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CIRCULAR PRESTRESSED CONCRETE COLUMNS SUBJECT TO CONCENTRIC FORCES, BENDING MOMENT AND TORSION

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BY

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CIRCULAR PRESTRESSED CONCRETE COLUMNS SUBJECT TO

CONCENTRIC FORCES, BENDING MOMENT AND TORSION

APPROVED BY

[signatures]

DISSERTATION COMMITTEE

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LIST OF SYMBOLS

A = area

A_c = Gross cross section area of prestressed column.

A_s = Area of prestressed strand.

C = Total interval compressive force in concrete.

c = Distance from neutral axis to extreme fiber in compression.

= Change in the strain of prestressed strand.

E_o = Modulus of elasticity of concrete.

E_s = Modulus of elasticity of prestressed strand.

f = stress in concrete

f_c = Compressive stress in concrete

f'_c = Cylinder strength in concrete.

f_{pe} = Compressive stress in concrete due to prestressing force after losses.

f_s = Calculated stress in prestressing steel at ultimate load.

f_{sy} = Nominal yield strength of prestressing strand

f_t = Tensile strength of concrete.

h = Actual unsupported length of column.

I = Moment of inertia of cross section area.

K_1, K_2 = Coefficients related to magnitude and position of internal compressive force in concrete.

M_b = Maximum bending moment.

M_u = Ultimate bending moment in flexure.

P_o = Internal force in prestressed strand.

P_r = Prestressing force after losses.

P_{uo} = Ultimate load of concentrically loaded column.

P_u = Ultimate load of eccentrically loaded column.

R = Radius of circular cross section

r = Radius of Gyration of gross concrete area of column.

σ_1 = Principal stress.

τ_p = Shearing stress due to torsional moment in prestressed concrete column.

T = Internal stress in prestressed strand.

T_o = Torsional strength of plain concrete section.

T_p = Torsional strength of a uniformly prestressed section.

T_{pu} = Ultimate torsional strength of a uniformly prestressed section

ε = Strain.

ε_s = Strain in prestressed strand.

ε_{so} = Strain in prestressed strand due to prestressing forces after losses.

ε_{su} = Ultimate strain in prestressed strand.

ε_u = Ultimate concrete strain.

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CIRCULAR PRESTRESSED CONCRETE COLUMNS SUBJECT TO CONCENTRIC FORCES, BENDING MOMENT AND TORSION

CHAPTER I

INTRODUCTION

Columns are basically compression members. A column that is compressed by its own prestress is not considered a compression member, rather the "column action" which it undergoes is the result of external compressive loads.

Axially loaded prestressed columns are not very common in practice since concrete, under many situations, can carry compressive loads better without being precompressed by prestressed strands. It has been difficult for the average engineer to conceive of the idea that the prestressing strands will help in carrying any compressive load. It is obvious that there truly is little justification for prestressing a short concentrically loaded column. On the other hand, in many compression members there is bending due either to load eccentricity, asymmetry in the section, or due to transverse loads, each tends to produce tension in the concrete or to make the column less stable. Prestressing in the foregoing becomes a practical solution. It might also be justifiable on the basis of economy, construction advantages, transportation and erection stresses.

Some research[4,11*] has been reported in the area of concentrically prestressed columns, both theoretical and experimental. It was concluded that prestressing a slender column does not decrease the concentric load that will cause the column to buckle and that the column will fail in compression if the prestressing stresses exceed the difference between the buckling stress and the ultimate strength of the concrete.

Most columns in practice are eccentrically loaded. Failure may be controlled by the material strength, buckling of the column, or the interaction of applied concentric load with bending. Prestressing may prove to be of considerable value in providing greater strength and stiffness to the column.

Prestressed columns under eccentric loads have been the subject of research for the past thirty years. Little progress, if any, has been made on circular sections. Most of the effort has been directed toward the study of rectangular columns. One exception is that of pile studies.[59] The research work on rectangular sections includes experimental work on the stability of slender prestressed columns;[4,39] the effect of partial prestressing on long columns;[60] and the effect of eccentrically loaded prestressed columns with hinged ends.[67] A thorough discussion of the literature is included in the next chapter.

The general conclusion is that cracking can be fairly well predicted by the elastic theory and that it occurs when the fiber stress reaches the modulus of rupture. Beyond cracking, the elastic theory is no longer applicable.

The ultimate strength of eccentrically prestressed slender

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*Superscript numbers refer to entries in the Bibliography.

columns was calculated on the conditions of static equilibrium and compatibility of strain.[67,70] Ultimate strength relationships were based on Hognestad's[2] stress-strain curve. It seems that some concern or caution is warranted here. Prestressing steel lacks a sharp and distinct yield point and since the prestress may modify the neutral axis location at failure, it is fairly clear that theoretical considerations based on a relationship developed for conventional reinforced concrete sections need further consideration.

In framed structures, columns undergo stresses due to bending moments, axial forces, and shear as well as torsional forces. Bending can be caused by eccentric loads, wind and earthquake forces or frame action. Columns may sustain severe torsional stresses due to earthquake loads or as a secondary effect of bending. Torsional stresses may not affect the design of columns but the combined effect of these forces and torsion is the least understood.

A considerable amount of research effort has been directed toward study of the behavior of prestressed concrete beams under torsion.[68,72] It is generally agreed that because of the high shear strength of concrete coupled with its low tensile strength, the failure of prestressed concrete beams in torsion seldom results from shearing stresses, but rather from principal tensile stresses produced by the shearing stresses and the compressive prestressing force. The general trend among researchers today is to represent the torsional strength of prestressed beams in a unitless interaction diagram or a surface in the case of combined torsion, bending, and shear and to derive an equation based on statistical inference. Even though test results are fairly well correlated by statistical equations, a well-defined failure theory for concrete under biaxial stresses is long overdue.

One failure criteria commonly used, governed by the maximum-stress theory,[12] compares quite favorably with test results for establishing the ultimate cracking moment for prestressed concrete sections. Equations derived for torsional failure of uniformly prestressed rectangular beams based on the bending mechanism are identical to equations derived by the writer based on Coulomb's stress distribution and maximum-stress theory for circular cross sections.

It was the objective of this study to investigate the relationship of circular slender prestressed concrete columns between the acting load and the bending moment for a given eccentricity when the load is increased from zero to collapse. This relationship will be represented graphically in interaction diagrams for both cracking and collapse failures.

The second objective is the study of the effect of torsion, the combined effect of torsion and concentric load, torsion and bending moment, and the interaction of torsion with concentric load and bending.

In order to fully understand the combined effect of concentric loads, torsion and bending moment, the height, cross section area, the amount of prestressing, the percentage of steel, and the eccentricity of the axial load will not be considered as variables, while the strength of concrete was an unintentional variable in this study.

A thorough review of the literature shows that the research here reported on circular prestressed concrete columns is unique and without precedent in the literature.

This research was motivated by the recommendations of the Conference on the Behavior of Structural Concrete Subject to Combined Loadings, West Virginia University, Morgantown, West Virginia, June 10-13, 1969.

CHAPTER II

HISTORICAL DEVELOPMENT

Research work on circular prestress concrete columns is non-existent. Some work has been reported on other shapes, mostly involving specimens of rectangular sections. Most of these tests involved the application of one type of load or another; however, research data is unavailable dealing with combined loads on prestressed concrete columns of any type of crossection. The review of historical development must of necessity be restricted to one type of loading condition at a time, while the study herein will give some insight into the interaction of the applied forces.

Torsion

Circular members are commonly used as compression members. Under earthquake loadings, columns may undergo severe torsional stresses.[62] There are no published data on the strength of reinforced or prestressed concrete columns undergoing torsion or the interaction effect of torsion with axial force or the combined interaction with bending or concentric forces. The writer is aware of only one exception and that is the study by Bishara[71] on reinforced concrete rectangular columns in torsion.

Pure Torsion

Plain concrete members subjected to pure torsion will fail along the plane of principal tension, at a spiral line 45 degrees to the axis. Spiral and longitudinal reinforcement will improve the column capability to withstand torsional stresses, but will not act until the concrete has cracked,[12,43] indicating that torsional reinforcement in ordinary reinforced concrete does not contribute to the elastic torsional strength of the concrete member. On the other hand prestressing delays the formation of tension cracks until the compression due to initial prestress has been offset, and, therefore, there is a great improvement in the torsional strength of concrete.[12]

Zia[72] states:

> Before cracking, the behavior of a concrete member in pure torsion can be closely predicted by the elastic theory, whether the member is of plain reinforced or prestressed concrete. Addition of reinforcement or prestressing into a plane concrete member, or both, has no appreciable effect on the member stiffness before cracking.

Cowan's[12] tests on prestress concrete members in pure torsion led him to believe that a state of pure shearing stress transforms into tensile and compressive principle stresses which combine with compressive prestress. The equations he derived showed good agreement with experimental data. Cowan noticed that failure of prestressed concrete in torsion follows immediately on the formation of the first crack. The prestress force is released by the spiral fracture, often in an explosive manner, accompanied by considerable noise and flying debris.

It is generally agreed that failure of prestressed concrete members in pure torsion occurs at initial cracking.[4,12,69] The mechanism of failure can be described in two different folds. First, the elastic theory applying Coulomb Theory for circular columns or Saint-Venant

Theory to rectangular cross sections, adding the compression effect of the prestressing, and second, the plastic theory exists in two different forms: One theory is based on Nadai's sand heap analogy assuming concrete to have infinite plasticity while the other theory considers a limited redistribution of stress according to the observed plastic behavior of concrete in compression. Since concrete does not possess the material properties as required by the three theories, and since concrete is no more elastic than it is plastic, all the above theories could be incorrect. The elastic theory will tend to underestimate the torsional strength while the plastic theory will overestimate the capacity of concrete in torsion.

Hsu[69] reported that all three theories may be incorrect based on his tests on rectangular sections. He also concluded that torsional failure of uniformly prestressed concrete members is due to a skewed bending concept. It should be kept in mind that all his tests were conducted on rectangular beams; he modified his equations to accommodate circular cross sections without obtaining test data.

It has been the trend of most researchers to account for the additional strength due to the presence of the prestressing force by modifying the equation for plain concrete in torsion by an appropriate factor to account for the apparent increase in strength.

In work done by Hsu,[69] he stated:

> The bending mechanism for torsional failure observed previously for plain concrete rectangular members was extended to uniformly prestressed rectangular members and was found to be applicable. The stress factor $\gamma=(1+10\sigma/f_c')$ established by the bending mechanism, is identical to that derived from the classical maximum tensile stress criterion of failure...The torsional strength of a prestressed concrete member is shown to be equal to that of a non-prestressed multiplied by a factor accounting for the effect of prestress.

Zia[68] also concluded:

> An accurate determination of the torsional strength of prestressed concrete is difficult because it requires not only a clear understanding of the torsional stress distribution, but also a well-defined failure theory for concrete under biaxial stresses.

Torsion Combined with Bending

Most columns in practice are eccentrically loaded and failure may be controlled by the material strength, buckling of the column, its torsional stiffness, or the interaction of torsion with bending and axial force as well as shear. Prestressing can prove to be of considerable value in providing greater strength and stiffness to the column.

Columns in practice are rarely loaded in torsion alone. If torsional moments exist they are generally accompanied by bending and axial forces. Columns with very large eccentricity can be considered as having no axial force effect.

Test programs on prestressed concrete columns subject to the combined bending and torsion are virtually non-existent. However, tests on prestressed beams subject to combined bending and torsion have been reported by Cowan[3] in 1953, Gardner[26] in 1960, Swamy,[40] Reeves[41] and Gersch[42] in 1962, Evsoy[51] and Swamy[57] in 1965, and Priya[74] in 1971. These tests involved specimens other than circular cross sections, both with and without reinforcement.

Torsion Combined with Concentric Force

Because of an assumed lack of practical significance, research in this area is untouched. Torsion has been considered a secondary type of failure and hence torsional stresses were not considered to affect the design of the column. On the other hand, prestressed columns may undergo severe torsional stresses due to earthquake loads and their combined effect with applied concentric force is not clear.

Bending Combined with Axial Load

The idea of prestressing a compression member might seem at first an anomaly. It might seem obvious that prestressing a concentrically loaded column would decrease the capacity of the member in spite of the advantages gained in economy and handling stresses.[12] However, for eccentrically loaded columns prestressing can be of great advantage, providing greater strength and stiffness in the column.[60]

Columns can be fully prestressed, partially prestressed, or tri-axially prestressed. The following discussion will pertain to the first two types of prestressed concrete columns.

Prestress concrete columns can fail by either material failure or by buckling,[24,61] depending upon the slenderness of the column. The ultimate load capacity of eccentrically prestressed concrete columns is affected by the following factors.

Effect of Prestress

One of the problems encountered in design, is to determine the required prestressing force to apply to a particular column in order to get its maximum load-carrying capacity.

Aroni[67] reports:

1- For columns of smallest eccentricity ($e/d=1/8$), low prestress is beneficial, up to a ratio of about $f_{cp}/f'_c=0.1$, where f_{cp} is the concrete tensile strength. Beyond this ratio, increase of prestress results in a significant drop in the value of the maximum critical buckling load. 2- For columns of medium eccentricity ($e/d=3/4$) acting as beam-column, the maximum critical buckling load P_{cr}, increases with prestress, with an apparent maximum value in the region of $f_{cp}/f'_c=0.3$ to 0.35. 3- For the largest eccentricity ($e/d=2$) prestress have no effect on columns of slenderness ratio of 20, whereas for the longer columns, P_{cr} shows some small increase with prestress.

Tests have been conducted by Zia and Moreadith[59] for columns with different prestressing levels and the same concrete strength. They concluded that for columns with e=0 heavy prestressing is detrimental to the load carrying capacity of the column especially for short columns. Prestressed columns of low strength concrete and subject to eccentric loads (e=0.5d) showed considerable advantage when compared with conventional reinforced concrete columns ranging from short columns (L/d=10) to slender columns (L/d=70). The effect was most pronounced in the short columns.

Effect of Eccentricity

Eccentricity in prestressed concrete columns plays an important role in determining the ultimate load capacity of the column. Zia and Moreadith[59] investigated this effect and found out, that under concentric loads (e=0) the column was controlled by material failure. With increase in the slenderness ratio, failure was due to instability. For eccentrically loaded columns, buckling failure was the primary factor even for short columns. For columns with large eccentricity, the column showed more beam action than column action even with a large slenderness ratio.

In tests conducted by Aroni,[67,70] he concluded that for a particular value of prestress, the maximum critical load decreases sharply with increase in eccentricity. Aroni found the effect to be more pronounced for the shorter columns. Higher prestress values showed a similar relationship.

Effect of Slenderness

The reduction in strength of prestressed columns loaded eccentrically was observed by Zia and Moreadity[59] and by Aroni;[70] tests results have shown that the column strength was reduced considerably due to increase in the slenderness ratio.

Aroni concluded from his test that:

> As expected, the critical load decreases with increase in slenderness. As the eccentricity becomes smaller, the member tends to behave more like an axially loaded column, and the effect of the slenderness is more pronounced. On the other hand, for large eccentricity, the behavior resembles more that of a beam and the influence of slenderness is much smaller. The same general relations are exhibited at higher prestress ratio.

Effect of Concrete Compressive Strength

Prestressed concrete columns with low compressive strength exhibited material failure even at high slenderness ratios. Tests[59] have shown that increase in concrete strength has increased the strength of shorter columns with small eccentricity and high prestress force.

CHAPTER III

TEST SPECIMENS AND INSTRUMENTATION

Materials and Fabrication

Concrete

Type III Portland Cement was used in all specimens. Aggregates were river sand and crushed gravel. The coarse aggregate had a maximum size of 0.275 inches. The sand was well graded having a sieve analysis shown in Appendix (C). Table 1 contains the compressive strength and modulus of elasticity for concrete in each column. Compressive strengths are based on 3.2 x 6.0 inch control cylinders. A 1:1.5:1.7, cement:sand:gravel mix by weight, with a water cement ratio of 0.5, was used for all concrete.

Concrete was mixed for 3 minutes in a drum-type mixer of 3 cu-ft capacity and was placed in the forms with the aid of a high frequency vibrator.

Prestressing Strand

A 3/8" diameter 7-wire prestress cable was used. The ultimate strength of the steel, as determined by the manufacturer was 270,000 lb per sq in. Appendix (C).

Casting and Curing

The columns were cast horizontally in order to avoid differentials

TABLE 1

PHYSICAL PROPERTIES OF COLUMN CONCRETE

Column	f'_{ci} (psi)	f'_c (psi)	$E \times 10^6$ (psi)
A-3	4000	4400	4.044
B-2	3800	4000	3.834
B-3	3800	4080	3.872
C-2	3800	4660	4.139
C-3	3800	4320	3.985
D-1	4200	5600	4.537
D-2	4200	5500	4.496
D-3	4200	5600	4.537
E-1	4210	5362	4.439
E-2	4210	5362	4.439
E-3	4210	5560	4.520
F-1	5950	6200	4.774
F-2	5950	6625	4.935
F-3	5950	6600	4.925
G-1	5900	6425	4.859
G-2	5900	6425	4.859
G-3	5900	6900	5.036

in concrete quality along the column length as will typically exist in a vertically cast column.[2,71] Horizontal casting,[2] on the other hand, will cause a strength differential across the cross section of the column. Therefore, the upper side of the column during casting was oriented to the compression zone under the applied load.

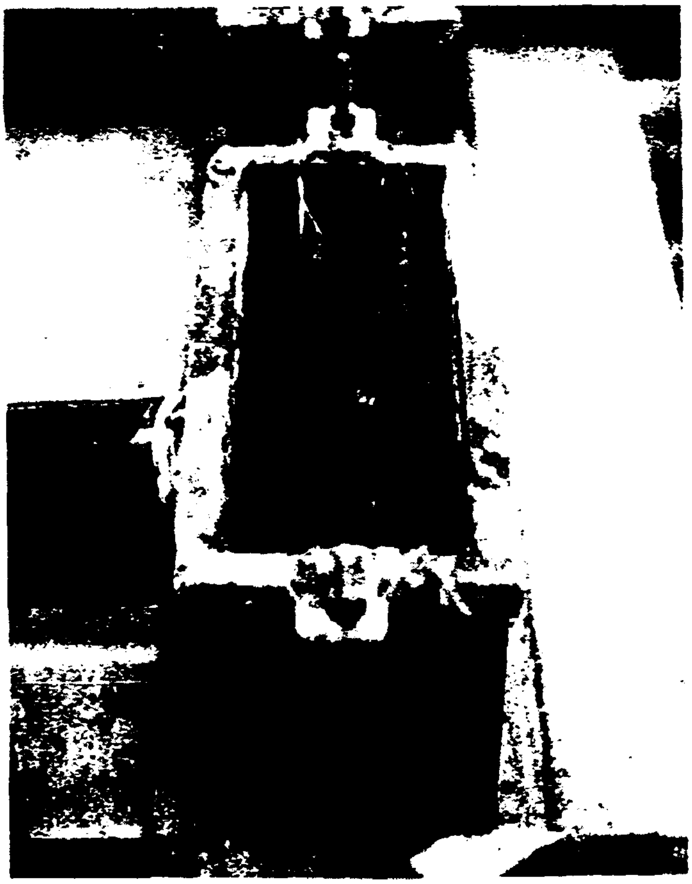
The columns were cast in plastic forms with the exception of the loading head which was made of wood (Figs. 1 and 2). The prestressing bed had an overall usable dimension of 28 ft. x 4 ft. This facilitated the casting of six columns in two rows with three columns sharing the same prestressing cable (Fig. 3). Each cable was stressed individually with an hydraulic jack[75] and the required prestressing force was read using a 50 kips calibrated load cell and also checked by measuring the total elongation of the cable. The cables were stressed at the time of casting.

To facilitate the load testing of the columns, a rectangular head having an overall dimension of 12 in. by 8 in. by 6 in. was cast integrally at each end of the column. The heads were reinforced in order to avoid local failure at the time of testing (Fig. 2).

The wooden forms for the heads were placed with their reinforcement positioned first, then the plastic pipe was inserted and taped securely to prevent leakage. The cables were then threaded through the forms, anchored at the east end of the bed while the west end of the cable was threaded through a special loading system that allowed the cable to twist freely during loading and facilitate the transfer of the load from the loading chair to the prestressing bed.[75]

Three batches of concrete were needed for the casting of the six columns and twenty-four cylinders. Out of each batch two columns and six test cylinders were cast. The concrete was poured through a one

Figure 1. Formwork



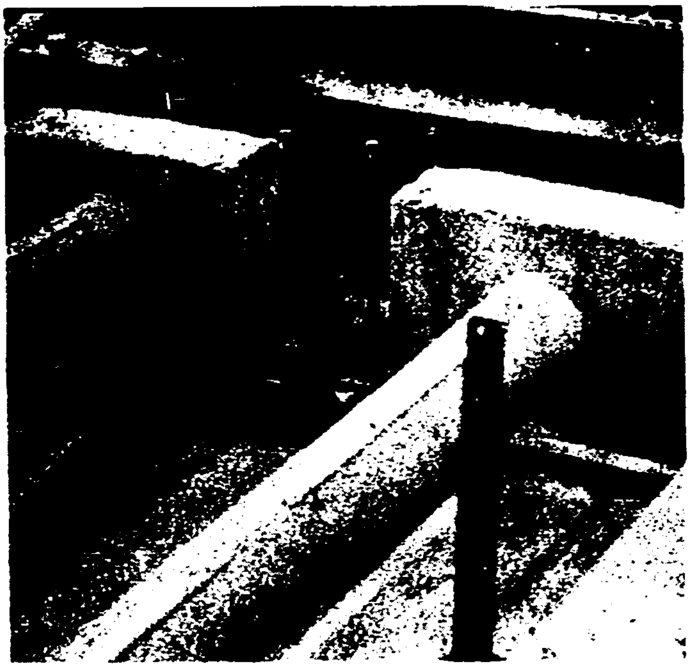


Figure 2. Formwork

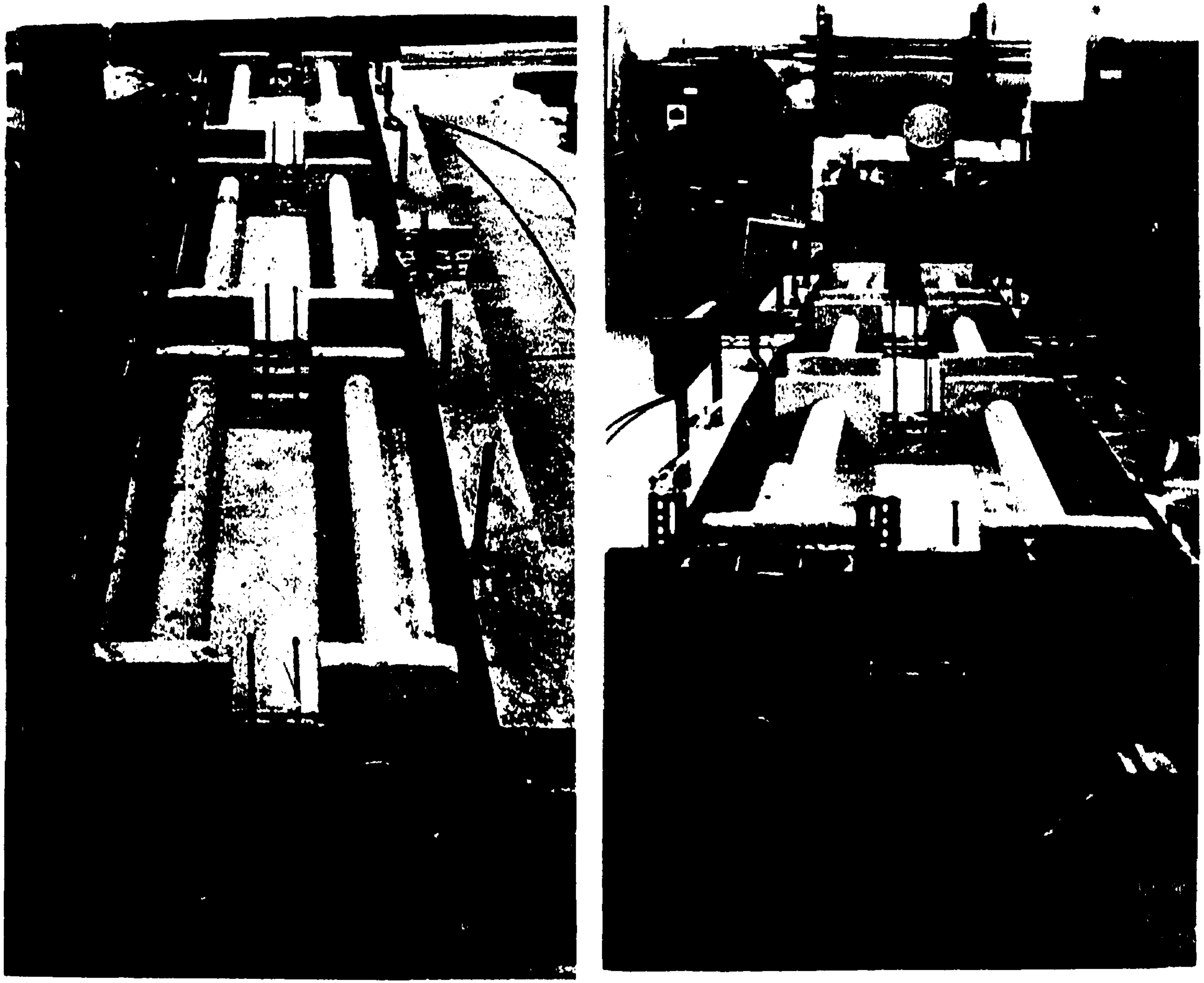


Figure 3. Casting and Prestressing System

inch opening in the plastic pipe, then it was thoroughly compacted in the forms using a pencil vibrator, and finished smoothly to the geometry of the form.

The specimens and the cylinders were cured in their forms for three days, then the forms were removed and were cured in open air. When the desired strength (f'_{ci} = 4000 psi) was attained, the prestressing force was transferred from the bed to the columns. The prestress cable was then cut at both sides of the middle column.

Concrete Control Specimen

Nine 3.2 inches by 6 inches cylinders were cast with each column. Three cylinders were tested initially to determine the concrete strength at transfer. The concrete strength at time of testing was obtained as an average of three cylinder strengths tested at the same time the corresponding column was loaded.

Column Configuration

Test specimens had a 3.75 inch diameter and a length of 75 inches, which models a prototype column of 15 inch diameter and 25 foot length with a scale factor of 1/4.[73]

The loading head on each end of the column was 12 in. by 8 in. by 6 in. and reinforced with six No. 3 bars to insure against failure of the head due to the externally applied loads. Four No. 6 bars were extended from the head and into the column as shown in Fig. 2 to insure against local end failure. This type of failure can result from the lack of prestress along the end development lengths in pretensioned members. In actual structures the end moment will come from the beams and since the beams can be monolithically prestressed with the column, end troubles can be minimized. In laboratory testing it was necessary to reinforce the loading head and ends of the column to prevent its end failure.

Test Procedure

Table 2 lists the type of tests for each of the seventeen columns. The following are several load arrangements to which test columns were subjected:

1. Pure bending moment.
2. Pure torsion.
3. Pure concentric load.
4. Bending moment and concentric load.
5. Concentric load and torsion.
6. Concentric load, bending moment and torsion.

A special testing frame was used for the column specimens. It was necessary to test the specimens horizontally, due to the lack of vertical clearance. The testing frame (Fig. 5) consisted of two beams 15 ft long separated at the ends by two columns 5 ft high. The different types of tests necessitated setting up two different sets of testing apparatus. One set up was for columns undergoing bending moment only, while the other testing apparatus sufficed for the other combination of loadings.

Bending Moment Apparatus

The specimens were simply supported on the bottom beam of the frame (Fig. 4) and were loaded on the third points through a steel section resting in half circular plates twenty-five inches apart. The load was applied in the middle of the steel section and the load was measured

TABLE 2

TEST ARRANGEMENTS

COLUMN NO.	TEST
B-2 B-3	Pure Concentric Force
G-1 G-2	Pure Moment
E-3 F-1 F-2	Eccentric Force
C-2 C-3	Pure Torsion
E-1 E-2	Concentric Force and Torsion
A-3 F-3	Bending Moment and Torsion
D-1 D-2 D-3 G-3	Torsion, Bending Moment and Concentric Force

