CONTROLLER GAIN SCHEDULING IN A MULTI-SPAN WEB LINE

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ABSTRACT

Gain scheduling is a common method of variable gain feedback control used to account for changes in the diameters of an unwind roll and a rewind roll during the operation of a web line. This paper focuses on three methods for gain scheduling in a multi-span web line with tension feedback. In an earlier paper the benefit of gain scheduling in the feedback control of tension in a web span entering a rewind roll was demonstrated for the case when the rewind roll had an increasing roll diameter during operation of the line [1]. This method is reviewed a background for this paper. One of the methods in this paper uses the same idea of pure tension control at the unwind and rewind. Simulations are used to compare a pure tension control method with control strategies that employ speed-based web tension control at the rewind and unwind. All three methods use the parameters of the Euclid Web Line in the Web Handling Research Center at Oklahoma State University. The line is controlled by Rockwell Automation drives.

The Euclid line has five control zones – unwind roll, s-wrap lead and follow rolls, pull roll, and rewind roll – and seven total controllers. Each controller uses Proportional plus Integral Control. Method 1 uses pure tension control in the unwind and rewind, while methods 2 and 3 employ speed-based web tension control systems [2]. The latter control technique uses an inner speed loop and an outer tension loop. Tension is measured with load cells. The other three control zones use simple speed controls. Roll speed is measured with rotary encoders in all five motors. During an unwind-rewind operation, the unwind and rewind roll inertias (roll diameters) vary with respect to time. The first method allowed the tension gains to vary with roll diameter. The second method allowed the gains of both the speed and tension controls to vary with roll diameter. In the third method, the tension gains were held constant while the speed gains were allowed to vary with the roll diameter.

A linearized models of the Euclid Web Line was tailored for each control method. Simulations using the models were conducted to determine the effect of varying inertia of the unwind and rewind rolls. Two cases were considered for each method: (i) fixed gains

in the tension loops and speed loops in the unwind and rewind controllers, and (ii) variable gains in both loops (Method 2) or in the speed loops only (Method 3) based on time varying inertias of the unwind and rewind rolls. Simulations show that for large changes in the inertias, the variable gain approach results in better system performance than that with a fixed gain approach.

INTRODUCTION

Web material has to travel through multiple control sections in order to be unwound from the parent roll, processed once or more times, and finished. Finishing could include cutting to length or winding onto another roll for storage until it is used in another process. In each section the web travels through, speed and tension should be monitored because excessive tension changes may cause blemishes in in the web material or rupture and loss. The defense against loss is controlling tension in the web.

Tension control has been the subject of many papers over the years. Only a few early papers are cited here, and these have been cited in most papers that have followed. Campbell developed a dynamic model for tension in a web assuming small strains and Hooke's Law, and discussed several methods of tension control when a web is transported [3]. King modeled a small portion of a newspaper press (roller, web span, nip) assuming a linear elastic web, and demonstrated that an unbalance in the roller results in oscillations of tension in the web span, the magnitude being dependent on the web speed [4]. Grenfell modeled a simple nipped roller- webspan-nipped roller system and showed the effects of disturbances in the roller speeds on the tension in the web [5]. Brandenburg developed dynamic models that take into account spatial variations of parameters to analyze web lines where print registration is critical [6]. Shelton developed models for use in web tension control, and compared two methods of tension control - torque control and velocity control of a roller or rewinding roll of material [7]. Shin developed the concept of "primitive elements" in a web line, and used them to model web lines [8].

Reid and Shin show for the rewind of a web line, that varying the gains in a PID controller as a function of the diameter of the rewind roll produces better performance than with a fixed gain controller [1]. The use of variable gains is referred to as "gain scheduling." Figure 1 is a block diagram of the system studied. The tension error is an input to a **P**roportional + **I**ntegral + **D**erivative (**PID**) controller G_1. The motor drive takes the output current of the controller and converts it to roll speed, while the plant uses the roll speed to calculate span tension and roll diameter. The method used to determine the gains for the PID controller will be discussed in a later section in this paper.

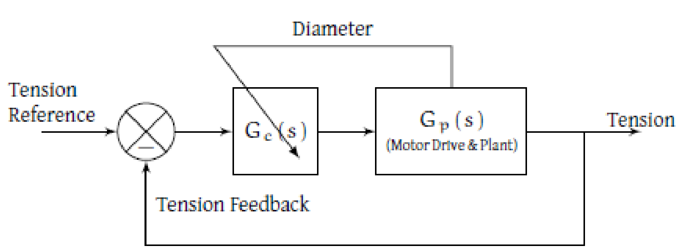

Figure 1 – Block diagram describing the gain scheduling control from [1]. The Plant has 2 outputs, tension and diameter. The diameter is used to vary the gains in the controller and the tension is the feedback.

Gain Scheduling

Reid and Shin describe a roller, span, driven roller system like that shown in Figure 2. A web tension measurement is compared to a reference tension and the difference (error e) is fed to a PID controller that drives the a rewind roll at speed V_2 [1]. The system is modeled by making assumptions that produce a linearized set of equations for the system.

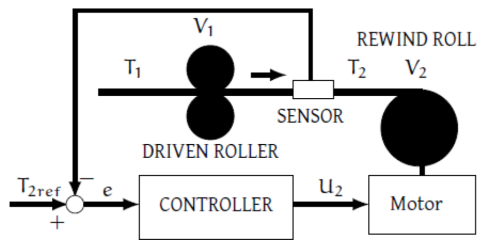

Figure 2 – Physical model of the rewind system considered in [1]

Three key equations were developed to represent the system in Figure 2. Application of the conservation of mass and Hooke's Law leads to the non-linear differential equation. Equation {1} results from linearizing the nonlinear equation about a steady-state operating point. Equation {2} is the model for a motor driven by an input current u_2. The controller used is a PID controller as shown in Eqn. {3}.

