## DESIGN OF A FREE WATER SURFACE TABLE

TO BE USED AS AN ANALOGY TO TWODIMENSIONAL COMPRESSIBLE

GAS FLOW


By
JOHN SAMOS
Bachelor of Science
Oklahoma State University
Stillwater, Oklahoma
1960

Submitted to the faculty of the Craduate School of the Oklahoma State University
in partial fulfillment of the requirements
for the degree of MASTER OF SCIENCE

May, 1960

# DESIGN OF A FREE WATER SURFACE TABLE <br> TO BE USED AS AN ANALOGY TO TWODIMENSIONAL COMPRESSIBLE <br> GAS FLOW 

## Report Approved:


$\qquad$

Dean of the Graduate School

## PREFACE

The purpose of this report is to present the design and construction details necessary for the construction of a free water surface table. A consideration was given to incorporate the desired features but to also maintain the cost to a minimum.

A short introduction to the analogy is given with a table correlating the properties and characteristics of two-dimensional compressible gas flow with those of liquid flow.

The author wishes to acknowledge his appreciation to Dr. G. W. Zumwalt for his excellent guidance throughout the period of the design.

To my wife Agnes and my two children, Harry and Effie, I can only say that I must spend a lifetime making up for the sacrifices that they have made the last 27 months that I might be free to take advantage of a wonderful opportunity.

TABLE OF CONTENTS
Chapter Page
I. INTRODUCTION ..... 1
II. EXPERIMENTAL METHODS ..... 4
Nozzle and Test Section ..... 4
Photographic Methods ..... 4
Measurement of Water Depths ..... 7
Choice of Water Height ..... 7
III. VALIDITY OF THE ANALOGY ..... 8
Vertical Motions ..... 8
Surface Tension ..... 8
Shock Wave ..... 9
IV. DESIGN CONSIDERATIONS ..... 10
Table Bottom and Sides ..... 10
Sluice-Type Nozzle ..... 11
Frame Assembly ..... 12
Sump Tank, Sluice Tank, and Piping ..... 12
Carriage Assembly ..... 13
Water Depth Gage and Pitot Tube ..... 13
Pump Selection ..... 14
V. SUMMARY AND CONCLUSIONS ..... 16
BIBLIOGRAPHY ..... 17
APPENDIX ..... 19

## LIST OF FIGURES

Figure Page

1. Photographic Techniques
a. Reflection of Diffuse Light From Water Surface ..... 5
b. Transmission of Diffuse Light Through Water Surface ..... 5
c. Transmission of Light Through Water Surface With Image Thrown On Screen ..... 6
2. Pitot Tube ..... 14
LIST OF PLATES
Plate Sheet ..... Page
11 Water Table Assembly ..... 20
1 2 Water Table - Section View ..... 21
2 1 Frame - Assembly ..... 22
2
2 Frame - Upper Section ..... 23
3 1 Sluice Assembly ..... 24
3 2 Sluice Details ..... 25
3 3 Channel Bottom Details ..... 26
4 1 Tank - Sump ..... 27
5 1 Carriage Assembly ..... 28
6 1 Sump Table ..... 29
7 1 Spout - Sluice Tank ..... 30
8 1 Tank - Sluice ..... 31

## LIST OF SYMBOLS

| Symbol | Meaning |
| :--- | :--- |
| a | Speed of sound |
| D | Diameter |
| d | Water Depth |
| E | Modulus of Elasticity |
| Fr | Froude Number |
| $f$ | Friction Factor |
| g | Acceleration of Gravity |
| $\mathrm{h}_{\mathrm{f}}$ | Head Loss |
| I | Moment of Inertia |
| M | Mach Number |
| P | Pressure |
| R | Reynolds Number |
| T | Absolute Temperature |
| V | Velocity |
| W | Weight |
| E | Roughness Height |
| $\gamma$ | Ratio of Specific Heats |
| $\nu$ | Kinematic Viscosity |
| $\rho$ |  |

Subscripts
Value of Stagnation
s
Value in Undisturbed Stream

## CHAPTER I

INTRODUCTION

An anology exists between pressure waves in a two-dimensional compressible gas flow and the gravity waves on the free surface of a liquid. The mathematics of this analogy has been known for a long time and the application of this analogy by means of experiments with rectangular water channels appears to have been first suggested by Jouguet (1). The application of this experimental method was implemented by Riabouchinsky (2), Von Karan (3) and Preiswerk (4). Additional investigations (5, 6, 7, 8, 9, 10) were stimulated by the high cost of supersonic wind tunnels which were required for the development of supersonic airplanes and vehicles.

