SYNTHESIS AND ANALYSIS OF SPATIAL MECHANISMS FOR TWO-PARAMETER TANGENT-PLANE ENVELOPE GENERATION

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CHAPTER I

INTRODUCTION

In the field of kinematics, there exists two problems in the design of mechanisms : synthesis and analysis. The fundamental problem in the kinematic synthesis is to determine the dimensions of linkages required to pass through several specified, finitely separated positions, infinitesimally separated positions, or mixed mode positions relative to another rigid body. Such specifications may include rigid body guidance, function generation, and path generation. The fundamental problem in kinematic analysis is to determine the relative motions of moving links where the linkage parameters are given.

The fundamental theories of kinematic synthesis and analysis of planar or three dimensional mechanisms appear to have evolved with the classical investigations of a rigid body motion. Such motion may be examined for one or more degrees of freedom or for one or more parameter motion. When large motion is of interest, finite motion theories for a rigid body are generally investigated. When, however, more precise motion in a local region is of interest, instantaneous motion theories for a rigid body are investigated.

Significant achievements have been made in recent years in understanding and applying the kinematics of one-parameter rigid body motion. For such one-parameter investigations, one may study a curve or a surface generated by a point, a line, or a plane moving with the coupler-link of a planar or a spatial mechanism. In planar motion, a line connected to the coupler-link of a mechanism will generate an envelope. This line is called the tangent-line. In space motion, a plane connected to the coupler-link of a mechanism will envelop a surface and the plane is called the tangent-plane.

Continuation of the study on the one-parameter motion of a plane in space is requested to investigate the kinematics of a two-parameter motion of a rigid body with a plane being considered as a moving element. In the following three sections, we will examine the significant contributions describing in a progressive manner the development of the key concepts lending to synthesis and analysis of spatial mechanisms, and the curvature theory of point, line, and plane trajectories in three-dimensional kinematics.

1.1 Synthesis of Spatial Mechanisms

For synthesis of finitely separated positions of a rigid body, moving relative to another rigid body, Wilson[1] developed an analytical procedure which used the analogy of planar kinematic synthesis problems. He introduced the rigid body guidance problem in spatial synthesis and also showed that function generation problem can be converted to a rigid body guidance problem by taking inversion about the input or output link. However, his procedure can be used only for Sphere-Sphere, Revolute-Sphere, Sphere-Revolute, and Revolute-Revolute cranks.

Roth[2] used screw theory and linear transformation to describe a rigid body through a series of finitely separated positions in order to determine those points which lie on a sphere, circle, plane, line or cylinder. Also, Roth applied these results for the synthesis of mechanisms. The parallel (plane) and intersecting (sphere) screws were presented as special cases. However, then applications are only for very simple mechanisms.

Roth[3] described the motion of a rigid body moving relative to another rigid body for up to five positions. He also extended the concepts of pole triangle and pole quadrangle into space. He obtained an infinite number of C-C cranks which displaces a rigid body through four finitely separated positions relative to another rigid body and obtained a finite number of C-C cranks for five finitely separated positions. These are found by intersecting the two cubic cones corresponding to two groups of four positions. These lines are the space analogs of the planar Burmester points.

Sandor[4] developped procedures by applying the quaterrions for kinematic synthesis of space mechanisms. He presented the space mechanism as general kinematic chains consisting of one or more loops of ball-jointed bar-slideball members. Sandor used the spatial circle-point theory to study four positions of a point of a rigid body which lies on a circle and verified Roth's[2] results that there can only be a maximum of four points on a circle in space.

Sandor and Bishop[5] applied the stretch-rotation tensor which is in a matrix form to present a general method of spatial kinematic synthesis. The method can be used to multi-loop linkages and to special cases.

Bottema, Koetsier and Roth[6] presented the procedure to find the smallest circle determined by three positions of a rigid body in space. It is shown that the minimum radius circle may arise when either the minimum circle is associated with a point which lies on a screw axis or it is associated with a more general point. The results can be applied for the design of the smallest Sphere-Revolute crank which will displace a rigid body through three finitely separated positions.

Chen and Roth[7,8], by using Roth's[2,9] results, presented a unified theory for the kinematic synthesis of finitely and infinitesimally separated positions of a rigid body moving relative to another rigid body.

Soni and Harrisberger [10] presented a criterion, based on the optimum transmission characteristics, for designing space mechanism.

Soni and Huang [11] extended the point position reduction to design spatial four-bar mechanisms by using the analogy of planar kinematic synthesis.

Rao, Sandor, Kohli, and Soni [12] developed a general closed-form synthesis procedure to synthesize function generators for the maximum number of precesion positions.

Tsai and Roth[13] presented a procedure by using screw triangle geometry to synthesize open-loop kinematic chain for completely and incompletely specified positions of a rigid body. They found the screws associated with these displacements and gave the constraining conditions for the design of cranks.

Roth[14] derived the constraining equations for any combination of revolute, prismatic, and cylindrical joints by using the screw triangle geometry method.

Tsai and Roth [15] presented the synthesis procedure, based on the equivalent screw triangle method, and rederived the constraining equation from which the design equation can be determined for any combinations of helical, cylindrical, revolute, spherical, and prismatic joints for both finitely and infinitesimally separated position problems in kinematic synthesis. The maximum number of finitely separated positions for each combination is shown in TABLE I.

TABLE I

MAXIMUM NUMBER OF FINITELY SEPARATED POSITIONS(FSP) FOR ANY COMBINATION OF BINARY LINKS DERIVED BY TSAI

Link-Combination	Max.FSP	Link-Combination	Max.FSP
R-R	3	P-R	2
R-P	2	P-P	2
R-C	3	P-C	2
R-H	3	P-H	2
R-S	4	P-S	3
C-R	3	H-R	3
C-P	2	H-P	2
C-C	5	H-C	4
C-H	4	H-H	3
C-S	8	H-S	5
S-R	4	S-H	5
S-P	3	S-S	7
S-C	8		

Suh[16,17] used 4 x 4 matrices for synthesis of space mechanisms where design equations are expressed as constraint equations in order to obtain the contrained motion.

Suh[18] discussed the R-R link and concluded that "the maximum number of positions for R-R link synthesis is three with no choice of papameter".

Suh[19], by using the finite-screw geometry, presented an analytical and geometrical proof establishing the duality of R-R crank for three positions. The proof is a geometrical one rather than an algebraic one in order to avoid the complexity in dealing with nonlinear algebraic equations, and to give it a simple and intuitive form.

Tsai and Roth [20], by using the constraining equations derived in [14], obtained a sixth degree polynomial for synthesis of R-R crank. The coefficients of the constraining equation are in explicit form and whose real roots give the direction cosines of lines in the moving frame of reference.

Sathyadev and Soni [37] introduced a new approach to synthesize the planar mechanism for coupler tangent-line generation based on the modification of the planar rigid body displacement matrix developed by Suh [38]. This leads to the concept of a curve being considered as a line-locus, the envelope of a set of its tangent-lines.

1.2 Analysis of Spatial Mechanisms

Kinematic analysis of space mechanisms was initiated by the significant contribution of Dimentberg[21,22] who presented the dual number and screw calculus to obtain closed-form displacement relationships of an RCCC and other spatial mechanisms. There are some other approaches have been applied to obtain the same closed-form displacement relationships of RCCC mechanism:

Denavit [23] used dual Euler angles.

Yang [24] used dual quaternions.

Chace [25] was the first used vector approach

Wallace and Freudenstein [26] also used vector approach to obtain closed-form displacement relationships of RRSRR and RRERR mechanisms.

Yang [27] presented a general formulation using dual number for displacement analysis of RCRCR spatial mechanism.

Soni and Pamidi [28] used the 3 x 3 dual matrix to obtain closed-form desplacement relations of RCCRR mechanisms.

Yuan [29] applied screw coordinates to obtain closed-form displacement relations for RRCCR and other spatial mechanisms.

Jenkins and Crossley [30], Sharma and Torfason [31], Dukkipati and Soni [32] used the method of generated surfaces applying the analysis of single-loop mechanisms containing a spheric pair.

Hartenberg and Denavit [33] using 4 x 4 matrix for displacement analysis of spatial mechanism.

Soni and Harrisberger [34] presented an iterative approach for performing kinematic analysis using 3 x 3 matrices with dual elements.

Kohli and Soni [35,36] used finite screws for displacement analysis and synthesis of single-loop and two-loop space mechanisms with revolute, prismatic, cylindrical, helical, and spherical joints.

1.3 Curvature Theory

For space point-path, Veldkamp [39,40] developed the fundamentals of the instantaneous invariants and applied them to study the point-path in space.

Siddhanty and Soni [41], Hsia and Yang [42] investigated the curvature theory of point trajectories in three-dimensional kinematics.

Yang, Roth, and Kirson [43,44] described the geometric properties of a ruled surface which generated by a line in a moving body as it moves in space may be examined either by applying the principle of transference to the results of the point-path trajectories on sphere.

McCarthy and Roth [45] studied the motion of a line in space.

Ting and Soni [46,47], and Veldkamp [48] investigated the one-parameter, instantaneous motion of rigid body where the moving element is a plane.

Schonemann [49] and Mannheim [50] contributed the first theorem of instantaneous two-parameter kinematics.

Blaschke [51] investigated the first-order property of two-parameter motion with line as moving element by using dual numbers and quarternions.

Bottema [52] studied the first- and second-order properties of two-parameter spatial motion with points as moving elements. He developed the analytical expressions for the Gaussion curvature of the point trajectory surface.

1.4 Present Study

The survey of literature mentioned in the previous three sections show that most of studies of synthesis and analysis of spatial mechanisms are devoted to the one-parameter rigid body guidence and two-parameter motion of a rigid body with points as a moving elements. Problems such as generating a surface in space by using a plane as moving element with two-paramter motion still remained unknown.

Just as point in two-parameter motion generates a surface, so does a plane in two-parameter motion. In general, in space geometry, a point and a plane are dual constructs and a line is dual to itself. For any geometric figure consisting of points, lines, and planes, its dual configuration is obtained by replacing every point by a plane, every line by a line, and every plane by a point. In a two-parameter spatial kinematics the dual of a point-trajectory surface is the trajectory of a plane which envelops a surface. Since each plane corresponds to a point on a surface, the study of a plane motion is analogous to the study of a point-trajectory surface.

This manner of investigation provides an insight into the dual relationships between the trajectory of a plane and the point-trajectory surface. The kinematic significance of this duality and its potential applications that generally follow in mechanism synthesis and analysis are of fundamental importance in the mechanism science.

The two-parameter motion of a rigid body may be investigated further by examining the moving element which may be a point, a line, or a plane. Because of the duality between a point and a plane, a study of plane-path with two-parameter variation is expected to provide better insight into the two-parameter rigid body motion.

The objective of the present study is to provide a general method of synthesizing and analyzing the spatial mechanisms for two-parameter tangent-plane envelope generation. The proposed method can be used for finitely separated positions, infinitesimally separated positions, and mixed mode positions. The synthesis procedure is based on the Homogenerous Transformation Matrix which developed by Hartenberg and Denavit.

In chapter II, the parametric discription of a surface enveloped by tangent-plane is described. This is a brief discussion how the tangent-plane envelops a given two-parameter surface from the geometric point of view.

In chapter III, the Homogenerous Transformation Matrix method, based on the Hartenberg-Denavit notation, is derived.

In chapter IV, the synthesis and analysis procedure of dyads for finitely separated positions generated by tangent-plane having two-parameter motion with any combination of Revolute, Prismatic, and Helical joints are derived.

In chapter V, the synthesis and analysis procedure of dyads for infinitesimally and mixed mode separated positions generated by tangent-plane having two-parameter motion with any combiantion of Revolute, Prismatic, and Helical joints are presented. Also, the first- and higher-order motion are discussed in this chapter.

In chapter VI, the synthesis procedure of two-parameter motion, two degree-of-freedom spatial mechanisms carrying a rigid body with a tangent-plane as a moving element having six, five, and four components of motion are derived.

Chapter VII presents the summary, conclusions, and recommendations for further research.

Numerical examples are presented in chapter IV, V, and VI to illustrate the proposed synthesis and analysis procedure. Also, the computer programs of kinematic synthesis and analysis are presented in appendix A and B.

CHAPTER II

PARAMETRIC DISCRIPTION OF A SURFACE DEVELOPING BY THE TANGENT PLANE

The design specification in this category of synthesis and analysis problems require a tangent-plane attached to the a moving rigid body enveloping a given surface.

Generally, The surface to be enveloped is expressed in the vector form along with the precision points which approximate the given surface. Therefore, in this chapter, we will simply discribe the paramatric dispription of a two-parameter surface enveloping by the tangent plane.

We note that the vector equation of the type

$$\mathbf{R}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k} \qquad (2\text{-}1)$$

is in the single parameter t describe space curves. The parametric representation of the space curves is

$$\mathbf{X} = x(t), \quad \mathbf{Y} = y(t), \quad \mathbf{Z} = z(t) \qquad (2\text{-}2)$$

Surfaces, in general, are described by the parametric equations of the type

$$\mathbf{X} = x(u,v), \quad \mathbf{Y} = y(u,v), \quad \mathbf{Z} = z(u,v) \qquad (2\text{-}3)$$

where u and v are unique parameters.

If v is fixed (i.e., v=C, a constant), then Eq(2-3) becomes a one-parameter expression, which describes a space curve along which u varies. This is the curve designated by v=C. Thus for each v, there exists a space curve. Similarly, a space curve can be obtained when v varies along the curve u=C. The locus of all the curves v=C and u=C forms a surface S. The parameters u and v are called the curvilenear coordinates of the point P on the surface, and the u-curves and v-curves are called parametric curves as shown in figure 1.

If the terminal point of the position vector **R** generates the surface **S**, then Eq(2-3) can be rewritten as

$$\mathbf{S}(u,v) = x(u,v)\,\mathbf{i} + y(u,v)\,\mathbf{j} + z(u,v)\,\mathbf{k} \qquad (2\text{-}4)$$

let $\mathbf{S}_u = \delta\mathbf{S} / \delta u$ and $\mathbf{S}_v = \delta\mathbf{S} / \delta v$ represent the tangent vectors to the curve u and v respectively.

Hence,

$$\mathbf{S}_u = \frac{\delta\mathbf{S}}{\delta u} = \frac{\delta x}{\delta u}\,\mathbf{i} + \frac{\delta y}{\delta u}\,\mathbf{j} + \frac{\delta z}{\delta u}\,\mathbf{k} \qquad (2\text{-}5)$$

$$\mathbf{S}_v = \frac{\delta\mathbf{S}}{\delta v} = \frac{\delta x}{\delta v}\,\mathbf{i} + \frac{\delta y}{\delta v}\,\mathbf{j} + \frac{\delta z}{\delta v}\,\mathbf{k} \qquad (2\text{-}6)$$

A point P(u,v) on a surface **S** is called a singular point if $\mathbf{S}_u \times \mathbf{S}_v = 0$; otherwise, it is called a non-singular point. Therefore,if $\mathbf{S}_u$ and $\mathbf{S}_v$ are continuous, the plane

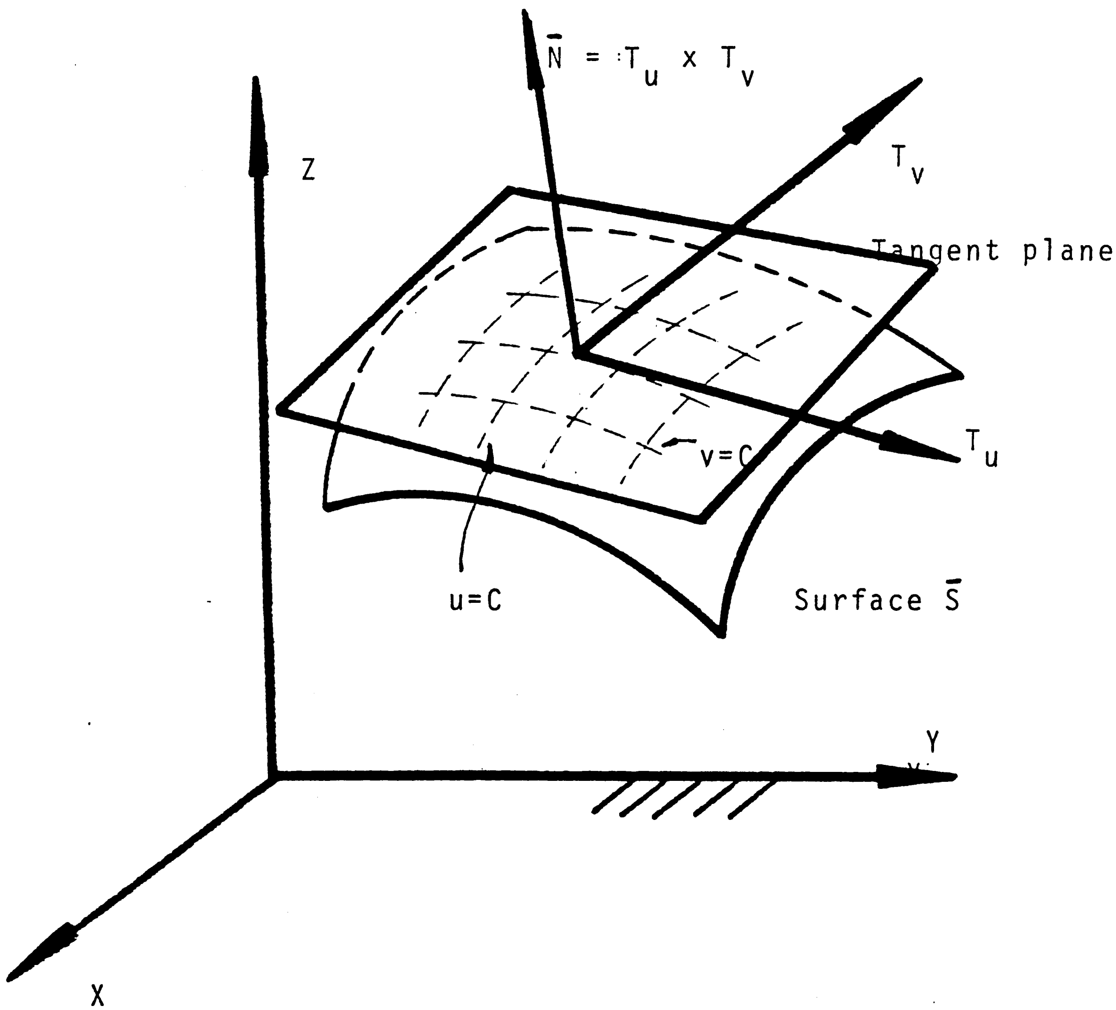


Figure 1. Parametric Discription of a Surface (Two-Parameter Surface)

through P parallel to $\mathbf{S}_u$ and $\mathbf{S}_v$ at point **P** is call the tangent plane to surface **S** at point P. Thus, the tangent planes exist only at the nonsingular points and can be defined by those two tangent vectors $\mathbf{S}_u$ and $\mathbf{S}_v$. Also, every nonzero vector linearly dependent upon $\mathbf{S}_u$ and $\mathbf{S}_v$ is the tangent vector of some curve through point **P**.

