

ASTRONOMICAL APPLICATIONS OF THE LAW
OF UNIVERSAL GRAVITATION AND OF
THE THEORY OF RELATIVITY

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TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
II. THE EARTH-MOON TEST OF THE LAW OF GRAVITATION . . .	4
III. DERIVATION OF KEPLER'S LAWS OF PLANETARY MOTION FROM THE LAW OF UNIVERSAL GRAVITATION	9
The First Law—Law of Elliptical Paths	10
The Second Law—Law of Equal Areas	15
The Third Law—Harmonic Law	19
IV. DETERMINATION OF THE MASSES OF ASTRONOMICAL BODIES .	20
V. ELEMENTARY THEORY OF TIDES	26
VI. ELEMENTARY THEORY OF THE EQUATORIAL BULGE OF PLANETS	33
VII. THE DISCOVERY OF NEW PLANETS	36
VIII. ASTRONOMICAL TESTS OF THE THEORY OF RELATIVITY . . .	40
BIBLIOGRAPHY	45

LIST OF FIGURES

Figure	Page
1. Fall of the Moon Toward the Earth	6
2. Newton's Calculation of the Earth's Gravitational Effect on the Moon	8
3. Relation Between the Unit Vectors of Rectangular Coordinates and Those of Polar Coordinates	13
4. Newton's Drawing Illustrating His Proof of the Law of Equal Areas	16
5. Illustration of the Relation Between the Infinitesimal Area Swept Out by the Radius Vector in Moving Through the Infinitesimal Angle $d\theta$	18
6. Motion of the Two Bodies About Their Common Center of Mass	23
7. Relation Between the Wavelength Shifts of the Light From Two Revolving Bodies Over One Period	24
8. Tide-Raising Forces at Opposite Points	27
9. The Two Tidal Bulges	28
10. Spring and Neap Tides	29
11. Effect of the Earth's Rotation on a Body at Its Surface	34
12. Apparent Displacement of a Star Near the Sun's Limb	41

CHAPTER I

INTRODUCTION

The purpose of this report is to describe some of the simpler applications of the law of universal gravitation to the field of astronomy, and to discuss some of the tests of the theory of relativity that were made possible through astronomical observations. Since the material presented is intended for use by the high school teacher in stimulating student interest in physics and astronomy, there is no attempt to use rigorous mathematical methods that are out of reach of the high school student—except in the case of the derivation of Kepler's first law. However, the gifted student may be able to benefit from the mathematical treatment given in this case. For the most part, the material is treated in a descriptive manner rather than quantitatively.

Sir Isaac Newton (1642–1727) published his Philosophiae Naturalis Principia Mathematica, commonly known as Newton's Principia, at London in 1687. This work has been acknowledged as probably the greatest single book in the history of science, the appearance of which probably marks the greatest forward step ever made in physical science. A portion of the Principia is devoted to the derivation of Newton's law of universal gravita-

tion which, as stated by Duncan, is as follows:

Every particle of matter in the universe attracts every other particle with a force that varies inversely as the square of the distance between them and directly as the product of their masses.¹

To express the law by means of symbols let m_1 and m_2 be the masses of any two particles, d the distance between them, F the force of their attraction, and G a constant²; thus:

$$F = G \frac{m_1 m_2}{d^2} .$$

The law of gravitation has been the basis of celestial mechanics, the study of the motion of planets and stars under gravity. This study was founded by Newton with his calculations of the motions of the bodies belonging to the solar system. Celestial mechanics has not been essentially changed since the time of Newton, but the applications of his principles have been expanded to include bodies beyond the solar system. Several of the topics covered in this report were discussed thoroughly by Newton in his Principia. The Earth-Moon test of the law of gravitation, the derivation of Kepler's laws, and the theories pertaining to the tides and the equatorial bulge of planets were done originally by Newton.

The theory of relativity, regarded by many scientists as the greatest advance in physical theory since Newton, was developed

¹John Charles Duncan, Astronomy (5th ed., New York, 1955), p. 234.

²The constant G is called the gravitational constant. The value of G is 6.670×10^{-11} newtons-m² kg⁻².

principally by Albert Einstein (1879-1955). Einstein has shown that the Newtonian laws are applicable only at comparatively small velocities (small, that is, compared with the velocity of light). The following doublet by Pope with the addition of a pendant by J. C. Squire is illustrative of the relation between Newton's work and Einstein's.

Nature and nature's law lay hid in night
 God said, Let Newton be, and all was light
 — Pope³

It did not last. The devil howling: Ho!
 Let Einstein be! restored the status quo.
 — Squire³

The success of Newtonian mechanics is a result of the fact that most of the velocities of the heavenly bodies are small compared with the velocity of light. However, there are a few astronomical phenomena that can be used to verify the predictions of relativity theory. Three of these verifications are described in the latter part of the report.

³Quoted in Cecilia Payne-Gaposchkin, Introduction to Astronomy, (New York, 1954), p. 168.

CHAPTER II

THE EARTH-MOON TEST OF THE LAW OF GRAVITATION

According to the well-known story, Sir Isaac Newton was sitting in his garden, reflecting upon the force that holds the Moon and planets in their orbits, when an apple fell from a nearby tree. It then occurred to him that the fall of the apple and the divergence of the motions of the Moon and planets from straight lines might all be produced by the same force. He became convinced that this was true and was eventually led to his law of universal gravitation. As a means of testing the law he compared the fall of the apple with the fall, or in other words deviation from straight line motion, of the Moon. This may be done as follows.

We will consider the fall of the Moon as though it begins its motion from a position of rest and falls toward the Earth. In a subsequent paragraph this imagined motion is correlated with the actual motion of the Moon. Let M and r represent the mass and radius of the Earth, m the mass of the Moon, R the distance between the centers of the Earth and the Moon, a the acceleration of the Moon in its fall toward the Earth, and m' and a' the mass and acceleration of the falling apple. Then, from Newton's second law of motion and the law of gravitation,

$$ma = G \frac{Mm}{R^2} \quad \text{and}$$

$$m'a' = G \frac{Mm'}{r^2} \quad .$$

Dividing out the masses common to both sides of the equations gives

$$a = G \frac{M}{R^2} \quad \text{and}$$

$$a' = G \frac{M}{r^2} \quad .$$

From the above equations we see that the acceleration of a falling body is independent of its mass. Dividing one of the equations by the other produces

$$\frac{a}{a'} = \frac{r^2}{R^2} \quad , \quad \text{or}$$

$$a = a' \frac{r^2}{R^2} \quad .$$

The value of a' has been determined by experiment to be approximately 32.2 feet per second per second. The distance between the Earth and the Moon is about 60 Earth-radii, so that $R = 60r$. Putting these values into the last equation, we have $a = 32.2/3600$ or $a = 0.00894$ feet per second per second. In the first second of the Moon's fall its velocity would change from zero to 0.00894 feet per second, and its average velocity during the first second would be $0.00894/2$ or 0.00447 feet per second. Hence, the distance that the Moon would fall in one second is approximately 0.00447 feet.

