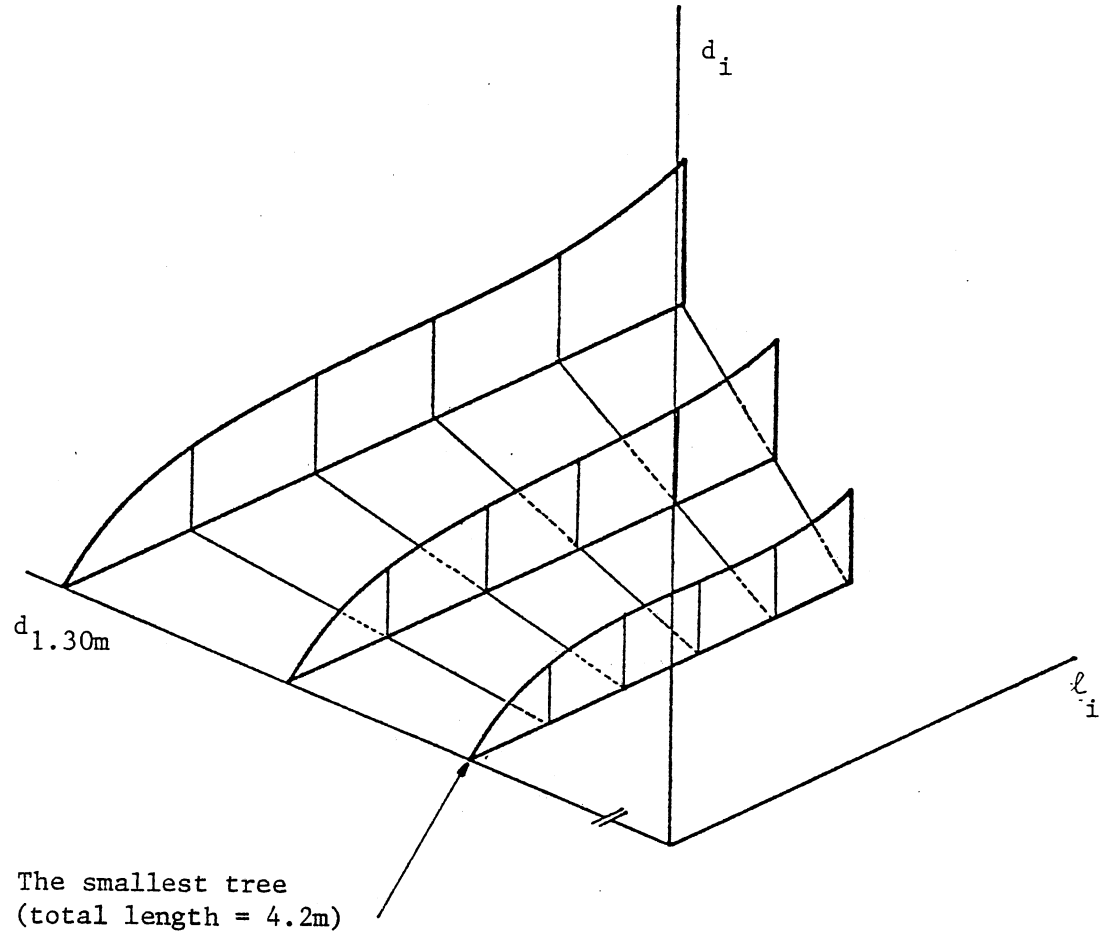
$\ell$ = 0.10L, 0.30L, ... , 0.967L

Figure 3.  Schematic Representation of Normalized Diameter
Data Used in the Third-Degree Polynomial
Regression Analysis

were normalized. After a taper model was satisfactorily fitted to these standardized values, the function was expanded back to the original data by using a conversion factor. The assumption on which this procedure was based is that, regardless of size, all trees have exactly the same form, and that the inflection point is located at a constant relative position.

That proved to be a wrong assumption. Volumes were overestimated for small trees and underestimated for larger trees. In trying to find alternatives to fit the data, an important conclusion was drawn: a second degree polynomial, or parabola, could do as well. The addition of the third power element to the model does provide a better fit for the butt region of the stems of many species. However, loblolly pine stems within this age range of 5 to 19 years old, do not have a pronounced butt swell and trees are more cylindrical at the base. In addition all measurements were done above a 15.0 cm stump height and this source of taper, if present, was partially eliminated. This conclusion was later useful in the selection of a model for the principal component analysis of the data.

## Principal Component Analysis

The principal aim of multivariate statistical methods is to have a better understanding of the underlying structure of the data represented with a joint frequency distribution of several variates (Marriot, 1974).

Among several statistical techniques under the general expression of multivariate methods, principal component analysis is useful in the ordination of variables as an aid for the interpretation of multivariate data.

It must be emphasized that principal component analysis is not directly used for hypothesis testing. It is just an exploratory technique for an efficient evaluation of the dimensions of variability. This is achieved with the development and analysis of orthogonal components.

Suppose a data vector of p random variables is measured for n independent observations so that an p x n data matrix X is obtained:

$$\text{Col. } j$$

$$X \doteq \text{Row } i \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ & \cdot & \\ \cdot & & \cdot \\ \cdots & x_{ij} & \\ \cdot & & \\ x_{n1} & \cdots & x_{np} \end{bmatrix}$$

$$i = 1 \ldots n$$
$$j = 1 \ldots p$$

The method of principal component analysis consists of transforming a set of correlated variables $X_1 \ldots X_p$ to a new set of uncorrelated variables or orthogonal components $C_1 \ldots C_p$, observing the following basic properties (Marriot, 1974 and, Isebrands and Crow, 1975):

a.  each new variable $c_k$ is a linear combination of the original variables $X_j$, $j = 1, \ldots p$.

$$c_k = a_{k1} X_1 + a_{k2} X_2 + \ldots + a_{kp} X_p$$

b.  the sum of squares of the coefficients equals unity. That is,

$$\sum_{j=1}^{p} a_{kj}^2 = 1, \text{ for each } k = 1, \ldots, p.$$

c.  the new variables are uncorrelated with each other.

d.  the total variation among the new variables is equal to the total variation among the original variables.

e.  the variance of each new variable decreases in order, i.e. of all possible linear combinations, $C_1$ has the largest variance.

Of all possible linear combinations uncorrelated with $c_1$, the one with the largest variance is $c_2$, and so forth.

With this linear and orthogonal transformation, a new set of p variables is obtained, uncorrelated with each other and arranged in order of decreasing variance (Marriot, 1974). The value of this new matrix however, is that a relatively small number of new variables or principal components may account for most of the total variance of the original data. Hence, the vector of coefficients or eigenvector associated with these new variables may be considered separately to express the multidimensional structure of the original data.

The solution for this algebraic procedure is equivalent, in matrix algebra, to the determination of the eigenvalues (characteristic roots or latent roots) and the associated eigenvectors (characteristic vectors) of the p x p variance-covariance matrix developed from the original data. The eigenvalues turn out to be the variances of the respective principal components, while the eigenvector elements provide the coefficients for obtaining the principal components.

Taper Development

After the polynomial approach produced unsatisfactory results, principal component analysis was used to investigate the stem form structures of the data and to define a mathematical expression of that structure. This multivariate statistical technique was applied following basically the same outline of those studies referred to in the review of previous work done on the subject.

Because the taper function was ultimately converted by integration to a volume function, all the computations involved in the taper model

development were done using radii inside bark instead of diameters

inside bark.

Having prepared the data in a two-dimensional array of seven posi-

tional radii (diameters) measured along each of 188 stems, a symmetric

7 x 7 sum of squares and cross-products matrix was obtained using PROC

CORR (Statistical Analysis System, SAS/1979 edition).

$$
\begin{array}{c}
D = \\
\text{(Data Matrix)}
\end{array}
\qquad
\begin{array}{c}
\text{Stem no.} \\
1 \\
2 \\
\cdot \\
\cdot \\
\cdot \\
188
\end{array}
\quad
\overset{\displaystyle \text{Radius no.}}{\overset{\displaystyle R_1 \quad R_2 \quad \cdots \quad R_7}{
\begin{bmatrix}
X_{1,1} & X_{1,2} & \cdots & X_{1,7} \\
X_{2,1} & X_{2,2} & \cdots & X_{2,7} \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \cdot \\
X_{188,1} & X_{188,2} & \cdots & X_{188,7}
\end{bmatrix}
}}
$$

$$
\begin{array}{c}
P = \\
\text{(sum of squares} \\
\text{and cross-} \\
\text{products matrix)}
\end{array}
\qquad
\begin{array}{c}
R1 \\
R2 \\
\cdot \\
\cdot \\
\cdot \\
R_7
\end{array}
\quad
\overset{\displaystyle R_1 \qquad R_2 \qquad \cdots \qquad R_7}{
\begin{bmatrix}
SS_{R1} & CP_{R1,R2} & & CP_{R1,R7} \\
 & SS_{R2} & & \\
 & & \cdot & \cdot \\
 & & \cdot & \cdot \\
 & & \cdot & \cdot \\
 & & & SS_{R7}
\end{bmatrix}
}
$$

where $SS_{R1}$ is the sum of squares for R1, $CP_{R1,R2}$ is the sum of cross-

products of $R_1$ and $R_2$, and so forth.

The solution for the eigenequation $|P - \lambda I| = 0$ was computed by

using PROC MATRIX and the SAS commands EIGEN, VALS and VECS. This

produced a diagonal matrix of eigenvalues ranked in decreasing order and,

an associated matrix of eigenvectors, each column vector being related

to an eigenvalue.

It was observed that the first eigenvalue by itself accounted for 99.72 percent of the total variance. This was considered to be a very strong evidence that the first eigenvalue and its associated eigenvector were extremely useful in explaining the latent structure of the data. Following suggestions of previous studies (Fries and Matern, 1966; Liu, 1973) and for interpretation purposes, the eigenvector elements associated with the three first ranked eigenvalues were plotted against the corresponding relative position u along the stem (Figure 4). After observing the trend of these three eigenvectors it was concluded that only the first eigenvalue was sufficient to explain and describe the average stem form of the stems considered in the analysis.

Polynomial regression analysis was then used to fit a model to the data represented by the seven elements (ELT) of the first eigenvector as a function of their relative position (u) along the stem of length unit. PROC RSQUARE was used to obtain all possible models with the following independent variables: $u^{1/4}$, $u^{1/3}$, $u^{1/2}$, $u$, $u^{3/2}$, $u^2$, $u^{5/2}$ and $u^3$. Among all possible combinations of those variables, five models were selected because of their very high coefficient of determination, and PROC GLM was used with each model; once with the intercept $b_o$ included in the least squares solution, and a second run forcing the model to pass through the origin. All ten models thus obtained were similar in their eigenvector elements prediction ability, but the following model was selected:

$$\hat{ELT} = F(u) = b_o + b_1 u^{1/4} + b_2 u^{1/3} + b_3 u^{1/2} + b_4 u \qquad (3.1)$$

Hence, among all models evaluated, equation 3.1 was the best function to fit the average taper represented with the eigenvector elements