The construction and operation of a water table is a very inexpensive way of studying two-dimensional compressible gas flow. Particularly, phenomena occuring in air at speeds too high for visual observations could be observed at very low speeds ( 1 to 3 fps ) in a water channel. Constructing the bottom and sides of the channel with plate glass, not only provides the smooth surface required, but also lends itself to a wide variety of photographio techniques.

In order to stimulate thinking in the application of a water table, the summary to the analogy is given in table 1 , where $Y=2$ is the equivalent value of $Y$ peculiar to a rectangular cross-sectional shape of the water channel.

## TABLE I

ANALOGOUS QUANTITTIES IN COMPRESSIBLE GAS FLOW AND LIQUID FLOW

| Significant quantities and characteristics of twodimensional gas flow $Y=2$ | Corresponding values in analogous liquid flow |
| :---: | :---: |
| Temperature ratio $\frac{T}{T_{0}}$ | Water-depth ratio $\frac{d}{d_{0}}$ |
| Density ratio $\frac{P}{\rho_{0}}$ | Water-depth ratio $\frac{d}{d_{0}}$ |
| Pressure ratio $\frac{P}{P_{0}}$ | Square of water depth $\left(\frac{d}{d}\right)^{2}$ |
| Velocity of sound $\quad \begin{aligned} & \frac{Y p}{p}\end{aligned}$ | Wave velocity, $c=\sqrt{\text { gd }}$ |
| Mach Number $M=\frac{V}{a}$ | Froude Number $\operatorname{Fr}=\frac{\mathrm{V}}{\mathrm{c}}=\frac{\mathrm{V}}{\sqrt{\mathrm{gd}}}$ |
| Shock wave | Hydraulic jump |
| Subsonic flow | Streaming water |
| Supersonic flow | Shooting water |

Streaming water - velocity of the liquid is less than $\sqrt{\mathrm{gd}},(\mathrm{M}<1)$

Shooting water = velocity of the liquid is greater than $\sqrt{\mathrm{gd}_{,}}(\mathrm{M}>1)$

The Mach number or Froude number in the liquid can be easily computed from

$$
F r=\left[\frac{2\left(d_{0}-d_{s}\right)}{d_{s}}\right]^{\frac{1}{2}}
$$

from which it can be seen that the $\operatorname{Fr}=1$ at points where the depth is two-thirds of the total head.

Although it appears that the free surface water table will not generally yield quantitative information that may be used directly in design,
it has been successful in reproducing most qualitative aspects of highspeed flows. It gives the operator the opportunity to view various flow patterns which can result in the development of original ideas. Its other advantages are its low cost, simplicity and rapid operation.

## CHAPTER II

## EXPERIMENTAL METHODS

## Nozzle and Test Section

The desire to produce supersonic flow in the analogy suggests the use of a converging-diverging nozzle. Since a uniform flow, free of vertical motion, is desired, the use of the converging-diverging nozzle is possible. This nozzle is quite long, however, and tends to produce small oblique disturbance waves. The method devised to overcome the unwanted waves is the use of a sluzcectype nozzle. Tests have shown that a very smooth surface is produced when a well-rounded sluice nozzle is used. In addition, the aluice nozzle permits the Mach number to be varied by merely controlling the head upstream of the sluice gate. Also, the flow depth can be varied by adjusting the sluice gate opening. The Laval nozzle not only creates vertical waves, but is also restricted to a single Mach number.

## Photographic Methods

The use of plate glass in the construction of the table permits a wide variety of photographic and group observation techniques. Figures $l a, b$ and $c$ are some suggested methods for photography (11).

The glass bottom and sides also present a smooth finish to the water, thereby creating a minimum of flow friction. The slight friction that is present can be approximately cancelled by tilting the table a small amount. This allows the forces of gravity to somewhat compensate for the friction


Fig. la. Reflection of Diffuse Light From Water Surface


Fig. Ib. Transmission of Diffuse Light Through Water Surface


Camera


Fig. 1c. Transmission of Light Through Water Surface With Image Thrown On Screen
present. The proper tilt is probably best attained through trial and error.