$$L_2 \frac{dT_2}{dt} = EA(V_2 - V_1) + \left(V_{1,0}T_1 - V_{2,0}T_2\right) \quad \{1\}$$

$$J_2 \frac{dV_2}{dt} = -(B_2 + C_{m2})V_2 + R_2 K_{m2} u_2 - \frac{R_2^2 T_2}{GR_2} \quad \{2\}$$

$$G_c = \left(K_{pt} + K_{it}\int dt + K_{dt}\frac{d}{dt}\right) \quad \{3\}$$

Taking the Laplace transform of each of these three equations and combining the resulting equations, leads to a closed loop transfer function of the form (see Figure 1)

$$\frac{T_2(s)}{T_{2ref}(s)} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_P(s)} \quad \{4\}$$

or written in terms of the parameters in Eqns. {1},{2}, and {3},

$$\frac{T_2(s)}{T_{2ref}(s)} = \frac{\beta_0 K_{dt}s^2+\beta_0 K_{pt}s+\beta_0 K_{it}}{s^3+(\alpha_1+\beta_0 K_{dt})s^2+(\alpha_0+\beta_0 K_{pt})s+\beta_0 K_{it}} \quad \{5\}$$

where

$$\alpha_0 = \left(V_{2,0}(B_{f2} + C_{m2}) + AER_2^2\right)/L_2 J_2 \quad \{6\}$$

$$\alpha_1 = V_{2,0}/L_2 + (B_{f2} + C_{m2})/J_2 \quad \{7\}$$

$$\beta_0 = K_{m2}\, AER_2/L_2 J_2 \quad \{8\}$$

By selecting a damping ratio, ζ, and natural frequency, ω_n, for the desired response, the values of the derivative gain, K_{dt}, the proportional gain, K_{pt}, and the integral gain, K_{it} can be found. The characteristic equation for Eqn. {5} can be factored into

$$(s + k_r\zeta\omega_n)(s^2 + 2\zeta\omega_n s + \omega_n^2) \tag{9}$$

where k_r determines a real pole. If $k_r > 10$, the effect of the first order root ($s = -k_r\zeta\omega_n$) is small compared to the effect of the roots of $(s^2 + 2\zeta\omega_n s + \omega_n^2)$ [9]. Comparing coefficients of the characteristic equations of {5} and {9}, the values of K_{dt}, K_{pt}, and K_{it} are found as follows:

$$K_{it} = (k_r\zeta\omega_n^3)/\beta_0 \tag{10}$$

$$K_{pt} = \frac{1}{\beta_0}\left((2\zeta\omega_n\beta_0 K_{it}/\omega_n^2) + \omega_n^2 - \alpha_0\right) \tag{11}$$

$$K_{dt} = \frac{1}{\beta_0}\left((\beta_0 K_{it}/\omega_n^2) + 2\zeta\omega_n - \alpha_1\right) \tag{12}$$

In the earlier paper, a "build-up ratio," R_b, was defined to indicate the amount of web material on the roll. The moment of inertia is stated in terms of the build-up ratio in Eqn. {14}. Equations {15}, {16}, and {17} are restatements of Eqns. {6}, {7}, and {8} as functions of build-up ratio.

$$R_b = R_2/R_{2,0} \tag{13}$$

$$J_2 = J_{2,0}R_b^4 \tag{14}$$

$$\alpha_0 = \frac{V_{2,0}(B_{f2}+C_{m2})}{L_2J_2}\frac{1}{R_b^4} + \frac{AER_{2,0}^2}{L_2J_2}\frac{1}{R_b^2} \tag{15}$$

$$\alpha_1 = \frac{V_{2,0}}{L_2} + \frac{B_{f2}+C_{m2}}{J_2}\frac{1}{R_b^4} \tag{16}$$

$$\beta_0 = K_{m2}\frac{AER_{2,0}}{L_2J_2}\frac{1}{R_b^3} \tag{17}$$

Using the gains defined in Eqns. {10}, {11}, and {12} along with Eqns. {1} and {2} expressed in the Laplace Domain, lead to three transfer functions (or three plants), G_{p1}, $G_{p1.5}$, and $G_{P1.75}$. These three transfer functions represent three build-up ratios, 1, 1.5, and 1.75. Using the definitions for the gains with the build-up ratios, three different controller gain sets were found, G_{c1}, $G_{c1.5}$, and $G_{c1.75}$.

Using the parameters in [1], and for step changes in the tension reference, responses were calculated for the closed loop system in Figure 1 for each set of controller and plant gains. Figure 3 shows results for each of the three plants G_{p1}, $G_{p1.5}$, and $G_{P1.75}$. The results on the left side of Figure 3 are for the controller gain G_{c1} fixed at a build-up ratio of 1. The overshoot in tension increases as the build-up ratio increases. The figure shows that for a build-up ratio of 1.75, there is more than 20% overshoot.

The right plot of Figure 3 shows the results of a step input applied to three plants created by using gains G_{c1} with plant G_{p1}, gains $G_{c1.5}$ with plant $G_{p1.5}$, and gains $G_{c1.75}$ with plant $G_{P1.75}$. The three system step responses are virtually the same. Therefore, scheduling the controller gains to account for the changing diameter leads to improved performance.

The controller considered in [1] is a pure tension controller, i.e., the output of the controller is the input to the motor. The Euclid web line uses a different control scheme described in the next section.

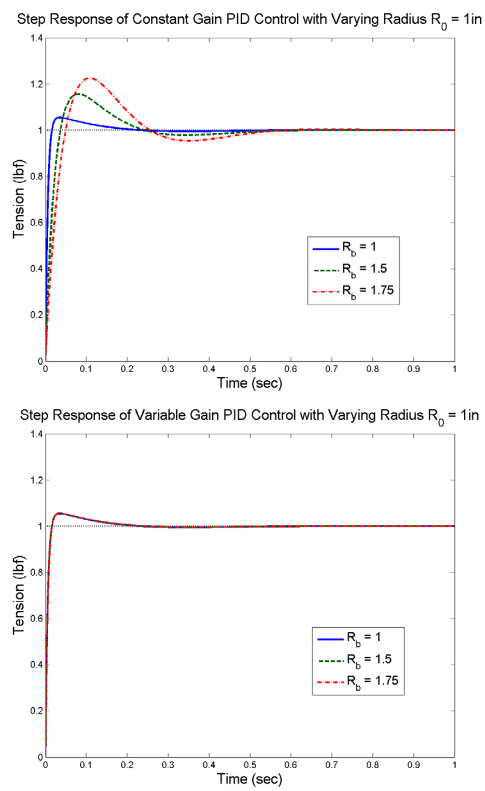

Figure 3 – Fixed P&I gains used to control the tension in the span just previous to a rewind roll do not maintain the performance goals as the roll diameter increases (left). Variable gains taking into account inertia changes maintain the performance goals of $\omega_n = 10rad/s$ and $\zeta = 0.7$ (right).

EUCLID REWIND CONTROL

The control method in the rewind of the Euclid Web Line is a speed-based tension control system as shown in Figure 4. The controllers in the forward path of both the speed loop and tension loop are Proportional + Integral controllers. The speed control gains are varied with respect to the roll diameter, but the tension loop is implemented with constant gains. Also, the tension loop creates an adjustment to the linear speed of the web, not the rotating rate of the motor. The "conv." block in Figure 4 serves the purpose of scaling the linear speed of the web to a rotating speed expected by the motor drive. It is assumed that the field-oriented control of the motor happens fast enough to allow the torque control aspects of the Rockwell drive in the Euclid line to be considered instantaneous.