PLOT OF EIGENVC1*U     SYMBOL USED IS *  ——————————   EIGENVECTOR NO. 1
PLOT OF EIGENVC2*U     SYMBOL USED IS +  ————  ————   EIGENVECTOR NO. 2
PLOT OF EIGENVC3*U     SYMBOL USED IS #  ——  ——  ——   EIGENVECTOR NO. 3
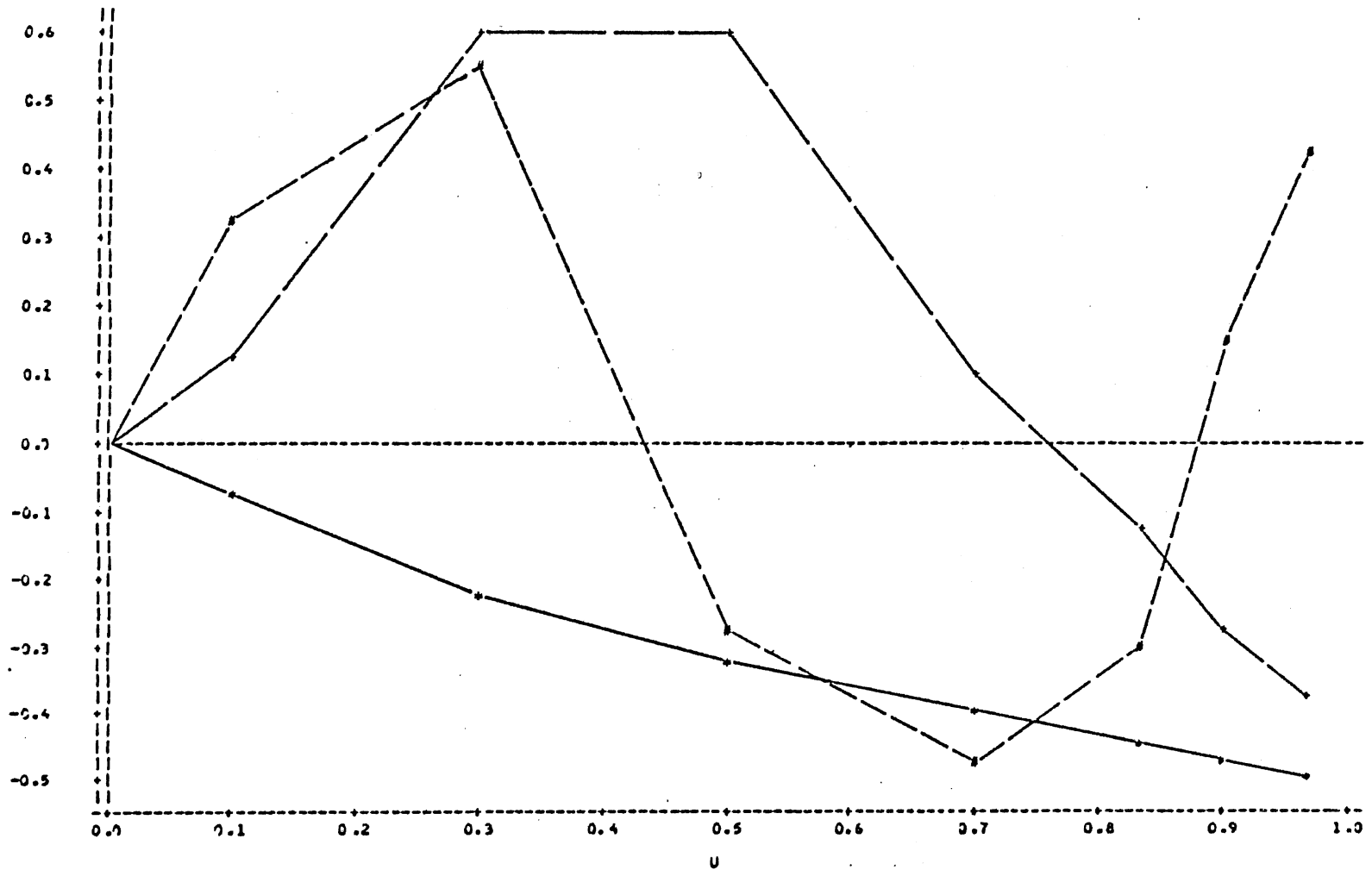
Figure 4.   Plot of Eigenvector Elements for the Three First Eigenvalues Against Relative
            Position Along the Stem

associated with the first eigenvalue. Because of the computational

nature of the procedure however, that model will only estimate relative

radii of a stem of length unit, i.e. related to the relative position

of the eigenvector elements. In order to have that model expressing

predicted radii (diameters) as a function of actual length, a transfor-

mation of the independent variables was introduced and the model became:

$$\hat{r}(\ell) = F(\ell/L) = b_0 + b_1(\frac{\ell}{L})^{1/4} + b_2(\frac{\ell}{L})^{1/3} + b_3(\frac{\ell}{L})^{1/2} + b_4(\frac{\ell}{L}) \quad (3.2)$$

This is the taper model for the prediction of a radius $\hat{r}(\ell)$ (diameter)

at the absolute position $\ell$ from the tip of a stem with average form and

total length L above a 15.0 cm stump height.

The objective of this study implies however the development of a

taper model for the prediction of inside bark of radii (diameters) along

the stem of any tree considered in the analysis and not only those with

stem of average form. Therefore, suppose a tree has a radius $r_{1.30}$

measured at 1.30 m from the ground, and has a stem of length L measured

above a 15 cm stump height. The previous model needs an adjustment

factor that will force the function to pass through the point of the

dbh. This factor is the ratio between the inside bark actual radius at

the dbh, $r_{1.30}$, and the predicted radius at the same point $\hat{r}(\ell_{1.30})$.

The final taper model obtained was:

$$\hat{r}(\ell) = F(\ell) = \frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \cdot \hat{F}(\ell/L) \quad (3.3)$$

where $\hat{r}(\ell)$ = predicted radius inside bark of an absolute position
from the tip, in meters

$r_{1.30} = d_{1.30}/2$ (inside bark, in meters)

$\ell_{1.30} = L - 1.15$ m

$$\hat{r}(\ell_{1.30}) = b_0 + b_1\ell_{1.30}^{1/4} + b_2\ell_{1.30}^{1/3} + b_3\ell_{1.30}^{1/2} + b_4\ell_{1.30}$$

$\hat{F}(\ell/L) = $ equation 3.2

## Volume Estimation

Total volume inside bark was predicted by integration of equation 3.3 as follows:

$$\hat{V} = \int_0^L \pi \left(\hat{r}(\ell)\right)^2 d\ell$$

$$\hat{V} = \int_0^L \pi \cdot \left[ \frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \cdot \hat{F}(\ell/L) \right]^2 d\ell$$

$$\hat{V} = \pi \cdot \left[ \frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \right]^2 \cdot \int_0^L \left[ \hat{F}(\ell/L) \right]^2 d\ell \tag{3.4}$$

In order to have a measure of the prediction ability of the volume function (equation 3.4) obtained by integration of the taper function (equation 3.3), total volume inside bark was also estimated by using a "traditional procedure". That was the so-called Hohenadl's method of stem volume estimation.

The data used for the taper development was originally collected according to Hohenadl's method, and Appendix B contains a detailed description of the field and office procedures involved in that method. Formulas for the volume estimation are also given.

Total volume inside bark was estimated for all 188 trees by using both procedures: the taper derived volume function and Hohenadl's method. Analysis of correlation, PROC CORR, was applied to observe the degree of association between these two volume estimates. Residuals of the volumes predicted with the taper-derived equation over those esti- mated with Hohenadl's method were also obtained and analyzed.

CHAPTER IV

RESULTS AND DISCUSSION

Eigenvalue and Eigenvector Analysis

The solution for the principal component analysis - PCA, requires
an interpretation of the eigenvalues and associated eigenvectors.

Because the data matrix consisted essentially of diameter measure-
ments along the stems of loblolly pine sample trees, the latent or
hidden structure of that matrix is that of stem form.  PCA assumes that
the observable or manifest variates can be expressed in a smaller number
of latent factor variates, the principal components (Morrison, 1976).
In this study there were seven manifest variates, the original seven
radius (diameter) measurements, thus seven orthogonal components were
obtained, of which ideally just one or very few are of interest, the
principal components.  The importance and utility of a principal compon-
ent can be measured by the proportion of the total variance attributable
to it.

The eigenvalues or variances of the seven orthogonal components
related to the average stem form are listed in Table II.  The associated
eigenvectors are recorded in Table III.  Because of the orthogonality
property, each eigenvalue can be considered separately.  Observing the
cumulative percentage of variance explained, it is obvious that after
the contribution of the first eigenvalue, 99.72 percent, the additional
variance accounted for by all other eigenvalues is negligible.  That is

30

TABLE II

EIGENVALUES OF THE SUM OF SQUARES AND
CROSS-PRODUCTS MATRIX

| EIGENVECTOR NUMBER | EIGENVALUE | EIGENVALUE PERCENT | CUMULATIVE EIGENVALUE PERCENT |
|---|---|---|---|
| 1 | 10.0236 | 99.72 | 99.72 |
| 2 | 0.0167 | 0.17 | 99.89 |
| 3 | 0.0041 | 0.04 | 99.93 |
| 4 | 0.0024 | 0.02 | 99.95 |
| 5 | 0.0019 | 0.02 | 99.97 |
| 6 | 0.0015 | 0.02 | 99.99 |
| 7 | 0.0012 | 0.01 | 100.00 |
| TOTAL | 10.0514 | 100.00 | |

TABLE III

EIGENVECTORS ASSOCIATED WITH THE EIGENVALUES OF THE
SUM OF SQUARES AND CROSS-PRODUCTS MATRIX

| ELEMENT NUMBER | EIGENVECTOR NUMBER | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | −0.073747 | 0.127024 | 0.313448 | 0.296600 | 0.405945 | −0.790901 | 0.043459 |
| 2 | −0.213371 | 0.611514 | 0.555317 | −0.426355 | 0.155088 | 0.257049 | −0.015540 |
| 3 | −0.330984 | 0.599204 | −0.282757 | 0.531880 | −0.376848 | 0.029875 | 0.160156 |
| 4 | −0.409884 | 0.102944 | −0.465203 | −0.378004 | 0.054499 | −0.277370 | −0.618217 |
| 5 | −0.447714 | −0.131427 | −0.312069 | −0.276361 | 0.329024 | 0.001063 | 0.707290 |
| 6 | −0.464723 | −0.273530 | 0.150432 | 0.468774 | 0.430824 | 0.439653 | −0.296533 |
| 7 | −0.505038 | −0.384944 | 0.420703 | −0.091333 | −0.610169 | −0.193077 | 0.042851 |

to say that the first eigenvalue is sufficient to explain stem form of
the data. That evaluation however, involves some subjectivity, and a
stronger basis for that conclusion was obtained with the analysis of the
plot of the eigenvector elements associated with the three first ranked
eigenvalues. It can be observed in Figure 4, that the trend of the
eigenvector elements associated with the first eigenvalue was closely
related to stem taper, while those of the second and third eigenvalues
had no apparent meaning for this purpose.

After using the first eigenvector to explain stem form of white
birch, Scots pine and lodgepole pine, Fries (1965) found the elements of
the second eigenvector correlated with crown/stem length ratio, and the
third eigenvector elements explaining dbh. Liu (1973) tried to improve
the results of his study by applying PCA to groups of trees classified
by size. After the solution of the eigenequation, he found no need to
separate trees into diameter classes because of the small improvement
on the cumulative percentage of the eigenvalues.

The alternative of grouping the data by tree size was not used in
this study given that the taper variability explained with the first
eigenvalue was considered to be extremely high. Thus a taper model
developed only upon the first eigenvalue would produce the average stem
form, applicable to all of the trees within the data range considered.

## Model Building

Because of the high proportion of taper variability explained with
the first eigenvalue, the elements of its associated eigenvector can be
analyzed as being in the same proportion as the radii (diameters) of a
tree with average form. Multiple regression analysis was used to fit a

polynomial function to the elements (ELT) of that first eigenvector

(Figure 4), and among the models tested, the following was found to be

a good relative diameter predictor:

$$\hat{ELT} = b_o + b_1 u^{1/4} + b_2 u^{1/3} + b_3 u^{1/2} + b_4 u \qquad (3.1)$$

where $\hat{ELT}$ = predicted eigenvector element or relative radius for a tree
of unit length and average form.

$u$ = relative position along a stem of unit length.
(0.10, 0.30 ... 0.967)

$b_0$ = 4.3106887 E-06

$b_1$ = 3.10149948

$b_2$ = -6.65976505

$b_3$ = 4.68992834

$b_4$ = -0.62829584

The observed coefficient of determination ($R^2$) was 0.9987. Relative

radius prediction ability was simulated and that model produced a good

fit all along the stem of unit length, including the butt region, where

most of the previous studies have shown biased estimates.