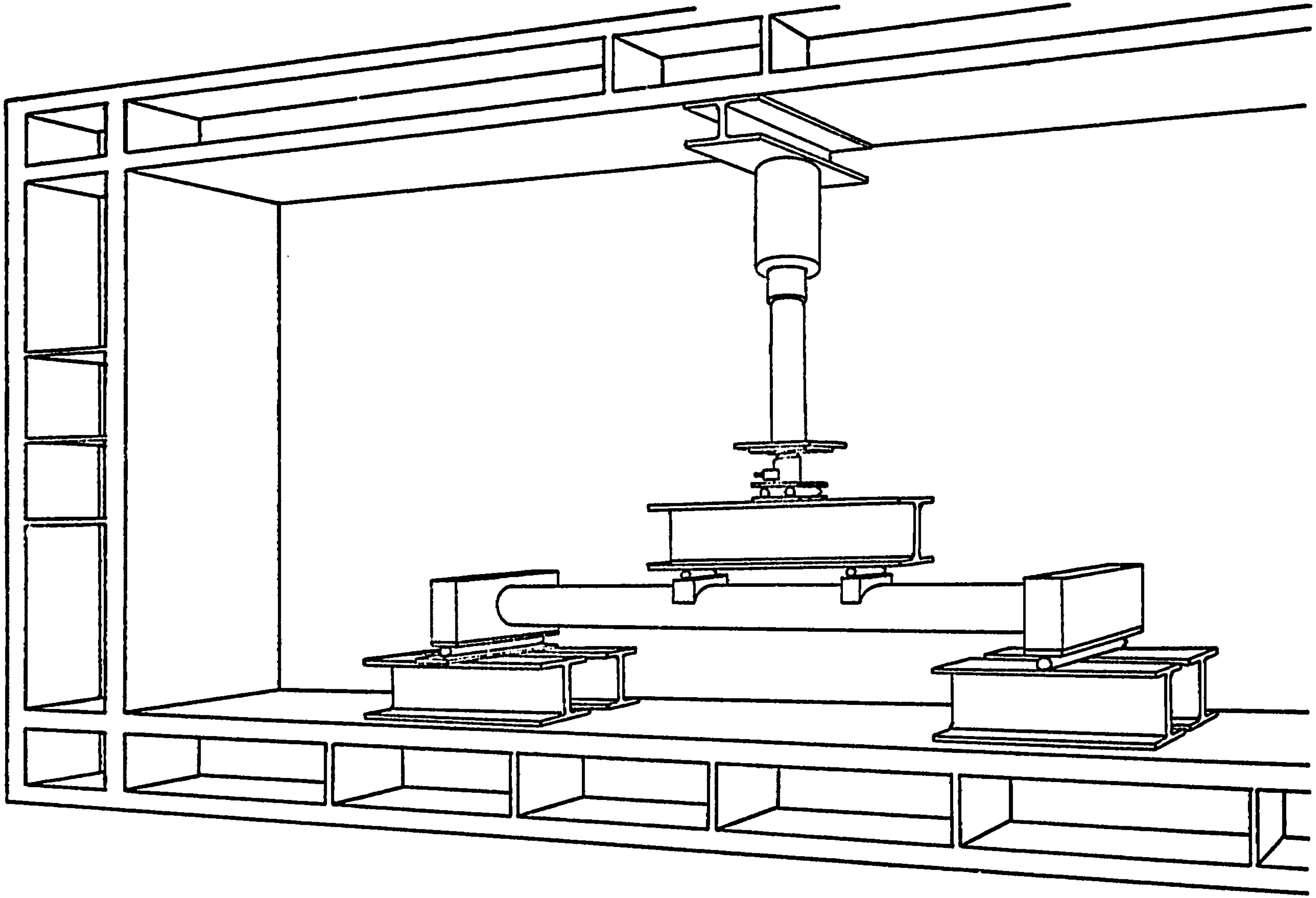


Figure 4. Pure Bending Apparatus

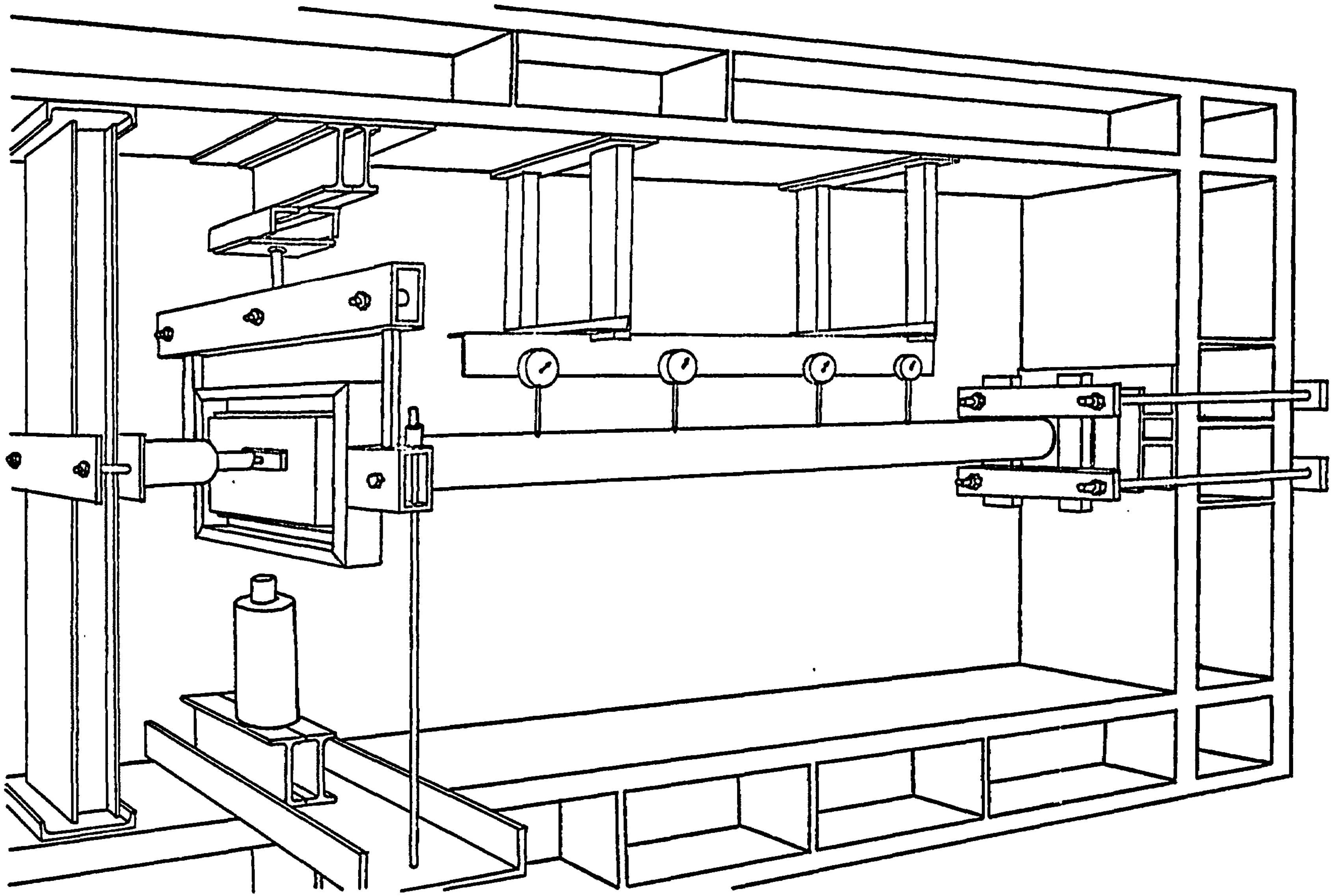


Figure 5. Torsion Rig and Eccentric Load Apparatus

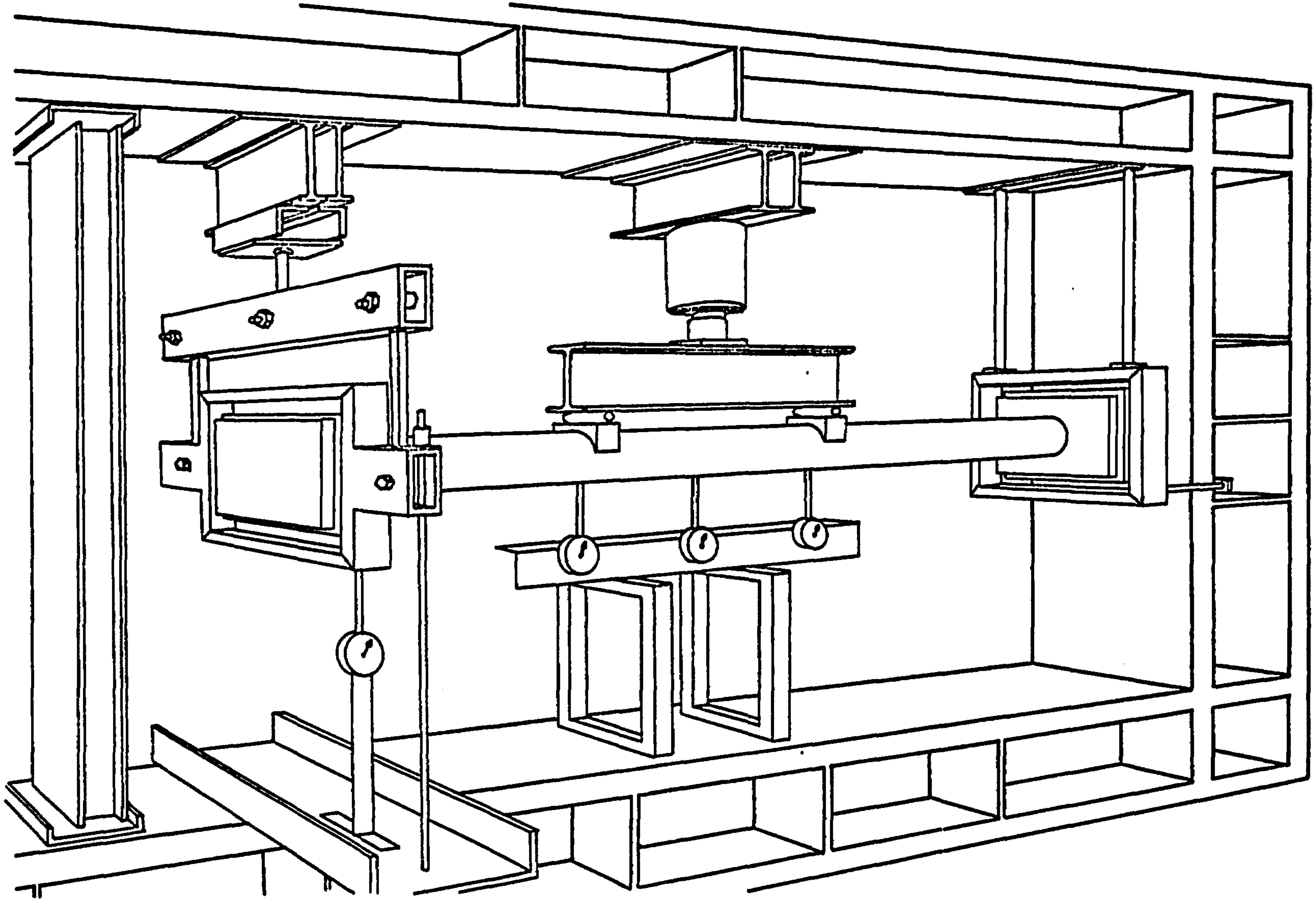


Figure 6. Torsion Rig and Bending Movement Apparatus

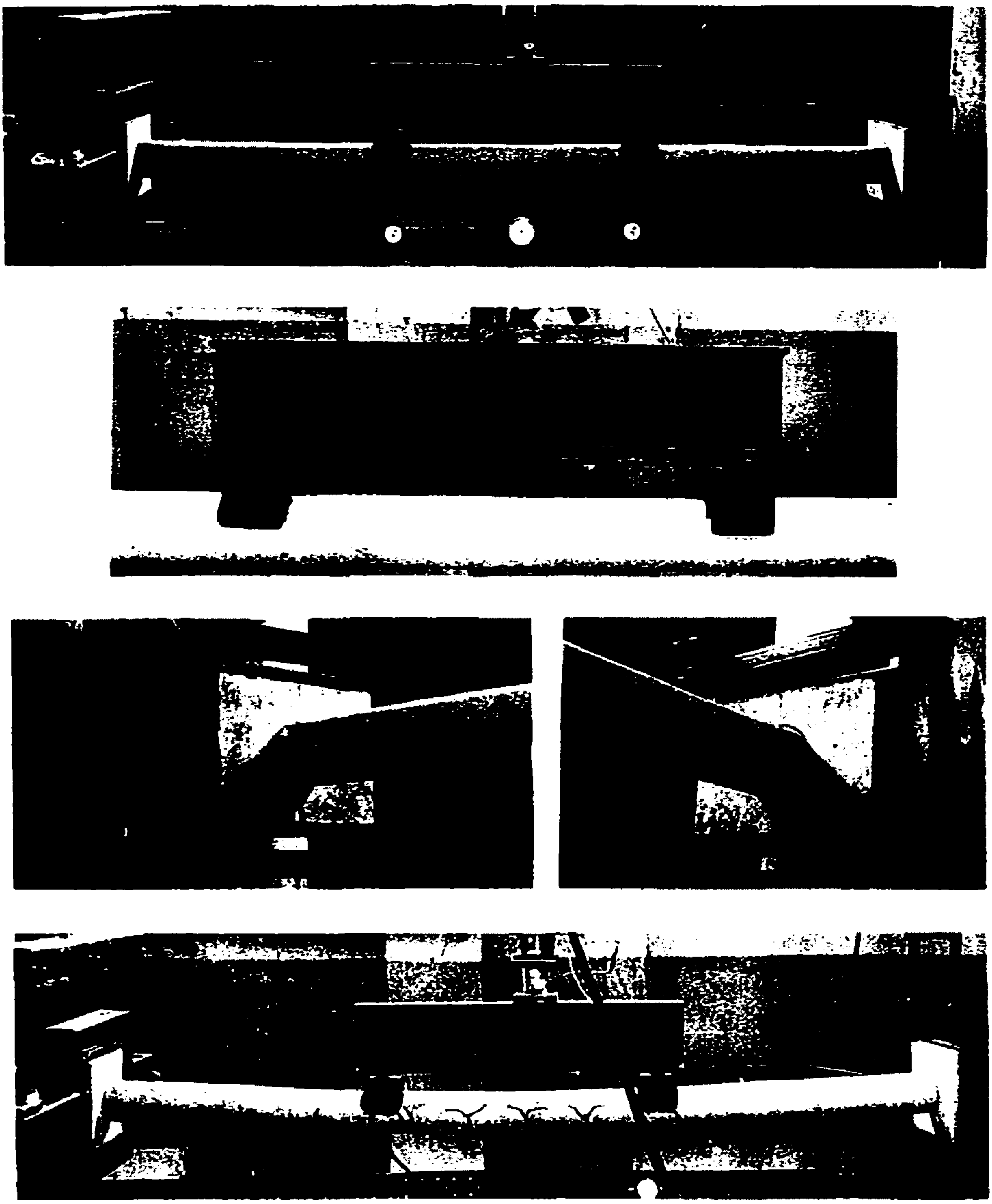


Figure 7. Prestressed Concrete Columns under Torsion and Bending

by a 50,000 lb. capacity load cell. The load was applied by a 50 ton jack mounted to the top beam.

The Torsion Apparatus

The simultaneous combination of three types of loading, at one time, bending moment, concentric load and torsion, necessitates the construction of a rig that will perform the application of these loads individually or collectively.

The torsion rig (Fig. 5 and 6) consisted of a rectangular frame that fitted around the head of the column. The frame itself was connected to two bearings allowing the head of the column to rotate freely. The other side of the bearings was welded to another frame through which the torsion moment was applied. The concentric load was applied through a hydraulic jack mounted on a movable plate in order to facilitate the change of eccentricity of the load. The movable plate in turn was mounted on a column on the west side of the specimen.

The outside frame of the torsion rig was designed in such a way that the torsion moment was applied through one hydraulic jack on the south side of the specimen (Fig. 6).

Instrumentation

The prestressing force in each cable was determined by a load cell and checked by measuring the total elongation of the cable. Four load cells were used to measure the different applied loads, they have 50,000, 20,000, 18,000, and 3000 lb. capacity. All the load cells were connected to a Budd portable digital strain indicator model p-350. They were calibrated before testing and their calibration was checked during the entire testing period.

For testing, a column was inserted in the proper position and fixed by its head in the west side of the frame. The torsion rig then was inserted in the other head of the specimen in the east side of the frame. A very small load, 20 to 30 lb., was applied and all initial readings taken. An axial shortening was applied to the column and the load read on the strain indicator. There was an initial increase in the load which followed by a relaxation and load drop. Equilibrium was assumed to exist when the load drop was less than 50 lb. per 30 sec. At that state of equilibrium all readings were taken and the column was examined for cracks. Another increment of load was applied and the process was repeated until material failure occurred. The increments were decreased at the final stages, and a total of 15 to 20 increments were obtained and tests on the average lasted 2 hr. 30 min.

The deflected shape during loading was measured by four dial gages, and the initial deflected shape of the column was determined by measuring, to an accuracy of 1/64 inches, from a thin string stretched between the column ends.

CHAPTER IV

PRESENTATION AND DISCUSSION OF TEST RESULTS

Introduction

A column is a structural member under compression parallel to its longitudinal axis. Most frequently bending moment is induced either through compression load eccentricity or laterally applied forces. In addition, transverse shear as well as torsion may be present. Columns will bend in a uniaxial or biaxial manner, depending on the location of the eccentric load. Biaxial bending is a complex problem, and it will not be dealt with in this research program.

Concrete columns prestressed or conventionally reinforced are in general eccentrically loaded. When the eccentricity is extremely large the column reduces to a "beam like" column and the case approaches that of a pure flexure. Columns concentrically loaded are rare and such specifying groups as ACI require designs for at least a minimum eccentricity. Concentric loads were considered in this study because they constitute the limiting case at one extreme of the interaction diagram for axial load and the bending moment.

Columns in general, prestressed or not, are affected by other factors beside external loads. Slenderness ratio and end conditions will control the column's ultimate capacity and its collapse mechanism. A prestressed column that is very slender will collapse through instability which is caused by out of plane motion rather than material failure. All columns tested in this study had the same slenderness ratio (h/4 = 80), that ensured material failure. It was not intended to study the collapse of long prestressed columns through instability, although this problem should be considered in further research. Also, in the case of bending moment combined with concentric load, the moment will be magnified due to the deflected shape of the column. Although the deflection will be less in the case of a prestressed column compared to a conventionally reinforced one, having the same end restraints, nevertheless, this magnification is very sensitive to the slenderness of the column. Hence an estimate of the slenderness effect on the ultimate capacity of the column should be studied throughout the entire range of h/r.

The effective length is not the same as the unbraced length of the column. Its value should depend on the nature and the degree of the restraint present at both ends of the columns. Rational methods give provisions for calculating the effective length for reinforced concrete columns.[76] Such provisions for prestressed concrete columns are yet to come and need further research.

Columns tested in this study were fixed at one end and restrained from translation at the other end, where the loads were applied. This choice in end conditions facilitates testing the specimens separately or under combined loads with minimum amount of change in the testing apparatus.

It was not intended in this study to vary the strength of concrete; nonetheless the cylinder strength of concrete ranged from 4000 to

6900 psi with an average of 5500 psi. In combine loading it was necessary to account for this change by multiplying torsion and axial load by $\sqrt{5500/f'_c}$ and bending moment by $(5500/f'_c)$. Although this was practiced by many previous researchers[51,75] and could be debatable, nevertheless, this was necessary as an attempt to eliminate f'_c as a variable in this investigation.

Stress losses due to transfer of prestressing force, shrinkage, elastic compression, relaxation in the steel tendons and creep were considered and were estimated at 20% of the prestressing force.[60]

Concentric Load

Test data for the two columns tested under concentric loads are given in Table 3. Most of the columns developed a small initial curvature before testing and hence no specimen was tested with truly axial load. Figure 8 shows conditions at ultimate for a realistic and idealized prestressed concrete column under axial load.

A circular section with prestressing steel in the center, loaded as shown in Figure 8a must satisfy the conditions of force equilibrium and strain compatibility:

Equilibrium of forces: $C = P_{uo} + T$

$$P_{uo} = K_3 f'_c K_1 A - f_s A_s$$

$$P_{uo} = K_3 f'_c K_1 A - A_s E_s (\varepsilon_{so} - \Delta\varepsilon_s) \qquad (1)$$

Equilibrium of moments: $M_b = K_3 f'_c K_1 A \, (R - K_2 c) \qquad (2)$

Compatibility equation: $\Delta\varepsilon_s = \frac{R}{c} (\varepsilon_u) \qquad (3)$

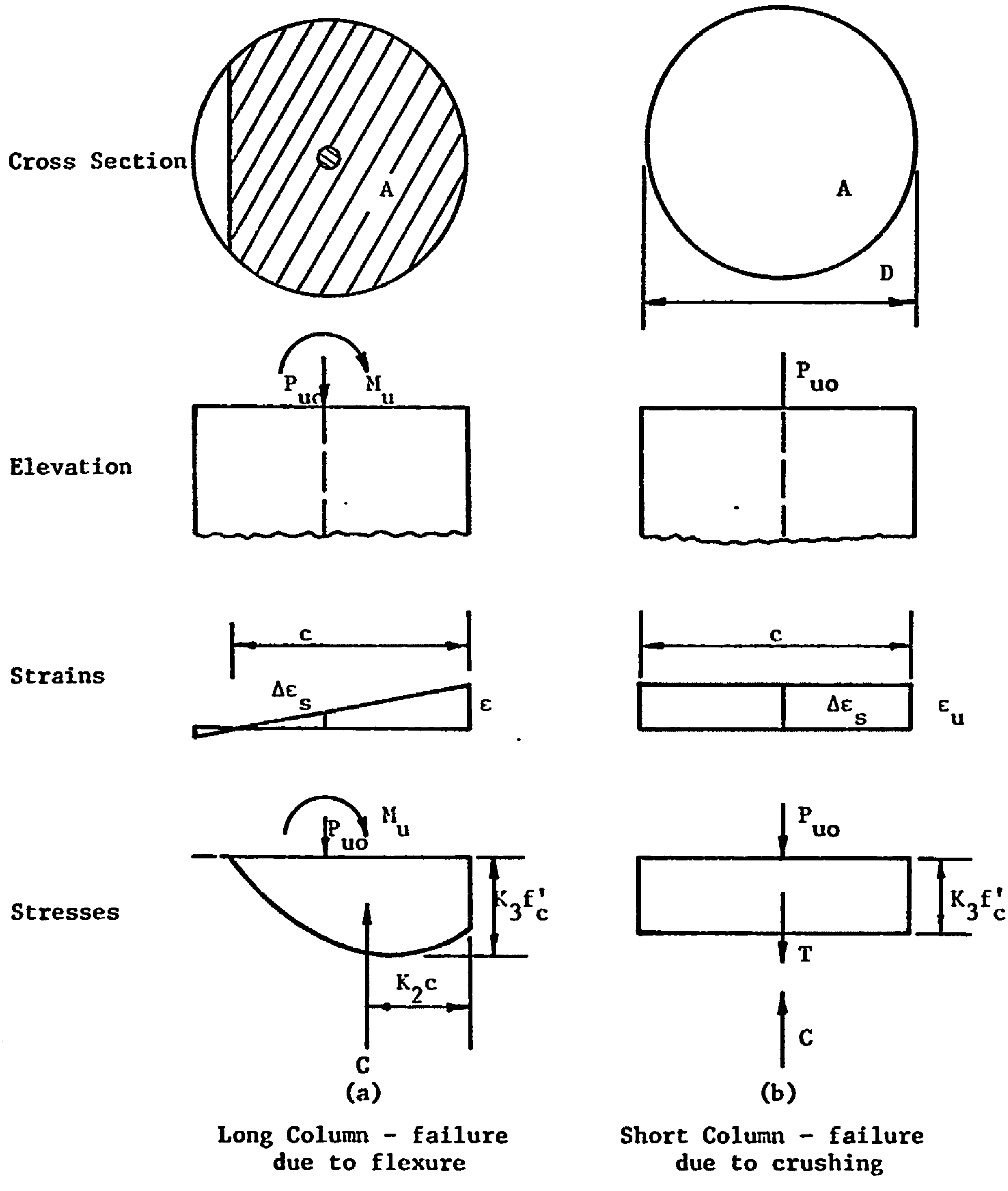


Figure 8. Conditions at Ultimate in Concentrically Loaded Prestressed Concrete Columns

Total strain in the prestressing strand: $\varepsilon_s = \varepsilon_{so} - \Delta\varepsilon_s$ and

$$\varepsilon_s = \frac{P_r}{A_s E_s} - \frac{R}{c}(\varepsilon_u) \tag{4}$$

where

C = total internal compressive force in concrete

P_{uo} = ultimate load of concentrically loaded column

K_1, K_2 = coefficients related to magnitude and position of internal compressive force in concrete

K_3 = 0.85

T = internal force in prestressed cable

R = radius of circular cross section

A = area of concrete in compression

M_b = maximum bending moment

$\Delta\varepsilon_s$ = change in the strain of prestress cable

ε_{so} = strain in prestressed strand due to prestressing forces after losses

ε_u = ultimate compressive strain

P_r = prestressing force after losses.

Results of previous tests[2] indicated that the values of K_1 and K_2 decrease as f'_c increases. The value 0.85 for K_3 was used, adopting Hognestad idealized stress-strain diagram for concrete. Although these constants were obtained considering rectangular cross sections, they were used here as a suitable approximation.

The area of the cross section in compression, A, can be readily evaluated from the following expression and Figure 9.

$$\text{Cos } \theta = \frac{c-R}{R}$$

$$A = R^2 (\pi - \theta) + R\,(c-R)\ \text{Sin } \theta \tag{5}$$

TABLE 3

COLUMN TEST RESULTS FOR CONCENTRIC LOADING

Column	f'_c (psi)	Ultimate Axial Force (lb) Theory*	Ultimate Axial Force (lb) Exp.	$\frac{P_{uo}\ (\text{Exp.})}{P_{uo}\ (\text{Theory})}$	Ultimate Strain in Concrete in/in	Initial Deflection (inches)	Failure Distance From Loaded End (inches)
B-2	4000	29,600	29,500	0.996	0.00516	0.156	65
B-3	4080	30,200	29,400	0.974	0.00448	0.235	65 & 24

*Equations 1-8.

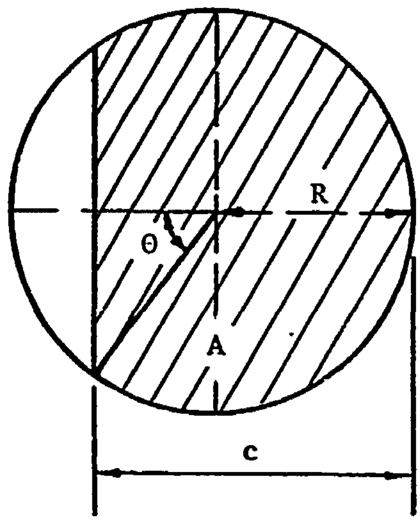


Figure 9. Properties of a circle segment

The moment M_b in Equation (2) is related to short columns with initial curvature or to long columns with substantional lateral deflections at ultimate axial load, with or without initial curvature. It was observed that even the slightest preloading deflection magnified the moment considerably in long prestressed columns and hence reduced the ultimate load capacity.

Conventional reinforcing steel used in the heads extended ten inches into the column to insure against local failure between the head and the end of the column. It is clear that the maximum moment will be at the fixed end of the column, but the presence of the reinforcing steel forces collapse of the column to some other high moment zone but with a lesser column moment of inertia. This was evident in both columns tested where the failure plane was at the cut off points of the reinforcing steel. Figures B-1 and B-2 show the variation of lateral deflections at various points along the column with the applied load. Measured deflections were used in evaluating the moments at points of collapse.

Column B-2, shown in Figure 10, collapsed in compression in two different places simultaneously. The ultimate axial loads were the same,

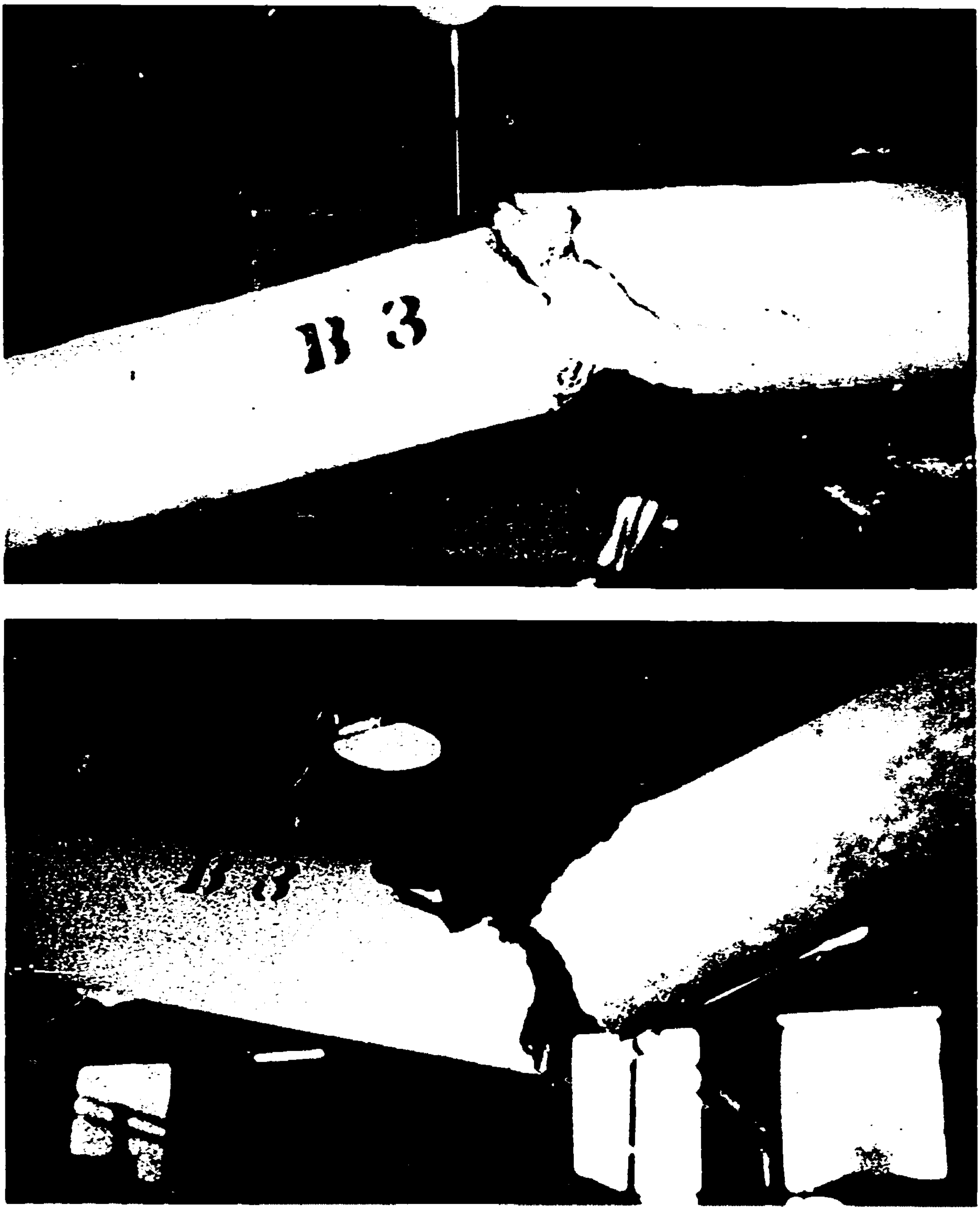


Figure 10. Crack Pattern for Axially Loaded·Column

but the moments were opposite in direction causing one plane of failure to look as an inverse of the other.

A prestressed concrete short column, loaded axially and exhibiting no pre-loading curvature, will have conditions at ultimate as shown in Figure 8b. Such a column must satisfy two conditions; (1) the prestressing stresses are low and uniform, (2) the column has a small h/r ratio to be efficient as a compression member. With reference to Figure 11, the following equations are available.

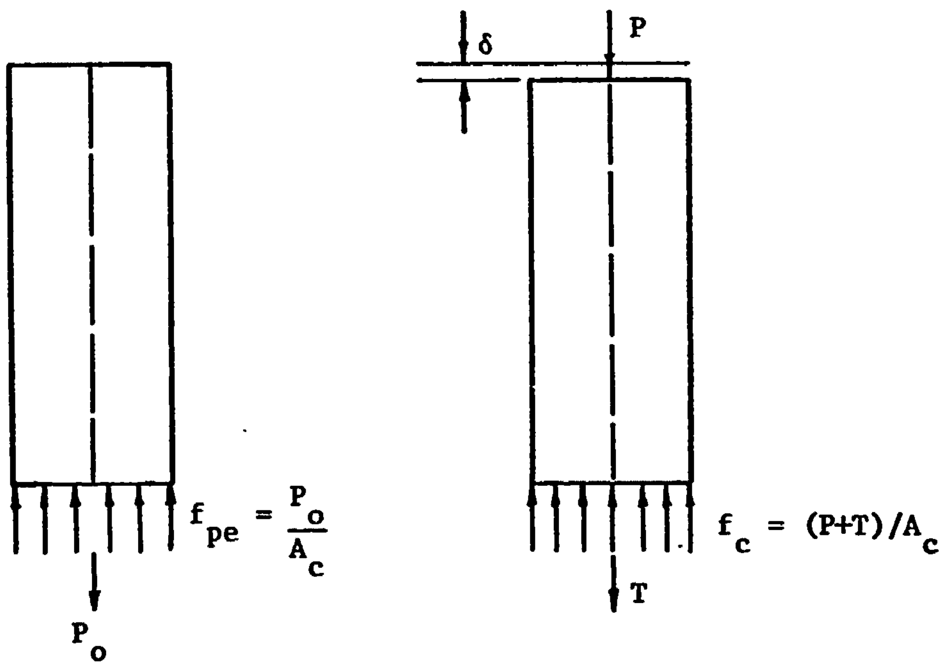


Figure 11. Prestress loss due to concentric load

$$T = P_o - \Delta\varepsilon E_s A_s$$

$$T = P_o - \frac{A_s E_s}{E_c} (f_c - f_{pe})$$

where $$\Delta\varepsilon = \frac{f_c - f_{pe}}{E_c},$$

Then $$T = P_o - \frac{E_s A_s}{E_c A_c} (P + T - P_o)$$

and $$T = P_o - \frac{K}{K+1} P .$$

In the above $$K = \frac{E_s A_s}{E_c A_c} , \qquad (6)$$

and $$\frac{f_c f_{pe}}{E_c} < \varepsilon_u . \qquad (7)$$

Note that $$T = 0 \text{ when } \frac{K}{K+1} P = P_o \qquad (8)$$

where

A_s = area of prestressed strand

A_c = gross cross section area of prestressed column

P_o = prestressing force after losses

P = axial load

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of prestressed strand

f_{pe} = compressive stress in concrete due to prestressing force after losses

f_c = compressive stress in concrete.

Eccentric Load

These tests consisted of three columns. Two of the columns had an initial curvature before testing, hence all three columns had different eccentricity at time of testing. All three test specimens, shown in Figure 12, collapsed due to crushing of the concrete along nearly identical planes of failure. The strain in the prestressed steel had an average of 40% of its ultimate. The ultimate loads and moment are shown in Table 4. Eccentricity values included measurements of the initial deflections.

TABLE 4

COLUMN TEST RESULTS FOR ECCENTRIC LOADING

(1) Column	(2) F'_c	(3) First Crack Axial Load (lbs)	(3) First Crack Moment (in-lb)	(4) Eccentricity e (in)	(5) Ultimate Moment (in-lb) Theory*	(5) Ultimate Moment (in-lb) Exp	(6) Ultimate Axial Load Exp. (lbs)	(7) Ultimate Axial Strain (in/in) Concrete	(7) Ultimate Axial Strain (in/in) Steel	(8) Initial Deflection (in)	(9) Failure Distance From Loaded End (in)
E-3	5,560	6000	10,600	2.075	21,200	19,400	11,000	0.0074	0.00301	0.075	12
F-1	6,200	6000	11,600	2.156	22,400	23,400	11,700	0.0073	0.00331	0.156	11
F-2	6,625	7000	11,110	2.0	25,000	28,350	15,400	0.0083	0.00293	0	12

*Equations 9-11

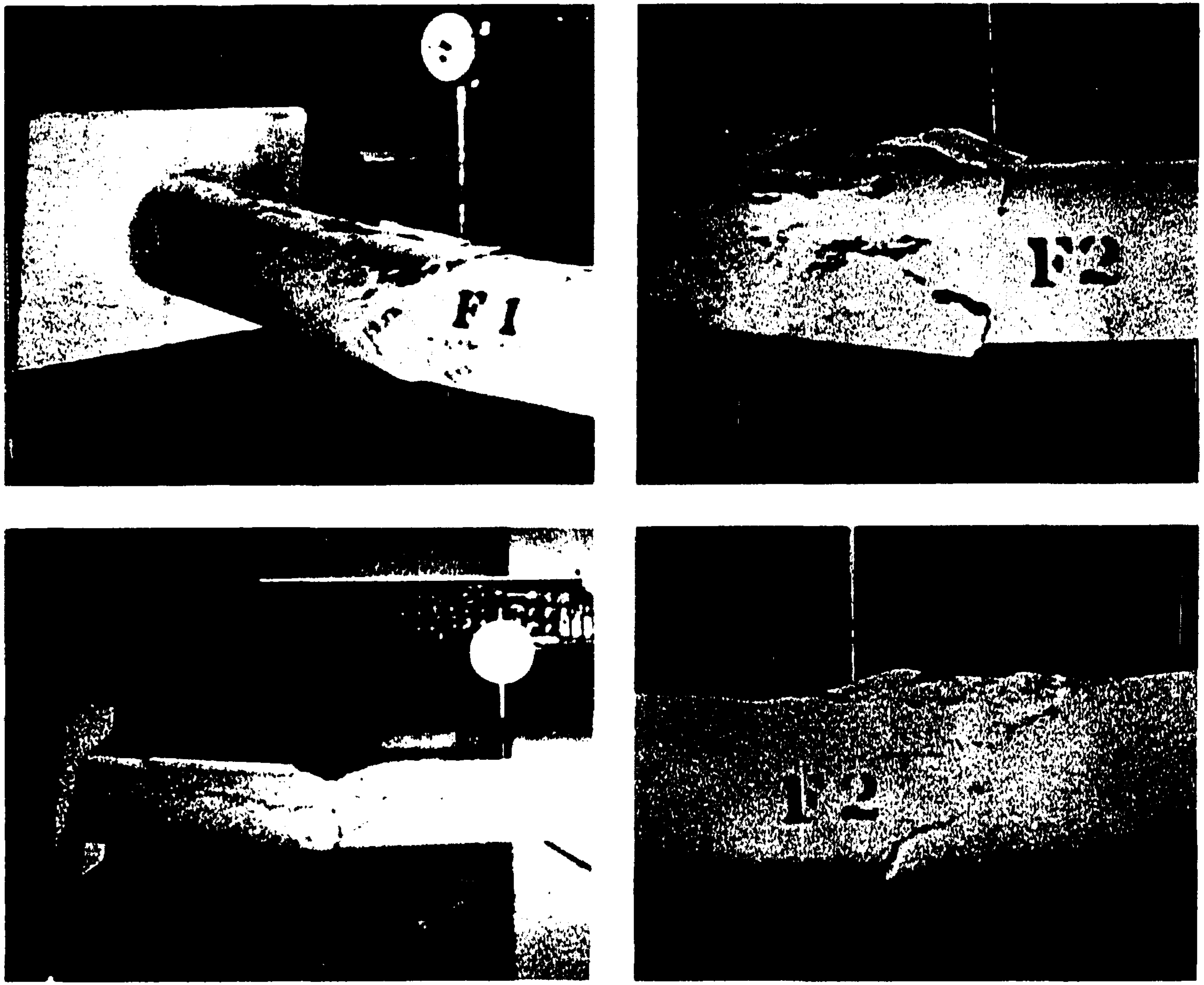


Figure 12. Crack Pattern for Eccentrically Loaded Columns

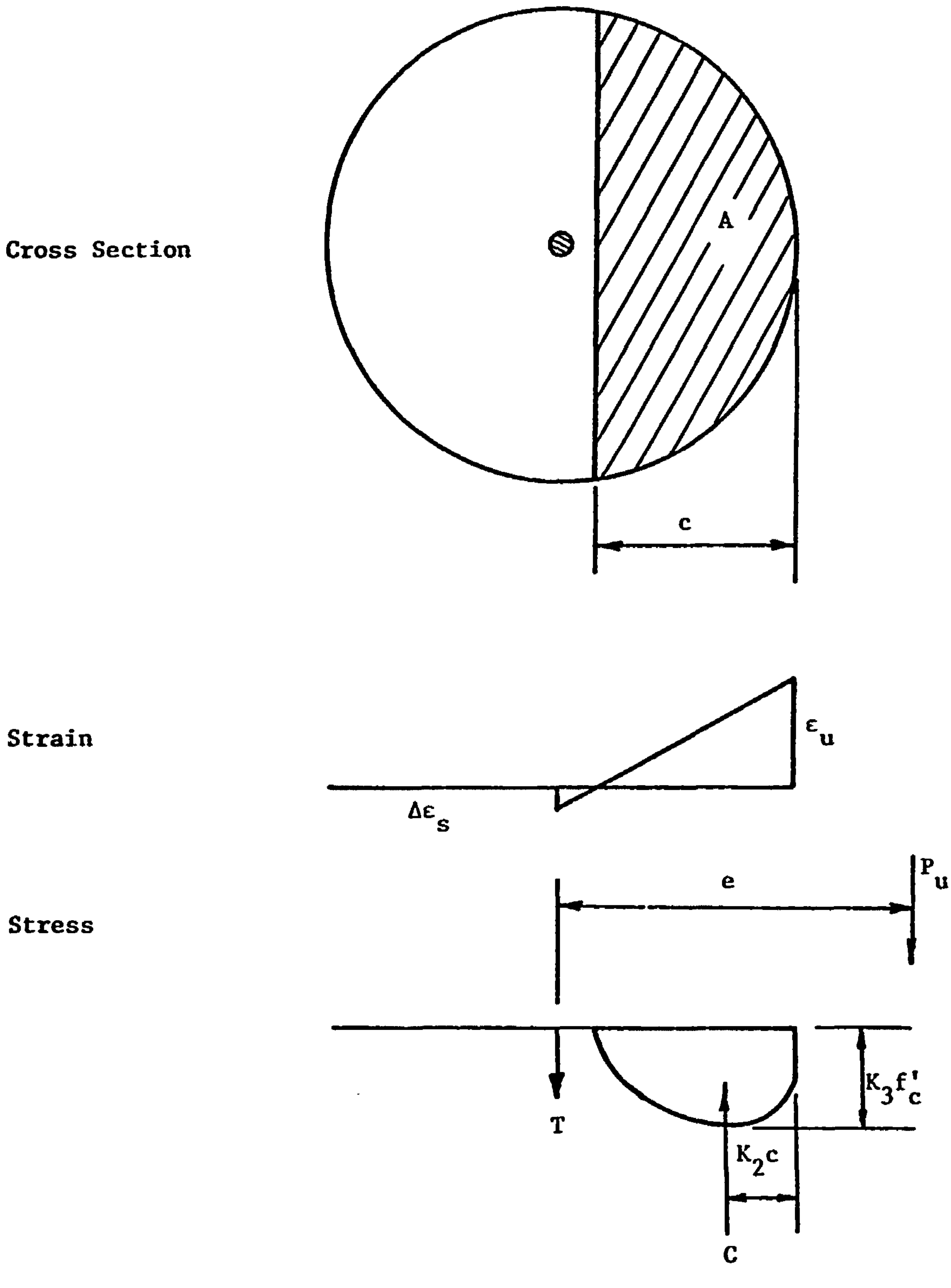


Figure 13. Conditions at Ultimate in an Eccentrically Loaded Prestressed Concrete Column

Conditions at ultimate are shown in Figure 13 and the equations of equilibrium and compatibility are as follows:

Equilibrium of forces: $C = P_u + T$

$$P_u = K_3 f'_c K_1 A = f_s A_s \quad \text{and}$$

$$P_u = K_1 K_3 f'_c A - E_s A_s (\varepsilon_{so} + \Delta\varepsilon_s) \tag{9}$$

Equilibrium of moments: $M_b = K_1 K_3 f'_c A \; (R - K_2 c)$ (10)

Compatibility equation: $\Delta\varepsilon_s = \frac{R - c}{c} \; (\varepsilon_u)$ (11)

Total strain in the prestressed strand, ε_s, is

$$\varepsilon_s = \varepsilon_{so} + \Delta\varepsilon_s \quad \text{or}$$

$$\varepsilon_s = \frac{P_r}{A_s E_s} + \frac{R - c}{c} \; (\varepsilon_u) \tag{12}$$

The area of the section in compression, A, can be readily evaluated from the following expression and Figure 14.

