EXPLORING THE STRAIN TRANSPORT FORMULA

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ABSTRACT

The strain transport formula relates the velocity of a moving web to its strain. It is based on the conservation of mass which, when applied to webs, becomes the conservation of relaxed length. Simply stated it says, the velocity of a web divided by one plus its strain is a constant throughout the web path.

This paper reviews many of the useful applications of the strain transport formula. While it is commonly used for strain and tension calculations, it can also be used to optimize a web path to minimize strain or tension variation, to calculate the elastic modulus of a running web, to calculate wound-in tension at an unwind or at a winder, and many other useful things.

NOMENCLATURE

A	Cross section area of web
B	Basis weight of web, force/area units
E	Young's modulus
h	Web thickness
l	Length of web
L	Span Length
r_i, r_o	Inside, outside radius of web on roller
S	Sag in web span at midpoint
T	Tension, force units
V	Surface velocity of roller
W	Width of web
ε	Strain
ω	Angular velocity

Subscripts

damage	Strain where damage occurs
n	Span and roller number index
ref	Reference velocity and strain
relaxed_web	Relaxed condition

BACKGROUND

The strain transport formula

$$\frac{V_n}{1+\varepsilon_n} = \frac{V_{ref}}{1+\varepsilon_{ref}} \quad \{1\}$$

states that the strain of the web, ε_n, and its velocity, V_n, are related. It is based on steady state mass flow as the web travels through the web path. For a web, this means that the length of relaxed web entering a span in a given amount of time must be balanced by an equal length of relaxed web leaving during that time. In steady state, web neither accumulates nor depletes in a span. The quantity of relaxed web exiting a span is equal to the velocity of its downstream roller divided by 1 plus the strain in the web approaching the roller, $V/(1+\varepsilon)$. This quantity of relaxed web enters the next span and must be matched, in the next span, by an equal quantity of relaxed web leaving that span.

In a typical web path, there is at least one roller that follows the line reference only, and is not controlled by a dancer or tension measuring idler. The first of these rollers establishes the reference velocity, V_{ref}. The roller that sets V_{ref} is typically called the master or pacer roller. In some cases, the roller labeled as the “master roller” drives an upstream roller, a bowed axis roller for example, at a fixed ratio. This upstream roller is then the true master and reference velocity. The strain in the span approaching the reference roller is the reference strain, ε_{ref}. The web that enters the reference roller is typically controlled by a dancer or tension measuring idler controlling the reference tension. The strain is the tension force in the web, T, divided by the product of Young’s modulus and the cross section area of the web, A, $\varepsilon = T/(EA)$. Normally, the reference velocity and strain are controlled in the unwind zone with the unwind dancer or tension measuring idler controlling the tension and the first line reference driven roller controlling the velocity. In a completely draw controlled line, the reference strain is the strain in the outer lap of the parent roll and the reference velocity is the surface velocity of the roll, and is either fixed or an operator adjusted variable.

There can be only one velocity and strain reference per web. Upstream of the reference roller, all driven rollers must be tension or torque controlled. In the case of a draw controlled line, the parent roll is the reference roller and there is nothing upstream. Downstream of the reference roller, driven rollers can be tension or torque controlled, or draw controlled to follow the reference roller. Be they tension controlled, draw controlled, or undriven, all rollers, including idlers, must conform to the strain transport formula and their velocity divided by one plus their strain is determined by the reference velocity and strain.

Good web handling practice is to carefully control the reference velocity and tension since these two variables affect every tension and strain in downstream spans. Ideally, the tension control method is applied as close to the reference roller as is practical. Idlers and other process devices located between the reference tension control and the reference velocity control are sources of variation that become variation in all strains and tensions downstream.

USES OF THE STRAIN TRANSPORT FORMULA

Tension or Strain Control

Strain and tension are the most important properties in a web path and the strain transport formula is the means by which they are calculated. A typical web path is shown in Figure 1.