However, the objective of this study required a model for the pre-

diction of actual diameter inside bark (radius) for trees of any actual

length and with taper that deviates from the average stem form (within

the data range). Hence, two transformations were included in equation

3.1. First, the independent variables were expressed as a ratio of the

absolute position of the diameter being predicted, and stem total length

(Equation 3.2). Next, the form adjustment was done by multiplying the

radius predicted for a tree of average form, by the ratio between actual

$r_{1.30}$ and the predicted $r_{1.30}$ for that tree (Equation 3.3).

The resulting equation thus obtained (Equation 3.4), is basically a taper function for diameter inside bark ($d_\ell$) prediction at any position $\ell$ from the tip of stems with total length L = h − 0.15 m. Obviously this statement is true only for the species, dbh and h range, and site considered for this study.

## Volume Estimation

Volume estimates were obtained for all 188 sample trees with the volume function (Equation 3.4) developed by integration of the taper function. Volumes were also predicted using Hohenadl's method. Correlation analysis was carried out in order to observe how consistent both procedures were in their volume prediction ability. The observed Pearson's coefficient of linear correlation was equal to 0.9927, which reflects an extremely high degree of association for the two variables being considered. However, that only says that the distribution of both estimates is consistent over the data range. It could happen that volume estimates obtained with both procedures "consistently" diverged over the data range, and the coefficient of linear correlation would still be the same.

In order to help in the interpretation of such associative pattern, both volume estimates were simultaneously plotted against $d_{1.30}$, a measure of stem size (Figure 5). The distribution of predicted volumes showed no evidences that two populations could be identified. However, although helpful, that is just a graphical representation of the volume estimates obtained, and a numerical analysis of the residuals was considered to be adequate in the evaluation of the prediction ability of the taper-derived volume function.
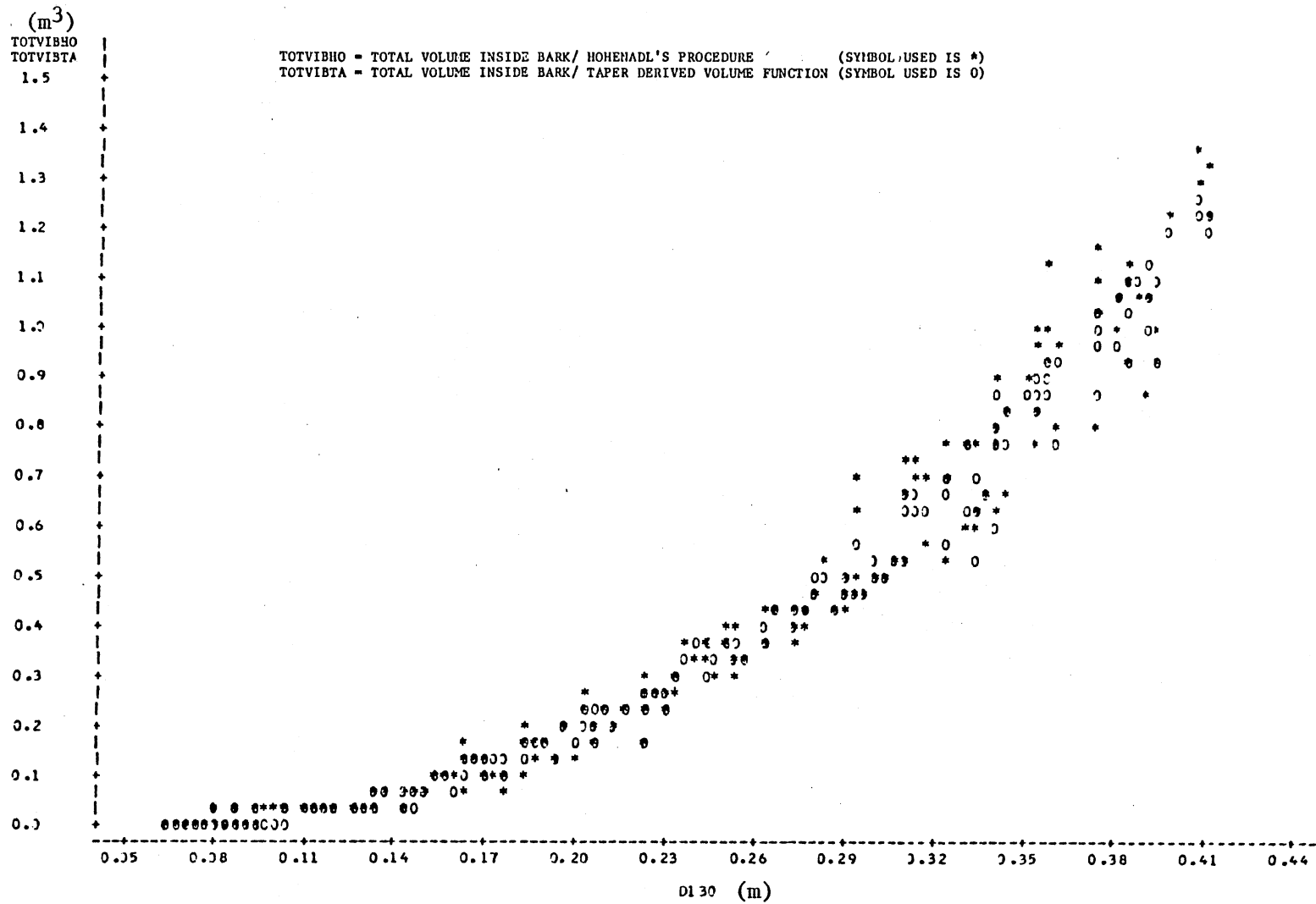
Figure 5. Plot of Total Volume Inside Bark Estimates Obtained With the Taper-Derived Volume Function and With Hohenadl's Procedure Against dbh

Residuals were analyzed by expressing the differences between the two volume estimates as a percent of the volume predicted with Hohenadl's method. The distribution of the percent residuals thus obtained over the diameter at breast height range is displayed with Figure 6. A similar trend was also observed with the distribution of residuals over the height range. It is noticeable (Figure 6) that for small trees, those with $d_{1.30}$ between 6.0 and 12.0 cm, volumes were overestimated with the taper-derived volume function as compared to Hohenadl's method. No such divergence was observed for the predicted volume of trees with $d_{1.30}$ larger than 12.0 cm. In an overall analysis the percent residuals ranged from -30.60 to 28.50, which was considered adequate for the objectives of this study. If more diameter measurements were obtained during sampling, then more accurate and reliable estimates would be determined with both methods, thus reducing the percent residual range.

It is worthwhile to note the fact that volume estimates obtained with Hohenadl's method will always be more accurate with decreasing stem height. The smaller the sections into which a stem is divided the more accurate will that method be in the total volume estimation.

On the other hand, the taper function obtained through principal component analysis was the expression of average stem form. The inclusion of an adjustment factor was done in order to have that taper function fitting all stems sampled. The solution for the eigenequation led to the conclusion that the statistical procedure was efficient to express stem form structure of the data.

These conflicting measures of efficiency observed for both procedures allows us to interpret the diverging volume estimation of small trees as being a result of stem form that seriously deviated from average
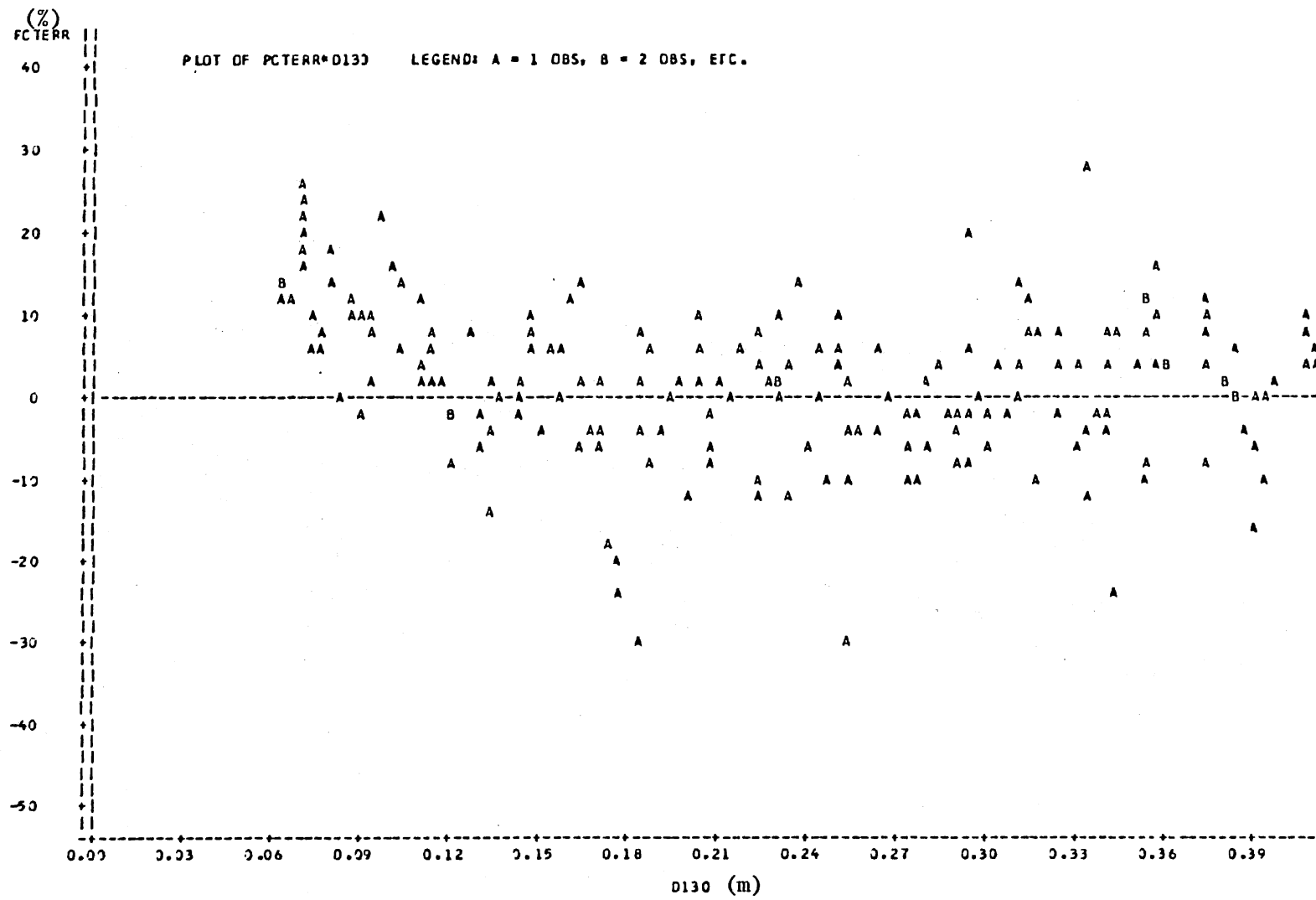
Figure 6.   Distribution of Residuals of Taper-Derived Total Volume Inside Bark Estimates

stem form. However, the value of the system must be evaluated mostly in regard to those trees considered in a size range of economical importance. In this respect, more relevant for practical purposes, the taper-derived volume function was found to be an adequate predictor that agreed well with volume estimates obtained using Hohenadl's method.

A complete listing of the volume estimates determined with both Hohenadl's method and the taper derived function is given in Appendix C. Field measurements of all sampled stems are also displayed.

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of this study was essentially the investigation of stem form of planted loblolly pine and its analytical expression with a mathematical function. By definition, integration of such a developed taper model provides a volume function.

Data were obtained in a loblolly pine plantation located in southern Brazil. A modified Hohenadl's method of tree stem measurement was used to collect the data needed for the analysis. Diameters inside bark were measured at seven relative positions along each of 188 stems.

Previous stem studies were mostly based on simple models or on the use of polynomials. Several of those investigators tried to define a universal taper model, but conflicting results were reported. On the other hand, those few studies on which principal component analysis was used, have demonstrated consistency in their conclusions. This refers to the algorithm, not to the structure of the final equation itself.