In order to derive the tangent plane equation, we need to define the unit normal vector of the tangent plane as:

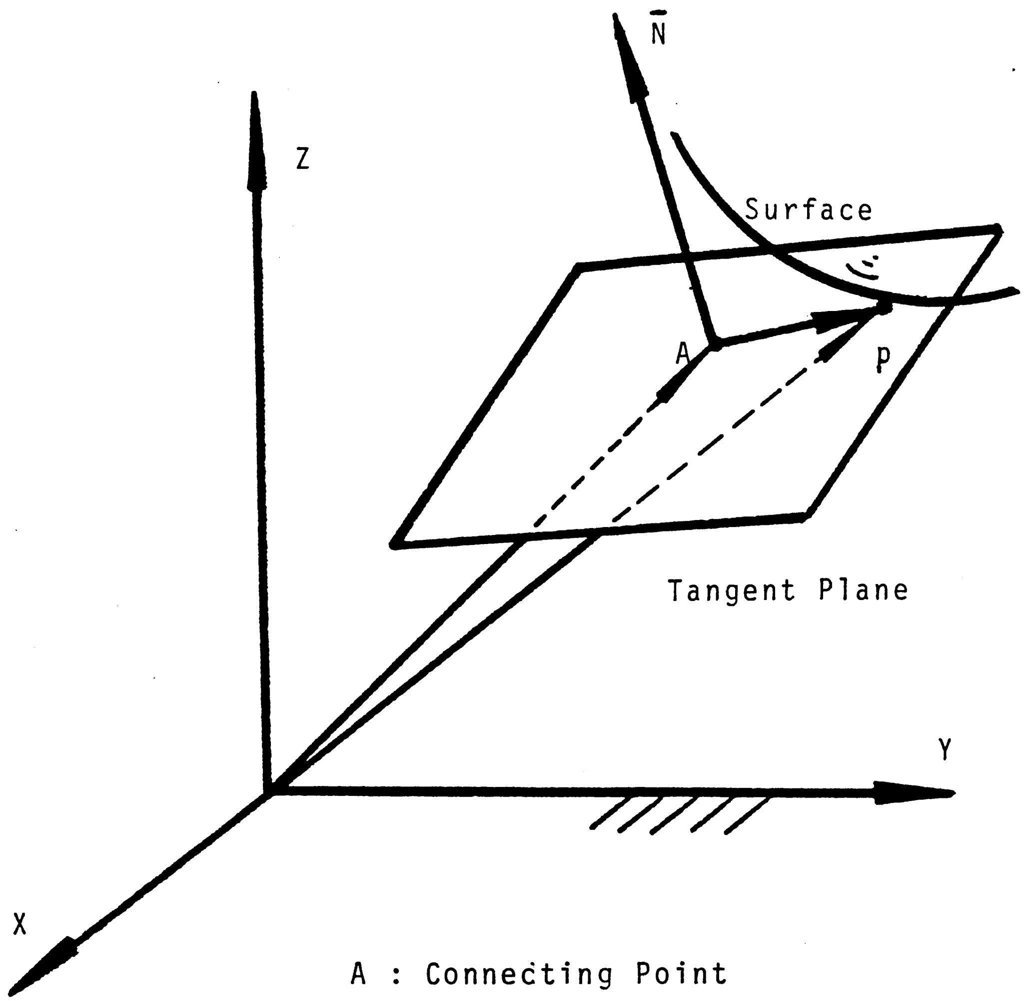
$$\mathbf{N} = \frac{\mathbf{S}_u \times \mathbf{S}_v}{\| \mathbf{S}_u \times \mathbf{S}_v \|} \tag{2-7}$$

In figure 2, it is shown that point **P** (x_0, y_0, z_0) is a point on the tangent-plane tangents to the surface. Therefore, the tangent-plane equation can be obtained by taking the dot product of the vector from an arbitrary point **A** (note that point **A** is also called a connecting point of tangent plane and mechanism) to **P** and unit normal vector of the tangent-plane.

Hence, we obtain

$$(\mathbf{X}-\mathbf{x}_0,\ \mathbf{Y}-\mathbf{y}_0,\ \mathbf{Z}-\mathbf{z}_0)\cdot\mathbf{N} = 0 \tag{2-8}$$

Since **N** can be expressed as (N_x, N_y, N_z), by taking dot product and rearranging Eq(2-8), yields



A : Connecting Point

P : Tangential Point

N̄ : Unit Normal Vector

Figure 2. Vector Expression of the Tangent Plane

$$N_x\mathbf{X} + N_y\mathbf{Y} + N_z\mathbf{Z} = C \qquad (2\text{-}9)$$

where $C = N_x x_0 + N_y y_0 + N_z z_0$

The tangent-plane equation Eq(2-8) or Eq(2-9) can be used to derive the synthesis equation for spatial mechanisms with the tangent-plane as a moving element carried by the rigid body of a mechanisms.

CHAPTER III

HOMOGENEOUS TRANSFORMATION MATRIX METHOD

For synthesis of planar mechanisms, Suh [53] derived the planar displacement matrix which expressed the orientation and position of the moving link in (3 x 3) matrix. For synthesis of spatial mechanisms, Wilson [1] was the first developed (3 x 3) matrix to define the motion of a body in space. Roth [2] also derived the (3 x 3) screw matrix by using the linear transformation and screw algebra. Denavit and Hartenberg[38] developed a new symbolic notation and derived (4 x 4) matrix for spatial mechanism based on the homogeneous transformation. This notation is called Denanit-Hartenberg notation (D-H notation) and the matrix is called D-H matrix. Because of the sufficient for the description of the complete kinematic properties of lower-pair mechanisms, D-H notation can be used for kinematic synthesis and analysis problems of spatial mechanisms to obtain the result which is more compact.

There are four parameters defined in D-H notation as shown in figure 3 and stated in the following:

a_i = link length, the common normal along X_{i+1} between Z_i and Z_{i+1}.

α_i = link twist angle, relative orientation of the

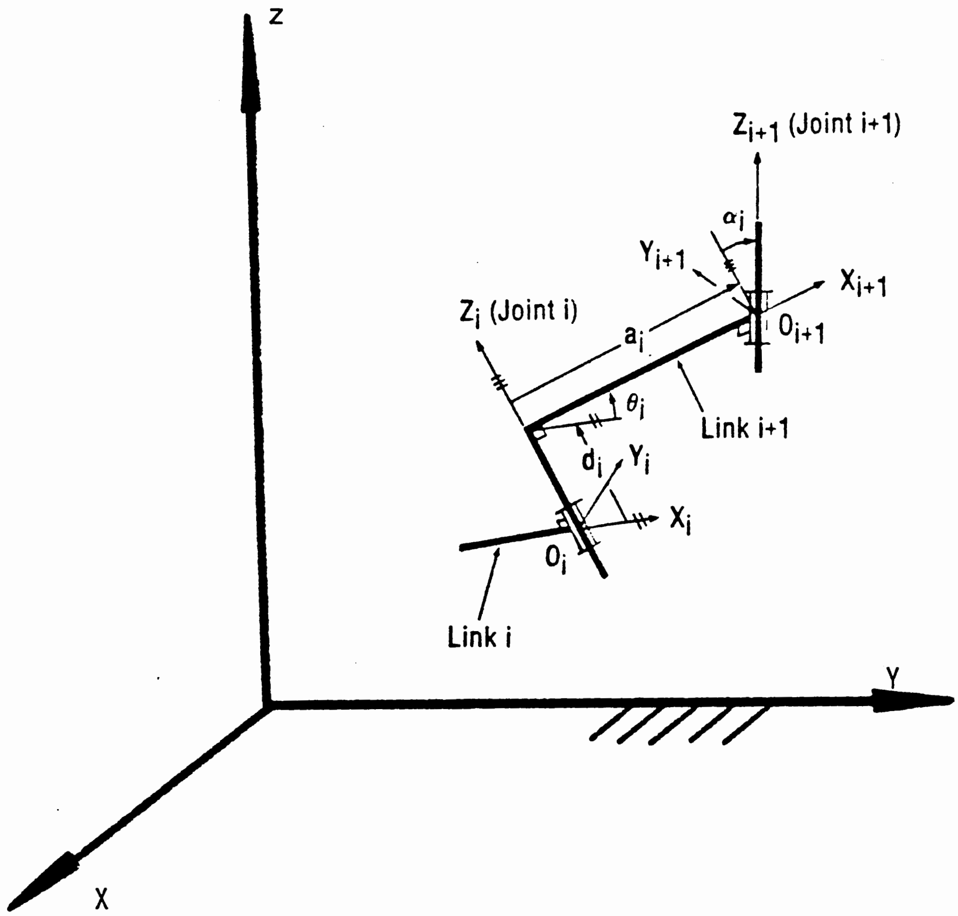


Figure 3. Notation of Homogeneous Transformation Matrix

kinematic pair, obtained by rotating Z_i to Z_{i+1} about X_{i+1}. The sign of rotation is given by the right-hand screw rule.

d_i = offset distance, the common normal along Z_i between X_i and X_{i+1}. The sign of distance can be positive or negative. d_i is positive when measured to the positive Z_i direction.

Θ_i = link angle, obtained by rotating X_i to X_{i+1} about Z_i. The sign of rotation is given by the right-hand screw rule.

Also, the coordinates are defined as : The Z_i axis is along the axis of motion or rotation of the (i+1) joint. The X_i axis in the direction of normal to both Z_i and Z_{i+1} axis,point away from the Z_i axis. The Y_i axis is chosen so as to make the coordinate X_i, Y_i , and Z_i following the Right-Hand screw.

Once the D-H coordinate system for each link is established, a homogeneous transformation matrix can be developed relating the $i+1^{th}$ coordinate frame to the i^{th} coordinate frame as shown in fig(3). It is clear that a point P expressed in the $i+1^{th}$ coordinate system may be expressed in the i^{th} coordinate system by performing the following successive transformations:

1. Rotate about the Z_i axis by an angle Θ_i to align the X_i and X_{i+1}, $Rot(Z_i, \Theta_i)$.
2. Translate along the Z_i axis a distance d_i to bring the X_i and X_{i+1} axes into coincidence,$Tran(Z_i, d_i)$.

3. Translate along the Z_i axis a distance a_i to bring the two origins into coincidence, $Tran(X_i, a_i)$.
4. Rotate about the X_{i+1} axis an angle α_i to bring the two coordinate systems to completely coincide, $Rot(X_i, \alpha_i)$.

Let the coordinates of a point P expressed in the i^{th} coordinate system be (p_{xi}, p_{yi}, p_{zi}) and in the $i+1^{th}$ coordinate system be $(p_{xi+1}, p_{yi+1}, p_{zi+1})$. Then the vectors P_i and P_{i+1} can be written in the (4 x 1) matrix forms as follows:

$$P_i = \begin{bmatrix} p_{xi} \\ p_{yi} \\ p_{zi} \\ 1 \end{bmatrix} \qquad P_{i+1} = \begin{bmatrix} p_{xi+1} \\ p_{yi+1} \\ p_{zi+1} \\ 1 \end{bmatrix} \qquad (3\text{-}1)$$

The complete transformation of link i+1 with respect to link i or joint i+1 with respect to joint i can be expressed as :

$$[A_i] = Rot(Z_i, \theta_i)\ Tran(Z_i, d_i)\ Tran(X_i, a_i)\ Rot(X_i, \alpha_i) \qquad (3\text{-}2)$$

Thus, we can obtain the homogenerous tramsformation matrix $[A_i]$ from $i+1^{th}$ frame to the i^{th} frame

$$[A_i] = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3\text{-}3)$$

where

$C\theta_i = \cos\theta_i$, $S\theta_i = \sin\theta_i$,

$C\alpha_i = \cos\alpha_i$, $S\alpha_i = \sin\alpha_i$.

Also, the transformation of coordinate frome the $i+1^{th}$ system to the i^{th} system will be

$$P_i = [A_i]\, P_{i+1} \qquad (3\text{-}4)$$

and the inverse transformation exists :

$$P_{i+1} = [A_i]^{-1}\, P_i \qquad (3\text{-}5)$$

where

$$[A_i]^{-1} = \begin{bmatrix} C\theta_i & S\theta_i & 0 & -a_i \\ -S\theta_i C\alpha_i & C\theta_i C\alpha_i & S\alpha_i & -d_i S\alpha_i \\ 0 & S\alpha_i & C\alpha_i & -d_i C\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3\text{-}6)$$

By applying the matrix transforamtion to each joint of coordinate frame from the last joint coordinate frame to the first joint coordinate frame, we yield

$$P_1 = [A_1][A_2]\ldots[A_n]P_{n+1} = [A_{eqv}]P_{n+1} \qquad (3\text{-}7)$$

The equivalent transformation matrix $[A_{equ}]$ defines the relationship between the coordinates of any point in the last frame P_{n+1} and that of the same point expressed in the first frame, P_1

Denavit and Hartenberg [38] developed a kinematic notation for lower-pair mechanisms including revolute, prismatic, cylindrical, helical, and spherical joints based on (4 x 4) matrices. For a revolute joint, d_i, a_i, and α_i are all constant, while θ_i varies as link i rotates about the axis of joint i. For a prismatic joint θ_i, a_i, and α_i are constant while d_i varies as link i translates along the axis of joint i. For a cylindrical joint, it can be considered as equivalent to a coaxial revolute and presmatic joints. therefore, the joint variables are : θ_i varies as link i rotates about the axis of joint i and d_i varies as link i translates along the axis of joint i. For a helical joint, both parameters θ_i and d_i vary, being related by the lead L_i as

$$\frac{\delta\theta}{2\pi} = \frac{\delta d_i}{L_i} \qquad \text{where } \theta \text{ : radian} \tag{3-8}$$

When L_i is constant, either θ_i or d_i varies. Once δd_i is obtained, d_i can be solved by

$$d_i = d_0 + \delta d_i \tag{3-9}$$

where d_0 : original link distance

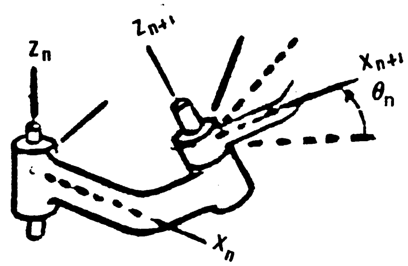
The spherical joint is equivalent to a combination of three revolute joints whose axes are mutually perperdicular at a common point of intersection (i.e., the joint variable θ become θ_1, θ_2, and θ_3). The joint motion parameters are summarized as shown in TABLE II and in figure 4.

TABLE II

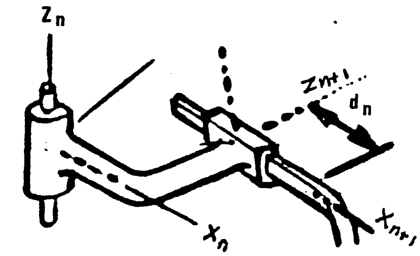
JOINT MOTION PARAMETERS FOR R,P, H,C,AND S JOINTS

Type of Joint	Motion Parameter
1. Revolute joint	θ_i
2. Prismatic joint	d_i
3. Helical joint	θ_i or d_i
4. Cylindrical joint	θ_i and d_i
5. Spherical joint	3-revolute joint(θ_1,θ_2, and θ_3, and $d_i = a_i = 0$)

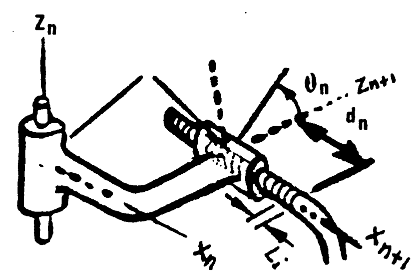
In order to synthesize and analyze spatial mechanisms by using homogeneous transformation matrix method, we can separate the transformation matrix $[A_n]$ into two submatrices: one is called joint-motion matrix $[A_v]$ in terms of joint motion and the other is called linkage-parameter matrix $[A_c]$ in terms of linkage parameters. When synthesis procedure is used, the joint-motion matrix $[A_v]$ becomes a



(A) Revolute joint



(B) Prismatic Joint



(C) Helical Joint

Figure 4. Joint Motion Parameter of Revolute, Prismatic, and Helical Joints.

constant and the linkage-parameter matrix $[A_c]$ becomes an unknown matrix and vice versa when analysis procedure is used. Therefore, we obtain

$$[A_i] = [A_{vi}][A_{ci}] \tag{3-10}$$

where

$$[A_{vi}] = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{(for revolute and spherical joint)} \tag{3-11}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{(for cylindrical, and helical joint)} \tag{3-12}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{(for prismatic joint} \tag{3-13}$$

$$[A_{ci}] = \begin{bmatrix} a_{i11} & a_{i12} & a_{i13} & a_{i14} \\ a_{i21} & a_{i22} & a_{i23} & a_{i24} \\ a_{i31} & a_{i32} & a_{i33} & a_{i34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3-14}$$

$$= \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(for revolute and spherical joint)} \qquad (3\text{-}15)$$

$$= \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(for cylindrical and helical joint)} \qquad (3\text{-}16)$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(for prismatic joint)} \qquad (3\text{-}17)$$

and expressing $[A_{si}]^{-1}$ as :

$$[A_{ci}]^{-1} = \begin{bmatrix} b_{i11} & b_{i12} & b_{i13} & b_{i14} \\ b_{i21} & b_{i22} & b_{i23} & b_{i24} \\ b_{i31} & b_{i32} & b_{i33} & b_{i34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3\text{-}18)$$

where $[a_{ni,j}]$ and $[b_{ni,j}]$ are in terms of linkage parameters. Once the coordinate systems are established, the synthesis and analysis procedure of spatial mechanisms can be obtained by using the transformation matrix with the help of the tangent-plane equation.

CHAPTER IV

SYNTHESIS AND ANALYSIS OF DYADS FOR FINITELY SEPATATED POSITIONS GENERATED BY TANGENT-PLANE

In this chapter, a new synthesis and analysis procedure is developed, based on the tangent plane equation presented in chapter II and the homogeneous transformation matrix method presented in chapter III, for finitely separated position with any combination of revolute, prismatic, and helical joints to envelop a given surface by a tangent plane carried by the moving rigid body.

The advantages of the proposed procedure can be briefly stated as:

1. Taking a plane as the tracing element for path-generation to envelop a surface.
2. Solving for any combination of binary links with R, P, and H joints.
3. Obtaining the closed-form solution.
4. Increasing the number of precesion positions.

The term "dyad" used in this thesis refers to a two-link chain (a fixed link and a moving binary coupling link) which is used to guide a third member through several design positions. With two-parameter motion, we can investigate a mechanism having two degrees of freedom (i.e. a dyad). There are nine combinations of dyads composed of mixed revolute, prismatic, and helical joints as shown in TABLE III.

In the next section, we will examine the synthesis and analysis procedure for each combination of binary links.

TABLE III

TWO DEGREES OF FREEDOM BINARY LINKS
WITH AND COMBINATION OF R, P,
AND H JOINTS

(1) R-R	(4) P-R	(7) H-R
(2) R-P	(5) P-P	(8) H-P
(3) R-H	(6) P-H	(9) H-H

Figure 5 shows a binary link kinematically connects a moving rigid body which carries the tangent-plane to a fixed coordinate frame. The joint connecting to the tangent plane is called the moving joint and the joint connecting to the fixed frame is called the fixed joint. P is the point where the plane tangent to the surface. In figure 5, we establish four coordinates frame as :

{X,Y,Z} : fixed coordinate frame.

{X1,Y1,Z1} : coordinate frame on the fixed joint **J1**.

{X2,Y2,Z2} : coordinate frame on the moving joint **J2**.

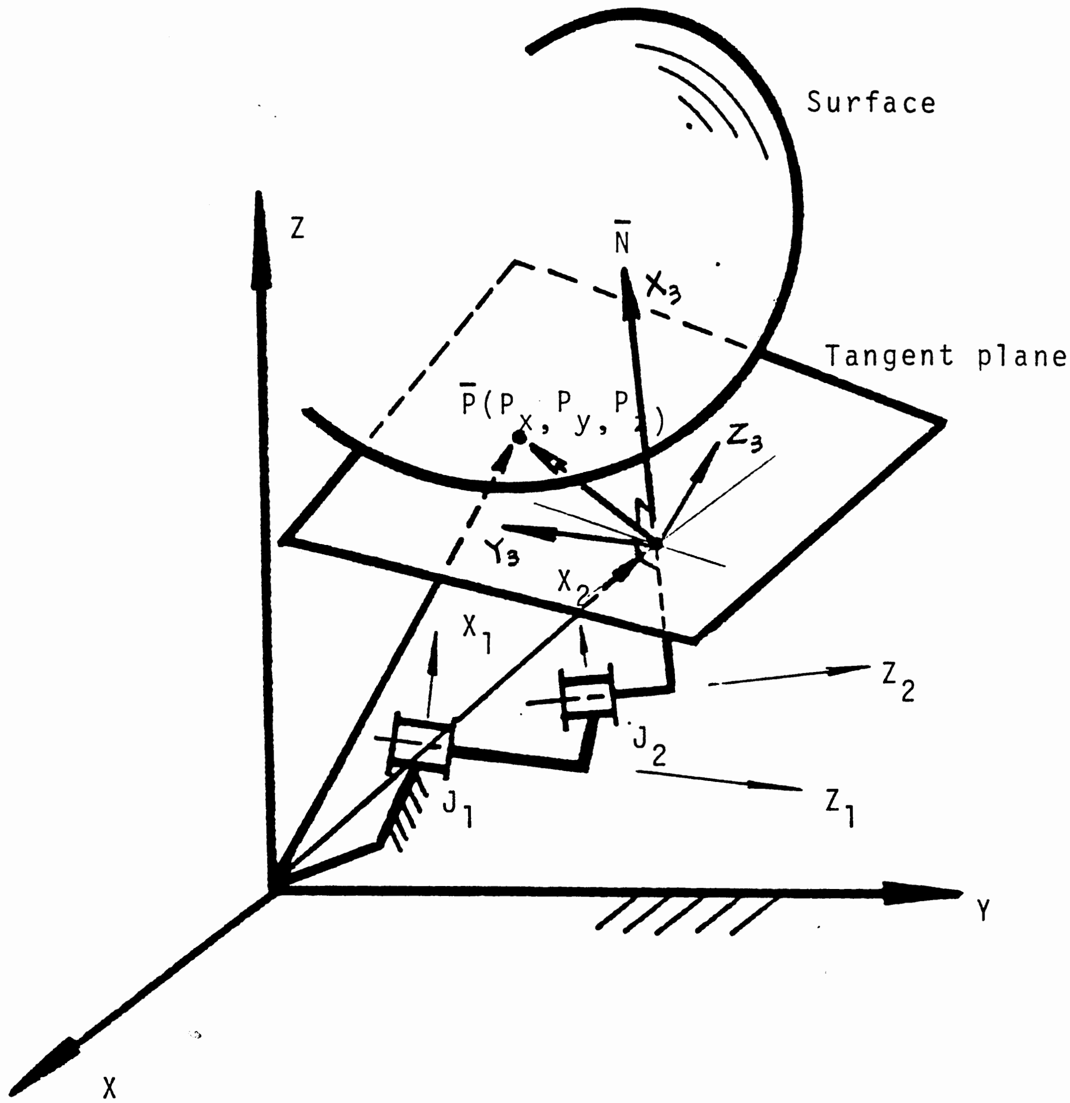


Figure 5. Binary Link with a Plane Attached To the Moving Joint Tangential to to a Surface

{**X3,Y3,Z3**} : coordinate frame on the given tangent plane at connecting point P.