Measurement of Water Depths

The simplest and most convenient method for determining the depth seems to be a needle probe that is mounted on a transverse mechanism capable of being moved in any of three rectangular coordinate directions. The tip of the probe should be constructed of platinum or other non-corrosive metal. The depth is determined from a vernier scale that should be capable of . 001 inches readability. The moment the tip of the probe makes contact with the water, a capillary wave will be clearly seen and the water depth can be read.

## Choice of Water Height

The water height selected must be such that the effects of capillary waves will be minimized and also the radius of curvature of the free surface is large compared with the water depth so that vertical motion can be made negligible except in the vicinity of a hydraulic jump.

The effects of capillary waves can be minimized by either using a small flow depth of about 0.25 inches which will result in the capillary waves behaving like gravity waves, or by using flow depths in excess of oze foot. The large flow depths will give capillary waves so large that the effects become secondary.

## CHAPTER III

## VALIDITY OF THE ANALOGY

Some assumptions of the analogy will be reviewed to help evaluate the significance of data obtained from a water table.

Vertical Motions .

The basis of the analogue is that water flow is two-dimensional. This assumption may immediately create the impression that the analogue has no validity when vertical motions exist. Although two-dimentional flow rules out any vertical motion, the analogy refers to a limiting case where the vertical components of velocity and acceleration are negligibly small. To maintain the validity, the slope of the water surface must be very small. This suggests the use of large models compared with water depth. In the case of shocks where there are rapid changes in water depth, the vertical motions may effect the analogy considerably. It is believed that the descrepancies that exist between wind tunnel and water table tests are largely due to ignoring of vertical motions in the analogy. In order to maintain the vertical motion to a minimum, a flow of about one-fourth inch in the "supersonic region" is recommended for best results.

## Surface Tension

The effects of capillary waves impose a problem on shadowgraph observation and photography. The capillary waves are in many instances more prominent than the phenomena that is attempted to be studied. Attempts
have been made to reduce the capillary effects by the adding of wetting agents to the water. This is an effective way of reducing them; but it introduced such unstable shocks, similar to those found in the mixed flow region, that its disadvantages exceeded its benefits. The choice of the water height is a suggested method of overcoming capillary waves.

## Shock Wave

The hydraulic jump and shock wave are considered analogous but unfortunately this analogy is not quantitatively exact. The error that exists varies directly with the Mach number, but when the Mach number is not much greater than one, the error is quite small.

The analogy is summarized in the introduction. The entire theory of the water-gas analogy is given in much greater detail in reference (4).

## CHAPTER IV

## DESIGN CONSIDERATIONS

Table Bottom and Sides

Plate glass will be used for the bottom and sides of the water table to allow for a wide variety of photographic techniques.

In the selection of the glass, the thickness must be considered so that it will not sag. If the glass sags, it would vary the depth of the water at different locations resulting in a different flow pattern. The variation of the depth would not be known to the operator from the method that is available for depth measurement and the results would be unreliable and the analogy invalid.

In determining the glass thickness, the weight of the water, the weight of the nozzle, and the weight of the model were considered for a maximum depth of one inch.

From the following calculations, the minimum thickness of the glass to be used was determined:
Maximum deflection $D=\frac{5 \mathrm{WL}^{3}}{384 \mathrm{EI}} \quad I=\frac{\mathrm{bd}^{3}}{12}$
E for plate glass $=10 \times 10^{6} \mathrm{psi}$


$$
\begin{array}{ll}
\text { Wt. of water }=(2.87 \times l x 1 / 12)(62.4) & =15 \\
\text { Assumed Nozzle wt. } & =10 \\
\text { Assumed wt. of Model } & =10 \\
\text { Wt. of } 3 / 8 \text { inch plate glass }(3 \times l x \operatorname{l} / 48)(160) & =\frac{10}{45} \mathrm{H} / \mathrm{ft}
\end{array}
$$