In this research study, a first attempt was made using a third-degree polynomial. However, results were biased and discouraging. Alternatives to improve that taper equation were initially considered, such as the inclusion of an additional independent variable to express the inflection point, or grouping the data by size. Nevertheless, that procedure was abandoned and a multidimensional and more flexible approach was finally used.

The objective of this study was successfully achieved with the use of principal component analysis, a multivariate exploratory technique.

The procedure consisted of solving the eigenequation of the sum of squares and cross-products matrix obtained from the data matrix of seven positional radii (diameters) and 188 stems. The solution for the eigenequation provided seven eigenvalues or latent roots, and seven associated column eigenvectors. Performing that matrix solution is algebraically equivalent to transforming a set of p correlated variables into a new set of p uncorrelated variables.

In interpreting the seven eigenvalues obtained, only the first was considered significant to explain stem form, the latent structure of the data. Thus the eigenvector of seven coefficients or elements associated with that first eigenvalue was considered to be an expression of average stem form for stems of unit length. Polynomial regression analysis was used to fit a model to those elements of the first eigenvector. The taper function thus obtained was adjusted to fit the taper of trees with any length and any dbh. The final developed model was;

$$\hat{r}(\ell) = \frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \cdot \hat{F}(\ell/L) \tag{3.3}$$

where

$\hat{r}(\ell)$ = predicted radius inside bark at any absolute position $\ell$ from the tip, in meters.

$r_{1.30}$ = $d_{1.30}/2$ (inside bark, in meters)

$\ell_{1.30}$ = $L - 1.15$ m

$\hat{r}(\ell_{1.30}) = b_o + b_1\ell_{1.30}^{1/4} + b_2\ell_{1.30}^{1/3} + b_3\ell_{1.30}^{1/2} + b_4\ell_{1.30}$

$\hat{F}(\ell/L) = b_o + b_1(\frac{\ell}{L})^{1/4} + b_2(\frac{\ell}{L})^{1/3} + b_3(\frac{\ell}{L})^{1/2} + b_4(\frac{\ell}{L})$

L = stem total length (meters) above a 15.0 cm stump height.

Total volume inside bark was predicted for all 188 sample stems by using both procedures: the volume function obtained by integration of the taper model and with Hohenadl's method.

Percent residual of the volume estimates obtained with the taper-derived function over those determined with Hohenadl's procedure, ranged from -30.60 to 28.50. The magnitude of the residuals as well as their consistent distribution over most of the data size range, were considered adequate for the purpose of this study.

After having evaluated the results obtained it was concluded that principal component analysis has proven to be an efficient means for the investigation of stem form. The results indicated that the objective has been met satisfactorily. Accuracy of the prediction ability of the taper-volume system developed can be significantly improved by measurement of diameters at a larger number of relative positions, perhaps 10 or 20.

Actually only total volume inside bark was computed. However, the main justification for the development of a taper-volume system is its flexibility in regard to the volume prediction for any section of the stem. By that means, merchantable volume estimates between any two length limits may be accordingly determined.

Among the previous stem form studies observed in the literature, some investigators tried to develop a "universal taper function". However, such a model must take in account all possible sources of stem form variability: among others, species, age, site, stand density and past management practices such as pruning and thinning. It is evident that this is an extremely difficult task. Rather than seeking a "cooking

formula", this author believes that much better and efficient results will be obtained by grouping the data as much as is possible and convenient. This must be done by considering both the cost of the taper-volume system development and the benefits of its use in practice. In this context, principal component analysis is a technique of great potential when the reduction of data dimensionality is desirable.

A universal taper function will not be produced by principal component analysis. In the taper model building process, the number and nature of the independent variables may vary with each new data set. However, if properly applied and interpreted, this multivariate exploratory technique may be considered as an adequate algorithm for the investigation of stem form under most of the circumstances.

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APPENDICES

APPENDIX A

COMMON AND SCIENTIFIC NAME OF THE CITED SPECIES

| | |
|---|---|
| Black spruce | (Picea mariana Mill.) |
| Gmelina | (Gmelina arborea L.) |
| Loblolly pine | (Pinus taeda L.) |
| Lodgpole pine | (Pinus contorta Dougl.) |
| Monterey pine | (Pinus radiata D. Dom.) |
| Norway spruce | (Picea excelsa Link) |
| Parana pine | (Araucaria angustifolia (Bert) O. Ktze) |
| Red alder | (Alnus rubra Bong.) |
| Red spruce | (Picea rubens Sarg) |
| Scots pine | (Pinus silvestris L.) |
| Slash pine | (Pinus elliottii Engelm.) |
| Sugar maple | (Acer saccharum Marsch.) |
| White birch | (Betula papyrifera var. communata Fern.) |

APPENDIX B

THE HOHENADL'S METHOD

Hohenadl (1936) suggested a procedure for stem volume estimation. This appendix will introduce the original method proposed, review some studies related to that procedure and describe the modified method used in this study.

## The Original Method

The procedure as it was first proposed, divides the stem into five or ten sections of equal length. The volume of each section is estimated according to Huber's formula, each section being treated as a cylinder and the corresponding volume obtained by:

$$V = L_H \cdot g_m = L_H \cdot \frac{\pi}{4} \cdot d_m^2 \tag{B.1}$$

where $g_m$ and $d_m$ are respectively the basal area and the diameter at the midpoint of a section of length $L_H$.

For the total stem volume above stump height, five sections being considered (Prodan, 1951), the procedure is defined with the model:

$$V = 0.2L \cdot \frac{\pi}{4} \cdot (d_{0.1L}^2 + d_{0.3L}^2 + d_{0.5L}^2 + d_{0.7L}^2 + d_{0.9L}^2) \tag{B.2}$$

where 0.2L is the length of each section or 2/10 of the total stem length, and $d_{0.1L} \cdot \cdot \cdot d_{0.9L}$ are the diameters measured at the midpoint of each section. Length is measured from top downward. Both inside and outside bark volumes may, evidently, be computed by using inside or outside bark diameter measurements.

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Comments on the Method

In a description of a study conducted by Zimmerle (1949) with stems

profile of Norway spruce, Assman (1970) pointed out that by using

Hohenadl's method, all diameters are taken at the same relative position

of the stem. An extremely regular pattern of form for stems of different

length was observed. An additional advantage was a very low variability

of the natural form-factor among trees (based on $d_{0.9L}$) when compared to

the artificial form-factor (determined with $d_{1.30m}$). The same opinion

is reported by Loetsch et al. (1973).

> Sectioning into equal relative lengths has the advantage that
> stems of different absolute length can be compared, which is
> not the case with sections of equal absolute length. It is an
> advantage of the latter method that long (which means mostly
> also big) trees will be measured more accurately than short
> trees. For accurate form investigations however, the volume
> determination of sections of equal relative length is prefer-
> able (p. 147).

Working with Parana pine in southern Brazil, Hoogh, et al. (1978)

observed the same trend and stated that the artificial form-factor $f_{1.30m}$,

depends on the $d_{1.30m}$ and consequently on the height of the tree.

Heger (1965) described Hohenadl's original method as well as the

study of Krenn and Prodan (1944) where the method is also analyzed.

Heger pointed out that the method represents a synthesis of both a

graphical and a numerical approach of stem form and stem volume estima-

tion. The natural form-factor is not just a reduction factor used for the

estimation of stem volume, but is a valid criterion for the description

of the geometrical shape of the stem. The method was found to be an

efficient and accurate means for the study of form and volume.

If Hohenadl's original procedure is applied by using only five sec-

tions, a considerable bias is obtained in the lowest fifth (the butt

section). Loetsch et al. (1973) reported that this bias can be reduced
by measurement of one or several additional diameters in the butt section.
Division of the total stem into ten or twenty sections of equal relative
length, will evidently also increase the accuracy of the method.

<center>The Modified Hohenadl's Method</center>

Following the suggestion reported by Loetsch et al. (1973), Hoogh
et al. (1978) introduced a modification on Hohenadl's original method
by dividing the butt section into three additional subsections, measure-
ment of diameters at the midpoint of each one of the subsections and
computation of a weighted average, giving a double weight for the
diameter measured on the third, closest to the ground subsection. It is
believed that an improved and more accurate procedure results.

This modified Hohenadl's method was used for the collection of the
data needed for this research study. It was applied to each sample tree
and consisted of the following steps:

- measurement of the outside bark $d_{1.30m}$ on the standing tree

- after felling the tree with a chainsaw and observing a standardized

    0.15m stump height, the total stem length above stump height was

    measured and sectioned into five equal length sections (Figure 7).

    Due to butt swell, the fifth section was subdivided into three

    additional subsections.

- diameter and bark thickness were measured on the midpoint of the four

    upper sections and of the three subsections of the butt region.

    These measurements were done across the upper face as the stem lay

    on the ground.
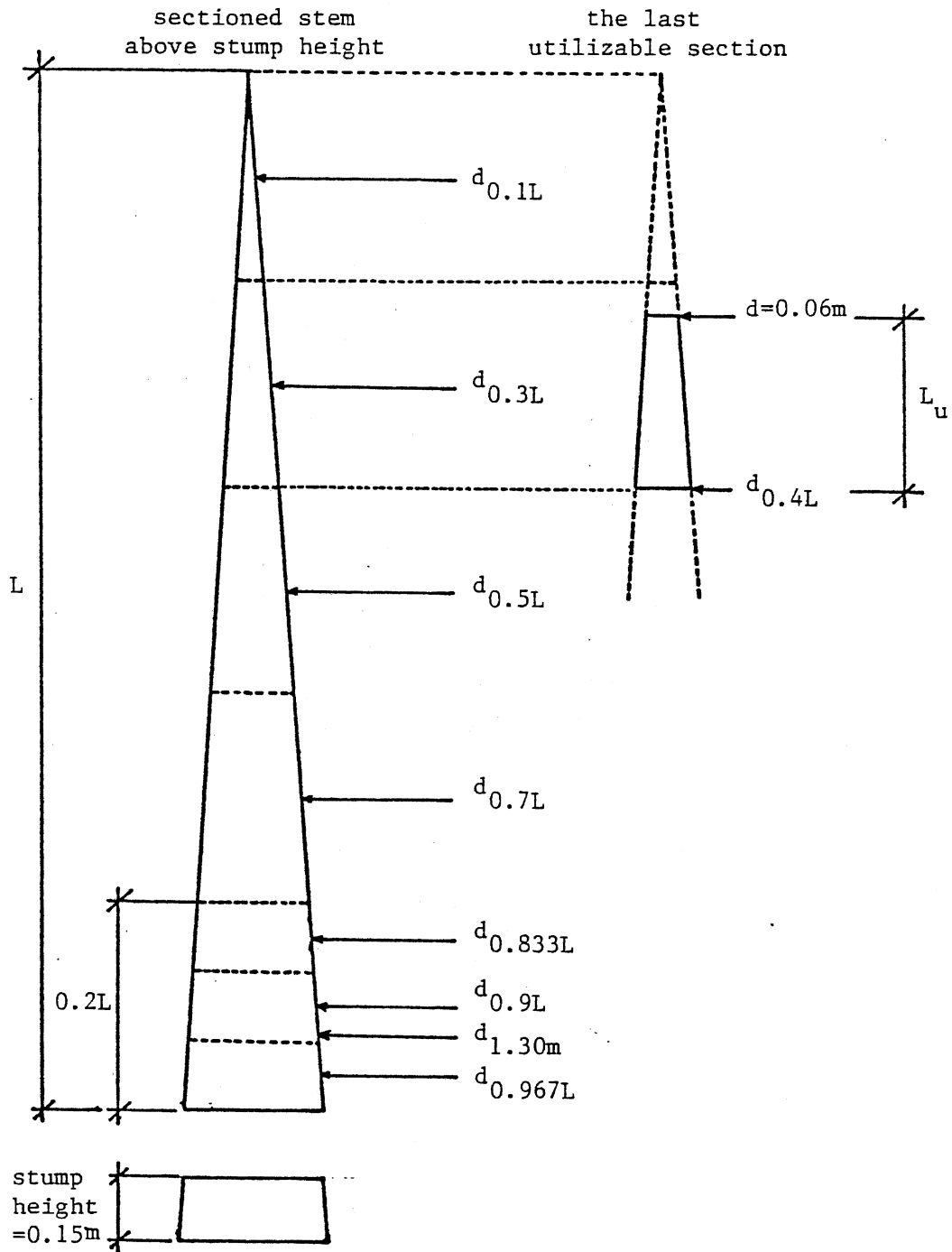
Figure 7. Diameter Measurements Along the Stem According to Modified Hohenadl's Method

- the position of a 6.0cm inside bark diameter limit of utilization

    was determined on the stem, and the length of the last utilizable

    section was measured.

