A SELECTED BIBLIOGRAPHY OF MATERIALS ON GEOMETRY USEFUL TO HIGH SCHOOL TEACHERS AND STUDENTS OF MATHEMATICS

By

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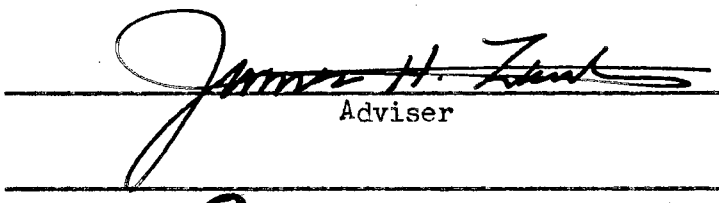

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PREFACE

This report is primarily intended to aid teachers or prospective teachers of elementary or secondary school mathematics in the selection of appropriate geometrical material to be used as supplementary exercises for the student or as self-instructional material for themselves.

The author is indebted to his adviser, Dr. James H. Zant, for his helpful assistance and guidance.

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CHAPTER I

INTRODUCTION

This bibliography is for the specific purpose of acquainting mathematics teachers with various sources of reference material on the subject of geometry. It not only contains a source of a particular work, but it also includes a short description of the work itself and other information which might be useful to the teacher.

Three principal methods were used in determining what books to include in this work. A preliminary list was obtained by asking several leading mathematics educators from various colleges and universities throughout the United States to suggest books and other materials that should be included in an undertaking of this nature. A second list was obtained from suggested mathematical materials obtained from various educational publications. A final list was selected by the author. This bibliography was then prepared from these three listings. Not all the materials found in the above three lists are included in this work. After a review of each article or book was taken, those which seemed most useful in enabling the teacher to teach and the student to learn fundamental mathematical concepts were selected to be included in this report. Several of the books listed here are strictly for the professional.

The need for such a collection of data as is compiled in this report is justified by the ever-increasing emphasis that has been placed in recent

years and is continuing to be placed on the teaching of the right kind of mathematics in the upper elementary and secondary schools of our country. Much of the material included in this work is rather recent and contains to a large extent articles concerned with the modern approach to mathematics. This is not to imply that our traditional courses are no longer valuable but to point out that they need to be at least supplemented and in many cases revised if they are to meet the ever-changing needs of our society. It follows that in a revised program of this sort, a very important asset to any teacher would be not only an adequate knowledge of the new approach itself, but also a wide source of reference material on the subject. A source of material is included here; the other asset is not included but it may be acquired through proper self-instruction.

It was this idea of self-instruction and individual effort that Professor G. H. Hardy stressed in 1925 when in the Presidential Address to the Mathematical Association he said, "In mathematics there is one thing only of primary importance, that a teacher should make an honest attempt to understand the subject he teaches as well as he can, and expound the truth to his pupils to the limits of their patience and capacity."

The need for a bibliography of this type can be further justified by the inadequate preparation of many of the instructors teaching mathematics in our schools today. Many are teaching subjects for which they were not prepared to teach. Others, who were supposedly prepared to teach mathematics, find themselves not yet ready to do a good job due to the fact that the courses they received in college were not sufficient or of the proper type for present-day ideas. This might have been

brought about by many schools leaving the training of mathematics teachers too much to the control of the education departments where emphasis was placed on methods rather than on the fundamental concepts and structure of mathematics.

Perhaps this was what Professor G. H. Hardy had reference to when, in the same Presidential Address as mentioned above, he set forth the following ideas.

"It is obvious that a great part of what is taught in schools and universities under the title of geometry is not geometry, or at any rate mathematical geometry, at all, but physics or perhaps philosophy."

"There is not, and cannot be, any question of a mathematician proving anything about the physical world; there is one way only in which we can possibly discerm its structure, that is to say, the laboratory method, the method of direct observation of the facts."

"School geometry is not a well-defined subject, a rational exposition of a particular geometrical system, but a collection of miscellaneous scraps, a selection of airs from different pieces, strung together in the manner which experience shows to be the most enlivening....."

"I think that it is time that teachers of geometry became a little more ambitious. It seems to me regrettable that students are not given the opportunity, while still at school, of learning a good deal more about the subject-matter out of which modern geometrical systems are built. It is probably easier and certainly vastly more instructive than a great deal of what they are actually taught."