To complete the test we must now consider the observed

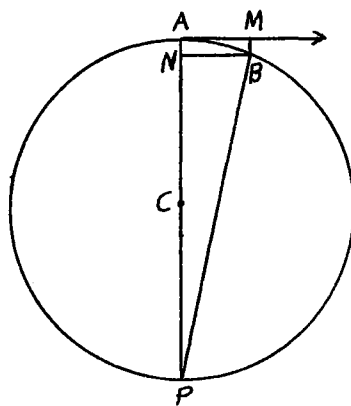


Figure 1. Fall of the Moon
Toward the Earth.¹

deviation of the Moon's path from the straight line that it would follow if it were not attracted by the Earth. In the course of one second the Moon moves through such a small arc that its deviation from a straight line may be regarded as a fall toward the Earth. In Figure 1 let the circle, of radius R , represent the orbit of the Moon, and suppose that the Moon moves from A to B in one second. The diameter AP is drawn through the Earth at the center C . The line BN is drawn at right angles to AP . The fall of the Moon toward the Earth in one second is the distance BM or its equivalent AN . The arc AB is so short that it may be taken as a straight line, being one side of a right triangle whose hypotenuse is AP . Since the triangles ABP and ANB are similar,

$$\frac{AN}{AB} = \frac{AB}{AP} \quad , \quad \text{or}$$

¹Duncan, p. 235.

$$AN = \frac{(AB)^2}{AP} .$$

The arc AB, the distance traveled along the orbit in one second, is $2\pi R/T$, where T is the number of seconds required for the Moon to complete one trip around its orbit; and, as $AP = 2R$,

$$AN = \frac{2\pi^2 R}{T^2} .$$

From astronomical measurements the value of R is approximately 1.26×10^9 feet, and T is about 2.36×10^6 seconds. Using these values in the above equation we have AN approximately equal to 0.00446 feet, which is in good agreement with the value calculated from the law of gravitation. After making similar calculations Newton stated in his Principia, "And therefore the force by which the moon is retained in its orbit is that very same force which we commonly call gravity"².

Figure 2, found on the following page, is based on a drawing by Thiel which summarizes very completely the content of this chapter. Notice in the drawing that the breadth of the arrows representing the acceleration of gravity increases as the Earth is approached.

²Florian Cajori, Newton's Principia, a revision of Motte's translation of Sir Isaac Newton's Mathematical Principles of Natural Philosophy (Berkeley, California, 1946), p. 408.

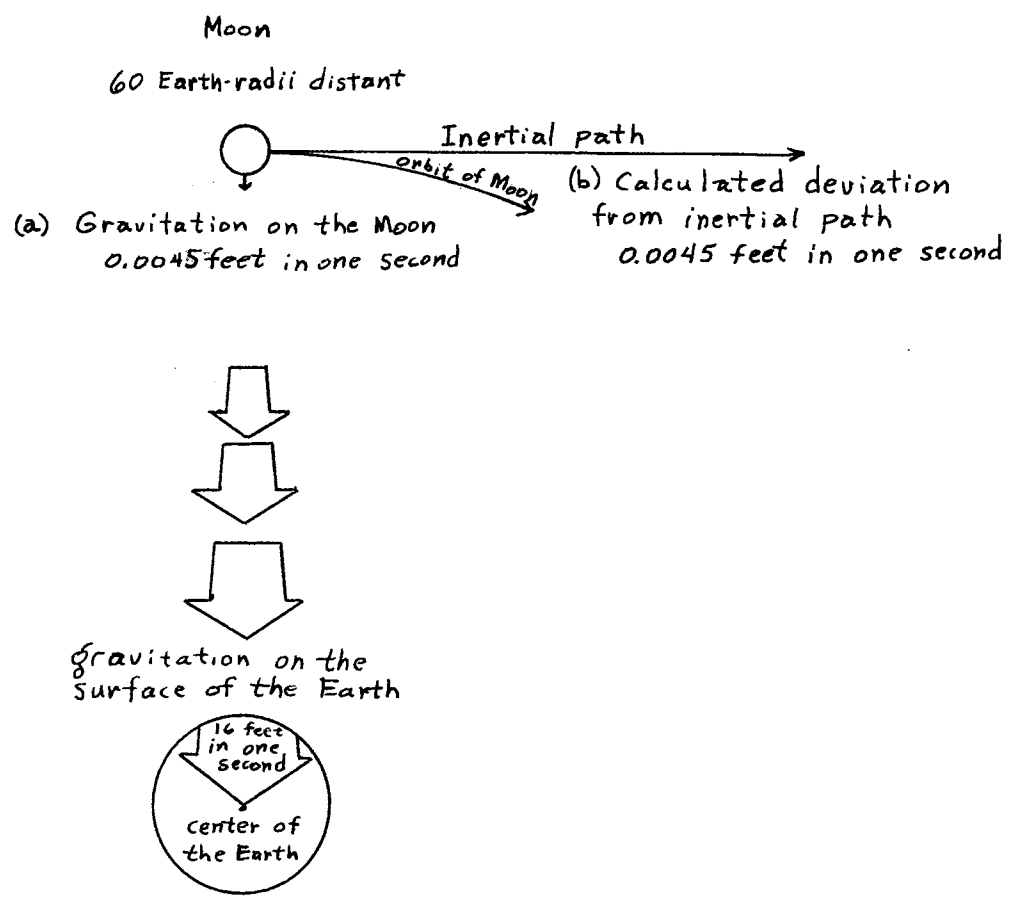


Figure 2. Newton's Calculation of the Earth's Gravitational Effect on the Moon.³

³Rudolf Thiel, And There Was Light, translated by Richard and Clara Winston (New York, 1960), p. 180.

CHAPTER III

DERIVATIONS OF KEPLER'S LAWS OF PLANETARY MOTION FROM THE LAW OF UNIVERSAL GRAVITATION

As stated by Holton, Kepler's three laws of planetary motion are as follows:

(1) Law of Elliptical Paths—"Planets move in elliptical paths, with the Sun at one focus of the ellipse."¹

(2) Law of Equal Areas—"During a given time interval a line from the planet to the Sun sweeps out an equal area anywhere along its elliptical path."²

(3) Harmonic Law—"If T be the sidereal period³ of any planet, and R be the mean radius of the orbit of that planet, then

$$T^2 = K R^3,$$

where K is a constant having the same value for all planets."⁴

These three laws are of strictly empirical nature. Tycho Brahe (1546-1601), the Great Danish astronomer, spent nearly a

¹Gerald Holton, Introduction to Concepts and Theories in Physical Science (Reading, Mass., 1952), p. 150.

²Ibid., p. 155.

³The sidereal period of a planet is the time it takes the planet to make a circuit of the sky from one star back to the same star.

⁴Holton, p. 156-157.

lifetime patiently observing and recording planetary motion with utmost precision. The accuracy of his measurements is indeed remarkable when it is noted that the telescope had not been invented in his time. After the death of Tycho, his German assistant, Johannes Kepler (1571-1630), continued the observations and reduction of the voluminous data. After years of work with Tycho's data, Kepler arrived at his famous three laws. Years later, Newton, in his Principia, showed that Kepler's laws follow from the law of universal gravitation. Later in this chapter an example of Newton's geometrical methods is given by the derivation of the law of areas. Otherwise, more familiar analytical methods are used.

The First Law—Law of Elliptical Paths

The purpose of this section is to show that the orbits of planets are ellipses with the Sun at one focus. In order to arrive at this result it is necessary to make use of the principles of Newtonian mechanics and the calculus. In the discussion that follows, capital letters represent vector quantities and lower case letters represent absolute magnitudes of quantities. Specifically, the following symbols represent the quantities indicated.

R - the radius vector (vector drawn from the origin of the coordinate system to the planet in question);

r - the length of the radius vector;

θ - the angle between the radius vector and a reference line;

U_x - the unit vector in the direction of the positive x-axis in rectangular coordinates;

U_y - the unit vector in the direction of the positive y-axis in rectangular coordinates;

U_r - the unit vector in the direction of the radius vector;

U_θ - the unit vector perpendicular to the radius vector and in the direction of increasing θ .

It is necessary to be able to write the equations of motion of a body moving under the influence of a force. To do this the components of the acceleration of the body must be obtained. In the case of a central force⁵, where the gravitational force is a special case, the motion is restricted to a plane. Hence, if we use a rectangular coordinate plane which contains the orbit of the planet, the vector expression for the acceleration A is given by

$$A = \frac{d^2x}{dt^2} U_x + \frac{d^2y}{dt^2} U_y .$$

To shorten the notation let symbols with a single dot immediately above them represent the first derivatives with respect to time of the quantities represented by the symbols. Similarly, let symbols with two dots above them represent the second derivatives with respect to time. Thus,

$$\frac{dx}{dt} = \dot{x} \quad \text{and} \quad \frac{d^2x}{dt^2} = \ddot{x} .$$

In this system of notation,

$$A = \ddot{x} U_x + \ddot{y} U_y .$$

⁵A central force is one which is always directed toward a particular point in space.