- also the diameter inside bark at the base of this last utilizable

    section was measured and recorded.

- the tree height was determined by adding the stump height to the

    stem length above stump.

The instruments used for the measurements were an aluminum caliper

for the diameters, a Swedish bark gauge for the bark thickness and a

steel tape for the determination of the several lengths. All measure-

ments and the computations were done by using the metric system.

An estimate of the volume of interest may be obtained by using the

following formulas: (actually only total volume inside bark was com-

puted). Total volume outside or inside bark: (outside or inside bark

diameter measurements must be used)

$$V = 0.2L \cdot \frac{\pi}{4} \cdot (d_{0.1L}^2 + d_{0.3L}^2 + d_{0.5L}^2 + d_{0.7L}^2) +$$

$$0.2L \cdot \frac{\pi}{16} \cdot (d_{0.833L}^2 + d_{0.9L}^2 + 2d_{0.967L}^2) \qquad (B.3)$$

Merchantable volume: since the 6.0 cm merchantable diameter limit is

located at a variable position along the stem, the merchantable volume

is the sum of the volume of those completely utilized sections and the

last partially utilizable portion defined by $L_u$. For this last section

or portion of a section, volume is obtained by Smalian's formula. The

following equation exemplifies the procedure for a stem where the first

section, beginning from the tip, was not utilized (Figure 7).

$$V = 0.2L \cdot \frac{\pi}{4} \cdot (d_{0.5L}^2 + d_{0.7L}^2) +$$

$$0.2L \cdot \frac{\pi}{16} \cdot (d_{0.833L}^2 + d_{0.9L}^2 + 2 \cdot d_{0.967L}^2) +$$

$$\cdot \quad L_u \cdot \frac{\pi}{4} \cdot (\frac{0.06 + d_{0.4L}}{2})^2 \qquad (B.4)$$

APPENDIX C

FIELD MEASUREMENTS AND VOLUME ESTIMATES

FOR THE SAMPLED LOBLOLLY PINE STEMS

List of Variables Reported and Definitions

TREE  =  Tree or stem number

D01 . . . D0967  =  Diameter inside bark measured at 0.1 . . .

0.967 of stem total length, beginning from the

tip (Figure 7).  (in meters)

D130  =  Diameter at breast height, outside bark, measured at

1.30 m from the ground.  (in meters)

HEIGHT  =  Tree total height (in meters)

TOTVIBHO  =  Total volume inside bark estimate, according to

Hohenadl's method.  (cubic meters)

TOTVIBTA  =  Total volume inside bark estimate, according to the

taper-derived volume function.  (cubic meters)

RESIDUAL  =  TOTVIBTA - TOTVIBHO  (cubic meters)

PCTERR  =  $\dfrac{\text{RESIDUAL}}{\text{TOTVIBHO}}$  .  100  (%)

| OBS | TREE | D01 | D03 | D05 | D07 | D0833 | D09 | D0957 | D130 | HEIGHT | TOTVI8H0 | TOTVI8TA | RESIDUAL | PCTERR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.028 | 0.102 | 0.169 | 0.206 | 0.230 | 0.242 | 0.254 | 0.278 | 19.65 | 0.434868 | 0.444666 | -0.00980 | -2.253 |
| 2 | 2 | 0.030 | 0.100 | 0.192 | 0.315 | 0.340 | 0.337 | 0.377 | 0.382 | 21.35 | 0.916926 | 0.919735 | -0.00281 | -0.306 |
| 3 | 3 | 0.036 | 0.076 | 0.126 | 0.151 | 0.170 | 0.190 | 0.188 | 0.212 | 16.95 | 0.210247 | 0.211211 | -0.00096 | -0.459 |
| 4 | 4 | 0.029 | 0.066 | 0.128 | 0.155 | 0.176 | 0.181 | 0.194 | 0.211 | 17.55 | 0.227943 | 0.223692 | 0.00425 | 1.855 |
| 5 | 5 | 0.032 | 0.080 | 0.119 | 0.180 | 0.200 | 0.220 | 0.218 | 0.246 | 18.55 | 0.290153 | 0.318919 | -0.01877 | -9.914 |
| 6 | 6 | 0.031 | 0.092 | 0.168 | 0.203 | 0.220 | 0.225 | 0.242 | 0.262 | 21.05 | 0.436294 | 0.410798 | 0.02551 | 5.546 |
| 7 | 7 | 0.038 | 0.091 | 0.155 | 0.192 | 0.222 | 0.229 | 0.254 | 0.279 | 19.25 | 0.384938 | 0.422319 | -0.03738 | -9.711 |
| 8 | 8 | 0.026 | 0.064 | 0.100 | 0.127 | 0.148 | 0.158 | 0.168 | 0.187 | 16.05 | 0.141687 | 0.153343 | -0.01216 | -8.580 |
| 9 | 9 | 0.034 | 0.111 | 0.167 | 0.195 | 0.239 | 0.234 | 0.256 | 0.286 | 19.05 | 0.417174 | 0.429640 | -0.01247 | -2.988 |
| 10 | 10 | 0.036 | 0.098 | 0.163 | 0.194 | 0.221 | 0.228 | 0.243 | 0.273 | 19.15 | 0.337496 | 0.428238 | -0.04079 | -10.527 |
| 11 | 12 | 0.026 | 0.048 | 0.082 | 0.108 | 0.132 | 0.146 | 0.152 | 0.175 | 15.45 | 0.102393 | 0.119994 | -0.01760 | -17.189 |
| 12 | 13 | 0.029 | 0.101 | 0.176 | 0.236 | 0.262 | 0.263 | 0.294 | 0.304 | 18.65 | 0.539665 | 0.490370 | 0.01929 | 3.766 |
| 13 | 14 | 0.032 | 0.092 | 0.155 | 0.177 | 0.203 | 0.220 | 0.216 | 0.256 | 18.35 | 0.324794 | 0.339181 | -0.01439 | -4.430 |
| 14 | 15 | 0.021 | 0.047 | 0.080 | 0.110 | 0.131 | 0.144 | 0.162 | 0.178 | 15.75 | 0.107198 | 0.129540 | -0.02234 | -20.843 |
| 15 | 16 | 0.026 | 0.075 | 0.115 | 0.135 | 0.150 | 0.161 | 0.159 | 0.183 | 17.55 | 0.175298 | 0.171264 | 0.00403 | 2.301 |
| 16 | 17 | 0.031 | 0.105 | 0.184 | 0.230 | 0.264 | 0.267 | 0.284 | 0.325 | 19.85 | 0.539416 | 0.555243 | -0.01583 | -2.934 |
| 17 | 18 | 0.032 | 0.077 | 0.104 | 0.131 | 0.147 | 0.152 | 0.167 | 0.188 | 17.05 | 0.159419 | 0.150901 | 0.00852 | 5.406 |
| 18 | 19 | 0.036 | 0.105 | 0.163 | 0.208 | 0.238 | 0.241 | 0.265 | 0.290 | 20.45 | 0.465366 | 0.479055 | -0.01367 | -2.937 |
| 19 | 20 | 0.032 | 0.079 | 0.128 | 0.235 | 0.240 | 0.249 | 0.269 | 0.279 | 20.45 | 0.452220 | 0.437835 | -0.02567 | -5.553 |
| 20 | 21 | 0.035 | 0.098 | 0.147 | 0.248 | 0.273 | 0.291 | 0.334 | 0.345 | 22.35 | 0.650897 | 0.819478 | -0.15358 | -23.995 |
| 21 | 22 | 0.031 | 0.081 | 0.125 | 0.151 | 0.168 | 0.248 | 0.230 | 0.254 | 19.35 | 0.293448 | 0.379867 | -0.08642 | -29.449 |
| 22 | 23 | 0.027 | 0.085 | 0.137 | 0.172 | 0.184 | 0.189 | 0.200 | 0.227 | 18.35 | 0.259456 | 0.263220 | 0.00524 | 2.314 |
| 23 | 24 | 0.022 | 0.060 | 0.089 | 0.112 | 0.133 | 0.142 | 0.148 | 0.170 | 17.05 | 0.119364 | 0.125432 | -0.00607 | -5.084 |
| 24 | 25 | 0.038 | 0.090 | 0.127 | 0.169 | 0.178 | 0.188 | 0.206 | 0.230 | 16.75 | 0.240437 | 0.233781 | 0.00666 | 2.768 |
| 25 | 26 | 0.027 | 0.071 | 0.112 | 0.135 | 0.151 | 0.153 | 0.163 | 0.190 | 17.25 | 0.164860 | 0.172699 | -0.00784 | -4.755 |
| 26 | 27 | 0.033 | 0.065 | 0.090 | 0.107 | 0.117 | 0.125 | 0.135 | 0.153 | 15.55 | 0.100564 | 0.094617 | 0.00595 | 5.914 |
| 27 | 28 | 0.023 | 0.075 | 0.133 | 0.160 | 0.178 | 0.189 | 0.206 | 0.233 | 18.65 | 0.254308 | 0.284571 | -0.03026 | -11.900 |
| 28 | 29 | 0.026 | 0.067 | 0.091 | 0.115 | 0.135 | 0.131 | 0.148 | 0.170 | 15.25 | 0.110221 | 0.114784 | -0.00456 | -4.139 |
| 29 | 30 | 0.035 | 0.089 | 0.145 | 0.183 | 0.198 | 0.212 | 0.227 | 0.242 | 21.35 | 0.367847 | 0.366521 | 0.00133 | 0.360 |
| 30 | 31 | 0.034 | 0.093 | 0.168 | 0.214 | 0.238 | 0.245 | 0.278 | 0.295 | 21.75 | 0.514483 | 0.556726 | -0.04224 | -8.211 |
| 31 | 32 | 0.038 | 0.083 | 0.154 | 0.178 | 0.195 | 0.202 | 0.210 | 0.234 | 18.25 | 0.299924 | 0.289883 | 0.01004 | 3.346 |
| 32 | 33 | 0.033 | 0.093 | 0.138 | 0.170 | 0.199 | 0.205 | 0.221 | 0.240 | 20.85 | 0.333313 | 0.350798 | -0.01749 | -5.246 |
| 33 | 34 | 0.035 | 0.074 | 0.133 | 0.154 | 0.159 | 0.172 | 0.188 | 0.205 | 18.25 | 0.228345 | 0.216505 | 0.01184 | 5.135 |
| 34 | 35 | 0.021 | 0.063 | 0.103 | 0.140 | 0.160 | 0.169 | 0.178 | 0.207 | 17.05 | 0.169901 | 0.183467 | -0.01357 | -7.935 |
| 35 | 36 | 0.034 | 0.104 | 0.170 | 0.208 | 0.235 | 0.263 | 0.274 | 0.294 | 20.25 | 0.462459 | 0.475896 | -0.01344 | -2.906 |
| 36 | 37 | 0.032 | 0.064 | 0.109 | 0.140 | 0.147 | 0.153 | 0.160 | 0.184 | 18.65 | 0.184865 | 0.169594 | 0.01527 | 9.251 |
| 37 | 38 | 0.037 | 0.100 | 0.156 | 0.191 | 0.208 | 0.218 | 0.240 | 0.265 | 18.35 | 0.353591 | 0.367448 | -0.01386 | -3.919 |
| 38 | 39 | 0.031 | 0.101 | 0.180 | 0.244 | 0.270 | 0.291 | 0.315 | 0.329 | 19.55 | 0.595533 | 0.630476 | -0.03494 | -5.858 |
| 39 | 40 | 0.028 | 0.118 | 0.172 | 0.228 | 0.260 | 0.263 | 0.285 | 0.301 | 18.55 | 0.497156 | 0.502614 | -0.00546 | -1.098 |
| 40 | 41 | 0.028 | 0.094 | 0.157 | 0.196 | 0.218 | 0.227 | 0.250 | 0.272 | 18.45 | 0.359950 | 0.339547 | -0.01960 | -5.297 |
| 41 | 42 | 0.033 | 0.092 | 0.175 | 0.194 | 0.211 | 0.213 | 0.254 | 0.250 | 19.05 | 0.396152 | 0.360026 | 0.03593 | 9.059 |
| 42 | 43 | 0.029 | 0.090 | 0.138 | 0.171 | 0.197 | 0.198 | 0.214 | 0.230 | 18.05 | 0.280124 | 0.275130 | 0.00499 | 1.753 |
| 43 | 44 | 0.029 | 0.085 | 0.138 | 0.180 | 0.199 | 0.205 | 0.231 | 0.255 | 19.25 | 0.319815 | 0.350067 | -0.03025 | -9.459 |
| 44 | 45 | 0.031 | 0.123 | 0.174 | 0.215 | 0.222 | 0.232 | 0.284 | 0.284 | 20.95 | 0.518501 | 0.499098 | 0.01940 | 3.742 |
| 45 | 46 | 0.037 | 0.104 | 0.161 | 0.200 | 0.219 | 0.229 | 0.255 | 0.275 | 20.75 | 0.439165 | 0.446998 | -0.00783 | -1.724 |
| 46 | 47 | 0.038 | 0.111 | 0.170 | 0.211 | 0.233 | 0.244 | 0.273 | 0.280 | 19.95 | 0.475567 | 0.470504 | 0.00506 | 1.055 |
| 47 | 48 | 0.036 | 0.092 | 0.173 | 0.189 | 0.206 | 0.214 | 0.235 | 0.250 | 19.35 | 0.377234 | 0.362331 | 0.01490 | 3.930 |
| 48 | 49 | 0.030 | 0.070 | 0.138 | 0.174 | 0.178 | 0.193 | 0.201 | 0.218 | 16.85 | 0.242790 | 0.226314 | 0.01648 | 6.785 |
| 49 | 50 | 0.027 | 0.066 | 0.103 | 0.129 | 0.148 | 0.161 | 0.189 | 0.184 | 17.55 | 0.159872 | 0.177613 | -0.01774 | -9.987 |
| 50 | 51 | 0.029 | 0.075 | 0.145 | 0.162 | 0.184 | 0.201 | 0.210 | 0.229 | 17.30 | 0.254169 | 0.254238 | -0.00007 | -0.027 |
| 51 | 52 | 0.032 | 0.102 | 0.161 | 0.194 | 0.217 | 0.225 | 0.235 | 0.255 | 19.60 | 0.388090 | 0.383047 | 0.00504 | 1.299 |
| 52 | 53 | 0.031 | 0.064 | 0.133 | 0.148 | 0.169 | 0.179 | 0.195 | 0.202 | 18.80 | 0.239557 | 0.236794 | 0.00276 | 1.153 |
| 53 | 54 | 0.033 | 0.087 | 0.137 | 0.163 | 0.179 | 0.184 | 0.193 | 0.205 | 19.25 | 0.267304 | 0.242957 | 0.02435 | 9.108 |
| 54 | 55 | 0.040 | 0.097 | 0.150 | 0.181 | 0.195 | 0.204 | 0.225 | 0.243 | 18.85 | 0.327498 | 0.306449 | 0.02105 | 6.427 |
| 55 | 56 | 0.038 | 0.089 | 0.181 | 0.193 | 0.209 | 0.212 | 0.223 | 0.237 | 19.25 | 0.379215 | 0.324467 | 0.05475 | 14.437 |