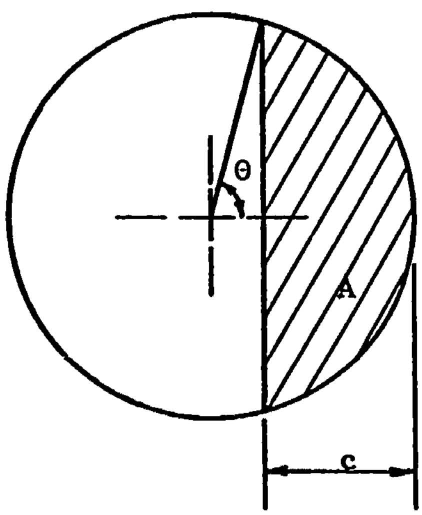


Figure 14. Properties of a circle segment

$$\cos\theta = \frac{R - c}{R}$$

and

$$A = \frac{\theta}{\pi} A_c - \frac{R^2}{2} \sin 2\theta \tag{13}$$

The moment M_b in equation (10) is the maximum moment developed in the column due to the eccentric load at collapse. Appendix figures B-3, B-4, and B-5 show the variation of column deflections taken at several points along the column as load was applied. These measured deflections together with applied loading were used to arrive at the maximum moment developed in the column. It is obvious that the moment should be the largest at the loaded end, but these columns were designed and reinforced in both heads up to a distance of eleven inches inside the column to insure that no "end" failures would occur. Therefore, the column would collapse in the next weakest section undergoing the maximum moment in the area where the prestressed cable is the sole reinforcement. This was evident from the test results in that all columns collapsed at essentially the same point and under the maximum stress, where change in moment of inertia, I, is considered.

Pure Moment

The tests consisted of two columns. Figure 15 represents conditions at ultimate and the equations of equilibrium and compatibility are as follows:

Equilibrium of forces: $C = T$

$$K_1 K_3 f'_c A = A_s f_s$$

$$K_1 K_3 f'_c A = E_s A_s (\varepsilon_{so} + \Delta\varepsilon_s) \tag{14}$$

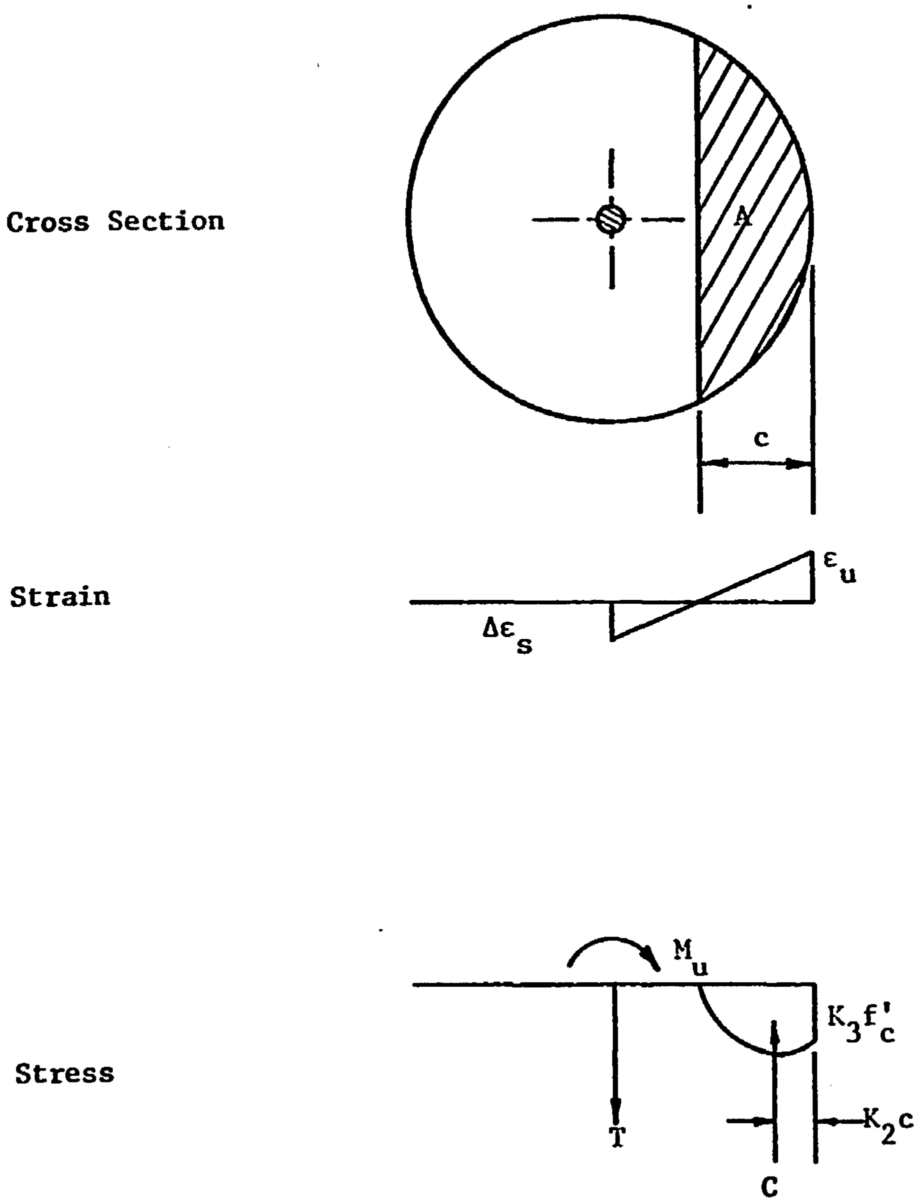


Figure 15. Conditions at Ultimate in a Prestressed Concrete Column Subject to Pure Bending Moment

TABLE 5

COLUMN TEST RESULTS FOR PURE BENDING

Column	f'_c (psi)	Cracking Moment (in-lb) Theory*	Cracking Moment (in-lb) Exp.	Ultimate Moment M_u (in-lb)	M. Theory / M. Test	Ultimate Strain (in/in) Concrete	Ultimate Strain (in/in) Steel	Failure Distance From Left End (in)	Initial Midheight Deflection (in)
G-1	6,425	6,100	6,520	17,600	.87	0.0097	0.00814	29	0.375
G-2	6,425	6,100	7,500	17,600	1.08	0.0097	0.00814	37.5	0

*Equation 15.

Equilibrium of moments: $M_u = K_1K_3f'_cA\ (R - K_2c)$ (15)

Compatibility equation: $\Delta\varepsilon_s = \frac{R - c}{c}\ (\varepsilon_u)$ (16)

From Equation (13): $A = \frac{\theta A_c}{\pi} - \frac{R^2}{2}\ \text{Sin}\ 2\theta$ (17)

In a prestressed concrete column, undergoing pure bending moment, collapse can occur when either the prestressing steel or the concrete fails. In practice, however, the presence of large amounts of prestressing steel will prevent the failure of steel before the crushing of concrete.[73] Hence, practically speaking, collapse in a prestressed concrete column will be the result of the crushing of the concrete in the compressive zone.

The strain in the prestressed steel will be

$$\varepsilon_s = \Delta\varepsilon_s + \varepsilon_{so} < \varepsilon_{su}$$

$$T = A_sE_s\varepsilon_s\ , \tag{18}$$

where

ε_{su} = ultimate strain in prestressed strand

if $\varepsilon_s > \varepsilon_{su}$.

Hence $T = \varepsilon_{su}f_{sy}$ (Tension failure)

where f_{sy} = nominal yield strength of prestressing strand.

From Equation (14)

$$K_1K_3f'_cA = A_sf_{sy}$$

and

$$A = \frac{A_sf_{sy}}{K_1K_3f'_c} \tag{19}$$

then c can be found from Equation (13) and

$$\Delta\varepsilon_s = \varepsilon_{su} - \varepsilon_{so} \tag{20}$$

$$\Delta\varepsilon_s = \frac{1}{E}(f_{su} - f_{pe}) \tag{21}$$

Substituting in Equation (16)

$$\varepsilon_u = \frac{c}{R - c}\,\Delta\varepsilon_s \tag{22}$$

Cracking moments based on elastic theory and ultimate moment are shown in Table 5. Figures B-6 and B-7 show the variation of mid-height deflections with applied moment. The columns collapsed by failure of the concrete in the compressive zone. The calculated strains for the prestressed steel indicated that the prestressed strand was close to its ultimate strain value.

Interaction Diagram of Concentric Load and Moment

Prestressed concrete columns can be concentrically or eccentrically loaded. Concentrically loaded columns are more likely to collapse by crushing of the concrete, or by instability if the column is a slender member. On the other hand, eccentrically loaded columns will collapse due to the crushing of the concrete in the compressive zone or the strain in the prestressing steel reaches its ultimate strain value.

The relationship between the ultimate moment and the ultimate load, called the interaction diagram, gives a graphical representation of the entire range of load moment relationship. This relationship will hold for any given eccentricity and a certain level of prestressing if the following assumptions are made:

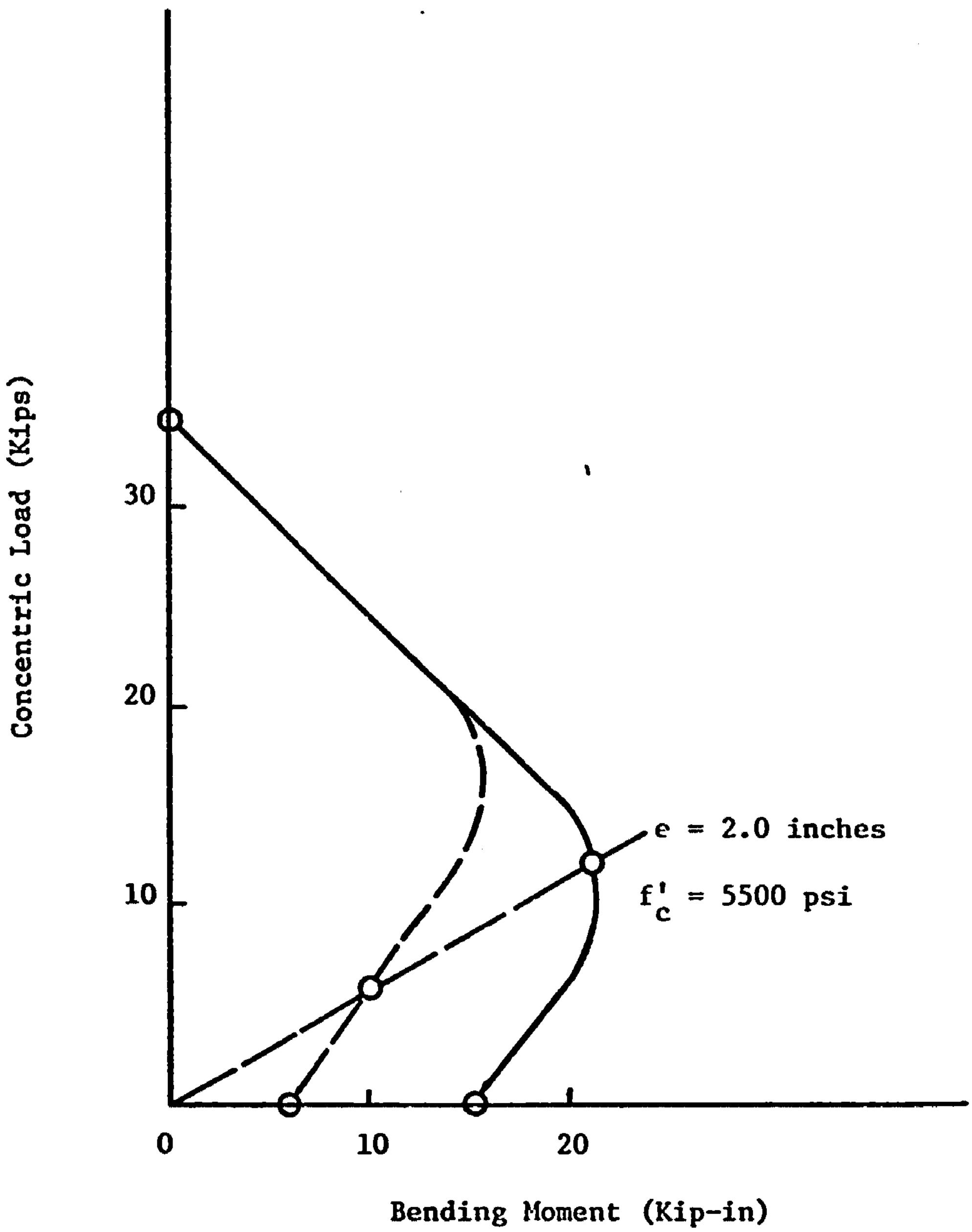


Figure 16. Interaction Diagram of a Prestressed Concrete Column

1. The stress-strain relationship for both concrete and prestressed steel are known.
2. Concrete will exhibit a linear strain distribution with depth in the compression zone.
3. The section will collapse when the strain in the concrete in its extreme fiber reaches ε_u or the prestressing steel reaches ε_{su}.

The solid curve in Figure 16 represents the ultimate interaction diagram at collapse while the broken line represents the conditions at cracking. The points plotted are those previously discussed in the preceding three sections of this chapter. For this diagram to have any reasonable accuracy more tests are necessary.

The interaction diagram presented lacks the sharp curvature at the balance point that is characteristic of the interaction diagram of reinforced concrete sections. Prestressed steels are high-strength and lack a definite yield point which accounts for the absence of a definite balance point and results instead in a less shapr curvature of the diagram.

Pure Torsion

Diagonal tension cracks occur when the principal stress, σ_1, exceeds the tensile strength of plain concrete. The principal tensile stress can be equated as:

$$\sigma_1 = f_t = KT_o$$

where

T_o = torsional strength of plain concrete section

K = constant, a function of the cross section

f_t = tensile strength of concrete.

In a uniformly prestressed section, the minor principal stress due to the combined action of the prestressing and the torsional shear, τ_p, is given by

$$\sigma_1 = \tfrac{1}{2}\sqrt{f_{pe}^2 + 4\tau_p^2} - \tfrac{1}{2} f_{pe}$$

where

f_{pe} = effective compressive stress due to uniform prestressing

τ_p = KT_p, the maximum shearing stress due to torsion

T_p = torsional strength of uniformly prestressed section.

Hence,

$$\sigma_1 = KT_o = \tfrac{1}{2}\sqrt{f_{pe}^2 + 4K^2T_o^2} - \tfrac{1}{2} f_{pe}$$

and $$(f_{pe} + 2KT_o)^2 = f_{pe}^2 + 4K^2T_p^2$$

and $$T_p^2 = T_o^2 + \frac{Tf_{pe}}{K}$$

or $$T_p^2 = T_o^2 \left(1 + \frac{f_{pe}}{T_o K}\right)$$

and since $$KT_o = f_t$$

then $$T_p = T_o\sqrt{1 + \frac{f_{pe}}{f_t}} \quad . \tag{23}$$

The initial cracking moment for columns subject to torsion is shown in Table 6. The cracks, as soon as they were formed, propagated on both sides of the column in a spiral manner with an angle smaller than 45°, which is a function of the prestress compressive stress (Figure 17). As the torsional moment increased, it was followed immediately by a

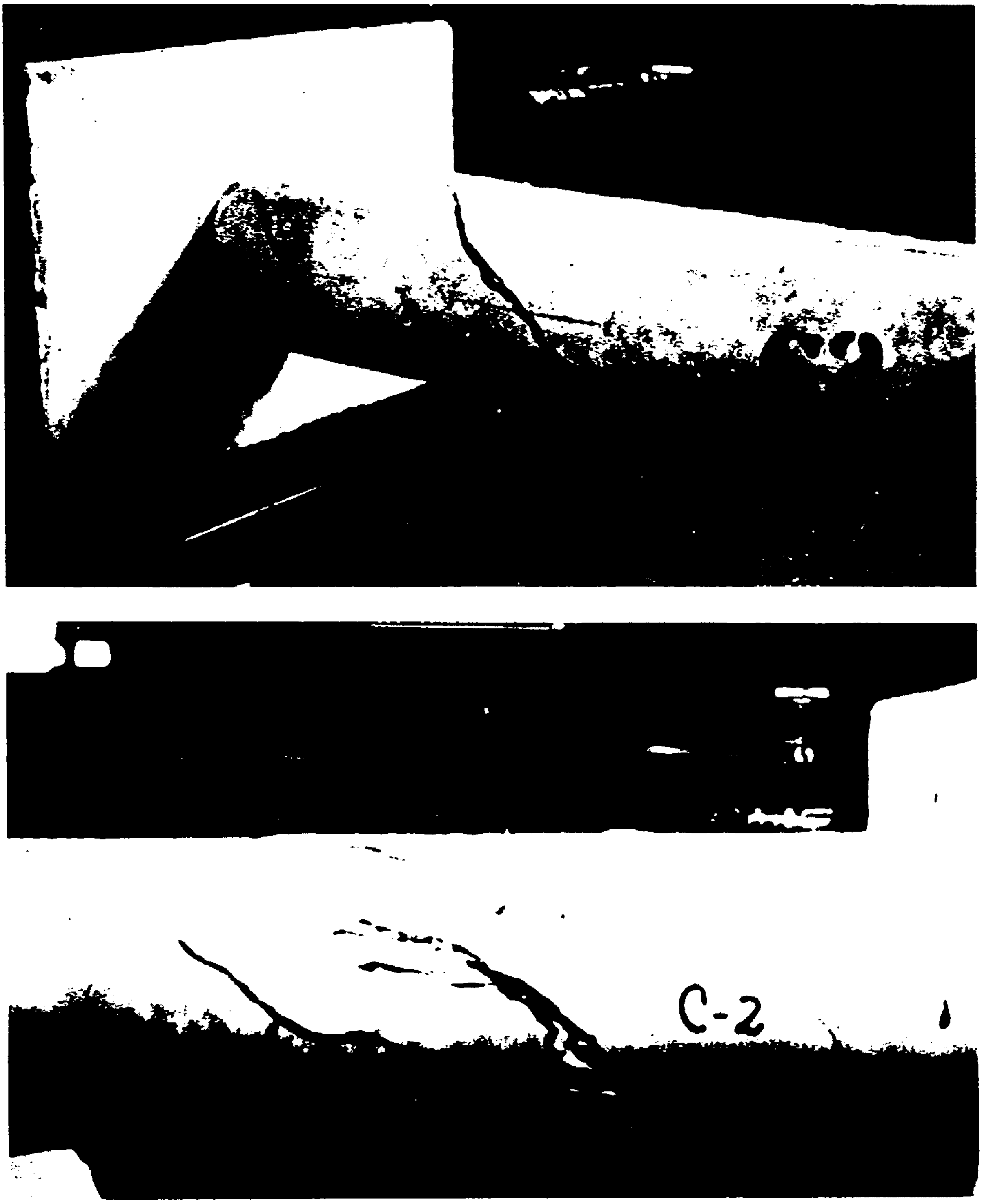


Figure 17. Crack Pattern in a Prestressed Concrete Column Subject to Pure Torsion

TABLE 6

COLUMN TEST RESULTS FOR PURE TORSION

Column	f'_c (psi)	Cracking Moment (Tp) (in-lb)		Ratio	Ultimate Torsional Moment (T_{pu}) (in-lb)		Ratio	Angle of Twist (Degree)	
		Theory*	Exp.	$\frac{Tp\ (Exp.)}{Tp\ (Theo.)}$	Plastic Theo.**	Exp.	$\frac{T_{pu}\ (Exp.)}{T_{pu}\ (Theo.)}$	Theo.	Exp.
C-2	4,660	4,180	4,275	1.02	5,700	5,344	.94	.546	0.60
C-3	4,320	4,000	4,061	1.01	5,340	5,130	.96	.568	0.55

*Equation 23.

**Equation 27.

complete collapse. The mode of failure was characterized by a clean, granular appearing crack, the result of diagonal tension and collapse was not accompanied by any noise.

Torsion Combined with Concentric Load

The initial cracking moment for the two columns subject to concentric load and torsion is shown in Table 7. The cracking torsional moment can again be computed from Equation (23) where f_{pe} can be replaced by f_c, the compressive stress in the concrete due to the axial load and the effective stress induced by prestressing.

In reality, the compressive stress f_c can be the result of prestressing or the combined effect of prestressing and axial load so long as the strain in compression does not exceed the ultimate compressive strain of the concrete.

Previous test results[69] on rectnagular sections indicated that f_{pe} can be as large as 0.60 f'_c (see Fig. 19). Any further increase in the prestressing level will decrease the strength of the section to resist torsion. A straight line was drawn between the cut-off point and P/Pu = 1 (broken line in Fig. 19) in order to close the interaction diagram. This is an approximation and should be verified by further research in that region of the diagram.

When an axial force P is added, the compressive stress becomes

$$f_c = \frac{P + T}{A_c} \quad . \tag{24}$$

Substituting in Equation (6) and simplifying . .

$$f_c = \frac{1}{A_c} (P_o + \frac{P}{K + 1}) \tag{25}$$

TABLE 7

TEST RESULTS, COMBINED AXIAL LOAD AND TORSION

Column	f'_c (psi)	Cracking Moment T_p in-lb Theo.*	Cracking Moment T_p in-lb Exp.	Ratio $\frac{T_p(\text{Exp})}{T_p(\text{Theo})}$	Ultimate Moment T_{pu} (in-lb) Theo.**	Ultimate Moment T_{pu} (in-lb) Exp.	Ratio $\frac{T_{pu}(\text{Exp})}{T_{pu}(\text{Theo})}$	Angle of Twist at Failure (Degree) Theo.	Angle of Twist at Failure (Degree) Exp.
E-1	5,362	6,000	5,980	0.996	7,960	6,626	.834	0.716	0.56
E-2	5,362	6,900	7,050	1.020	9,200	8,764	.954	0.840	1.10

*Equation (26).

**Equation (27).

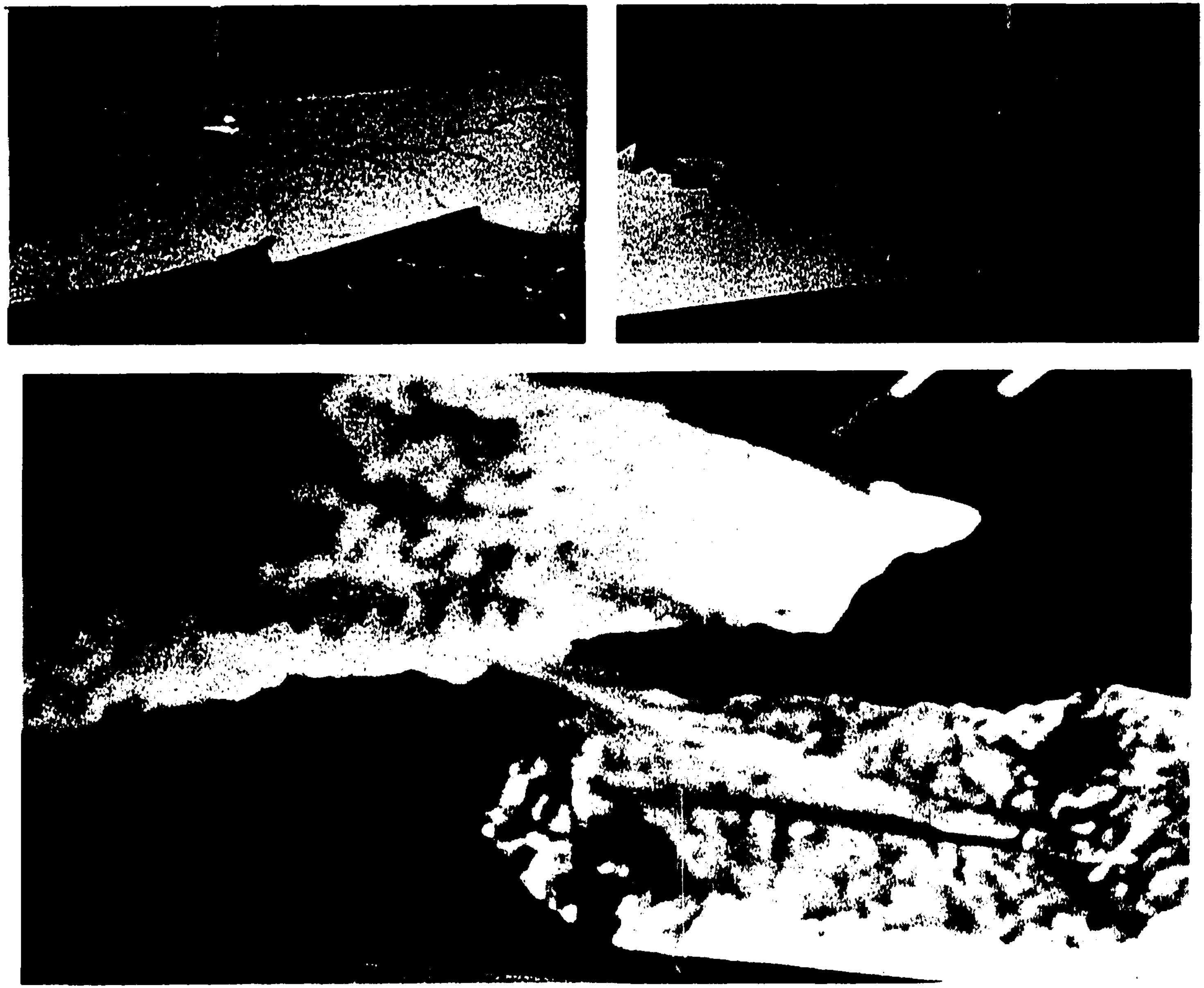


Figure 18. Crack Pattern in Prestressed Concrete Columns Subject to Combined Torsion and Concentric Load

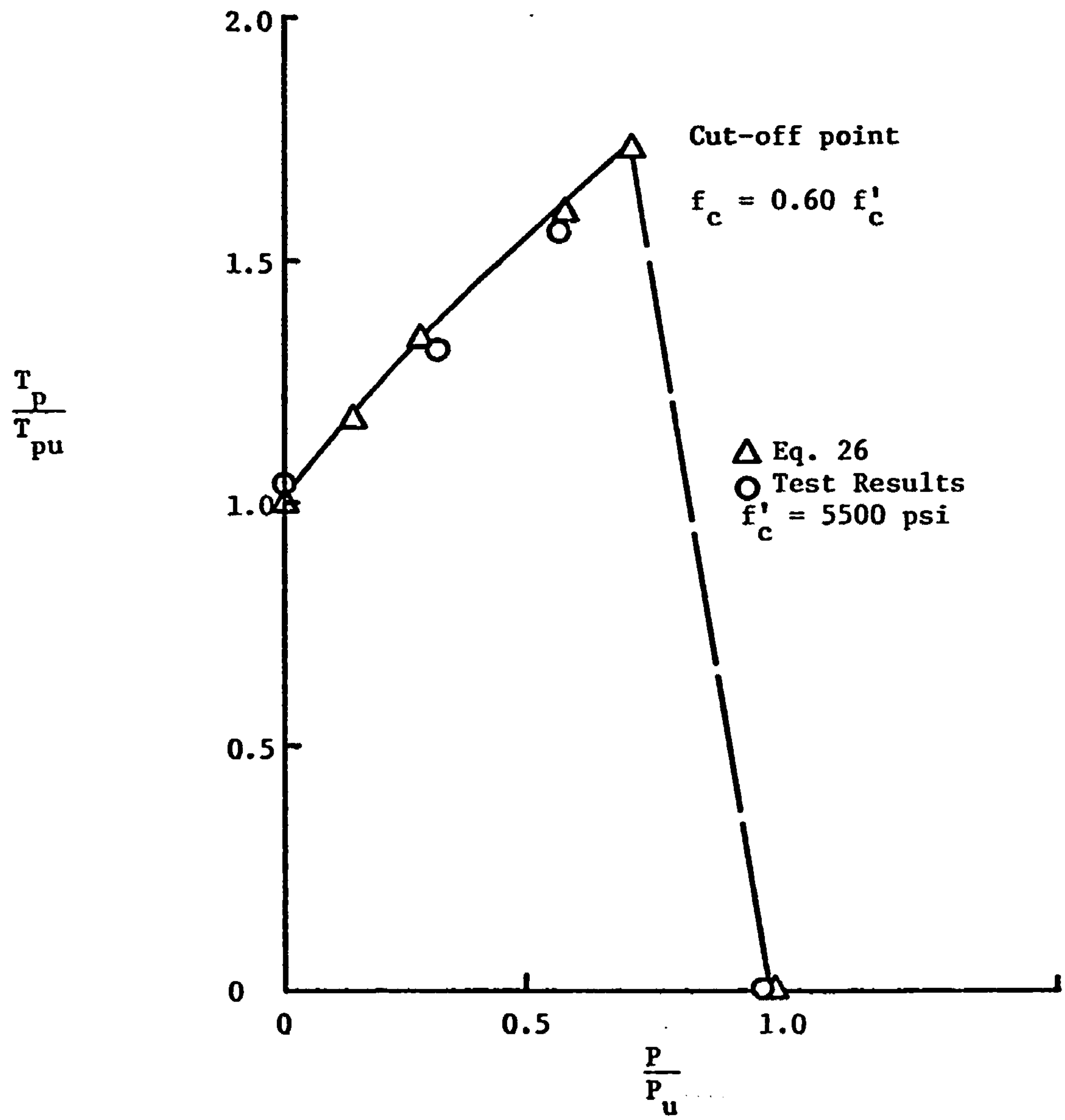


Figure 19. Interaction Diagram for a Prestressed Concrete Column under Combined Concentric Load and Torsion

where

$$K = \frac{E_s A_s}{E_c A_c}$$

and Equation (23) will change to

$$T_p = T_o \sqrt{1 + \frac{f_c}{f_t}} \quad . \tag{26}$$

The maximum shearing stress, τ_p, in a prestressed column can be evaluated as follows:

$$\tau_p = \frac{2T_p}{\pi R^3}$$

and the plastic theory gives the ultimate torsional moment as

$$T_{pu} = \tau_p \cdot \frac{2\pi R^3}{3} \quad ,$$

hence

$$T_{pu} = \frac{4}{3} T_p \quad . \tag{27}$$

The mode of failure of prestressed columns under the combined action of torsion and concentric load was the same as in the case under pure torsion. The cracks developed in a spiral manner but with a sharper angle of inclination with the longitudinal axis of the column depending on the compressive force applied. Collapse was sudden and accompanied with noise and debris (Fig. 18).

Torsion Combined with Bending

In a prestressed concrete column cracks develop due to torsion when the ultimate tensile stress is reached. The angle at which they

TABLE 8

COLUMN TEST RESULTS FOR TORSION COMBINED WITH BENDING

Column	f'_c	Torsion (in-lb)	Bending Moment (in-lb)		Ratio $\frac{M_b \text{ (Exp.)}}{M_b \text{ (Theo.)}}$	Mode of Failure	Torsion (in-lb)	Bending Moment (in-lb)		Ratio $\frac{M_b \text{ (Exp.)}}{M_b \text{ (Theo.)}}$	Mode of Collapse
			Theo.*	Exp.				Theo.**	Exp.		
A-3	4,400	2,138	3,310	3,700	1.12	Diagonal Cracks	2,138	11,400	10,950	0.96	Crushing of Concrete
F-3	6,600	2,993	3,190	3,000	0.94	Diagonal Cracks	5,130	5,440	5,000	0.92	Diagonal Cracks

*Equation 28.

**Equation 29.

propagate will be less than 45 degrees, depending upon the prestressing level. The angle at which flexural cracks develop in bending are approximately 90 degrees. Therefore, one can expect cracks to form in combined bending and torsion some place between 45 and 90 degrees. The Maximum Stress Theory of failure seems most logical for this situation.

The principal tensile stress is

$$\sigma_1 = \tfrac{1}{2}f + \sqrt{\tfrac{1}{4}f^2 + \tau_p^2}$$

where $$f = K'M_b - f_{pe}$$

The value of f is positive when in tension; M_b is the bending moment; K' is a constant and a function of the cross section in flexure.