Also, the parameters involve in fig(5) are:

$\{\alpha_1, \alpha_2, \alpha_3\}$: the twist angles between the pair axes.

$\{a_1, a_2, a_3\}$: the link length.

$\{s_1, s_2, s_3\}$: the offset distances.

$\{\theta_1, \theta_2, \theta_3\}$: the rotation angles at each joint.

P_i : the vector measured from origin of i^{th} coordinate frame to the point P. where i = 1..4.

Since the origin of (**X3,Y3,Z3**) coordinate frame is on the connecting point of tangent plane and moving joint. Thus, we obtain the matrix transformation measured from the origin of (**X3,Y3,Z3**) coordinate frame to the fixed coordinate frame as :

$$\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \\ 1 \end{bmatrix} = [A_1][A_2][A_3] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4-1}$$

$$= [A_{v1}][A_{c1}][A_{v2}][A_{c2}][A_{v3}][A_{c3}] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4-2}$$

Figure 6 shows that the finite displacement of a tangent plane attached to a moving rigid body displaces from i^{th} to its j^{th} position. We note that the number of possible synthesis positions depends on the number of unknown parameters and the constraint equations. Once the constraint equations are derived, we can determine the maximum number of allowable synthesis positions.

4.1. Synthesis of Dyads For Any Combination of Revolute, Prismatic, and Helical Joints

(1) Synthesis of R-R crank:

It has been shown by Suh[19] and Tsai[20] that, for rigid body guidance problems, the maximum number of design positions for R-R cranks is three with no free choice of design parameters. However, by using the tangent-plane envelope generation presented in this thesis, the maximum number of synthesis positions can be obtained is nine.

By substituting Eq(4-2) into Eq(2-9), we obtain the synthesis equation of R-R crank with tangent-plane attached on the moving joint.

$$\begin{aligned}
&(n_x C\theta_1 + n_y S\theta_1)\,(C\theta_2 a_3 C\theta_3 - S\theta_2 C\alpha_2 a_3 S\theta_3 + S\theta_2 S\alpha_2 d_3 + a_2 C\theta_2) \\
&+ (n_y C\theta_1 C\alpha_1 + n_z S\alpha_1 - n_x S\theta_1 C\alpha_1)(S\theta_2 a_3 C\theta_3 + C\theta_2 C\alpha_2 a_3 S\theta_3 - \\
&C\theta_2 S\alpha_2 d_3 + a_2 S\theta_2) + (n_x S\theta_1 S\alpha_1 - n_y C\theta_1 S\alpha_1 + n_z C\alpha_1)(S\alpha_2 a_3 S\theta_3 \\
&+ C\alpha_2 d_3 + d_2) + n_x a_1 C\theta_1 + n_y a_1 S\theta_1 + n_z d_1 \\
&= n_x\, p_x + n_y\, p_y + n_z\, p_z \qquad (4\text{-}3)
\end{aligned}$$

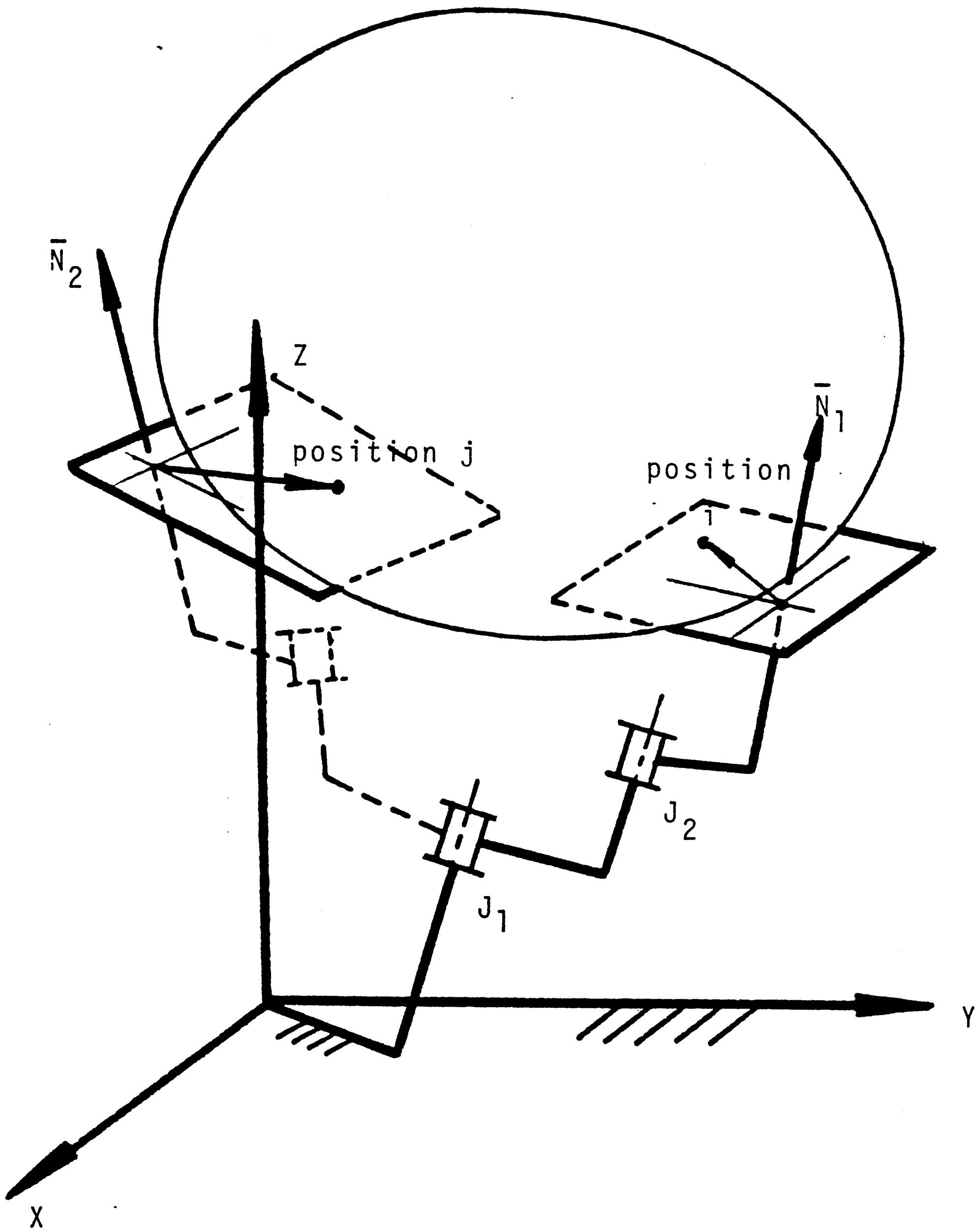


Figure 6. Finite Displacement of Binary Link

We note that the fundamental problem in the kinematic synthesis is to determine the dimensions of linkages required to pass through a series of points in space. Therefore, the unknown variables in Eq(4-3) will be :

$$\left\{ \begin{matrix} a_1, & a_2, & a_3 \\ d_1, & d_2, & d_3 \\ \alpha_1, & \alpha_2, & \alpha_3 \end{matrix} \right\}$$

Since θ_1 is a fixed angle in this configuration, we can assume θ_1 as an unknown variable rather than as a known joint motion variable. Also, α_3 is free from Eq(4-3). Hence, the unknown linkage parameters are :

$$\left\{ \begin{matrix} a_1, & a_2, & a_3 \\ d_1, & d_2, & d_3 \\ \alpha_1, & \alpha_2, & \\ \theta_1 & & \end{matrix} \right\}$$

Let N be the maximum number of finitely separated positions. There are nine unknowns in Eq(4-3). Thus, we obtain

$$N = 9 \tag{4-4}$$

Therefore, the maximum number of finitely separated positions for R-R crank can be obtained by given nine joint motions. Then Eq(4-3) can be rewritten as:

$$(n_{x_n}C\theta_1 + n_{y_n}S\theta_1)\ (C\theta_{2n}a_3C\theta_{3n} - S\theta_{2n}C\alpha_2a_3S\theta_{3n} +$$
$$S\theta_{2n}S\alpha_2d_3 + a_2C\theta_{2n}) + (n_{y_n}C\theta_1C\alpha_1 + n_{z_n}S\alpha_1 -$$
$$n_{x_n}S\theta_1C\alpha_1)(S\theta_{2n}a_3C\theta_{3n} + C\theta_{2n}C\alpha_2a_3S\theta_{3n} -$$

$$C\theta_{2n}S\alpha_2 d_3 + a_2 S\theta_{2n}) + (n_{x_n}S\theta_1 S\alpha_1 - n_{y_n}C\theta_1 S\alpha_1 \tag{4-5}$$

$$+ n_{z_n}C\alpha_1)(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_2) + n_{x_n}a_1 C\theta_1$$

$$+ n_{y_n}a_1 S\theta_1 + n_{z_n}d_1$$

$$= n_{x_n}p_{x_n} + n_{y_n}p_{y_n} + n_{z_n}p_{z_n}$$

where

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$

$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n = 1..9$$

From Eq(4-4), if θ_1, α_1, and α_2 are chosen as known value, Thus, we yield a linear equation in six unknowns.

$$K_1 a_3 + K_2 a_2 + K_3 a_1 + K_4 d_3 + K_5 d_2 + K_6 d_1 = K \tag{4-6}$$

where

$$K_1 = (n_{x_n}C\theta_1 + n_{y_n}S\theta_1)\ (C\theta_{2n}C\theta_{3n} - S\theta_{2n}C\alpha_2 S\theta_{3n}) +$$

$$(n_{y_n}C\theta_1 C\alpha_1 + n_{z_n}S\alpha_1 - n_{x_n}S\theta_1 C\alpha_1)(S\theta_{2n}C\theta_{3n} +$$

$$C\theta_{2n}C\alpha_2 S\theta_{3n}) + (n_{x_n}S\theta_1 S\alpha_1 - n_{y_n}C\theta_1 S\alpha_1 + n_{z_n}C\alpha_1)$$

$$(S\alpha_2 S\theta_{3n})$$

$$K_2 = (n_{x_n}C\theta_1 + n_{y_n}S\theta_1)\ C\theta_{2n} +$$

$$(n_{y_n}C\theta_1 C\alpha_1 + n_{z_n}S\alpha_1 - n_{x_n}S\theta_1 C\alpha_1)\ S\theta_{2n}$$

$$K_3 = (n_{x_n}C\theta_1 + n_{y_n}S\theta_1)$$

$$K_4 = (n_{x_n}C\theta_1 + n_{y_n}S\theta_1)\ S\theta_{2n}S\alpha_2 +$$

$$(n_{y_n}C\theta_1 C\alpha_1 + n_{z_n}S\alpha_1 - n_{x_n}S\theta_1 C\alpha_1)(-C\theta_{2n}S\alpha_2)$$

$$+ (n_{x_n}S\theta_1 S\alpha_1 - n_{y_n}C\theta_1 S\alpha_1 + n_{z_n}C\alpha_1)\ C\alpha_2$$

$$K_5 = (n_{x_n}S\theta_1 S\alpha_1 - n_{y_n}C\theta_1 S\alpha_1 + n_{z_n}C\alpha_1)$$

$$K_6 = n_{z_n}$$

$$K = n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n}$$

Hence, the closed-form solution of R-R crank for six finitely separated positions can be obtained by assuming θ_1, α_1 and α_2.However, because of nonlinearity and complication, the solution of nine synthesis positions of R-R crank can not be obtain easily. The proposed synthesis procedure provide an effective way to solve for nine separated positions by first obtaining the closed-form solution for six finitely separated positions and assuming θ_1, α_1, and α_2 as arbitrary values. Once the closed-form solution for six finitely seaprated positions is obtained, we can proceed to solve for seven, eight, and nine positions.

In TABLE IV, we summarize the synthesis procedure of R-R crank for nine finitely separated positions.

TABLE IV

SYNTHESIS PROCEDURE OF A R-R CRANK FOR NINE POSITIONS

Given :

1) The parametric equation of the surface to be enveloped by a tangent-plane attached to the moving joint of R-R cranks and nine precision points which approximate the surface.

2) the rotational angle (joint motion) of each joint

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}}$$

Objective : Design a R-R crank, a tangent-plane attached to a moving joint in which envelopes a given surface at the precision points.
(i.e., determine the linkage parameters a_1, a_2, a_3, α_1, α_2, θ_1, s_1, s_2, s_3.)

Procedure :

1) Calculate the normal vector of each precision points from

$$\mathbf{N}_n = \mathbf{S}_{u_n} \times \mathbf{S}_{v_n} \qquad \text{where } n = 1..9$$

2) Establish the synthesis equations with the help of the tangent-plane equations

$$(\mathbf{C}_n - \mathbf{P}_n)\,\mathbf{N}_n = 0$$

where

$$\mathbf{P} = p_x\,\mathbf{i} + p_y\,\mathbf{j} + p_z\,\mathbf{k}$$

$$\mathbf{C} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = [A_1][A_2][A_3] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

3) Obtain the closed-form solution for 6 positions by taking θ_1, α_1, and α_2 as a guessing value.

4) Obtain the numerical solution of seven, eight, and nine positions by using the synthesis equation of R-R crank :

$$(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\,(C\theta_{2n} a_3 C\theta_{3n} - S\theta_{2n} C\alpha_2 a_3 S\theta_{3n} + S\theta_{2n} S\alpha_2 d_3 + a_2 C\theta_{2n}) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - n_{x_n} S\theta_1 C\alpha_1)(S\theta_{2n} a_3 C\theta_{3n} + C\theta_{2n} C\alpha_2 a_3 S\theta_{3n} - C\theta_{2n} S\alpha_2 d_3 + a_2 S\theta_{2n}) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_2) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 = n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n}$$

Similarly, the synthesis procedure for the other cranks with any combination of R, P, and H joints can be derived by using the proposed procedure.

(2) Synthesis of R-P crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_2, \theta_1, \theta_3$)

Given joint motion = θ_{2i}, d_{3i}

Maximum number of positions = 9

Maximum number of positions for closed-form solution= 5

The synthesis equation is :

$$\begin{aligned}
&(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\ (C\theta_{2n} a_3 C\theta_3 - S\theta_{2n} C\alpha_2 a_3 S\theta_3 + \\
&S\theta_{2n} S\alpha_2 d_{3n} + a_2 C\theta_{2n}) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - \\
&n_{x_n} S\theta_1 C\alpha_1)(S\theta_{2n} a_3 C\theta_3 + C\theta_{2n} C\alpha_2 a_3 S\theta_3 - C\theta_{2n} S\alpha_2 d_{3n} \\
&+ a_2 S\theta_{2n}) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1) \\
&(S\alpha_2 a_3 S\theta_3 + C\alpha_2 d_3 + d_2) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 \\
&= n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \qquad (4\text{-}7)
\end{aligned}$$

where

$$\begin{aligned}
\theta_{2_n} &= \theta_{2_{n-1}} + \delta\theta_{2_{n-1}} \\
d_{3_n} &= d_{3_{n-1}} + \delta d_{3_{n-1}} \qquad n=1..9
\end{aligned}$$

(3) Synthesis of R-H crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_2, d_3, \theta_1$)

Given joint motion = θ_{2i}, θ_{3i}(or d_{3i}) (Also, L_3 lead of helical joint is provided)

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 6

(Note : for helical joint the joint motion variable can be θ_i or d_i)

The synthesis equation is :

$$(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\ (C\theta_{2n} a_3 C\theta_{3n} - S\theta_{2n} C\alpha_2 a_3 S\theta_{3n} + S\theta_{2n} S\alpha_2 d_3 + a_2 C\theta_{2n}) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - n_{x_n} S\theta_1 C\alpha_1)(S\theta_{2n} a_3 C\theta_{3n} + C\theta_{2n} C\alpha_2 a_3 S\theta_{3n} - C\theta_{2n} S\alpha_2 d_3 + a_2 S\theta_{2n}) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_2) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 = n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \tag{4-8}$$

where

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n=1..9$$
$$d_3 = d_0 + \delta d_3$$

(4) Synthesis of P-R crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_3, \theta_1, \theta_2$)

Given joint motion = d_2, θ_3

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 5

The synthesis equation is :

$$(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\ (C\theta_2 a_3 C\theta_{3n} - S\theta_2 C\alpha_2 a_3 S\theta_{3n} + S\theta_2 S\alpha_2 d_3 + a_2 C\theta_2) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - n_{x_n} S\theta_1 C\alpha_1)(S\theta_2 a_3 C\theta_{3n} + C\theta_2 C\alpha_2 a_3 S\theta_{3n} - C\theta_2 S\alpha_2 d_3 + a_2 S\theta_2) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_{2n}) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 = n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \tag{4-9}$$

where

$$d_{2_n} = d_{2_{n-1}} + \delta d_{2_{n-1}}$$

$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n=1..9$$

(5) Synthesis of P-P crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, \theta_1, \theta_2, \theta_3$)

Given joint motion = d_2, d_3

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 4

The synthesis equation is :

$$\begin{aligned}
&(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\ (C\theta_2 a_3 C\theta_3 - S\theta_2 C\alpha_2 a_3 S\theta_3 + \\
&S\theta_2 S\alpha_2 d_{3n} + a_2 C\theta_2) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - \\
&n_{x_n} S\theta_1 C\alpha_1)(S\theta_2 a_3 C\theta_3 + C\theta_2 C\alpha_2 a_3 S\theta_3 - C\theta_2 S\alpha_2 d_{3n} \\
&+ a_2 S\theta_2) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1) \\
&(S\alpha_2 a_3 S\theta_3 + C\alpha_2 d_{3n} + d_{2n}) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 \\
&= n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n}
\end{aligned} \tag{4-10}$$

where

$$d_{2_n} = d_{2_{n-1}} + \delta d_{2_{n-1}}$$
$$d_{3_n} = d_{3_{n-1}} + \delta d_{3_{n-1}} \qquad n=1..9$$

(6) Synthesis of P-H crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_3, \theta_1, \theta_2$)

Given joint motion = d_2, θ_3

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 5

The synthesis equation is :

$$\begin{aligned}
&(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\ (C\theta_2 a_3 C\theta_{3n} - S\theta_2 C\alpha_2 a_3 S\theta_{3n} + \\
&S\theta_2 S\alpha_2 d_3 + a_2 C\theta_2) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - \\
&n_{x_n} S\theta_1 C\alpha_1)(S\theta_2 a_3 C\theta_{3n} + C\theta_2 C\alpha_2 a_3 S\theta_{3n} - C\theta_2 S\alpha_2 d_3
\end{aligned}$$

$$+ a_2 S\theta_2) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)$$
$$(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_{2n}) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1$$
$$= n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \qquad (4\text{-}11)$$

where

$$d_{2_n} = d_{2_{n-1}} + \delta d_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n=1..9$$
$$d_3 = d_0 + \delta d_3$$

(7) Synthesis of H-R crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_2, d_3, \theta_1$)

Given joint motion = θ_2, θ_3

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 6

The synthesis equation is :

$$(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\ (C\theta_{2n} a_3 C\theta_{3n} - S\theta_{2n} C\alpha_2 a_3 S\theta_{3n} +$$
$$S\theta_{2n} S\alpha_2 d_3 + a_2 C\theta_{2n}) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 -$$
$$n_{x_n} S\theta_1 C\alpha_1)(S\theta_{2n} a_3 C\theta_{3n} + C\theta_{2n} C\alpha_2 a_3 S\theta_{3n} - C\theta_{2n} S\alpha_2 d_3$$
$$+ a_2 S\theta_{2n}) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)$$
$$(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_2) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1$$
$$= n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \qquad (4\text{-}12)$$

where

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n=1..9$$
$$d_2 = d_0 + \delta d_2$$

(8) Synthesis of H-P crank:

Number of Unknowns = 9 ($\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_2, \theta_1, \theta_3$)

Given joint motion = θ_2, d_3.