| OBS | TREE | D01 | D03 | D05 | D07 | D0833 | D09 | D0967 | D130 | HEIGHT | TOTVIBHO | TOTVIBTA | RESIDUAL | PCTERR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 56 | 57 | 0.030 | 0.093 | 0.139 | 0.210 | 0.237 | 0.237 | 0.288 | 0.289 | 19.75 | 0.438805 | 0.474647 | -0.035842 | -8.168 |
| 57 | 58 | 0.030 | 0.100 | 0.166 | 0.200 | 0.213 | 0.222 | 0.240 | 0.249 | 19.55 | 0.398956 | 0.371229 | 0.027727 | 6.950 |
| 58 | 59 | 0.044 | 0.137 | 0.232 | 0.289 | 0.309 | 0.318 | 0.350 | 0.360 | 22.75 | 0.953002 | 0.919039 | 0.033963 | 3.564 |
| 59 | 60 | 0.035 | 0.079 | 0.148 | 0.173 | 0.181 | 0.189 | 0.196 | 0.229 | 18.55 | 0.277891 | 0.249569 | 0.028322 | 10.192 |
| 60 | 61 | 0.034 | 0.093 | 0.136 | 0.158 | 0.176 | 0.194 | 0.207 | 0.224 | 19.35 | 0.276990 | 0.266463 | 0.010527 | 3.801 |
| 61 | 62 | 0.035 | 0.086 | 0.141 | 0.176 | 0.199 | 0.199 | 0.215 | 0.254 | 19.25 | 0.306788 | 0.318468 | -0.011680 | -3.807 |
| 62 | 63 | 0.034 | 0.101 | 0.169 | 0.197 | 0.222 | 0.233 | 0.252 | 0.267 | 20.35 | 0.432710 | 0.434108 | -0.001398 | -0.323 |
| 63 | 64 | 0.032 | 0.077 | 0.122 | 0.146 | 0.153 | 0.166 | 0.179 | 0.197 | 17.45 | 0.196483 | 0.194077 | 0.002407 | 1.225 |
| 64 | 65 | 0.030 | 0.071 | 0.092 | 0.164 | 0.176 | 0.189 | 0.217 | 0.222 | 17.40 | 0.220888 | 0.246330 | -0.025442 | -11.518 |
| 65 | 66 | 0.024 | 0.067 | 0.095 | 0.120 | 0.137 | 0.144 | 0.153 | 0.167 | 16.75 | 0.130561 | 0.134668 | -0.004107 | -3.146 |
| 66 | 67 | 0.020 | 0.084 | 0.123 | 0.154 | 0.155 | 0.181 | 0.195 | 0.207 | 16.95 | 0.211933 | 0.217366 | -0.005433 | -2.564 |
| 67 | 68 | 0.039 | 0.104 | 0.144 | 0.175 | 0.189 | 0.201 | 0.203 | 0.225 | 17.95 | 0.288921 | 0.266613 | 0.022308 | 7.721 |
| 68 | 69 | 0.029 | 0.077 | 0.099 | 0.114 | 0.138 | 0.143 | 0.151 | 0.169 | 16.55 | 0.130971 | 0.127836 | 0.003135 | 2.393 |
| 69 | 70 | 0.030 | 0.073 | 0.097 | 0.118 | 0.135 | 0.144 | 0.155 | 0.165 | 17.15 | 0.137444 | 0.134559 | 0.002884 | 2.099 |
| 70 | 71 | 0.044 | 0.082 | 0.111 | 0.127 | 0.139 | 0.139 | 0.154 | 0.164 | 17.05 | 0.155638 | 0.135055 | 0.020583 | 13.225 |
| 71 | 72 | 0.021 | 0.043 | 0.075 | 0.105 | 0.120 | 0.133 | 0.146 | 0.144 | 8.15 | 0.047979 | 0.049324 | -0.001345 | -2.803 |
| 72 | 73 | 0.027 | 0.047 | 0.078 | 0.112 | 0.128 | 0.141 | 0.164 | 0.148 | 9.05 | 0.051625 | 0.056464 | 0.005161 | 8.375 |
| 73 | 74 | 0.023 | 0.042 | 0.073 | 0.108 | 0.129 | 0.123 | 0.152 | 0.144 | 8.45 | 0.051395 | 0.051643 | -0.000248 | -0.482 |
| 74 | 75 | 0.024 | 0.049 | 0.072 | 0.104 | 0.121 | 0.134 | 0.150 | 0.147 | 9.15 | 0.054253 | 0.049139 | 0.005114 | 9.427 |
| 75 | 76 | 0.014 | 0.040 | 0.068 | 0.086 | 0.094 | 0.103 | 0.114 | 0.111 | 7.35 | 0.028473 | 0.025207 | 0.003266 | 11.469 |
| 76 | 77 | 0.029 | 0.052 | 0.071 | 0.094 | 0.109 | 0.122 | 0.132 | 0.135 | 14.45 | 0.073733 | 0.072655 | 0.001076 | 1.452 |
| 77 | 78 | 0.029 | 0.063 | 0.102 | 0.149 | 0.188 | 0.207 | 0.228 | 0.225 | 12.55 | 0.151579 | 0.177906 | -0.016327 | -10.104 |
| 78 | 79 | 0.023 | 0.063 | 0.113 | 0.154 | 0.179 | 0.179 | 0.199 | 0.206 | 14.05 | 0.157594 | 0.179110 | -0.011415 | -6.807 |
| 79 | 80 | 0.022 | 0.050 | 0.079 | 0.101 | 0.117 | 0.131 | 0.149 | 0.143 | 12.35 | 0.073280 | 0.071262 | 0.002018 | 2.754 |
| 80 | 81 | 0.018 | 0.057 | 0.091 | 0.114 | 0.134 | 0.138 | 0.152 | 0.148 | 11.75 | 0.083184 | 0.078460 | 0.004724 | 5.679 |
| 81 | 82 | 0.020 | 0.040 | 0.067 | 0.073 | 0.089 | 0.101 | 0.112 | 0.120 | 11.05 | 0.038730 | 0.039461 | -0.000731 | -1.887 |
| 82 | 83 | 0.020 | 0.054 | 0.085 | 0.111 | 0.135 | 0.142 | 0.149 | 0.176 | 12.15 | 0.082108 | 0.102411 | -0.020303 | -24.727 |
| 83 | 84 | 0.022 | 0.062 | 0.092 | 0.118 | 0.136 | 0.150 | 0.163 | 0.159 | 11.75 | 0.091561 | 0.081244 | 0.010317 | 11.268 |
| 84 | 85 | 0.017 | 0.051 | 0.078 | 0.109 | 0.126 | 0.141 | 0.160 | 0.162 | 12.15 | 0.080288 | 0.085775 | -0.005486 | -5.833 |
| 85 | 86 | 0.027 | 0.062 | 0.112 | 0.137 | 0.160 | 0.179 | 0.199 | 0.201 | 12.75 | 0.138736 | 0.154190 | -0.015455 | -11.140 |
| 86 | 87 | 0.018 | 0.041 | 0.061 | 0.080 | 0.092 | 0.098 | 0.100 | 0.110 | 10.55 | 0.034597 | 0.033584 | 0.001013 | 2.927 |
| 87 | 88 | 0.011 | 0.039 | 0.053 | 0.062 | 0.070 | 0.075 | 0.085 | 0.081 | 9.35 | 0.021010 | 0.018085 | 0.002925 | 13.924 |
| 88 | 89 | 0.010 | 0.031 | 0.050 | 0.067 | 0.080 | 0.080 | 0.090 | 0.092 | 8.85 | 0.020909 | 0.020338 | 0.000571 | 2.730 |
| 89 | 90 | 0.013 | 0.039 | 0.056 | 0.070 | 0.076 | 0.080 | 0.084 | 0.087 | 9.30 | 0.023425 | 0.021021 | 0.002404 | 10.262 |
| 90 | 91 | 0.019 | 0.041 | 0.095 | 0.118 | 0.137 | 0.142 | 0.151 | 0.157 | 12.60 | 0.090203 | 0.089906 | 0.000297 | 0.330 |
| 91 | 92 | 0.014 | 0.047 | 0.070 | 0.091 | 0.109 | 0.122 | 0.130 | 0.134 | 10.75 | 0.053113 | 0.055405 | -0.002293 | -4.316 |
| 92 | 93 | 0.017 | 0.047 | 0.072 | 0.094 | 0.106 | 0.107 | 0.122 | 0.128 | 10.00 | 0.045846 | 0.042418 | 0.003428 | 7.477 |
| 93 | 94 | 0.021 | 0.063 | 0.097 | 0.131 | 0.159 | 0.168 | 0.192 | 0.195 | 12.35 | 0.120326 | 0.121284 | -0.000958 | -0.796 |
| 94 | 95 | 0.016 | 0.057 | 0.083 | 0.124 | 0.141 | 0.150 | 0.161 | 0.185 | 12.20 | 0.093364 | 0.121961 | -0.028596 | -30.629 |
| 95 | 96 | 0.017 | 0.045 | 0.076 | 0.085 | 0.100 | 0.105 | 0.111 | 0.120 | 10.55 | 0.041350 | 0.041919 | -0.000569 | -1.376 |
| 96 | 97 | 0.016 | 0.035 | 0.063 | 0.078 | 0.099 | 0.110 | 0.131 | 0.113 | 9.75 | 0.038588 | 0.036636 | 0.001953 | 5.051 |
| 97 | 98 | 0.015 | 0.041 | 0.067 | 0.087 | 0.096 | 0.113 | 0.105 | 0.113 | 10.75 | 0.041581 | 0.037993 | 0.003588 | 8.629 |
| 98 | 99 | 0.023 | 0.046 | 0.077 | 0.097 | 0.107 | 0.115 | 0.134 | 0.136 | 10.25 | 0.052560 | 0.052614 | -0.000054 | -0.102 |
| 99 | 100 | 0.017 | 0.039 | 0.056 | 0.084 | 0.107 | 0.113 | 0.126 | 0.130 | 7.15 | 0.028582 | 0.030340 | -0.001757 | -6.149 |
| 100 | 101 | 0.018 | 0.029 | 0.054 | 0.077 | 0.094 | 0.099 | 0.109 | 0.102 | 5.80 | 0.013291 | 0.015662 | 0.002629 | 14.373 |
| 101 | 102 | 0.018 | 0.029 | 0.051 | 0.068 | 0.076 | 0.082 | 0.083 | 0.090 | 5.15 | 0.012396 | 0.012672 | -0.000277 | -2.233 |
| 102 | 103 | 0.014 | 0.031 | 0.051 | 0.083 | 0.093 | 0.102 | 0.103 | 0.098 | 6.00 | 0.019255 | 0.014982 | 0.004272 | 22.189 |
| 103 | 104 | 0.016 | 0.028 | 0.052 | 0.064 | 0.073 | 0.079 | 0.080 | 0.078 | 5.20 | 0.011052 | 0.010247 | 0.000805 | 7.282 |
| 104 | 105 | 0.020 | 0.026 | 0.045 | 0.058 | 0.065 | 0.069 | 0.073 | 0.073 | 4.85 | 0.008399 | 0.007930 | 0.000468 | 5.577 |
| 105 | 106 | 0.017 | 0.038 | 0.056 | 0.079 | 0.098 | 0.103 | 0.110 | 0.114 | 6.55 | 0.022331 | 0.021896 | 0.000435 | 1.949 |
| 106 | 107 | 0.016 | 0.025 | 0.051 | 0.086 | 0.108 | 0.113 | 0.123 | 0.121 | 7.15 | 0.026918 | 0.029094 | -0.002176 | -8.035 |
| 107 | 108 | 0.014 | 0.022 | 0.045 | 0.060 | 0.062 | 0.067 | 0.075 | 0.065 | 4.35 | 0.007390 | 0.006324 | 0.001065 | 14.417 |
| 108 | 109 | 0.019 | 0.026 | 0.057 | 0.075 | 0.080 | 0.083 | 0.090 | 0.090 | 6.25 | 0.015561 | 0.015034 | 0.001526 | 9.215 |
| 109 | 110 | 0.015 | 0.029 | 0.056 | 0.079 | 0.088 | 0.090 | 0.097 | 0.103 | 6.10 | 0.017859 | 0.016794 | 0.001065 | 5.955 |
| 110 | 111 | 0.016 | 0.024 | 0.044 | 0.059 | 0.054 | 0.067 | 0.077 | 0.065 | 4.75 | 0.008208 | 0.006979 | 0.001229 | 14.971 |