In order to facilitate the mathematical operations, the components of the acceleration should be in terms of polar coordinates. The transformation from rectangular to polar coordinates is achieved by the use of the equations

$$x = r \cos \theta \quad \text{and} \quad (1)$$

$$y = r \sin \theta . \quad (2)$$

The components of the acceleration along the x and y axes are given by the second derivatives of Equations (1) and (2) respectively. Hence,

$$\ddot{x} = \sin \theta (-r \ddot{\theta} - 2 \dot{r} \dot{\theta}) + \cos \theta (-r \dot{\theta}^2 + \ddot{r}) , \text{ and}$$

$$\ddot{y} = \sin \theta (-r \dot{\theta}^2 + \ddot{r}) + \cos \theta (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) .$$

Vector addition of these two components gives the vector expression for the acceleration;

$$\begin{aligned} A = \ddot{x} U_x + \ddot{y} U_y = (\ddot{r} - r \dot{\theta}^2)(\cos \theta U_x + \sin \theta U_y) \\ + (r \ddot{\theta} + 2 \dot{r} \dot{\theta})(-\sin \theta U_x + \cos \theta U_y) . \end{aligned}$$

To get the expression for A in the form desired, U_x and U_y must be replaced by U_r and U_θ . From Figure 3, on the following page, it is seen that

$$U_r = \cos \theta U_x + \sin \theta U_y , \text{ and}$$

$$U_\theta = -\sin \theta U_x + \cos \theta U_y .$$

Therefore, in terms of the unit vectors U_r and U_θ ,

$$A = (\ddot{r} - r \dot{\theta}^2) U_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) U_\theta .$$

From Newton's second law of motion,

$$F = m A ,$$

where m is the mass of the body in question, F is the force acting

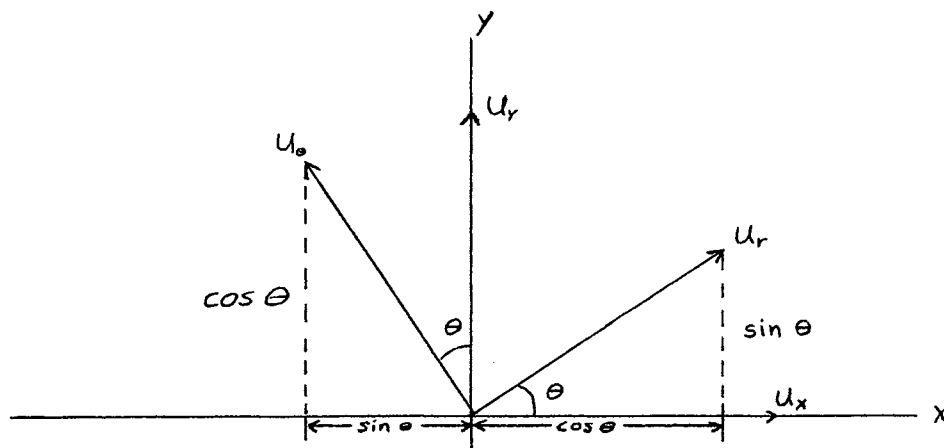


Figure 3. Relation Between the Unit Vectors of Rectangular Coordinates and Those of Polar Coordinates.

on the body, and A is the acceleration experienced by the body as a result of the force. Since the force being dealt with is a central force there is no component of the force in the direction of U_{θ} . Hence, the magnitude, f , of the force is given by

$$f = m (\ddot{r} - r \dot{\theta}^2) , \text{ and} \quad (3)$$

$$m (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) = 0 . \quad (4)$$

Considering Equation (4) notice that

$$m (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) = \frac{m}{r} \frac{d}{dt} (r^2 \dot{\theta}) = 0 .$$

Multiplying by r gives

$$m \frac{d}{dt} (r^2 \dot{\theta}) = 0 .$$

Integrating the above equation, we have

$$m r^2 \dot{\theta} = \text{constant} = j , \quad (5)$$

where j is called the angular momentum.

In the expression for the law of universal gravitation let

$k = GMm$. Then the force of Equation (3) is $-k/r^2$, where the negative sign is used to indicate that the force is directed oppositely to the direction of the radius vector drawn from the attracting center to the body in question. Making this substitution for the force f , Equation (3) becomes

$$m (\ddot{r} - r \dot{\theta}^2) = -\frac{k}{r^2} .$$

$\dot{\theta}$ may be eliminated from the above equation by the use of Equation (5), where $\dot{\theta} = j/mr^2$. This substitution gives

$$\ddot{r} - \frac{j^2}{m^2 r^3} = -\frac{k}{m r^2} . \quad (6)$$

Equation (6) is in terms of r and t , where the equation desired is one in terms of r and θ which would be the differential equation of the orbit in polar coordinates. To make this transition note that

$$\dot{r} = \frac{dr}{d\theta} \dot{\theta} = \frac{dr}{d\theta} \frac{j}{m r^2} .$$

Then,

$$\ddot{r} = \frac{d\dot{r}}{d\theta} \dot{\theta} = \frac{j^2}{m^2 r^4} \left[\frac{d^2 r}{d\theta^2} - \frac{2}{r} \left(\frac{dr}{d\theta} \right)^2 \right] .$$

Making use of the last expression, Equation (6) assumes the form

$$\frac{d^2 r}{d\theta^2} - \frac{2}{r} \left(\frac{dr}{d\theta} \right)^2 - r = -\frac{k m r^2}{j^2} , \quad (7)$$

which is the differential equation of the orbit in the desired form.

To solve Equation (7) we make the substitution $r = 1/u$. Then,

$$\frac{dr}{d\theta} = -\frac{1}{u^2} \frac{du}{d\theta} , \text{ and}$$

$$\frac{d^2 r}{d\theta^2} = \frac{2}{u^3} \left(\frac{du}{d\theta} \right)^2 - \frac{1}{u^2} \frac{d^2 u}{d\theta^2} .$$

Employing these in Equation (7), the differential equation takes the form

$$\frac{d^2 u}{d\theta^2} + u = \frac{k m}{j^2} .$$

The solution of this equation is seen by inspection to be

$$u = a \cos (\theta + \phi) + \frac{k m}{j^2} ,$$

where a is an arbitrary constant to be determined by the boundary conditions and ϕ is the phase angle. The equation for r is

$$r = \frac{1}{u} = \frac{1}{(km/j^2) + a \cos (\theta + \phi)} .$$

The above expression is the equation of a conic section. The conic sections consist of the circle, ellipse, parabola, and the hyperbola. The nature of the orbit for a particular body is dependent upon the total energy (potential energy plus kinetic energy) of the body. The planets possess energies in the proper range for their orbits to be of elliptical nature.

The Second Law—Law of Equal Areas

In deriving the law of equal areas, two methods are used. The first method is that undertaken by Newton in his Principia. The second method is done with use of the calculus.