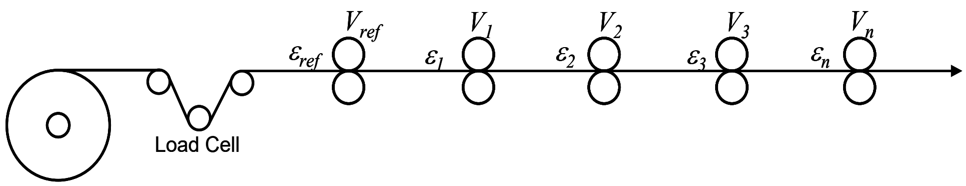

Figure 1 – A Typical Web Path

When solved for strain, ε_n, as shown in Formula 2, the strain of any span can be found if its driven roller velocity, V_n, is known along with the reference velocity and strain.

$$\varepsilon_n = \frac{V_n}{V_{ref}}\left(1+\varepsilon_{ref}\right)-1 \qquad \{2\}$$

The strain in any span is controlled by the references, V_{ref} and ε_{ref}, and its downstream roller velocity, V_n, only. If there are no transformations upstream of a span of interest, the velocity of upstream rollers have no effect on its strain. In this case, the tension can be calculated using $T_n = \varepsilon_n EA$. If transformations occur upstream, the references and downstream velocity still control the absolute strain, but the changes in relaxed length, modulus and cross section area of the web must be known to calculate tension.

In some cases, it is important to achieve a target strain in a span. The strain transport formula, arranged as in Formula 3, can be used to calculate the velocity of a driven roller required to achieve a target strain in the span approaching that roller.

$$V_n = \frac{1+\varepsilon_n}{1+\varepsilon_{ref}}V_{ref} \qquad \{3\}$$

Length of Relaxed Web Past a Roller

Calculating the length of relaxed web that is wound into a parent roll or that travels past a roller in a given time is a common need. Relaxed web has zero strain and the strain transport formula can be used to calculate a relaxed web velocity. Any roller in the path where the velocity and strain are known can be used for the references.

$$\frac{V_{relaxed\_web}}{1+0}=\frac{V_n}{1+\varepsilon_n}=\frac{V_{ref}}{1+\varepsilon_{ref}}$$

$$\frac{l_{relaxed\_web}/time}{1+0}=\frac{l_n/time}{1+\varepsilon_n}=\frac{V_n}{1+\varepsilon_n}=\frac{V_{ref}}{1+\varepsilon_{ref}} \qquad \{4\}$$

$$l_{relaxed\_web}=\frac{l_n}{1+\varepsilon_n}=\left(\frac{V_n}{1+\varepsilon_n}\right)time=\left(\frac{V_{ref}}{1+\varepsilon_{ref}}\right)time$$

Formula 4 applies if there is no permanent deformation created in the web. Permanent deformation may result from a transformation to the web downstream of the references or from permanent viscoelastic strain. If there is 1% permanent deformation created downstream of the velocity reference, then the relaxed length will need to be increased by 1%. A roller and strain downstream of a deformation producing transformation could be used as new references to reset the strain transport formula for calculation downstream of the transformation.

Length of a Cut Web

The cut length of a product, $l_{relaxed\_web}$, is the length of relaxed web that passes the cut off device in the time to make one product.

$$\frac{V_{relaxed\_web}}{1+0}=\frac{V_{ref}}{1+\varepsilon_{ref}}$$

$$\frac{V_{relaxed\_web}}{CPT}=\frac{V_{ref}}{1+\varepsilon_{ref}}*\frac{1}{CPT} \qquad \{5\}$$

$$l_{relaxed\_web}=\frac{V_{ref}}{1+\varepsilon_{ref}}*\frac{1}{CPT}$$

Where CPT = Cuts Per Time

Again, if there is permanent deformation downstream of the velocity reference roller, this must also be applied to the final relaxed cut length.

Strain Sensitive Processes

By arranging the strain transport formula as shown in Formula 6, we see that the lower the reference tension, T_{ref}, the less influence the web's modulus, width or thickness will have on the strain in downstream spans. If your process is strain sensitive, T_{ref} should be as low as possible! If the reference tension is zero, the term T_{ref}/EA will be zero and the modulus will have no influence on the downstream strain. While zero is not possible, lower is better. The strain in any downstream spans can be set at some higher value using V_n to control ε_n.

$$\varepsilon_n = \frac{V_n}{V_{ref}}\left(1+\varepsilon_{ref}\right) - 1 = \frac{V_n}{V_{ref}}\left(1+\frac{T_{ref}}{EA}\right) - 1 \qquad \{6\}$$

Some webs can tolerate very low tension in a free span and a slack loop is run entering the reference roller. The amount of sag in the span is measured with a displacement sensor and its output is used to control the speed of the parent roll to maintain constant sag. The resulting tension is that which is needed to maintain a sag and can be calculated from

$$T_{ref} = \frac{L^2 BW}{8S} \qquad \{7\}$$

Any unnecessary idlers or other devices between the dancer or tension measuring idler and the reference roller will cause the tension to be higher at the reference roller and therefore increase the path's sensitivity to modulus. In addition, these extra devices introduce additional sources of variation to the reference tension due to variation of their bearing and inertial drag. These variations are passed on to the downstream rollers because of their influence on T_{ref}.