Design for . 01 in . deflection, considering the total weight to be uniforly distributed:

$$
\begin{gathered}
D=\frac{5 \mathrm{WL}^{3}}{384 \mathrm{E} \frac{\mathrm{bd}}{} \mathrm{~d}^{3}}=\frac{5 \mathrm{WL}^{3}}{32 \mathrm{E} \mathrm{bd}^{3}} \\
\mathrm{~d}^{3}=\frac{5 \mathrm{WL}^{3}}{32 \mathrm{EbD}}=\frac{5(45)(2.625)^{3}}{32\left(10 \times 10^{6}\right) 144(2.87) \frac{.01}{12}} \\
\mathrm{~d}^{3}=3.68 \times 10^{-5} \\
d=.033 \mathrm{ft} .=.366 \text { inches }
\end{gathered}
$$

A three-eights inch plate glass will suffice for a deflection of .01 inches with one inch water flow, but to decrease the defleotion and also allow additional weight for the model and noszle, one-half inoh plate glass should be used. This is readily available commoroially.

Table sides: The sides of the table will not be subjected to any alenim ficant weight, but they should be thiok onough to withatand acoldental taps. A one-fourth inch plate glass.vill ba.used.

## Sluice-Type Noare

A sluice-type nozzle will be used primarlly to produce a toat paction flow with a smooth surface.

The sluice nozzle will be constructed of one-half inch plexiglass and can be moved upward at a $45^{\circ}$ angle to increase the water flow depth. Refer to Plate 3, sheet 2.

## Frame Assembly

The frame assembly will be constructed of $2 \times 2 \times \frac{1}{4}$ inch steel angles. This will have sufficient strength to support the table, sluice tank, sump tank, piping, motor and other auxiliary equipment. The floor of the frame will consist of white pine and will be bolted to the bottom of the frame. Refer to Plate $1_{9}$ sheet 2. Attached to the bottom of the frame will be four casters, swivel type, with rubber wheels. The casters will allow the table to be mobile which will increase its usefullness by locating the table in class meeting areas or wherever else desired. Adjacent to each wheel on the ends of the frame is an adjustable "floor truck lock" to use in leveling the table. When the table is moved to the desired location, the looks will be adjusted to lift the table off the wheels and give the needed slope to the table to overcome the friction of the water on the table, thereby giving constant flow in the test section.

## Sump Tank, Sluice Tank and Piping

The tanks and the piping will be constructed of material that will resist corrosion or hold corrosion to a minimum. Galvanized iron, aluminum, copper, brass and rubber were considered. It was decided to use gaivanized iron for constructing the tanks rather than aluminum, since it would be easier to work with and may be easily soldered. Piping will be galvanized iron, with rubber hosing used for facilitating connections. The rubber hosing will also decrease sump noise and dampen pump vibrations as well as to facilitate the installation of piping.

An angle valve will be located downstream of the pump to regulate the water flow to the sluice tank, which will determine the head. The regulation of the head in the sluice tank will determine the rate of
wa:er flow on the table which, in turn, determines the Mach number in the anaicgy.

A hose drain and valve will be installed below the sluice tank in order to drain the water from the system when it is not in use.

## Carriage Assembly

The assembly for the overhead carriage will be constructed of alumirim with fiber wheels which will be machined to fit snugly over tracks tade of onjominch aluminum rod, Plate 5.

Longitudinal one-inch rods will function as tracks so supported as to prevent any deflection. Since the ability to determine the water depth very accurately is a prime factor, care was taken to prevent any sag in the tracks as well as the glass bottom.

Attached to the carriage will be a mount in which the depth gage or an impact tube may be placed. The mount will ride on transverse seveneighths inch rods and can be moved by the cable and pulley arrangement, shown on Plate 5, across the width of the table. The depth gage (Starrett ( 44.4 B ) and probe will be capable of vertical movement. The combination of the carriage, mount and gage movement will result in the ability to survey the tabie in all three rectangular coordinate directions.

## Water Depth Gage and Pitot Tube

A needle probe with platinum tip mounted on the carriage assembly which may be moved in any of three rectangular coordinate directions will be used for surveying the water depth.