It is evident that the time for better understanding of the fundamental concepts of mathematics on the part of both teacher and student has come and it is hoped that this bibliography will be of some value to

all people interested in the study of mathematics and in particular to high school teachers and students of geometry.

CHAPTER II

BIBLIOGRAPHY

The references included in this report are divided into specific categories for ease in the selection of a particular work. It is realized that in many cases the textbooks and articles included in this bibliography may be improperly placed, but the exact classification of each work seemed rather difficult and was considered a minor detail with respect to the over-all purposes of this report. Even though all of the works listed in this bibliography can offer much toward the improvement of general understanding of geometry, those which seem to offer most in terms of present-day needs of teachers and students of mathematics are indicated by an asterisk.

1. History

*Eves, Howard. An Introduction to the History of Mathematics. New York: Rinehart and Company, Incorporated, 1953, 422 p.

This is an excellent book on the history of mathematics and it is especially good for teacher-education purposes. Only a knowledge of elementary mathematics is needed to understand this book which points out in an interesting manner the chronological growth of mathematics from its primitive form to the possibilities of the modern mathematics superstructure.

*Gould, S. H. "Origins and Developments of Concepts of Geometry." National Council of Teachers of Mathematics Yearbook, XXII, 1957, pp. 273-305.

As the title indicates, the author gives a fine account of the origin and development of geometry from Euclid to the non-Euclidean geometry of Gauss, Lobachevski, and Bolyai and the finite but unbounded space concept of Riemann.

Kline, Morris. Mathematics In Western Culture. New York: Oxford University Press, 1953, 484 p.

This book illustrates the cultural force that mathematics has made in Western civilization. Such a fine book as this should be available to all energetic students of mathematics.

*Reves, George E. "Outline of the History of Geometry." School Science and Mathematics, III, 1952, pp. 299-309.

This article contains a short descriptive outline of the history of the development of geometry from 3100 B.C. to the present. Mention is made of various individuals or groups of people and their contribution to the field of geometry.

Smith, David Eugene. History of Mathematics. New York: Dover Publications, Incorporated, 1925.

Here is a chronological development of mathematics from number concepts to the calculus. This should be very beneficial at the senior high school level.

2. Plane and Solid Euclidean Geometry

Anderson, Raymond. Romping Thru Mathematics. New York: A. A. Knopf, 1947, 152 p.

Many explanatory drawings, diagrams, and charts to make clear various topics on geometry, algebra, and other phases of mathematics are included in this work.

*Birkhoff, George David and Ralph Beatley. Basic Geometry. New York: Chelsea Publishing Company, 1959, 288 p.

This is one of the better textbooks in print today concerned with a modern approach to Euclidean geometry. This course, designed for high school students, is based on only five fundamental postulates, seven basic theorems, and nineteen other theorems together with seven on loci. To accommodate the student who will not take another course, some modern, analytic, and solid geometry is included.

*Brumfiel, Charles F., Robert E. Eichols, and Merrill E. Shanks. Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1960. 281 p.

Here is one of the most recent texts on geometry designed for the high school level. This book is well-written, has plenty of material for a full years work in plane geometry, and will certainly challenge the scholar. This text is based on Hilbert's Axioms. (See Hilbert below.)

College Entrance Examination Board. Report of the Commission on Mathematics. New York: College Entrance Examination Board, % Educational Testing Service, Box 592, Princeton, New Jersey, 1959. pp. 108-174.

This report contains nine short chapters dealing with various phases of geometry from reasons for modifying the traditional treatment of geometry to indirect proofs, geometrical transformations, order relations, and many others. This should prove very valuable as a part of the teacher's mathematics library.

Court, Nathan A. College Geometry. New York: Barnes and Noble, Incorporated, 1952, 313 p.
This has been a standard text in many colleges for several years. The material is arranged in a very systematic manner. It contains many helpful hints which make it quite useful to students.

*Custis, C. W., P. H. Daus and R. J. Walker. Euclidean Geometry Based on Ruler and Protractor Axioms. School Mathematics Study Group, Box 2029, Yale Station, New Haven, Connecticut, 1959, 145 p.
This is a study written for high school teachers, of the approach to Euclidean geometry which is being used in the SMSG tenth grade textbooks.