<u>Tension Sensitive Process</u>

The arrangement of the strain transport formula in Formula 8 shows that if a downstream roller has the same velocity as the reference roller, the span approaching the downstream roller will have the same tension as the span approaching the reference roller. Since the tension approaching the reference roller is controlled by a dancer or load cell, it will not be influenced by the modulus of the web and neither will be the tension approaching the downstream roller.

$$\begin{aligned}
\varepsilon_n &= \frac{V_n}{V_{ref}}\left(1+\varepsilon_{ref}\right) - 1 \\
\frac{T_n}{EA} &= \frac{V_n}{V_{ref}}\left(1+\frac{T_{ref}}{EA}\right) - 1 \\
\frac{T_n}{EA} &= 1\left(1+\frac{T_{ref}}{EA}\right) - 1 \qquad \text{if} \quad V_n = V_{ref} \qquad \{8\} \\
\frac{T_n}{EA} &= \frac{T_{ref}}{EA} \\
T_n &= T_{ref}
\end{aligned}$$

If your process is tension sensitive, V_n should equal V_{ref} ! Then $T_n = T_{ref}$.

Tension Troubleshooting

The strain transport formula arrangement in Formula 9 shows that the ratio of the velocity of a downstream driven roller to the velocity of the reference roller can change how modulus affects tension.

$$T_n = \frac{V_n}{V_{ref}} T_{ref} + EA\left(\frac{V_n}{V_{ref}} - 1\right) \qquad \{9\}$$

The first term in Formula 9 is insensitive to modulus. In the second term, the modulus is multiplied by the velocity ratio minus one. In this term, if the velocity of the downstream roller is greater than the reference roller, then the ratio will be greater than one and the second term will be added to the first as a positive value, i.e., if modulus increases, tension increases. If the ratio is less than one, the second term will be added to the first as a negative value, i.e., if the modulus increases, the tension will decrease. If the velocities are equal, there is no affect as was shown with Formula 8.

If $V_n > V_{ref,}$ when EA increases, T_n increases.
If $V_n < V_{ref,}$ when EA increases, T_n decreases.
If $V_n = V_{ref,}$ when EA increases, T_n is unchanged.

In paths with a tension controlled unwind zone and draw controlled zones downstream, it is common to have some rollers faster than the reference, some slower, and some the same. In this path, when modulus changes, you would see some tensions increase while others decrease and some not change at all. All at the same time! In tension controlled equipment, some rollers will speed up, some slow down and some not change. Without understanding Formula 9, this could lead to troubleshooting confusion.

Parent Roll *WOT* at Winding

The tension in the outer layer of a parent roll during winding, T_2, can be calculated if the velocity of the parent roll surface, V_2, is known along with the tension and velocity at one other point in the path. This is the case for both surface and center winding.

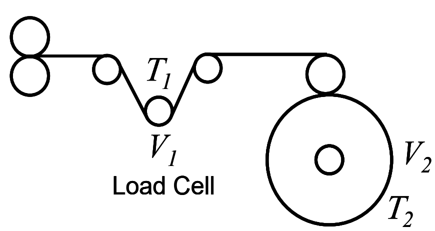

Figure 2 – Winding a Parent Roll

$$
\begin{aligned}
\varepsilon_2 &= \frac{V_2}{V_1}(1+\varepsilon_1)-1 \\
\frac{T_2}{EA} &= \frac{V_2}{V_1}\left(1+\frac{T_1}{EA}\right)-1 \\
T_2 &= EA\left(\frac{V_2}{V_1}\left(1+\frac{T_1}{EA}\right)-1\right) \\
T_2 &= \frac{V_2}{V_1}(EA+T_1)-EA
\end{aligned}
\qquad \{10\}
$$

Using a surface wheel or velocimeter to measure parent roll surface velocity can be used understand and manage the winding process.

Parent Roll *WOT* at Unwind

In a similar fashion, the tension in the outer layer of the parent roll can be estimated using Formula 10 as shown above.