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 5

The synthesis equation is :

$$(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\,(C\theta_{2n} a_3 C\theta_3 - S\theta_{2n} C\alpha_2 a_3 S\theta_3 + S\theta_{2n} S\alpha_2 d_{3n} + a_2 C\theta_{2n}) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - n_{x_n} S\theta_1 C\alpha_1)(S\theta_{2n} a_3 C\theta_3 + C\theta_{2n} C\alpha_2 a_3 S\theta_3 - C\theta_{2n} S\alpha_2 d_{3n} + a_2 S\theta_{2n}) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)(S\alpha_2 a_3 S\theta_3 + C\alpha_2 d_{3n} + d_2) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 = n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \qquad (4\text{-}13)$$

where

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n=1..9$$
$$d_2 = d_0 + \delta d_2$$

(9) Synthesis of H-H crank:

Number of Unknowns = 9 $(\alpha_1, \alpha_2, a_1, a_2, a_3, d_1, d_2, d_3, \theta_1)$

Given joint motion = θ_2, θ_3

Maximum number of positions = 9

Maximum number of positions for closed-form solutions= 6

The synthesis equation is :

$$(n_{x_n} C\theta_1 + n_{y_n} S\theta_1)\,(C\theta_{2n} a_3 C\theta_{3n} - S\theta_{2n} C\alpha_2 a_3 S\theta_{3n} + S\theta_{2n} S\alpha_2 d_3 + a_2 C\theta_{2n}) + (n_{y_n} C\theta_1 C\alpha_1 + n_{z_n} S\alpha_1 - n_{x_n} S\theta_1 C\alpha_1)(S\theta_{2n} a_3 C\theta_{3n} + C\theta_{2n} C\alpha_2 a_3 S\theta_{3n} - C\theta_{2n} S\alpha_2 d_3 + a_2 S\theta_{2n}) + (n_{x_n} S\theta_1 S\alpha_1 - n_{y_n} C\theta_1 S\alpha_1 + n_{z_n} C\alpha_1)(S\alpha_2 a_3 S\theta_{3n} + C\alpha_2 d_3 + d_2) + n_{x_n} a_1 C\theta_1 + n_{y_n} a_1 S\theta_1 + n_{z_n} d_1 = n_{x_n} p_{x_n} + n_{y_n} p_{y_n} + n_{z_n} p_{z_n} \qquad (4\text{-}14)$$

where

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}} \qquad n=1..9$$
$$d_2 = d_0 + \delta d_2$$
$$d_3 = d_0 + \delta d_3$$

4.2. Analysis of Dyads For Any Combination of Revolute, Prismatic, and Helical Joints

(1) Analysis of R-R crank:

The fundamental problem in the kinematic analysis is to datermine the relative motions of moving links where the linkage parameters are given. Therefore, the unknown variables in this category are θ_2 and θ_3 while the linkage parameters $a_1, a_2, a_3, \alpha_1, \alpha_2, \alpha_3, s_1, s_1, s_1$, and θ_1 are provided as known values.

The point P on which plane tangential to the surface can be expressed in the fixed coordinate frame as:

$$\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \\ 1 \end{bmatrix} = [A_1][A_2][A_3] \begin{bmatrix} P_{4x} \\ P_{4y} \\ P_{4z} \\ 1 \end{bmatrix} \qquad (4-15)$$

Let the X-axis of the (**X3,Y3,Z3**) coordinate frame normal to the tangent-plane (i.e., in the same direction of unit normal vector of tangent plane), then we obtain

$$P_{4x} = 0 \tag{4-16}$$

Hence, from Eq(4-15), we yield four unknowns (i.e., θ_2, θ_3, P_{4y} and P_{4z}) in three equations.

$$P_{1x} = [A_{equ}]_1 P_{4x} \tag{4-17}$$

$$P_{1y} = [A_{equ}]_2 P_{4y} \tag{4-18}$$

$$P_{1z} = [A_{equ}]_3 P_{4z} \tag{4-19}$$

where $[A_{equ}]_i$ is the i_{th} row of $[A_{equ}]$.

With the help of tangent-plane, we obtain the tangent-plane equation as

$$(\mathbf{X}-x_0,\ \mathbf{Y}-y_0,\ \mathbf{Z}-z_0)\mathbf{N} = 0 \tag{4-20}$$

where

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = [A_1][A_2][A_3] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4-21}$$

by rearranging Eq(4-20), yields

$$(n_xC\theta_1 + n_yS\theta_1)\ (C\theta_2a_3C\theta_3 - S\theta_2C\alpha_2a_3S\theta_3 + S\theta_2S\alpha_2d_3 + a_2C\theta_2)$$
$$+ (n_yC\theta_1C\alpha_1 + n_zS\alpha_1 - n_xS\theta_1C\alpha_1)(S\theta_2a_3C\theta_3 + C\theta_2C\alpha_2a_3S\theta_3 -$$
$$C\theta_2S\alpha_2d_3 + a_2S\theta_2) + (n_xS\theta_1S\alpha_1 - n_yC\theta_1S\alpha_1 + n_zC\alpha_1)(S\alpha_2a_3S\theta_3$$
$$+ C\alpha_2d_3 + d_2) + n_xa_1C\theta_1 + n_ya_1S\theta_1 + n_zd_1$$

$$= n_x \; p_x + n_y \; p_y + n_z \; p_z \tag{4-22}$$

Thus, from Eq(4-17) to Eq(4-19), and Eq(4-22),we obtain four unknowns in four equations. The analysis procedure of R-R crank is summarize in TABLE V.

4.3. Numerical Examples

In this section, numerical examples of synthesis of R-R crank for six and nine finitely separated positions generating different surfaces are presented in TABLE VI, VII, VIII, and IX. Also, numerical examples of analysis of R-R crank by using the given parameters which derived in TABLE VI and VIII are presented in TABLE X and XI.

TABLE V

PROCEDURE OF ANALYSIS OF R-R CRANK

Given :1) The parametric equation of the surface to be enveloped by a tangent-plane attached to the moving joint of R-R cranks and precision points which on the surface.

2) the linkage parameters of R-R links :

a_i, d_i, α_i, and θ_1 where $i=1..3$

Objective : Determine the joint motion of R-R links.

(i.e., calculate θ_2 and θ_3)

Procedure :

1) Calculate the normal vector of each precision points from $N = S_u \times S_v$

2) Derive the analysis equations for the tangent-plane motion

$$P_{1x} = [A_{equ}]_1 \, P_{4x}$$

$$P_{1y} = [A_{equ}]_2 \, P_{4y}$$

$$P_{1z} = [A_{equ}]_3 \, P_{4z}$$

$$(C - P) \, N = 0$$

where $P = p_x i + p_y j + p_z k$

$$\mathbf{C} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = [A_1][A_2][A_3] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

TABLE VI

NUMERICAL EXAMPLE OF SYNTHESIS OF R-R CRANK FOR SIX FINITELY SEPARATED POSITIONS (CLOSED-FORM SOLUTION)

Given: 1) The surface is given as a torus :

$$\mathbf{S} = (20+10\cos V)\cos U\ \mathbf{i} + (20+10\cos V)\sin U\ \mathbf{j} + 10\sin V\ \mathbf{k}$$

2) the six finitely separated positions is given as :

$P_1 = (30,\ 0,\ 0)$ $\quad$ $P_2 = (24.82,\ 14.33,\ 5)$

$P_3 = (12.5,\ 21.65,\ 8.66)$ $\quad$ $P_4 = (0,\ 20,\ 10)$

$P_5 = (-7.5,\ 12.99,\ 8.66)$ $\quad$ $P_6 = (-9.82,\ 5.67,\ 5)$

3) the joint motion is given as :

$$\theta_{21} = 0°, \quad \theta_{31} = 0°$$

$\delta\theta_{21} = 30°,\ \delta\theta_{31} = 30°$ $\quad$ $\delta\theta_{22} = 30°,\ \delta\theta_{32} = 30°$

$\delta\theta_{23} = 30°,\ \delta\theta_{33} = 30°$ $\quad$ $\delta\theta_{24} = 30°,\ \delta\theta_{34} = 30°$

$\delta\theta_{25} = 30°,\ \delta\theta_{35} = 30°$

4) the choice of linkage parameters

$$\theta_1 = 0°\ , \quad \alpha_1 = 90°, \quad \alpha_2 = 90°$$

Result :

$a_1 = 28.583969,\quad a_2 = 13.956359,\quad a_3 = -12.540329$

$s_1 = -3.956347,\quad s_2 = -3.344426,\quad s_3 = -15.720272$

TABLE VII

NUMERICAL EXAMPLE OF SYNTHESIS OF R-R CRANK FOR NINE FINITELY SEPARATED POSITIONS

[Given] :1) The surface is the same as in TABLE VI.

2) the six finitely separated positions is given as

P_1 = (30, 0, 0)

P_2 = (29.8479, 2.6113, 0.8715)

P_3 = (29.3946, 5.1831, 1.7365)

P_4 = (28.6486, 7.6764, 2.5882)

P_5 = (27.6241, 10.0543, 3.4202)

P_6 = (26.3401, 12.2826, 4.2262)

P_7 = (24.8205, 14.3301, 5)

P_8 = (23.0932, 16.1700, 5.736)

P_9 = (21.1891, 17.7798, 6.4279)

3) the joint motion is given as : (θ : degree)

$\theta_{21} = 0°$, $\theta_{31} = 0°$

$\delta\theta_{21} = 5°$, $\delta\theta_{31} = 5°$ $\delta\theta_{22} = 5°$, $\delta\theta_{32} = 5°$

$\delta\theta_{23} = 5°$, $\delta\theta_{33} = 5°$ $\delta\theta_{24} = 5°$, $\delta\theta_{34} = 5°$

$\delta\theta_{25} = 5°$, $\delta\theta_{35} = 5°$ $\delta\theta_{26} = 5°$, $\delta\theta_{36} = 5°$

$\delta\theta_{27} = 5°$, $\delta\theta_{37} = 5°$ $\delta\theta_{28} = 5°$, $\delta\theta_{38} = 5°$

Result :

$\theta_1 = 45°$, $\alpha_1 = 30°$, $\alpha_2 = 27.662°$

$a_1 = 6.26973343$, $a_2 = 22.55064583$, $a_3 = -6.54820347$

$s_1 = 1.39241803$, $s_2 = -45.4885788$, $s_3 = 50.77313232$

TABLE VIII

NUMERICAL EXAMPLE OF SYNTHESIS OF R-R CRANK FOR SIX FINITELY SEPARATED POSITIONS (SURFACE IS GIVEN AS A SPHERE)

[Given] :

1) The surface is given as a sphere :

$$\mathbf{S} = 20\cos U\sin V\ \mathbf{i} + 20\sin U\sin V\ \mathbf{j} + 20\cos V\ \mathbf{k}$$

2) the six finitely separated positions is given as :

$P_1 = (8.66,\ 5.0,\ 17.32)$

$P_2 = (9.848,\ 8.26,\ 15.321)$

$P_3 = (9.848,\ 11.736,\ 12.856)$

$P_4 = (8.66,\ 15.0,\ 10.0)$

$P_5 = (6.428,\ 17.66,\ 6.84)$

$P_6 = (3.42,\ 19.4,\ 3.473)$

3) the joint motion is given as :

$$\theta_{21} = 30°,\ \theta_{31} = 30°$$

$\delta\theta_{21} = 10°,\ \delta\theta_{31} = 10°$ $\delta\theta_{22} = 10°,\ \delta\theta_{32} = 10°$

$\delta\theta_{23} = 10°,\ \delta\theta_{33} = 10°$ $\delta\theta_{24} = 10°,\ \delta\theta_{34} = 10°$

$\delta\theta_{25} = 10°,\ \delta\theta_{35} = 10°$

4) the choice of linkage parameters

$$\theta_1 = 45°,\quad \alpha_1 = 30°,\quad \alpha_2 = 30°$$

Result :

$a_1 = 1.40560913,\quad a_2 = 39.28113174,\quad a_3 = -3.97064018$

$s_1 = 45.46696472,\quad s_2 = 13.25787354,\quad s_3 = -73.80399323$

TABLE IX

NUMERICAL EXAMPLE OF SYNTHESIS OF R-R CRANK FOR NINE FINITELY SEPARATED POSITIONS (SURFACE IS GIVEN AS A SPHERE)

[Given] :

1) The surface is the same as in TABLE VIII.

2) the six finitely separated positions is given as :

P_1 = (3.42, 0.6031, 19.6962)

P_2 = (5.0, 1.3397, 19.3185)

P_3 = (6.4279, 2.3396, 18.7939)

P_4 = (7.6604, 3.5721, 18.1262)

P_5 = (8.6603, 5.0, 17.3205)

P_6 = (9.3969, 6.5798, 16.383)

P_7 = (9.8481, 8.2635, 15.3209)

P_8 = (10.0, 10.0, 14.1421)

P_9 = (9.8481, 11.7365, 12.8558)

3) the joint motion is given as : (θ : degree)

$\theta_{21} = 10°$, $\theta_{31} = 10°$

$\delta\theta_{21} = 5°$, $\delta\theta_{31} = 5°$ $\delta\theta_{22} = 5°$, $\delta\theta_{32} = 5°$

$\delta\theta_{23} = 5°$, $\delta\theta_{33} = 5°$ $\delta\theta_{24} = 5°$, $\delta\theta_{34} = 5°$

$\delta\theta_{25} = 5°$, $\delta\theta_{35} = 5°$ $\delta\theta_{26} = 5°$, $\delta\theta_{36} = 5°$

$\delta\theta_{27} = 5°$, $\delta\theta_{37} = 5°$ $\delta\theta_{28} = 5°$, $\delta\theta_{38} = 5°$

Result :

$\theta_1 = 30.0°$, $\alpha_1 = 45.0°$, $\alpha_2 = 33.2497°$

$a_1 = -8.63857841$, $a_2 = 16.09467697$, $a_3 = -0.94676548$

$s_1 = 33.69320679$, $s_2 = -13.00296783$, $s_3 = -22.09254456$

TABLE X

NUMERICAL EXAMPLE 1 OF ANALYSIS OF R-R CRANK

[Given]:

1) The surface is given in the TABLE VI.

2) The point on the surface is given as

$P = (30,0,0)$

3) The linkage parameters are given as:

$a_1 = 28.584$, $a_2 = 13.956$, $a_3 = -12.540$

$s_1 = -3.956$, $s_2 = -3.344$, $s_3 = -15.720$

$\theta_1 = 0°$, $\alpha_1 = 90°$, $\alpha_2 = 90°$

[Result]:

$\theta_2 = 30.0351°$ $\theta_3 = 29.9664°$

TABLE XI

NUMERICAL EXAMPLE 2 OF ANALYSIS
OF R-R CRANK

[Given]:

1) The surface is given in the TABLE VIII.

2) The point on the surface is given as

$P = (0,0,20)$

3) The linkage parameters are given as:

$a_1 = 35.468$, $a_2 = 61.683$, $a_3 = 0$

$s_1 = 0$, $s_2 = -130.773$, $s_3 = -186.392$

$\theta_1 = 45°$, $\alpha_1 = 30°$, $\alpha_2 = 30°$

[Result]:

$\theta_2 = 30.0021°$ $\theta_3 = 30.3053°$

CHAPTER V

SYNTHESIS OF DYADS FOR INFINITESIMALLY AND MIXED MODE SEPATATED POSITIONS GENERATED BY TANGENT-PLANE

Generally, motion of a rigid body can be described in a number of ways. Sometimes, it is required of the tangent plane to move with a given velocity, acceleration, jerk, etc. (higher-order properties of motion) which generate the given surface. Design methods to satisfy such requirements will be developed in this chapter. Such design procedures are also referred to as design for infinitesimally separated position or mixed mode position synthesis. Infinitesimally separated positions synthesis procedure differ from mixed mode separated position in that only one position of the tangent-plane is considered or we have only one finitely separated position involved in the design. Infinitesimally separated position design can be considered as a degenerate case of mixed position design.

In the previous chapter, we developed the synthesis procedures for finitely separated positions of dyads composed of Revolute, Prismatic, and Helical joints. In this chapter, we will develop the first-order and higher-order synthesis procedures for infinitesimally separated and mixed mode positions of dyads with any combination of Revolute, Prismatic, and Helical joints.

The infinitesimally separated displacements of a rigid body tangential to any surface, is described by the properties of the rigid body as it approaches the surface at the tangential point. These properties may be the velocity, acceleration, jerk, time rate of change of jerk(kerk) etc. Hence, the instantaneous angular motion of the tangent plane involving infinitesimal changes in angular displacements can be described with respect to changes in time by specifying $d\theta/dt$, $d^2\theta/dt^2$, $d^3\theta/dt^3$, $d^3\theta/dt^3$ (or θ, θ, θ)etc.

Mixed mode position synthesis is more in touch with reality involving concepts familiar to a mechanical eangineer rather than the esoteric ideas of theoretical kinematics. In general it has two or more finitely separated positions with the design requirements being velocity, acceleration etc. at each finite position. A different way of describing mixed position synthesis would be define it as designing for finitely separated positions with having to satisfy infinitesimal position requirements at one or more of the finite positions.

The synbolic notation proposed by Tesar for mixed position synthesis will be made use of in this study to represent the design situation. The symbol P represents a single position of the tangent-plane. The combination P-P represents two finitely separated positions, and PP represents tow infinitesimally separated positions. The combination P-P-P-P-P represnets a five finitely separated position (five precisions point) problem. The combination P-PP-PPP-PP-P represents a five finitely separated position problem with higher order motion requirements at the second, third, and fourth positions.

5.1. First Order Infinitesimally and Mixed Mode Separated Positions

The synthesis equations derived in the previous chapter will be used here to derive the synthesis equations for the first order infinitesimally and any combination of mixed mode separated positions.

(1) R-R Crank

(A) Synthesis of R-R Crank for Infinitesimally And Mixed Mode Separated Positions

The synthesis equation for the first order infinitesimal separated position is obtained by differentiating the basic form of the synthesis equation (4-3) for finitely separated positions. Therefore, for synthesis of infinitesimally separated positions of R-R crank, we need to take the rate of change of joint motion variables θ_2 and θ_3.

By taking the derivative of Eq(4-3) with respect to time t, we obtain

$$(n_xC\theta_1+n_yS\theta_1)(\ -S\theta_2a_3C\dot{\theta}_3\theta_2 - C\theta_2a_3S\theta_3\dot{\theta}_3 + C\theta_2C\alpha_2a_3S\theta_3\dot{\theta}_2 +$$
$$S\theta_2C\alpha_2a_3C\theta_3\dot{\theta}_3 + C\theta_2S\alpha_2d_3\dot{\theta}_2 - a_2S\theta_2\dot{\theta}_2) + (n_yC\theta_1C\alpha_1 + n_zS\alpha_1 -$$

$$n_x S\theta_1 C\alpha_1)(-S\theta_2 a_3 C\theta_3 \dot{\theta}_2 + C\theta_2 a_3 S\theta_3 \dot{\theta}_3 - S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 + C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_3 - S\theta_2 S\alpha_2 d_3 \dot{\theta}_2 + a_2 C\theta_2 \dot{\theta}_2) + (n_x S\theta_1 S\alpha_1 - n_y C\theta_1 S\alpha_1 + n_z C\alpha_1)(S\alpha_2 a_3 C\theta_3 \dot{\theta}_3) = 0 \qquad (5-1)$$

Hence, by adding Eq(4-3), we yield two synthesis equations of R-R crank for each infinitesimally separated position.

(B) Synthesis of One First Order Mixed Mode Separated Positions

Since the maximum number of synthesis positions for finitely separated positions is nine, the possible combinations of synthesis of mixed mode separated positions with the first order motion requirement can be listed as in TABLE XII. The procedure can be best explained by example.