Descartes, Rene (tr. by David Eugene Smith and Marcia L. Latham). Geometry. New York: Dover Publications, Incorporated, 1954, 243 p.
This amounts to a translation of a classic in mathematics with a facsimile of the original French edition.

Eaves, David R. Modern College Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1949, 232 p.
This is a course in synthetic geometry built directly upon a first course in plane Euclidean geometry. It is designed primarily for teachers of high school mathematics. It deals with such topics as loci, fundamental theorems of advanced Euclidean geometry, harmonic range, inversion, and constructions in Euclidean geometry.

*Forder, Henry George. The Foundations of Euclidean Geometry. New York: Dover Publications, Incorporated, 1958, 349 p.
Here is a connected and rigorous account of Euclidean geometry in the light of modern investigations. While this work should be very valuable to the teacher, it will prove rather stiff for a high school student.

Frame, J. S. Solid Geometry. New York: McGraw-Hill, 1948, 339 p.
This is not a conventional textbook. It contains an original approach to three dimensional space relations, maps, and projections.

Hart, W. W., V. Schult, and H. Swain. Plane Geometry and Supplements. Boston: D. C. Heath and Company, 1959, 464 p.
This is a revision of a favorite text and provides a basic course in plane geometry together with supplementary enrichment material from solid and analytic geometry.

Heath, Thomas L. The Thirteen Books of Euclid's Elements. New York:

Dover Publications, Incorporated, 1956, 349 p.
This is a very fine translation of Euclid's original work. It is for the professional.

Hemmerling, Edwin M. College Plane Geometry. New York: Wiley and Sons, Incorporated, 1958, 310 p.
In this work the student is introduced to reasoning by induction, by deduction, by analogy, and by indirect methods. It stresses the relationship of the abstract materials of geometry to experiences in daily life.

Herberg, Theodore and Joseph B. Orleans. A New Geometry for Secondary Schools. Boston: D. C. Heath and Company, 1958, 274 p.
This is a semi-modern text on plane geometry for secondary schools. Formal proofs are limited in number and many corollaries, formulas, and theorems are treated by inductive methods. Adequate material is presented for a good college-preparatory course. Problems range in difficulty in order to stimulate the better student.

*Hilbert, David. Foundations of Geometry. La Salle: The Open Court Publishing Company, 1938, 143 p.
This is a different type of Euclidean geometry developed by the author in an attempt to choose for geometry a simple set of independent axioms and to deduce from them the more important geometrical theorems. This is a fine work illustrating the true nature of mathematics. The better high school student should be able to interpret a large portion of this material.

Hollerberg, A. E. "Geometry of the Fixed Compass." The Mathematics Teacher, LII (April, 1959), pp. 230-44.
This is an interesting article which should prove interesting to the better students. It will provide the teacher with a means of making construction work more challenging to the student.

Horblit, Marcus and Kay L. Nielsen. Plane Geometry Problems. New York: Barnes and Noble, Incorporated, 1947, 197 p.
This contains various types of problems which may be useful as review for advanced students or as supplementary problems.

Kern, W. F. and J. H. Bland. Solid Mensuration With Proofs. New York: Wiley and Sons, Incorporated, 1938, 138 p.
Here is a practical and rather complete volume on traditional secondary school geometry.

Klein, Felix. Famous Problems of Elementary Geometry. New York: Dover Publications, Incorporated, 1956, 92 p.
Here is a fairly simple and easily understood account of three famous problems and the proof that these cannot be solved by ruler and compass. The proofs, however, are rather advanced.

*Lieber, Lillian R. The Education of T. C. Mitts. New York: W. W. Norton and Company, Incorporated, 1944, pp. 129-167.

This is a short, light-hearted account of the development of the geometries from Euclid to the present. Designed to strengthen the appreciation of the general public for mathematics, this work will offer much to the high school student.

Loomis, Elisha. The Pythagorean Theorem. Berea: Mohler Printing Company, 1927, 214 p.

This book contains a collection of over 167 algebriac and geometric proofs of the famous proposition of Pythagoras.

Mallory, Virgil S., Bruce E. Meserve, and Kenneth C. Skeen. A First Course In Geometry. Syracuse: The L. W. Singer Company, 1959, 557 p.

A more or less traditional text designed to meet the needs of students at all levels of development, this work, printed in color, introduces the student to the study of geometry by means of review of elementary constructions, which may serve as an aid in the study of geometric definitions, postulates, and axions. In chapter two, two congruence theorems are postulated. Chapter three goes into proof. Theorems are kept to a minimum in this book. This fine work integrates algebra and arithmetic very well in its geometrical development.