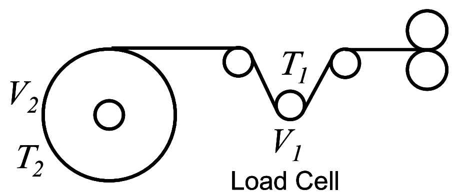

Figure 3 – Unwinding a Parent Roll

Formula 11 may be more useful as it can identify a difference between parent rolls. A plot of the velocity ratio from outside to inside of the roll is an indication of how the roll is wound. A different profile, either offset or angled, would indicate a difference between rolls.

$$
\begin{aligned}
\varepsilon_2 &= \frac{V_2}{V_1}(1+\varepsilon_1)-1 \\
1+\varepsilon_2 &= \frac{V_2}{V_1}(1+\varepsilon_1) \\
\frac{1+\varepsilon_2}{(1+\varepsilon_1)} &= \frac{V_2}{V_1} \\
\frac{V_2}{V_1} &= \frac{1+\varepsilon_2}{1+\frac{T_1}{EA}} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{EA+T_2}{EA+T_1}
\end{aligned}
\qquad \{11\}
$$

Assuming EA varies little over the short time of making the parent roll, the variable driving the velocity ratio is the wound in tension or strain.

Modulus Measurement

If you know two tensions and two velocities, you can calculate modulus.

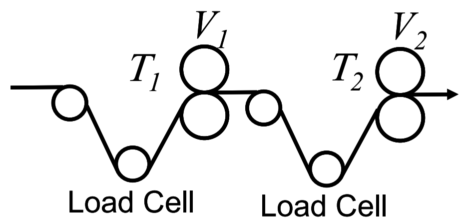

Figure 4 – Measuring Modulus

$$\frac{V_1}{1+\frac{T_1}{EA}} = \frac{V_2}{1+\frac{T_2}{EA}}$$

$$V_1\left(1+\frac{T_2}{EA}\right) = V_2\left(1+\frac{T_1}{EA}\right)$$

$$V_1 + V_1\frac{T_2}{EA} = V_2 + V_2\frac{T_1}{EA}$$

$$V_1 - V_2 = V_2\frac{T_1}{EA} - V_1\frac{T_2}{EA} \qquad \{12\}$$

$$V_1 - V_2 = \frac{V_2T_1}{EA} - \frac{V_1T_2}{EA}$$

$$EA = \frac{V_2T_1 - V_1T_2}{(V_1 - V_2)}$$

$$V_1 \neq V_2$$

If T_2 is zero, as in slack loop feed control mentioned above, only one tension and the two velocities are all that is needed.

The results of Formula 12, i.e., the real time, online calculation of *EA*, can be substituted into many of the preceding formulas to make predictions and enable control of important results. When substituted into Formula 4 to calculate the reference strain, for example, a prediction of the feed rate can be made and V_{ref} adjusted manually or automatically to deliver a constant feed rate in spite of varying modulus.

Smallest Bending Radius

What is the smallest radius for a roller or edge to avoid overstraining the outer edge of the web?

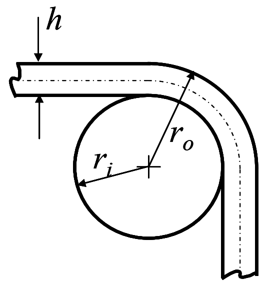

Figure 5 – Smallest Bending Radius

The web velocity on the inner surface of the web on the roller is $r_i\omega$. On the outer surface, it is $(r_i+h)\omega$. These velocities can be used in the strain transport formula to calculate the strain on the outer and inner surfaces of the web over the roller.

$$\begin{aligned}
\varepsilon_{damage} &> \varepsilon_o = \frac{(r_i+h)\varpi}{V_{ref}}\left(1+\varepsilon_{ref}\right)-1 \\
\varepsilon_{damage} &> \frac{(r_i+h)\frac{V_1}{r_i}}{V_{ref}}\left(1+\varepsilon_{ref}\right)-1 \\
\varepsilon_{damage} &> \frac{V_1}{V_{ref}}\left(1+\frac{h}{r_i}\right)\left(1+\varepsilon_{ref}\right)-1 \\
\frac{1+\varepsilon_{damage}}{1+\varepsilon_{ref}} &> \frac{V_1}{V_{ref}}\left(1+\frac{h}{r_i}\right) \\
\frac{1+\varepsilon_{damage}}{1+\varepsilon_{ref}}\frac{V_{ref}}{V_1} &> 1+\frac{h}{r_i} \\
r_{i_\text{minimum}} = r_i &> \frac{h}{\frac{1+\varepsilon_{damage}}{1+\varepsilon_{ref}}\frac{V_{ref}}{V_1}-1}
\end{aligned} \qquad \{13\}$$

At radii less than the minimum, the outer surface of the web can be damaged by overstraining. A similar formula can be derived to avoid compressing failure of the inner surface of the web.