The first method is taken directly from a translation of the Principia which was written originally in Latin. As stated in the translation, the law of areas is as follows: "The areas which revolving bodies describe by radii drawn to an immovable

centre of force do lie in the same immovable planes, and are proportional to the times in which they are described."⁶ Since the gravitational force is a central force it satisfies the statement of this proposition. The proof proceeds as follows:

For suppose the time to be divided into equal parts, and in the first part of that time let the body by its innate force describe the right line AB. In the second part of that time, the same would (by Law I)⁷, if not hindered, proceed directly

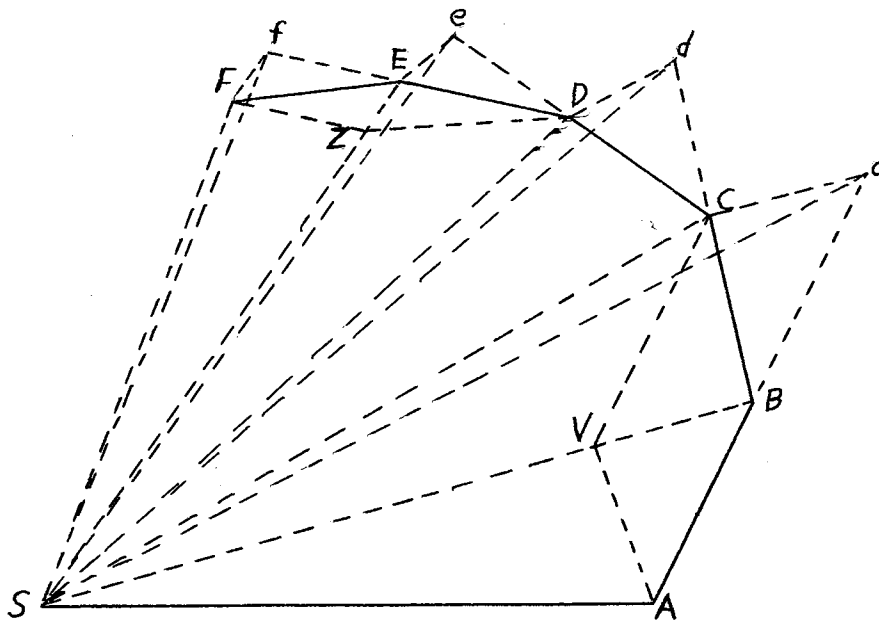


Figure 4. Newton's Drawing Illustrating His Proof of the Law of Equal Areas.

⁶Cajori, p. 40.

⁷Law I states: "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it." Ibid., p. 13.

to c , along the line Bc equal to AB ; so that by the radii AS , BS , cS , drawn to the centre, the equal areas ASB , BSc , would be described. But when the body is arrived at B , suppose that a centripetal force acts at once with a great impulse, and, turning aside the body from the right line Bc , compels it afterwards to continue its motion along the right line BC . Draw cC parallel to BS , meeting BC in C ; and at the end of the second part of the time, the body (by Cor. I of the Laws)⁸ will be found in C , in the same plane with the triangle ASB . Join SC , and, because SB and Cc are parallel, the triangle SBC will be equal to the triangle SBc , and therefore also to the triangle SAB . By the like argument, if the centripetal force acts successively in C , D , E , etc., and makes the body, in each single particle of time, to describe the right lines CD , DE , EF , etc., they will all lie in the same plane; and the triangle SCD will be equal to the triangle SBC , and SDE to SCD , and SEF to SDE . And therefore, in equal times, equal areas are described in one immovable plane: and, by composition, any sums $SADS$, $SAFS$, of those areas, are to each other as the times in which they are described. Now let the number of those triangles be augmented, and their breadth diminished in infinitum; and their ultimate perimeter ADF will be a curved line: and therefore the centripetal force, by which the body is continually drawn back from the tangent of this curve, will act continually; and any described areas $SADS$, $SAFS$, which are always proportional to the times of description, will, in this case also, be proportional to those times. Q.E.D."⁹

Now the same result will be obtained through the use of the calculus. Consider Figure 5 in which a particle of mass m is traversing the trajectory SS' under the influence of a central force the center of which is at O . The area of the infinitesimal triangle OBB' is

$$dA = \frac{1}{2} r r d\theta = \frac{1}{2} r^2 d\theta .$$

Dividing by dt , an infinitesimal element of time, gives

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} .$$

⁸Corollary I of the Laws states: "A body, acted on by two forces simultaneously, will describe the diagonal of a parallelogram in the same time as it would describe the sides by those forces separately." Ibid., p. 14.

⁹Ibid., pp. 40-41.

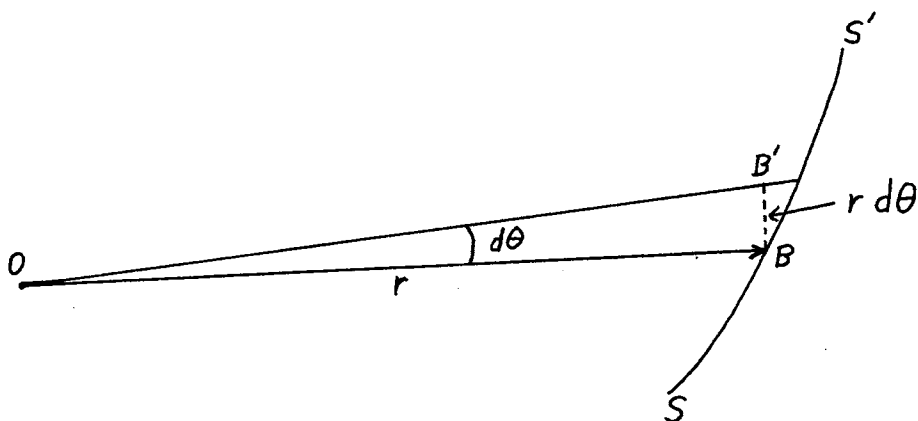


Figure 5. Illustration of the Relation Between the Infinitesimal Area Swept Out by the Radius Vector in Moving Through the Infinitesimal Angle $d\theta$.¹⁰

It has been shown on page 13 that

$$m r^2 \frac{d\theta}{dt} = \text{constant} = j .$$

Hence,

$$\frac{dA}{dt} = \frac{1}{2} \frac{j}{m} .$$

From the above equation we see that dA/dt is a constant depending upon the mass of the body in question. dA/dt is called the areal velocity and is the rate at which area is swept out by the radius vector drawn from the force center to the body.

¹⁰Robert A. Becker, Introduction to Theoretical Mechanics (New York, 1954), p. 225.

The Third Law—Harmonic Law

In this section we are to show that $T^2 = KR^3$, where T is the sidereal period of any planet, R is the mean radius of the orbit of that planet, and K is a constant having the same value for all planets.

To arrive at this result let us simplify the conditions by assuming that the orbits are circular rather than elliptical. This is very nearly true for some of the planets. For circular orbits, the centripetal force is given by mv^2/R , where m is the mass of the planet, v is its speed, and R is the radius of its orbit. The attracting center is assumed to be at rest. Then

$$\frac{m v^2}{R} = G \frac{M m}{R^2},$$

where M is the mass of the attracting body. If T is the time required for one complete revolution of the planet, that is the sidereal period, then

$$v = \frac{2 \pi R}{T}, \text{ and}$$

$$\frac{4 \pi^2 R^2}{T^2} = \frac{G M}{R}, \text{ or}$$

$$T^2 = \frac{4 \pi^2 R^3}{G M}.$$

For the case of the planets of the solar system, M is the mass of the Sun and $4\pi^2/GM$ is constant for all planets. Of course, the planets themselves exert forces upon each other, but for most purposes these forces are negligible when compared with the force exerted by the Sun.