Maxwell, E. A. Geometry For Advanced Pupils. New York: Oxford University Press, 1949, 176 p.

As the title indicates, this book can be utilized by the advanced student or the teacher as a reference book. It is highly recommended for the teacher's private library or as a school library reference book. This work, intended not to be used as a text book, contains such things as material on the nine point circle, the theorems of Ceva and Menelaus, incentres and excribed centres, and many others.

*Meserve, Bruce E. Fundamental Concepts of Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1955, 321 p.

Especially well-adapted for the perspective or in-service teacher of geometry, this book should broaden the reader's concept of what geometry really is. The content of the text is as follows: Foundations of geometry. Affine geometry. Euclidean plane geometry. Evolution of geometry. Non-Euclidean geometry. Topology.

Mock, Alex J. "Trisecting an Angle." The Mathematics Teacher, LII (April, 1959), pp. 245-46.

A direct proof of the impossibility of the trisection of any angle is offered in a simple and understandable fashion. An indirect proof follows.

Moshan, B. "Primitive Pythagorean Triplets." The Mathematics Teacher, LII (November, 1959), pp. 541-45.

This consists of an exercise dealing with the 8, 15, 17 and 861, 620, 1061 right triangles. This article might be used effectively in conjunction with the 3, 4, 5 triangle.

National Council of Teachers of Mathematics. The Growth of Mathematical Ideas Grades K-12. Washington, D. C. National Council of Teachers of Mathematics, 1959, pp. 111-179.
Mathematical proof is dealt with very adequately in these few pages.

Nyberg, Joseph A. Fundamentals of Solid Geometry. New York: American Book Company, 1947, 261 p.
This consists of a concise treatment of the logical relation of lines, planes, and simple curved surfaces in space.

Pickett, H. "Length of Chords and Their Distances From the Center." The Mathematics Teacher, L (May, 1957), pp. 325-26.
Here is a stimulating series of problems which may be used as supplementary exercises for the better student.

*Polya, G. How to Solve It. Garden City: Doubleday and Company, 1957, 253 p.
Setting forth a definite and precise guide for the solution of problems, this book illustrates clear thinking and analytical reasoning. This should be a fine text for the high school reference library. It can be used by both students and teachers.

Ransom, William R. Geometry Growing. Washington, D. C. : National Council of Teachers of Mathematics, 1959. 517 p.
Presenting bits of geometry from Pythagoras to Newton, this book should provide valuable enrichment material for the high school student.

Ravielli, Anthony. An Adventure In Geometry. New York: The Viking Press, 1957. 117 p.
This is not a textbook, but a work designed to create an interest in the beauty of geometry on the part of students. Containing many fine pictures and other illustrations, this book will prove very satisfactory for special reading in an introductory geometry course.

Robinson, Gilbert de B. Foundations of Geometry. Toronto: University of Toronto Press, 1952, 168 p.
Outlining the classification and reduction to axiomatic form of geometrical relations, this should be a very valuable book to those with a background in high school plane and solid geometry.

Robusto, C. C. "Trisecting An Angle." The Mathematics Teacher, LII (May, 1959), pp. 358-60.
By means of an algebriac and trigonometric approach, this author illustrates the impossibility of trisecting an angle by ordinary means.

*Rosskopf, Myron F. Mathematics: A Second Course. New York: McGraw-Hill Book Company, 1952.

Here is a relatively new high school textbook with a modern approach to the teaching of plane geometry. The central theme of this work is the logical presentation of geometry. It assumes the high school student can master and enjoy demonstrative geometry. Two mastery tests are found at the end of each chapter. One is geometric; the other is non-geometric. Since the approach of this text is fairly new, it is accompanied by a manual for the teacher.

Roudebush, Elizabeth. Laboratory Geometry. Englewood Cliffs: Prentice-Hall, Incorporated, 1960, 198 p.

This book covers the fundamentals of plane geometry through the use of practical examples. It also touches upon some exercises from solid geometry and trigonometry. Many crawings are used to clarify the work.

Schnell, Leroy H. and Mildred G. Crawford. Plane Geometry. New York: McGraw-Hill Book Company, Incorporated, 1953, 436 p.