TABLE XII

NINE SYNTHESIS POSITIONS WITH ONE, TWO, THREE, AND FOUR FIRST ORDER MOTIONS

One First Order	Tow first order	Three First Order
PP-P-P-P-P-P-P-P	PP-PP-P-P-P-P-P	PP-PP-PP-P-P-P
P-PP-P-P-P-P-P-P	PP-P-PP-P-P-P-P	PP-PP-P-PP-P-P
P-P-PP-P-P-P-P-P	PP-P-P-PP-P-P-P	PP-PP-P-P-PP-P
P-P-P-PP-P-P-P-P	PP-P-P-P-PP-P-P	PP-PP-P-P-P-PP
P-P-P-P-PP-P-P-P	PP-P-P-P-P-PP-P	PP-P-PP-PP-P-P
P-P-P-P-P-PP-P-P	PP-P-P-P-P-P-PP	PP-P-PP-P-PP-P
P-P-P-P-P-P-PP-P		PP-P-PP-P-P-PP
P-P-P-P-P-P-P-PP	P-PP-PP-P-P-P-P	PP-P-P-PP-PP-P
	P-PP-P-PP-P-P-P	PP-P-P-PP-P-PP
Four First Order	P-PP-P-P-PP-P-P	PP-P-P-P-PP-PP
	P-PP-P-P-P-PP-P	P-PP-PP-PP-P-P
PP-PP-PP-P-PP	P-PP-P-P-P-P-PP	P-PP-PP-P-PP-P
PP-PP-P-PP-PP		P-PP-PP-P-P-PP
PP-P-PP-PP-PP	P-P-PP-PP-P-P-P	P-PP-P-PP-P-PP
P-PP-PP-PP-PP	P-P-PP-P-PP-P-P	P-PP-P-P-PP-PP
PP-PP-PP-PP-P	P-P-PP-P-P-PP-P	P-P-P-PP-PP-PP
	P-P-PP-P-P-P-PP	P-P-PP-P-PP-PP
	P-P-P-PP-PP-P-P	P-P-PP-PP-PP-P
	P-P-P-PP-P-PP-P	P-P-PP-PP-P-PP
	P-P-P-PP-P-P-PP	.
	P-P-P-P-PP-PP-P	.
	P-P-P-P-PP-P-PP	etc.
	P-P-P-P-P-PP-PP	

Consider a nine position problem. The design equation for a nine position finitely separated problem is given by Eq(4-3). Also, the first-order infinitesimally separated equation is obtained from Eq(5-1). Supposed we are synthesizing for a PP-PP-PP-PP-P type of problem. They are five finitely separated positions with a first-order (velocity) requirement at four of the finite position. With the five finite position, five synthesis equations can be formed. A nine-position requires nine equations to solve for the nine unknowns. The remaining four equations of synthesis are the first order infinitesimal synthesis equations at the first, second, third, and fourth finite positions. Hence, Eq(5-1) can be rewritten as

$$\begin{aligned}
&(n_{x_n}C\theta_1+n_{y_n}S\theta_1)(-S\theta_{2n}a_3C\theta_{3n}\dot{\theta}_{2n} - C\theta_{2n}a_3S\theta_{3n}\dot{\theta}_{3n} + \\
&C\theta_{2n}C\alpha_2a_3S\theta_{3n}\dot{\theta}_{2n} + S\theta_{2n}C\alpha_2a_3C\theta_{3n}\dot{\theta}_{3n} + C\theta_{2n}S\alpha_2d_3\dot{\theta}_{2n} - \\
&a_2S\theta_{2n}\dot{\theta}_{2n}) + (n_{y_n}C\theta_1C\alpha_1 + n_{z_n}S\alpha_1 - n_{x_n}S\theta_1C\alpha_1) \\
&(-S\theta_{2n}a_3C\theta_{3n}\dot{\theta}_{2n} + C\theta_{2n}a_3S\theta_{3n}\dot{\theta}_{3n} - S\theta_{2n}C\alpha_2a_3S\theta_{3n}\dot{\theta}_{2n} + \\
&C\theta_{2n}C\alpha_2a_3C\theta_{3n}\dot{\theta}_{3n} - S\theta_{2n}S\alpha_2d_3\dot{\theta}_{2n} + a_2C\theta_{2n}\dot{\theta}_{2n}) + (n_{x_n}S\theta_1S\alpha_1 \\
&- n_{y_n}C\theta_1S\alpha_1 + n_zC\alpha_1)(S\alpha_2a_3C\theta_{3n}\dot{\theta}_{3n}) = 0 \qquad (5\text{-}2)
\end{aligned}$$

where

$$\theta_{2_n} = \theta_{2_{n-1}} + \delta\theta_{2_{n-1}}$$
$$\theta_{3_n} = \theta_{3_{n-1}} + \delta\theta_{3_{n-1}}$$

and

$$n = 1..4$$

and the other five equations can be obtained from Eq(4-3) when n=1..5.

Similarly, the synthesis procedure for first-order infinitesimally and mixed mode separated positions of the other types of dyads composed of revolute, prismatic, and helical joint can be obtained by using the proposed synthesis procedure of R-R crank.

5.2. Higher-Order Infinitesimally, Mixed Mode Separated Positions

Recapitulating, a tangent-plane can be designed for a maximum of nine positions for R-R crank. With the help of Eq(4-3) it is now possible to design satisfying higher order motion requirements at certain position. For example, P-PP-PPP-PP-P, PPP-PPPP-PP, PP-PPP-PPP-P-P, etc. The second order synthesis equation in obtained by differentiating Eq(4-3) twice or diffenentiating Eq(5-1) once. The basic form of the second order synthesis equation is given by

$$(n_xC\theta_1+n_yS\theta_1)(-C\theta_2a_3C\theta_3\dot{\theta}_2^{\,2} + S\theta_2a_3S\theta_3\dot{\theta}_2\dot{\theta}_3 - S\theta_2a_3C\theta_3\ddot{\theta}_2 -$$
$$C\theta_2a_3C\theta_3\dot{\theta}_3^{\,2} + S\theta_2a_3S\theta_3\dot{\theta}_3\dot{\theta}_2 - C\theta_2a_3S\theta_3\ddot{\theta}_3 + C\theta_2C\alpha_2a_3C\theta_3\dot{\theta}_2\dot{\theta}_3 -$$
$$S\theta_2C\alpha_2a_3S\theta_3\dot{\theta}_2^{\,2} + C\theta_2C\alpha_2a_3S\theta_3\ddot{\theta}_2 + C\theta_2C\alpha_2a_3C\theta_3\dot{\theta}_3\dot{\theta}_2 -$$
$$S\theta_2C\alpha_2a_3S\theta_3\dot{\theta}_3^{\,2} + S\theta_2C\alpha_2a_3C\theta_3\ddot{\theta}_3 - S\theta_2S\alpha_2d_3\dot{\theta}_2^{\,2} + C\theta_2S\alpha_2d_3\ddot{\theta}_2 -$$
$$a_2C\theta_2\dot{\theta}_2^{\,2} + a_2S\theta_2\ddot{\theta}_2) + (n_yC\theta_1C\alpha_1 + n_zS\alpha_1 - n_xS\theta_1C\alpha_1)$$
$$(-C\theta_2a_3C\theta_3\dot{\theta}_2^{\,2} + S\theta_2a_3S\theta_3\dot{\theta}_2\dot{\theta}_3 - S\theta_2a_3C\theta_3\ddot{\theta}_2 + C\theta_2a_3C\theta_3\dot{\theta}_3^{\,2}$$
$$- S\theta_2a_3S\theta_3\dot{\theta}_2\dot{\theta}_3 + C\theta_2a_3S\theta_3\ddot{\theta}_3 - C\theta_2C\alpha_2a_3S\theta_3\dot{\theta}_2^{\,2} -$$
$$S\theta_2C\alpha_2a_3C\theta_3\dot{\theta}_2\dot{\theta}_3 - S\theta_2C\alpha_2a_3S\theta_3\ddot{\theta}_2 - S\theta_2C\alpha_2a_3C\theta_3\dot{\theta}_2\dot{\theta}_3 -$$
$$C\theta_2C\alpha_2a_3S\theta_3\dot{\theta}_3^{\,2} - C\theta_2C\alpha_2a_3C\theta_3\ddot{\theta}_3 - C\theta_2S\alpha_2d_3\dot{\theta}_2^{\,2} - S\theta_2S\alpha_2d_3\ddot{\theta}_2 -$$

$$a_2 S\theta_2 \dot{\theta}_2^{\,2} + a_2 C\theta_2 \ddot{\theta}_2) + (n_x S\theta_1 S\alpha_1 - n_y C\theta_1 S\alpha_1 + n_z C\alpha_1)$$
$$(-S\alpha_2 a_3 S\theta_3 \dot{\theta}_3^{\,2} + S\alpha_2 a_3 C\theta_3 \ddot{\theta}_3) = 0 \qquad (5\text{-}3)$$

A second order design requirement is to specify the acceleration of the tangent-plane. A second-order infinitesimally separated position synthesis requires the simultaneous solution of two equation - the first order and the second order synthesis equation, that is Eq(5-1) and Eq(5-3). It is now possible to synthesis for design requirements of the types P-PPP-P-P-P-P-P, P-PPP-PP-PPP, PPP-PPP-PPP, etc.

Considering a nine position problem - PPP-PP-PPP-P. There are four finitely separated positions with a first-order (velocity) requirement at one of the finite position and with a second-order (acceleration) requirement at two of the finite position. With the four finite position, four synthesis equations can be formed. We know that a nine-position requires nine equations to solve for the nine unknowns. The remaining five equations of synthesis are the first infinitesimal synthesis equations at the first, second, and third finite positions, and the second infinitesimal synthesis equations at the first and third finite positions.

The synthesis equation of third-order infinitesimally and mixed mode separated positions can be obtained by differentiating Eq(5-3).

$$(n_x C\theta_1 + n_y S\theta_1)(\, S\theta_2 a_3 C\theta_3 \dot{\theta}_2^{3} + C\theta_2 a_3 S\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 - 2C\theta_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_2$$

$$+ C\theta_2 a_3 S\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 + S\theta_2 a_3 C\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} + S\theta_2 a_3 S\theta_3 \ddot{\theta}_2 \dot{\theta}_3 +$$

$$S\theta_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_3 - C\theta_2 a_3 C\theta_3 \ddot{\theta}_2 \dot{\theta}_2 + S\theta_2 a_3 S\theta_3 \ddot{\theta}_2 \dot{\theta}_3 - S\theta_2 a_3 C\theta_3 \dddot{\theta}_2 +$$

$$S\theta_2 a_3 C\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} + C\theta_2 a_3 S\theta_3 \dot{\theta}_3^{3} - 2C\theta_2 a_3 C\theta_3 \dot{\theta}_3 \ddot{\theta}_3 +$$

$$C\theta_2 a_3 S\theta_3 \dot{\theta}_3 \dot{\theta}_2^{2} + S\theta_2 a_3 C\theta_3 \dot{\theta}_3^{2} \dot{\theta}_2 + S\theta_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_2 + S\theta_2 a_3 S\theta_3 \ddot{\theta}_3 \dot{\theta}_2$$

$$+ S\theta_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_3 - C\theta_2 a_3 C\theta_3 \dot{\theta}_3 \ddot{\theta}_3 - C\theta_2 a_3 S\theta_3 \dddot{\theta}_3 -$$

$$S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 - C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} + C\theta_2 C\alpha_2 a_3 C\theta_3 \ddot{\theta}_2 \dot{\theta}_3 +$$

$$C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_3 - C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2^{3} - S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3$$

$$- 2S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_2 - S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_2 + C\theta_2 C\alpha_2 a_3 C\theta_3 \ddot{\theta}_2 \dot{\theta}_3 +$$

$$C\theta_2 C\alpha_2 a_3 S\theta_3 \dddot{\theta}_2 - S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_3 \dot{\theta}_2^{2} - C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_3^{2} \dot{\theta}_2 +$$

$$C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_3 \ddot{\theta}_2 + C\theta_2 C\alpha_2 a_3 C\theta_3 \ddot{\theta}_3 \dot{\theta}_2 - C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} -$$

$$S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_3^{3} - 2S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_3 + C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_3 -$$

$$S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_3 + S\theta_2 C\alpha_2 a_3 C\theta_3 \dddot{\theta}_3 - C\theta_2 S\alpha_2 d_3 \dot{\theta}_2^{3} -$$

$$2S\theta_2 S\alpha_2 d_3 \dot{\theta}_2 \ddot{\theta}_2 - S\theta_2 S\alpha_2 d_3 \dot{\theta}_2 \ddot{\theta}_2 + C\theta_2 S\alpha_2 d_3 \dddot{\theta}_2 +$$

$$a_2 S\theta_2 \dot{\theta}_2^{3} - 2a_2 C\theta_2 \dot{\theta}_2 \ddot{\theta}_2 + a_2 C\theta_2 \dot{\theta}_2 \ddot{\theta}_2 + a_2 S\theta_2 \dddot{\theta}_2) + (n_y C\theta_1 C\alpha_1 +$$

$$n_z S\alpha_1 - n_x S\theta_1 C\alpha_1)(S\theta_2 a_3 C\theta_3 \dot{\theta}_2^{3} + C\theta_2 a_3 S\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 -$$

$$2S\theta_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_2 + C\theta_2 a_3 S\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 + S\theta_2 a_3 C\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} +$$

$$S\theta_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_3 + S\theta_2 a_3 S\theta_3 \ddot{\theta}_2 \dot{\theta}_3 - C\theta_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_2 + S\theta_2 a_3 S\theta_3 \ddot{\theta}_2 \dot{\theta}_3 -$$

$$S\theta_2 a_3 C\theta_3 \dddot{\theta}_2 - S\theta_2 a_3 C\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} - C\theta_2 a_3 S\theta_3 \dot{\theta}_3^{3} + 2C\theta_2 a_3 C\theta_3 \dot{\theta}_3 \ddot{\theta}_3$$

$$- C\theta_2 a_3 S\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 - S\theta_2 a_3 C\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} - S\theta_2 a_3 S\theta_3 \ddot{\theta}_2 \dot{\theta}_3 -$$

$$S\theta_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_3 - S\theta_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_3 + C\theta_2 a_3 C\theta_3 \dot{\theta}_3 \ddot{\theta}_3 + C\theta_2 a_3 S\theta_3 \dddot{\theta}_3 +$$

$$S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2^{3} - C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 - 2C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_2 -$$

$$C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 + S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} + S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_3 +$$

$$S\theta_2 C\alpha_2 a_3 C\theta_3 \ddot{\theta}_2 \dot{\theta}_3 - C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \ddot{\theta}_2 - S\theta_2 C\alpha_2 a_3 C\theta_3 \ddot{\theta}_2 \dot{\theta}_3 -$$

$$S\theta_2 C\alpha_2 a_3 S\theta_3 \dddot{\theta}_2 - C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2^{2} \dot{\theta}_3 + S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} -$$

$$S\theta_2 C\alpha_2 a_3 C\theta_3 \ddot{\theta}_2 \dot{\theta}_3 - S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_3 + S\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_2 \dot{\theta}_3^{2} -$$

$$C\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_3^{3} - 2C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_3 + S\theta_2 C\alpha_2 a_3 C\theta_3 \dot{\theta}_2 \ddot{\theta}_3 +$$

$$C\theta_2 C\alpha_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_3 - C\theta_2 C\alpha_2 a_3 C\theta_3 \dddot{\theta}_3 + S\theta_2 S\alpha_2 d_3 \dot{\theta}_2^{\,3} -$$
$$2C\theta_2 S\alpha_2 d_3 \dot{\theta}_2 \ddot{\theta}_2 - C\theta_2 S\alpha_2 d_3 \dot{\theta}_2 \ddot{\theta}_2 + S\theta_2 S\alpha_2 d_3 \dddot{\theta}_2 -$$
$$a_2 C\theta_2 \dot{\theta}_2^{\,3} - 2a_2 S\theta_2 \dot{\theta}_2 \ddot{\theta}_2 - a_2 S\theta_2 \dot{\theta}_2 \ddot{\theta}_2 + a_2 C\theta_2 \dddot{\theta}_2) + (n_x S\theta_1 S\alpha_1$$
$$- n_y C\theta_1 S\alpha_1 + n_z C\alpha_1)(-S\alpha_2 a_3 C\theta_3 \dot{\theta}_3^{\,3} - 2S\alpha_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_3$$
$$- S\alpha_2 a_3 S\theta_3 \dot{\theta}_3 \ddot{\theta}_3 + S\alpha_2 a_3 C\theta_3 \dddot{\theta}_3) = 0 \tag{5-4}$$

Similarly, the synthesis procedure for higher-order infinitesimally and mixed mode separated positions of the other types of dyads composed of revolute, prismatic, and helical joint can be obtained by using the proposed synthesis procedure of R-R crank.

5.3. Numerical Examples

In this section, numerical examples of synthesis of R-R crank for first-order and higher-order infinitesimally separated positions are presented in TABLE XIII and XIV.

TABLE XIII

NUMERICAL EXAMPLE OF SYNTHESIS OF R-R CRANK FOR FIRST-ORDER INFINITESIMALLY SEPARATED POSITIONS(PP-PP-PP-PP-P)

Given:1) The surface is given the same as in TABLE VI.

2) the five finitely separated positions are given as

P_1 = (30, 0, 0)

P_2 = (24.82, 14.33, 5)

P_3 = (12.5, 21.65, 8.66)

P_4 = (0, 20, 10)

P_5 = (-7.5, 12.99, 8.66)

3) the joint motion is given as :(θ:degree, θ:rad/sec)

$\theta_{21} = 0°$,	$\theta_{31} = 0°$	$\theta_{21} = 1$, $\theta_{31} = 1$
$\delta\theta_{21} = 30°$,	$\delta\theta_{31} = 30°$	$\theta_{22} = 1$, $\theta_{32} = 1$
$\delta\theta_{22} = 30°$,	$\delta\theta_{32} = 30°$	$\theta_{23} = 1$, $\theta_{33} = 1$
$\delta\theta_{23} = 30°$,	$\delta\theta_{33} = 30°$	$\theta_{24} = 1$, $\theta_{34} = 1$
$\delta\theta_{24} = 30°$,	$\delta\theta_{34} = 30°$	

Result :

$\theta_1 = 67.31°$,	$\alpha_1 = 23.56°$,	$\alpha_2 = 147.58°$
$a_1 = 18.3476$,	$a_2 = 113.349$,	$a_3 = -62.329$
$s_1 = -243.637$,	$s_2 = -13.346$,	$s_3 = -14.202$

TABLE XIV

NUMERICAL EXAMPLE OF SYNTHESIS OF R-R CRANK FOR HIGHER-ORDER MIXED MODE SEPARATED POSITIONS (P-PPP-PP-PPP)

Given: 1) The surface is given as the same as in TABLE VI.

2) the four finitely separated positions is given as :

$P_1 = (30,\ 0,\ 0)$

$P_2 = (24.82,\ 14.33,\ 5)$

$P_3 = (12.5,\ 21.65,\ 8.66)$

$P_4 = (0,\ 20,\ 10)$

3) the joint motion is given as :

(θ : degree, θ : rad/sec, and θ : red/sec^2)

$\theta_{21} = 0°$, $\theta_{31} = 0°$

$\delta\theta_{21} = 30°$, $\delta\theta_{31} = 30°$ $\theta_{22} = 1$, $\theta_{32} = 1$

$\delta\theta_{22} = 30°$, $\delta\theta_{32} = 30°$ $\theta_{23} = 1$, $\theta_{33} = 1$

$\delta\theta_{23} = 30°$, $\delta\theta_{33} = 30°$ $\theta_{24} = 1$, $\theta_{34} = 1$

$\theta_{22} = 0$, $\theta_{32} = 0$

$\theta_{24} = 0$, $\theta_{34} = 0$

Result :

$\theta_1 = 132.86°$, $\alpha_1 = 26.27°$, $\alpha_2 = 48.56°$

$a_1 = 21.376$, $a_2 = 14.59$, $a_3 = 12.29$

$s_1 = 23.37$, $s_2 = 23.46$, $s_3 = 17.42$

CHAPTER VI

SYNTHESIS OF TWO-PARAMETER-MOTION TWO-DEGREE-OF-FREEDOM SPATIAL MECHANISMS CARRYING A RIGID BODY WITH A TANGENT PLANE AS MOVING ELEMENT TO HAVE SIX, FIVE, AND FOUR COMPONENTS OF MOTION

In the previous chapters, the synthesis procedures of finitely, infinitesimally, and mixed mode separated positions are derived by using the tangent plane equations and homogeneous transformation matrix for open loop chains (dayds composed of mixed revolute, prismatic, and helical joints) with two degrees of freedom. It is desired that the synthesis procedures can be made use of in the synthesis of closed-loop spatial mechanisms. One of the singificant advantage of the proposed synthesis procedures is that it also can be applied for two degree-of-freedom spatial closed-loop mechanisms with two-parameter motion having six, five, and four components of motion.

In this chapter, we will synthesize closed-loop spatial mechanisms by using the proposed synthesis procedures developed in Chapter IV.