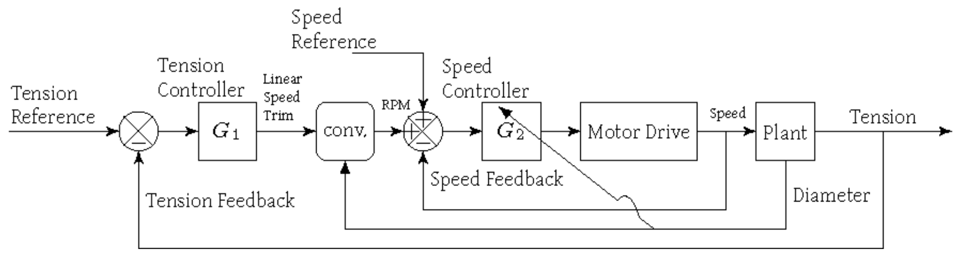

Figure 4 – Euclid Line controller block diagram for the unwind and rewind rolls. The Plant has two outputs: the web tension and the unwind or rewind roll diameter. The tension is fed back to correct the speed reference and the diameter is used to convert the linear speed to RPM and vary the gains in the speed controller.

The basic-model of the rewind in the Euclid line differs from that in [1] because of the internal speed control loop. Referring to the block diagram in Figure 4, the speed controller is designated as G_2. The PI controller is of the form $(K_{ps}s + K_{is})/s$. Assuming one span precedes the rewind roll, it can be shown that the transfer function for the system is Eqn. {18}.

$$\frac{T_n}{T_{ref}} = \frac{K_{mn}AE/J_nL_n\begin{pmatrix}K_{ps}K_{dt}s^2+(K_{pt}K_{ps}+K_{dt}K_{is})s^2\\ +(K_{it}K_{ps}+K_{pt}K_{is})s+K_{it}K_{is}\end{pmatrix}}{\begin{gathered}s^4+\left(\frac{K_{mn}AE}{J_nL_n}K_{ps}K_{dt}+\frac{B_{fn}+C_{mn}}{J_n}+K_{ps}\frac{K_{mn}}{J_n}+\frac{V_{n+1,0}}{L_n}\right)s^3\\ +\begin{bmatrix}\frac{K_{mn}AE}{J_nL_n}(K_{ps}K_{pt}+K_{dt}K_{is})\\ +K_{is}\frac{K_{mn}}{J_n}+\frac{V_{n+1,0}}{L_n}\left(\frac{B_{fn}+C_{mn}}{J_n}+K_{ps}\frac{K_{mn}}{J_n}\right)\end{bmatrix}s^2\\ +\left(\frac{K_{mn}AE}{J_nL_n}(K_{ps}K_{it}+K_{pt}K_{is})+\frac{V_{n+1,0}}{L_n}K_{is}\frac{K_{mn}}{J_n}\right)s\\ +\frac{K_{mn}AE}{J_nL_n}K_{it}K_{is}\end{gathered}} \qquad \{18\}$$

The index n is 25 for all the parameters in Eqn. {18} except for L_n, which is assumed to be equal to $L_{19} + \cdots + L_{24}$. The characteristic equation is 4th order, which leads to difficulty in finding numerical values for the gains K_{ps}, K_{is}, K_{pt}, K_{it}, and K_{dt}.

<u>Gain Selection for the Rewind Controller of the Euclid Web Line</u>

Earlier in this paper it was assumed that there was only one span in the rewind. This was done to facilitate a comparison with the system considered in [1]. In actuality there are 6 spans between the load cell and the rewind roll as shown in Figure 5.

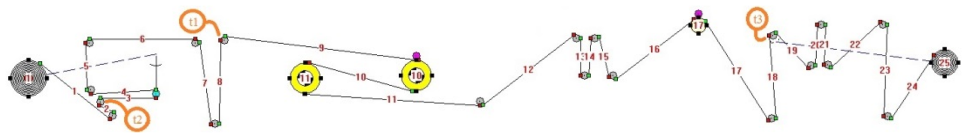

Figure 5 – The Euclid Web Line with rollers and spans numbered. The load cell locations are indicated with orange circles.

There are functions within the drives of the Euclid Web Line that determine the speed loop gains for a given diameter of the rewind roll. This diameter is continuously measured and updated during operation of the line. The Euclid Web line drives utilize Eqn. {19} to calculate the proportional speed gain.

$$K_{ps} = \frac{J_{25}(1772)\cdot 15}{307.487(44.4)} \qquad \{19\}$$

where 1772 is the rated speed of the motor in RPM, 44.4 is the rated torque of the motor in lbf-ft, and the constant 15 is a gain.

The Euclid Web line uses a logic function to ensure that K_{ps} does not become too large with the periodic recalculation of the inertia. The integral gain is calculated from the proportional gain. The Euclid line drives use the following equation to calculate the Integral gain:

$$\mathrm{K_{is}} = \frac{15\mathrm{K_{ps}}}{4(1.1)^2} \qquad \{20\}$$

Equations {19} and {20} can be linked to performance goals like damping ratio and rise time by first conducting a simulation of the system for the case of a step input in the tension reference, and then estimating a damping ratio and natural frequency from the step response. This method will work reasonably well if the step response behavior is sufficiently like that of a second order system.

Simulations of the Rewind Controller on the Euclid Web Line

In this paper, the rewind control system of the Euclid Line is investigated in order to show a comparison with the investigation of the hypothetical system in [1] and shown in Figure 2. The systems differ substantially. The hypothetical system in [1] uses a PID controller. Only tension feedback is used and there is only one span leading into the rewind roll. The rewind zone of the Euclid Line uses a speed-based tension control system that has an inner speed loop and an outer tension loop. A simulation with three build-up ratios was conducted for the model described in Eqn. {18} using the parameters from the Euclid Web line (see Table 1)

In the fixed gains case, the speed gains were fixed at values calculated for a roll diameter of 3 inches. But, the inertias used in the simulations were for the 12 inch and 18 inch diameter rolls. In the variable gains case, the speed gains were recalculated with each new inertia. The simulation was conducted for a step input in the tension reference for the rewind. The tension gains were fixed at $K_{pt} = 10$ and $K_{it} = 30$ (found by trial and error to work well by a previous researcher that used the Euclid Line for experimental studies [10]). The speed gains were calculated using Eqns. {19} and {20}.