This text presents a clear thinking approach to the study of geometry. Such topics as constructions, fundamental geometric concepts, assumptions, and the meaning of proofs are throughly discussed before any attempt is made to utilize theorems, definitions, axioms, or postulates in any type of proof.

*School Mathematics Study Group. Junior High School Mathematics Units. School Mathematics Study Group, Box 2029, Yale Station, New Haven, Connecticut, 1959, 100 p.

This is volume II, "Geometry", of a series of three volumes on junior high mathematics prepared by the SMSG. In this series of experimental units, sets are introduced early along with many other more modern concepts. This volume on geometry may be obtained separately from the other two volumes.

*School Mathematics Study Group. Mathematics For High School Geometry. School Mathematics Study Group, Box 2029, Yale Station, New Haven, Connecticut, 1959, 650 p.

This is an experimental undertaking prepared for the purpose of perhaps giving future authors of new textbooks a source of suggestions for an improved curriculum. This book is devoted primarily to plane geometry, with a few chapters on solid geometry, and a short introduction to analytic geometry at the end. Since the traditional Euclidean postulates are not logically sufficient for geometry and since this work is designed especially as a course in elementary mathematics, the basic scheme in the postulates in this text is that of G. D. Birkhoff. (See Birkhoff above.) This book has been used very successfully during the school year 1959-60 and will be revised during the summer of 1960. A very helpful commentary for teachers is available.

Spiller, Lee R. Today's Geometry. Englewood Cliffs: Prentice-Hall, Incorporated, 1960. 321 p.

A fairly traditional course in plane geometry, this text features the practical applications of geometry to present-day life.

Emphasis is placed on vocabulary and basic concepts.

Welchons, A. M. and W. R. Krickenberger. New Solid Geometry. New York: Ginn and Company, 1955, 326 p.
This is a very fine traditional course in solid geometry, consisting of a large number of exercises with many of them designed for the weaker student.

Wiseman, J. D. "A B C's of Geometry." The Mathematics Teacher, L (May, 1957), pp. 327-29.
The advantages of the "if-then" pattern of reasoning is illustrated in this article.

*Yates, R. C. "Euclidean Constructions." The Mathematics Teacher, XLVII (April, 1954), pp. 231-33.
This short article contrasts original Euclidean constructions with those being performed today.

3. Plane and Solid Analytic Geometry

Albert, Abraham Adrian. Solid Analytic Geometry. New York: McGraw-Hill Book Company, 1949, 162 p.
Modern algebriac techniques are used very regularly in this text in treating with the basic topics of solid analytic geometry. The final chapter contains a brief presentation of the elements of projective geometry.

Fuller, Gordon. Analytic Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1954, 205 p.
Departing slightly from traditional courses, the author tries to dwell on the topics that will be most beneficial to the pre-calculus student. The later two chapters deal with the elements of solid analytic geometry.

Oakley, C. O. Analytic Geometry Problems. New York: Barnes and Noble, 1958, 253 p.
This book gives in each chapter concisely explained principles and formulas with 341 problems with solutions so as to illustrate each principle. This text could prove valuable for reference purposes.

*Sisam, Charles H. Concise Analytic Geometry. New York: Henry Holt and Company, 1946, 155 p.
This text covers well the basic topics and important applications usually covered in analytical geometry. There is an abundance of problems of varying degrees of difficulty. Answers are given to the odd-numbered problems.

Smith, Edward S., Meyer Salkover, and Howard K. Justice. Analytic Geometry. New York: Wiley and Sons, Incorporated, 1939, 167 p.

Both the applied and theoretical sides of analytical geometry are utilized in this work. Analytical proof is discussed very well.

Smith, Percy F., Arthur S. Gale, and John H. Neelley. New Analytic Geometry. Boston: Ginn and Company, 1928, 323 p.
Here is a proven textbook which presents material in a readable manner. It covers the regular topics of analytic geometry with three chapters on space geometry at the end.

Thomas, George B. Elements of Calculus and Analytic Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1959, 576 p.
This si a fine book designed for use in accelerated courses offered in secondary schools or in college-preparatory programs.

Underwood, R. S. and Fred W. Sparks. Analytic Geometry. Boston: Houghton-Mifflin Company, 1956, 280 p.
Containing the usual features of analytic geometry, this text places great emphasis on clarity as evidenced by the many examples before each group of exercises.