| OBS | TREE | D01 | D03 | D05 | D07 | D0833 | D09 | D0967 | D130 | HEIGHT | TOTVIBHO | TOTVIBTA | RESIDUAL | PCTERR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 111 | 112 | 0.015 | 0.027 | 0.047 | 0.058 | 0.064 | 0.068 | 0.075 | 0.069 | 4.85 | 0.00850 | 0.00690 | 0.00160 | 18.637 |
| 112 | 113 | 0.018 | 0.029 | 0.042 | 0.070 | 0.076 | 0.080 | 0.082 | 0.084 | 5.09 | 0.01105 | 0.01115 | -0.00010 | -0.910 |
| 113 | 114 | 0.014 | 0.032 | 0.048 | 0.062 | 0.059 | 0.067 | 0.077 | 0.069 | 4.55 | 0.00874 | 0.00705 | 0.00169 | 19.359 |
| 114 | 115 | 0.012 | 0.030 | 0.058 | 0.075 | 0.086 | 0.091 | 0.096 | 0.093 | 5.45 | 0.01545 | 0.01376 | 0.00169 | 10.952 |
| 115 | 116 | 0.018 | 0.030 | 0.047 | 0.061 | 0.070 | 0.072 | 0.075 | 0.074 | 5.01 | 0.00953 | 0.00855 | 0.00098 | 10.292 |
| 116 | 117 | 0.020 | 0.031 | 0.052 | 0.055 | 0.071 | 0.077 | 0.080 | 0.086 | 5.55 | 0.01207 | 0.01060 | 0.00147 | 12.198 |
| 117 | 118 | 0.016 | 0.025 | 0.045 | 0.059 | 0.064 | 0.068 | 0.077 | 0.069 | 4.35 | 0.00751 | 0.00573 | 0.00188 | 24.737 |
| 118 | 119 | 0.020 | 0.046 | 0.061 | 0.084 | 0.094 | 0.108 | 0.112 | 0.109 | 6.55 | 0.02482 | 0.02373 | 0.00109 | 4.395 |
| 119 | 120 | 0.020 | 0.030 | 0.052 | 0.084 | 0.112 | 0.117 | 0.142 | 0.132 | 3.00 | 0.03416 | 0.03862 | -0.00446 | -13.067 |
| 120 | 121 | 0.018 | 0.036 | 0.061 | 0.084 | 0.097 | 0.107 | 0.116 | 0.117 | 7.95 | 0.02982 | 0.02904 | 0.00079 | 2.634 |
| 121 | 122 | 0.015 | 0.038 | 0.059 | 0.085 | 0.083 | 0.090 | 0.093 | 0.099 | 5.90 | 0.01847 | 0.01559 | 0.00288 | 15.594 |
| 122 | 123 | 0.017 | 0.031 | 0.057 | 0.075 | 0.082 | 0.088 | 0.094 | 0.094 | 5.25 | 0.01740 | 0.01612 | 0.00128 | 7.368 |
| 123 | 124 | 0.018 | 0.027 | 0.049 | 0.067 | 0.076 | 0.078 | 0.083 | 0.080 | 4.85 | 0.01060 | 0.00875 | 0.00185 | 17.444 |
| 124 | 125 | 0.016 | 0.028 | 0.048 | 0.066 | 0.059 | 0.073 | 0.078 | 0.077 | 4.95 | 0.01000 | 0.00943 | 0.00057 | 5.667 |
| 125 | 126 | 0.018 | 0.027 | 0.047 | 0.059 | 0.061 | 0.067 | 0.074 | 0.071 | 4.45 | 0.00779 | 0.00610 | 0.00169 | 21.730 |
| 126 | 127 | 0.018 | 0.029 | 0.046 | 0.060 | 0.066 | 0.067 | 0.071 | 0.070 | 4.60 | 0.00812 | 0.00598 | 0.00213 | 26.270 |
| 127 | 128 | 0.016 | 0.023 | 0.040 | 0.055 | 0.060 | 0.061 | 0.058 | 0.054 | 4.59 | 0.00666 | 0.00583 | 0.00084 | 12.535 |
| 128 | 129 | 0.012 | 0.025 | 0.043 | 0.059 | 0.058 | 0.063 | 0.072 | 0.068 | 4.75 | 0.00760 | 0.00665 | 0.00095 | 12.514 |
| 129 | 130 | 0.017 | 0.027 | 0.044 | 0.058 | 0.054 | 0.072 | 0.078 | 0.069 | 5.25 | 0.00788 | 0.00788 | 0.00147 | 15.743 |
| 130 | 131 | 0.020 | 0.036 | 0.060 | 0.087 | 0.102 | 0.110 | 0.126 | 0.129 | 5.75 | 0.02740 | 0.02789 | -0.00049 | -1.773 |
| 131 | 132 | 0.043 | 0.139 | 0.246 | 0.294 | 0.302 | 0.314 | 0.327 | 0.372 | 26.13 | 1.09792 | 1.01115 | 0.08677 | 7.903 |
| 132 | 133 | 0.047 | 0.147 | 0.221 | 0.294 | 0.311 | 0.320 | 0.341 | 0.380 | 25.61 | 1.06786 | 1.05110 | 0.01676 | 1.570 |
| 133 | 134 | 0.040 | 0.140 | 0.200 | 0.247 | 0.247 | 0.259 | 0.288 | 0.339 | 24.70 | 0.75469 | 0.77226 | -0.01757 | -2.328 |
| 134 | 135 | 0.033 | 0.104 | 0.188 | 0.220 | 0.232 | 0.233 | 0.280 | 0.335 | 24.25 | 0.61281 | 0.69027 | -0.07746 | -12.640 |
| 135 | 136 | 0.046 | 0.123 | 0.203 | 0.239 | 0.254 | 0.257 | 0.275 | 0.318 | 24.29 | 0.70540 | 0.64471 | 0.06069 | 8.604 |
| 136 | 137 | 0.032 | 0.119 | 0.169 | 0.207 | 0.227 | 0.248 | 0.273 | 0.317 | 23.89 | 0.56726 | 0.62070 | -0.05344 | -9.421 |
| 137 | 138 | 0.060 | 0.170 | 0.210 | 0.255 | 0.263 | 0.295 | 0.327 | 0.333 | 20.75 | 0.76357 | 0.54599 | 0.21758 | 28.495 |
| 138 | 139 | 0.083 | 0.152 | 0.227 | 0.271 | 0.288 | 0.303 | 0.336 | 0.392 | 23.55 | 0.93766 | 0.93083 | 0.00683 | 0.728 |
| 139 | 140 | 0.039 | 0.119 | 0.196 | 0.271 | 0.311 | 0.293 | 0.326 | 0.390 | 24.63 | 0.87027 | 1.01277 | -0.14250 | -16.374 |
| 140 | 141 | 0.049 | 0.124 | 0.181 | 0.232 | 0.251 | 0.257 | 0.285 | 0.332 | 22.63 | 0.52585 | 0.64952 | -0.02367 | -3.783 |
| 141 | 142 | 0.042 | 0.139 | 0.206 | 0.269 | 0.285 | 0.290 | 0.299 | 0.359 | 22.79 | 0.73919 | 0.76474 | 0.02445 | 3.098 |
| 142 | 143 | 0.057 | 0.127 | 0.194 | 0.234 | 0.255 | 0.258 | 0.283 | 0.341 | 22.50 | 0.64847 | 0.59225 | 0.05623 | 8.671 |
| 143 | 144 | 0.053 | 0.155 | 0.229 | 0.267 | 0.293 | 0.298 | 0.319 | 0.351 | 23.41 | 0.89554 | 0.85624 | 0.03931 | 4.339 |
| 144 | 145 | 0.051 | 0.162 | 0.261 | 0.318 | 0.333 | 0.356 | 0.406 | 0.407 | 24.60 | 1.30547 | 1.24029 | 0.06518 | 4.593 |
| 145 | 146 | 0.043 | 0.154 | 0.238 | 0.295 | 0.312 | 0.330 | 0.356 | 0.390 | 23.91 | 1.06568 | 1.06568 | -0.00413 | -0.339 |
| 146 | 147 | 0.056 | 0.155 | 0.254 | 0.307 | 0.332 | 0.361 | 0.367 | 0.409 | 25.38 | 1.24207 | 1.19724 | 0.04483 | 3.610 |
| 147 | 148 | 0.042 | 0.142 | 0.218 | 0.245 | 0.253 | 0.279 | 0.297 | 0.331 | 23.75 | 0.77494 | 0.75074 | 0.02420 | 3.123 |
| 148 | 149 | 0.040 | 0.184 | 0.271 | 0.333 | 0.351 | 0.369 | 0.382 | 0.408 | 24.27 | 1.36156 | 1.22012 | 0.14144 | 10.358 |
| 149 | 150 | 0.055 | 0.175 | 0.246 | 0.277 | 0.294 | 0.315 | 0.334 | 0.375 | 24.03 | 1.02437 | 0.97766 | 0.04671 | 4.560 |
| 150 | 151 | 0.043 | 0.157 | 0.243 | 0.299 | 0.347 | 0.325 | 0.352 | 0.382 | 24.35 | 1.11534 | 1.04568 | 0.06967 | 6.246 |
| 151 | 152 | 0.040 | 0.142 | 0.228 | 0.286 | 0.305 | 0.315 | 0.359 | 0.393 | 23.95 | 1.00209 | 1.10815 | -0.10606 | -10.594 |
| 152 | 153 | 0.056 | 0.153 | 0.228 | 0.283 | 0.323 | 0.331 | 0.350 | 0.390 | 24.85 | 1.06052 | 1.12227 | -0.06174 | -5.822 |
| 153 | 154 | 0.053 | 0.173 | 0.257 | 0.311 | 0.345 | 0.351 | 0.386 | 0.409 | 25.45 | 1.32067 | 1.23815 | 0.08252 | 6.248 |
| 154 | 155 | 0.052 | 0.149 | 0.209 | 0.255 | 0.265 | 0.294 | 0.325 | 0.372 | 22.60 | 0.79405 | 0.86312 | -0.06907 | -8.698 |
| 155 | 156 | 0.059 | 0.162 | 0.227 | 0.298 | 0.301 | 0.303 | 0.319 | 0.355 | 23.80 | 0.99018 | 0.87523 | 0.11496 | 11.610 |
| 156 | 157 | 0.050 | 0.158 | 0.245 | 0.304 | 0.329 | 0.334 | 0.371 | 0.396 | 25.67 | 1.21733 | 1.19918 | 0.01815 | 1.491 |
| 157 | 158 | 0.051 | 0.161 | 0.243 | 0.287 | 0.291 | 0.299 | 0.310 | 0.352 | 24.85 | 1.01463 | 0.89299 | 0.12164 | 11.989 |
| 158 | 159 | 0.060 | 0.203 | 0.260 | 0.310 | 0.338 | 0.337 | 0.386 | 0.408 | 25.63 | 1.36065 | 1.26006 | 0.10060 | 7.393 |
| 159 | 160 | 0.045 | 0.132 | 0.231 | 0.286 | 0.318 | 0.329 | 0.345 | 0.388 | 23.55 | 1.06313 | 1.11373 | -0.05060 | -4.750 |
| 160 | 161 | 0.048 | 0.164 | 0.240 | 0.275 | 0.291 | 0.295 | 0.339 | 0.356 | 23.99 | 0.98417 | 0.87819 | 0.10599 | 10.769 |
| 161 | 162 | 0.051 | 0.169 | 0.239 | 0.290 | 0.316 | 0.325 | 0.356 | 0.385 | 24.90 | 1.11688 | 1.10759 | 0.00929 | 0.832 |
| 162 | 163 | 0.053 | 0.148 | 0.256 | 0.301 | 0.315 | 0.315 | 0.339 | 0.373 | 24.15 | 1.08544 | 0.95142 | 0.13403 | 12.348 |
| 163 | 164 | 0.052 | 0.148 | 0.210 | 0.265 | 0.285 | 0.306 | 0.324 | 0.341 | 24.39 | 0.89531 | 0.85523 | 0.04007 | 4.476 |
| 164 | 165 | 0.053 | 0.159 | 0.259 | 0.296 | 0.310 | 0.316 | 0.326 | 0.357 | 25.13 | 1.11796 | 0.93902 | 0.17894 | 16.006 |
| 165 | 166 | 0.051 | 0.106 | 0.199 | 0.244 | 0.248 | 0.268 | 0.280 | 0.322 | 23.90 | 0.69206 | 0.66478 | 0.02727 | 3.941 |