Figure 6 shows the step responses in tension for the Euclid rewind at the three build-up ratios. For the plot on the left, the speed gains were not allowed to vary with increased inertia. Between build-up ratios of 1 and 4, very little difference is seen in the responses, but afterward the overshoot increases with increases in build-up ratio. For the plot on the right, the speed gains were allowed to vary with the roll inertias. In this case, the responses overlay one another for all three build-up ratios. As was the case with the system considered in [1], scheduling the controller gains to account for the changing diameter leads to improved performance.

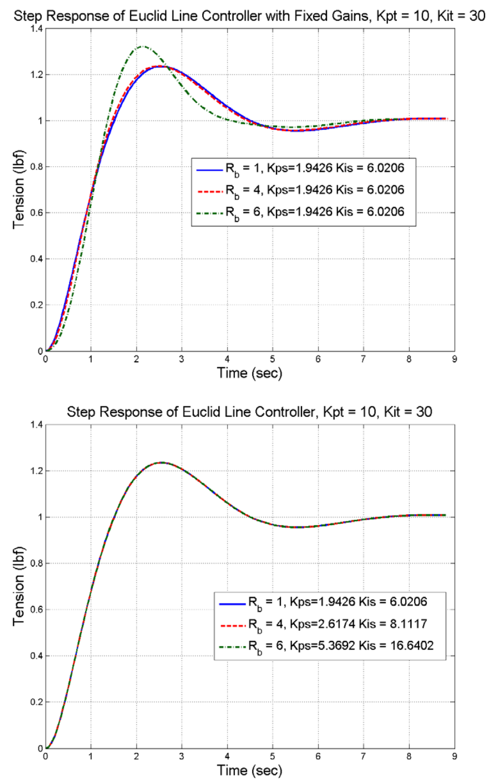

Figure 6 – Euclid Web line rewind tension responses to a step input in reference tension. Using the build-up ratio idea, the blue line is a bare shaft, the red dashed line is a 12 inch diameter, and the green dash-dotted line is an 18 inch diameter. The left plot shows the response of the rewind with fixed gains as the roll radius increases. The overshoot increases as the diameter increases. The right plot shows results with variable speed gains.

SIMULATION OF THE FULL EUCLID WEB LINE

Simulations of the full Euclid Web line were conducted to evaluate three different methods of control at the unwind and rewind. The system consists of 25 rollers, 24 spans, 3 speed controlled (constant inertia) motors and two speed-based tension controlled motors (variable inertia). The elements were modeled with generalized versions of Eqns. {1}, {2}, and {3}. A load cell was used as the feedback device in both unwind and the rewind sections. The model was complex and linearization was necessary. Since this model is for the full Euclid line, web that is unwound from the unwind has to be wound on the rewind. The simulations were done with three roll diameters for the rewind roll and three for the unwind roll. The rewind roll diameter takes one of the following values: 3 inches, 12 inches, and 18 inches. The unwind roll takes one of the following values: 18.5 inches, 14 inches, and 6.6 inches. Figure 7 shows the inertia progression curve for both rolls.

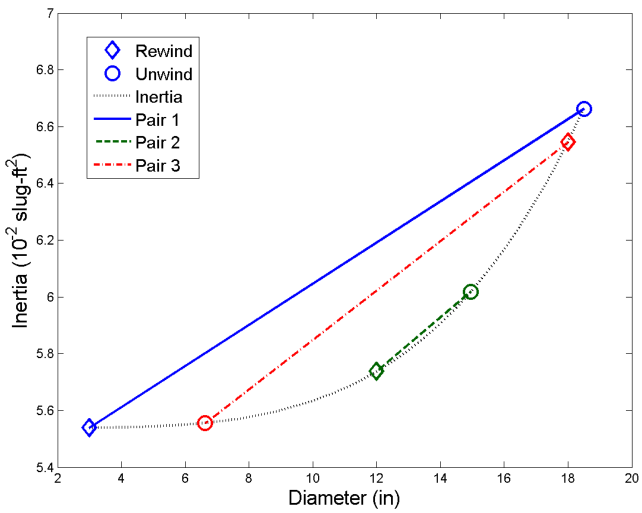

Figure 7 – Inertia for rewind and unwind rolls follows a 4th order curve (dotted line). Each simulation uses a pair of inertias for the unwind and rewind rolls. The bold lines link the pairs. The unwind starts large and decreases, and the rewind starts small and grows. The inertia increases rapidly toward the larger diameters.

The two cases for the simulations were fixed gains and variable gains. For the fixed gains case, the gains are calculated for the initial diameter pair of an 18.5 inch unwind and a 3 inch rewind. This pair corresponds to just beginning to unwind from a full unwind roll (initial dia. 18.5 in.) to winding the web onto a rewind roll (initial 3 in. dia.). The gain for the initial diameter of the unwind was used as the gain for each of the other two diameters of the unwind. The gain for the initial diameter of the rewind was used as the gain for the other two diameters of the rewind. The variable gains case begins the same way, but the second and third diameters are used to update the inertias, and these inertias are used to calculate separate gains for each inertia.

The simulations were started from a steady-state web speed of 100 FPM and a steady-state tension of 10 lbf. At 0.5 seconds, the reference tension was increased by 1 lbf to 11 lbf. The tension feedback and the velocity of the rolls were recorded.

Euclid Web Line Using Pure Tension Control in the Unwind and Rewind – Method 1

Method 1 considered pure tension control at the unwind and rewind rolls. A speed feedback loop was not included in the control of the unwind or rewind. This is the general idea used in [1], but the Euclid Line model is used in Method 1. When the gains are calculated, Eqns. {10}, {11}, and {13} – {17} are used. The other three driven rolls in the line are under velocity control.

Figure 8 shows the rewind tension response to a step in the reference tension from 10 lbf to 11 lbf while the line runs at 100 FPM. For the fixed gain case, the gain is calculated for a 3 inch diameter roll. This gain was also used for the 12 and 18 inch diameter rolls. The character of the latter two traces are similar to the first, but in each case, the responses are highly damped. For the variable gain case, the inertias were calculated for the 3 in., 12 in., and 18 in. diameter rolls. The responses are very "sluggish" and far from the responses expected. In fact, all the responses for both the fixed gain and variable gain cases are very sluggish. These results suggest that the proportional gains need to be much higher than calculated, but stability may be problematic.