6.1. RRSS Spatial Four-Link Mechanism

The synthesis procedure of spatial closed-loop mechanism can be obtained by using the similar precedures presented in chapter IV. A RRSS spatial four-link mechanism is shown in figure 7. Since the tangent plane attached to the coupler link connecting R_2 and S_2 joints, we can synthesize RRSS mechanism by separating it into two open-loop chains: right hand side open-loop chain and left hand side open-loop chain. From figure 7, the right hand side open-loop chain is SS crank and the left hand side open-loop chain is RR crank. From RR crank, we establish four coordinate frames presented in chapter IV. From SS crank, we establish another four coordinate frames. Therefore, the linkage parameters involve in this mechanism are :

RR crank	SS crank
a_1, a_2, a_3	a_4, a_5, a_6
d_1, d_2, d_3	d_4, d_5, d_6
$\alpha_1, \alpha_2, \alpha_3$	$\alpha_4, \alpha_5, \alpha_6$
$\theta_1, \theta_2, \theta_3$	$\theta_4, \theta_5, \theta_6$

For a spherical joint, Denavit and Hartenberg presented a notation which can be considered as three revolute joints intersect in one point and perpendecular to each other. Hence, θ_5 will become three joint motion parameters $\theta_{51}, \theta_{52}, \theta_{53}$. Also, θ_6 becomes $\theta_{61}, \theta_{62}, \theta_{63}$. Since RRSS mechanism is a two degrees of freedom mechanism, we can

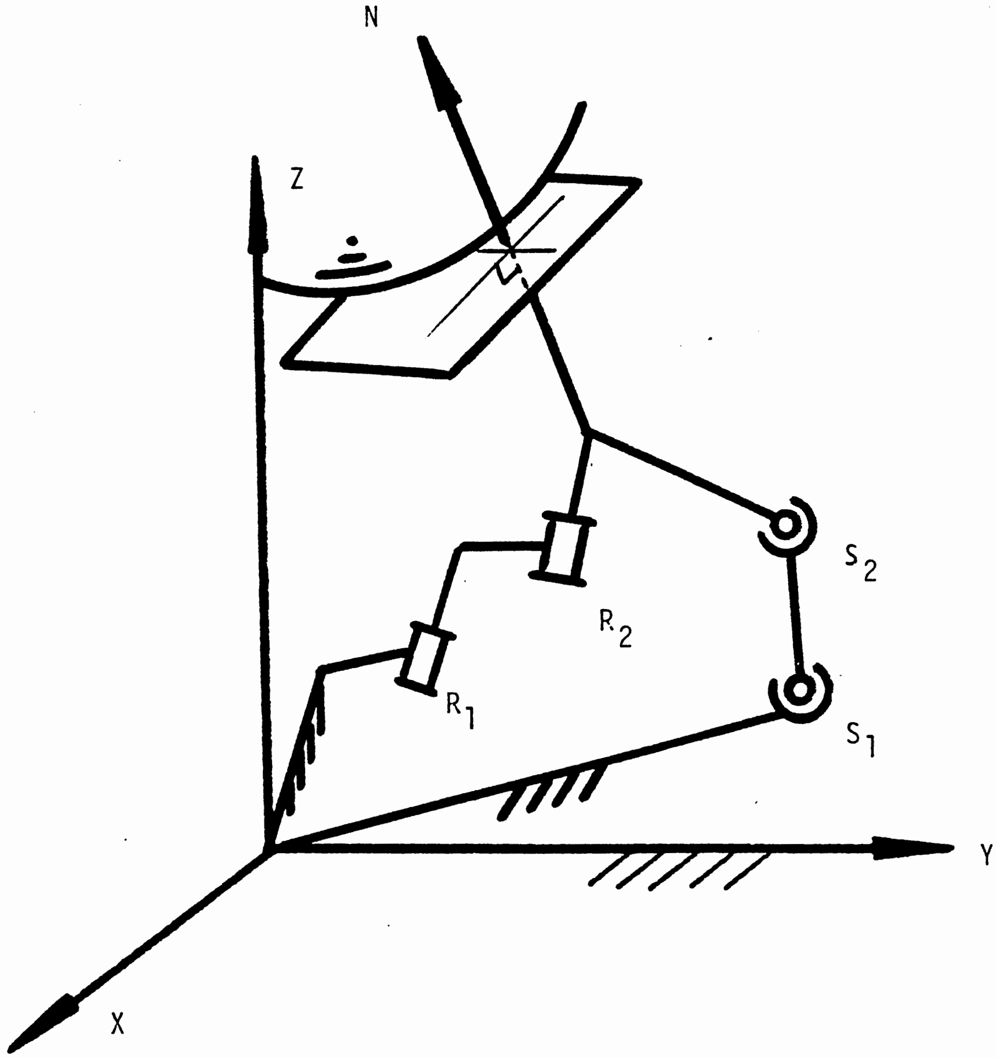


Figure 7. RRSS Spatial Mechanism

assume joints R_1 and R_2 as two driving input. Hence, θ_2 and θ_3 are given as the input motion parameters. Also, a_3 is equal to a_6. α_6 can be assumed as 90 from figure 7. Hence, there are 14 unknowns in right hand side and 10 unknowns in left hand side.

In order to derive the synthesis equations of RSSR mechanism, we know that we can obtain two tangent plane equations from R-R crank and S-S crank.

$$(X_R-x_0, Y_R-y_0, Z_R-z_0)\ N = 0 \qquad (6\text{-}1)$$

$$(X_L-x_0, Y_L-y_0, Z_L-z_0)\ N = 0 \qquad (6\text{-}2)$$

where $\{x_0,\ y_0,\ z_0\}$ = a point tangential to the surface.

N = unit normal vector.

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \\ 1 \end{bmatrix} = [A_1][A_2][A_3] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (6\text{-}3)$$

= transformation of the right hand side open-loop mechanism

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \\ 1 \end{bmatrix} = [A_4][A_5][A_6] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (6\text{-}4)$$

= transformation of the left hand side open-loop mechanism

Since we establish the connecting point as the origin of coordinate frame on the tangent plane and the X-axis is normal to the tangent plane, then the point on the tangent plane can be expressed as $P = \{ 0, P_y, P_z \}$. The constraint equation can be obtained by equating both side and by using transformation matrix.

$$[A_1][A_2][A_3]\ P_4 = [A_4][A_5][A_6]\ P_4 \qquad (6\text{-}5)$$

Eq(6-3) can be rewritten as three component equations

$$[A_{equ\ right}]_1\ P_4 = [A_{equ\ left}]_1\ P_4 \qquad (6\text{-}6)$$

$$[A_{equ\ right}]_2\ P_4 = [A_{equ\ left}]_2\ P_4 \qquad (6\text{-}7)$$

$$[A_{equ\ right}]_3\ P_4 = [A_{equ\ left}]_3\ P_4 \qquad (6\text{-}8)$$

where $[A_{equ\ right}]_i$ = the i_{th} row of $[A_{equ}]$ on the right hand side.

$P_4 = \{ 0, P_{4y}, P_{4z} \}$

For each separated position, we can obtain two umknowns from P_{4y} and P_{4z}, and five synthesis equations. Hence

Number of synthesis equations = 5N

Number of unknown = 10 + 14 + 2N

Maximum number of positions = 8

Number of free choice parameter = 0

Thus, the maximum number of positions of RRSS spatial mechanism is eight with no free choise of parameter. The result derived here is consistent with eight synthesis finitely separated positions for path-generation of RRSS mechanism presented by Suh[59].

6.2. RHCRC Spatial Five-Link Mechanism Having Four Components Of Motion

From the synthesis procedure of RRSS mechanism, we can derive the similar procedure for RHCRC spatial five-link mechanism having four components of motion. A RHCRC spatial five-link mechanism having four components of motion is shown in figure 8. By separating RHCRC mechanism into two open loop chains, we obtain RH link on the left hand side and RRC link on the right hand side. the total linkage parameters of RHCRC are

RH link	CRC link
a_1, a_2, a_3	a_4, a_5, a_6, a_7
d_1, d_2, d_3	d_4, d_5, d_6, d_7
$\theta_1, \theta_2, \theta_3$	$\theta_4, \theta_5, \theta_6, \theta_7$
$\alpha_1, \alpha_2, \alpha_3$	$\alpha_4, \alpha_5, \alpha_6, \alpha_7$

The procedures of obtaining the synthesis equation of RHCRC mechanism is similar to RRSS mechanism.

From Eq(6-1) and Eq(6-2), we obtain two synthesis

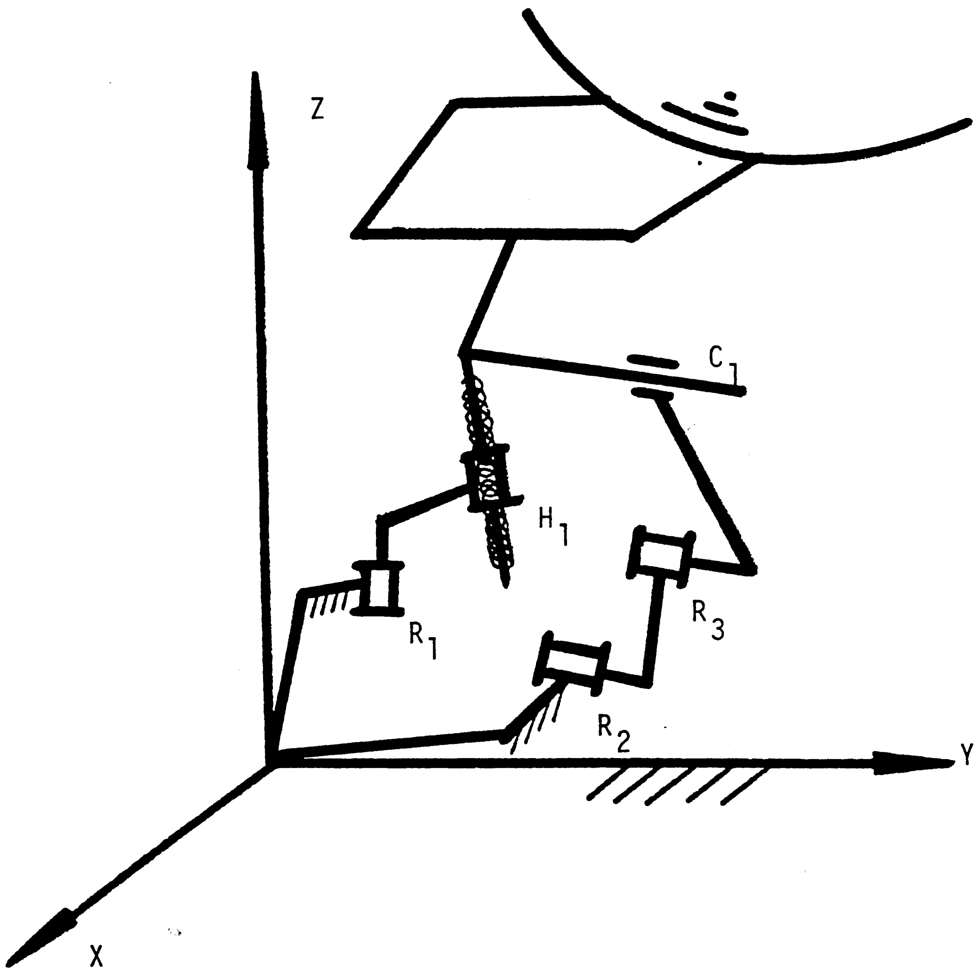


Figure 8. RHCRR Spatial Mechanism Having Four Components of Motion

equations.

$$(X_R - x_O, Y_R - y_O, Z_R - z_O)\ N = 0$$

$$(X_L - x_O, Y_L - y_O, Z_L - z_O)\ N = 0$$

where $\{x_O,\ y_O,\ z_O\}$ = a point tangential to the surface.

N = unit normal vector.

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \\ 1 \end{bmatrix} = [A_1][A_2][A_3][A_4] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (6\text{-}9)$$

= transformation of the right hand side open-loop mechanism

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \\ 1 \end{bmatrix} = [A_5][A_6][A_7] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (6\text{-}10)$$

= transformation of the left hand side open-loop mechanism

The constraint equation can be obtained by equating both side and by using transformation matrix.

$$[A_1][A_2][A_3][A_4]\ P_4 = [A_5][A_6][A_7]\ P_4 \qquad (6\text{-}11)$$

Eq(6-9) can be rewritten as three component equations the same as Eq(6-6)-Eq(6-8). Hence

Number of synthesis equations	= 5N
Number of unknown	= 10 + 15 + 2N
Maximum number of positions	= 8
Number of free choice parameter	= 1

6.3. RCCRR Spatial Five-Link Mechanism Having Five Components Of Motion

From the synthesis procedure of RRSS mechanism, we can derive the similar procedure for RCCRR spatial five-link mechanism having five components of motion. A RCCRR spatial five-link mechanism having four components of motion is shown in figure 9. By separating RCCRR mechanism into two parts, we obtain RC link and CRR link. Hence,

Number of synthesis equations	= 5N
Number of unknown	= 10 + 15 + 2N
Maximum number of positions	= 8
Number of free choice parameter	= 1

6.4. RCCCR Spatial Five-Link Mechanism Having Six Components Of Motion

From the synthesis procedure of RRSS mechanism, we can derive the similar procedure for RCCCR spatial five-link

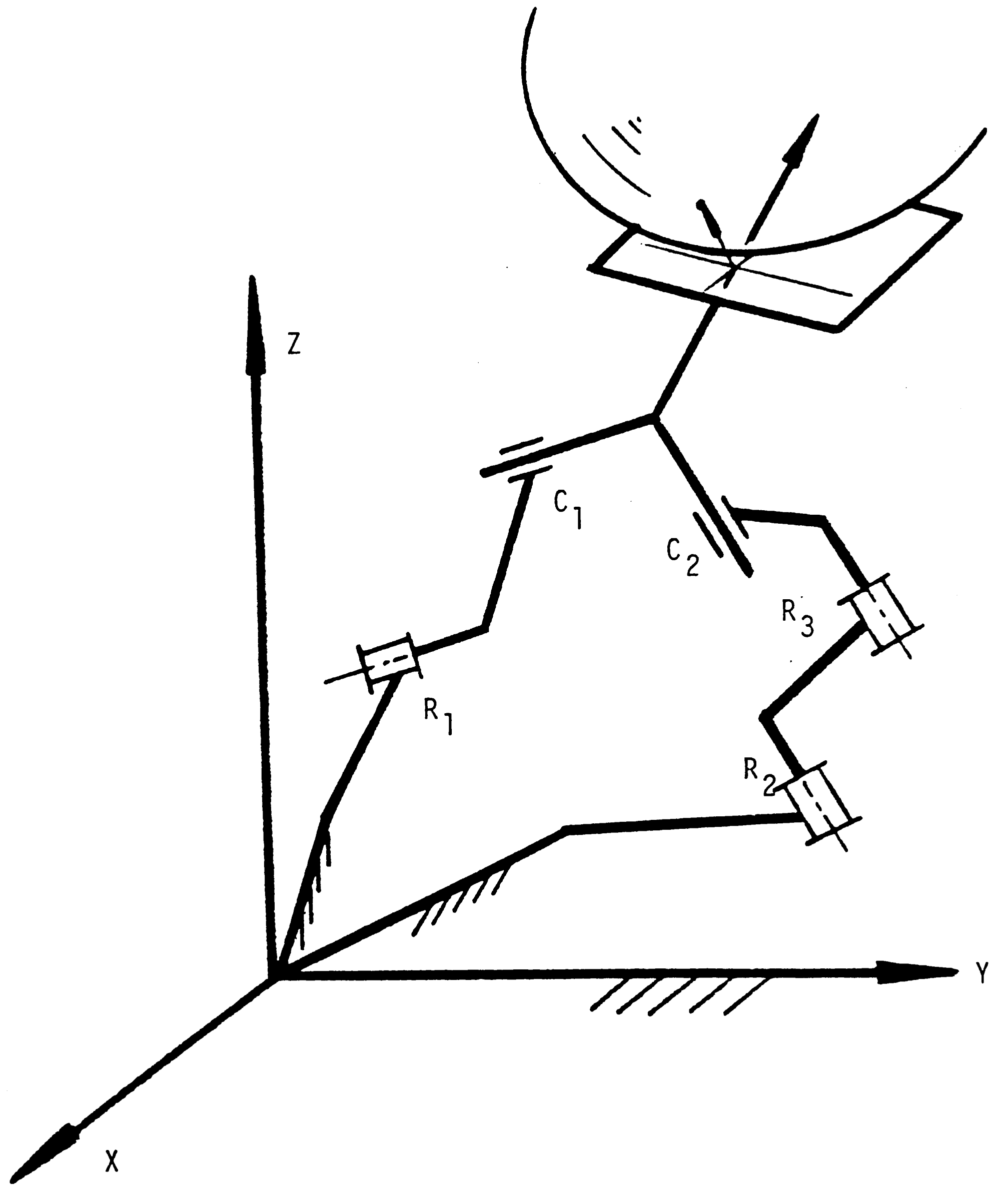


Figure 9. RCCRR Spatial Mechanism Having
Five Components of Motion

mechanism having six components of motion. A RCCCR spatial five-link mechanism having four components of motion is shown in figure 10. By separating RCCCR mechanism into two parts, we obtain RC link and CCR link. Hence,

Number of synthesis equations = 5N

Number of unknown = 10 + 15 + 2N

Maximum number of positions = 8

Number of free choice parameter = 1

6.5. Numerical Example

In this section, numerical example of synthesis of RCCCR mechanism for three finitely separated positions is presented.

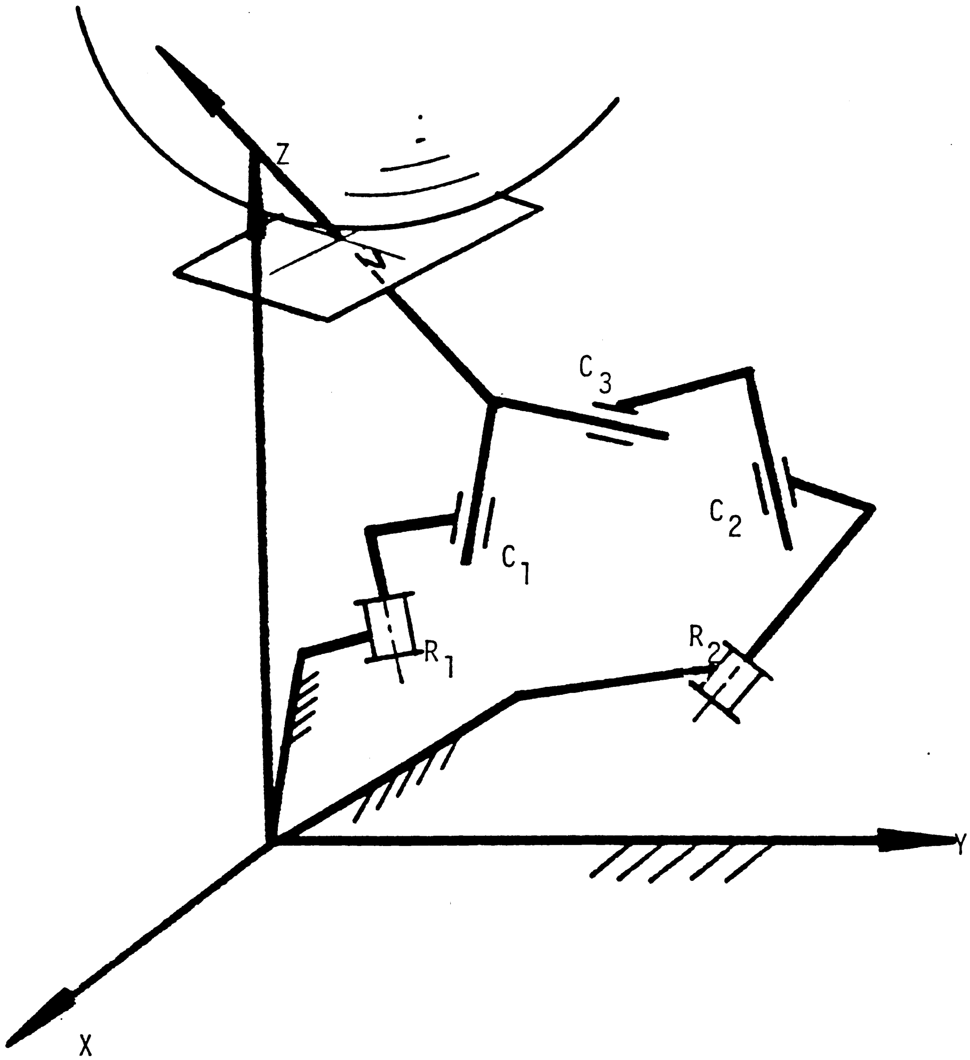


Figure 10. RCCCR Spatial Mechanism Having Six Components of Motion

TABLE XV

NUMERICAL EXAMPLE OF SYNTHESIS OF RCCCR MECHANISM FOR EIGHT FINITELY SEPARATED POSITIONS

Given: 1) The surface is given as the same in TABLE VI.

2) the three finitely separated positions is given as :

$P_1 = (30, 0, 0)$ $P_2 = (24.82, 14.33, 5)$

$P_3 = (12.5, 21.65, 8.66)$ $P_4 = (28.65, 7.68, 2.59)$

$P_5 = (27.62, 10.05, 3.42)$ $P_6 = (26.34, 12.28, 4.22)$

$P_7 = (24.82, 14.33, 5)$ $P_8 = (21.19, 17.78, 6.43)$

3) the joint motion is given as :

$\theta_{21} = 0°, \theta_{31} = 0°$

$\delta\theta_{21} = 30°, \delta\theta_{31} = 30°$ $\delta\theta_{22} = 30°, \delta\theta_{32} = 30°$

$\delta\theta_{23} = 30°, \delta\theta_{33} = 30°$ $\delta\theta_{24} = 30°, \delta\theta_{34} = 30°$

$\delta\theta_{25} = 30°, \delta\theta_{35} = 30°$ $\delta\theta_{26} = 30°, \delta\theta_{36} = 30°$

$\delta\theta_{27} = 30°, \delta\theta_{37} = 30°$

4) the choice of linkage parameters

$\theta_1 = 0°$

Result :

$a_1 = 8.839$, $a_2 = 3.539$, $a_3 = 112.409$, $a_4 = 126.649$,

$a_5 = 27.249$, $a_6 = 12.509$, $d_1 = 3.347$, $d_2 = 45.446$,

$d_3 = 18.569$, $d_4 = 53.639$, $d_5 = -8.64$, $d_6 = 23.62$,

$d_7 = -2.24$, $\alpha_1 = 33.346°$, $\alpha_2 = 15.722°$, $\alpha_4 = 48.379°$,

$\alpha_5 = 123.559°$, $\alpha_6 = 172.329°$, $\alpha_7 = -32.26°$,

$\theta_4 = 43.647°$, $\theta_5 = 75.226°$, $\theta_6 = 217.36°$, $\theta_7 = -25.45°$

CHAPTER VII

SUMMARY AND CONCLUSIONS

Based on the traditional approach to path-generation problems, the coupler-curve is viewed as a set of discribe points. From the geometric points of view, a curve or a surface may be generated in general by a point, a line, or a plane embedded in a moving rigid body. It is known that a point and a plane are dual concept in space geometry as well as a point and a line are considered as dual elements in planar projective geometry. This leads to a new concept of a surface being considered as plane-envelop which is a set of its tangent plane, i.e., the surface is considered to be defined by a set of tangent planes.