CHAPTER IV

DETERMINATION OF THE MASSES OF ASTRONOMICAL BODIES

To determine the masses of astronomical bodies we make use of Kepler's harmonic law. In deriving the harmonic law in the preceding section it was assumed that the attracting body was at rest. This is approximately true if the mass of the attracting body is very large as compared with the mass of the attracted body. Actually, both bodies experience an acceleration toward the center of mass of the two. Hence, if the masses of the bodies are comparable the accelerations of both bodies must be considered. If we let a_1 be the acceleration toward the center of mass of the body of mass m_1 , and a_2 be the acceleration of the body of mass m_2 , then the acceleration observed is $a_1 + a_2$. From Newton's second law of motion and the law of gravitation we can write two equations,

$$m_1 a_1 = \frac{G m_1 m_2}{R^2} \quad \text{and}$$

$$m_2 a_2 = \frac{G m_1 m_2}{R^2} ,$$

where R is the distance between the centers of the bodies, assuming that the bodies are spherical. Dividing out the masses common to both sides of the equations produces

$$a_1 = \frac{G m_2}{R^2} \quad \text{and}$$

$$a_2 = \frac{G m_1}{R^2} .$$

Adding these two equations gives

$$a_1 + a_2 = \frac{G}{R^2} (m_1 + m_2) .$$

If we have orbital motion both bodies revolve about the center of mass. However, we can regard the motion as though one body is fixed and the other revolves about the fixed body. Let the observed acceleration, $a_1 + a_2$, be denoted by a . Let v represent the orbital speed of one body about the other. Then $a = v^2/R$, if the orbit is circular. This is very nearly true in some cases. Since $v = 2\pi R/T$, where T is the period of the motion, we have that

$$a = \frac{4 \pi^2 R}{T^2} = \frac{G}{R^2} (m_1 + m_2) , \quad \text{or}$$

$$m_1 + m_2 = \frac{4 \pi^2 R^3}{G T^2} . \quad (8)$$

For planets having satellites, and double stars that are sufficiently resolvable, the quantities R and T can be measured, and the sum of the masses of the two bodies under consideration may be calculated. If one of the bodies is very large compared to the other, the mass of the large body may be taken as approximately equal to the sum of the two masses. It is possible to do this for some of the planets of the solar system. In the consideration of double stars this approximation usually cannot be made. However, there is a method by which the ratio of the

two masses may be obtained if the plane of the motion of the two bodies is parallel to the line of sight. Then there will be observed a shift in the wavelengths of the light from each of the bodies toward either the red or violet depending upon whether the body is moving toward or away from the observer in its motion about the center of mass. This wavelength shift is an example of the Doppler-Fizeau principle, which may be stated as follows:

"When the distance between an observer and a source of light is increasing, the lines of the spectrum lie farther to the red than their normal positions, and when the distance is diminishing they lie farther to the violet, the displacement being proportional to the relative velocity of recession or approach."¹

Therefore, the magnitude of the shift in wavelength depends upon the component of velocity of the body that is directed toward or away from the observer. Hence, the magnitude of the shift reaches a maximum when the line between the centers of the bodies is perpendicular to the line of sight and reaches a minimum when the line of centers is parallel to the line of sight.

In Figure 6 let c be the position of the center of mass of the system, M the mass of the more massive body, R the distance from the center of the mass M to c , m the mass of the less massive body, and r the distance from the center of the mass m to c . Then, from the law of moments, $MR = mr$. Since M is greater than m , it follows that R is less than r .

At the position where the line of centers is perpendicular to the line of sight the component of the velocity parallel to

¹Duncan, p. 168.

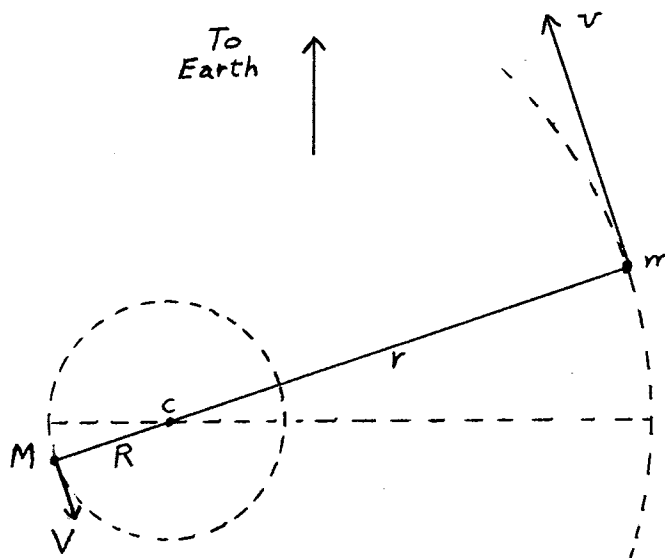


Figure 6. Motion of Two Bodies About Their Common Center of Mass.

the line of sight is the only component of the velocity, and the magnitude of the shifts in the wavelengths of the observed light is greater than at any other position. Let V represent the orbital speed of the mass M about the center of mass and v represent the orbital speed of the mass m about the center of mass. Then

$$V = \frac{2 \pi R}{T} \quad \text{and} \quad v = \frac{2 \pi r}{T} ,$$

where T is the period of the orbital motion of the bodies about the center of mass. Note that T is the same for the motion of both bodies. Dividing one equation by the other gives

$$\frac{V}{v} = \frac{R}{r} .$$

Since

$$\frac{R}{r} = \frac{m}{M},$$

we have that

$$\frac{V}{v} = \frac{m}{M}.$$

If we let K be the magnitude of the maximum wavelength shift of the light from mass M and k be the magnitude of the maximum wavelength shift of the light from the mass m , then, since the magnitude of the shift is proportional to the velocity,

$$\frac{V}{v} = \frac{K}{k} = \frac{m}{M}. \quad (9)$$

Figure 7 shows the relation between the wavelength shifts of the light from the two bodies for one period, where the beginning of the period is taken to be when the line of centers is parallel to the line of sight.

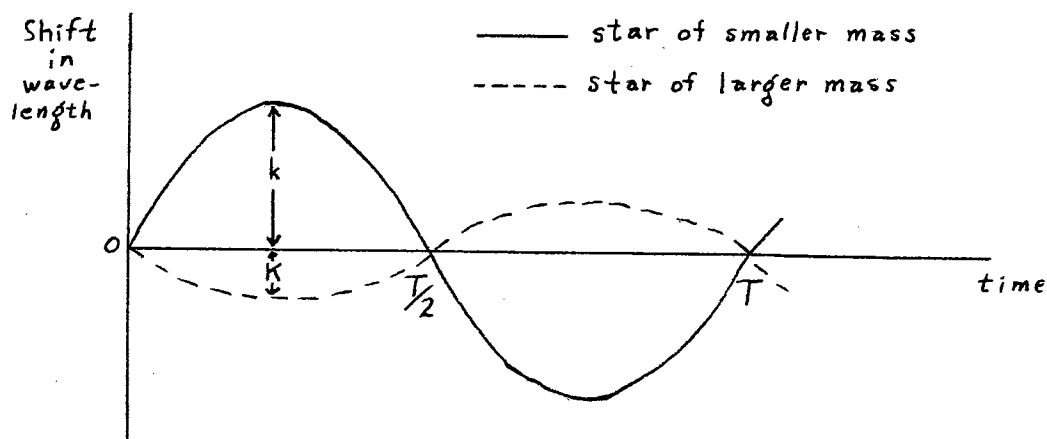


Figure 7. Relation Between the Wavelength Shifts of the Light From Two Revolving Bodies Over One Period. The figure is drawn for a mass ratio of 3 to 1.

If K/k is known, we have from Equation (9) that $m = MK/k$ which may be used to replace m in Equation (8) so that M may be determined. In an analogous manner the mass m may be calculated.

It is sometimes of interest to know the surface gravity of a planet. By the surface gravity of a planet is meant its attraction for bodies at its surface. It can be computed when the body's mass and radius are known. The surface gravity may be conveniently expressed as the ratio of the acceleration of a body falling upon the planet and the acceleration due to gravity at the surface of the Earth. Let M and r be the mass and radius of the Earth, M' and r' be the mass and radius of the planet, g and g' be the acceleration due to gravity at the surface of the Earth and the planet respectively, and m be the mass of the falling body. From Newton's second law of motion and the law of gravitation we have

$$m g = G \frac{M m}{r^2} \quad \text{and}$$

$$m g' = G \frac{M' m}{r'^2} .$$

Dividing one equation by the other gives

$$\frac{g'}{g} = \frac{M' r^2}{M r'^2} .$$

That is, the planet's surface gravity is numerically equal to its mass divided by the square of its radius if each of these quantities is expressed in terms of the Earth's mass and radius as units.