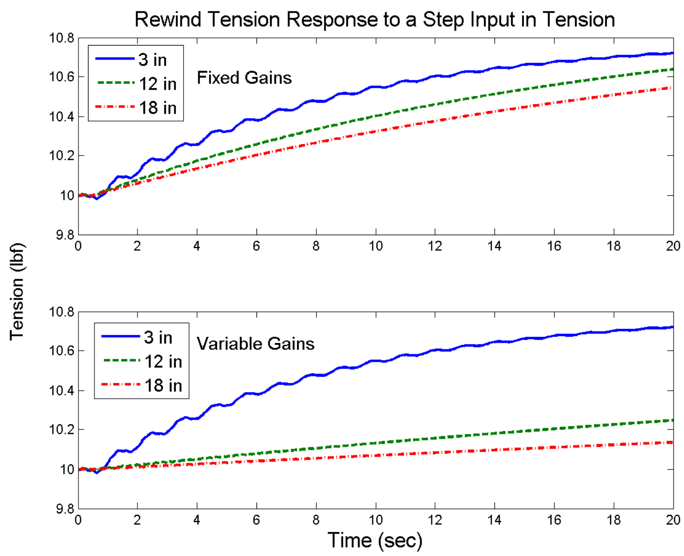

Figure 8 – The rewind tension responses to a step in reference tension for pure tension control (Method 1). The top plot is for the fixed gain case and the character of the responses are similar. The bottom plot shows the variable gain case and the responses change in character greatly with each increase in inertia.

Figure 9 shows the unwind section tension response to the same step in reference tension as the rewind section. The responses are very "sluggish" and far from the responses expected. In fact, all the responses for both the fixed gain and variable gain cases are very sluggish. These results suggest that the proportional gains need to be much higher than calculated, but stability is problematic.

Speed-Based Tension Control with Variable Tension Gains – Method 2

The Euclid Web line unwind and rewind motors are under speed-based tension control. The gain calculation method is more involved and follows the process described in a companion paper [11]. Four gain values have to be determined for each motor. The velocity loop gains are calculated first, and then the closed loop transfer functions for the speed loops are obtained and put into the forward path of the tension control systems. Expressions for the open loop tension dynamics are then found. These may be 3rd order or higher. The Routh Approximation method is used to reduce the open loop transfer functions to second order [12] [13]. Then the tension control gains are calculated from Eqns. {5} and {9}.

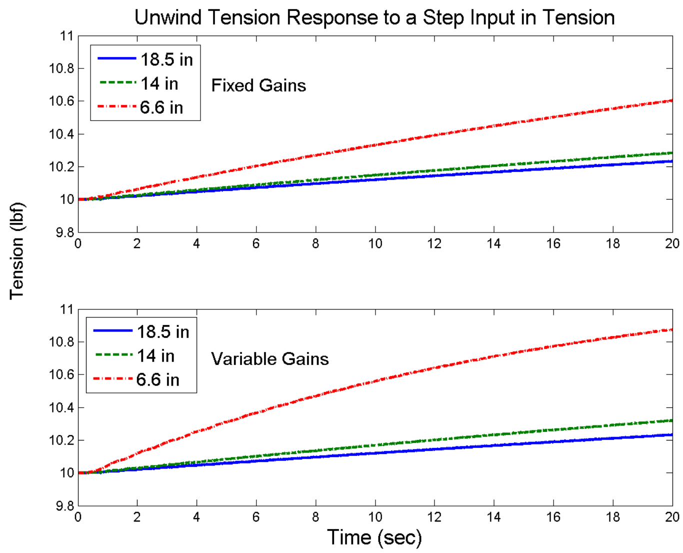

Figure 9 – The unwind tension responses to a step in reference tension for pure tension control (Method 1). The top plot shows the fixed gain case where the inertia starts out large and decreases. The bottom plot shows the variable gain case. Again, none of the traces meets the performance goals of 0.9 damping ratio and 0.3 second rise time.

Figure 10 shows the speed responses to the step change in reference tension. The top plot shows simulations of the unwind roll speed under fixed gain control for three unwind roll diameters. The gains were calculated for the 18.5 in. diameter roll (blue line). The green dashed line is for a 14 in. diameter roll, but the gains were calculated for an 18.5 in. diameter roll. This simulation is barely visible from the 18.5 inch trace. The red dash-dotted line shows the simulation for the 6 in. diameter roll, but the gains were calculated for an 18.5 inch roll. This last simulation is unstable. The bottom plot shows the variable gain case. The controller gains are calculated for each diameter value. The oscillation frequency remains almost the same for all three trace, but the magnitude of the speed oscillation increases.

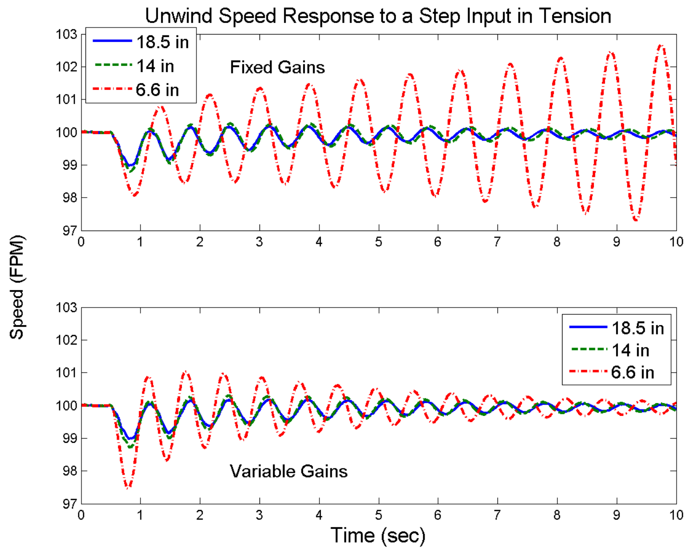

Figure 10 – The unwind speed responses for a step in the tension reference (Method 2). The fixed gain case (top) becomes unstable as the diameter of the roll increases. The bottom plot shows the same progression of three diameters, but the system is stable.

Figure 11 shows the rewind speed responses to a step increase in the tension reference. The upper plot is the fixed gain case where the controller gains were calculated for a roll diameter of 3 inches. Those same gains were used for the 12 in. and the 18 in. diameter simulations. The oscillations start out large, but decrease with each diameter, and the frequency shifts slightly. The bottom plot shows the variable gain case. The oscillation magnitude is about the same as the fixed gain case, but the frequency is consistent across all diameters.

Figure 12 shows the unwind tension responses to a step increase in the tension reference. The upper plot is the fixed gain case where the gains were calculated for an unwind roll of 18.5 in. diameter. The gains are fixed at these values for the other two simulations. The frequency of the tension oscillations shifts with each new diameter. The system goes unstable at the smallest diameter simulation. The lower plot shows the case of variable gains. The tension responses change from those of a damped system to a less damped system, and the frequency shifts with each diameter. However, the system is stable, unlike the fixed gain case.