Previously σ_1 was equated as

$$\sigma_1 = f_t = KT_o \quad ,$$

therefore, $$KT_o = \tfrac{1}{2}f + \sqrt{\tfrac{1}{4}f^2 + K^2T_p^2}$$

hence $$(KT_o - \tfrac{1}{2}f)^2 = \tfrac{1}{4}f^2 + K^2T_p^2$$

or $$KT_p^2 + T_of = KT_o^2$$

and so $$\frac{T_p^2}{T_o^2} + \frac{f}{KT_o} = 1 \tag{28}$$

The maximum shearing stress due to torsion occurs at points most remote from the center of the cross section. Therefore, points that lie on the periphery of the cross section have the same stress intensity. Bending will cause compressive stress at the top and tensile stress on the bottom for the load configuration used here. The presence of the compressive stress on the top will help in resisting the torque applied,

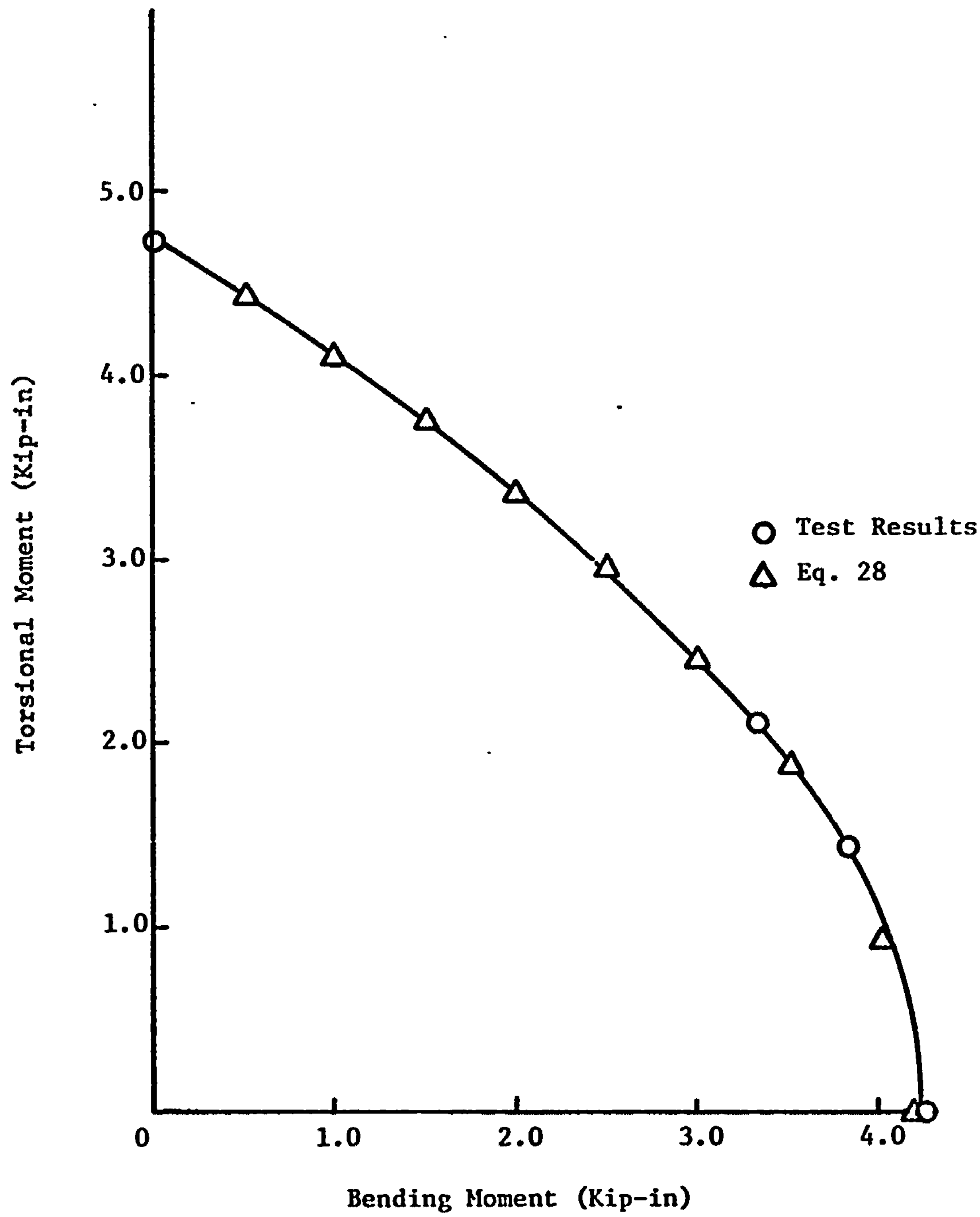


Figure 20. Torque vs. Bending Moment

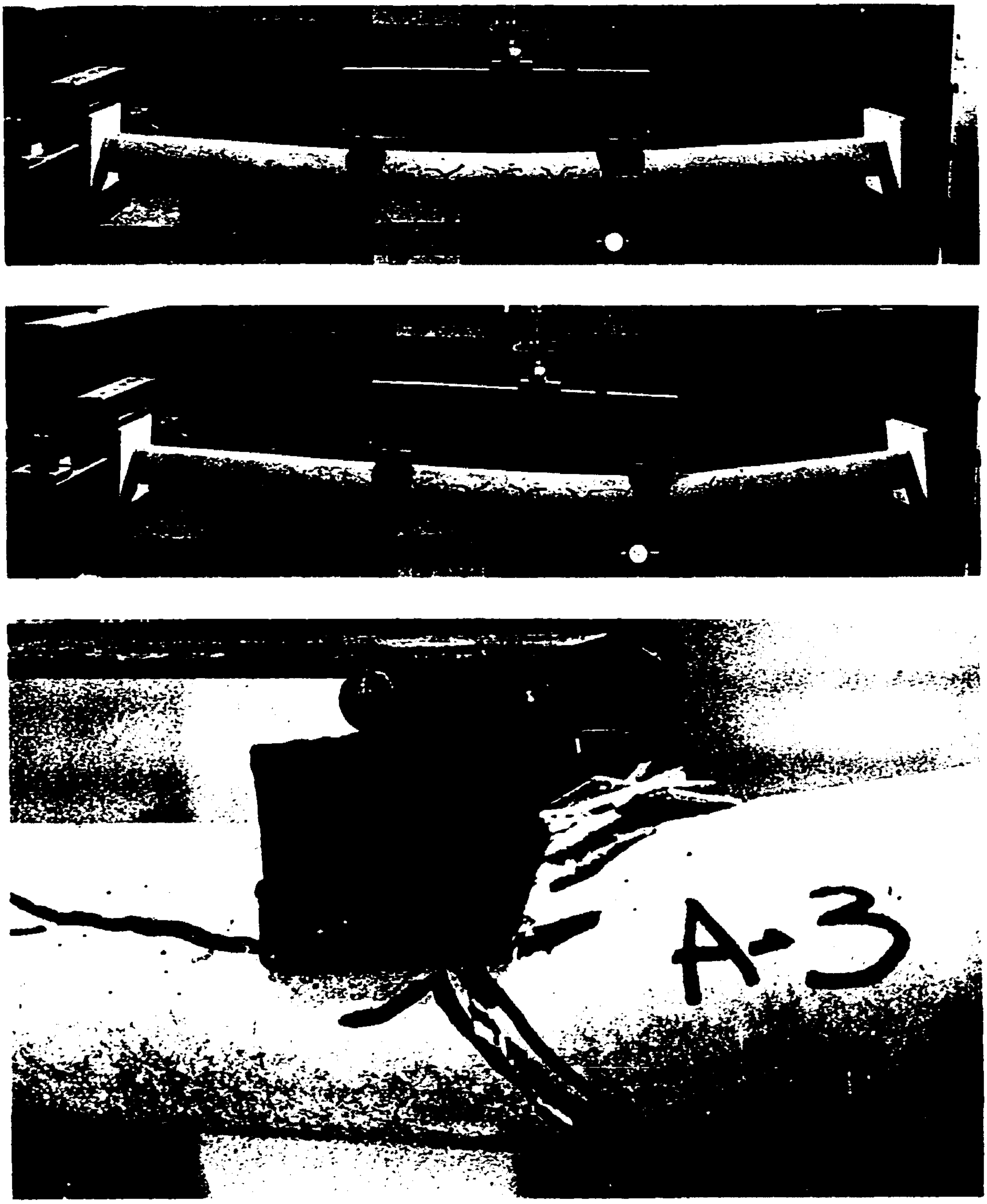


Figure 21. Crack Pattern in a Prestressed Concrete Column under Combined Bending Moment and Torsion

up to a certain point, while the tension in the bottom will reduce the section capability to resist the torsional moment. Hence, the torsional capacity of the section is a function of the bending moment and the prestressing level. Therefore, it is not surprising to see that Reeves'[41] test results showed a substantial increase in torque capacity due to an addition of bending. Reeves' test specimens were prestressed eccentrically and therefore delayed the formation of tensile stress, until additional moment was applied. Therefore, the apparent increase in torque capacity was due mainly to eccentricity of the prestressed steel.

Test results in Table 8 were found to give good agreement between measured and computed quantities. Cowan's[12] equation showed good agreement with test results for both cracking and ultimate moments as follows:

$$\left(\frac{T_p}{T_{pu}}\right)^2 + \frac{M_b}{M_u} = 1 \quad , \tag{29}$$

where M_u is the ultimate bending moment in flexure.

Test results showed that failure was characterized by the formation of diagonal cracks (Figure 21) in apparent disregard of the torque moment ratio. On the other hand, the collapse criteria were characterized by crushing of the concrete at low ratios of torque/moment, and by diagonal cracking for high ratios of torque/moment. Between these two ratios there is obviously a point where one failure mode succumbs to the other. This point must be determined by further research.

Torsion Combined with Eccentric Load

Three columns were tested in combined torsion and eccentric load. In the first column, (D-2), the eccentric load was first applied then the torque was added until collapse. The second column (F-3) was loaded similarly except that the eccentric load was increased substantially before the torsional moment was applied. The loading was reversed in the third test (G-3) where the torsion was applied first and held constant; then the eccentric load was imposed until collapse. Test results are shown in Table 9.

The mode of failure was the same up to the cracking point, regardless of the ratio of torque to bending moment. Failure was indicated by the presence of diagonal cracks differing from cracks observed in torsion tests in that they did not propagate in a spiral form. It appeared reasonable to assume that failure occurred when the principal tensile stress exceeded the ultimate tensile strength of concrete.

In prestressed concrete columns under low ratio of torsional moment to bending moment, collapse was gradual and the column retained much reserve strength and ductility. Collapse was the eventual result of crushing of the concrete. When the ratio of torque to bending moment was high the column collapsed due to a significant increase in the width of diagonal cracks and it was very similar to the case of prestressed concrete columns under pure torsion (see section under Pure Torsion).

A theoretical determination for the cracking or the cracking capacity of the section requires a well defined theory of failure. A biaxial state of stress exists due to the combined axial load, effective prestressing stress, bending and torsion. The appearance of a diagonal crack at failure might indicate that failure could be explained by the maximum stress theory. An equation was derived on the basis of the maximum stress theory which predicted the test results with fair accuracy,

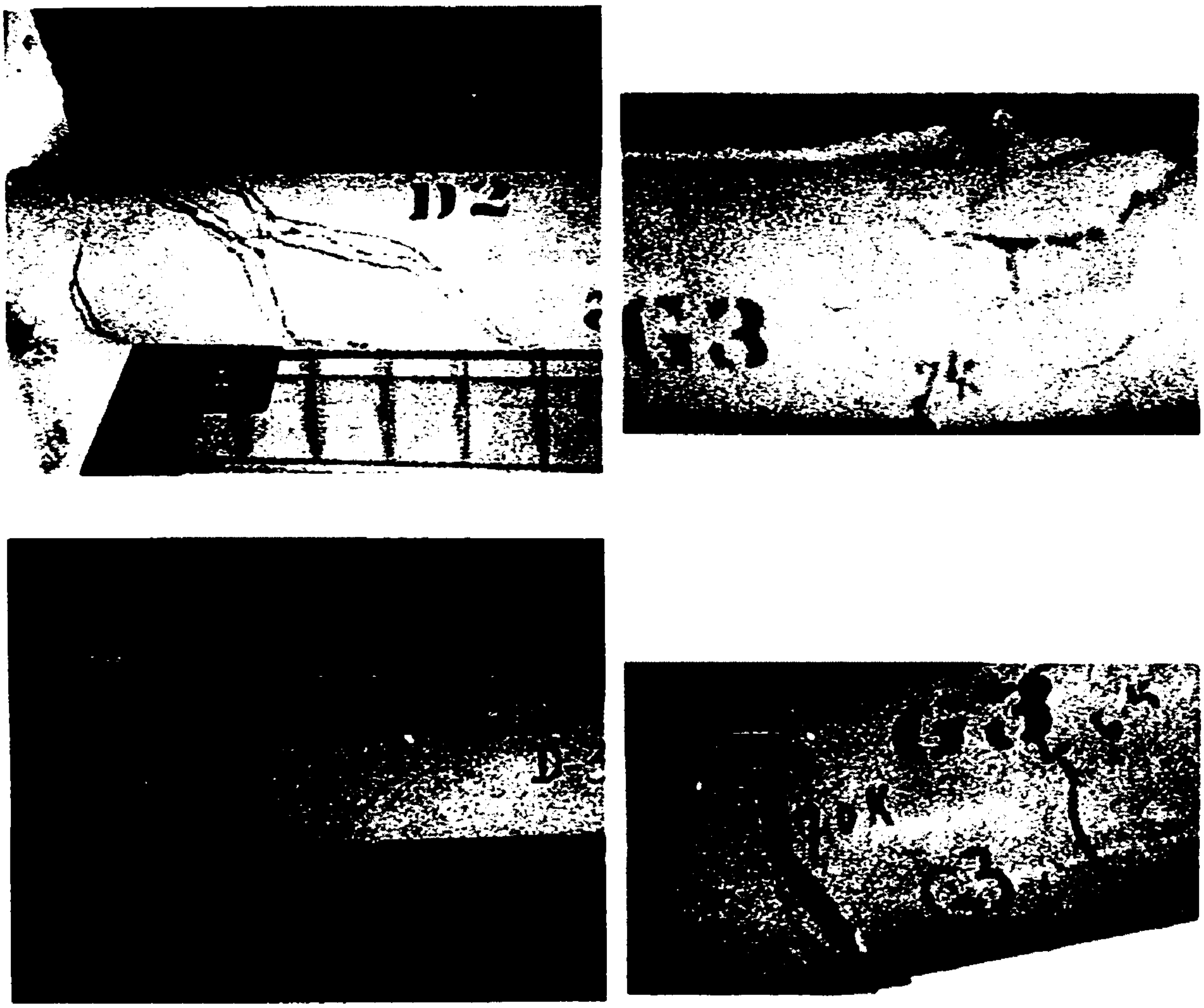
TABLE 9

COLUMN TEST RESULTS FOR TORSION COMBINED WITH ECCENTRIC LOAD

Column	f'_c	Load (lbs)	Eccentric Load e = 2 in Moment (in-lb)	Cracking Torque Theo.* (in-lb)	Cracking Torque Exp. (in-lb)	Ratio $\frac{T_{pu}\ (exp)}{T_{pu}\ (theo)}$	Ultimate Torque Theo.** (in-lb)	Ultimate Torque Exp. (in-lb)	Ratio $\frac{T_{pu}\ (exp)}{T_{pu}\ (theo)}$
D-2	5,600	3,000	6,000	3,300	3,206	0.97	5,700	5,130	0.90
D-3	5,500	5,000	10,000	1,420	1,500	1.05	5,100	5,000	0.98
G-3	6,900	10,600	21,000	--	--	--	1,980	2,138	1.08

*Equation 30.

**Equation 31.



a. Eccentrically Loaded then Torque Superimposed

b. Torque Applied then Eccentric Load Superimposed

Figure 22. Crack Pattern in Eccentrically Loaded Prestressed Concrete Columns Combined with Torsion

at low ratios of bending moment to torque. At high ratios of bending moment to torque the experimental results did not correlate whatsoever with the theoretical results based on the maximum stress theory.

A statistical approach was considered as an alternative to the lack of an accurate theory of failure encompassing the entire range of torque to bending moment ratios. In a prestressed concrete column under combined torsion and eccentric load the load and the moment are related to each other by the eccentricity of the applied load:

$$T_p = f\ (M_b, P) \quad ,$$

where $$M_b = Pe \quad ,$$

and, therefore, $$T_p = F(P) \quad .$$

A polynomial regression analysis using a least-squares criteria was performed using the test data. The method consisted of generating powers of an independent variable $(\frac{M_b}{M_u} + \frac{P}{P_u})$ to calculate polynomials of successively higher degree. When there is no reduction in the residual sum of squares between two successive degrees of polynomial, the polynomial with degree (n-1) is used as a function of the dependent variable, $(\frac{T_p}{T_{pu}})$. Here n is the degree of the highest polynomial and is limited to that value where there is no improvement in the sum of the squares term if it increases by one.

The following equation was obtained at cracking of the column:

$$\frac{T_p}{T_{pu}} = 0.977 - 0.074\ (\frac{P}{P_u} + \frac{M_b}{M_u}) - 0.174\ (\frac{P}{P_u} + \frac{M_b}{M_u})^2 \quad . \tag{30}$$

Multiple correlation coefficient between Equation (30) and test data herein

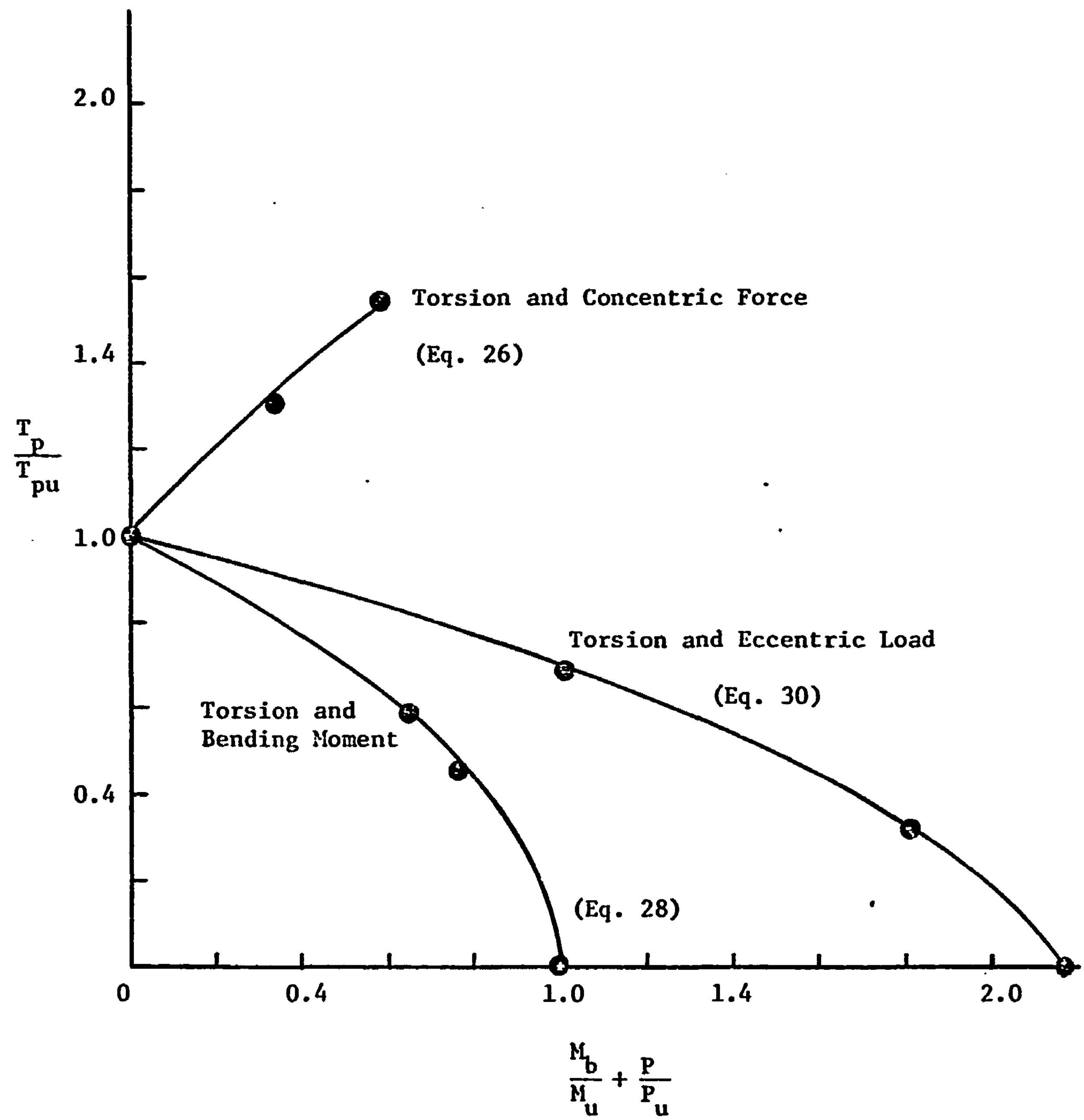


Figure 23. Torsion, Concentric Load and Bending Moment Interaction Diagrams for Prestressed Concrete Columns at Cracking

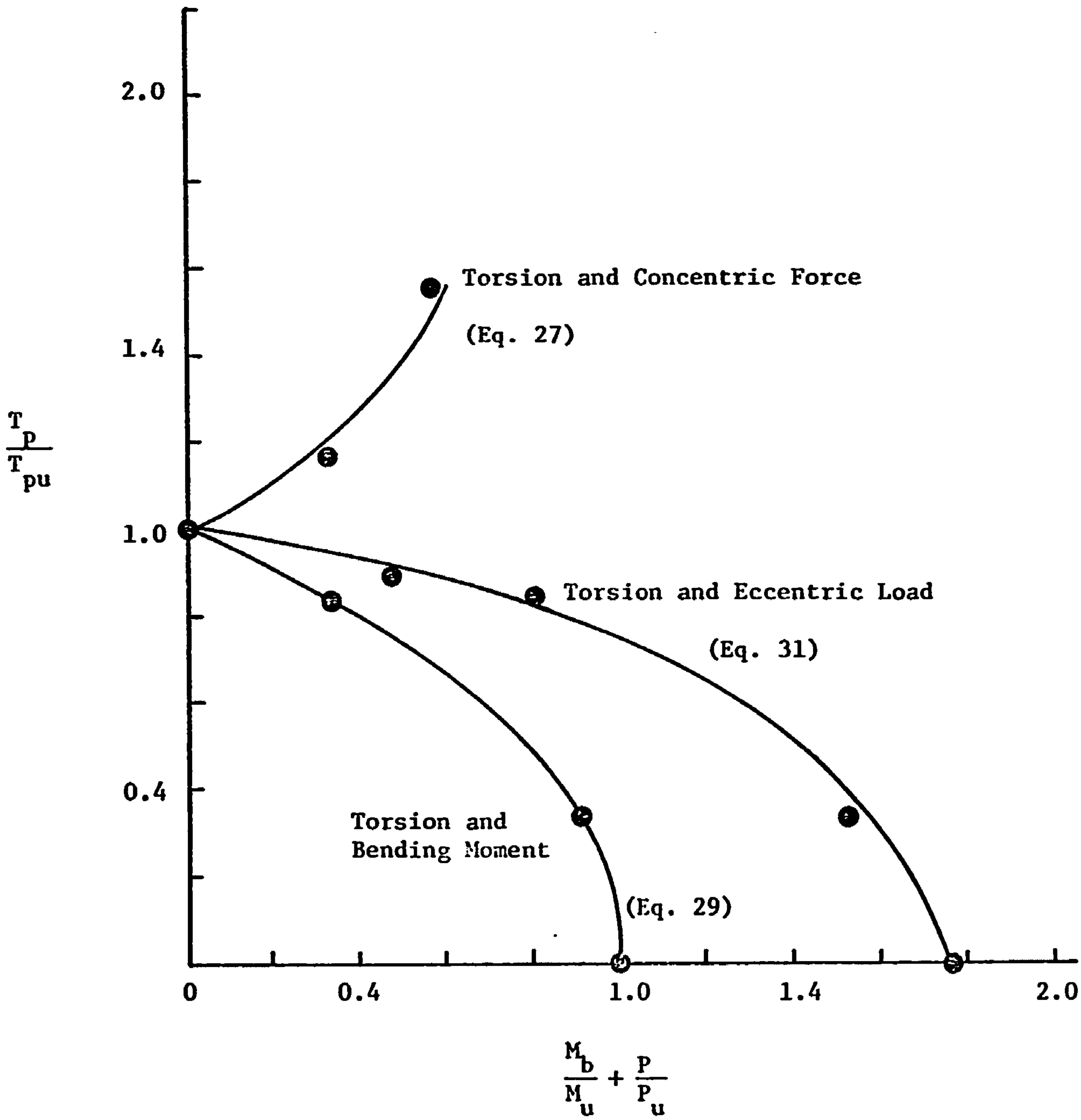


Figure 24. Torsion, Concentric Load and Bending Moment Interaction Diagrams for Prestressed Concrete Columns at Collapse

is 0.997 with a standard error of the estimate equal to 0.039. A graph representation of test data and Equation (30) are shown in Figure 23.

For failure by collapse the following equation was obtained:

$$\frac{T_p}{T_{pu}} = 0.997 + 0.20268\left(\frac{P}{P_u} + \frac{M_b}{M_u}\right) - 0.4223\left(\frac{P}{P_u} + \frac{M_b}{M_u}\right)^2 . \qquad (31)$$

Multiple correlation coefficient between Equation (31) and test data is 0.998 and standard error of estimate is 0.037. Figure 24 shows the test data and Equation (31).

CHAPTER V

INTERACTION SURFACE FOR PRESTRESSED CONCRETE COLUMNS SUBJECT TO COMBINED TORSION, CONCENTRIC LOAD AND BENDING MOMENT

The objective of this study was to investigate the effect of torsion combined with bending and axial force. Three orthogonal axes were chosen to represent the three loading conditions. The x-axis represents bending moment, the y-axis the concentric force and the z-axis represents the torsional moment. The interaction between torsion, bending and the concentric force is a surface, being the locus of all individual points that represent the possible combinations of the three applied loads. This surface will describe the relationship between torsion, bending and concentric force for a certain prestressing level, cross section area, percentage of steel, and strength of concrete. A family of surfaces can be drawn to consider other values of these variables.

Two surfaces are considered here; one surface will represent conditions at cracking, while the other surface will describe conditions at collapse. Both surfaces are plotted to unitless axes by dividing the torsion, concentric load or moment value by its ultimate value. Table 10 gives these values for both cracking and collapse.

TABLE 10

TORSION, CONCENTRIC FORCE, AND MOMENT CAPACITIES

Failure	T_{pu} (Kip - in)	M_u (Kip - in)	P_u (Kip)
Cracking	4.625	6.0	34.4
Collapse	5.8	15.0	34.4

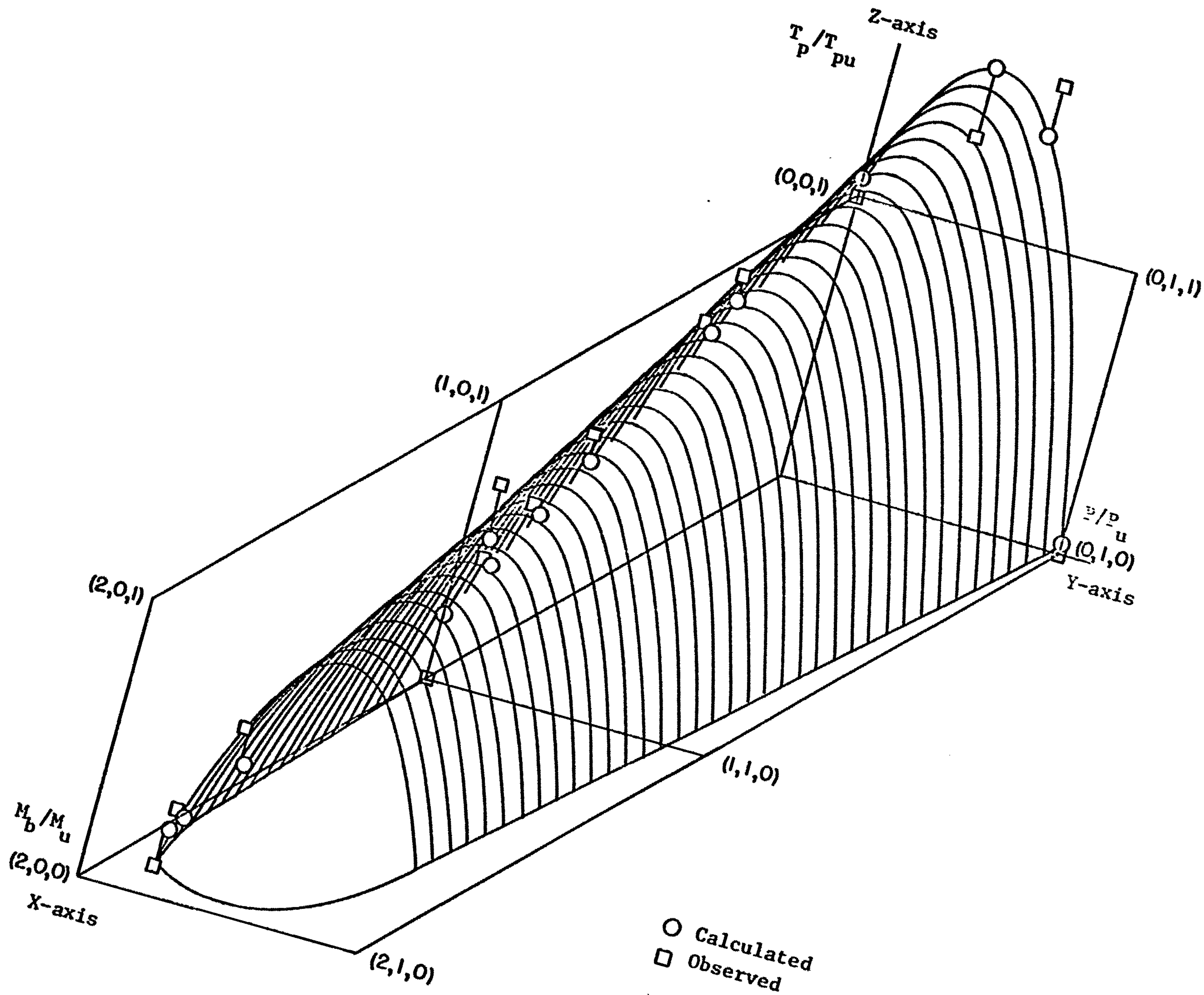


Figure 25. Torsion, Concentric Load and Bending Moment Unitless Surface for Prestressed Concrete Columns at Cracking

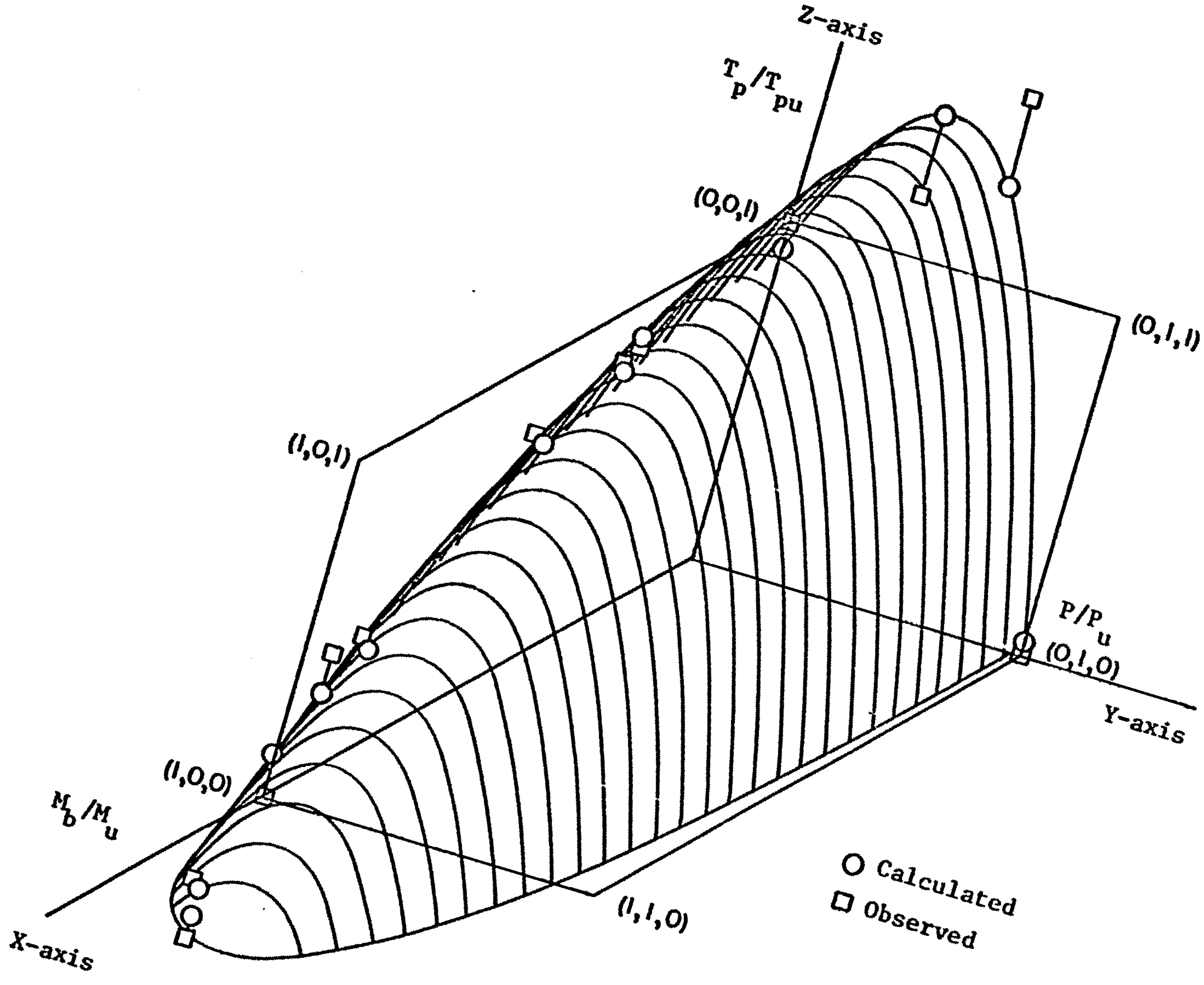


Figure 26. Torsion, Concentric Load and Bending Moment Unitless Surface for Prestressed Concrete Columns at Collapse

The surface equations were established statistically with the aid of an IBM 360-50 computer at the University of Oklahoma using the program "BMD02R Stepwise Regression," a sub-program of the University of California Biomedical Computer Package.*

> This program computes a sequence of multiple linear regression in a stepwise manner. At each step one variable is added to the regression equation. The variable added is the one which makes the greatest reduction in the error sum of squares. Equivalently, it is the variable which has highest partial correlation with the dependent variable partialed on the variables which have already been added; and equivalently it is the variable which, if it were added, would have the highest F value.

The following equation was obtained for cracking failure of the concrete:

$$\frac{T_p}{T_{pu}} = 1.06 - 0.83\left(\frac{M_b}{M_u}\right) + 2.697\left(\frac{P}{P_u}\right) + 0.92746\left(\frac{M_b}{M_u}\right)\left(\frac{P}{P_u}\right) - 3.728\left(\frac{P}{P_u}\right)^2. \quad (32)$$

Multiple correlation coefficient between Equation (32) and test data is 0.964 and the standard error of estimate is 0.158. A surface representation of test data and Equation (32) is shown in Figure 25.

For collapse failure the following equation was obtained:

$$\frac{T_p}{T_{pu}} = 0.9077 - 0.254\left(\frac{M_b}{M_u}\right) + 2.783\left(\frac{P}{P_u}\right) - 0.497\left(\frac{M_b}{M_u}\right)^2 - 3.644\left(\frac{P}{P_u}\right)^2. \quad (33)$$

The multiple correlation coefficient between Equation (33) and the test data is 0.968 and standard error of estimate is 0.152. A surface plot of test data and Equation (33) is shown in Figure 26.

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*Dixon, W. J., University of California Publications in Automatic Computation Number 2, Biomedical Computer Programs, University of California Press, Berkeley, Los Angeles, London 1970.