CHAPTER V

ELEMENTARY THEORY OF TIDES

The periodic rise and fall of the water of the ocean, known as tides, is caused by the action of the Moon and Sun. The Moon's tide-raising force at a point on the Earth's surface is the difference between the gravitational force exerted by the Moon at the distance of the Earth's center and the force exerted at the point. The Sun's tide-raising force is of a similar nature. The body of the Earth, which is almost perfectly rigid, responds only slightly to this difference in force, but the fluid oceans are more affected. Since the gravitational attraction between two bodies decreases as the distance between the bodies becomes greater, the Moon's or Sun's attraction is more than average for that part of the ocean nearest the Moon or Sun and less than average for the most distant part. Though much more massive than the Moon, the Sun is so much more distant that its tide-raising force is only about one-third that of the Moon.

For simplicity, set us consider for the purpose of the following discussion only the action of the Moon in producing tides. To further simplify, let us suppose that the whole Earth is covered by very deep water. In Figure 8, let the center of the Earth be at C and that of the Moon at M, and consider the action of the Moon on drops of water situated at A and B. Since the

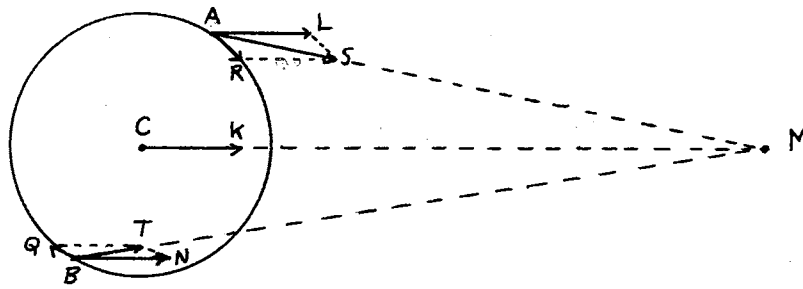


Figure 8. Tide-Raising Forces at Opposite Points.¹

body of the Earth is almost perfectly rigid, it may be considered to move as a single body and the Moon's effect on it is the same as if its mass were concentrated at C. But the drops of water in the ocean are free to move relatively to the body of the Earth. Let CK represent the acceleration produced by the Moon's attraction in the solid Earth, and let AS and BT be the accelerations produced in A and B. Since A is nearer the Moon than is C, AS will be longer than CK. Similarly, since B is farther from the Moon than is C, BT will be shorter than CK. Now resolve AS and BT into component vectors so that the components AL of AS and BN of BT have the same length as CK and also the same direction as CK. Then the differences in acceleration of the drops of water and the solid Earth are represented by AR and BQ. It is important to note that, since the diagonal of the lower parallelogram is shorter than its horizontal side, the other side is here di-

¹Duncan, p. 253.

directed toward the left; that is, the tide-raising force is directed away from the Moon. The tide-raising acceleration in different parts of the Earth's circumference is shown by the arrows in Figure 9, by which it is seen that the tidal forces of the Moon tend to heap the water up in two places on the Earth's surface situated at opposite ends of a diameter—one directly under the Moon and the other on the opposite side of the Earth.

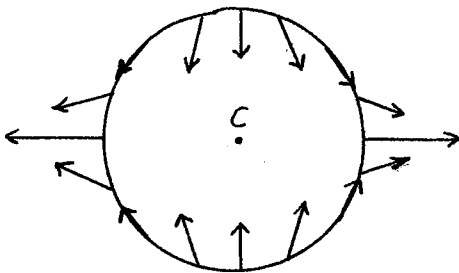


Figure 9. The Two Tidal Bulges.²

Now let us consider the combined effects of the Moon and Sun in producing tides. The two sets of tides may be considered as operating independently so that the separate effects of the two add together to produce the resultant tides. Let us consider two special cases of this combined tidal action—spring tide and

²Ibid.

neap tide. The spring tide occurs when the Moon is new or full. Since the Moon and Sun are then attracting from the same or opposite directions, lunar and solar tides reinforce each other; the high tide is highest and the low tide is lowest. The neap tide occurs when the Moon is at either quarter phase. Then the Moon and Sun are 90° apart, so that one set of tides is partly neutralized by the other. Figure 10 illustrates the addition of the effects of the Sun and Moon to produce spring and neap tides. The heights of the tides are greatly exaggerated in the drawing.

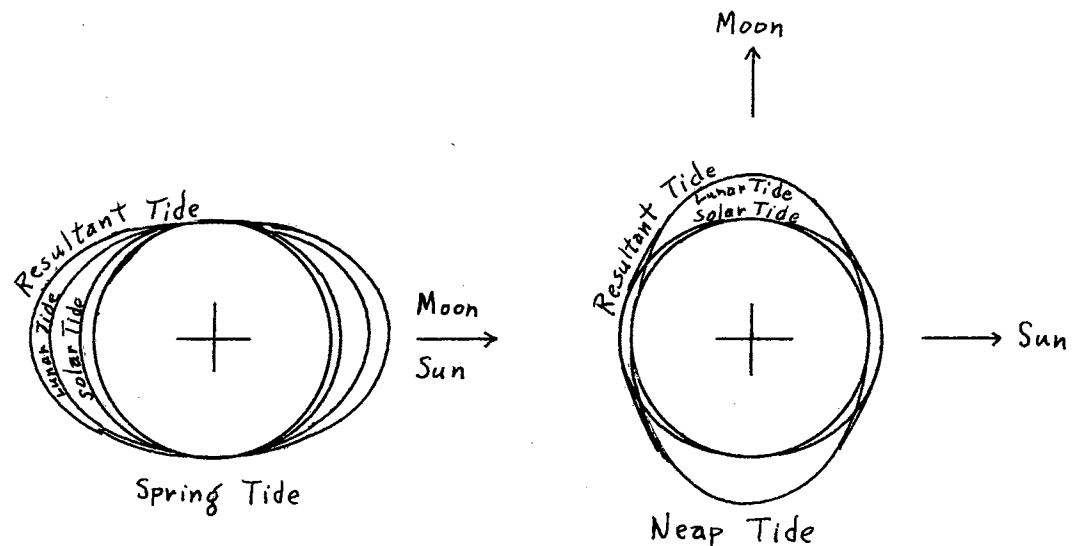


Figure 10. Spring and Neap Tides.³

The average interval of time between successive passages of

³Payne-Gaposchkin, p. 146.

the Moon across the meridian is 24 hours and 50 minutes. Hence, on the average, high tides should occur every 12 hours and 25 minutes and low tides at equal intervals halfway between. In the actual case, however, the water of the ocean is not frictionless, and the tidal bulge is partly carried forward by the Earth's rotation. Also, the bulge has a tendency to travel as a wave with a velocity which depends upon the depth, and the progress of the tidal wave around the Earth is vastly complicated by the varying depth of the sea, the presence of continents and islands, and the irregularity of coast lines. Therefore, high tide and the transit of the Moon are generally far from simultaneous. The actual prediction of the time and height of high water cannot be made from astronomical data alone, but depends also upon local observations of the tides.

Although there is disagreement as to how the Earth-Moon system was formed in the beginning, it is generally agreed that a few billion years ago the rotation of the Earth was very rapid and that the Moon was much closer to the Earth than it now is. The nearness of the Moon would have produced excessively high tides, and the friction between the water and the uneven floors of the ocean produced by the Earth rotating under the tides would have slowed the rotation of the Earth. But the same friction would tend to drag the tides slightly ahead of the line joining the centers of the Earth and Moon, and the gravitational pull of this unsymmetrical tide would increase the Moon's orbital velocity and cause it to spiral out into a larger orbit. Since the angular momentum of the Earth-Moon system must remain constant, any rota-

tional energy lost by the Earth would have to be transformed into orbital energy of the Moon. Thus the length of the day on the Earth has slowly increased to its present value of 24 hours, and the Moon has spiraled out to its distance of 238,000 miles from the Earth.