A pitot tube can be used to measure the "stagnation pressure" or total head. The total head is composed of two parts, the static head, $h_{0}$, and the dynaaic head, $\Delta h$, expressed in length of a column of the flowing

## fluid (Figure 2)



Fig. 2. Pitot tube

Bernoulli's equation written between points 1 and 2 is

$$
\frac{v^{2}}{2 g}+\frac{p_{1}}{\bar{\gamma}}=\frac{p_{2}}{\bar{\gamma}}=h_{o}+\Delta h
$$

since both points are at the same elevation. As $\frac{p_{1}}{\gamma}=h_{0}$ the equation reduces to

$$
\frac{\nabla^{2}}{2 g}=\Delta h \quad \text { or } \quad v=\sqrt{2 g \Delta h}
$$

which gives the velocity of the fluid.

Pump Selection

As previously mentioned in Chapter III, the analogy is more valid when the flow depth is limited so that vertical influence would be negligible.

About a one-fourth inch depth gives the most accurate results, and the depth should be held to a maximum of one inch even at low speeds. Since the analogy loses its validity in depths exceeding one inch, the pump calculations were based on a Froude number of one at a one-inch flow depth. After arriving at the minimum required horse power, the pump was selected from the Deming Company catalog using the total head and the flow rate.

The minimum pump size was determined from the following calculations:

Using two-inch piping

$$
\begin{aligned}
F r=\frac{V}{\sqrt{g d}} & V=1(\sqrt{g d})=1 \sqrt{32.2(1 / 12)} \\
& V=1.637 \approx 1.64 \mathrm{fps}
\end{aligned}
$$

Flow $=(1 / 12 \times 3 \times 1.64)=.41 \mathrm{cu} \mathrm{ft} / \mathrm{sec}$

$$
\begin{aligned}
& =.41(448.8)=184 \mathrm{gal} / \mathrm{min} \\
\mathrm{~V} & =\frac{Q}{\mathrm{~A}}=\frac{.41}{\frac{\pi(1 / 6)^{2}}{4}}=18.76 \mathrm{fps} \\
\nu & =1 \times 10^{-5} @ 70 \text { degrees } \\
R & =\frac{\mathrm{VD}}{\nu}=\frac{18.76(1 / 6)}{1 \times 10^{-5}}=3.13 \times 10^{5} \\
\epsilon & =.0005 \mathrm{ft}(\mathrm{galv} . \text { iron }) \\
\frac{\epsilon}{D} & =\frac{.0005}{\frac{1}{6}}=.003
\end{aligned}
$$

From the Moody Diagram $f=.03$ (Ref. 14)

## Equivalent length of pipe

Three two-inch standard elbows $=3(5.2)=15.6$
One two-inch angle valve $=6.3$
Three foot length of straight pipe
$=\frac{3}{24.9}$

$$
h_{f}=f \frac{L}{D} \frac{V^{2}}{2 g}=.03 \frac{24.9}{\frac{1}{6}} \frac{(18.76)^{2}}{64.4}=24.5
$$

head less $=24.5$
Elevation head $=1.5$

$$
\begin{aligned}
& \mathrm{h}_{\mathrm{f}}=24.5+1.5=26 \text { feet } \\
& \mathrm{BHP}=\frac{Q \mathrm{~h}_{\mathrm{f}}}{3960 \text { eff }}=\frac{184(26)}{3960(.70)}=1.73 \mathrm{BHP}
\end{aligned}
$$

The motor pump unit selected is a Deming Unit No. 31 S , 3 hp, at 1750
rpm. It will be mounted as shown on Plate 1 , sheet 1 , with a flex mount to reduce transmission of motor vibrations.


## CHAPTER V

## SUMMARY AND CONCLUSIONS

The primary objective of this report was to design an apparatus to be used for class demonstration and experiments utilizing the water analogy to two-dimensional compressible gas flow.

The unit was so designed to be self-contained, mobile, inexpensive to construct and operate, yet contain all the features desired in such an apparatus.

Care was taken to allow for adjustments of all parts whose position is critical. This would eliminate the necessity of construction to extreme tolerances.

During the construction of the water table, care should be taken to isolate motor vibrations. In the event that vibration effects cannot be isolated, the present pump mount can be easily removed and a portable mount with its base on the room floor be substituted or provision made for lifting the presently located motor free from the support frame.