| OBS | TREE | DO1 | DO3 | DO5 | DO7 | DO833 | DO9 | DO967 | D130 | HEIGHT | TOTVI3HO | TOTVI8TA | RESIDUAL | PCTERR |
|-----|------|-----|-----|-----|-----|-------|-----|-------|------|--------|----------|----------|----------|--------|
| 166 | 167 | 0.066 | 0.161 | 0.256 | 0.299 | 0.316 | 0.322 | 0.348 | 0.372 | 25.17 | 1.16588 | 1.04700 | 0.125873 | 10.7964 |
| 167 | 168 | 0.054 | 0.168 | 0.234 | 0.268 | 0.301 | 0.310 | 0.324 | 0.353 | 24.35 | 0.97650 | 0.90003 | 0.075472 | 7.3313 |
| 168 | 169 | 0.036 | 0.116 | 0.192 | 0.223 | 0.245 | 0.249 | 0.269 | 0.311 | 20.35 | 0.53343 | 0.53368 | -0.000251 | -0.0471 |
| 169 | 170 | 0.043 | 0.138 | 0.198 | 0.231 | 0.253 | 0.264 | 0.278 | 0.310 | 23.40 | 0.67756 | 0.65470 | 0.022857 | 3.3735 |
| 170 | 171 | 0.035 | 0.150 | 0.197 | 0.234 | 0.255 | 0.249 | 0.290 | 0.313 | 23.10 | 0.68890 | 0.62833 | 0.060566 | 8.7917 |
| 171 | 172 | 0.040 | 0.155 | 0.235 | 0.281 | 0.295 | 0.305 | 0.334 | 0.380 | 24.29 | 0.98818 | 0.96824 | 0.019934 | 2.0172 |
| 172 | 173 | 0.056 | 0.147 | 0.220 | 0.273 | 0.305 | 0.309 | 0.327 | 0.356 | 23.99 | 0.93027 | 0.90218 | 0.028086 | 3.0192 |
| 173 | 174 | 0.052 | 0.156 | 0.207 | 0.224 | 0.244 | 0.239 | 0.247 | 0.295 | 25.50 | 0.71569 | 0.57500 | 0.140697 | 19.6588 |
| 174 | 175 | 0.053 | 0.144 | 0.206 | 0.249 | 0.266 | 0.275 | 0.308 | 0.340 | 23.75 | 0.73594 | 0.81095 | -0.025015 | -3.1828 |
| 175 | 176 | 0.054 | 0.147 | 0.204 | 0.226 | 0.255 | 0.260 | 0.285 | 0.336 | 22.25 | 0.65399 | 0.67749 | -0.013503 | -2.0336 |
| 176 | 177 | 0.055 | 0.162 | 0.204 | 0.239 | 0.254 | 0.246 | 0.269 | 0.310 | 24.25 | 0.74019 | 0.63923 | 0.100959 | 13.6397 |
| 177 | 178 | 0.039 | 0.130 | 0.203 | 0.249 | 0.274 | 0.271 | 0.318 | 0.355 | 25.25 | 0.82529 | 0.89905 | -0.073751 | -8.9353 |
| 178 | 179 | 0.044 | 0.138 | 0.200 | 0.242 | 0.263 | 0.277 | 0.308 | 0.353 | 23.75 | 0.75421 | 0.82504 | -0.070831 | -9.3914 |
| 179 | 180 | 0.050 | 0.133 | 0.231 | 0.258 | 0.274 | 0.279 | 0.310 | 0.343 | 23.45 | 0.82859 | 0.77035 | 0.058241 | 7.0289 |
| 180 | 181 | 0.043 | 0.139 | 0.199 | 0.254 | 0.252 | 0.283 | 0.310 | 0.322 | 23.45 | 0.77049 | 0.71204 | 0.058453 | 7.5854 |
| 181 | 182 | 0.046 | 0.146 | 0.215 | 0.248 | 0.259 | 0.262 | 0.291 | 0.313 | 23.15 | 0.74942 | 0.66679 | 0.082633 | 11.0263 |
| 182 | 183 | 0.027 | 0.103 | 0.176 | 0.228 | 0.255 | 0.256 | 0.280 | 0.301 | 19.59 | 0.50732 | 0.53274 | -0.025411 | -5.0089 |
| 183 | 184 | 0.039 | 0.103 | 0.174 | 0.224 | 0.247 | 0.258 | 0.271 | 0.295 | 18.55 | 0.45590 | 0.46282 | 0.003076 | 0.5602 |
| 184 | 185 | 0.036 | 0.120 | 0.173 | 0.216 | 0.235 | 0.247 | 0.257 | 0.290 | 20.19 | 0.48592 | 0.50486 | -0.018937 | -3.8972 |
| 185 | 186 | 0.037 | 0.117 | 0.173 | 0.226 | 0.234 | 0.257 | 0.272 | 0.308 | 20.93 | 0.53289 | 0.53961 | -0.006722 | -1.2614 |
| 186 | 187 | 0.040 | 0.119 | 0.191 | 0.230 | 0.251 | 0.259 | 0.290 | 0.293 | 22.21 | 0.62274 | 0.58071 | 0.042030 | 6.7493 |
| 187 | 188 | 0.022 | 0.056 | 0.082 | 0.104 | 0.129 | 0.136 | 0.141 | 0.151 | 12.80 | 0.07925 | 0.08257 | -0.003313 | -4.1801 |
| 188 | 189 | 0.022 | 0.065 | 0.103 | 0.124 | 0.134 | 0.141 | 0.146 | 0.158 | 13.48 | 0.10639 | 0.09927 | 0.007123 | 5.5954 |

2

VITA

Sergio Ahrens

Candidate for the Degree of

Master of Science

Thesis:   A MATHEMATICAL EXPRESSION OF STEM FORM AND
         VOLUME OF LOBLOLLY PINE IN SOUTHERN BRAZIL

Major Field:   Forest Resources

Biographical:

   Personal Data:   Born in Curitiba, State of Parana, Brazil, February
        26, 1951, the son of Mr. and Mrs. A. E. Ahrens.

   Education:   High School graduation from Colegio Militar de Curitiba
        (Curitiba Military Academy) in December, 1969; received degree
        of Engenheiro Florestal from Universidade Federal do Parana,
        Curitiba, PR, Brazil, in December, 1973; completed requirements
        for the Master of Science degree at Oklahoma State University
        in December, 1980.

   Professional Experience:   Block-forester with Jari Florestal e
        Agropecuaria Ltda, Para, Brazil, 1973-1974; researcher with
        Moosmayer Associados - Consultores de Recursos Florestais
        Ltda, Parana, Brazil, 1975; researcher with Project for
        Forestry Development and Research, UNDP-FAO/BRA-45, Parana,
        Brazil, 1976-1977; researcher with Empresa Brasileira de
        Pesquisa Agropecuaria-EMBRAPA, Parana, Brazil, 1978-present.