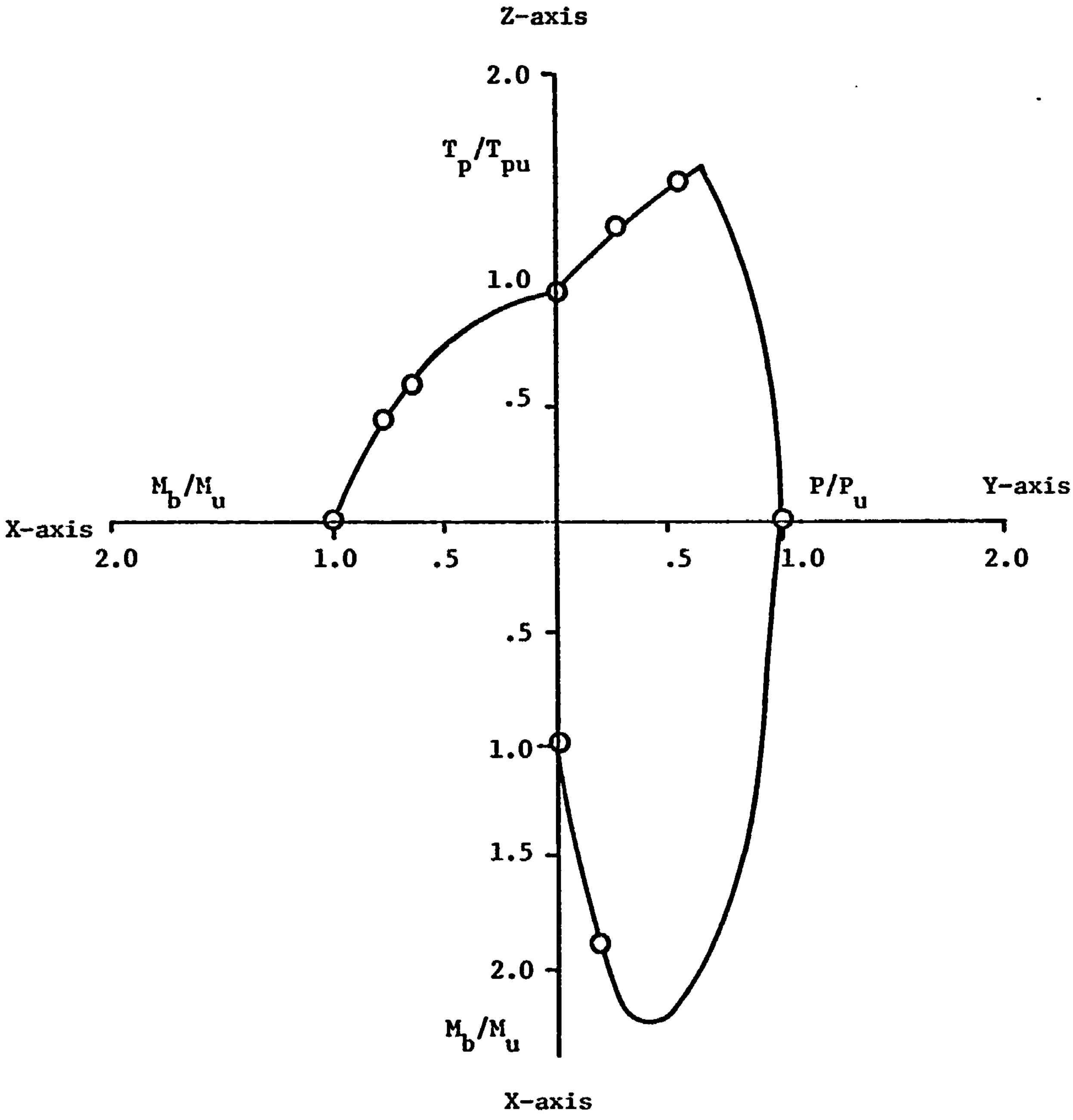

Figure 27. Torque-Concentric Force-Bending Moment Interaction Diagrams at Cracking

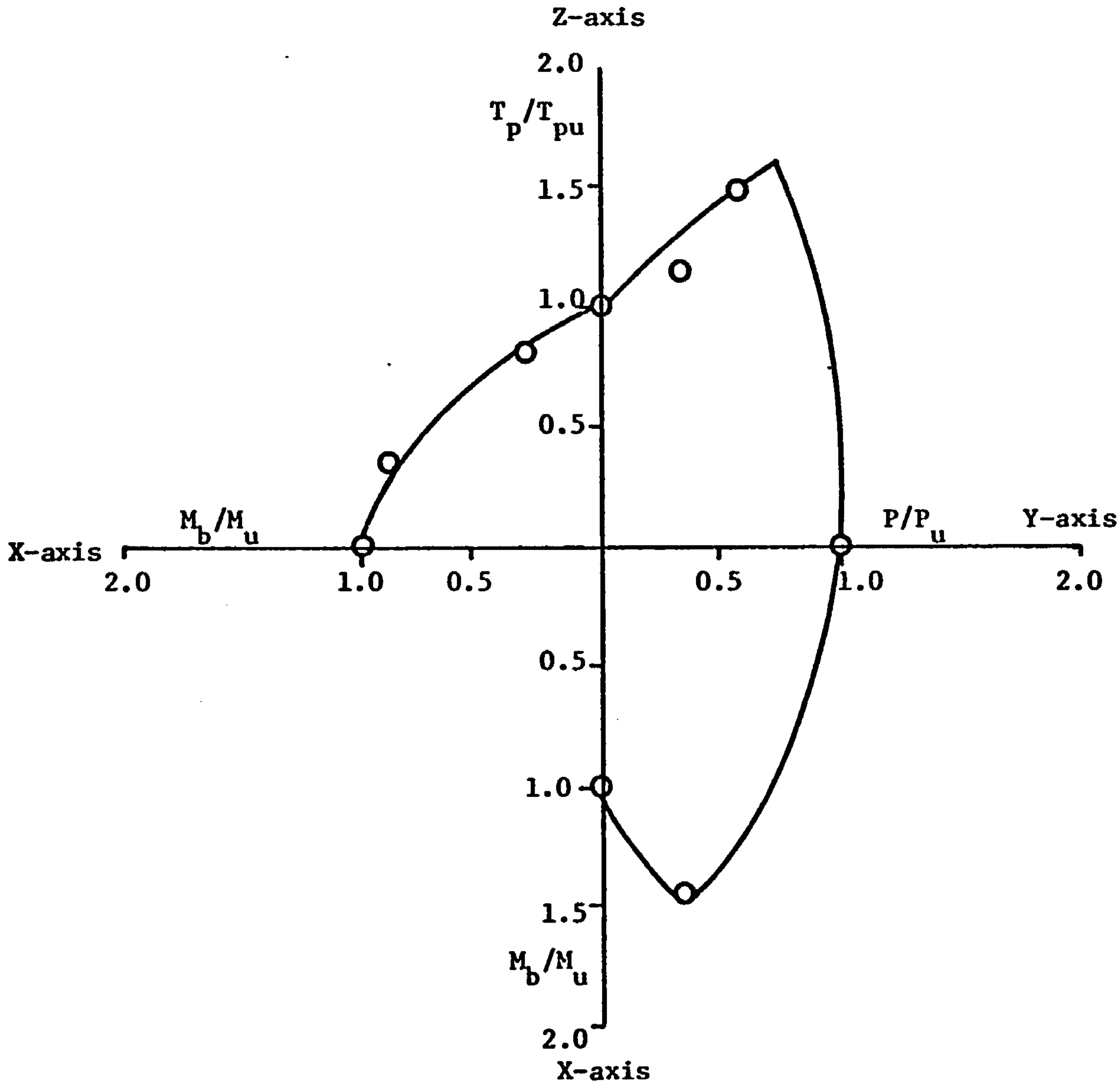

Figure 28. Torque-Concentric Force-Bending Moment Interaction Diagrams at Collapse

The surface will intersect the X-Y plane, representing the interaction diagram for the concentric load and the bending moment. The interaction diagram for torsion combined with concentric force will result from the intersection of the surface with the Y-Z plane. A third interaction diagram for torsion combined with bending moment will result from the intersection of the surface with the X-Z plane. All three interaction diagrams were previously established in Chapter IV. Figures 27 and 28 can be shown to represent these interaction diagrams for both cracking and collapse conditions respectively. One can rotate the X-axis 90 degrees counter clockwise until it coincides with the Z-axis. Then, the X-axis will be rotated upward until it is perpendicular to the plane of the paper. The comparison of these plots, based on test data, with the intersections of the two surfaces shown in Figures 25 and 26 will lead to the conclusion that the surface equations not only predicted the test data with reasonable accuracy, but they also gave a fair description of the interaction diagrams for the three different combinations of loading.

CHAPTER VI

SUMMARY

Seventeen circular prestressed concrete columns with an $h/r = 80$ were tested under pure torsion, concentric load, bending moment and all combinations of such. The cross section, the level of prestress, the percentage of steel, and the eccentricity of the applied force were all constants throughout the entire study. All columns were concentrically prestressed. Therefore, conclusions drawn here will pertain to the above constants only.

Conclusions

1. Prestressing a slender concrete column does not affect its ultimate concentric load capacity, and will reduce its deflection as compared with a conventionally reinforced column up to the cracking point. This reduces the magnification in the moment caused by the interaction of the applied load and the deflected shape of the column.

2. The cracking load of a prestressed concrete column under an eccentric load can be predicted closely by the usual elastic theory. The ultimate collapse load can be predicted closely by ultimate theory, taking into consideration the effect of cracking of concrete on the column deflections.

3. The non-dimensional interaction curve of prestressed concrete columns under combined bending and concentric force resembles the interaction

curve for the conventionally reinforced column, except that the prestressed diagram lacks a definite yield point which accounts for the absence of a definite balance point.

4. The cracking torsion can be satisfactorily predicted by the principal stress theory, Equation (23), while the ultimate torsion at collapse can be easily predicted by the plastic theory.

5. The torsional capacity of a concentrically prestressed column is greater than that of a conventionally reinforced column. Its torsional capacity will increase with increase in the prestressing level up to a certain limit which must yet be established by further research.

6. The application of a concentric force will increase the torque capacity of the prestressed concrete column and can be predicted up to cracking by Equation (26) and at collapse by Equation (27).

7. Bending moment will decrease the capacity of a prestressed column to resist torsion, while the bending moment capacity will simultaneously be impaired by the presence of torque. This is true for all ratios of torque to moment. The relationship between torque and bending moment can be predicted by Equations (28) and (29).

8. The addition of torque to moment, and concentric force will reduce the ultimate bending moment capacity of the column below its corresponding capacity, when the column is under the action of concentric load and bending moment only. Hence, the presence of torsion is detrimental to eccentrically loaded prestressed concrete columns, as shown by Equations (32) and (33).

9. (a) Eccentrically loaded prestressed concrete columns, will exhibit earlier cracking failure upon the application of torque. Cracking

failure is caused by diagonal cracks for all ratios of torque to moment.

(b) In prestressed concrete columns under low ratio of torque to moment, collapse will result in crushing of the concrete while for high ratio of torque to moment, collapse will occur only after significant increase in the width of the diagonal cracks.

Recommendations

1. The slenderness effect on the ultimate capacity of the prestressed concrete column should be studied throughout the entire range of h/r.

2. The effective change of the prestressing level on the slenderness ratio and their combined effect with the change of the eccentricity of the applied load should be established before prestressed columns can be used safely.

3. The effect of initial curvature, since this will be a problem in long prestressed columns, should be investigated.

4. The combined effect of transverse shear with concentric load, bending and torsion should also be thoroughly investigated.

5. Future experimental work is required to determine the stress-strain relationship of prestressed concrete and particularly accounting for the history of creep from time of release of the prestressing force to time of testing. This work is most needed for high strength concrete.

6. The percentage of steel and at what prestressing level to apply to a column to get the maximum load carrying capacity are questions that research must answer soon if prestressed columns are to be designed effectively.

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APPENDIX A

DETAIL DISCUSSION OF EACH SPECIMEN

Axial Load

Column B-2

Column B-2 was tested under pure compression. The load was applied axially on one end of the column, while the other end was fixed. The column had an initial deflection of 0.156 inches at 0.25 L and 0.75 L while the middle deflection was 0.203 inches.

The axial load was applied at an increment of 1,000 pounds and the column was checked for cracks. The increment of loading was decreased to 500 pounds when 20,000 pounds was reached.

Due to the initial curvature of the column, the column deflected downward. No cracks were observed until the column collapsed. The collapse was sudden and violent accompanied by spalling of the concrete. Failure started at the bottom and moved upward in a plane 90 degrees to the longitudinal axis of the column. The plan of failure was 65 inches from the loaded end and the collapse load was 29,500 pounds.

Column B-3

Column B-3 was tested in pure compression. The load was applied axially on one end of the column, while the other end was fixed. The column had an initial deflection of 0.1875 inches at 0.25 L from the loaded end and a deflection of 0.312 inches at 0.75 L. The center line deflection was 0.375 inches.

The axial load was applied at an increment of 2,000 pounds up to 10,000 pounds. The load then incremented at a 1,000 pounds up to 20,000 pounds. The increment of loading then decreased gradually until collapse.

Due to the initial curvature of the column, the column deflected downward. No cracks were observed until the column collapsed. The collapse

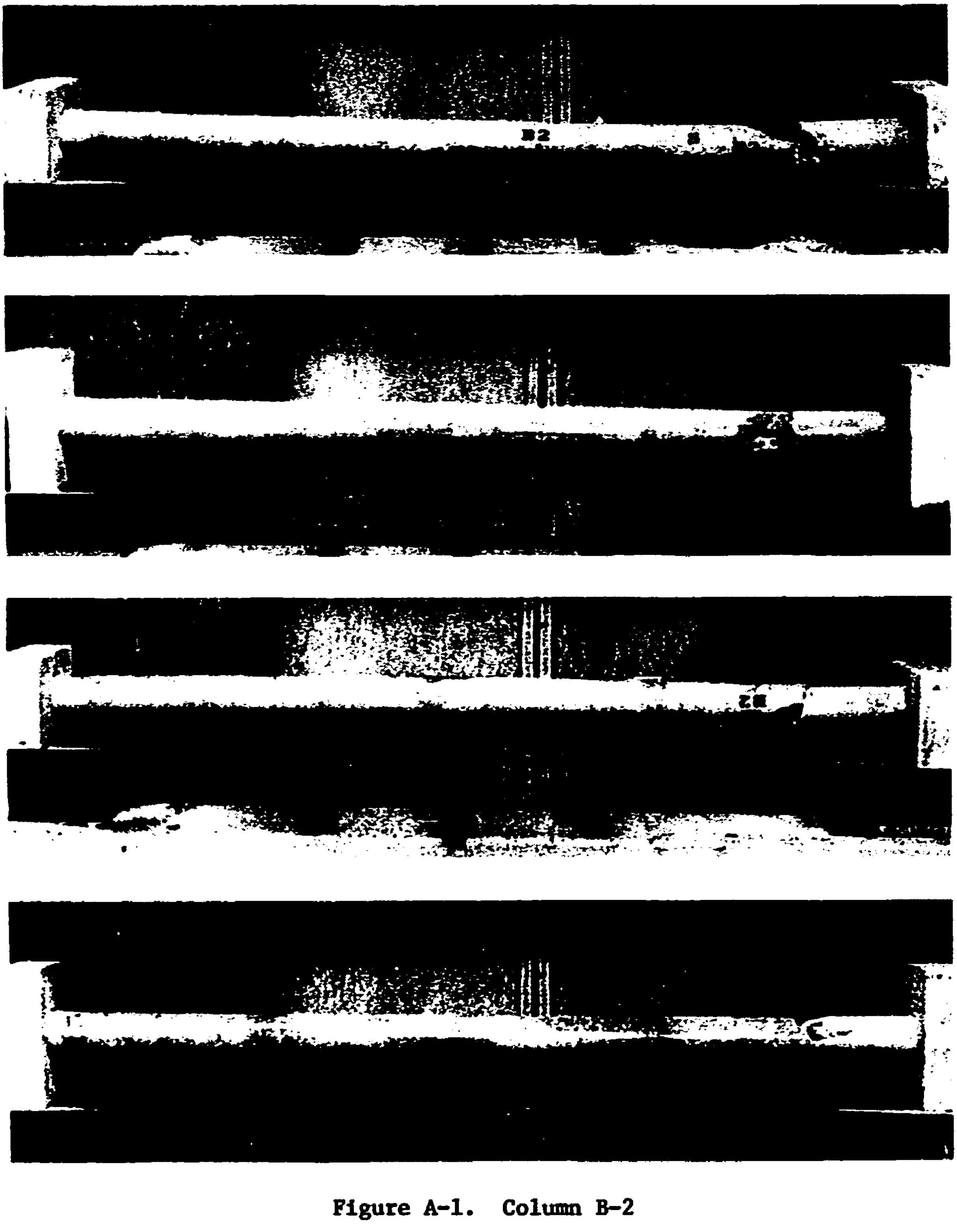

Figure A-1. Column B-2

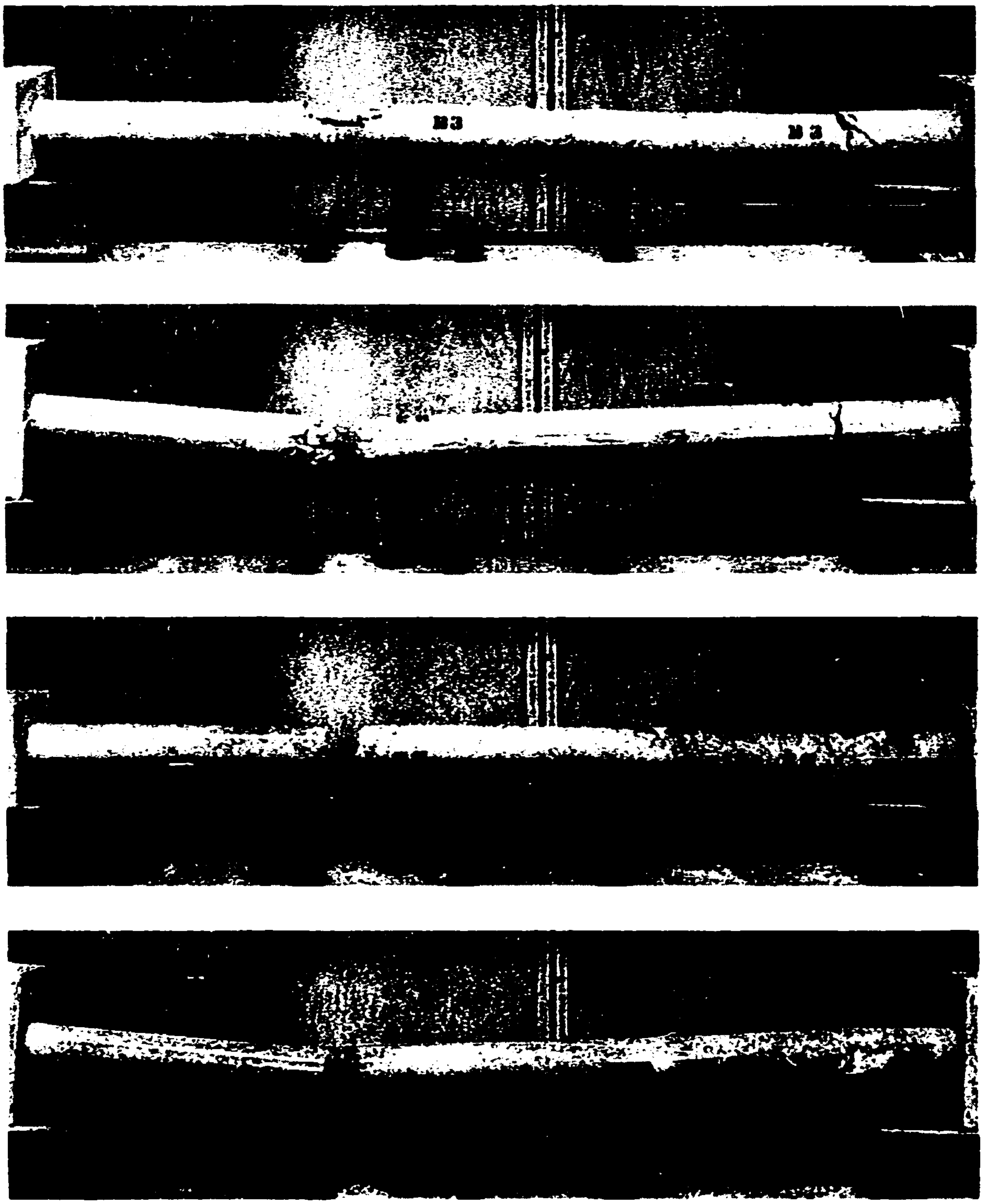

Figure A-2. Column B-3

was sudden, violent and accompanied by spalling of the concrete. Failure occurred at two places in the same time. One was at 24 inches and the other at 65 inches from the loaded end of the column. Failure started at the top side of the column 24 inches from the loaded end while the other started at the bottom. Both planes of failure were identical, but inverse to each other and both made an angle of 90 degrees with the longitudinal axis of the column. The collapse load was 29,400 pounds.

Eccentrically Loaded

Column E-3

Column E-3 was tested under eccentric load. The load was applied with an eccentricity of two inches at one end of the column, while the other end was fixed. The column had an initial deflection of 0.156 inches at 0.25 L from the loaded end and a deflection of 0.282 inches at 0.75 L. The center line deflection was 0.282 inches.

The load was applied at a 1,000 pound increment up to 10,000 pounds then it was decreased gradually until collapse.

Two cracks developed 7.5 and 2.5 inches from the loaded end in the bottom part of the cross section. The load then was 6,000 pounds. At 7,000 pounds the two cracks increased in width and no new cracks were observed. Two more cracks developed at 8,000 pounds, one at 5.5 inches and the other at 9.75 inches from the loaded end all on the bottom side of the cross section. The initial two cracks developed further up on both sides. At 20,000 pounds, four new cracks developed on the bottom; they were at distances of 12.0, 13.75, 16.00 and 18.5 inches from the loaded end. When the load was 11,000 the crack at twelve inches increased

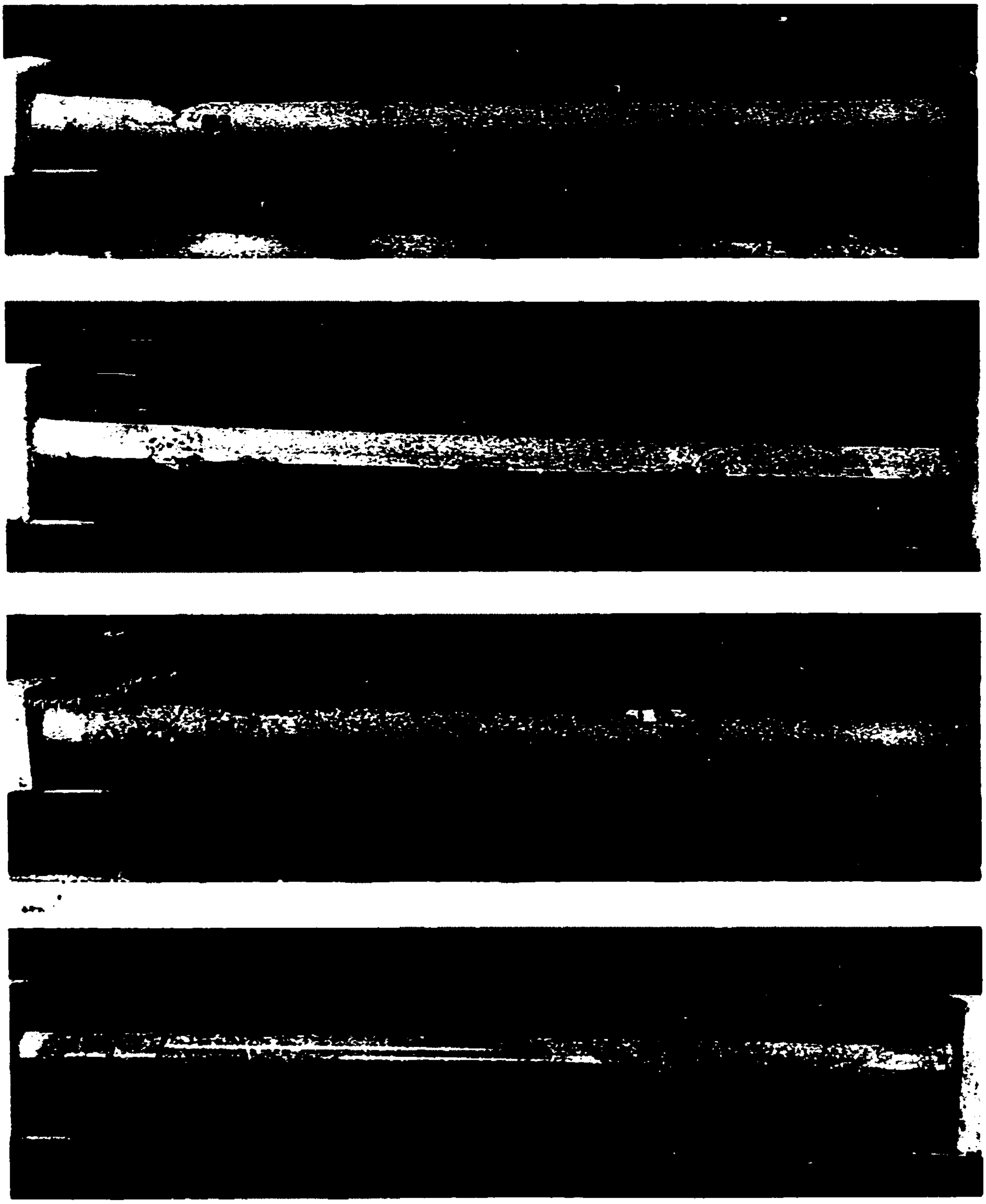

Figure A-3. Column E-3

significantly in width and developed upward perpendicular to the longitudinal axis of the column up to about the bottom third of the cross section, then the crack changed direction and developed with an angle of 30 degrees with the longitudinal axis of the column up to the prestressing strand. Then, it split into two cracks, one developed upward while the other propagated toward the loaded end in a plane parallel to the prestressing steel. Collapse occurred at an ultimate load of 11,000 with spalling of the concrete on the top side of the cross section.

Column F-1

Column F-1 was tested with eccentricity of two inches. The column had an initial deflection of 0.1875 inches at the middle of the column. One end of the column was fixed while the simply supported end was under the eccentric load.

The load was applied at a 1,000 pound increment up to 8,000 pounds. The increments were decreased to 500 pounds then after until 10,000 pounds were reached. The load then was incremented at 100 pounds at a time.

The first crack occurred at a distance of 3.5 inches from the loaded end and under a failure load of 6,000 pounds. As the load increased to 7,000 pounds, 8 cracks appeared at distances of 6.5, 20.5, 23, 31, 35, 37.5, 45, 47.5 inches from the loaded end. All were tension-type cracks and were located on the bottom. Similar cracks at 8,000 pounds developed at distances of 3.3, 11, 15.5, 27, 32.5 inches. At 8,500 pounds, two new tension cracks also appeared in the bottom side of the column. They were at 13 and 17.5 inches. While new cracks were developing, the first cracks were increasing in width and propagating upward in a plane 90 degrees to the longitudinal axis of the column.

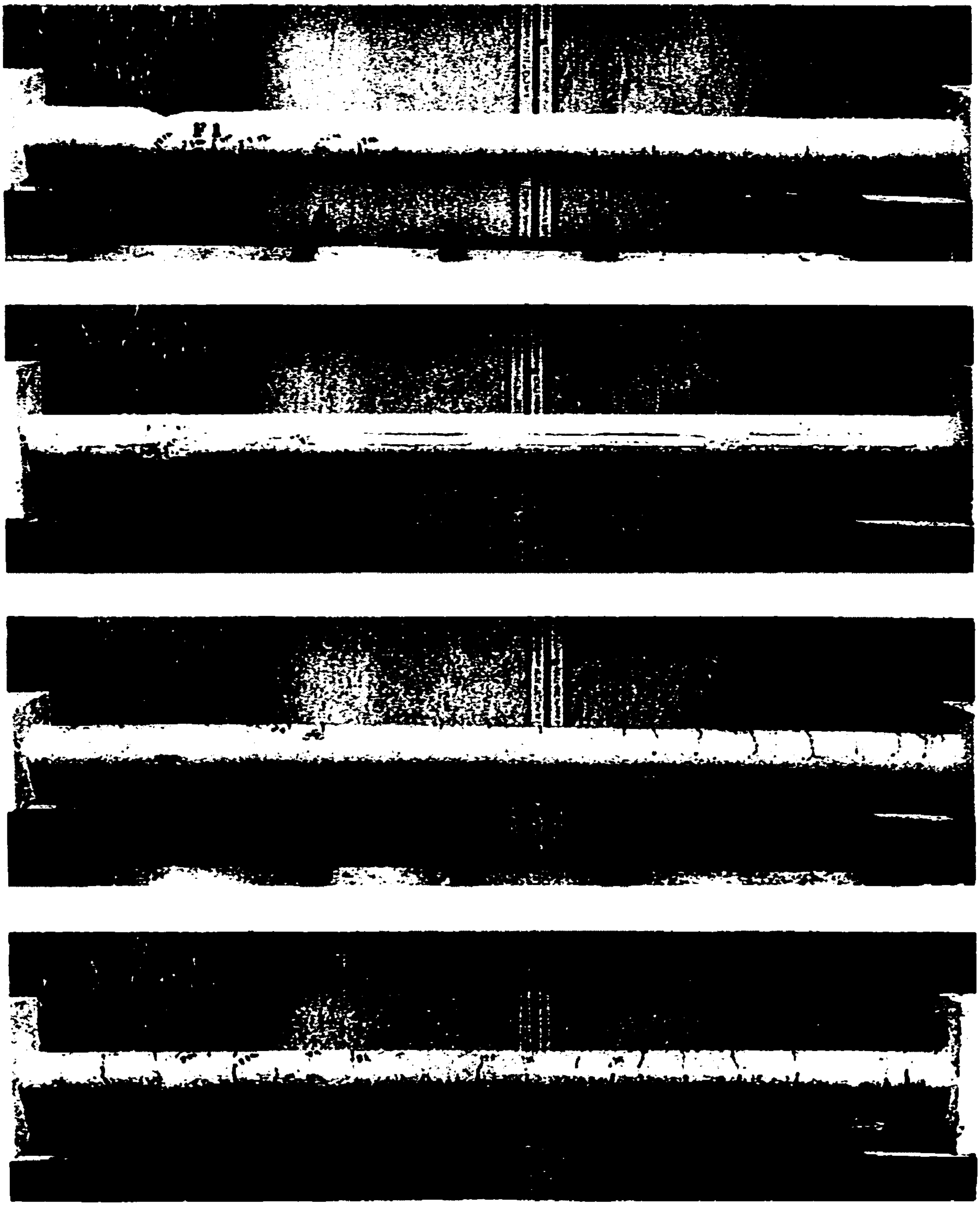

Figure A-4. Column F-1

The crack at 11 inches, from the loaded end, was increasing in width the most as the load was increased to 10,000 pounds. At 11,500 pounds, this crack suddenly propagated upward; at the upper two thirds of the cross section, it split into two cracks, one continued upward and the other propagated horizontally in a plane parallel to the prestressing steel. Collapse was sudden, at 11 inches from the loaded end. The eccentric load, then, was 11,700 causing spalling of the concrete on the top face of the column.

Column F-2

Column F-2 was tested with an eccentricity of two inches. The column had no initial curvature. One end of the column was fixed while the loaded side was free to rotate and translate longitudinally. The load was applied at an increment of 1,000 pounds, then it decreased to 500 pounds after cracking. Near the collapse load, the load increments were 100 pounds.

The first crack developed at 12 inches from the loaded end and the cracking load was 7,000 pounds, as the load increased to 10,000 pounds the first crack on the bottom side of the column increased in width and two new cracks at 2.75 and 5.25 inches were formed. The new cracks were similar to the first crack and all were tension-type cracks. The load increased to 11,000 pounds and three new cracks developed at 2.75, 5.50, and 13 inches from loaded end. As the load increased from 7,000 to 11,000, the initial crack propagated upward, then changed direction on plane inclined to the longitudinal axis of the column of about 30 degrees. At 12,000 pounds, a new tension crack developed at a distance

Figure A-5. Column F-2

of 14.5 inches. Two more cracks at a distance of 17 and 19.25 inches developed as the load reached 13,000 pounds. The load, then, was increased to 13,500 and a new crack was observed at a distance of 8 inches. The initial crack at 12 inches was significantly increasing in width on the tension side of the column. At a distance of 21 inches, a new tension crack developed as the load reached 15,200 pounds.

When the collapse load of 15,400 pounds was reached, the initial crack at 12 inches from the loaded end propagated past the prestressing steel into two directions making what looks like a V shape. The column collapsed with spalling the concrete on the top side of the column above the V shape.

Pure Moment

Column G-1

Column G-1 was tested with a two-point system symmetrical about the middle of the column. Both ends of the column were free to rotate. The column had an initial mid-deflection of 0.375 inches.

The initial crack was observed on the bottom side and at a distance of 29.2 inches from the left end of the column. The cracking moment was 6,250 inches-pounds and the crack was of a flexural tension type. At a moment of 6,880 inches-pound two cracks at a distance of 41.0 and 45.8 inches developed similar to the initial crack. When the moment increased to 7,500 inches-pound, two new cracks developed outside the constant moment area at a distance of 22.5 and 51.5 inches from the left end. Another crack at a distance of 36.0 inches developed. When the applied moment reached 9,375, the first crack propagated vertically until it

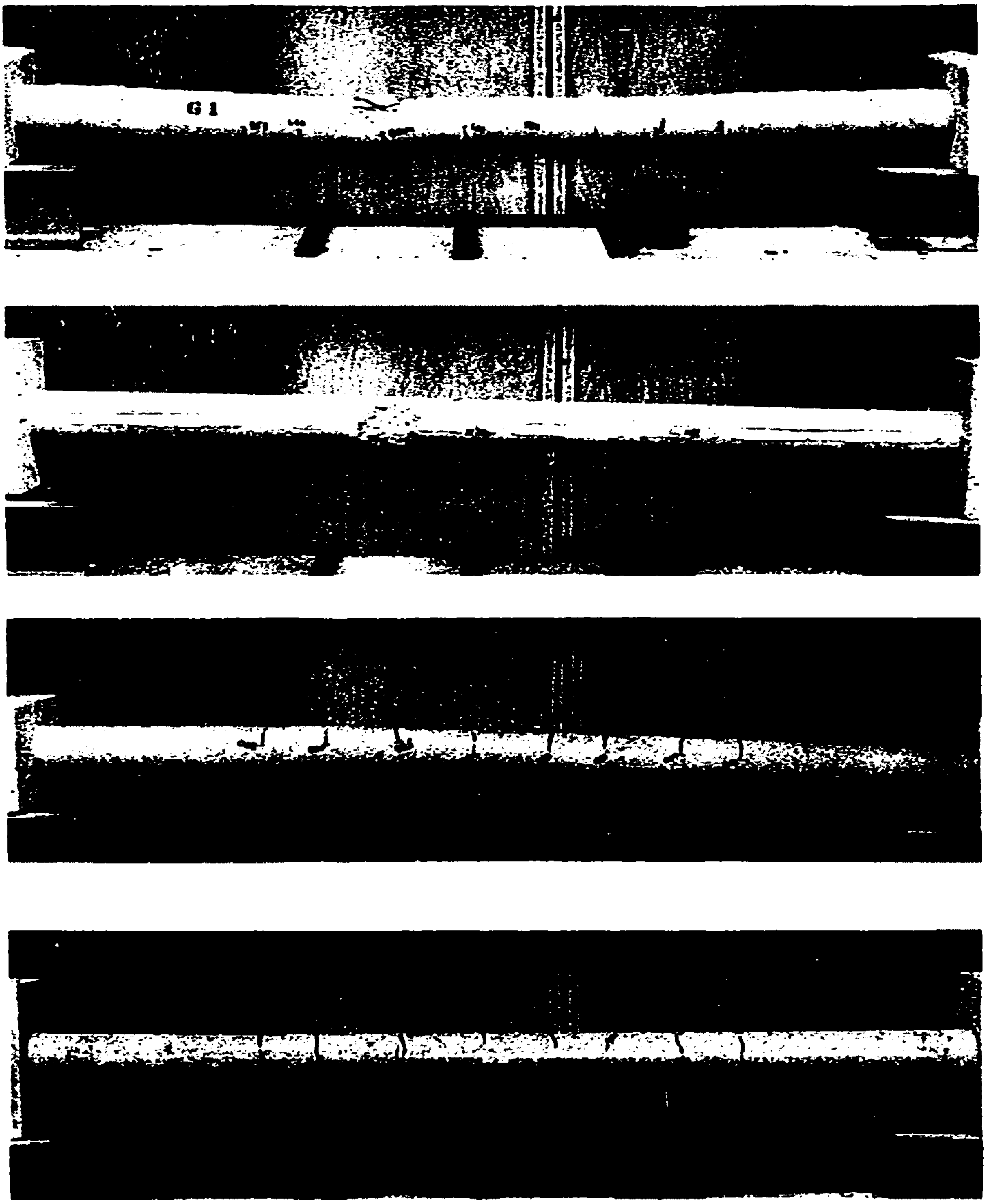

Figure A-6. Column G-1

reached the prestressing steel, then horizontally parallel to the longitudinal axis of the column. At the same moment, a new tension crack developed at a distance of 56.25 inches. When the moment reached 10,000 inches-pounds a new crack was observed at 18.5 inches while the cracks on both sides of the loading points were propagating in a 45 degree angle toward the loading point. At the same time, the middle two cracks were heading toward each other at 40 degree angles.