Figure 13 shows the rewind tension responses to a step in the tension reference. The upper plot shows the fixed gain case. The frequency of the tension oscillations shift with each change in diameter, and the response becomes more damped as the inertia increases. The bottom plot of the figure shows the variable gain case. The increase in damping of the system is even more evident than in the fixed gain case, but the frequency of the tension oscillations is consistent across the steps in diameter.

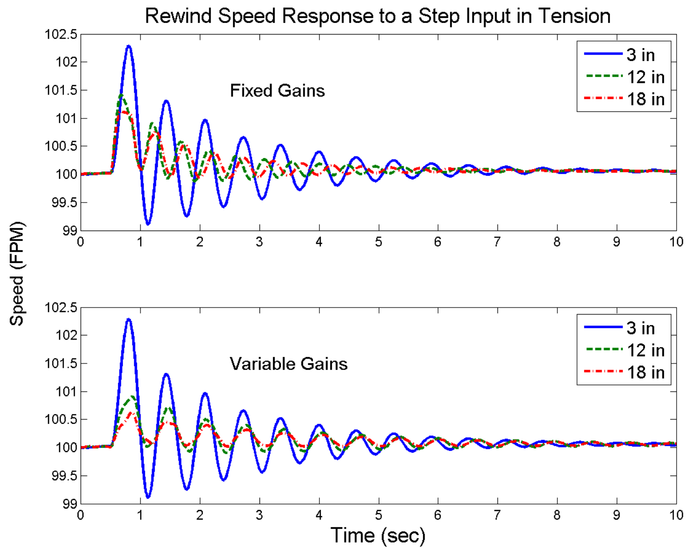

Figure 11 – The rewind speed responses to a step in the tension reference (Method 2). The upper plot shows the fixed gain case where the controller gains were fixed at the values calculated for a 3 in. diameter roll. The oscillations are larger with a small inertia and the frequency shifts with each increase in diameter. The lower plot shows the variable gain case.

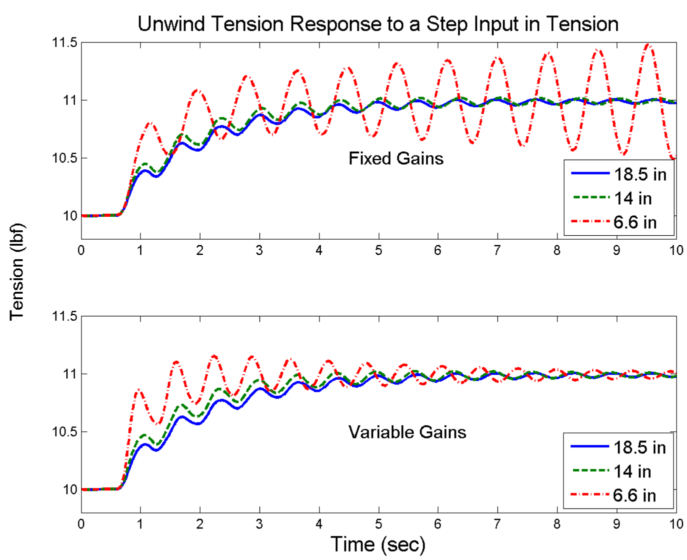

Figure 12 – The unwind tension responses to a step in the reference tension (Method 2). The upper plot shows the fixed gain case. The gains were calculated for an 18.5 in. diameter roll. At the smallest diameter, the system is unstable. The frequency of the tension oscillation is shifted slightly as the diameter decreases. The lower plot shows the

variable gain case where the system is stable and the frequency of the oscillations is more consistent across the changes in diameter.

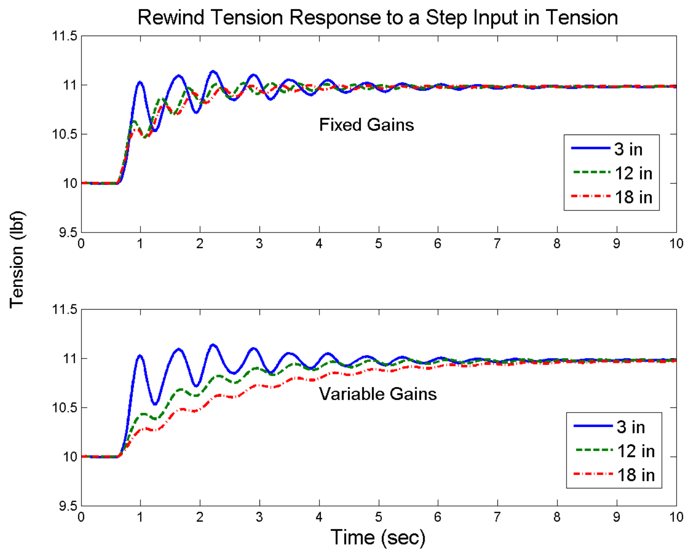

Figure 13 – The rewind tension responses to a step in the reference tension (Method 2). The upper plot shows the fixed gain case. The oscillations shift in frequency and the response becomes more damped as the diameter increases. The bottom plot shows the variable gain case. The increase in damping is more evident, but the frequency of oscillations is more consistent.

Speed-Based Tension Control with Constant Tension Gains – Method 3

The Euclid Web line unwind and rewind motors are under speed-based tension control. But when the simulations of the Euclid Web line were conducted for this method, the tension gains for the outer tension loop in both the unwind and rewind were held constant at all times. The only gains affected by the changing inertia of the unwind or rewind roll were the speed gains. The tension gains for the unwind were selected as if the motor was under pure tension control with a 15 in. diameter roll. The tension loop gains for the rewind were maintained at those calculated for the 3 in. diameter roll.

Figure 14 shows the unwind speed responses to a step in the reference tension starting at 0.5 seconds. The upper plot shows the fixed speed gains case, and the gains were based on a roll diameter of 15 inches. The unwind speed is unstable for the 6 inch diameter simulation. The lower plot shows the case with variable speed gains. The unwind speed response is stable.

Figure 15 shows the rewind speed responses to a step in the tension reference. The upper plot shows the fixed speed gains case. The speed loop gains and the tension loop gains were calculated for a 3 inch diameter roll and these values were used for the second two diameters. The bottom plot in Figure 15 is the variable speed gains case. Simulations with the 12 in. and 18 in. diameters show smaller amounts of overshoot than with the fixed gain case.