This process is still going on. The length of the day is increasing by about 0.0016 seconds per century. In the passage of 2,000 years the change will amount to about 3 hours. It has been calculated by G. H. Darwin that the tidal friction will continue to slow the Earth's rotation and increase the size of the Moon's orbit with a corresponding increase in its period of revolution until the day and month are of the same length and are both equal to about 47 of our present days. At this remote period in the future the Earth-Moon system will be internally stable, the Earth turning the same hemisphere always toward the Moon, just as the Moon now presents one hemisphere to the Earth. At this stage lunar tides cannot alter the system; but solar tides still operate on it, and they will slow down the Earth's rotation until the day is longer than the month and the Moon will rise in the west and set in the east. The lunar tides on the Earth will now exert a retarding force on the Moon's orbital motion, causing it to spiral in slowly toward the Earth's surface. However, instead of eventually colliding with the Earth, it seems more likely that the Moon will finally be disrupted by the tremendous tidal forces which would be exerted on it by the Earth as the Moon draws near the Earth. If this happens the Moon will be shattered and the fragments scattered around the Earth in a

ring comparable to the rings of Saturn.

Calculations made by E. A. Roche on the assumption of a liquid satellite of equal density with its primary planet showed that the satellite could not withstand tidal disruption if it were at a distance less than 2.44 times the planet's radius (the "Roche limit"). The outer edge of Saturn's ring is at a distance of 2.30 radii from the planet's center, and the nearest approach of a satellite is 3.11 radii. Thus it seems very possible that the rings are the remnants of one or more moons whose orbits carried them too close to the planet. This evidence seems to support the speculation that the Moon will be disrupted as it nears the Earth.

It is important to understand that it is dangerous, scientifically, to project very minute changes over exceedingly long periods of time. Unsuspected forces may alter the picture completely. It is quite likely that new and better theories will soon displace the theory of the Earth-Moon system as presented here.

CHAPTER VI

ELEMENTARY THEORY OF THE EQUATORIAL BULGE OF PLANETS

If a planet were stationary, with no force but gravitation acting on its parts, it would have the form of a perfect sphere. This is true because the gravitational force being directed toward the center of gravity of the system of particles results in these particles grouping themselves as near the center as possible. However, if the planet is rotating, every particle of its substance (except those exactly on its axis) revolves in a circle which is centered on a point on the axis and which lies in a plane parallel to the equator. Motion in a circle results in a tendency of the moving body to fly away from the center, a tendency which is known as centrifugal acceleration. The magnitude of the centrifugal acceleration, a_c , is given by

$$a_c = \frac{v^2}{r},$$

where v is the magnitude of the velocity of the particle, and r is the distance from the particle to the center of the circle. If the magnitude of the velocity is constant, then

$$v = \frac{2 \pi r}{T},$$

where T is the time for one revolution. Then,

$$a_c = \frac{4 \pi^2 r}{T^2}. \quad (10)$$

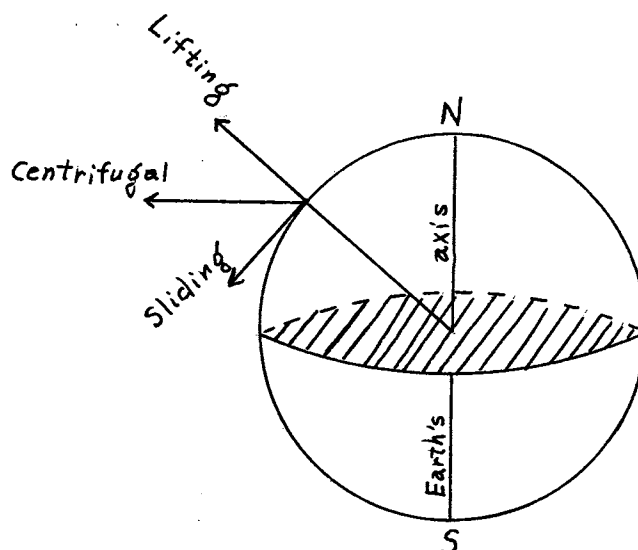


Figure 11. Effect of the Earth's Rotation on a Body at Its Surface.¹

Since the times of revolution of all the planets are practically constants, Equation (10) applies to their motion.

Let us consider the centrifugal acceleration of a particle at the surface of a planet. Then, in Equation (10), r is the perpendicular distance from the particle to the axis of rotation of the planet. Assuming that T is a constant for all positions on the surface, we see that a_c varies as we go from the pole to the equator so that a_c is zero at the poles and is a maximum value at the equator. As shown in Figure 11, the acceleration at any place may be regarded as the resultant of two accelerations operating at right angles to each other:

(1) The lifting effect is opposed to the planet's attraction and therefore diminishes the weight of an object at that

¹Robert H. Baker, Astronomy (6th ed., New York, 1955), p. 50.

place.

(2) The sliding effect of the planet's rotation is directed along the surface toward the equator. This effect has only the rigidity of the planet to resist it. The result is that the planets, notably the Earth, Jupiter, Saturn, and Uranus, instead of being exact spheres, are bulged at their equators and have approximately the form of oblate spheroids.

If we define the oblateness of a planet to be the difference between its equatorial and polar radii divided by its equatorial radius, then, from astronomical measurements, we have for the Earth an oblateness of 0.0034; for Jupiter, 0.06; for Saturn, 0.11; and for Uranus, 0.09.

CHAPTER VII

THE DISCOVERY OF NEW PLANETS

In 1781 William Herchel of Bath, England, an amateur astronomer, discovered the planet Uranus which at the time was the outermost known member of the solar system. By that time it was known how to compute the elliptic orbit of a planet from a few widely separated observations of its varying positions. Also, the expected small deviations from the true ellipse owing to the perturbing force of the other planets were accurately predictable on the basis of Newton's law of gravitation. Uranus' 84-year orbit was so mapped out for it by calculation, and for many years no irregularities were noticed. But by 1830 it became evident that there were unaccounted-for irregularities in its orbit.

This caused some people to suggest that perhaps Newton's theory did not hold at such immense distances. Uranus is nearly twice as far from the Sun as Saturn, which previous to the discovery of Uranus was the outermost known planet. Others suggested that perhaps there was a more distant, undiscovered planet that was causing additional perturbations in Uranus' path. The latter idea induced a young undergraduate at Cambridge University, John C. Adams, to undertake the immensely difficult mathematical task of locating the positions of the unknown planet solely from the observed motions of Uranus, using throughout the

law of gravitation. He finally obtained his result two years after his graduation and wrote to the Royal Observatory at Greenwich asking that their telescope search for the new planet at the predicted location. But, since Adams was a young unknown mathematician, he was not taken seriously enough to interrupt the current work at the observatory.

A short time later another young man, Leverrier in France, finished similar independent calculations which predicted almost the same position for the unknown planet as that predicted by Adams. Leverrier sent his own prediction to the head of the observatory at Berlin, who on the very evening of the letter's arrival himself searched for and recognized the planet at very nearly the suspected position. This occurred in 1846. The new planet was named Neptune. The discovery of Neptune served to fortify the position of the law of gravitation as indeed a universal law.

Even after the effects due to the presence of Neptune were taken into account, the observations of Uranus did not agree perfectly with theoretical calculations. The success of Adams and Leverrier naturally led to attempts to find another unknown planet that could be causing the discrepancies. The most determined and thorough of the investigators attacking this problem was Percival Lowell. He based his computations on the motion of Uranus for, in his opinion, Neptune could not be used because it had traversed less than half its orbit since its discovery and the available observations did not go far enough back. In 1905 he began a photographic search by his assistants at the Lowell Observatory based upon previously calculated estimates as to the

position of the suspected planet. In 1915, a year before his death, Lowell published the results of mathematical work on the subject which he had been conducting for years. His investigation disclosed two possible solutions which placed the planet in either of two opposite regions of the sky, with considerable uncertainty as to its exact location in either.

On January 21, 1930, C. W. Tombaugh, working at the Lowell Observatory under the direction of V. M. Slipher and C. O. Lamp-land, found on photographs taken by him in one of the predicted regions an object which in apparent path and rate of motion conformed approximately to Lowell's predictions of the trans-Neptunian planet. Further watching established its planetary character and its location beyond the orbit of Neptune. The new planet was named Pluto. The mass and brightness of Pluto proved to be less than that predicted by Lowell. The unexpected faintness of Pluto points to the skill and persistence with which Tombaugh examined his plates.