The moment was increased to 11,250 inches-pound, and one can observe significant width of about one millimeter in all the cracks. Concrete was spalling from all cracks. Apalling of concrete became more obvious as the moment increased and the middle cracks had a width of about two millimeters at a moment of 13,750 inches-pound.

At a moment of 15,300 inches-pound the column collapsed at the first crack with a center deflection of about 2.25 inches.

Column G-2

Column G-2 was tested with a two-point system symmetrical about the middle of the column. Both ends of the column were free to rotate. The column had no apparent initial curvature.

The first crack was observed at the middle of the column and of the flexural tension type at a moment of 7,500 inches-pound.

At a moment of 8,750 inches-pound five cracks were observed at a distance of 25, 28.75, 37.5, 43.5 and 50.5 inches from left end of column. All were tension cracks and extended vertically toward the prestressing steel.

The cracks were about one-half millimeter wide when the moment applied was 9,375 inches-pound. A new crack developed at 48.0 inches

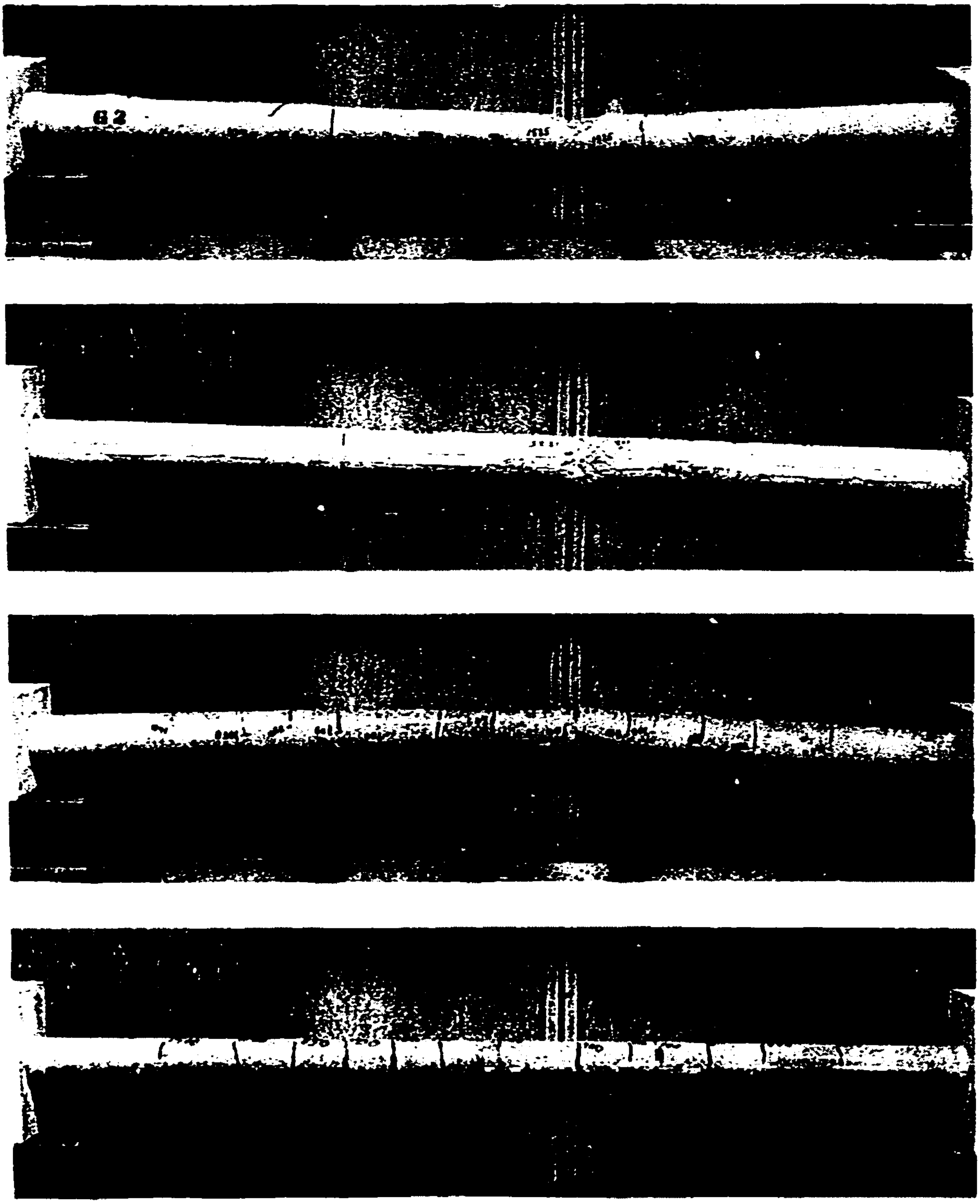

Figure A-7. Column G-2

and a moment of 10,000 inches-pound. As the moment reached 10,625 inches-pound, two new cracks were formed at 4 inches from the two loading points and outside the constant moment area.

As the moment reached 11,875 inches-pound, all cracks propagated vertically upward to the prestressing steel. Two more cracks developed as the moment became 13,125 inches-pound and were at a distance of 17.5 and 58.5 inches.

The first crack was about one millimeter wide at a moment of 14,375 inches while the two cracks at 29.0 and 41.0 inches were developing upward at an angle of 45 degrees. At a moment of 15,600 inches-pound, the first crack propagated over the prestressing steel in a horizontal manner. Another new crack was formed at 11 inches from left end while all other cracks increased in width and spalling of concrete was obvious. The moment then was 17,500 inches-pound. At a moment of 18,100 inches-pound the last crack was developed at a distance of 64.5 inches from left end.

The collumn collapsed at an ultimate moment of 19,000 inches-pound at the first crack.

Pure Torsion

Column C-2

Column C-2 was tested in pure torsion; one end of the column was fixed and the torsional moment was applied to the free end.

The torsional moment was applied gradually and the first crack appeared at a moment of 4,275 inches-pound and an angle of twist of 0.60 degrees. The initial crack developed at two inches from the loaded end

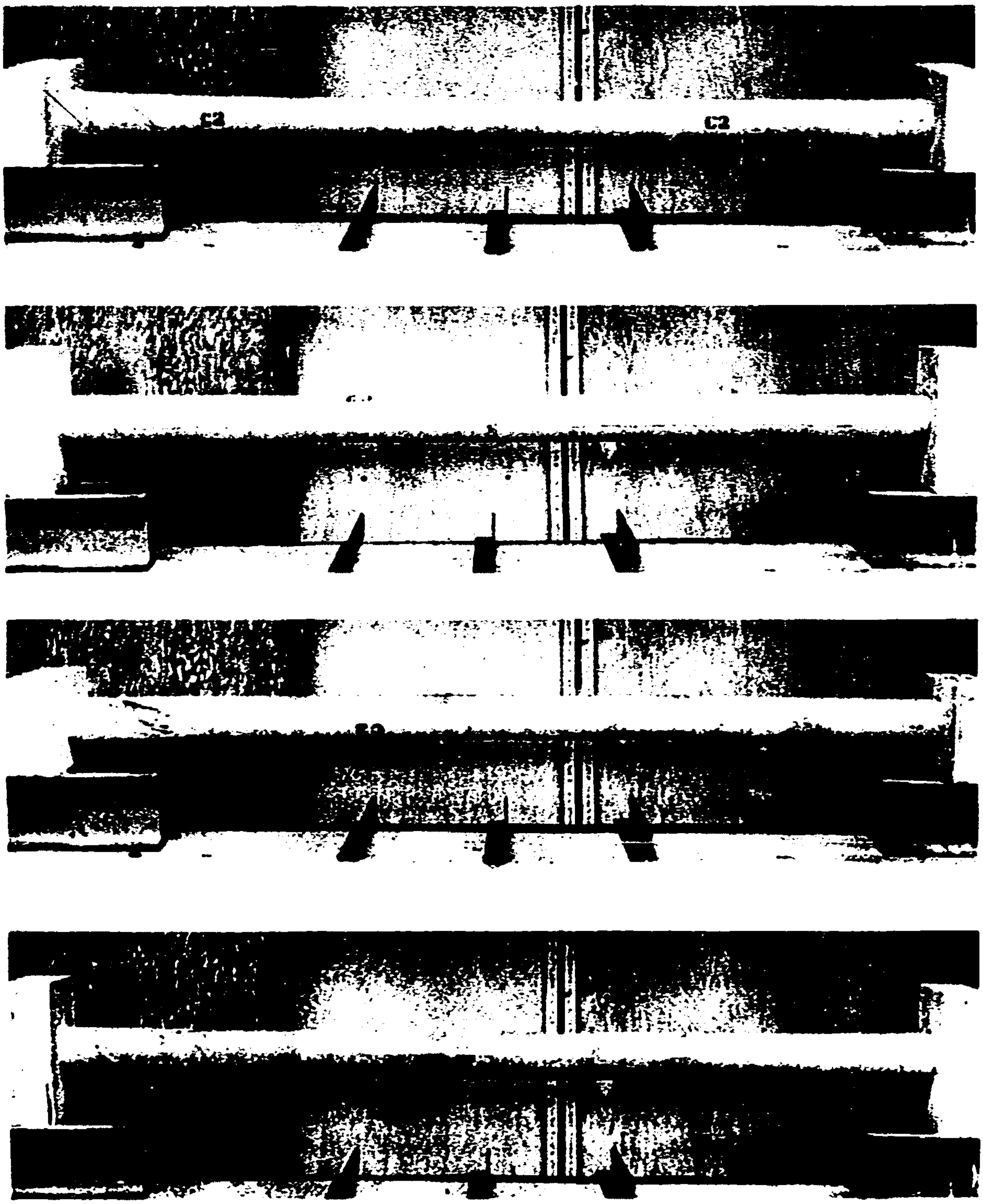

Figure A-8. Column C-2

in the bottom of the member and spread upward on both sides in a spiral way up to the upper third of the cross section. The crack made an angle of 39 degrees with the longitudinal axis of the column. As the crack developed further upward, with increase in the torsional moment, it split into two cracks. One continued upward in the same angle, while the other crack changed its angle of inclination and developed in the longitudinal direction.

Another crack developed as the torsional moment increased in the same fashion at about three inches from the first crack, while the first crack increased in width until the column collapsed at the first crack. The width of the crack at collapse was one-eighth inches wide and the torsional moment at collapse was 5,344 inches-pound and an angle of twist of 1.03 degrees. No cracks were observed at the fixed end of the column.

Column C-3

Column C-3 was tested in pure torsion. One end of the column was fixed and the torsional moment was applied to the free end.

The torsional moment was applied at an increment of 427.50 inches-pound until collapse. At every increment of load the angle of twist is recorded and the column is checked for cracks. The first crack in the specimen occurred at a torsional moment of 4,275 inches-pound and an angle of twist of 0.55 degrees.

The first crack was observed in the bottom side of the member and propagated upward on both sides in a spiral way at an angle of 39 degrees at a distance of two inches from the loaded end. When the torsional

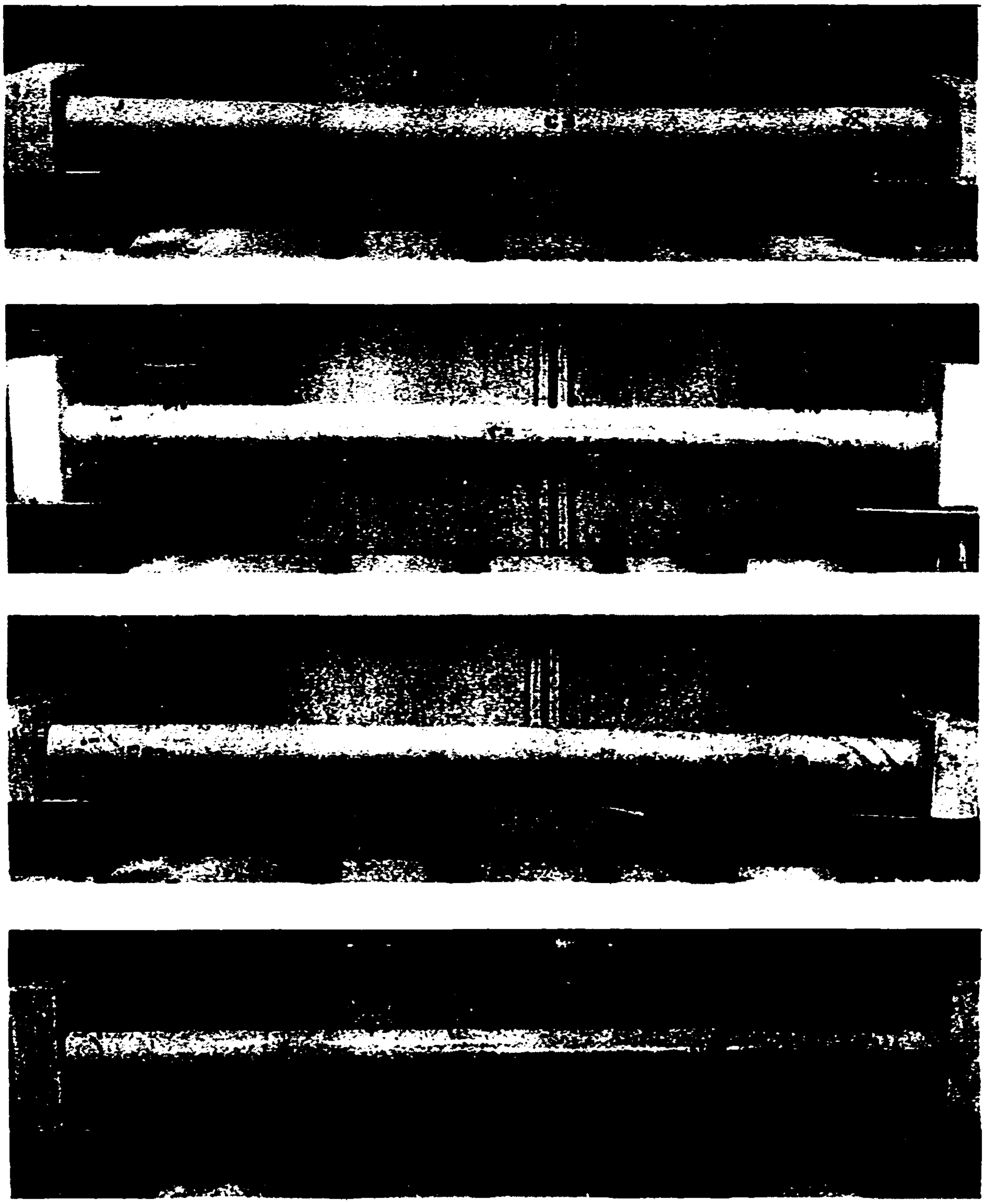

Figure A-9. Column C-3

moment was increased to 4,700 inches-pound, two more cracks were developed, one of them at three inches from the fixed end and the other at a distance of two and one-half inches from the initial crack. Both cracks were similar to the initial crack and have the same angle of inclination with the longitudinal axis of the column. As the torsional moment increased the crack at three inches from the fixed end increased in width to about one-eighth of an inch and the column collapsed there. The torsional moment at collapse was 5,130 inches-pound and the angle of twist was 0.85 degrees.

Concentric Load and Torsion

Column E-1

Column E-1 was tested in combined axial load and torsion. One end of the column was fixed while the other end was insetted in the torsional rig.

The axial load was applied first at increments of 1,000 pounds and the specimen was checked for cracks. A final axial load of 12,000 pounds was reached and no cracks were observed.

The torsional moment was applied gradually while the axial load was maintained at 12,000 pounds to collapse. The initial crack developed at four inches from the fixed end and spread upward on both sides in a spiral way up to the upper third of the cross section. The torsional moment was 6,000 inches-pound and the angle of twist was 0.58 degrees with the longitudinal axis of the column.

As the torsional moment increased to 6,412 inches-pound, and an angle of twist of 0.689 degrees, a second crack similar to the first

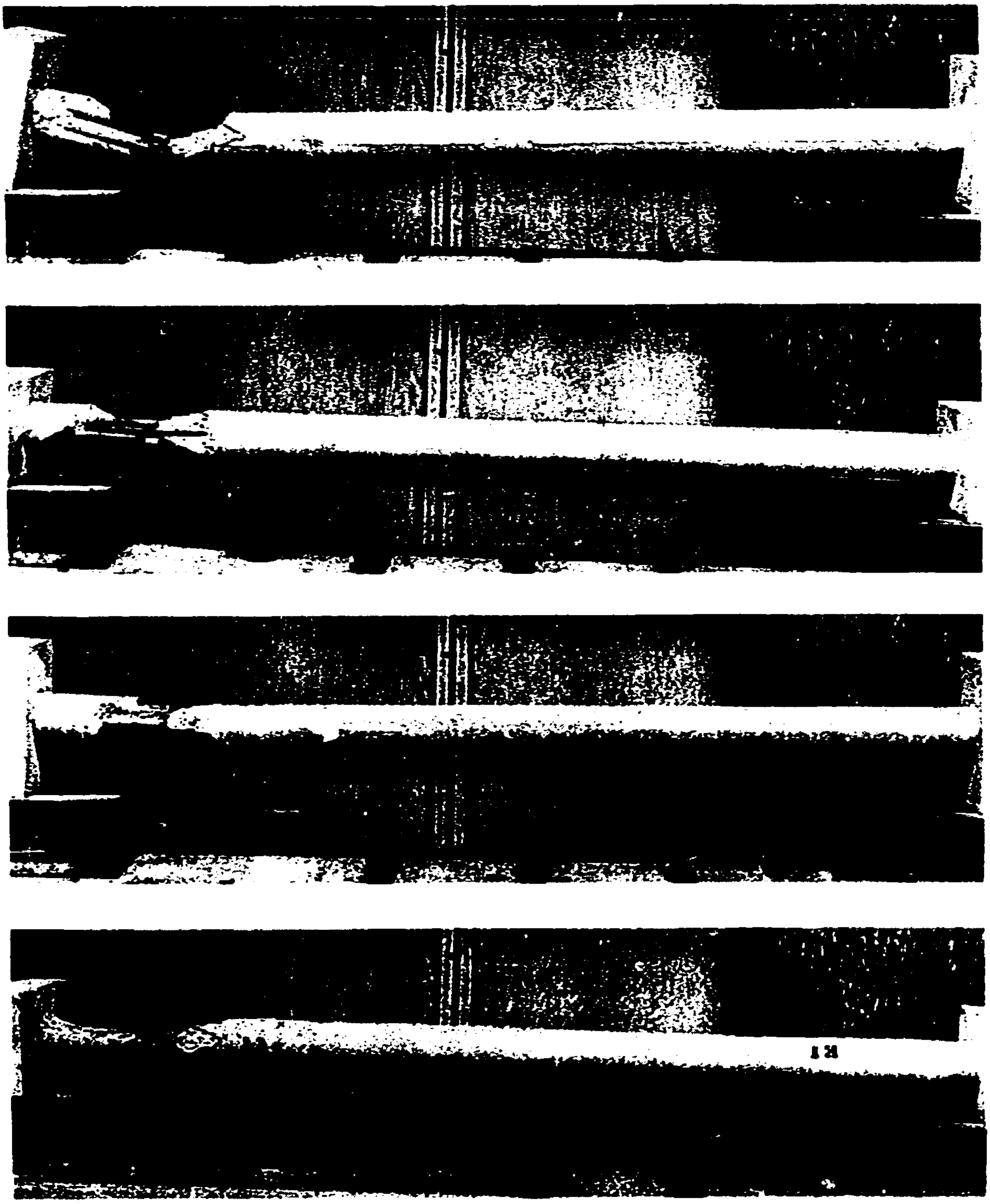

Figure A-10. Column E-1

crack developed at ten inches from the fixed end and had the same angle of inclination with the longitudinal axis of the column. The first crack split into two cracks, one continued upward in the same angle, while the other changed direction and developed horizontally toward the free end support.

An increase of the torsional moment to 6,626 inches-pound caused the column to collapse at the first crack with an angle of twist of 1.074 degrees. The width of the crack at collapse was 0.5 of an inch. Collapse was sudden and noisy.

Column E-2

Column E-2 was tested in combined axial load and torsion. One end of the column was fixed while the other end was attached to the torsional rig.

The axial load was applied first as increments of 2,000 pounds and the specimen was checked for cracks. A final axial load of 20,000 pounds was reached and no cracks were observed.

The torsional moment was applied gradually while the axial load was maintained at 20,000 pounds up to collapse. The initial crack developed at two inches from the fixed end, spread upward on both sides in a spiral way up to the middle of the cross section. The torsional moment was 6,900 inches-pound and the angle of twist was 1.12 degrees. The crack made an angle of 18 degrees with the longitudinal axis of the column.

As the torsional moment increased to 7,500 inches-pound, a second crack developed very similar to the first crack and at 12 inches from the

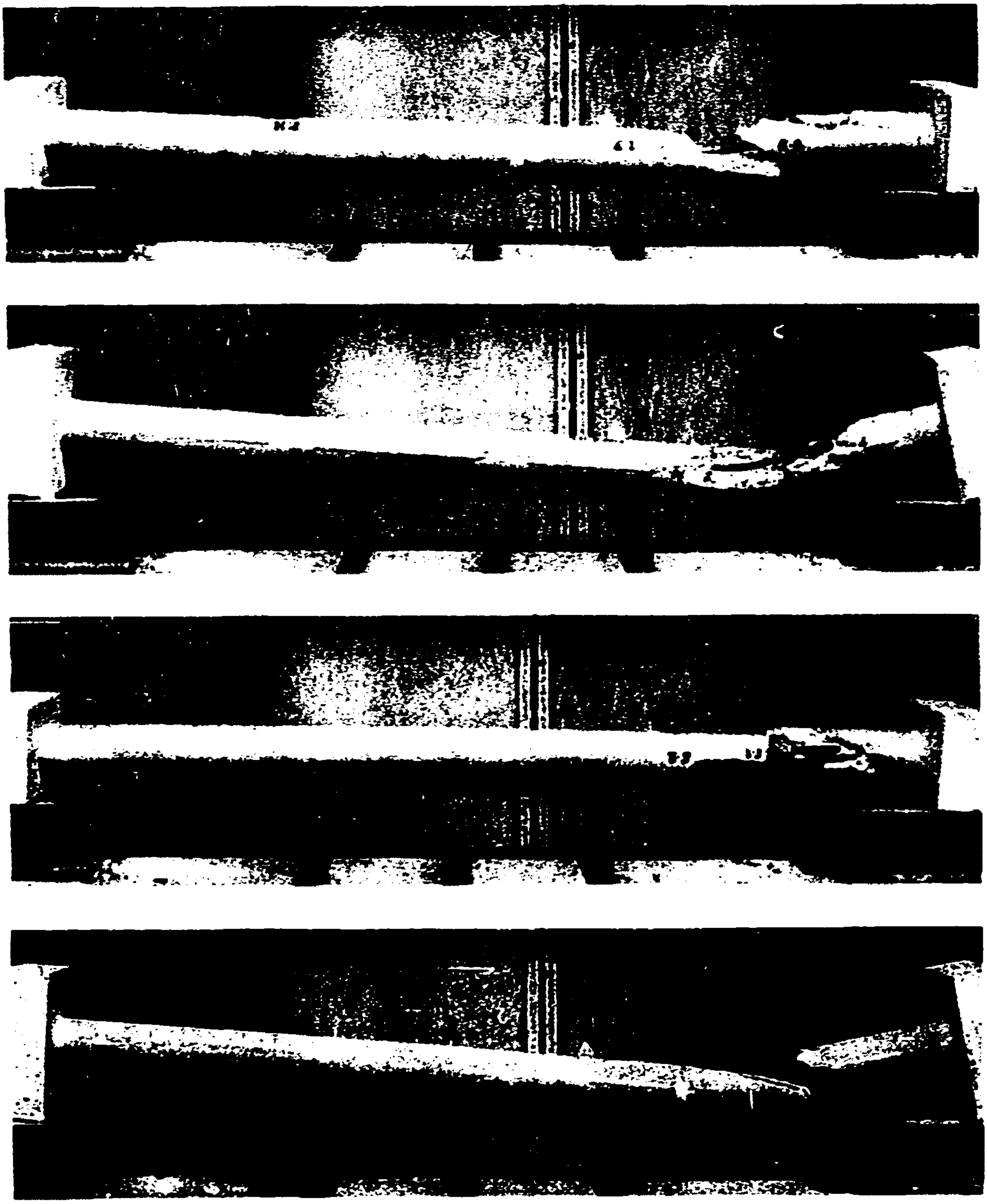

Figure A-11. Column E-2

fixed end with the same angle of inclination and an angle of twist of 1.159 degrees. The torsional moment then increased to 7,900 inches-pound and an angle of twist of 1.255 degrees, causing the second crack to split into two cracks; one continued in its spiral manner while the other developed horizontally under the prestressing cable toward the first crack.

As the torsional moment increased, the cracks got larger in width and the column collapsed at the second crack in a very violent manner and accompanied with debris. The torsional moment at collapse was 8,764 inches-pound with an angle of twist of 2.021 degrees.

Pure Bending and Torsion

Column A-3

Column A-3 was tested in combined bending and torsion. The torsional moment was applied through the torsion rig and the moment was implemented with a two point system symmetrical about the middle of the column.

The torsional moment was applied first until a moment of 2138 inches-pounds was reached (45% of ultimate). The flexural moment was applied next and torsional moment of 2138 inches-pounds was maintained until collapse.

The first crack was observed at a moment of 3,700 inches-pound and a distance of 28 inches from the left end of the column. The crack had a spiral pattern similar to the one developed in pure tension. The ratio of moment/torque was 1.73. The second crack developed similarly at a distance of 53 inches from left end. In the same time, the initial crack propagated toward the left point of loading. At a moment of 5945 inches-pounds another crack developed at a distance of 43 inches from left end, it was also a spiral type crack. A new crack that looks more like a flexure type crack developed at a distance of 33 inches from the left end and a moment of 6560 inches-pound. At a moment of 7500 inches-pounds and at a distance of 24 inches from the left end a new crack developed. All other cracks propagated in a spiral fashion on both sides of the column up to the plane of the prestressed steel.

At a moment of 8,750 inches-pound, the crack at 33 inches propagated horizontally toward the loading point at 50 inches from the left end. In the meantime, all other cracks were getting larger in width. At 10,000 inches-pound another crack developed at 19 inches from the left end, while the crack at 33 inches reached the loading point at 50 inches and joined another crack that started at 53 inches. The column collapsed at an ultimate moment 10,950 inches-pound in crushing of the concrete under the loading point 50 inches from the left end.

Column F-3

Column F-3 was tested in combined bending and torsion. The bending moment was applied through a two point system symmetrical about the middle of the column. The torsional rig was used to implement the torque.

The bending moment was applied first until a moment of 5000 inches-pound was reached (28% of ultimate). The torsional moment was applied next and the bending moment was maintained constant until collapse.

The first crack was observed at a torsional moment 2993 inches-pound

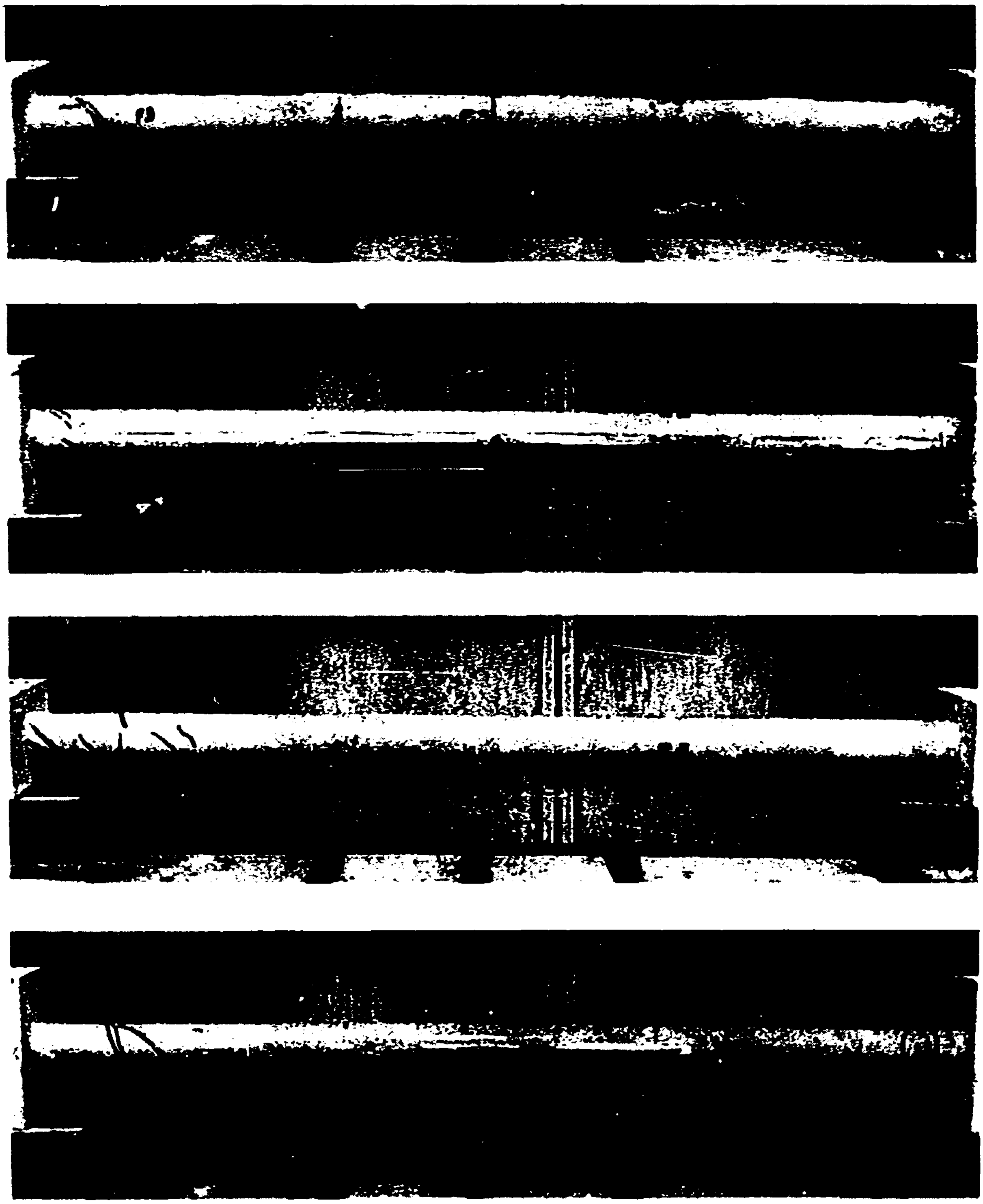

Figure A-12. Column F-3

at a ratio of torque/moment of 0.60. It developed at a distance of 15 inches from the left end. When the torque reached 4275 another crack developed in the bottom at a distance of 7.5 inches and propagated in a spiral fashion in the direction of the torsion rig. The end of the crack at the top was 4.5 inches from the left end. When the torque was 4,489 the second crack propagated on top side horizontally all the way to the left end. At a moment of 4,703 inches-pound the two cracks increased in width. Another crack at 4.5 inches from the left end, propagated from the middle back and moved away from the left end at almost 45 degrees angle with the longitudinal axis of the column. The torque then was 4917 inches-pound.

The column collapsed at torque of 5130 inches-pound and at the location of the second crack.

Torsion Combined with Eccentric Load

Column D-2

Column D-2 was tested in combined torsion and eccentric load. One end of the column was fixed while the other end was inserted in the torsion rig. The eccentric load was applied first and then the torsional moment.

The load was applied with an eccentricity of two inches. An increment of 3000 pounds was reached. Then this load was maintained constant throughout the application of the torque until collapse. The column was checked for cracks and found none.