## BIBLIOGRAPHY

1．Jouguet，E．，＂Quelquesproblèmes d＇hydrodynamique générale．＂J．math． pures app1．（8）3，1－63（1920）．

2．Riabouchinsky，D．，＂Mécanique des fluides－Sur l＇analogie hydraulique des mouvements d＇un fluide compressible．＂Compt．rend．195， 998－999（1932）．＂Mécanique des fluides－Quelques nouvelles remar－ ques sur l＇analogie hydraulique des mouvements d＇un fluide com－ pressible．＂Compt．rend。 199，632－634（1934）．

3．Kármán，Th．von．，＂Eine praktische Anwendung der Analogie zwishchen Überschallströmung in Gasen und überkritischer Strömung in offen Gerinnen．Z．angew．Math．u．Mech。 18，49－56（1938）．

4．Preiswerk，E．，＂Application of the Methods of Gas Dynamics to Water Flows with Free Surface．＂Part l：Flows with no energy dissi－ pation．Part 2：Flows with momentum discontinuities（hydraulic jumps）．NACA Tech．Mem． 234 and 235，1940．

5．Orlin，W．J．，Lindner，N．J．，and Bitterly，J．Go，＂Application of the Analogy Between Water Flow with a Free Surface and Two－Di－ mensional Compressible Gas Flow．＂NACA Tech．Note 1185， 1947.

6．Einstein，H．A．，and Baird，E．H．，＂Progress Report of the Analogy Between Surface Shock Waves on Liquids and Shocks in Compress－ ible Gases．＂Calif。 Inst．Technol．Hydrodynamics Lab。 Rept． $\mathrm{N}-54,1947$.

7．Bruman，J．Ro，＂Application of the Water Channel Compressible Gas Analogy．＂North Amer．Aviation Rept．NA－47－87， 1947.

8．Jacobs，D．H．a An Investigation of Water Channel Photography．＂North Amer．Aviation Rept．AL－129， 1947.

9．Johnson，R．H．，Nial，W．R．，and Witbeck，N．C．＂Water Analogy to Two－Dimensional Air Flow．＂Gen．Elec．Rept．55218， 1947.

10．Shapiro，A．H．，＂An Appraisal of the Hydraulic Analogue to Gas Dyna－ mics．＂Mass．Inst．Technol．Meteor Rept．34，1949．

11．Shapiro，A．H．，＂Free Surface Water Table．＂Article H，l，Physical Measurements in Gas Dynamics and Combustion，Princeton Univer－ sity Press， 1954.

12．Alcoa Structural Handbook－Aluminum Co．of America，Pittsburg，Pa． 1956.
13. Mantell, Charles L., Editor, Engineering Materials Handbook, McGrawHill Book Co., Inc., 1958 .
14. Streeter, V. L., Fluid Mechanics, Second edition, McGraw-Hill Book Co., Inc., 1958.
15. Flowers, J. V., "The Development of Experimental Techniques for the Study of Compressible Flow by the Hydraulic Analogy." Master of Science Thesis, Univ. of Calif., Burkeley, June, 1951.
16. Hedgecock, J. L., "A study of Detailed Shock Waves by the Water Channel - Compressible Gas Analogy." Master of Science Thesis, Univ. of Calif., Berkeley, Sept. 1950.


#### Abstract

APPENDIX


Design Plates





TOP VIEW - UPPER STRUCTURE









## VITA

Jobn Samos
Candidate for the Degree of
Master of Sciense

Reports: DESIGN OF A. FREE WATER SURFACE TABLE TO BE USED AS AN ANALOGY TO TWO DIMENSIONAL COMPRESSIBLE GAS FLOW

Mafor Finld: Meshanical Engineering
Bographtad:
Peracral Data: Born in New York City, New York, August 27, 1923, the son of Frank and Effie Samos.

Educations Attended grade s.hool in New Yorik, New York, graduated irom the High School of Commerce in New York, New York in 1942. Attended for varying periods. Aosdsmy of Aeronautias, New York, Penrsy"uazia State College, Saoramerto State College and the Urifersity of Houston. Recefved the Bachelor of Science Degrer fwom Oklahera State University, with a major in Mechanical Er... gitecring, in January, 1960\% completed the requirements for the Master of Sclerce degree in May, 1960, at Oklahoma State Uni... VEraity.

Prufsasickail Experienses Empluyed as a Projeot Engineer at NACA, Langioy Fiela, Virginia. from 1947 to 1949; is a Reserve Offic. Ner in the Uritted States Air Foree and presentiy serving in th. grade of Captain.