4. Modern and Non-Euclidean Geometry

*Bakst, Aaron. Space and Geometry. New York: New York University Book Store, 1950, 77 p.
In this mimeographed work is presented a brief survey of the fundamental ideas of geometry. A very good distinction is made between the various non-Euclidean geometries. The material in this pamplet was prepared especially for the teacher whose background in the various phases of geometry may be weak.

Bonola, Roberto. Non-Euclidean Geometry. New York: Dover Publications, Incorporated, 1955, 389 p.
This work amounts to a critical and historical study of the development of the field. Perhaps it is too advanced for the high school level.

Coxeter, H. S. M. Non-Euclidean Geometry. Toronto: University of Toronto Press, 1957, 309 p.
Presenting fundamental principles in a clear and readable manner, this text deals with the two families of "midlines" between two given lines, an elementary derivation of the basic formulae of sperical trigonometry and hyperbolic trigonometry, a computation of the Gaussian curvature of the elliptic and hyperbolic planes, and a proof of Schlafli's formula for the differential of the volume of a tetrahedron.

Davis, David R. Modern College Geometry. Reading: Addison-Wesley Press, Incorporated, 1949, 232 p.
This is a course in synthetic geometry built directly upon a first course in plane Euclidean geometry. It is designed primarily

for teachers of high school mathematics and deals with such topics as loci, fundamental theorems of advanced Euclidean geometry, harmonic range, inversion, and constructions in Euclidean geometry.

*Hempel, C. G. "Geometry and Empirical Science." American Mathematics Monthly, LII, 1945, pp. 7-17.
Here is a fine disucssion of the validity of the various geometrical theories in contrast to the calculations of empirical nature.

Hilbert, D. and S. Cohn-Vossen. Geometry and the Imagination. New York: Chelsea Publishing Company, 1952, 358 p.
This is not a text book, but it should prove very valuable as a part of the high school library. The theme of this book is insight. Not merely proofs, but proofs that offer insight into why they are true. Since this book is concerened with all phases of geometry and the relationship and usefulness of geometry in our environment, it will be useful to both the professional and the beginner.

Klein, Felix. Elementary Mathematics From an Advanced Stanpoint: Geometry. New York: Dover Publications, Incorporated, 2939, 214 p.
This is a comprehensive view of the whole field of geometry from a professional view.

*National Council of Teachers of Mathematics. Insights Into Modern Mathematics. Washington, D. C. : National Council of Teachers of Mathematics, 1957, 448 p.
This book, written especially for teachers, has as its primary goal the presentation of sufficient information about the major areas of mathematics so as to enable high school teachers to understand the common dependence of all areas of mathematics upon the fundamentals underlying high school mathematics. Many of the chapters are easilty read; others require study. While chapters 4, 6, and 9 should be of particular interest to both student and teacher of geometry, all other chapters will be very beneficial as an aid to anyone interested in nearly any phase of modern mathematics. Chapter 4 deals with deductive methods in mathematics. Chapter 6 relates the application of geometric vector analysis to high school mathematics problems. Chapter 9, "Origin and Development of Concepts of Geometry", is well-described by its title.

*Russell, Bertrand A. Foundations of Geometry. New York: Dover Publications, Incorporated, 1956, 201 p.
Here is a mathematical and philosophical analysis of the basic principles of geometry. This work should be very beneficial to the mathematics teacher who wishes to improve his or her basic ideas of geometry.

*Shively, Levi S. An Introduction to Modern Geometry. New York: Wiley and Sons, Incorporated, 1939, 167 p.

Designed as a college text in modern geometry, this book is primarily for the preparation of teachers in high schools. Of particular interest to teachers are such areas of investigation as the triangle, harmonic points and lines, cross ratio, and constructions with ruler and compass.

Sommerville, Duncan M'Loren Young. The Elements of Non-Euclidean Geometry. New York: Dover Publications, Incorporated, 1958, 274 p.

This book is made up of five well-written chapters entitled, "The historical development of geometry", "Elementary Hyperbolic geometry", "Elliptic Geometry", "Analytic geometry", and "Representation of non-Euclidean space."

Wolfe, Harold E. Non-Euclidean Geometry. New York: The Dryden Press, Incorporated, 1945, 244 p.