The torsional moment was applied at an increment of 428 inches-pound, then decreased gradually as collapse was approached. A spiral

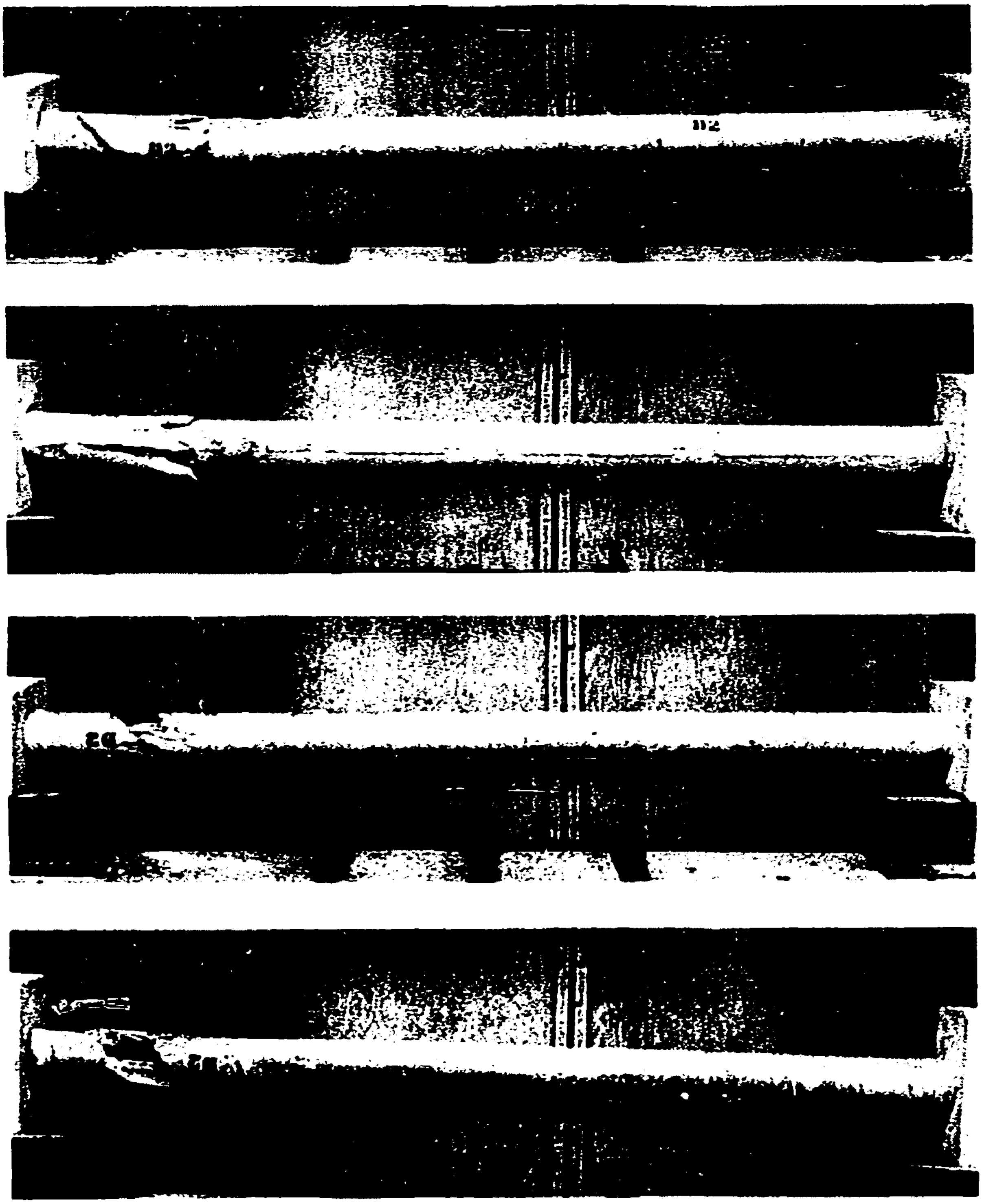

Figure A-13. Column D-2

type crack developed on the bottom and at a distance of 6.25 inches from the loaded end. The torque was 3206 inches-pound and the angle of twist was 0.397 degrees. As the torque reached 3420 inches-pound another crack developed at 2.25 inches from the loaded end. The first crack started to change angles from 30 degrees to about 40 degrees as it propagated upward; the torque then was 3850 inches-pound. The second crack was propagating upward toward the loaded head in about a 30 degree angle. At torques of 4,500 and 4,700, the first crack and the second crack widened considerably. As the torque approached 4,900 inches-pound, the first crack developed into a new crack about 2/3 up and propagated horizontally away from the loaded end.

The column collapsed at a torque of 5130 inches-pound and an angle of twist of 1.18 degrees; failure was gradual and accompanied by spalling of the concrete.

Column D-3

Column D-3 was tested in combined torsion and eccentric load. One end of the column was fixed while the other end was inerted in the torsional rig. The eccentric load was applied first and then the torque.

The eccentric load was applied first at increments of 500 pounds up to 5000. The load had an eccentricity of two inches. This load was maintained constant throughout the torque phase of loading until collapse occurred.

The torque was applied at an increment of 428 inches-pound, then decreased gradually as collapse was approached. The first crack developed at a distance of 6.25 inches from the loaded end and a torque of 1500 inches-pound and an angle of twist of 0.60 degrees. The crack was a

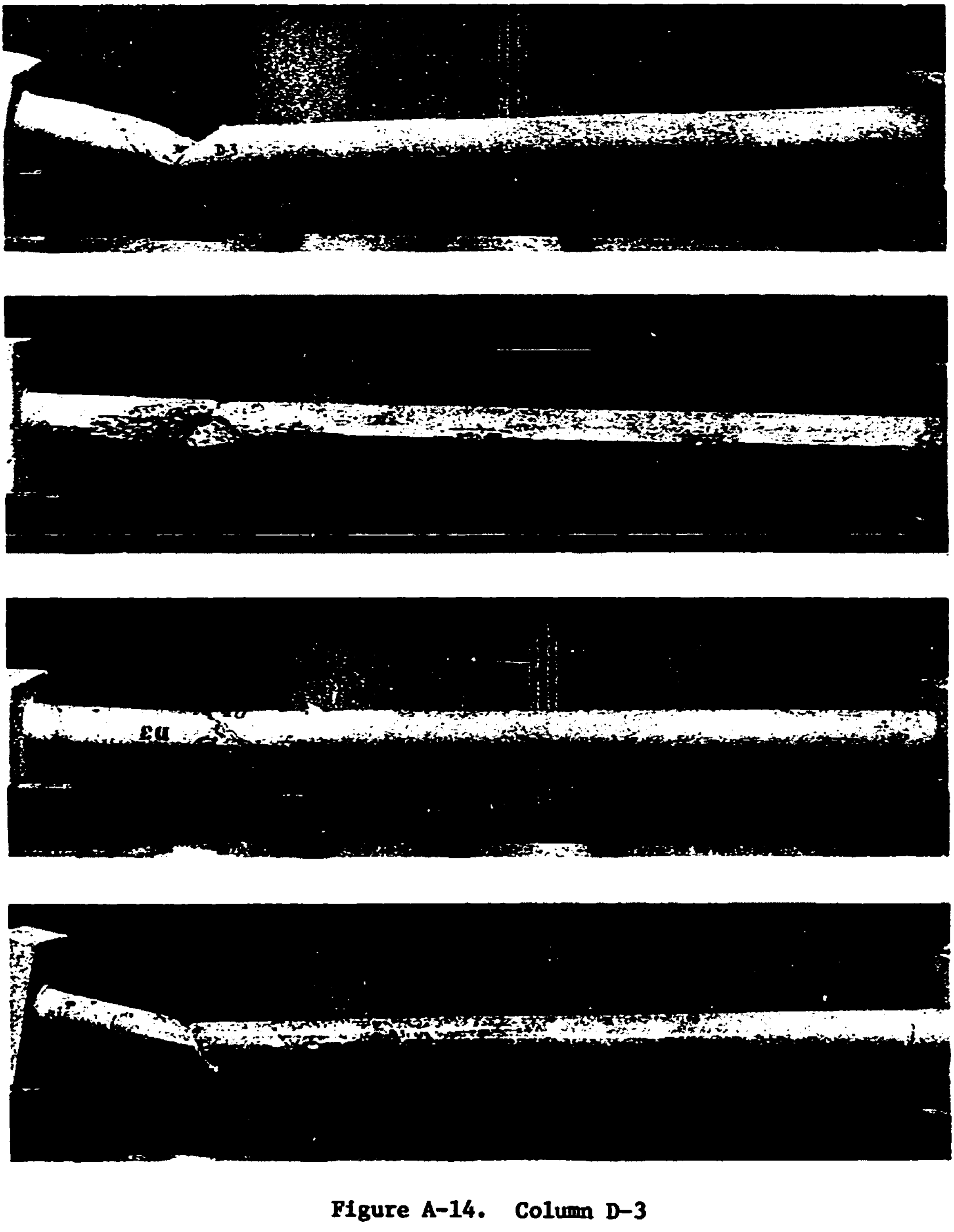

Figure A-14. Column D-3

spiral type and extended up on both sides with an angle of 30 degrees to about 1/4 of the cross section area.

At a torque 3420 inches-pound two new cracks were observed at distances of 1.75 inches and 4.0 inches from the loaded end. Both cracks were spiral type developed on the bottom and propagated upward to about 2/3 the area.

When the torque applied increased to 4702 inches-pound the cracks at 1.75 and 6.25 shifted direction and propagated toward the loaded head at an angle of 45 degrees and 30 degrees respectively. A new crack developed at a torque 4900 inches-pound at a distance 13 inches from the loaded end. The crack propagated spirally on one side with an angle of 30 degrees to 1/3 of area, while on the other side, it propagated to about 2/3 of area. At the same torque, another crack developed at 20 inches on the bottom, propagated on both sides with an angle of 35 degrees. When the torsional moment reached 5000 a new crack was observed on the bottom at a distance of 27.5 inches from the loaded end. All cracks were forming at a 7-inch pitch.

Collapse was gradual and accompanied by spalling of concrete at 13 inches from the loaded end. The collapse torque was 5000 inches-pound and the angle of twist was 0.912 degrees.

Column G-3

Column G-3 was tested in combined torsion and eccentric load. One end of the column was fixed while the other end was inserted in the torsion rig. The torque was applied first then the eccentric load was implemented with an eccentricity of two inches.

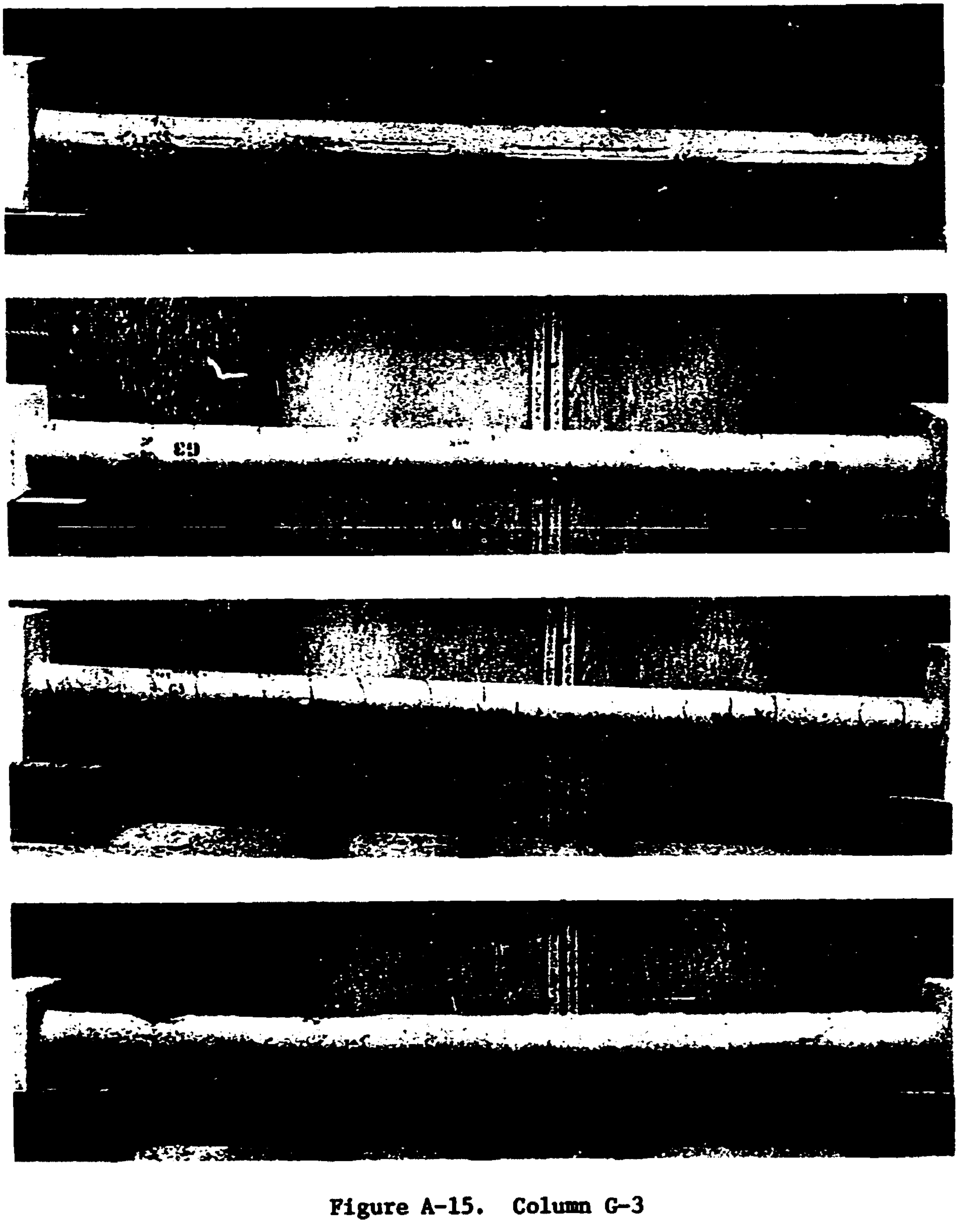

Figure A-15. Column G-3

The torque was applied at an increment of 214 inches-pound up to 2138 inches-pound and then maintained at that moment throughout the second phase of loading. The eccentric load was incremented at 500 pounds and decreased gradually when reached collapse.

The first crack developed at a load of 2000 pounds and a distance of 21.5 inches from loaded end. The crack was diagonal in shape and on the bottom side of the column. At a load of 3000 pounds three new cracks were observed, at distances of 14 inches, 21.5 inches, and 24 inches from the loaded end. All were developed on bottom and propagated diagonally when the load was 3100 when another similar crack was observed at 10 inches from the loaded end. Two similar cracks were observed at 27.5 inches and 32.5 inches from the loaded end at a load of 3200 pounds. When the load was 3500 the cracks at 17.5 inches and 21.5 inches ·ropagated diagonally toward the loaded head. At a load of 4000 pounds, a similar crack developed 42 inches from the loaded end. All other cracks were propagating diagonally at an angle of 40 degrees.

When the eccentric load was 4500, diagonal cracks developed near the fixed end on the bottom part. These cracks were 3.5 inches apart and extended a distance of 42 inches from the fixed end. At a load of 5000 pounds, two new cracks developed at distances of 27 inches and 37.5 inches. At 6000 pounds, another crack at 19 inches from the loaded end. Then, at 6500 pounds, cracks at 13.5 inches and 22.75 inches from the loaded end developed on the bottom and were flexure type cracks. It was also observed that the cracks between the loaded end and around the center of the column were increasing in width.

As the load increased to 7000 pounds, the cracks at 18.5, 27.0

and 42.0 were developing upward at an angle of 40 degrees.

At 8000 pounds, the crack at 17 inches was increasing in width, while the cracks near the fixed end were completely closed. At 9000 pounds, all cracks between loaded end and a distance of 27 inches had propagated up 90% of the area at an angle of about 40 degrees. While at 10,000 pounds all cracks from the fixed end and up to 14 inches from there have closed completely. Some of these cracks that closed had a width of about one millimeter. At 10,500 all cracks up to 23 inches from the loaded end closed.

The column collapsed by crushing of concrete at 17 inches from the loaded end and at a load of 10,600 pounds.