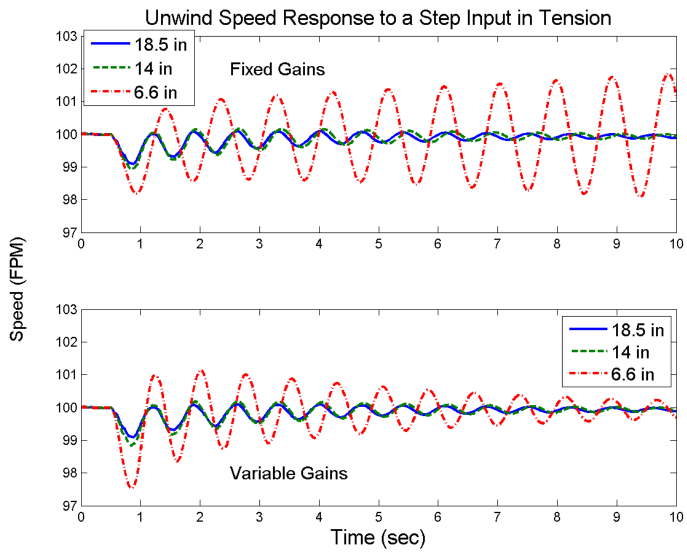

Figure 14 – Unwind roll speed responses to a step in the tension reference (Method 3). The top plot shows the responses with fixed speed and tension gains. The unwind is unstable at the smallest diameter in the fixed gains case. The lower plot shows the responses with variable speed gains. The unwind system is stable for all diameters.

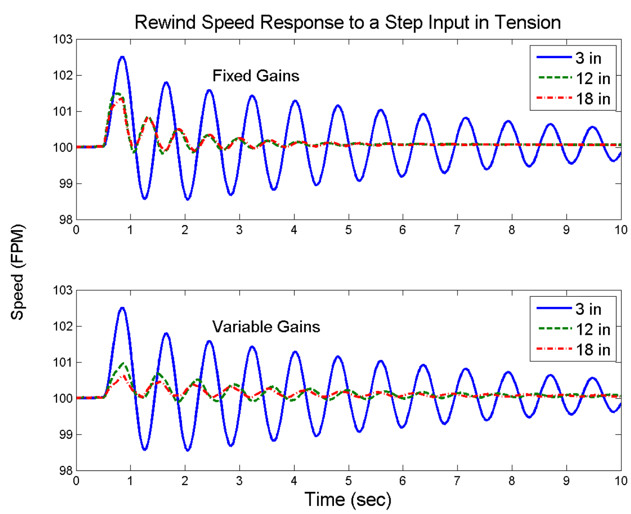

Figure 15 – The rewind speed responses to a step in the tension reference when the tension gains are held constant (Method 3). The upper plot shows the fixed speed gain case where the speed and tension gains were selected for a 3 inch diameter and not changed when the roll was simulated at 12 and 18 inches. The lower plot shows the variable speed gain case. The overshoot decreases with increases in diameter in both the fixed gain and variable gain cases.

Figure 16 shows the unwind tension responses to a step in the reference tension. The upper plot shows the fixed gains case. The simulation for the smallest inertia is unstable. The lower plot shows the variable speed gains case. All the simulations, including the one for the smallest inertia, are stable. The simulations at the two larger inertias in the variable gain case are very similar to those in the fixed speed gain case.

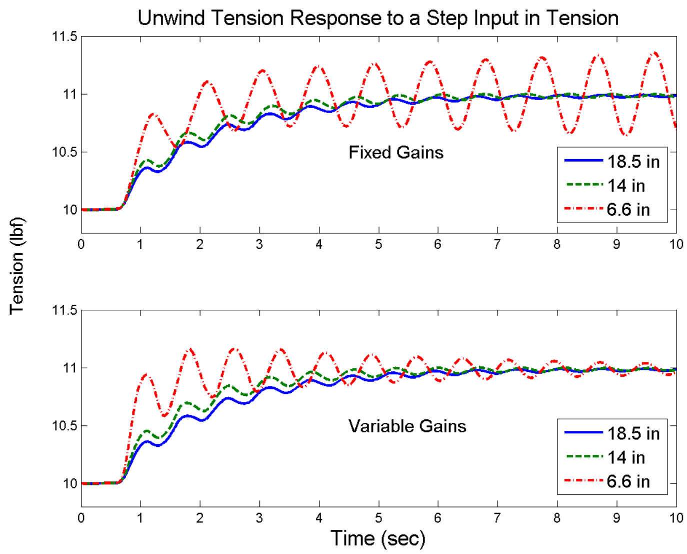

Figure 16 – The unwind tension responses to a step in the reference tension when the tension gains are held constant (Method 3). The upper plot is the fixed speed gains case and shows that the tension is unstable for the smallest inertia. The lower plot shows that with the variable speed gains case, and the tension simulation for the smallest inertia is stable.

Figure 17 shows the rewind tension responses to a step in the reference tension. The upper plot shows the fixed speed gains case. The lower plot shows the variable gains case where the speed gains were calculated for each diameter. At first glance it appears that the two sets of plots are interchanged. But they are not. The simulations for the fixed gains case are what was expected for the variable gains simulations. More investigation is needed.

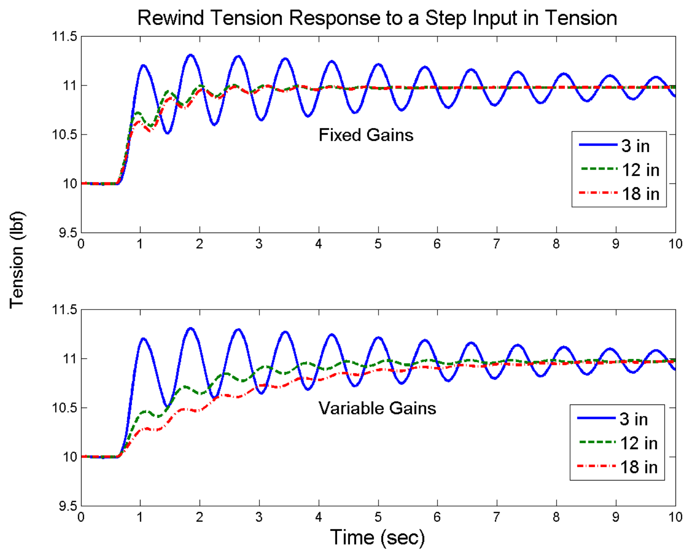

Figure 17 – The rewind tension response to a step in the reference tension when the tension gains are held constant (Method 3). The upper plot shows the fixed speed gains case where the speed gains were calculated for a 3 inch diameter roll, and is not changed when the roll increases in diameter. The lower plot displays the variable speed gains case. The simulations for the fixed gains case are what was expected for the variable gains simulations.