For the dyads with any combination of R, H, and P joints having two-parameter motion, the synthesis procedures are derived for nine finitely separated positions. Also, the synthesis procedures of first-order and higher-order infinitesimally and mixed mode separated positions are presented in chapter IV. For two degree-of-freedom closed-loop spatial mechanism having four, five, and six components of motion, the synthesis procedure is derived by separating it into two open-loop chains. We located on the coupler link the locus of points or the family of planes which generate surfaces with the desired local properties.

This thesis presents the extension study on the spatial mechanism having two-parameter motion. It extends the synthesis procedure from the tranditional one-parameter point-path motion and rigid-body guidance to two-parameter tengent-plane envelop. The synthesis and analysis procedure is based on the homogeneous transformation matrix method. The proposed theoretical developments of the two- parameter motion study followed their applications demonstrating synthesis of two degrees of freedom mechanisms carrying a rigid body with a plane as moving element and having six, five, and four components of motion. The proposed research on two-parameter motion of a plane in space motion contribute significantly in advancing the fundamentals of kinematic synthesis of rigid body motion having two degrees of freedom.

One of the important applications of present study is the robot hand with multiple fingers. There are two or three degrees of freedom for each finger. It is desirable to catch an arbitrary object by using the robot hand. the motion of each finger to touch the surface of object can be related to a two to three degrees of freedom mechanism carries a tangent plane as a moving element as shown in figure 11. Therefore, the present study provide the insight of kinematics for synthesis problems.

For the future study, the present study is expected to provide a significant contribution for the tangent plane

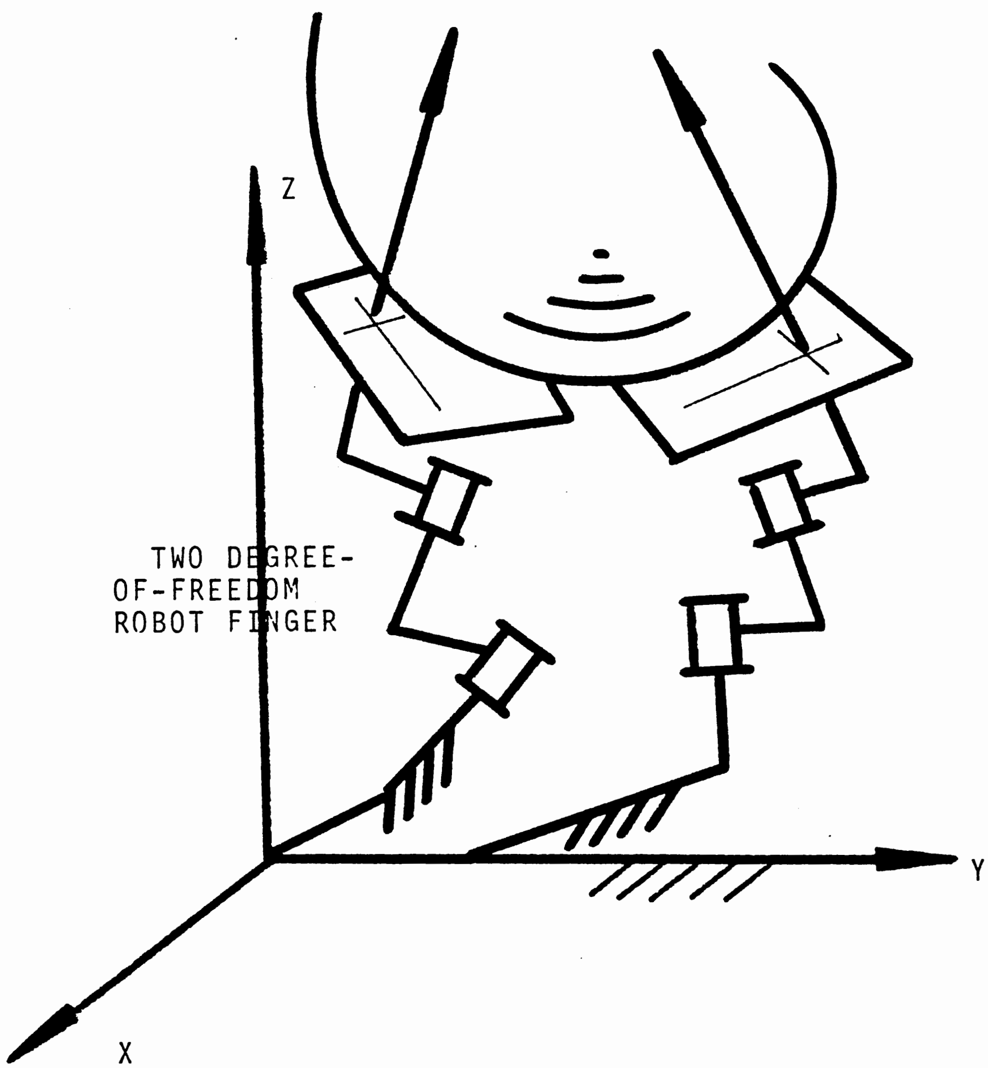


Figure 11. Two Tangent Plane Attached to Two
Two-Degree-of-Freedom Robot Fingers

envelope generation with more than two-parameter motion.

A general computer program is developed to carry out the synthesis and analysis procedure of open-loop and closed-loop mechanisms for finitely, infinitesimally, and mixed mode separated positions.

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APPENDIX A

COMPUTER PROGRAMS FOR FINITELY, INFINITESIMALLY, AND MIXED MODE SEPARATED POSITIONS

```
C       **************************************************
C       *
C       *  SYNTHESIS OF SIX FINITELY SEPARATED POSITIONS OF
C       *                       R-R CRANK
C       *
C       **************************************************

C       **************************************************
C       *
C       *  THIS PROGRAM IS SOLVE FOR THE LINEAR SOLUTION OF
C       *  LINKAGE PARAMETERS a1, a2, a3, d1, d2, d3
C       *
C       *  Synthesis of six positions
C       *
C       **************************************************

        dimension a(40,41),x(40)
c
        real nx(6),ny(6),nz(6),theta2(6),theta3(6),px(6),py(6),
       - pz(6)
        pi=3.14159/180.
c
        theta1=45.*pi
        alpha1=30.*pi
        alpha2=30.*pi
        theta2(1)=30.*pi
        theta3(1)=30.*pi
        theta2(2)=40.*pi
        theta3(2)=40.*pi
        theta2(3)=50.pi
        theta3(3)=50.*pi
        theta2(4)=60.*pi
        theta3(4)=60.*pi
        theta2(5)=70.*pi
        theta3(5)=70.*pi
        theta2(6)=80.*pi
        theta3(6)=80.*pi
c
        nx(1)=300.
        nx(2)=297.3435364
        nx(3)=289.4804993
        nx(4)=276.7246704
        nx(5)=259.5814209
        nx(6)=238.7224121
        ny(1)=0.
        ny(2)=24.0141697
        ny(3)=51.0431862
        ny(4)=74.1481018
        ny(5)=94.4798431
        ny(6)=111.3179932
```

```
      nz(1)=-173.2050018
      nz(2)=-196.9615021
      nz(3)=-196.9615936
      nz(4)=-173.2052612
      nz(5)=-128.5578156
      nz(6)=-68.4044037
c
      px(1)=8.6602507
      px(2)=9.8480759
      px(3)=9.8480797
      px(4)=8.6602621
      px(5)=6.4278908
      px(6)=3.4202201
      py(1)=4.9999933
      py(2)=8.2635078
      py(3)=11.7364693
      py(4)=14.9999857
      py(5)=17.6604328
      py(6)=19.3969193
      pz(1)=17.3205128
      pz(2)=15.3208952
      pz(3)=12.8557625
      pz(4)=10.0000143
      pz(5)=6.8404207
      pz(6)=3.4729831
      data n,eps/6,1.e-5/
      m=n+1
c      print*,'input the coefficienrt '
c      do 1 i=1,n
c      do 1 j=1,n+1
c     1   read*,a(i,j)
      do 100 i=1,6
      a(i,1)=(nx(i)*cos(theta1)+ny(i)*sin(theta1))
     -       *(cos(theta2(i))*cos(theta3(i))
     -         -sin(theta2(i))*cos(alpha2)*sin(theta3(i)))
     -       +(-sin(theta1)*cos(alpha1)*nx(i)+cos(theta1)*
     -        cos(alpha1)*ny(i)+sin(alpha1)*nz(i))*
     -        (sin(theta2(i))*cos(theta3(i))+cos(theta2(i))*
     -         cos(alpha2)*sin(theta3(i)))+(sin(theta1)*
     -         sin(alpha1)*nx(i)-cos(theta1)*sin(alpha1)
     -          *ny(i)+cos(alpha1)*nz(i))*(sin(alpha2)*
     -        sin(theta3(i)))
      a(i,2)=(nx(i)*cos(theta1)+ny(i)*sin(theta1))*
     -       cos(theta2(i))+(-sin(theta1)*cos(alpha1)*nx(i)
     -        +cos(theta1)*cos(alpha1)
     -         *ny(i)+sin(alpha1)*nz(i))*sin(theta2(i))
      a(i,3)=nx(i)*cos(theta1)+ny(i)*sin(theta1)
      a(i,4)=(nx(i)*cos(theta1)+ny(i)*sin(theta1))
     -       *(sin(theta2(i))*sin(alpha2))
     -       +(-sin(theta1)*cos(alpha1)*nx(i)+cos(theta1)*
     -       cos(alpha1)*ny(i)+sin(alpha1)*nz(i))*(-cos
     -       (theta2(i))*sin(alpha2)+(sin(theta1)*sin
     -       (alpha1)*nx(i)-cos(theta1)*sin(alpha1)
     -          *ny(i)+cos(alpha1)*nz(i))*cos(alpha2)
```

```
      a(i,5)=sin(theta1)*sin(alpha1)*nx(i)-cos(theta1)*
     -       sin(alpha1)*ny(i)+cos(alpha1)*nz(i)
      a(i,6)=nz(i)
100   a(i,7)=px(i)*nx(i)+py(i)*ny(i)+pz(i)*nz(i)
      do 111 ii=1,6
      print*,'nx=',nx(ii),'ny=',ny(ii),'nz=',nz(ii)
      print*,'px=',px(ii),'py=',py(ii),'pz=',pz(ii)
      theta2(ii)=theta2(ii)/pi
      theta3(ii)=theta3(ii)/pi
111   print*,'theta2=',theta2(ii),'theta3=',theta3(ii)
      kk=0
      jj=0
      do 10 i=1,n
      jj=kk+1
      ll=jj
      kk=kk+1
 20   if(abs(a(jj,kk))-eps)21,21,22
 21   jj=jj+1
      go to 20
 22   if(ll-jj)23,24,23
 23   do 25 mm=1,m
      atemp=a(ll,mm)
      a(ll,mm)=a(jj,mm)
 25   a(jj,mm)=atemp
 24   div=a(i,i)
      do 11 j=1,m
 11   a(i,j)=a(i,j)/div
      k=i+1
      if(k-m)12,13,13
 12   do 10 l=k,n
      amult=a(l,i)
      do 10 j=1,m
 10   a(l,j)=a(l,j)-a(i,j)*amult
 13   x(n)=a(n,m)
      l=n
      do 30 j=2,n
      sum=0.
      i=m+1-j
      do 31 k=1,n
 31   sum=sum+a(i-1,k)*x(k)
      l=l-1
 30   x(l)=a(i-1,m)-sum
      do 40 ii=1,n
 40     print*,'root(',ii,') = ',x(ii)
      stop
      end
```

```
C
C ******************************************************
C *
C *          SYNTHESIS OF NINE SYNTHESIS POSITIONS
C *
C ******************************************************
C
      real x(9),f(9),delta,xtol,ftol
      integer n,maxit,i
      external fcn
      data i,n,maxit,delta/0,9,800,0.001/
      data xtol,ftol/1.e-4,1.e-4/
c
      x(1)=6.26973343
      x(2)=22.55064583
      x(3)=-6.54820347
      x(4)=1.39241803
      x(5)=-45.4885788
      x(6)=50.77313232
      pi=3.14159/180.
      x(7)=0.48279029
      x(8)=0.52359837
      print*,'input x(9) ='
      read*,x(9)
      x(9)=x(9)*pi
      call nlsyst(fcn,n,maxit,x,f,delta,xtol,ftol,i)
c
      do 1 i=1,9
    1  print*,'x(',i,') = ',x(i),'  f(',i,') = ',f(i)
c
      stop
      end
c
c
      subroutine fcn(x,f)
c
c
      real x(9),f(9),nx1,nx2,nx3,nx4,nx5,nx6,nx7,ny7,nz7,
     -ny1,ny2,ny3,ny4,ny5,ny6,nz1,nz2,nz3,nz4,nx8,ny8,nz8,
     -nz5,nz6,nx9,ny9,nz9
      pi=3.14159/180.
c      theta1=45.*pi
c      alpha1=90.*pi
c      alpha2=90.*pi
      theta21=0.*pi
      theta31=0.*pi
      theta22=5.*pi
      theta32=5.*pi
```

```
theta23=10.*pi
theta33=10.*pi
theta24=15.*pi
theta34=15.*pi
theta25=20.*pi
theta35=20.*pi
theta26=25.*pi
theta36=25.*pi
theta27=30.*pi
theta37=30.*pi
theta28=35.*pi
theta38=35.*pi
theta29=40.*pi
theta39=40.*pi
nx2=297.3435364
nx3=289.4804993
nx4=276.7246704
nx5=259.5814209
nx6=238.7224121
nx7=214.9520264
nx8=189.1680756
ny8=132.4567719
nz8=161.6998138
nx1=300.
ny1=0.
nz1=0.
nx9=162.3182983
ny9=136.2010956
nz9=177.7979363
ny2=26.0141697
ny3=51.0431862
ny4=74.1481018
ny5=94.4798431
ny6=111.3179932
ny7=124.1024857
nz2=26.1135387
nz3=51.8306084
nz4=76.7637558
nz5=100.5433426
nz6=122.825798
nz7=143.3011627
p2x=29.8479328
p3x=29.3946190
p4x=28.6486454
p5x=27.6240768
p6x=26.3400993
p7x=24.8205147
p2y=2.6113539
p3y=5.1830606
p4y=7.6763749
p5y=10.0543346
p6y=12.2825794
p7y=14.3301182
p2z=0.8715568
```

```
      p3z=1.7364806
      p4z=2.5881884
      p5z=3.4201992
      p6z=4.2261796
      p7z=4.9999967
      p8x=23.0931511
      p8y=16.1699829
      p8z=5.7357602
      p1x=30.
      p1y=0.
      p1z=0.
      p9x=21.1891422
      p9y=17.7797832
      p9z=6.4278722
c
      f(1)=(nx1*cos(x(9))+ny1*sin(x(9)))*(cos(theta21)*x(3)*
     -cos(theta31)-sin(theta21)*cos(x(7))*x(3)*sin(theta31)+
     -sin(theta21)*sin(x(7))*x(6)+x(2)*cos(theta21))+(-sin
     -(x(9))*cos(x(8))*nx1+cos(x(9))*cos(x(8))*ny1+sin
     -(x(8))*nz1)*(sin(theta21)*x(3)*cos(theta31)+cos
     -(theta21)*cos(x(7))*x(3)*sin(theta31)-cos(theta21)*
     -sin(x(7))*x(6)+x(2)*sin(theta21))+(sin(x(9))*sin(x(8))
     -*nx1-cos(x(9))*sin(x(8))*ny1+cos(x(8))*nz1)*(sin(x(7))
     -*x(3)*sin(theta31)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx1+x(1)*sin(x(9))*ny1+x(4)*nz1-(p1x*nx1+p1y*ny1+p1z*
     -nz1)
c
c
      f(2)=(nx2*cos(x(9))+ny2*sin(x(9)))*(cos(theta22)*x(3)*
     -cos(theta32)-sin(theta22)*cos(x(7))*x(3)*sin(theta32)+
     -sin(theta22)*sin(x(7))*x(6)+x(2)*cos(theta22))+(-sin
     -(x(9))*cos(x(8))*nx2+cos(x(9))*cos(x(8))*ny2+sin
     -(x(8))*nz2)*(sin(theta22)*x(3)*cos(theta32)+cos
     -(theta22)*cos(x(7))*x(3)*sin(theta32)-cos(theta22)*
     -sin(x(7))*x(6)+x(2)*sin(theta22))+(sin(x(9))*sin(x(8))
     -*nx2-cos(x(9))*sin(x(8))*ny2+cos(x(8))*nz2)*(sin(x(7))
     -*x(3)*sin(theta32)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx2+x(1)*sin(x(9))*ny2+x(4)*nz2-(p2x*nx2+p2y*ny2+p2z*
     -nz2)
c
c
      f(3)=(nx3*cos(x(9))+ny3*sin(x(9)))*(cos(theta23)*x(3)*
     -cos(theta33)-sin(theta23)*cos(x(7))*x(3)*sin(theta33)+
     -sin(theta23)*sin(x(7))*x(6)+x(2)*cos(theta23))+(-sin
     -(x(9))*cos(x(8))*nx3+cos(x(9))*cos(x(8))*ny3+sin
     -(x(8))*nz3)*(sin(theta23)*x(3)*cos(theta33)+cos
     -(theta23)*cos(x(7))*x(3)*sin(theta33)-cos(theta23)*
     -sin(x(7))*x(6)+x(2)*sin(theta23))+(sin(x(9))*sin(x(8))
     -*nx3-cos(x(9))*sin(x(8))*ny3+cos(x(8))*nz3)*(sin(x(7))
     -*x(3)*sin(theta33)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx3+x(1)*sin(x(9))*ny3+x(4)*nz3-(p3x*nx3+p3y*ny3+p3z*
     -nz3)
c
c
```

```
      f(4)=(nx4*cos(x(9))+ny4*sin(x(9)))*(cos(theta24)*x(3)*
     -cos(theta34)-sin(theta24)*cos(x(7))*x(3)*sin(theta34)+
     -sin(theta24)*sin(x(7))*x(6)+x(2)*cos(theta24))+(-sin
     -(x(9))*cos(x(8))*nx4+cos(x(9))*cos(x(8))*ny4+sin
     -(x(8))*nz4)*(sin(theta24)*x(3)*cos(theta34)+cos
     -(theta24)*cos(x(7))*x(3)*sin(theta34)-cos(theta24)*
     -sin(x(7))*x(6)+x(2)*sin(theta24))+(sin(x(9))*sin(x(8))
     -*nx4-cos(x(9))*sin(x(8))*ny4+cos(x(8))*nz4)*(sin(x(7))
     -*x(3)*sin(theta34)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx4+x(1)*sin(x(9))*ny4+x(4)*nz4-(p4x*nx4+p4y*ny4+p4z*
     -nz4)
c
c
      f(5)=(nx5*cos(x(9))+ny5*sin(x(9)))*(cos(theta25)*x(3)*
     -cos(theta35)-sin(theta25)*cos(x(7))*x(3)*sin(theta35)+
     -sin(theta25)*sin(x(7))*x(6)+x(2)*cos(theta25))+(-sin
     -(x(9))*cos(x(8))*nx5+cos(x(9))*cos(x(8))*ny5+sin
     -(x(8))*nz5)*(sin(theta25)*x(3)*cos(theta35)+cos
     -(theta25)*cos(x(7))*x(3)*sin(theta35)-cos(theta25)*
     -sin(x(7))*x(6)+x(2)*sin(theta25))+(sin(x(9))*sin(x(8))
     -*nx5-cos(x(9))*sin(x(8))*ny5+cos(x(8))*nz5)*(sin(x(7))
     -*X(3)*sin(theta35)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx5+x(1)*sin(x(9))*ny5+x(4)*nz5-(p5x*nx5+p5y*ny5+p5z*
     -nz5)
c
c
      f(6)=(nx6*cos(x(9))+ny6*sin(x(9)))*(cos(theta26)*x(3)*
     -cos(theta36)-sin(theta26)*cos(x(7))*x(3)*sin(theta36)+
     -sin(theta26)*sin(x(7))*x(6)+x(2)*cos(theta26))+(-sin
     -(x(9))*cos(x(8))*nx6+cos(x(9))*cos(x(8))*ny6+sin
     -(x(8))*nz6)*(sin(theta26)*x(3)*cos(theta36)+cos(theta26)
     -*cos(x(7))*x(3)*sin(theta36)-cos(theta26)*sin(x(7))*
     -x(6)+x(2)*sin(theta26))+(sin(x(9))*sin(x(8))*nx6-cos
     -(x(9))*sin(x(8))*ny6+cos(x(8))*nz6)*(sin(x(7))*
     -x(3)*sin(theta36)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx6+x(1)*sin(x(9))*ny6+x(4)*nz6-(p6x*nx6+p6y*ny6+p6z*
     -nz6)
c
c
      f(7)=(nx7*cos(x(9))+ny7*sin(x(9)))*(cos(theta27)*x(3)*
     -cos(theta37)-sin(theta27)*cos(x(7))*x(3)*sin(theta37)+
     -sin(theta27)*sin(x(7))*x(6)+x(2)*cos(theta27))+(-sin
     -(x(9))*cos(x(8))*nx7+cos(x(9))*cos(x(8))*ny7+sin
     -(x(8))*nz7)*(sin(theta27)*x(3)*cos(theta37)+cos(theta27)
     -*cos(x(7))*x(3)*sin(theta37)-cos(theta27)*sin(x(7))*
     -x(6)+x(2)*sin(theta27))+(sin(x(9))*sin(x(8))*nx7-cos
     -(x(9))*sin(x(8))*ny7+cos(x(8))*nz7)*(sin(x(7))*
     -x(3)*sin(theta37)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx7+x(1)*sin(x(9))*ny7+x(4)*nz7-(p7x*nx7+p7y*ny7+p7z*
     -nz7)
c
c
      f(8)=(nx8*cos(x(9))+ny8*sin(x(9)))*(cos(theta28)*x(3)*
     -cos(theta38)-sin(theta28)*cos(x(7))*x(3)*sin(theta38)+
```

```
     -sin(theta28)*sin(x(7))*x(6)+x(2)*cos(theta28))+(-sin
     -(x(9))*cos(x(8))*nx8+cos(x(9))*cos(x(8))*ny8+sin
     -(x(8))*nz8)*(sin(theta28)*x(3)*cos(theta38)+cos(theta28)
     -*cos(x(7))*x(3)*sin(theta38)-cos(theta28)*sin(x(7))*
     -x(6)+x(2)*sin(theta28))+(sin(x(9))*sin(x(8))*nx8-cos
     -(x(9))*sin(x(8))*ny8+cos(x(8))*nz8)*(sin(x(7))*
     -x(3)*sin(theta38)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx8+x(1)*sin(x(9))*ny8+x(4)*nz8-(p8x*nx8+p8y*ny8+p8z*
     -nz8)
c
c
      f(9)=(nx9*cos(x(9))+ny9*sin(x(9)))*(cos(theta29)*x(3)*
     -cos(theta39)-sin(theta29)*cos(x(7))*x(3)*sin(theta39)+
     -sin(theta29)*sin(x(7))*x(6)+x(2)*cos(theta29))+(-sin
     -(x(9))*cos(x(8))*nx9+cos(x(9))*cos(x(8))*ny9+sin
     -(x(8))*nz9)*(sin(theta29)*x(3)*cos(theta39)+cos(theta29)
     -*cos(x(7))*x(3)*sin(theta39)-cos(theta29)*sin(x(7))*
     -x(6)+x(2)*sin(theta29))+(sin(x(9))*sin(x(8))*nx9-cos
     -(x(9))*sin(x(8))*ny9+cos(x(8))*nz9)*(sin(x(7))*
     -x(3)*sin(theta39)+cos(x(7))*x(6)+x(5))+x(1)*cos(x(9))
     -*nx9+x(1)*sin(x(9))*ny9+x(4)*nz9-(p9x*nx9+p9y*ny9+p9z*
     -nz9)
C
      return
      end
C
C
C
      SUBROUTINE nlsyst(fcn,n,maxit,x,f,delta,xtol,ftol,i)
C
C ********************************************************
C *
C * SUBROUTINE NLSYST :
C *      SOLVE FOR NINE NONLINEAR EQUATIONS
C *
**********************************************************
      real x(n),f(n),delta,xtol,ftol
      integer n,maxit,i
      real a(10,11),b(7),xsave(10),fsave(10)
      integer np,it,ivbl,itest,ifcn,irow,jcol
c
c
c -------------------------------------------------------
c
c  check validity of value of n
c
      if (n.lt.2 .or. n.gt.10) then
         i=-3
         print*,'n=',n
         print*
         return
      endif
c
c ------------------------------------------------------
```

```
c
c  begin iterations - save x values, then get f values
c
      np = n + 1
      do 100 it = 1,maxit
        do 10 ivbl = 1,n
          xsave(ivbl) = x(ivbl)
   10 continue
      call fcn(x,f)
c
c ------------------------------------------------------
c
c test f values and save them
c
      itest = 0
      do 20 ifcn = 1,n
        if (abs(f(ifcn)) .gt. ftol) itest = itest + 1
        fsave(ifcn) = f(ifcn)
   20 continue
      if ( i.eq.0) then
        print*,'it=',it,'  x= ',x
        print*
        print*,'          f=',f
        print*
      endif
c
c -------------------------------------------------------
c
c  see if ftol is met. if not, continue. if so, set i=2
c  and return.
c
      if (itest.eq.0) then
        i=2
        return
      endif
c
c ---------------------------------------------------------
c
c this double loop computes the partial derivatives of each
C function for each varivable and stores them in a
C coefficient array.
c
      do 50 jcol = 1,n
        x(jcol) = xsave(jcol) + delta
        call fcn(x,f)
        do 40 irow = 1,n
           a(irow,jcol) = (f(irow) - fsave(irow))/delta
   40 continue
c
c
c  reset x values for nest column of partials
c
      x(jcol)=xsave(jcol)
   50 continue
```

```
c
c -----------------------------------------------------------
c
c now we put negative of f values as right hand sides
C and call elim
c
      do 60 irow = 1,n
         b(irow)=-fsave(irow)
   60 continue
      call elim(a,b,n,1.e-6)
c
c -----------------------------------------------------------
c
c be sure that the coefficient matrix is not too ill-
C conditioned
c
      do 70 irow = 1,n
        if(abs(a(irow,irow)).le.1.e-10) then
        i = -2
        print*,'cannot solve system,matrix nearly singular'
        return
        endif
   70 continue
c
c
c ----------------------------------------------------------
c
c apply the corrections to the x values, also see if xtol
C is met
c
      itest = 0
      do 80 ivbl=1,n
        x(ivbl)=xsave(ivbl) + b(ivbl)
        if(abs(b(ivbl)).gt. xtol) itest = itest + 1
   80 continue
c
c
c -----------------------------------------------------------
c
c if xtol is met, print last values and return, else do
C another iteration
c
      if(itest .eq. 0) then
         i=1
         if(i.eq.0) print*,'it=',it,'   x=',x
         return
      endif
  100 continue
c
c -----------------------------------------------------------
c
c  when we have done maxit iterations , set i=-1 and return
      i = -1
      return
```

```
      end
c
c
      subroutine elim(a,b,n,eps)
c
      dimension a(10,10),b(10)
      do 1 i=1,n
      k=1
      if(i-n)21,7,21
   21 if(abs(a(i,i))-eps)6,6,7
    6 k=k+1
      b(i)=b(i)+b(k)
      do 23 j=1,n
   23 a(i,j)=a(i,j)+a(k,j)
      go to 21
    7 div=a(i,i)
      b(i)=b(i)/div
      l=i+1
      do 9 j=1,n
    9 a(i,j)=a(i,j)/div
      do 1 m=1,n
      delt=a(m,i)
      if(abs(delt)-eps)1,1,16
   16 if(m-i)10,1,10
   10 b(m)=b(m)-b(i)*delt
      do 11 j=1,n
   11 a(m,j)=a(m,j)-a(i,j)*delt
    1 continue
      return
      end
```

```
c     ************************************************
c     **                                            **
c     ** ANALYSIS OF R-R CRANK GENERATING A SURFACE  **
c     **                                            **
c     ************************************************
c
$      INCLUDE /usr/include/fgl.h
$      INCLUDE /usr/include/fdevice.h
c
       REAL dummy(4,4),a1(4,4),a2(4,4),a3(4,4),aeq(4,4),
      -theta2(200),theta3(200),p4(4,4),p1(4,4),acoor1(4,4),
      -acoor2(4,4)
       INTEGER I,J,kj,loop1,loop2

       OPEN(unit=9,file='data')
       OPEN(unit=8,file='data1')
       OPEN(unit=7,file='data2')
       loop1 = 45
       loop2 = 30

c ******************
c *
c * Point on the tangent-plane and the surface
c *
c ******************
       print*,'-------------------------------------------'
       print*,'input  Y from tangent point to connect point'
       print*,'-------------------------------------------'
       read*,p4(2,1)
       print*,'-------------------------------------------'
       print*,'input  Z from tangent point to connect point'
       print*,'-------------------------------------------'
       read*,p4(3,1)
       print*,'-------------------------------------------'
       print*,'input twist angle alpha(i)'
       print*,'-------------------------------------------'
       print*,'alpha(1) = '
       read*,alfa1
       print*,'alpha(2) = '
       read*,alfa2
       print*,'alpha(3) = '
       read*,alfa3
       print*,'------------------------------------------'
       print*,'input link length a(i)'
       print*,'------------------------------------------'
       print*,'a(1) = '
       read*,aa1
       print*,'a(2) = '
```

```
      read*,aa2
      print*,'a(3) = '
      read*,aa3
      print*,'--------------------------------------------'
      print*,'input link distane s(i)'
      print*,'--------------------------------------------'
      print*,'s(1) = '
      read*,s1
      print*,'s(2) = '
      read*,s2
      print*,'s(3) = '
      read*,s3
      print*,'---------------------------------------------'
      print*,'input theta(1)'
      print*,'---------------------------------------------'
      print*,'theta(1) = '
      read*,theta1
      p4(4,1)=1.
      pi=3.14159/180.
      theta1=theta1*pi
      theta2(1)=-90.
      theta3(1)=-90.
      alfa1=alfa1*pi
      alfa2=alfa2*pi
      alfa3=alfa3*pi
      on = 1
      CALL amatrix(alfa1,aa1,theta1,s1,a1)
      call multi (4,a1,p4,acoor1)
      x2=acoor1(1,1)
      y2=acoor1(2,1)
      z2=acoor1(3,1)
      write (8,101) x2,y2,z2
101   format (f12.6,f12.6,f12.6)
      DO 1 i=1,loop1
      theta2(i)=theta2(i)*pi
      CALL amatrix(alfa2,aa2,theta2(i),s2,a2)
      call multi (4,a1,a2,dummy)
      call multi (4,dummy,p4,acoor2)
      x3=acoor2(1,1)
      y3=acoor2(2,1)
      z3=acoor2(3,1)
      write (7,102) x3,y3,z3
102   format (f12.6,f12.6,f12.6)
      do 2 kj=1,loop2
      theta3(kj)=theta3(kj)*pi
      CALL amatrix(alfa3,aa3,theta3(kj),s3,a3)
      CALL multi(4,a1,a2,dummy)
      CALL multi(4,dummy,a3,aeq)
      CALL multi(4,aeq,p4,p1)
      dum1= p1(1,1)*80.0
      dum2= p1(2,1)*80.0
      dum3= p1(3,1)*80.0
      write (9,100) dum1,dum2,dum3
100   format (f12.6,f12.6,f12.6)
```

```
      theta3(kj)=theta3(kj)/pi
      theta3(kj+1)=theta3(kj)+16.
    2 CONTINUE
      theta3(1)=-90.
      p4(1,1)=0.
      theta2(i)=theta2(i)/pi
      theta2(i+1)=theta2(i)+8.
    1 CONTINUE
      stop
      end
c
c     *************************
c     *
c     *   subroutine Multiply
c     *
c     *************************
c
      SUBROUTINE amatrix(a,b,c,d,tt)
      dimension tt(4,4)
      tt(1,1)=cos(c)
      tt(1,2)=-sin(c)*cos(a)
      tt(1,3)=sin(c)*sin(a)
      tt(1,4)=b*cos(c)
      tt(2,1)=sin(c)
      tt(2,2)=cos(c)*cos(a)
      tt(2,3)=-cos(c)*sin(a)
      tt(2,4)=b*sin(c)
      tt(3,1)=0.
      tt(3,2)=sin(a)
      tt(3,3)=cos(a)
      tt(3,4)=d
      tt(4,1)=0.
      tt(4,2)=0.
      tt(4,3)=0.
      tt(4,4)=1.
      return
      end
c     *************************
c     *
c     *   subroutine Multiply
c     *
c     *************************
c
      SUBROUTINE multi(n,a,b,c)
      dimension a(4,4),b(4,n),c(4,n)
      do 1 i=1,4
      do 2 j=1,n
      c(i,j)=0.
      do 3 k=1,n
    3  c(i,j)=c(i,j)+a(i,k)*b(k,j)
    2 continue
    1 continue
      return
      end
```