APPENDIX B

LOAD-DEFORMATION CURVES

Figure

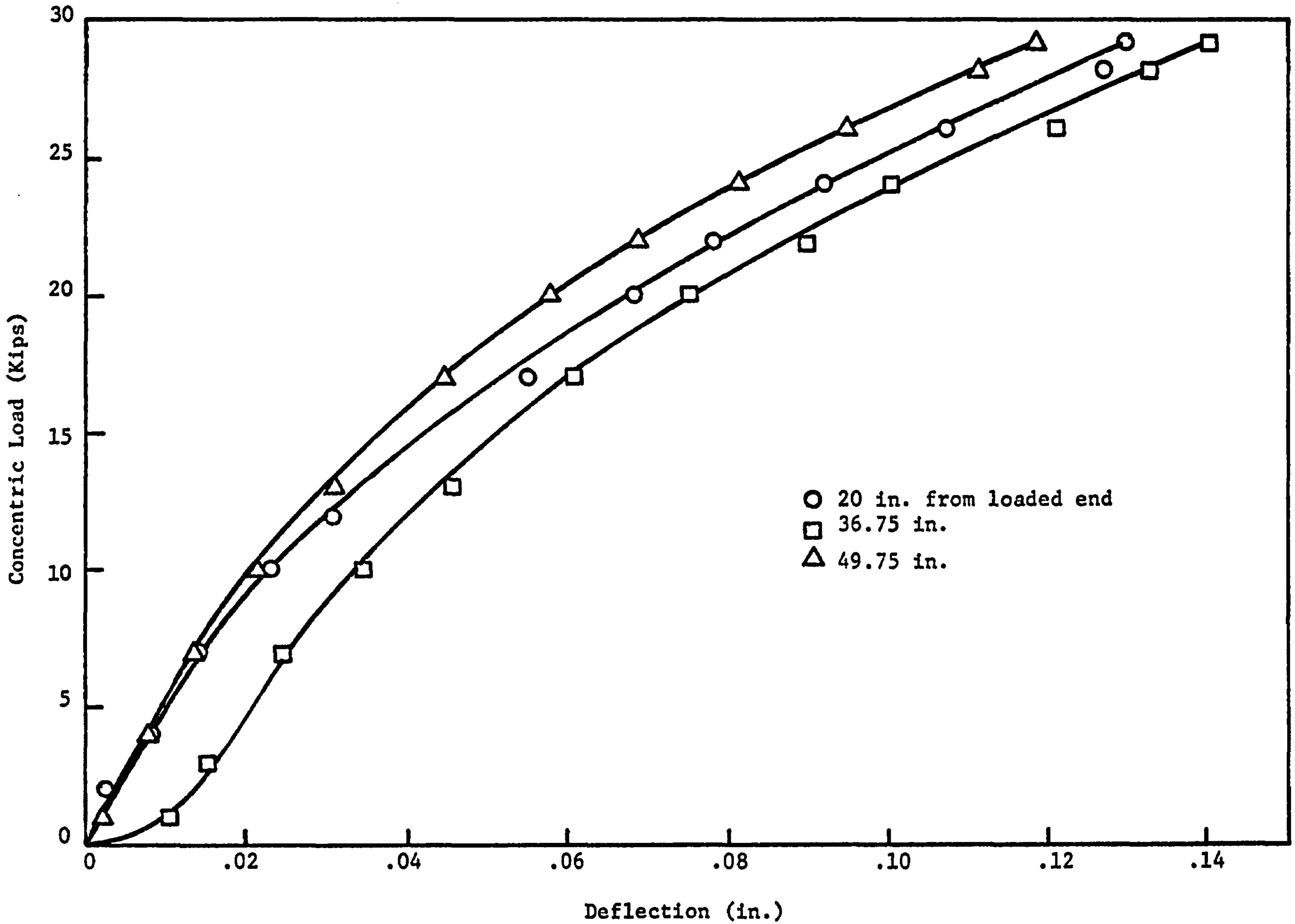

Figure B-1. Concentric Load vs. Deflection, Column B-2

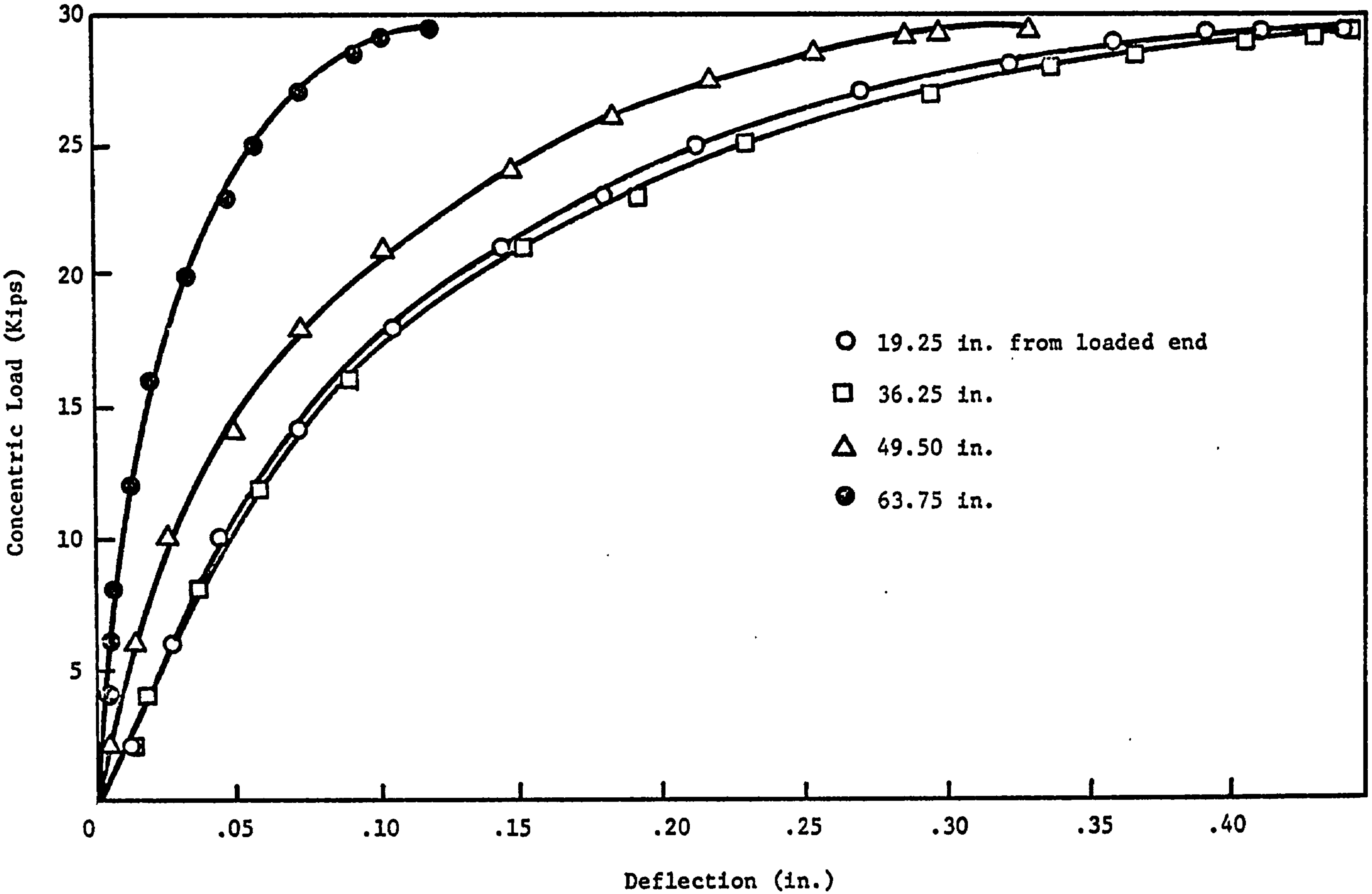

Figure B-2. Concentric Load vs. Deflection, Column B-3

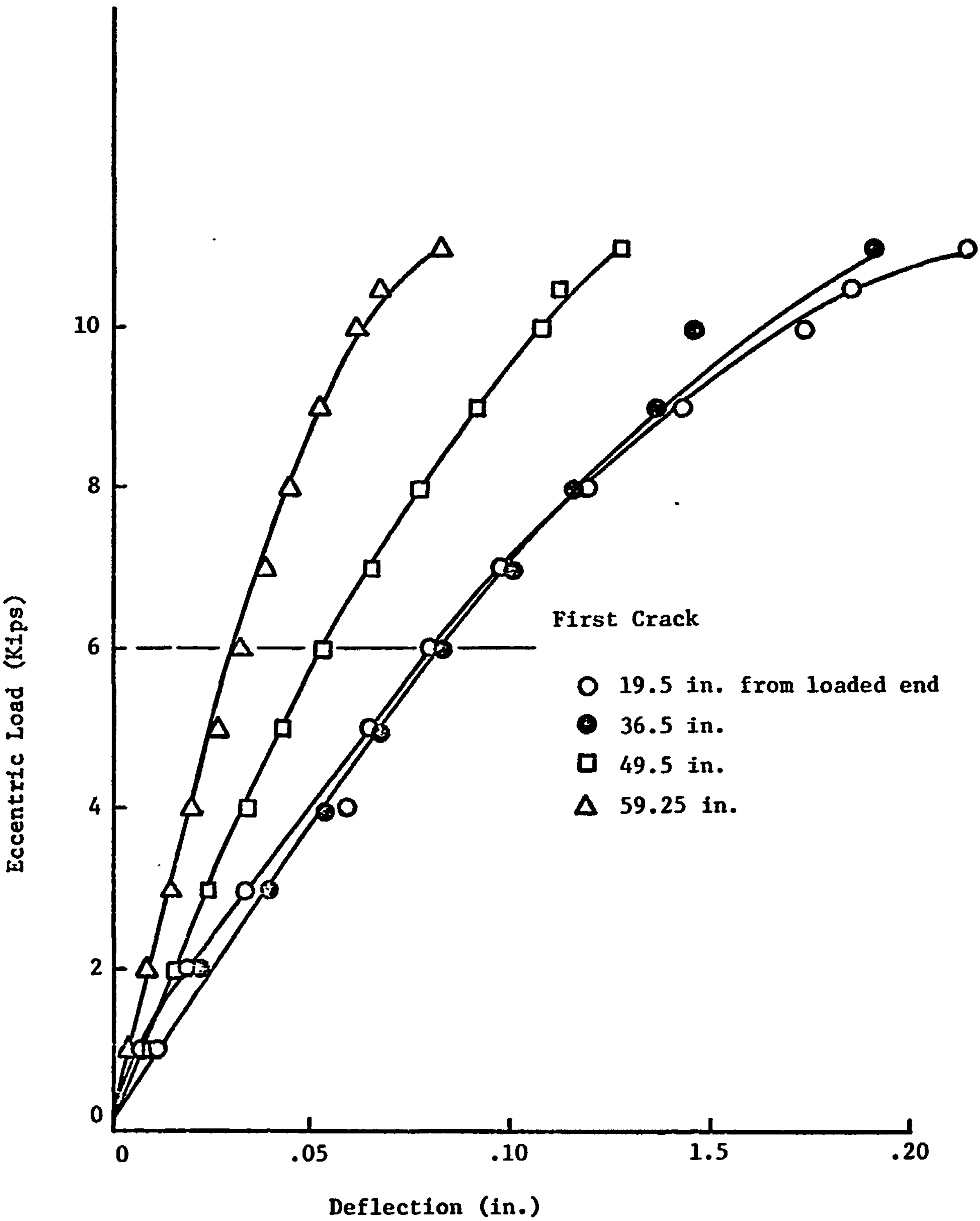

Figure B-3. Eccentric Load vs. Deflection, Column E-3

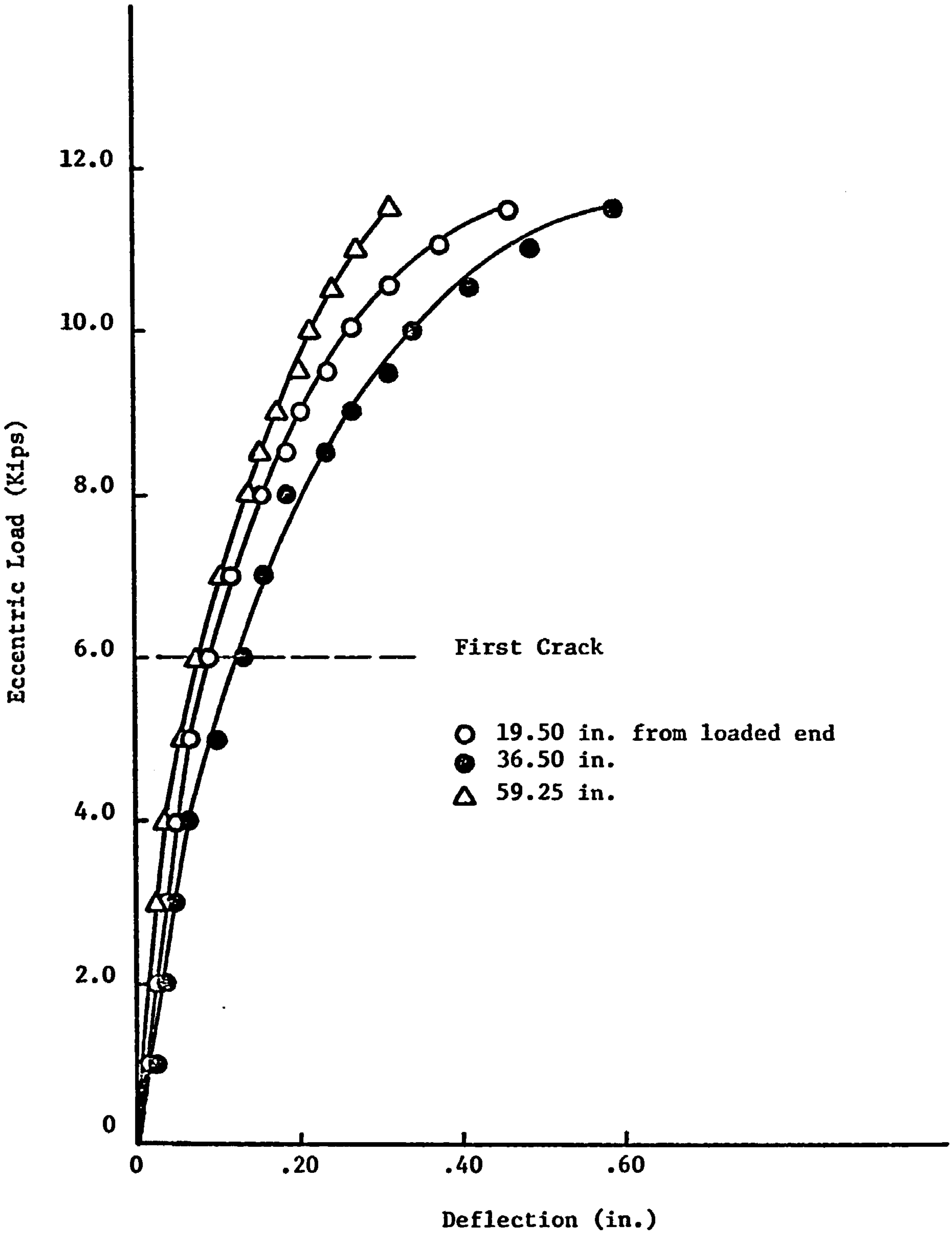

Figure B-4. Eccentric Load vs. Deflections, Column F-1

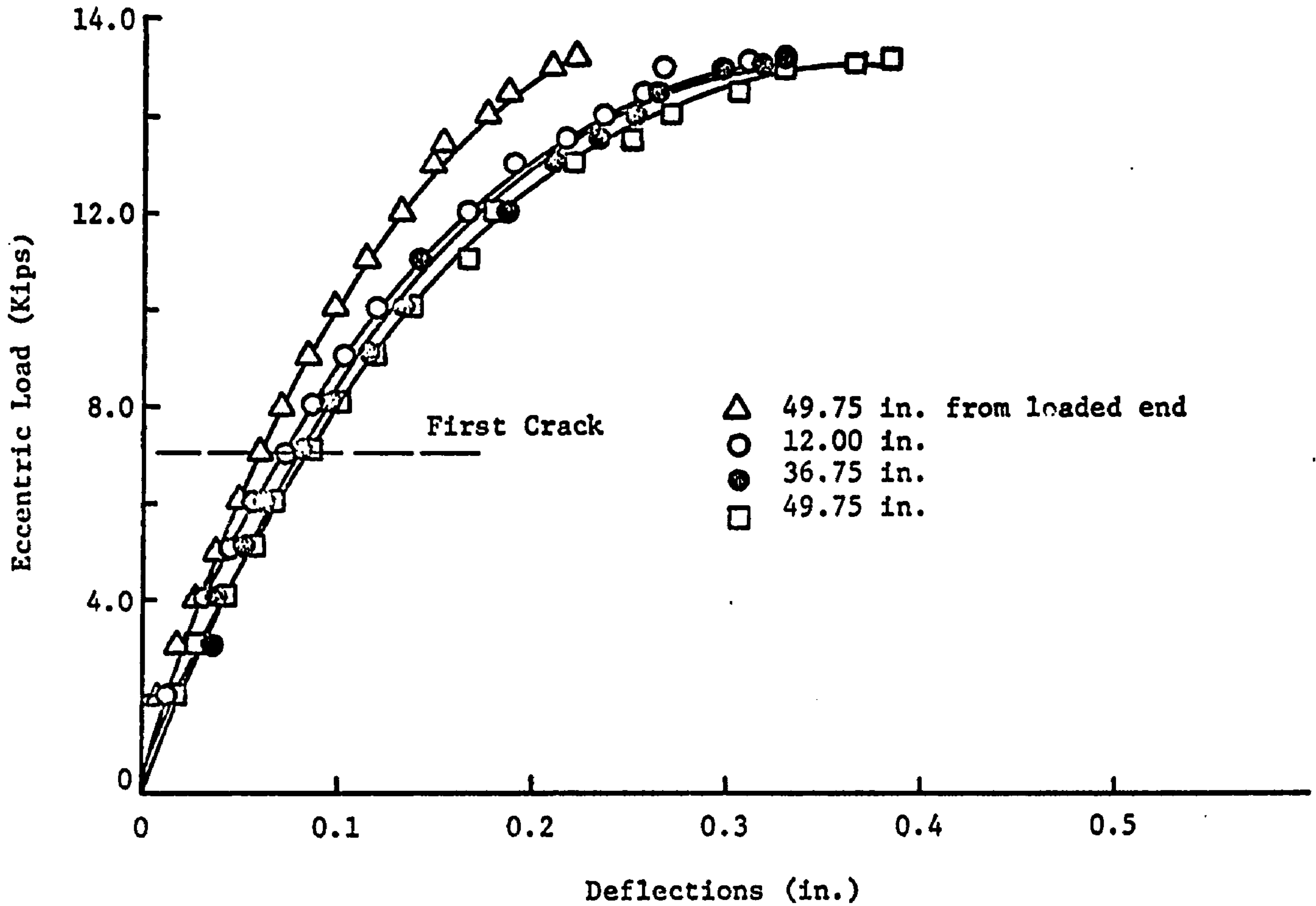

Figure B-5. Eccentric Load vs. Deflections, Column F-2

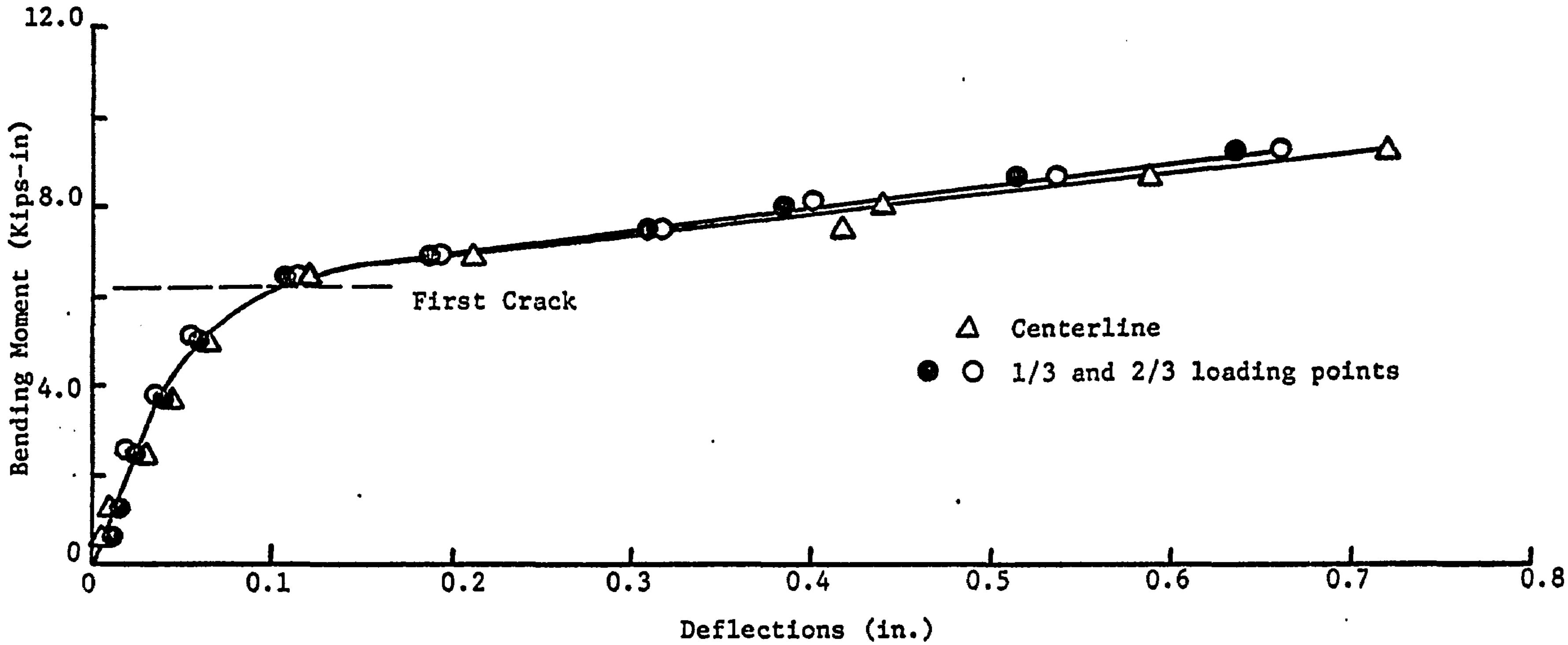

Figure B-6. Bending Moments vs. Deflections, Column G-1

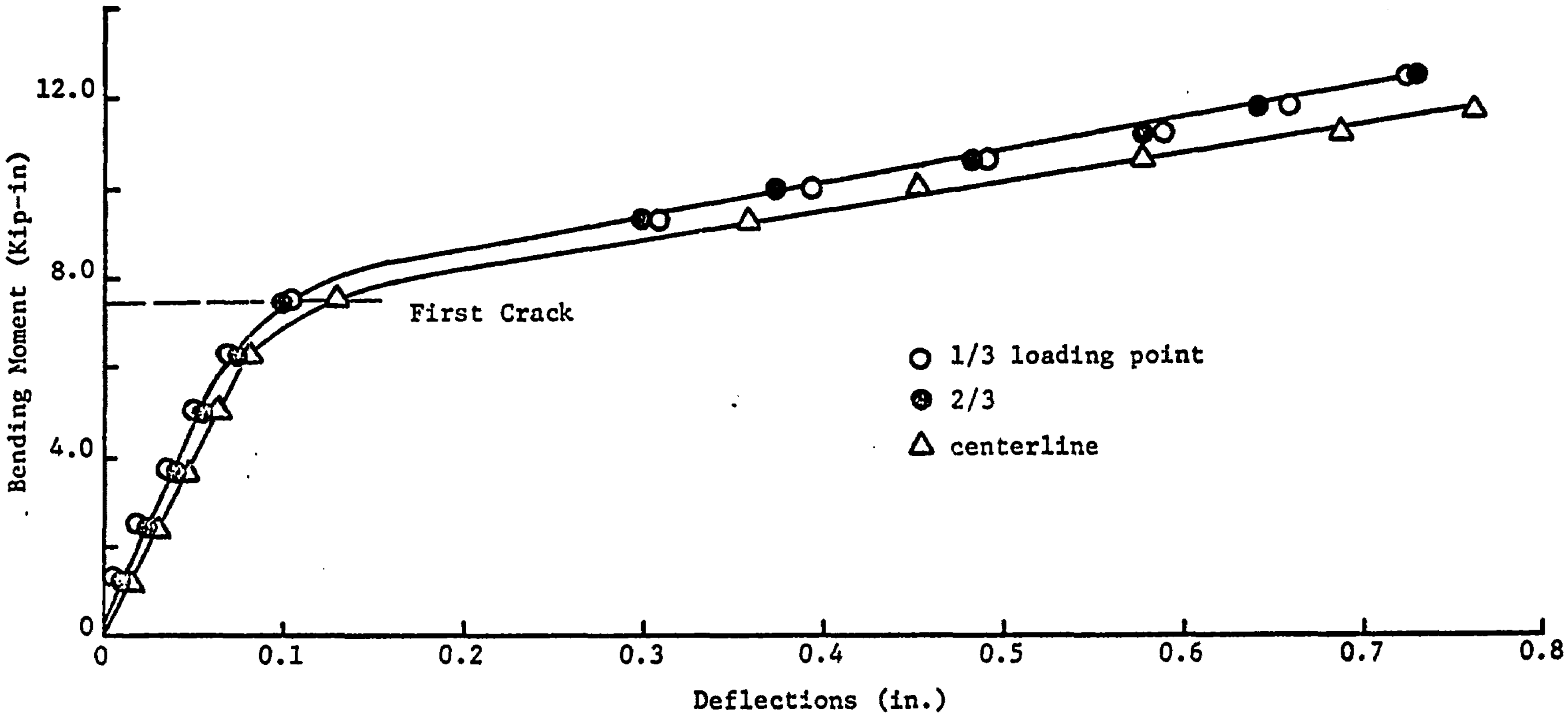

Figure B-7. Bending Moment vs. Deflections, Column G-2

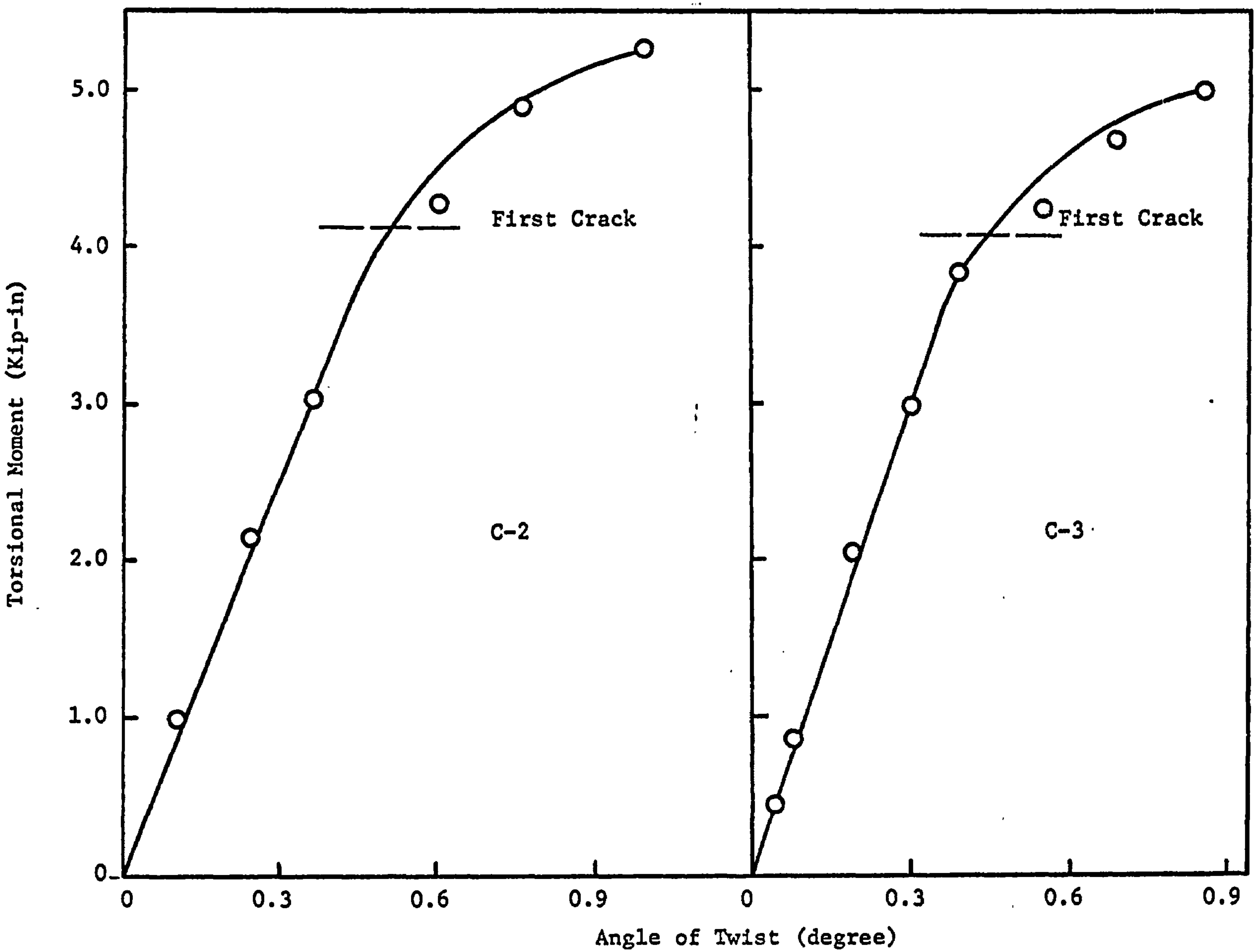

Figure B-8. Torque vs. Angle of Twist, Columns C-2 and C-3

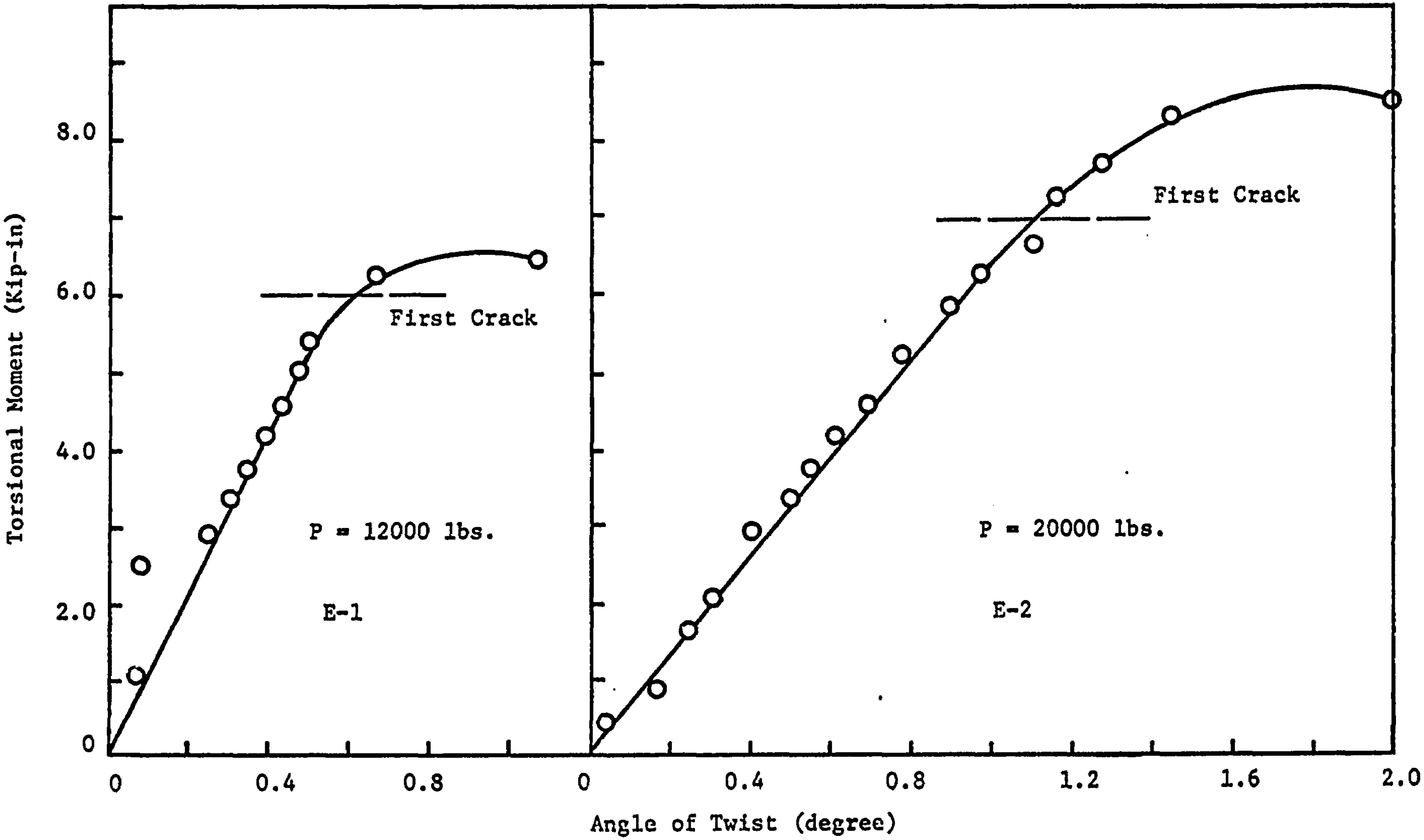

Figure B-9. Torque vs. Angle of Twist, Column E-1 and E-2 in Combined Concentric Load and Torsion

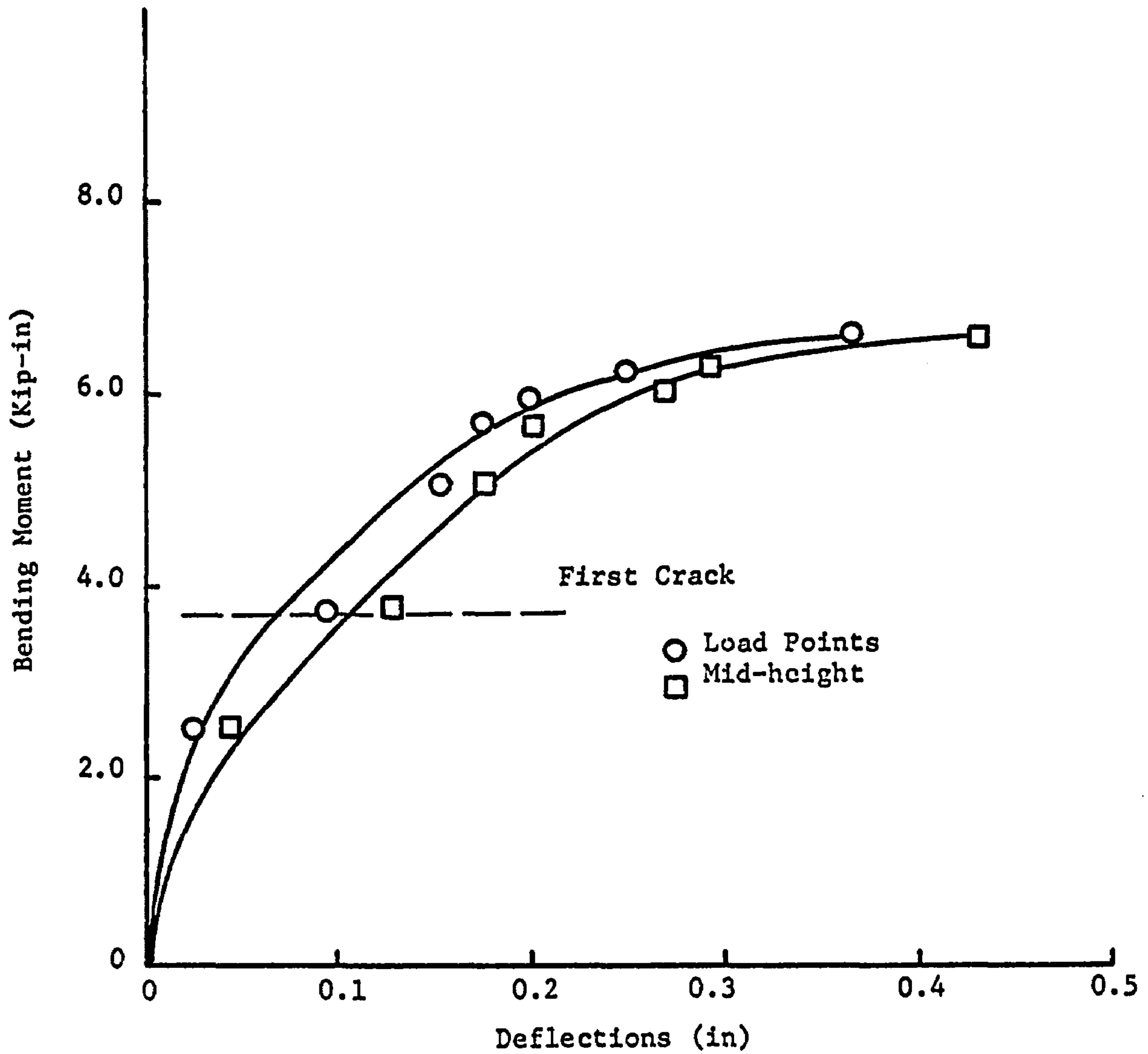

Figure B-10. Bending Moment vs. Deflections, Column A-3 in Combined Bending and Torsion

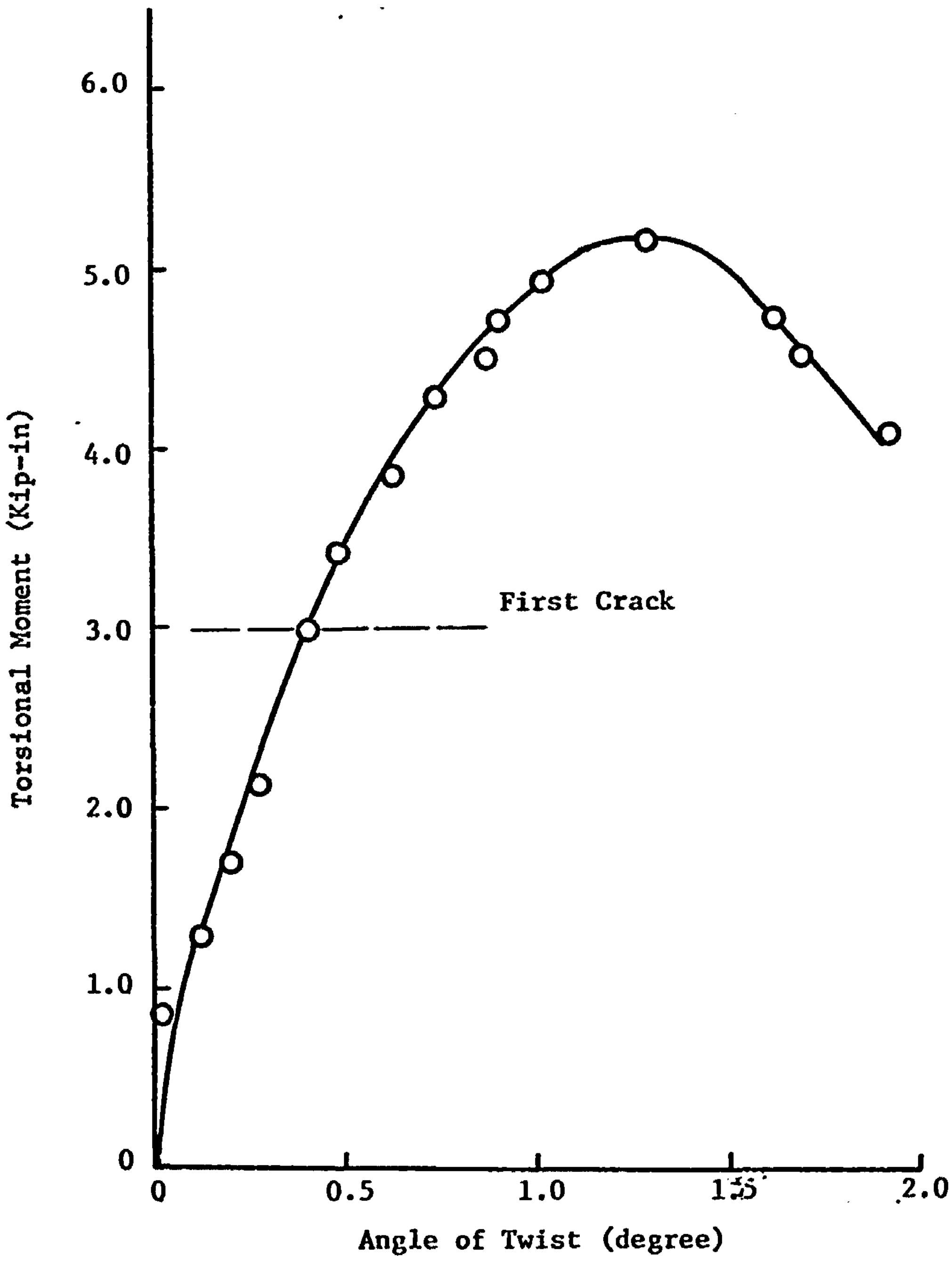

Figure B-11. Torque vs. Angle of Twist, Column F-3 in Combined Bending and Torsion

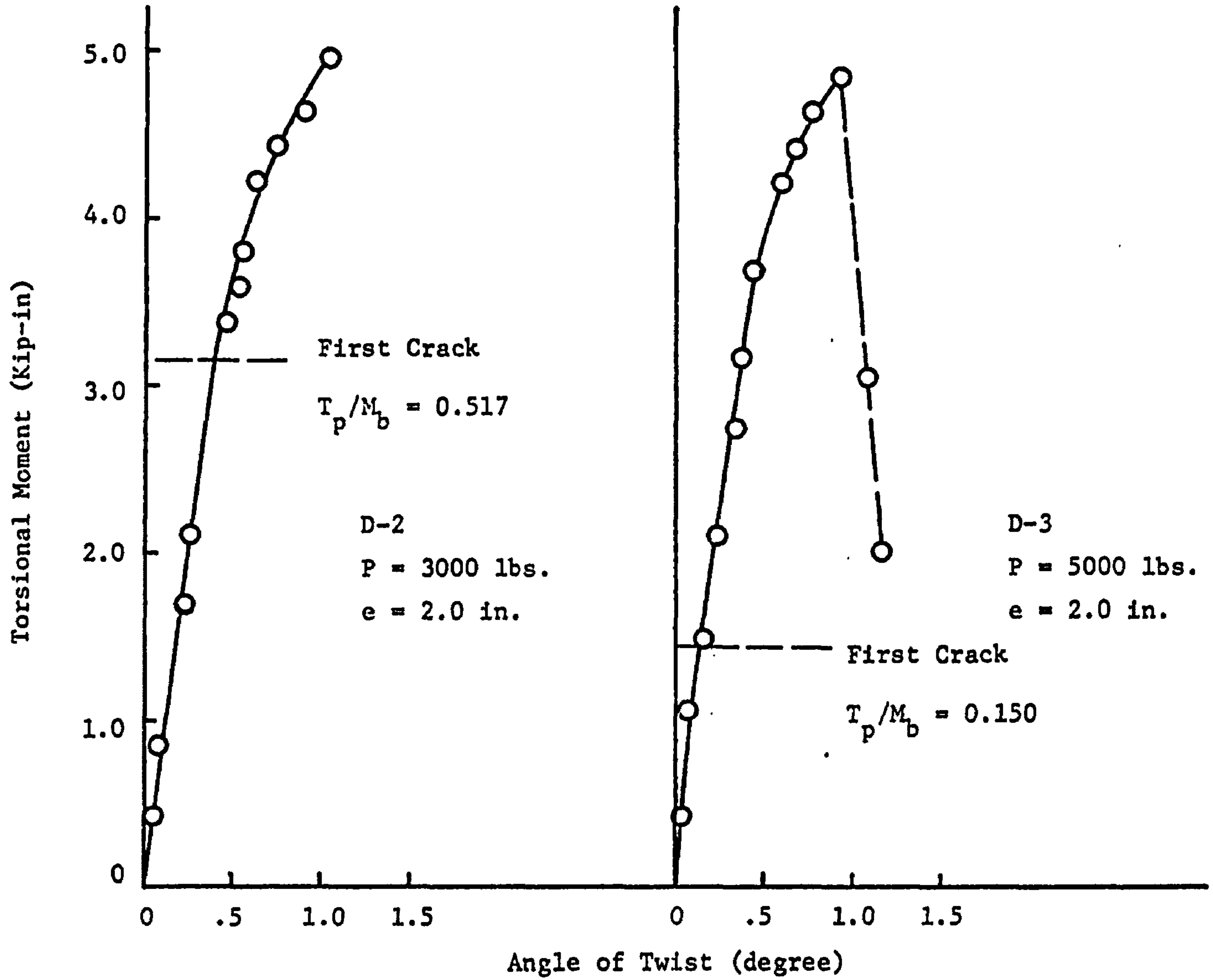

Figure B-12. Torque vs. Angle of Twist, Columns D-2 and D-3 in Combined Concentric Load, Bending Moment and Torsion

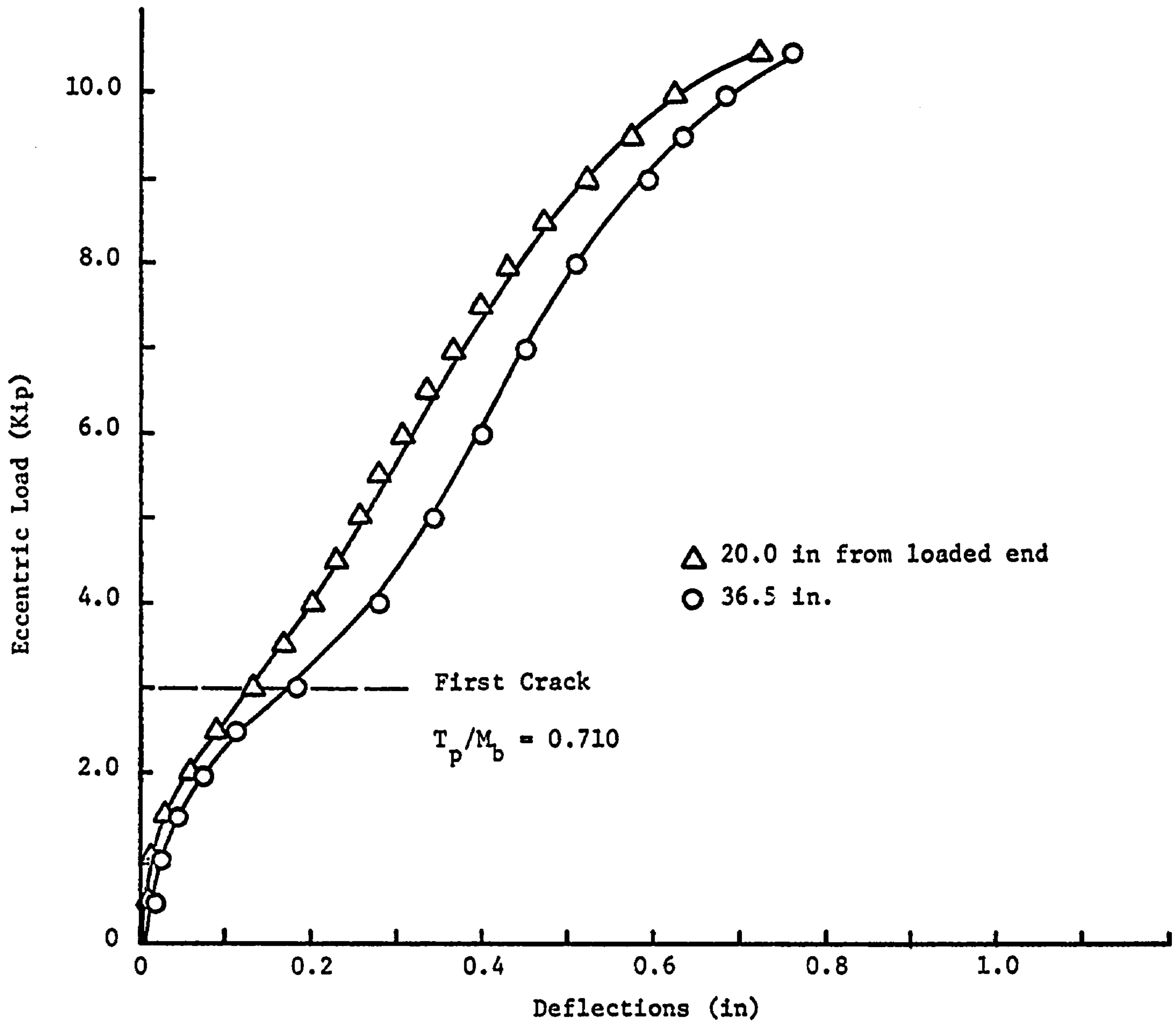

Figure B-13. Eccentric Load vs. Deflections, Column G-3 in Combined Concentric Load, Bending Moment and Torsion

APPENDIX C

MATERIAL PROPERTIES

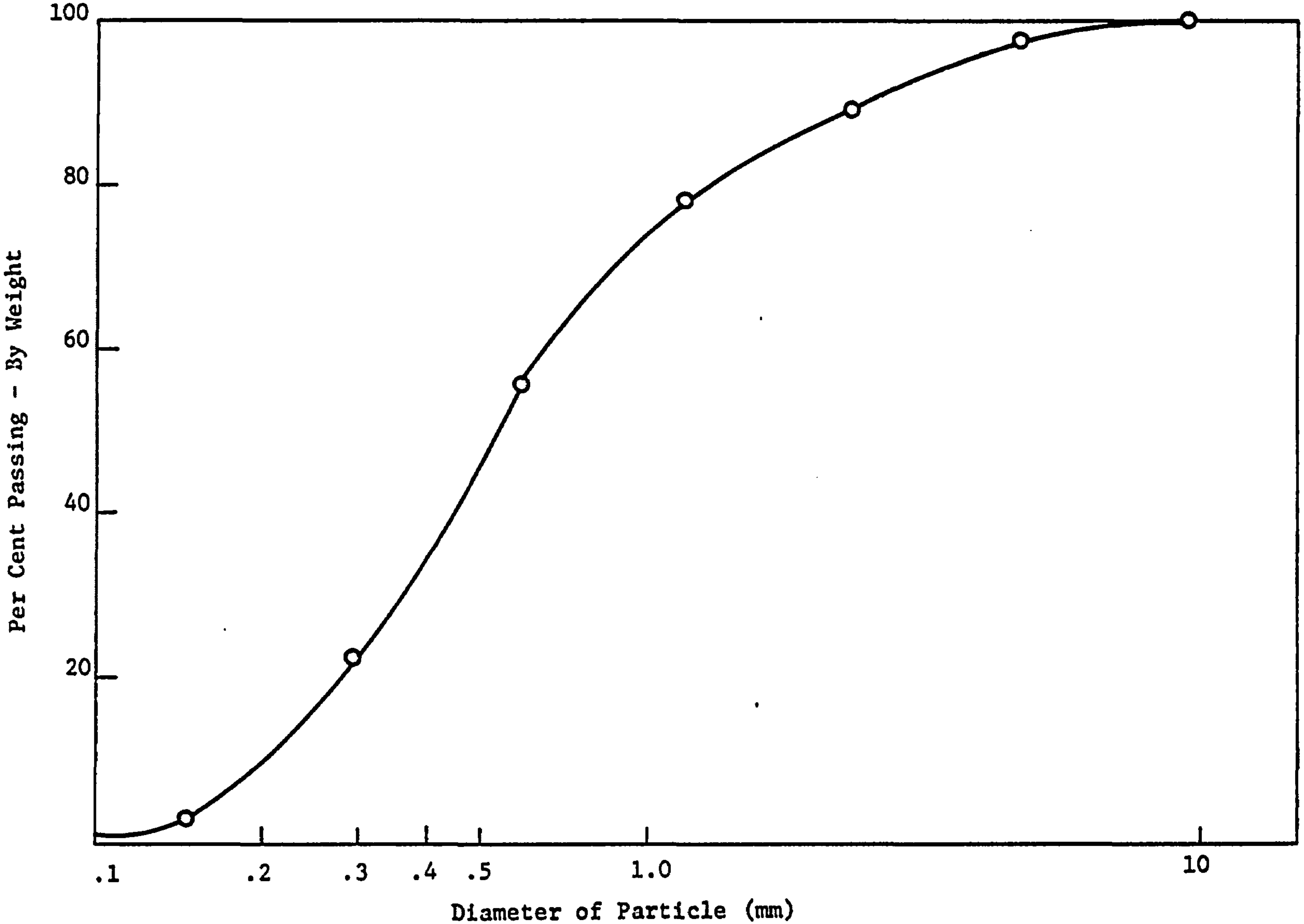

Figure C-1. Sieve Analysis for Sand

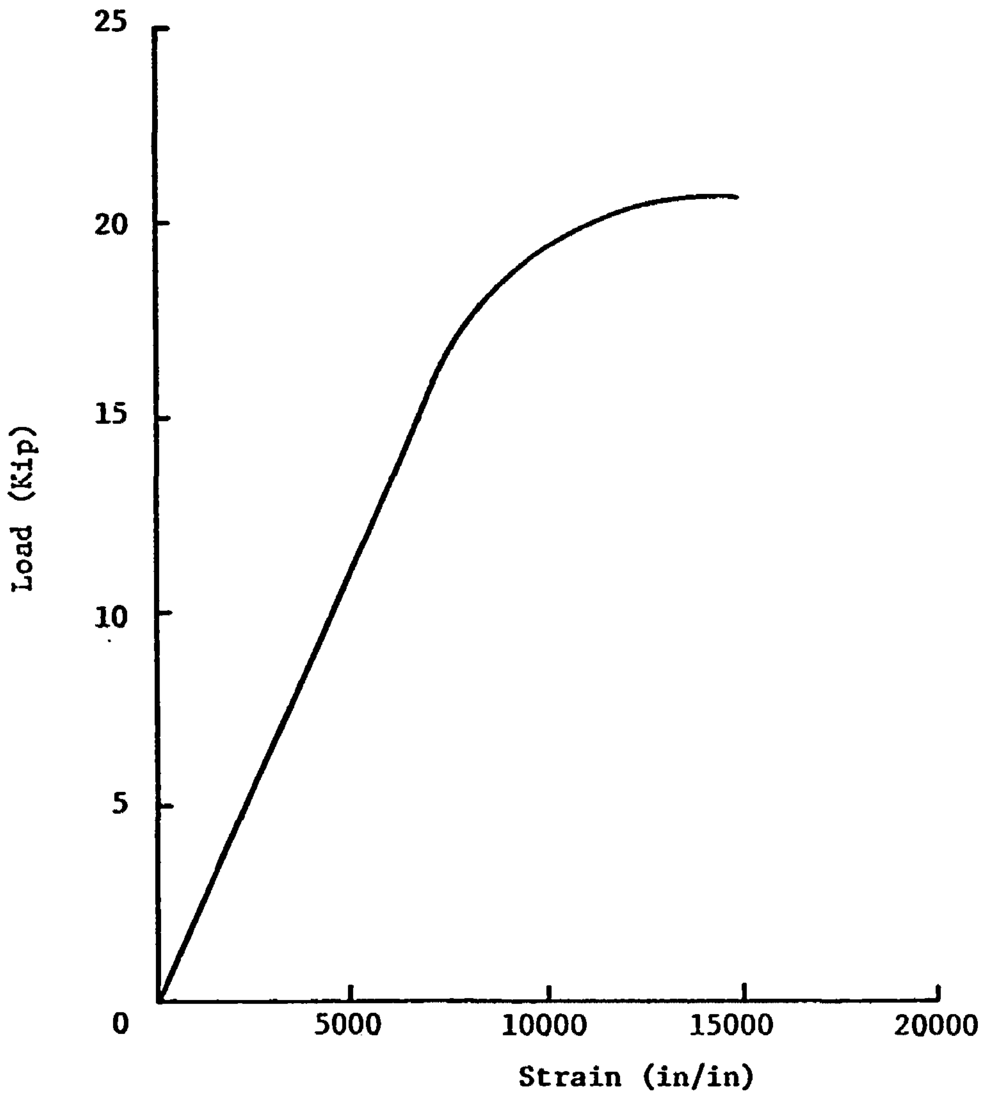

Figure C-2. Load-Strain Diagram for Prestressing Cable