SUMMARY AND CONCLUSIONS

Gain scheduling is a control process that accounts for the time varying parameters of a web line like the inertia of the unwinding or rewinding roll. In general, if a controller is tuned for one operating condition, it will not be tuned well for another operating condition unless the gains are changed to be consistent with that operating condition. This is the idea of gain scheduling.

The first system discussed in this paper was an earlier paper by Reid and Shin [1]. A physical representation of the system studied is shown in Figure 1. This simple system had one roller, one span, and a rewind roll. The parameter values were for an hypothetical system. The rewind controller structure was **P**roportional + **I**ntegral + **D**erivative. Two cases were considered for the rewind controller, fixed gain and variable gain based on rewind roll diameter. The web tension responses demonstrated that a controller that uses variable gains based on roll diameter (gain changes with diameter), gives better results than a fixed gain controller based on a single roll diameter.

The earlier paper by Reid and Shin was revisited to serve as a background for considering a gain scheduling in controlling the rewind section of the Euclid Web Line. The systems differ substantially in their structure and complexity. The hypothetical system in [1] uses tension feedback only and there is only one span leading into the rewind roll. A simplified model of the hypothetical system is a third order linear differential equation, which makes it straight forward to estimate gains based on a performance factor involving natural frequency and damping ratio. The Euclid Line rewind uses a speed-based tension control system that has an inner speed loop and an outer tension loop as shown in Figure 4. A simplified model of the system is a fourth order linear differential equation, which makes it difficult to estimate the two sets of gains.

Simulations of the Euclid Line **Rewind** were conducted using the simplified model. Three diameter build-up ratios and both fixed speed gains and variable speed gains were considered. The tension gains were fixed at values found by trial and error to work well by a previous researcher that used the Euclid Line for experimental studies [10]). The speed gains were calculated using Equations built into the Rockwell Automation drives associated with the Euclid Line. In the fixed gain case, the speed gains were not allowed to vary with unwind roll inertia, while in the variable gain case the speed gains were allowed to vary with unwind roll inertia. As was the case with the system considered in [1], scheduling the controller gains to account for the changing diameter leads to improved performance.

The remainder of the study discussed in this paper dealt with exploring three methods of control for the full Euclid Web Line during an unwind-rewind operation. Three methods for controlling both the unwind and rewind rolls were explored. Method 1 uses pure tension control in the unwind and rewind, while methods 2 and 3 employ speed-based web tension control systems in both. The control systems for the s-wrap rolls and the pull roll use simple speed controls. During an unwind-rewind operation, the unwind and rewind roll inertias (roll diameters) vary with respect to time. The first method allowed the tension gains to vary with roll diameter. The second method allowed the gains of both the speed and tension controls to vary with roll diameter. In the third method, the tension gains were held constant while the speed gains were allowed to vary with the roll diameter.

Linearized models of the Euclid Web Line were developed, one for each control method. The gains were determined using a Routh Approximation method discussed in a companion paper. Simulations were conducted to determine the effect of varying inertia. Two cases were considered for each method: (i) fixed gains in the tension loops (Method 1) and speed loops (no speed loops in Method 1) in the unwind and rewind controllers, and (ii) variable gains in both loops (Method 2) or in the speed loops only (Method 3) based on time varying inertias of the unwind and rewind rolls.

Simulations showed that in some cases, the variable gain approach results in the system being stable while it is not with the fixed gain approach. In most but not all cases, the variable gain approach results in better system performance than that with a fixed gain approach. In several cases involving the unwind control, the simulations for the fixed gains case are what was expected for the variable gains simulations. More investigation is needed.

Method 1, which used pure tension control in both the unwind and rewind, was inspired by the earlier work of Reid and Shin [1]. However, unlike the results in that earlier paper, the case is not made for gain scheduling except in one situation. The simulations show a significant change in the responses when the roll inertias are increased substantially from the initial condition.

For Method 2, where speed-based tension control was used for both the unwind and rewind, the results were more striking and supportive of gain scheduling. For the fixed gain case, the responses in unwind speed and unwind tension were both unstable when the smallest diameter was used. The variable gain case produced results which made the case for gain scheduling (Figures 10 and 12). For the fixed gain case, the responses in rewind speed and rewind tension are well behaved for the different diameters. These are the responses that were expected for the variable gain case. But, in the variable gain case the tension responses are very sluggish for the larger diameters and not supportive of gain scheduling. However, the rewind speed responses do support gain scheduling (Figures 11 and 13).

Method 3, where speed-based tension control also are used for both the unwind and rewind, represents the way the Euclid Web Line is normally configured. The primary difference between Method 2 and 3 are the tension gains used. In Method 2, the tension and speed gains are both varied with roller diameter. In Method 3, the tension gains are held constant and only the speed gains are varied with roller diameter. The simulation results for Method 3 are as striking as those in Method 2, and generally supportive of gain scheduling. For the fixed gain case in Method 3, the unwind speed response for the smallest diameter roll is unstable, while the response for the variable gain case is stable and reasonably well damped (Figure 14). The rewind speed responses are well behaved for both the fixed gain and variable gain case, and the positive effect of variable gains is demonstrated (Figure 15). For the fixed gain case, the responses in unwind tension were unstable when the smallest diameter was used, while the response was stable for the variable gain case. The variable gain case produced results which are supportive of gain scheduling (Figures 16). For the fixed gain case, the responses in rewind tension are well behaved for the different diameters. These are the rewind tension responses that were expected for the variable gain case. But, in the variable gain case the rewind tension responses are very sluggish for the larger diameters and not supportive of gain scheduling.

In summary, simulations show that for large changes in the inertias, the variable gain approach generally results in better system performance than that with a fixed gain approach, but not always.

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APPENDIX

Parameter	Value	Units
Motor Inertia	4.786E-02	slug-ft^2
Gear Ratio	2.635E-01	shaft rotations per motor rotation
Shaft inertia	3.680E-01	slug-ft^2
Inertia of wound web (14in dia.)	3.900E-02	slug-ft^2
Combined Inertia	1.096E+00	slug-ft^2
Bearing Friction	6.073E-04	lbf-ft-s
Motor Damping	0.000E+00	lbf-ft-s
Motor Constant	3.356E+00	lbf-ft/A
Roller radius	1.250E-01	ft
Young's modulus (Tyvek)	6.667E+06	lbf/ft^2
Web cross-sectional area	2.262E-04	ft^2
Web Density	5.586E-01	slug/ft^3
Span Length	1.141E+01	ft
Steady-state speed	6.667E+00	ft/s

Table 1 – Parameters used in the linearized simulation of the Euclid Web line's rewind section.