APPENDIX B

ISIS GRAPHIC COMPUTER PROGRAM

```
c *****************************
c *
c *   SURFACE GENERATION  -
c *
c *       IN THIS PROGRAM, THE SURFACE IS GENERATED
c *    FOR TWO-DEGREE-OF-FREEDOM OPEN LOOP MECHANISM
c *    BY USING IRIS COMPUTER PROGRAM
c *
c *
c *****************************

$include /usr/include/fgl.h
$include /usr/include/fdevice.h


      integer*2  i,j,aa1,ss1,twist,theta1,alfa1
      twist = 0
      aa1 = 0*80
      ss1 = 2*80
      call ginit( )
      call color(0)
      call clear( )
      call cursof( )
      call color (7)
      call recti(50,50,1000,700)
      call color(5)
      call rectfi(51,51,999,699)
      call color(7)
      call recti(100,100,950,650)
      call color(4)
      call rectfi(101,101,949,649)

c
c *****************************
c *
c *  object(2) ---
c *
c *      make the first page
c *
c *
c *****************************
c

      call makeob (2)
      call ortho2(50.0,1000.0,50.0,700.0)
      call color (7)
      call recti (190,510,850,560)
      call color (2)
```

```
      call rectfi (191,511,849,559)
      call color (1)
      call cmovi (210,530,0)
      call charst('Simulation of Spatial Mechanisms for Tang
     -ent-Plane Envelope Generation',72)
      call color (3)
      call recti (250,390,460,460)
      call color (1)
      call rectfi (251,391,459,459)
      call color (2)
      call cmovi (270,430,0)
      call charst('Part I  : Analysis',18)
      call cmovi (270,410,0)
      call charst('Part II : Synthesis',19)
      call color (7)
      call cmovi (310,320,0)
      call charst('Foo-Ming Fu',11)
      call cmovi (290,300,0)
      call charst('Graduate Student',16)
      call cmovi (310,270,0)
      call charst('A. H. Soni',10)
      call cmovi (290,250,0)
      call charst('Regents Professor',17)
      call cmovi (250,200,0)
      call charst('Oklahoma State University',25)
      call cmovi (270,180,0)
      call charst('Stillwater, Oklahoma',20)
      call closeo(2)
c
c *****************************
c *
c * close object(2)
c *
c *****************************
c
      call callob(2)
c
c ******************************
c *
c *  object(1) ---
c *
c *      make a surface on the first page
c *
c ******************************
c
      call ortho (-1600.0,800.0,-900.0,1500.0,-1200.0,1200.0)
      open (unit=9, file='datafile')
      call rotate (400,'x')
      call rotate (-400,'y')
      call axis ( )
      do 70 i=1,45
      do 70 j=1,30
      read (9,*)dum1,dum2,dum3
      if(j.eq.1)then
```

```
      dum4=dum1
      dum5=dum2
      dum6=dum3
      endif
      call color (2)
      call move (dum4,dum5,dum6)
      call draw (dum1,dum2,dum3)
      dum4=dum1
      dum5=dum2
      dum6=dum3
   70 continue
      close(unit=9,status='keep')
c
c *************************
c *
c * close object(1)
c *
c *************************
c
      call gexit( )
c
      do 60 i =1,39
      do 60 j =1,39
      a = cos(1.57)
  60  continue
      do 30 k=1,36
      open (unit=9, file='data')
      open (unit=8, file='data1')
      open (unit=7, file='data2')
      call ginit( )
      call color(0)
      call clear( )
      call cursof( )
      call color(7)
      call recti(50,50,1000,700)
      call color(4)
      call rectfi(51,51,999,699)
      call color(7)
      call recti(600,600,1000,700)
      call color(1)
      call rectfi(601,601,999,699)
      call color(2)
      call cmovi(660,680,0)
      call charst('SURFACE GENERATION OF R-R CRANK',31)
      call color(6)
      call cmovi(660,655,0)
      call charst('     --  by FOO-MING FU     --    ',31)
      call color(3)
      call cmovi(660,630,0)
      call charst(' alpha1=90, alpha2=90, alpha3=0',31)
      call cmovi(650,610,0)
      call charst(' a1=0, a2=0, a3=2, s1=2, s2=2, s3=2',35)
      call ortho (-600.0,600.0,-600.0,600.0,-600.0,600.0)
      call rotate (400,'x')
```

```
      call rotate (-400+twist,'y')
      call axis ( )
      read (8,*)x2,y2,z2
      x2=x2*80.
      y2=y2*80.
      z2=z2*80.
      ntheta1=0
      nalfa1=900
      call axis1 (x2,y2,z2,ntheta1,nalfa1)
      call color(6)
      call move (0.0,0.0,0.0)
      call draw (x2,y2,z2)
      call move (0.0,0.0,0.0)
      call draw (x2,y2,z2)
c      call makeob (1)
c      call rotate (900,'x')
c      call circf(0.0,y2,20.0)
c      call closeo()
c      call makeob(1)
      do 10 i=1,45
      do 10 j=1,30
      read (9,*)dum1,dum2,dum3
      if(j.eq.1)then
      dum4=dum1
      dum5=dum2
      dum6=dum3
      endif
      call color (2)
      call move (dum4,dum5,dum6)
      call draw (dum1,dum2,dum3)
      dum4=dum1
      dum5=dum2
      dum6=dum3
 10   continue
      close(unit=9,status='keep')
      close(unit=8,status='keep')
      close(unit=7,status='keep')
      twist=twist+100
      do 50 i =1,99
      do 50 j =1,99
      a = cos(1.57)
50    continue
 30   continue
      call gexit( )
      do 20 i =1,1199
      do 20 j =1,1199
      a = cos(1.57)
20    continue
      stop
      end
c
c
c *******************
c *
```

```
c *   sub axis
c *
c *******************
c
      subroutine axis ( )
      call color(3)
      call movei(0,0,0)
      call drawi(550,0,0)
      call cmovi (560,0,0)
      call  color (2)
      call charst ('X',1)
      call color(3)
      call movei(0,0,0)
      call drawi(0,550,0)
      call cmovi (0,560,0)
      call  color (2)
      call charst ('Y',1)
      call  color(3)
      call movei(0,0,0)
      call drawi(0,0,550)
      call cmovi (0,0,570)
      call color (2)
      call charst ('Z',1)
      call color (3)
      call movei (300,0,0)
      call drawi (270,-25,25)
      call movei (315,0,0)
      call drawi (285,-25,25)
      call movei (330,0,0)
      call drawi (300,-25,25)
      call movei (345,0,0)
      call drawi (315,-25,25)
      call movei (360,0,0)
      call drawi (330,-25,25)
      call movei (375,0,0)
      call drawi (345,-25,25)
      call movei (390,0,0)
      call drawi (360,-25,25)
      return
      end
c
c *******************
c *
c *   sub axis1
c *
c *******************
c
      subroutine axis1 (x2,y2,z2,theta1,alfa1)
      integer*2 theta1,alfa1
      call color(1)
      call move (x2,y2,z2)
      call draw (x2+150.0,y2,z2)
      call move (x2,y2,z2)
      call draw (x2+150.0,y2,z2)
```

```
call move (x2+60.0,y2,z2)
call draw (x2+80.0,y2,z2+30.0)
call move (x2+70.0,y2,z2)
call draw (x2+90.0,y2,z2+30.0)
call move (x2+80.0,y2,z2)
call draw (x2+100.0,y2,z2+30.0)
call move (x2+90.0,y2,z2)
call draw (x2+110.0,y2,z2+30.0)
call move (x2+100.0,y2,z2)
call draw (x2+120.0,y2,z2+30.0)
call move (x2+110.0,y2,z2)
call draw (x2+130.0,y2,z2+30.0)
call cmov (x2+160.0,y2,z2)
call color (7)
call charst ('X1',2)
call color(1)
call move (x2,y2,z2)
call draw (x2,y2,z2+150.0)
call cmov (x2,y2,z2+160.0)
call  color (7)
call charst ('Y1',2)
call  color(1)
call move (x2,y2,z2)
call draw (x2,y2-150.0,z2)
call cmov (x2,y2-160.0,z2)
call color (7)
call charst ('Z1',2)
return
end
```

VITA

FOO-MING FU

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