Included in this text, which is designed for an elementary course in non-Euclidean geometry, is very fine accounts of the foundations of Euclidean geometry, attempts to prove the fifth postulate, and the discovery of non-Euclidean geometry. Much space in the later part of this work is devoted to elliptic and hyperbolic geometry. The first three chapters of this text should be of particular interest to high school students, while the entire text will prove valuable to the teacher.

5. Advanced Geometry

Artin, E. Geometric Algebra. New York: Interscience Publishers, 1957, 224 p.

Departing slightly from the classical such as linear algebra, topology, differential and algebriac geometry, this text is designed to offer something distinct. It contains such topics as preliminary notions, affine and projective geometry, symplectic and orthogonal geometry, the general linear group, and the structure of symplectic and orthogonal groups.

Coxeter, Harold Scott Macdonald. The Real Projective Plane. New York: McGraw-Hill Book Company, 1949, 196 p.

Designed as a first course in projective geometry, this text requires no training beyond secondary school geometry and algebra.

Durell, C. V. Algebriac Geometry. London: G. Bell and Sons, Ltd., 1955, 380 p.

This text deals primarily with plane projective algebriac geometry. However, the last two chapters are concerned with the relation between projective and cartesian geometry. Chapter six deals with algebriac solid geometry.

Kreyszig, Erwin. Differential Geometry. Toronto: University of Toronto Press, 1959, 356 p.

This book is intended to meet the need for a text introducing advanced students in mathematics, physics, and engineering to the field of differential geometry. Only a knowledge of the calculus is necessary. The material is presented in a simple but rigouous manner. Many examples are used. The text covers such things as use of tensors, introduction to Riemannian geometry, modern presentation of the theory of curves and surfaces in three-dimensional Euclidean space, etc.

Lang, Serge. Introduction to Algebriac Geometry. New York: Interscience Publishers, 1958, 272 p.

The prupose of this book is to provide the graduate student, whose background in elementary algebra need only be roughly at the level of Galois theory, with a rapid, concise, and self-contained introduction to algebriac geometry. Included in this work are topics such as extension of places, Zariski's main theorem, linear systems, differential forms, normalization of varieties, the theory of albebriac and linear disjointers, and others.

Segre, B. Algebriac Geometry, New York: Stevens and Company Scientific Books, 1954, 67 p.

This is a series of eight lectures of algebriac geometry.

Slaby, Steve M. Engineering Descriptive Geometry. New York: Barnes and Noble, 1956, 347 p.

Here is an expertly prepared textbook stating the basic principles of the subject with each principle being illustrated by fully worked out problems. This text is especially well-suited to the student studying on his own without supervision.

Struik, D. J. Analytic and Projective Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1953, 291 p.

This text requires a fairly good understanding of calculus, analytic geometry, some vector algebra and space geometry. With these prerequisites, this book starts with affine and projective geometry concerning points on a line and lines through a point. Then the geometry of plane and space, leading to conics and quadrics, is developed inside the triple frame of Euclidean, affine, and projective geometry.

Struik, D. J. Differential Geometry. Reading: Addison-Wesley Publishing Company, Incorporated, 1950, 221 p.

This is a fine introductory textbook for courses in classical differential geometry employing vector methods.

Veblen, Oswald and John Wesley Young. Projective Geometry, Boston: Ginn and Company, 1918.

Here is a scholarly treatment of projective geometry designed for the professional.

Welchman, W. Gordon. Introduction to Algebriac Geometry. Cambridge: Cambridge University Press, 1950, 351 p.

The first three chapters of this text is concerned mostly with definition of terms. The later chapters deal with conics, configurations, invariants, covariants, etc.

*Young, John W. Projective Geometry. Chicago: The Open Court Publishing Company, 1930, 185 p.

The first five chapters of this text will be most valuable to teachers who have little background in projective geometry as they are treated in a fairly elementary manner. They start with the fundamental propositions of projective geometry and end with the theorems of Pascal and Brianchon and the polar system of a conic.

6. Enrichment and Professional Material

Abbott, Edwin A. Flatland. New York: Dover Publications, Incorporated, 1952, 128 p.

This fantasy in varied dimensions is a lesson in the development of an open mathematical mind. This should be suitable for the average high school student.

Bakst, Aaron. Mathematics: Its Magic and Mastery. Princeton: D. Van Nostrund Company, Incorporated, 1952, 790 p.