While there is no question of the honor due Lowell and his associates for the discovery of Pluto through systematic photographic search, the view that Lowell could be said to have discovered the planet mathematically has been disputed. W. H. Pickering also had worked on the problem using less elaborate methods than Lowell's but taking into account the irregularities in Neptune's motion as well as those of Uranus. He had predicted a position for the unknown planet that was nearly the same as that predicted by Lowell. In 1919 M. L. Humason photographed at Mount Wilson a region suggested by Pickering. However, his exam-

imation of the plates failed to reveal a new planet. Following Tombaugh's discovery, S. B. Nicholson re-examined Humason's plates and found the new planet. The orbit of Pluto as determined from old photographs and Lowell Observatory positions is in fair agreement with that predicted by Lowell and Pickering. The mass of Pluto, however, proves to be less than predicted, and it is held by some authorities that the discrepancies of motion of Uranus and Neptune could not possibly be due to the attraction of so small a body. If this is true, then it is an astonishing coincidence that the planet appeared so near the predicted position.

CHAPTER VIII

ASTRONOMICAL TESTS OF THE THEORY OF RELATIVITY

The theory of relativity was developed principally by the physicist Einstein in papers published in 1905 and 1915. On the basis of the theory of relativity Einstein derived a new law of gravitation with the use of general tensor analysis, which is too complicated for any example of it to be given here. It is important to understand that the law must be such that its consequences agree with those derived from Newton's law of gravitation as a first approximation, since this law describes the motions of the solar system with high accuracy; and it must also be in harmony with the predictions of the theory of relativity. Actually, there were available to Einstein several possible guesses as to the correct law of gravitation. But one stood out in contrast to all others as the simplest in mathematical form. He adopted this law as a tentative hypothesis and then proceeded to look for predictions based on it which could be tested by experiment.

From the new law of gravitation, Einstein made three predictions that might be tested by astronomical observations. One prediction was as follows: Rays of light passing near a massive body should be bent toward it. A ray of light just grazing the surface of the Sun should be deflected by 1.75 seconds of arc. Stars seen adjacent to the Sun during an eclipse should appear to

be displaced outward by this angular amount. This effect is shown diagrammatically in Figure 12.

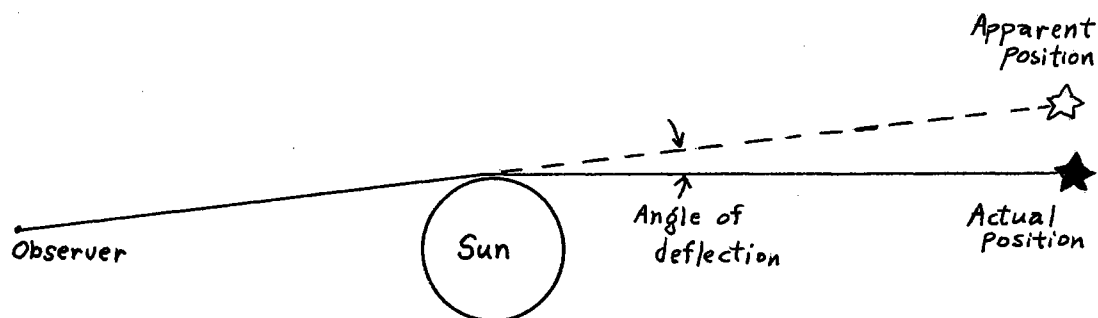


Figure 12. Apparent Displacement of a Star Near The Sun's Limb.

This effect was first tested in 1919 during a total solar eclipse that was visible in Africa and Brazil. This was an excellent opportunity for the test because the Sun at the time was among the bright stars of the cluster Hyades, so that their light had to pass near the Sun to reach the Earth. These stars were photographed during the eclipse by Sir Arthur Eddington and C. R. Davidson, who later photographed the same star field with the same instruments when the Sun was in another part of the sky. Comparison of the plates showed that the star positions were apparently shifted by an amount to verify Einstein's prediction. These tests have been repeated when the opportunity presented itself. At the Australian eclipse of 1922 W. W. Campbell confirmed the results of Eddington and Davidson. The test by G. van Biesbroeck at the eclipse of February 25, 1952, showed an average dis-

placement of 1.70 seconds at the Sun's edge, which is in good agreement with the prediction.

The second prediction to be considered was that physical processes in a region of high gravitational potential, when compared with similar processes at a point of low potential, should be found to take place more slowly. Consequently, atomic vibrations on the Sun should appear to be slowed down, and spectral lines observed in the spectrum of sunlight should be shifted slightly toward the red (longer wavelengths) as compared with lines emitted or absorbed by the same elements on the Earth.

Many of the spectral lines of sunlight are thus shifted, but the Einstein effect is so complicated with effects of pressure, radial velocity, etc., that its detection is very difficult. However, the star which is the companion of the star Sirius¹ affords an excellent test of the prediction. This star is known to have a diameter of 24,000 miles and a density of 53,000 times that of water. Its small diameter and tremendous density gives the companion of Sirius an incredibly great surface gravity so that the relativistic red shift is also large. In the case of most stars, the Einstein effect is too slight to be easily measured and is also combined with the shift of the spectrum lines due to the star's motion. However, the motion of the companion of Sirius is accurately known so that the shift due to the motion can be allowed

¹The companion of Sirius, although unseen, was first detected by F. W. Bessel in 1844 from its gravitational effects on the motion of Sirius. In 1862, during a test of a new 18-inch refractor, A. G. Clark observed the companion for the first time.

for. After allowing for this, there was found by W. S. Adams a residual redward displacement of 0.32 angstrom units, in almost perfect agreement with the Einstein displacement calculated by Eddington.

Einstein's third prediction was that the motion of the planets in their orbits should be slightly different from that predicted by Newton's law of gravitation. In particular, the major axis of the elliptical orbit of Mercury should be caused to rotate about the Sun at the rate of 43 seconds of arc per century.

The above prediction was made in 1915, but a discrepancy in the rate of precession of the major axis of Mercury's orbit had already been noted by Leverrier in 1845. By calculating the perturbing effects of Venus and other planets he obtained a predicted value for the rate of rotation of the axis which was about 40 seconds less than the observed rate of approximately 570 seconds per century. At this time there were several suggestions as to the cause of the irregularity. Some suggested that Newton's law of gravitation should be amended by substituting the exponent 2.00000016 for 2. But this suggestion was abandoned when it was noted that the calculated motion of the major axis of the orbit of Venus would then not agree with the observed motion. Leverrier concluded that this irregular motion of Mercury must be caused by an undiscovered planet revolving within the orbit of Mercury. However this hypothetical planet, which received the name Vulcan, has never been seen. Another idea was that the orbital motion is influenced by a cloud of meteoric matter. From such a cloud light should be reflected so that it should be observable. No such light

has been detected so that this suggestion has also been abandoned. Hence, after many years of conjecture, the new law of gravitation as proposed by Einstein was successful in explaining this disturbing matter. This was the first physical test of the theory of relativity.

It is also predicted by the theory of relativity that the major axes of the other planets should rotate at a rate different from that predicted by Newton's law of gravitation. However, Mercury is the only planet with the proper conditions of orbit and velocity to afford a good test. The magnitude of the Einstein effect on the rotation of the axis of the orbit is dependent on the orbital velocity of the planet. The velocity of Mercury is large enough to produce a measurable effect, and its orbit is highly elliptical so that the position of the major axis is easily located. The planet Venus has a large enough orbital velocity for the test, but its orbit is so nearly circular that the position of its major axis is difficult to determine. The orbit of Mars is satisfactorily elliptical, but its velocity is too small. The other planets are similarly unsuitable for the test. However, the effect is cumulative and should eventually be detected from the study of observations made over a long period of time.

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