Covering various fields of mathematics including informally such topics as algebra, trigonometry, and geometry, this book makes application of mathematics to other fields such as physics and astronomy.

Ball, W. W. Rouse. Mathematical Recreations and Essays. London: The McMillan Company, 1922, 266 p.

A rather lengthy work, this book contains both geometrical and arithemetical recreations.

Brandes, Louis Grant. Math Can Be Fun. J. Weston Walsh, Publisher, Box 1075, Portland, Maine, 1956, 200 p.

This book can be of particular interest to the teacher who is over-burdened with students of varying ability and interests. It contains many interesting problems, puzzles, tricks, self-tests, optical illusions, and other material to stimulate student interest.

Gardner, Martin. Mathematics, Magic, and Mystery. New York: Dover Publications, Incorporated, 1954, 176 p.

This book contains over 500 interesting tricks with cards, dice and coins; demonstrations with pure numbers, geometrical tricks making use of triangles and squares and many other interesting feats to make ones teaching more interesting.

Kasner, Edward and James Newman. Mathematics and the Imagination. New York: Simon and Schuster, 1958, 380 p.

Here is a fascinating treatment of all fields of mathematics. Of special interest is chapter four which discusses the development

of the various types of geometries and also chapter seven which is concerned with a discussion of topology.

Kinney, Lucien Blair and C. Richard Purdy. Teaching Mathematics In The Secondary School. New York: Rinehart and Company, 1952, pp. 100-135.
The aim and methods of teaching geometry are set forth in these few pages.

Ling, C. "Shall They Take Geometry." The Mathematics Teacher, XLVII (December, 1954), pp. 557-58.
Geometry counseling programs can prove very helpful in the large school as indicated by this article.

Logsdon, Mayme I. A Mathematician Explains. Chicago: University of Chicago Press, 1958, 189 p.
While this entire book is quite valuable, chapters 4, 5, and 9 take on a more geometrical aspect. The book can be easily read by the more advanced high school student. It offers mathematics as an interesting and understandable subject.

*Meserve, Bruce E. "Modern Geometry for Teachers." School Science and Mathematics, LVIII (June, 1958), pp. 437-41.
In a few short pages the author gives in summary form the type of college preparation geometry teachers should have. He also discusses what he feels should be the content of secondary geometry courses.

Polya, G. "On the Curriculum for Prospective High School Teachers." The American Mathematical Monthly, LXV, 1958, pp. 101-104.
The author sets forth some points he feels are characteristics of a good teacher of mathematics.

*Roskopf, Myron F. and Robert M. Exner. "Modern Emphasis in the Teaching of Geometry." The Mathematics Teacher, L, 1957, pp. 272-79.
This article deals with two aspects of modern mathematics that needs emphasis in the teaching of geometry. They are axiomatic structure and interpretations of such structures.

*Schnell, L. H. "New Emphasis in Teaching Geometry." National Council of Teachers of Mathematics Yearbook, XXII, 1955, pp. 269-76.
In this article the author gives a more or less historical development of geometry from Euclid to the present. This article explains why the traditional course remained unchanged so long. It also covers the more important points of the modern approach to the teaching of geometry.

Steinhaus, Hugo. Mathematical Snapshots. New York: Oxford University Press, 1950, 266 p.
This is a recreational work containing a tremendous amount of supplementary material of geometrical nature.

CHAPTER III

SUMMARY

High school mathematics textbooks today are in a process of being revised. New approaches to the study of mathematics are being employed. Emphasis is being placed on the fundamental concepts of mathematics and the real nature of mathematical systems.

These various changes have brought into play several problems. Foremost among them is that of improper preparation of teachers of mathematics in our elementary and secondary school systems. This situation has been recognized by various organizations and they have attempted to alleviate the conditions by means of financial grants of various types. These grants have been of great value, but the problem is far from being solved. While the primary function of this report is to supply teachers with a source of reference material on geometry, certainly a secondary objective is to point out the need for some sort of self-instructional program on the part of all teachers.

It is hoped that this bibliography may in some way enable instructors to better select the material on geometry which will aid them most in their particular teaching situation.

VITA

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Master of Science

Report: A SELECTED BIBLIOGRAPHY OF MATERIALS ON GEOMETRY USEFUL TO HIGH SCHOOL TEACHERS AND STUDENTS OF MATHEMATICS

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